Two Scalar mesons annihilation in to two rho mesons

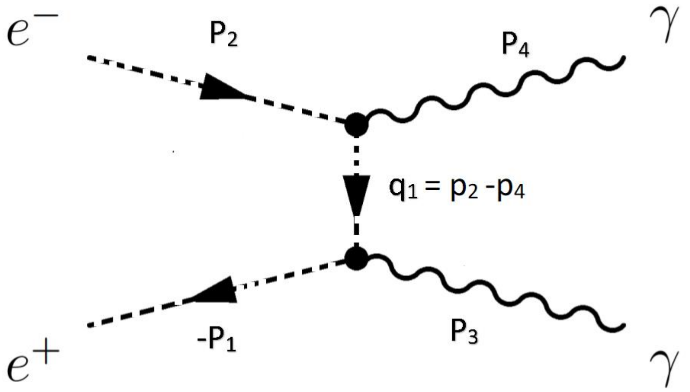
Group meeting

26th April 2019

Deepasika Dayananda

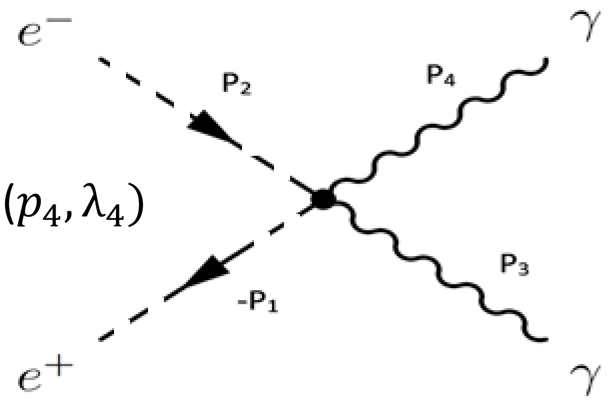
Lowest –order Covariant annihilation diagrams

t-Channel



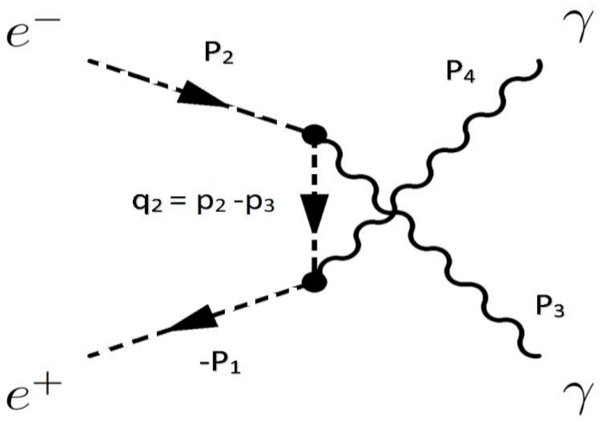
$$M_t = (-p_1 + q_1)^{\mu} \varepsilon^*_{\mu}(p_3, \lambda_3) \frac{1}{q_1^2 - m^2} (p_2 + q_1)^{\nu} \varepsilon^*_{\nu}(p_4, \lambda_4)$$

Seagull



$$M_{se} = -2 g_{\mu\nu} \epsilon^{*\mu}(p_3, \lambda_3) \varepsilon^{*\nu} (p_4, \lambda_4)$$

u-Channel



$$M_u = (-p_1 + q_2)^{\nu} \varepsilon^*_{\nu}(p_4, \lambda_4) \frac{1}{q_2^2 - m^2} (p_2 + q_2)^{\mu} \varepsilon^*_{\mu}(p_3, \lambda_3)$$

The interpolating photon polarization vectors

$$\epsilon_{\hat{\mu}}(P,+) = -\frac{1}{\sqrt{2}\,\mathbb{P}}\left(\mathbb{S}|\boldsymbol{p}_\perp|, \frac{P_1 P_{\hat{-}} - iP_2\mathbb{P}}{|\mathbf{p}_\perp|}, \frac{P_2 P_{\hat{-}} + iP_1\mathbb{P}}{|\mathbf{p}_\perp|}, -\mathbb{C}\,|\boldsymbol{p}_\perp|\right)$$

$$\epsilon_{\hat{\mu}}(P,-) = -\frac{1}{\sqrt{2}\,\mathbb{P}}\left(\mathbb{S}|\boldsymbol{p}_\perp|, \frac{P_1 P_{\hat{-}} + iP_2\mathbb{P}}{|\mathbf{p}_\perp|}, \frac{P_2 P_{\hat{-}} - iP_1\mathbb{P}}{|\mathbf{p}_\perp|}, -\mathbb{C}\,|\boldsymbol{p}_\perp|\right)$$

$$\epsilon_{\hat{\mu}}(P,0) = -\frac{P^{\hat{+}}}{m_\gamma \mathbb{P}}\left(P_{\hat{+}} - \frac{m_\gamma^2}{P^{\hat{+}}}, P_1, P_2, P_{\hat{-}}\right)$$

Constraints

$$\epsilon_{\hat{\mu}}(p,\lambda)p^{\hat{\mu}} = 0$$

$$\epsilon^*(p,\lambda).\,\epsilon(p,\lambda') = -\delta_{\lambda\lambda'}$$

Where $\mathbb{S} = Sin(2\delta)$

$\mathbb{C} = Cos(2\delta)$

$\mathbb{P} = \sqrt{{P_{\hat{-}}}^2 + \mathbb{C}{\mathbf{p}_\perp}^2} = \sqrt{(P^{\hat{+}})^2 - \mathbb{C}{m_\gamma}^2}$

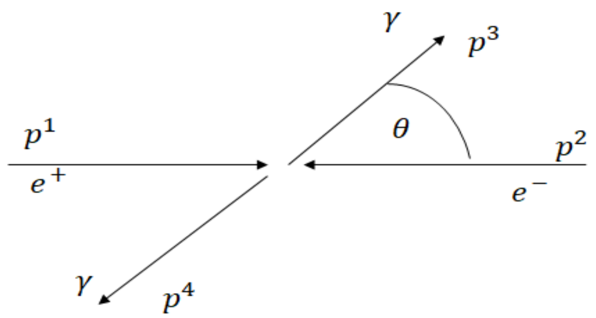
C-R ji,Ziyue Li and A.T.Suzuki ,Phy Rev D 91 06065020 (2014)

Covariant Feynman amplitude

$$M = M_t + M_u + M_{se}$$

$$\begin{aligned} M &= (-p_1 + q_1)^\mu \varepsilon^*_\mu(p_3,\lambda_3)\frac{1}{q_1^2-m^2}(p_2 + q_1)^\nu \varepsilon^*_\nu(p_4,\lambda_4) \\ &+ (-p_1 + q_2)^\nu \varepsilon^*_\nu(p_4,\lambda_4)\frac{1}{q_2^2-m^2}(p_2 + q_2)^\mu \varepsilon^*_\mu(p_3,\lambda_3) \\ &-2g_{\mu\nu}\epsilon^{*\mu}(p_3,\lambda_3)\varepsilon^{*\nu}(p_4,\lambda_4) \end{aligned}$$

Center of mass kinematics



$$p^1 = \{E_0, 0,0, P_e\}$$

$$p^2 = \{E_0, 0,0, -P_e\}$$

$$p^3 = \{E_0, P_\gamma Sin(\theta), 0, P_\gamma Cos(\theta)\}$$

$$p^4 = \{E_0, -P_\gamma Sin(\theta), 0, -P_\gamma Cos(\theta)\}$$

Lorentz Transformation

$$E = \sqrt{4E_0^2 + P_z^2} \qquad \alpha = \frac{E}{4E_0^2} \qquad \alpha\beta = \frac{P_z}{4E_0^2}$$

$$p_i'^0 = \alpha p_i^0 + \alpha\beta p_i^z$$

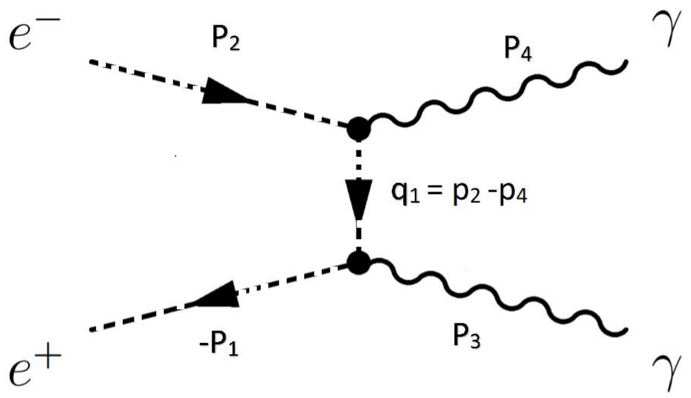
$$p_i'^z = \alpha p_i^z + \alpha\beta p_i^0$$

$$p_i'^\perp = p_i^\perp$$

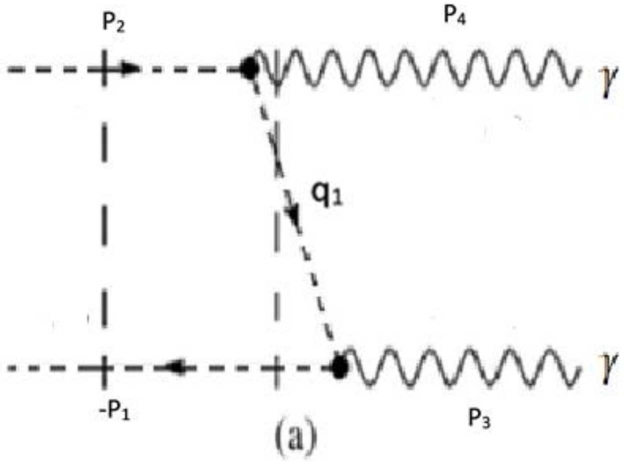
Time ordering in the interpolation dynamics

t-Channel

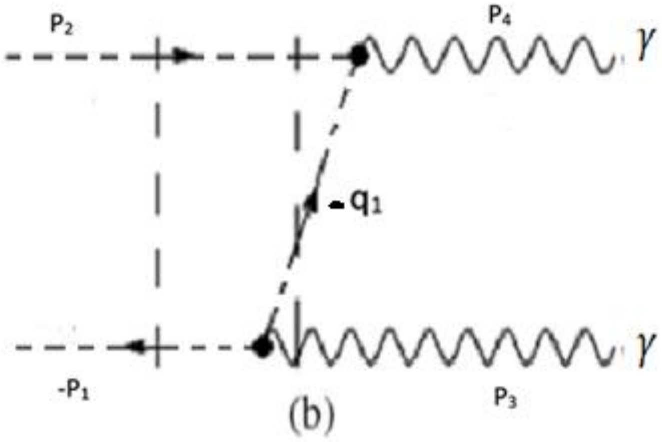
Covariant Propagator



Forward moving



Backward moving



$$\Sigma = \Sigma_a^\delta + \Sigma_b^\delta = \frac{1}{q_1^2 - m^2}$$

$$\Sigma_a^\delta = \frac{C}{2Q^{\hat{\mp}}\,(q^{\hat{\mp}} - Q^{\hat{\mp}})}$$

$$\Sigma_b^\delta = -\frac{C}{2Q^{\hat{\mp}}\,(q^{\hat{\mp}} + Q^{\hat{\mp}})}$$

Where: $C = Cos(2\delta)\,,\;\; S = Sin(2\delta),\;\; q^{\hat{\mp}} = p_2^{\hat{\mp}} - p_4^{\hat{\mp}},\;\;\; Q^{\hat{\mp}} = \pm\sqrt{Q_{\hat{=}}^2 + C(\vec{q}^{\,2}{}_{\perp} + m^2)}$

Observe symmetries between covariant Helicity amplitudes

Mt + +	=	Mt - -	Mt 0 +	=	-(Mt 0 -)	Mt 00
Mu + +	=	Mu - -	Mu 0 +	=	-(Mu 0 -)	Mu 00
Mse + +	=	Mse - -	Mse 0 +	=	-(Mse 0 -)	Mse 00
Mt + -	=	Mt - +	Mt + 0	=	-(Mt – 0)	
Mu + -	=	Mu - +	Mu + 0	=	-(Mu – 0)	
Mse + -	=	Mse - +	Mse + 0	=	-(Mse – 0)	

Mt ++	=	-(Mt + -)
Mu ++	=	-(Mu + -)

$$\theta = \frac{\pi}{2}$$

Mt + +	=	Mu ++
Mt 0 +	=	Mu + 0
Mu 0 +	=	Mt + 0
Mse 0 +	=	Mse+ 0
Mt 00	=	Mu 00

- Helicity amplitudes satisfy symmetry based on parity conservation

$$H(-s', -h', -s, -h) = (-1)^{s'+h'-s-h} H(s', h', s, h)$$

$$H(-h', -h) = (-1)^{h'-h} H(\, h', h)$$

- Corresponding time order amplitudes also satisfy these symmetries

C-R . Ji, B.L.G.Bakker , International Journal of Modern Physics,(2013)

Critical annihilation angle

$$q_1^+ = \frac{-\frac{pepz}{2E0} + \frac{pzp\gamma Cos[\theta]}{2E0}}{\sqrt{2}} + \frac{-\frac{pe\sqrt{4E0^2 + pz^2}}{2E0} + \frac{\sqrt{4E0^2 + pz^2}p\gamma Cos[\theta]}{2E0}}{\sqrt{2}}$$

$$q_2^+ = \frac{-\frac{pepz}{2E0} - \frac{pzp\gamma Cos[\theta]}{2E0}}{\sqrt{2}} + \frac{-\frac{pe\sqrt{4E0^2 + pz^2}}{2E0} - \frac{\sqrt{4E0^2 + pz^2}p\gamma Cos[\theta]}{2E0}}{\sqrt{2}}$$

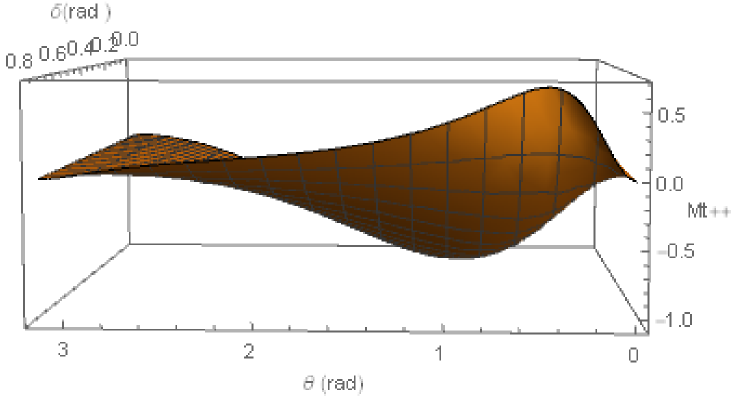
$q_1^+ > 0$ →Forward

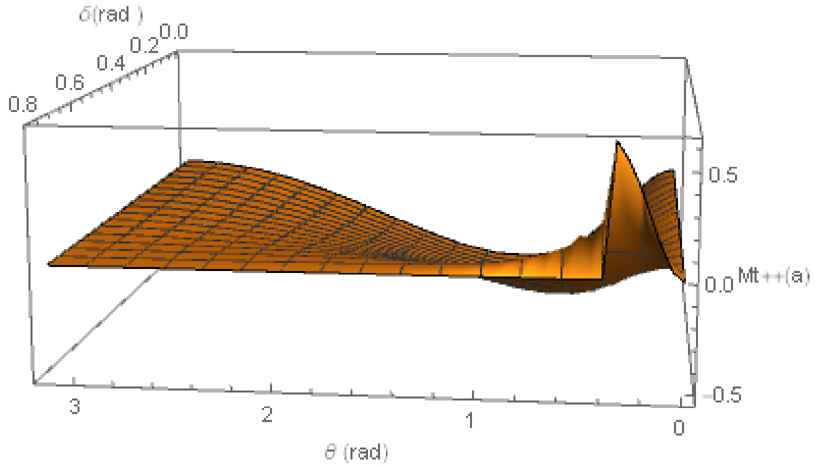
$q_1^+ < 0$ → Backward

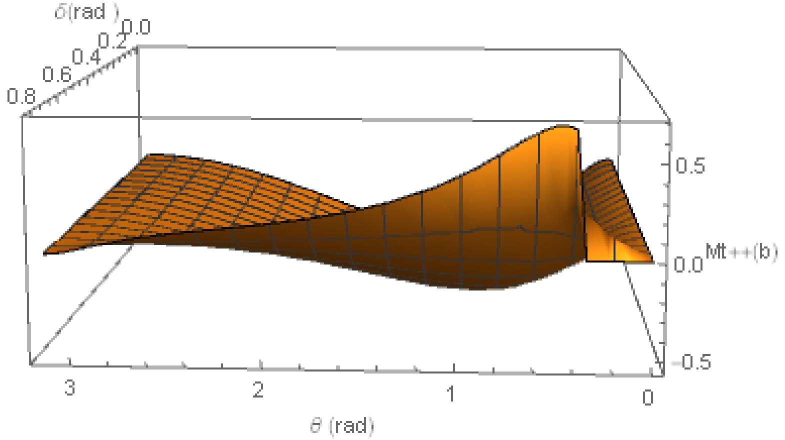
$q_1^+ = 0 \quad \rightarrow \quad \theta_{c,t} = ArcCos\left(\frac{P_e}{P_\gamma}\right)$

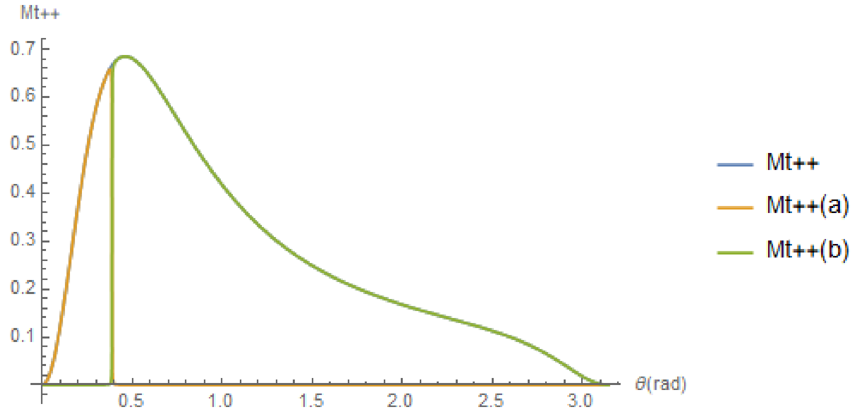
$q_2^+ = 0 \quad \rightarrow \quad \theta_{c,u} = -ArcCos\left(\frac{P_e}{P_\gamma}\right)$

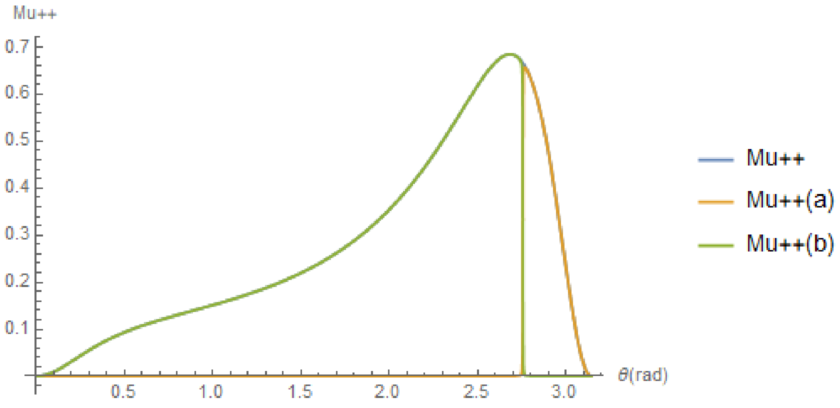
$\rightarrow \exists\ Cos(\theta_c) \leftrightarrow P_e < P_\gamma$











$$E_0 = 2m_e$$

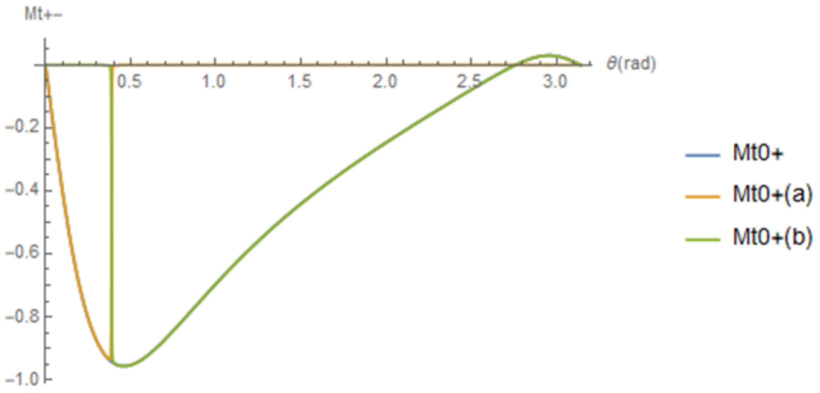
$$P_e = \sqrt{3}m_e$$

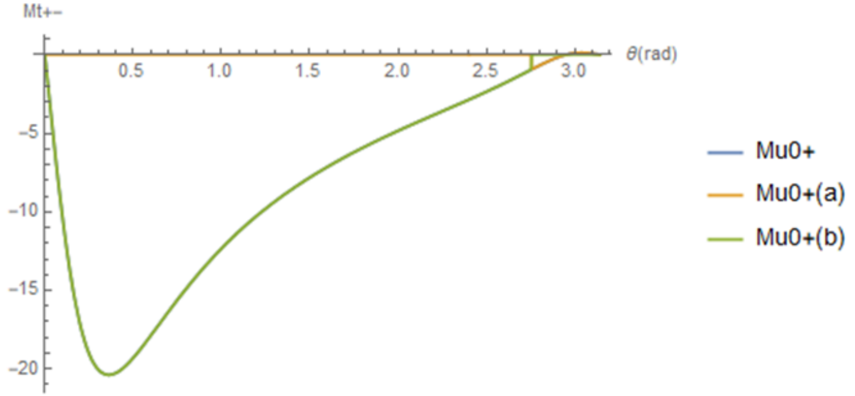
$$P_\gamma = \sqrt{3.5}m_e$$

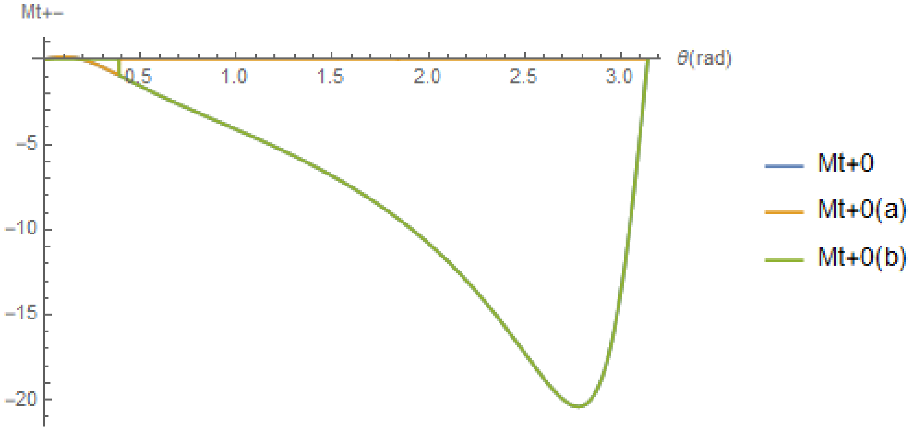
$$\delta = 0.785398 \sim \frac{\pi}{4}$$

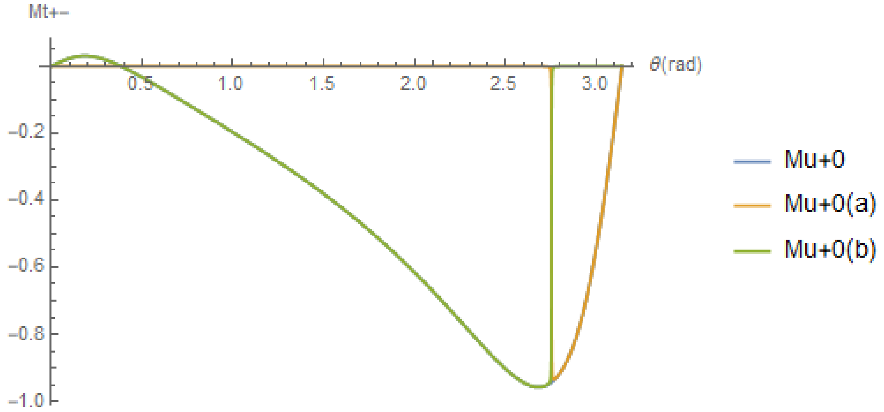
$$\theta_{c,t} = ArcCos\left(\sqrt{\frac{3}{3.5}}\right) = 0.387597$$

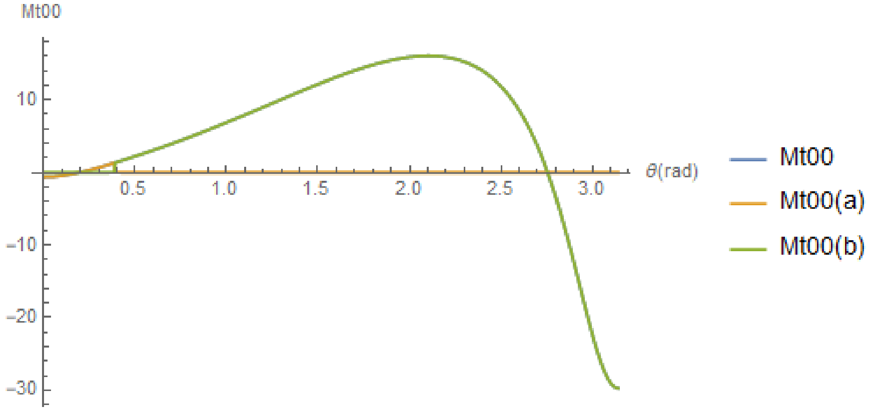
$$\theta_{c,u} = -ArcCos\left(\sqrt{\frac{3}{3.5}}\right) = 2.754$$

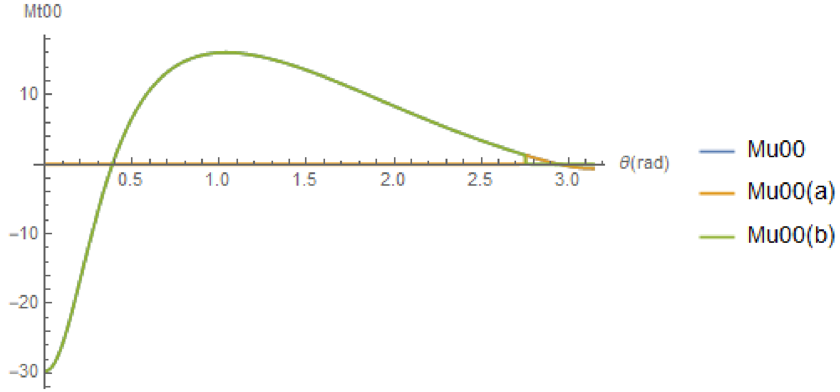

Mt+–
θ(rad)
Mt0+
Mt0+(a)
Mt0+(b)


Mt+–
θ(rad)
Mu0+
Mu0+(a)
Mu0+(b)


Mt+–
θ(rad)
Mt+0
Mt+0(a)
Mt+0(b)

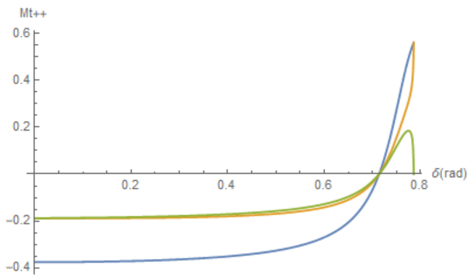

Mt+–
θ(rad)
Mu+0
Mu+0(a)
Mu+0(b)


Mt00
θ(rad)
Mt00
Mt00(a)
Mt00(b)

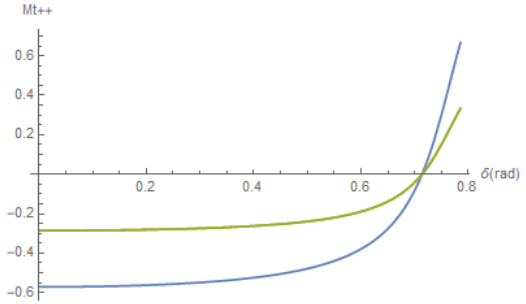

Mt00
θ(rad)
Mu00
Mu00(a)
Mu00(b)

Critical interpolation angles

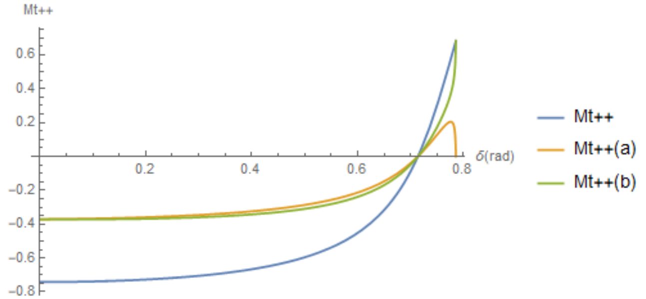
$\theta = \theta_{c,t} - 0.1$



$\theta = \theta_{c,t}$

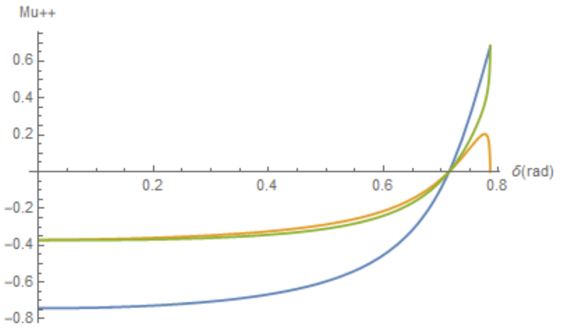


$\theta = \theta_{c,t} + 0.1$

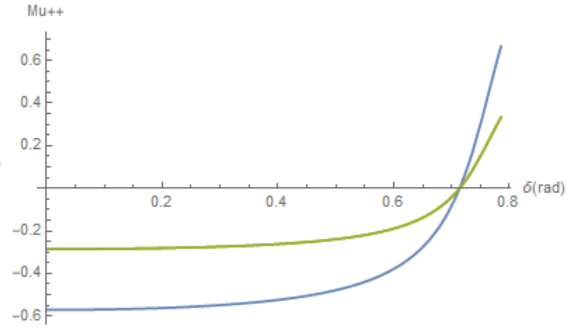


$$E_0 = 2m_e$$
$$P_\gamma = \sqrt{3.5}m_e$$
$$P_e = \sqrt{3}m_e$$

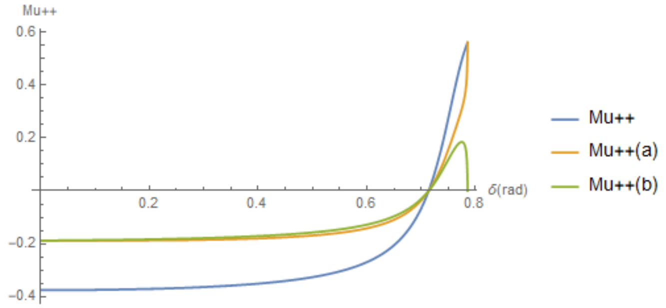
$\theta = \theta_{c,u} - 0.1$



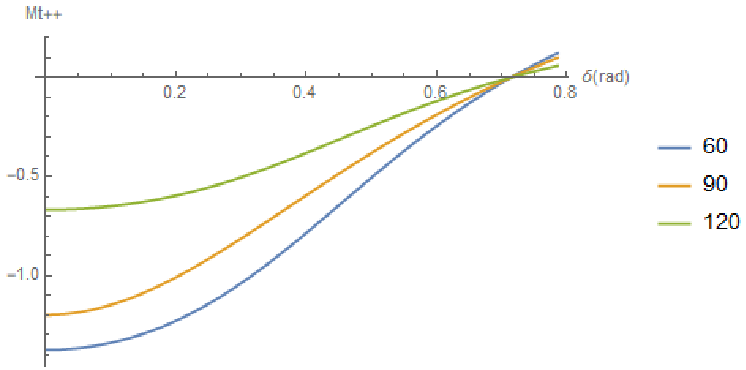
$\theta = \theta_{c,u}$

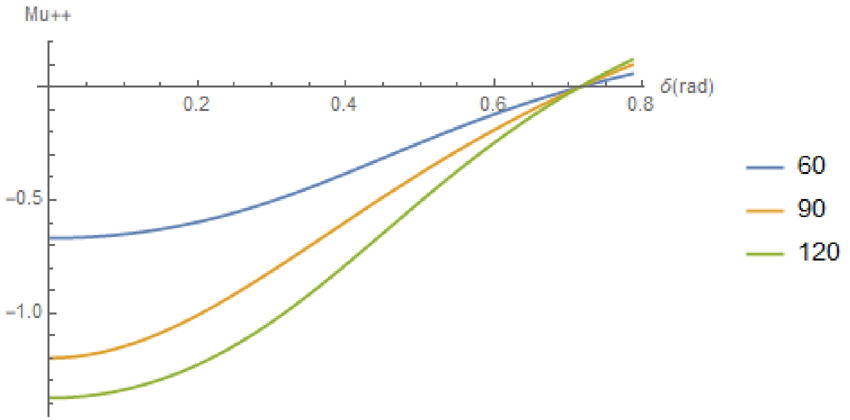


$\theta = \theta_{c,u} + 0.1$



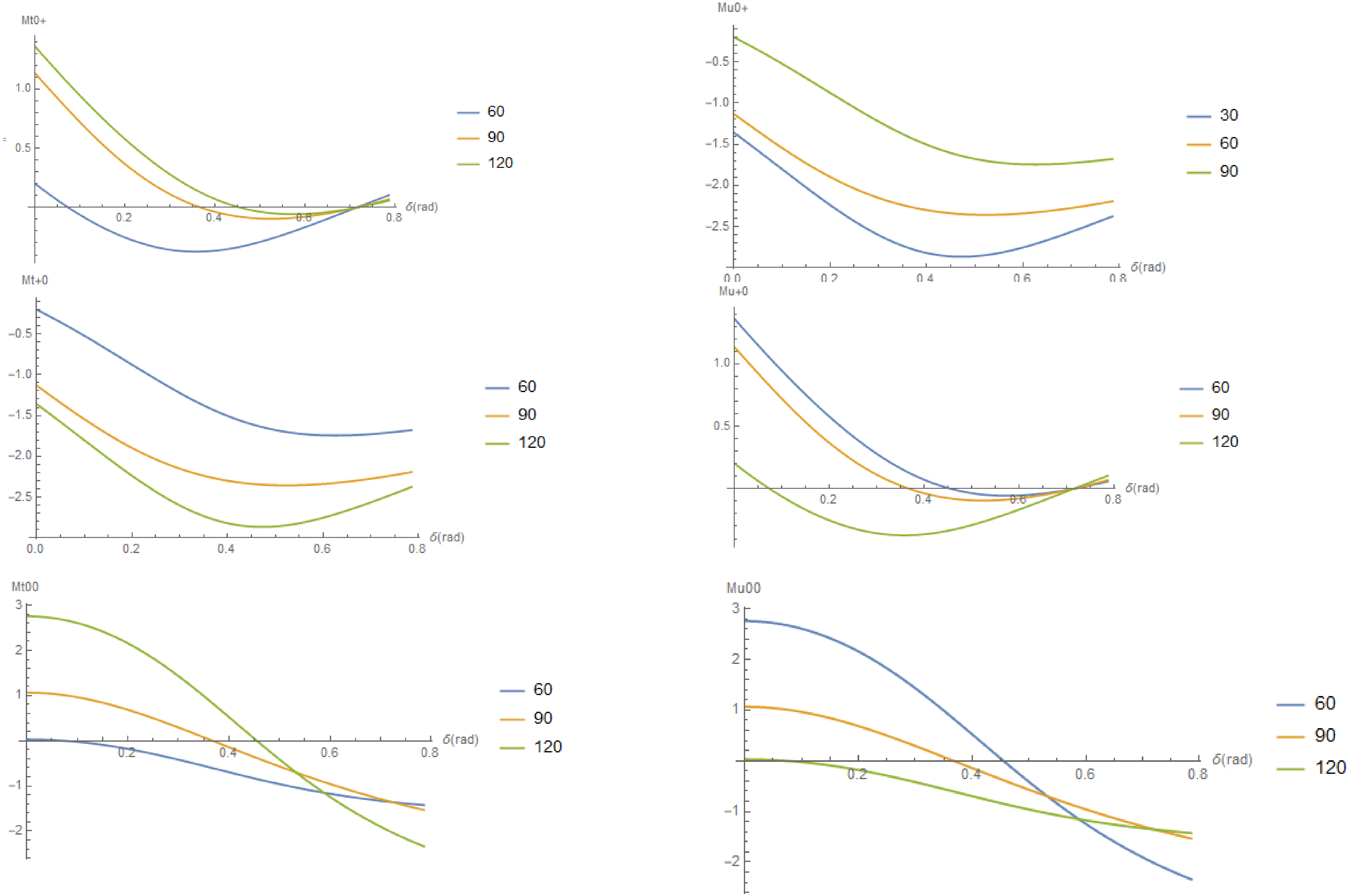
- When rho meson polarization vectors are transverse ,all δ_c values are independent of P_γ and θ .





$$P_\gamma = m_e$$

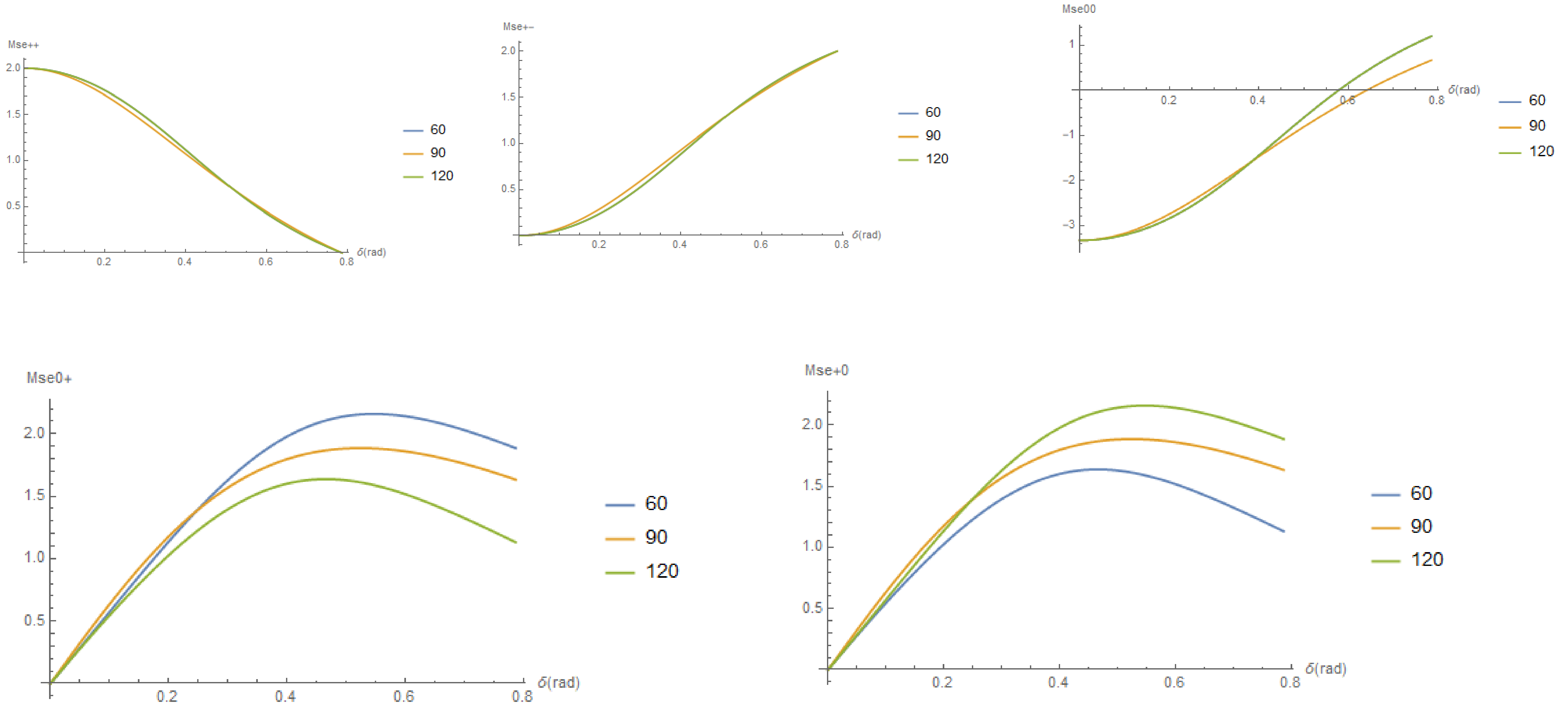
$$\delta_c = ArcTan\left(\frac{P_e}{E_0}\right)$$
$$= 0.713724$$



$$E_0 = 2m_e$$

$$P_e = \sqrt{3}m_e$$

$$P_\gamma = m_e$$


Mse++
Mse+−
Mse00
Mse0+
Mse+0
δ(rad)
60
90
120

Critical interpolation angles ($Pz \neq 0$)

$$Et = E0^2 p\gamma Cos[\theta] - pe(E0^2 - p\gamma^2 Sin[\theta]^2)$$
$$Pt = E0 p\gamma(p\gamma - peCos[\theta])$$
$$Eu = E0^2 p\gamma Cos[\theta] + pe(E0^2 - p\gamma^2 Sin[\theta]^2)$$
$$Pu = E0 p\gamma(p\gamma + peCos[\theta])$$

$$\delta_p^{\pm} = -ArcTan\left[\frac{\left(E0 * pz + pe * \sqrt{4E0^2 + pz^2}\right)}{\left(pe * pz + E0 * \sqrt{4E0^2 + pz^2}\right)}\right]$$

$$\delta_e^{\pm} = -ArcTan\left[\frac{\left(E0 * pz - pe * \sqrt{4E0^2 + pz^2}\right)}{\left(-pe * pz + E0 * \sqrt{4E0^2 + pz^2}\right)}\right]$$

$$\delta_{pt}^{00} = -ArcTan\left[\frac{\left(Et * pz + pt * \sqrt{4E0^2 + pz^2}\right)}{\left(pt * pz + Et * \sqrt{4E0^2 + pz^2}\right)}\right]$$

$$\delta_{et}^{00} = -ArcTan\left[\frac{\left(Et * pz - pt * \sqrt{4E0^2 + pz^2}\right)}{\left(-pt * pz + Et * \sqrt{4E0^2 + pz^2}\right)}\right]$$

$$\delta_{pu}^{00} = -ArcTan\left[\frac{\left(Eu * pz + pu * \sqrt{4E0^2 + pz^2}\right)}{\left(pu * pz + Eu * \sqrt{4E0^2 + pz^2}\right)}\right]$$

$$\delta_{eu}^{00} = -ArcTan\left[\frac{\left(Eu * pz - pu * \sqrt{4E0^2 + pz^2}\right)}{\left(-pu * pz + Eu * \sqrt{4E0^2 + pz^2}\right)}\right]$$

- Pt and Pu play the role of Pe
- Et and Eu play the role of E0

$$M_t = (-p_1 + q_1)^\mu \varepsilon^*_\mu(p_3,\lambda_3) \frac{1}{q_1^2 - m^2} (p_2 + q_1)^\nu \varepsilon^*_\nu(p_4,\lambda_4)$$

$$(-p_1 + q_1)^\mu \varepsilon^*_\mu(p_3,\lambda_3) = -2\,(p_1)^\mu \varepsilon^*_\mu(p_3,\lambda_3) \quad \rightarrow (\delta_p^\pm, \delta_{pt}^{00})$$

$$(p_2 + q_1)^\nu \varepsilon^*_\nu(p_4,\lambda_4) = 2(p_2)^\nu \varepsilon^*_\nu(p_4,\lambda_4) \quad \rightarrow (\delta_e^\pm, \delta_{et}^{00})$$

$$M_u = (-p_1 + q_2)^\nu \varepsilon^*_\nu(p_4,\lambda_4) \frac{1}{q_2^2 - m^2} (p_2 + q_2)^\mu \varepsilon^*_\mu(p_3,\lambda_3)$$

$$(-p_1 + q_2)^\nu \varepsilon^*_\nu(p_4,\lambda_4) = -2(p_1)^\nu \varepsilon^*_\nu(p_4,\lambda_4) \quad \rightarrow (\delta_p^\pm, \delta_{pu}^{00})$$

$$(p_2 + q_2)^\mu \varepsilon^*_\mu(p_3,\lambda_3) = -2\,(p_2)^\mu \varepsilon^*_\mu(p_3,\lambda_3) \quad \rightarrow (\delta_e^\pm, \delta_{eu}^{00})$$

Ex

- Positive scalar meson projection to the polarization vector of the rho meson give $(\delta_p^\pm, \delta_{pt}^{00})$ critical angles depending on whether polarization vectors are longitudinal or transverse

Summary

Covariant Amplitudes					δ_c	
Mt ++ =	Mt - - =	-(Mt + -) =	-(Mt + -)		$\delta_p^{\pm}$	$\delta_e^{\pm}$
Mu ++ =	Mu - - =	-(Mu + -) =	-(Mu + -)		$\delta_p^{\pm}$	$\delta_e^{\pm}$
Mt 0 + =	-(Mt 0 -)				$\delta_e^{\pm}$	δ_{pt}^{00}
Mu 0 + =	-(Mu 0 -)				$\delta_p^{\pm}$	δ_{pu}^{00}
Mt + 0 =	-(Mt – 0)				$\delta_p^{\pm}$	δ_{et}^{00}
Mu + 0 =	-(Mu – 0)				$\delta_e^{\pm}$	δ_{eu}^{00}
Mt 00					δ_{pt}^{00}	δ_{et}^{00}
Mu 00					δ_{pu}^{00}	δ_{eu}^{00}

δ_c= Critical interpolation angles in which covariant amplitudes are equal to zero

- Corresponding time order-amplitude also vanishes when δ->δ_c

Limitation

IFD ⟷ Interpolating Dynamic ⟷ LFD

0 δ $\frac{\pi}{4}$

$$0 \le Tan(\delta) \le 1$$

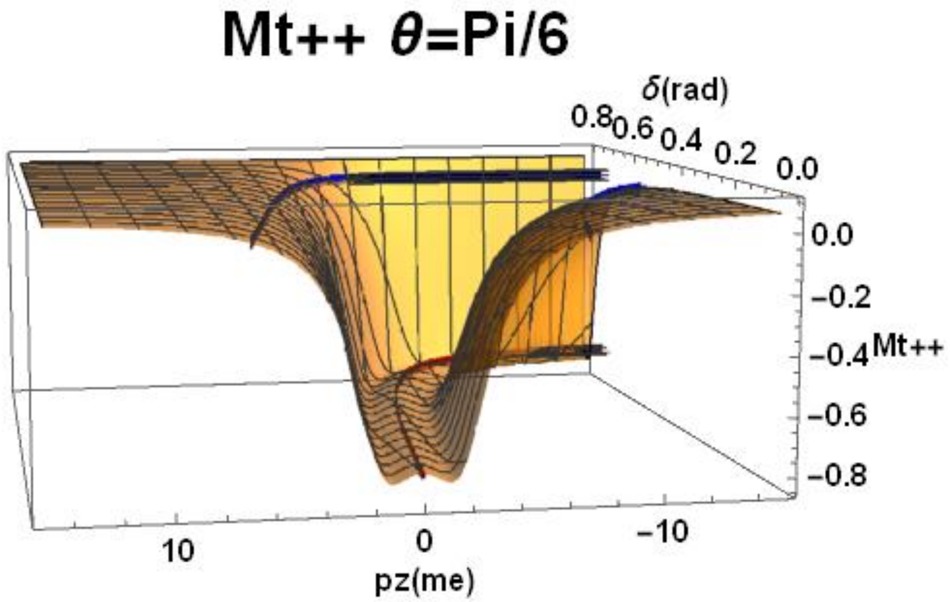
	$0 \le Tan(\delta)$	$Tan(\delta) \le 1$
$\delta_p^{\pm}$	-1≤R	-Rz ≥R
$\delta_e^{\pm}$	1≥R	Rz≤R
δ_{pt}^{00}	-1≤Rt	-Rz ≥Rt
δ_{et}^{00}	1≥Rt	Rz≤Rt
δ_{pu}^{00}	-1≤Ru	-Rz ≥Ru
δ_{eu}^{00}	1≥Ru	Rz≤Ru

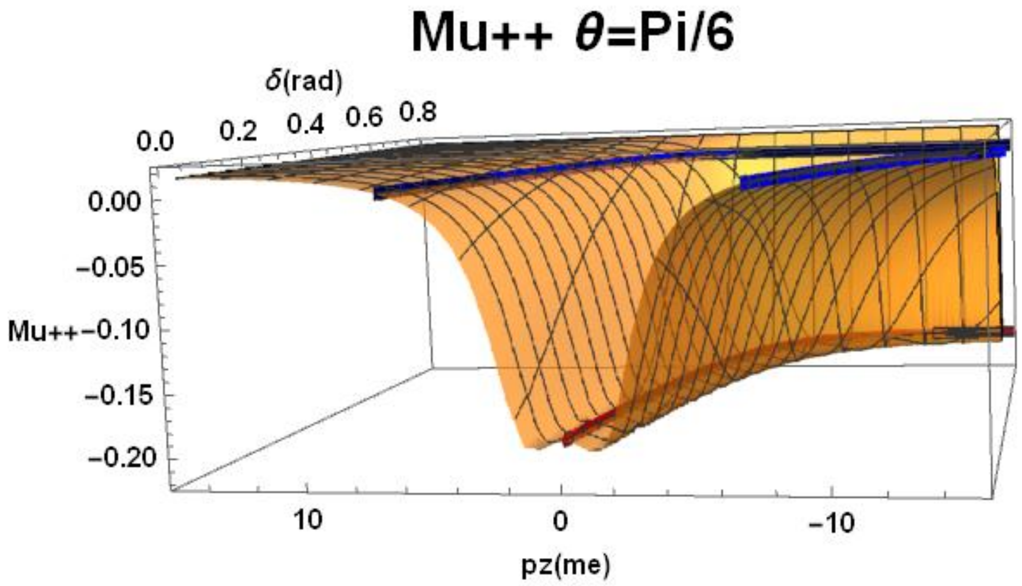
$$R = \frac{P_e}{E0} \qquad Rz = \frac{P_z}{E}$$

$$Rt = \frac{Pt}{Et} \qquad Ru = \frac{Pu}{Eu}$$

Total energy of the boosted frame =

$$E = \sqrt{4E_0^2 + P_z^2}$$

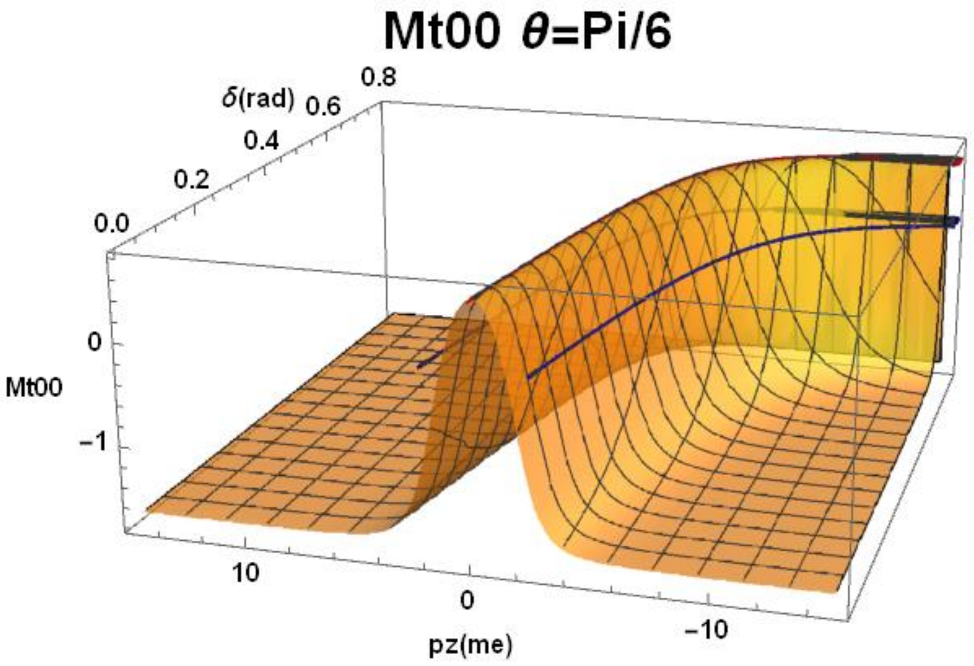


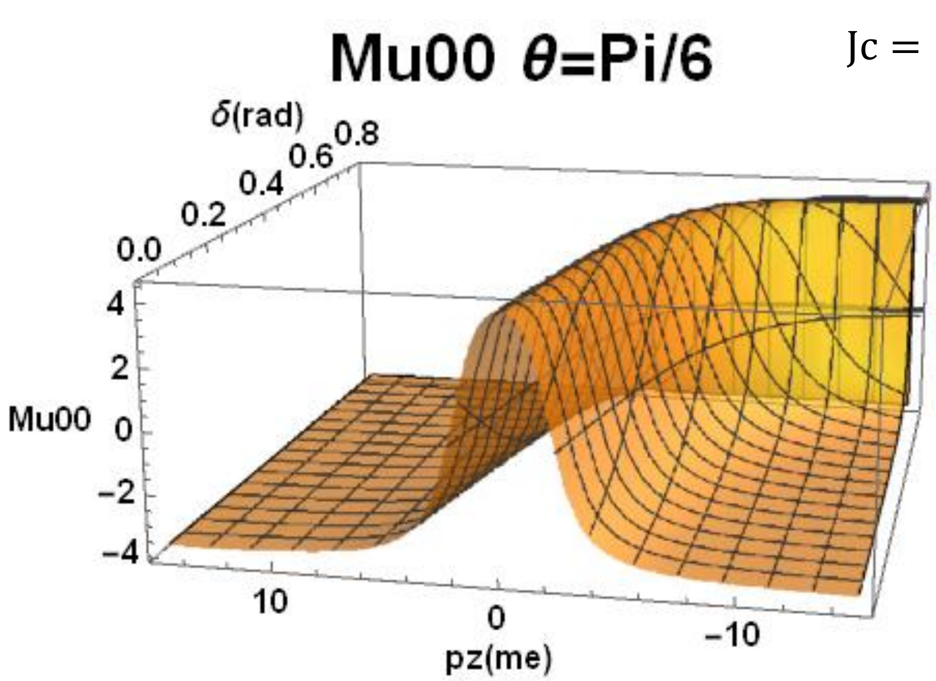


$$E_0 = 2m_e$$

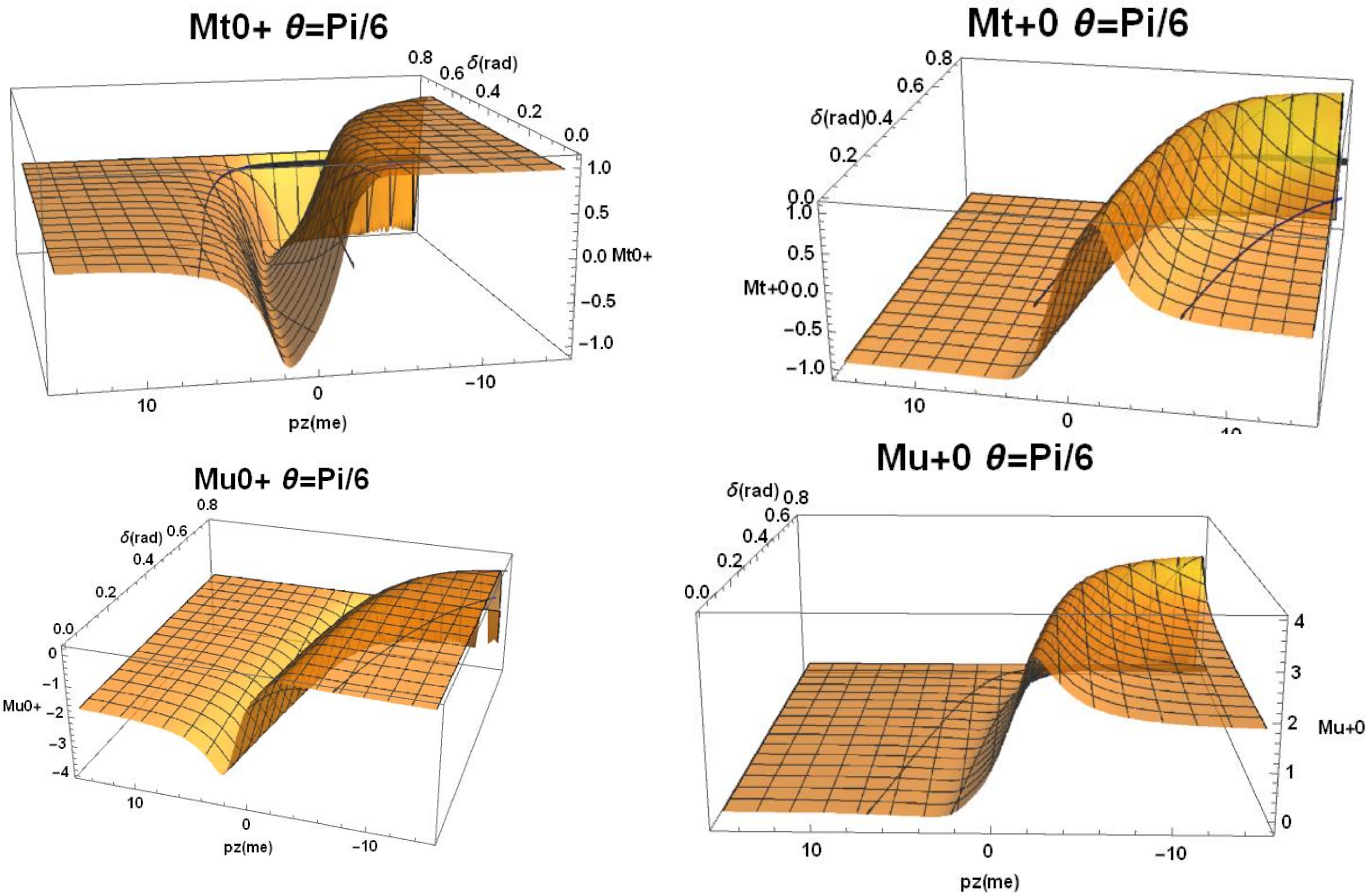
$$P_e = \sqrt{3}m_e$$

$$P_\gamma = m_e$$





$$\mathrm{Jc} = -\sqrt{\frac{(2\mathrm{E0})^2(1-\mathrm{Cos}[2\delta])}{2\mathrm{Cos}[2\delta]}}$$


Mt0+ θ=Pi/6
δ(rad)
0.8
0.6
0.4
0.2
0.0
1.0
0.5
0.0 Mt0+
−0.5
−1.0
10
0
−10
pz(me)
Mt+0 θ=Pi/6
0.8
0.6
δ(rad)0.4
0.2
0.0
1.0
0.5
Mt+0 0.0
−0.5
−1.0
10
0
Mu0+ θ=Pi/6
0.8
δ(rad) 0.6
0.4
0.2
0.0
0
−1
Mu0+ −2
−3
−4
10
0
pz(me)
−10
Mu+0 θ=Pi/6
δ(rad) 0.8
0.6
0.4
0.2
0.0
4
3
2 Mu+0
1
0
10
0
−10
pz(me)

Cross-Section of the processes using Mandelstam variable.

$$|M|^2 = \sum_{\lambda_1,\lambda_2} |M_t^{\lambda_1,\lambda_2} + M_u^{\lambda_1,\lambda_2} + M_{se}^{\lambda_1,\lambda_2}|^2$$

- Two Scalar mesons annihilation in to two rho mesons

$$|M_\rho^2| = \left[\left[\frac{2t + 2m_e^2 - m_\gamma^2}{(t - m_e^2)}\right]^2 + \left[\frac{2u + 2m_e^2 - m_\gamma^2}{(u - m_e^2)}\right]^2 + 2\left[\frac{\left[t + u + 2m_e^2 - 3m_\gamma^2\right]^2}{(t - m_e^2)(u - m_e^2)}\right]\right]$$

$$-4\left[\left[\frac{5t + u + 2m_e^2 - 4m_\gamma^2}{(t - m_e^2)}\right] + \left[\frac{5u + t + 2m_e^2 - 4m_\gamma^2}{(u - m_e^2)}\right]\right] + 16$$

- Scaler electron and proton annihilation in to two photons

$$|M|^2 = 4\left[\left[\frac{t + m_e^2}{t - m_e^2}\right]^2 + \left[\frac{u + m_e^2}{u - m_e^2}\right]^2 + 4\left[\frac{(t + m_e^2)(u + m_e^2) - 2tu}{(t - m_e^2)(u - m_e^2)}\right] + 4\right]$$

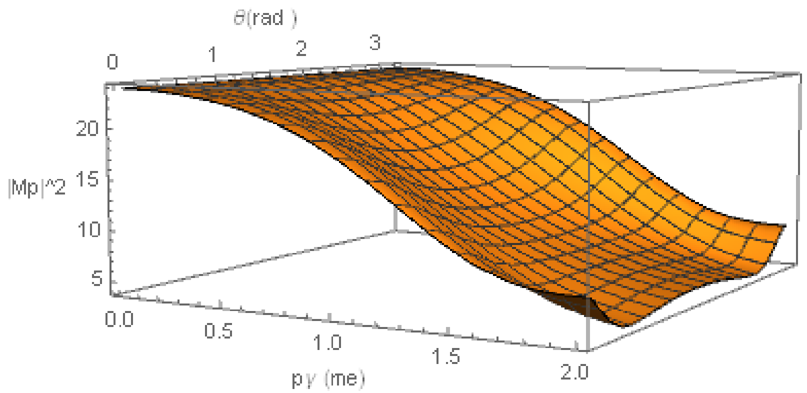
- Two Scalar mesons annihilation in to two rho mesons

$$|M_\rho^2| = 4\left[\left(1-\frac{2pe^2(E0^2+p\gamma^2)Sin[\theta]^2}{(E0^2+p\gamma^2)^2-4pe^2p\gamma^2Cos[\theta]^2}\right)^2+\frac{8pe^4(E0^2+p\gamma^2)^2Sin[\theta]^4}{\left((E0^2+p\gamma^2)^2-4pe^2p\gamma^2Cos[\theta]^2\right)^2}+\right]$$

$$4\left[\frac{(E0^2-p\gamma^2)^2(E0^2+p\gamma^2-4pe^2Cos[\theta]^2)^2}{\left((E0^2+p\gamma^2)^2-4pe^2p\gamma^2Cos[\theta]^2\right)^2}+\frac{8E0^2pe^4(E0^2-p\gamma^2)Sin[2\theta]^2}{\left((E0^2+p\gamma^2)^2-4pe^2p\gamma^2Cos[\theta]^2\right)^2}\right]$$

- Scaler electron and proton annihilation in to two photons

$$|M|^2 = 4\left[1+\left[1-\frac{2p_e^2Sin^2(\theta)}{E_0^2-p_e^2Cos^2(\theta)}\right]^2\right]$$



Thank You

Calculating cross section as a function of Mandelstam variable.

$$M \quad = (-p_1 \ + q_1)^{\mu} \varepsilon_{\mu}^{*}(p_3, \lambda_3) \frac{1}{q_1^2 - m^2} (p_2 \ + q_1)^{\nu} \varepsilon_{\nu}^{*}(p_4, \lambda_4)$$

$$+ (-p_1 \ + q_2)^{\nu} \varepsilon_{\nu}^{*}(p_4, \lambda_4) \frac{1}{q_2^2 - m^2} (p_2 \ + q_2)^{\mu} \ \varepsilon_{\mu}^{*}(p_3, \lambda_3 \ )$$

$$-2 g_{\mu\nu} \epsilon^{*\mu}(p_3, \lambda_3) \varepsilon^{*\nu} \ (p_4, \lambda_4)$$

$$s = (p_1 \ + p_2 \ )^2$$

$$t = (p_3 - p_1 \ )^2 = q_1^2$$

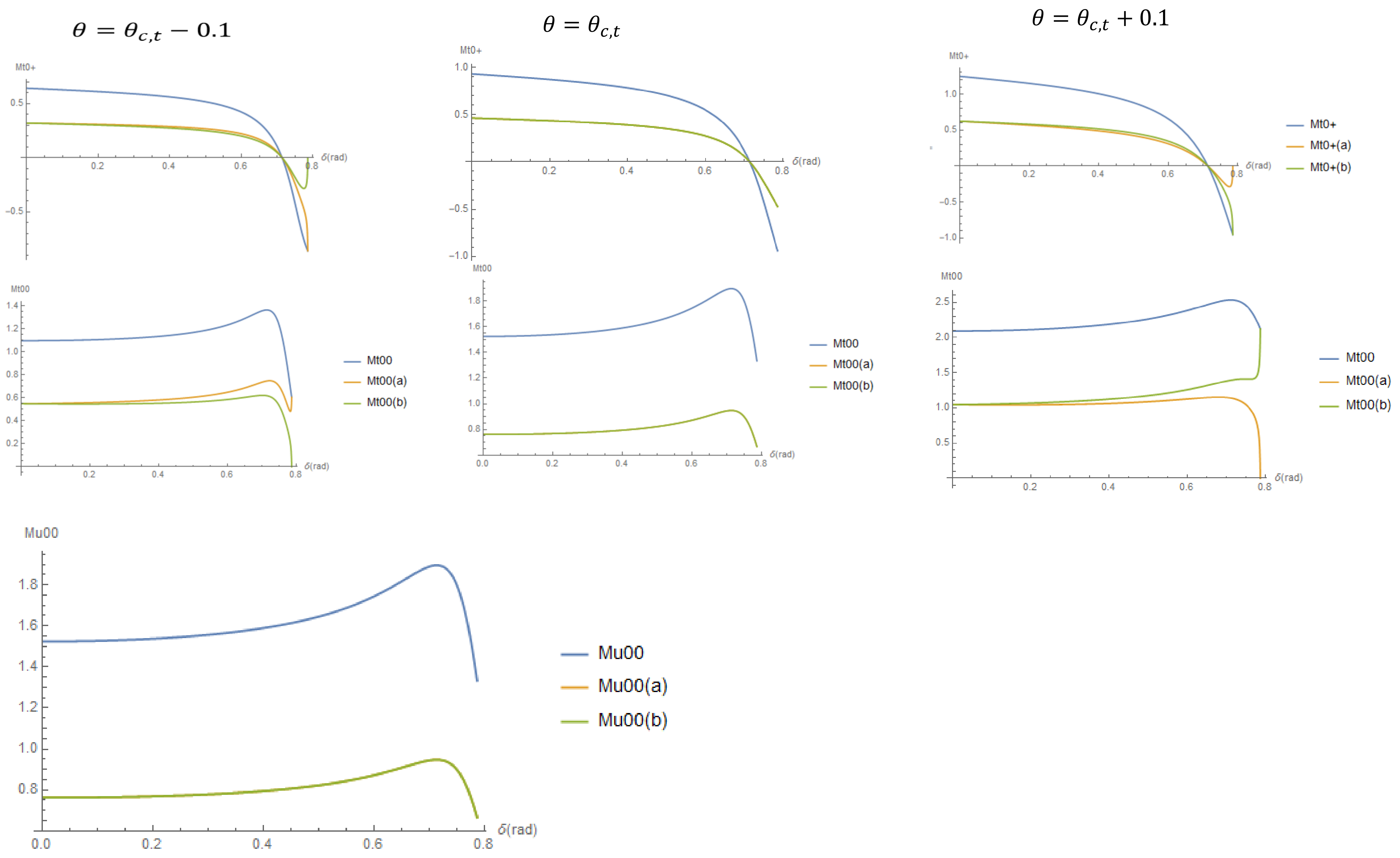
$$M = \varepsilon_{\mu}^{*}(p_3, \lambda_3) \varepsilon_{\nu}^{*}(p_4, \lambda_4) [J^{t_{\mu\nu}} + J^{u_{\mu\nu}} + J^{se_{\mu\nu}}]$$

$$u = (p_4 - p_1 \ )^2 = q_2^2$$

$$\sum_{\lambda} \varepsilon_{\mu}^{*}(p, \lambda) \epsilon_{\mu'}(p, \lambda) = -g_{\mu\mu'}$$

$$s + t + u = 2m_e^2 + 2m_{\gamma}^2$$

$$|M|^2 = [J_{\mu\nu}^{t} + J_{\mu\nu}^{u} + J_{\mu\nu}^{se}] \ [J^{t_{\mu\nu}} + J^{u_{\mu\nu}} + J^{se_{\mu\nu}}]$$



$\theta = \theta_{c,t} - 0.1$
$\theta = \theta_{c,t}$
$\theta = \theta_{c,t} + 0.1$
Mt0+
Mt0+(a)
Mt0+(b)
Mt00
Mt00(a)
Mt00(b)
Mu00
Mu00(a)
Mu00(b)
δ(rad)
0.0
0.2
0.4
0.6
0.8