
Parton distributions in hadrons

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JLab Angular Momentum collaboration

Outline

- *Aim*: understand internal quark-gluon structure of hadrons
- *Method*: extract parton distribution functions (PDFs) from global QCD analysis, using new Monte Carlo-based methods

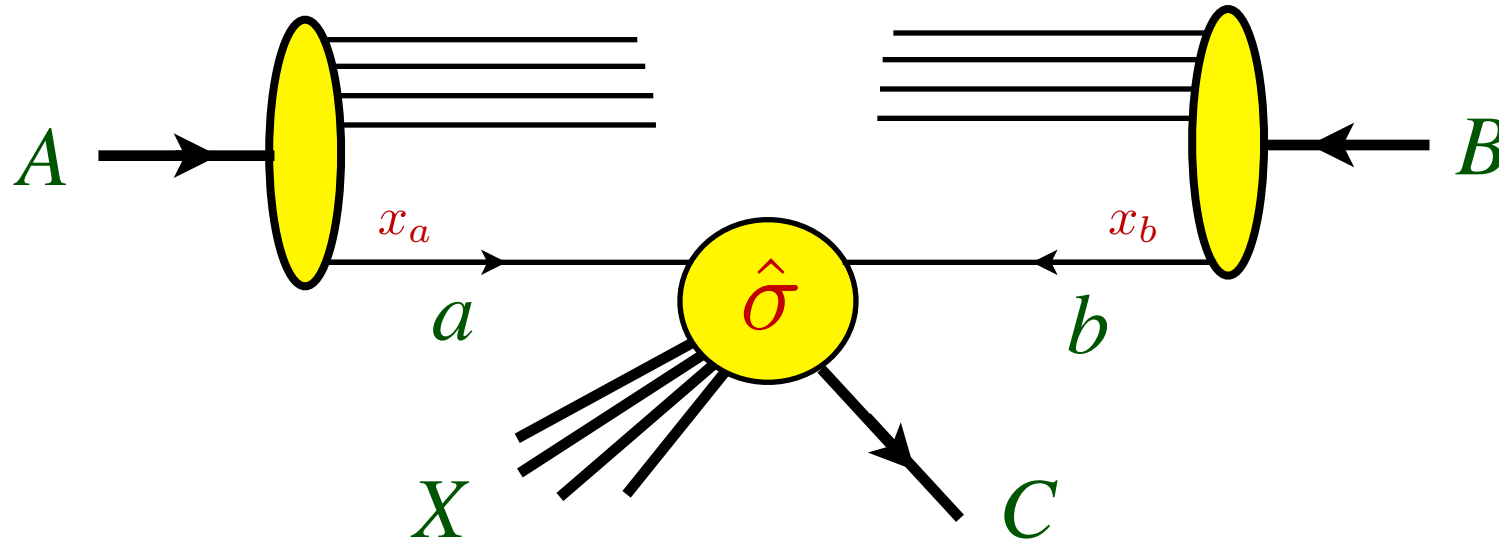
Recent highlights:

- Constraints from Fermilab & JLab data on unpolarized PDFs at high x
- First combined analysis of polarized DIS + SIDIS + SIA data, with *simultaneous* extraction of PDFs & fragmentation functions
- First extraction of pion PDFs from Drell-Yan and HERA leading neutron production data

Parton distributions in hadrons



Generic process: inclusive particle production $AB \rightarrow CX$



$$\sigma_{AB \rightarrow CX}(p_A, p_B) = \sum_{a,b} \int dx_a dx_b f_{a/A}(x_a, \mu) f_{b/B}(x_b, \mu) \times \sum_n \alpha_s^n(\mu) \hat{\sigma}_{ab \rightarrow CX}^{(n)}(x_a p_A, x_b p_B, Q/\mu)$$

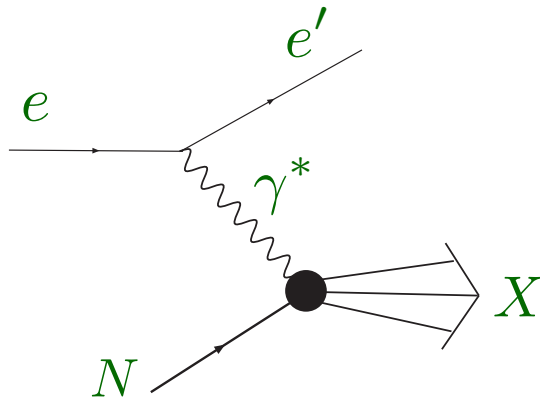
“factorization”

Collins, Soper, Sterman (1980s)

→ universal functions $f_{a/A}$ characterize internal structure of bound state A

Parton distributions in hadrons

- Most information on parton distribution functions obtained from inclusive deep-inelastic scattering (DIS)

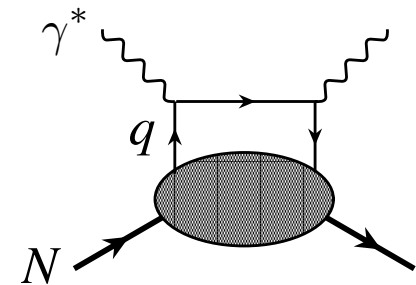


$$\frac{d^2\sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2 \cos^2 \frac{\theta}{2}}{Q^4} \left(2 \tan^2 \frac{\theta}{2} \frac{F_1}{M} + \frac{F_2}{\nu} \right)$$

$$\begin{aligned} \nu &= E - E' \\ Q^2 &= \vec{q}^2 - \nu^2 \\ W^2 &= M^2 + Q^2 \frac{(1-x)}{x} \end{aligned} \quad x = \frac{Q^2}{2M\nu}$$

- At leading order (LO) in pQCD, structure functions given in terms of charge-weighted sums of PDFs

$$F_2(x, Q^2) = x \sum_q e_q^2 q(x, Q^2)$$

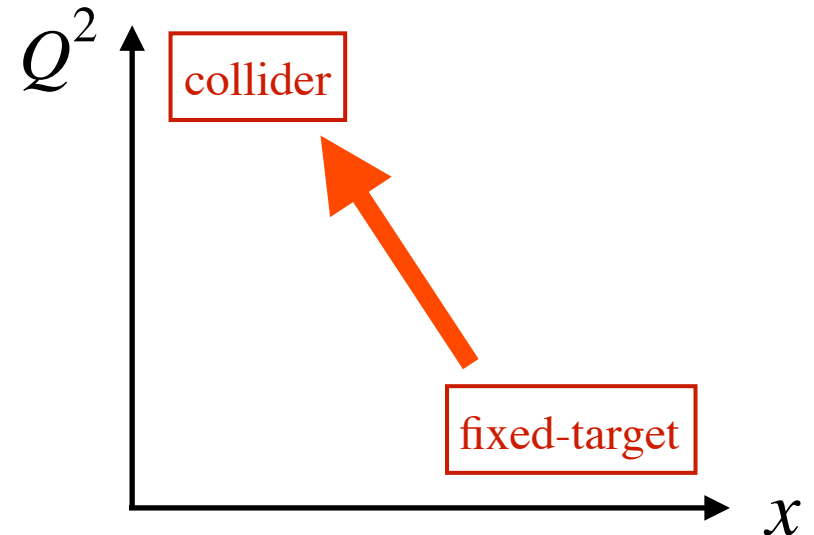


Parton distributions in hadrons

● Precision PDFs needed to

- (1) understand basic structure of QCD bound states
- (2) compute backgrounds in searches for BSM physics

→ Q^2 evolution feeds
low x , high Q^2 (“LHC”)
from high x , low Q^2 (“JLab”)



● Information on PDFs obtained from

- (1) nonperturbative approaches (low-energy models, DSE, χ EFT)
- (2) lattice QCD
- (3) global QCD analysis

Global PDF analysis

- Universality of PDFs allows data from different processes (DIS, SIDIS, jet production, Drell-Yan ...) to be analyzed simultaneously
- Several dedicated global efforts to extract PDFs using factorization theorems + pQCD at a given order in α_s
 - CTEQ, MRS/MMHT, HERAPDF, DSSV, ...
use standard maximum likelihood methods (χ^2 minimization)
 - NNPDF, JAM
use Monte Carlo methods (neural networks, nested sampling)
- Typically PDF parametrizations are nonlinear functions of PDF parameters, *e.g.* $xf(x, \mu) = Nx^\alpha(1-x)^\beta P(x)$ where P is a polynomial, neural net, ...
 - *multiple local minima* present in the χ^2 function
 - thoroughly scan over sufficiently large parameter space

Global PDF analysis

- A major challenge has been to characterize PDF *uncertainties*, especially in the presence of *tensions* among data sets
- Previous attempts sought to address tensions in data sets by introducing
 - “tolerance” factors (artificially inflating PDF errors)
 - “neural net” parametrization (instead of polynomial parametrization), together with MC techniques
- However, to address the problem in a more statistically rigorous way, one requires going *beyond* the standard χ^2 minimization paradigm
 - utilize modern techniques based on Bayesian statistics

Bayesian approach to global analysis

- Analysis of data requires estimating expectation values E and variances V of “observables” \mathcal{O} (functions of PDFs) which are functions of parameters

$$E[\mathcal{O}] = \int d^n a \mathcal{P}(\vec{a}|\text{data}) \mathcal{O}(\vec{a})$$

$$V[\mathcal{O}] = \int d^n a \mathcal{P}(\vec{a}|\text{data}) [\mathcal{O}(\vec{a}) - E[\mathcal{O}]]^2$$

“Bayesian master formulas”

- Using Bayes’ theorem, probability distribution \mathcal{P} given by

$$\mathcal{P}(\vec{a}|\text{data}) = \frac{1}{Z} \mathcal{L}(\text{data}|\vec{a}) \pi(\vec{a})$$

in terms of the likelihood function \mathcal{L}

Bayesian approach to global analysis



Likelihood function

$$\mathcal{L}(\text{data}|\vec{a}) = \exp\left(-\frac{1}{2}\chi^2(\vec{a})\right)$$

is a Gaussian form in the data, with χ^2 function

$$\chi^2(\vec{a}) = \sum_i \left(\frac{\text{data}_i - \text{theory}_i(\vec{a})}{\delta(\text{data})} \right)^2$$

with priors $\pi(\vec{a})$ and “evidence” Z

$$Z = \int d^n a \mathcal{L}(\text{data}|\vec{a}) \pi(\vec{a})$$

→ Z tests if *e.g.* an n -parameter fit is statistically different from $(n+1)$ -parameter fit

Bayesian approach to global analysis

- Standard method for evaluating E, V via maximum likelihood

→ maximize probability distribution

$$\mathcal{P}(\vec{a}|\text{data}) \rightarrow \vec{a}_0$$

→ if \mathcal{O} is linear in parameters, and if probability is symmetric in all parameters

$$E[\mathcal{O}(\vec{a})] = \mathcal{O}(\vec{a}_0), \quad V[\mathcal{O}(\vec{a})] \rightarrow \text{Hessian} \quad H_{ij} = \frac{1}{2} \frac{\partial^2 \chi^2(\vec{a})}{\partial a_i \partial a_j} \Big|_{\vec{a}=\vec{a}_0}$$

- In practice, since in general $E[f(\vec{a})] \neq f(E[\vec{a}])$, maximum likelihood method often fails

→ need more robust (Monte Carlo) approach

$$E[\mathcal{O}] \approx \frac{1}{N} \sum_k \mathcal{O}(\vec{a}_k), \quad V[\mathcal{O}] \approx \frac{1}{N} \sum_k [\mathcal{O}(\vec{a}_k) - E[\mathcal{O}]]^2$$

Monte Carlo methods

- First group to use MC for global PDF analysis was NNPDF, using neural network to parametrize $P(x)$ in

Forte et al. (2002)

$$f(x) = N x^\alpha (1 - x)^\beta P(x)$$

— α, β are fitted “preprocessing coefficients”

- Iterative Monte Carlo (IMC), developed by JAM Collaboration, variant of NNPDF, tailored to non-neutral net parametrizations

N. Sato et al. (2016)

- Markov Chain MC (MCMC) / Hybrid MC (HMC)
— recent “proof of principle” analysis, ideas from lattice QCD

Gbedo, Mangin-Brinet (2017)

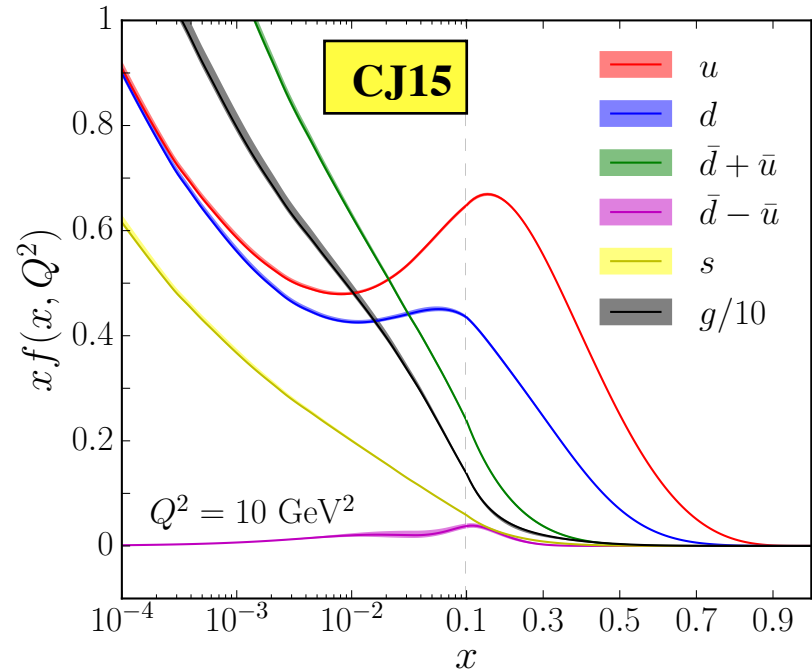
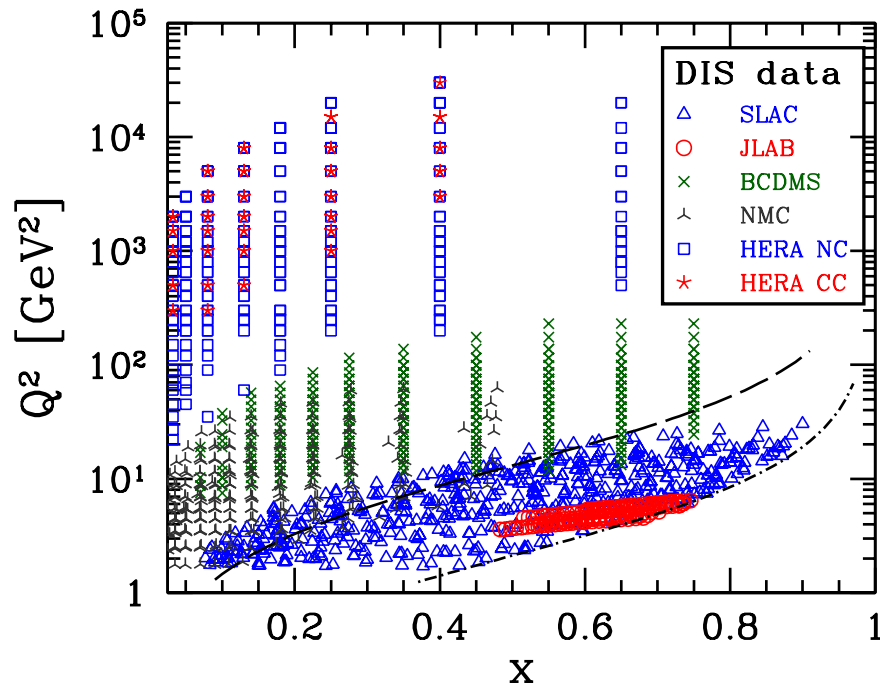
- Nested sampling (NS) — computes integrals in Bayesian master formulas (for E , V , Z) explicitly

Skilling (2004)

Unpolarized Nucleon PDFs

Unpolarized PDFs

- Ubiquity of proton F_2 data (SLAC, BCDMS, NMC, HERA, JLab, ...) provides strong constraints on u -quark PDF over large x range



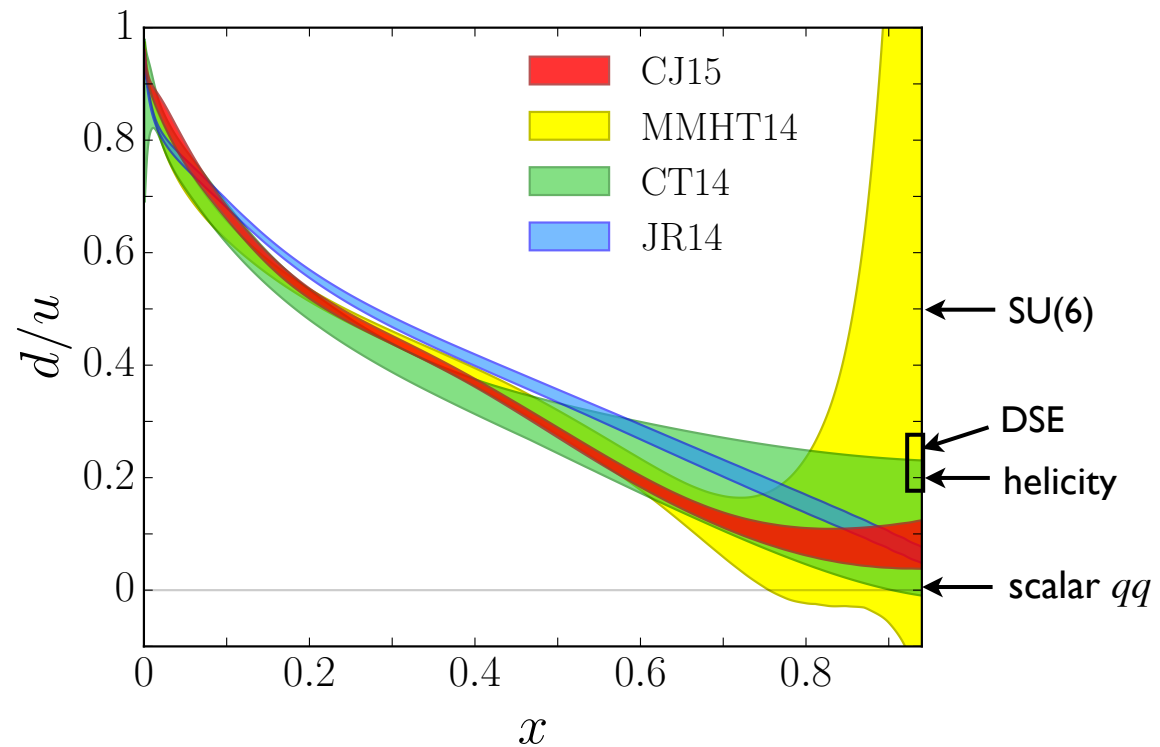
- Absence of free-neutron data and smaller $|e_q|$ of d quarks limit precision of d -quark PDF, especially at high x
 - nuclear effects in deuterium obscure free-neutron structure

Unpolarized PDFs

Valence d/u ratio at high x of particular interest

→ testing ground for nucleon models in $x \rightarrow 1$ limit

- $d/u \rightarrow 1/2$
SU(6) symmetry
- $d/u \rightarrow 0$
 $S = 0$ qq dominance
(color-hyperfine interaction)
- $d/u \rightarrow 1/5$
 $S_z = 0$ qq dominance
(perturbative gluon exchange)
- $d/u \rightarrow 0.18 - 0.28$
DSE with qq correlations



→ considerable uncertainty at high x from deuterium corrections (no free neutrons!)

$$F_2^d(x, Q^2) = \int_x^1 dy f(y, \gamma) F_2^N(x/y, Q^2)$$

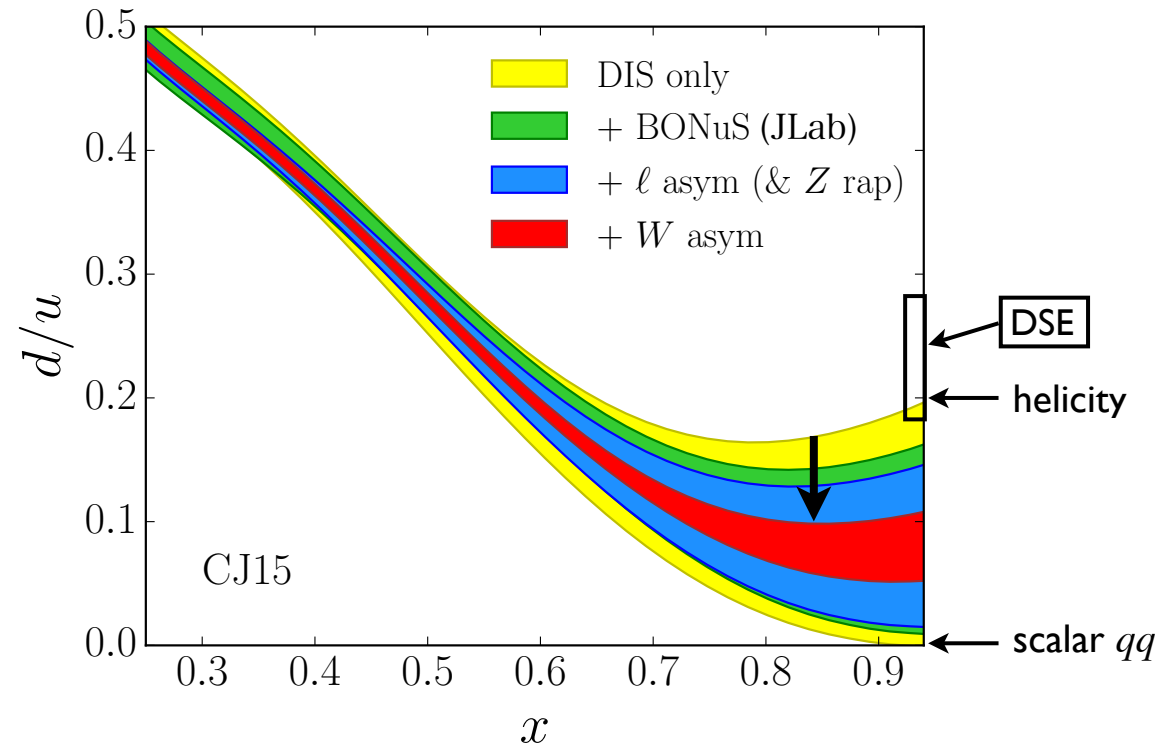
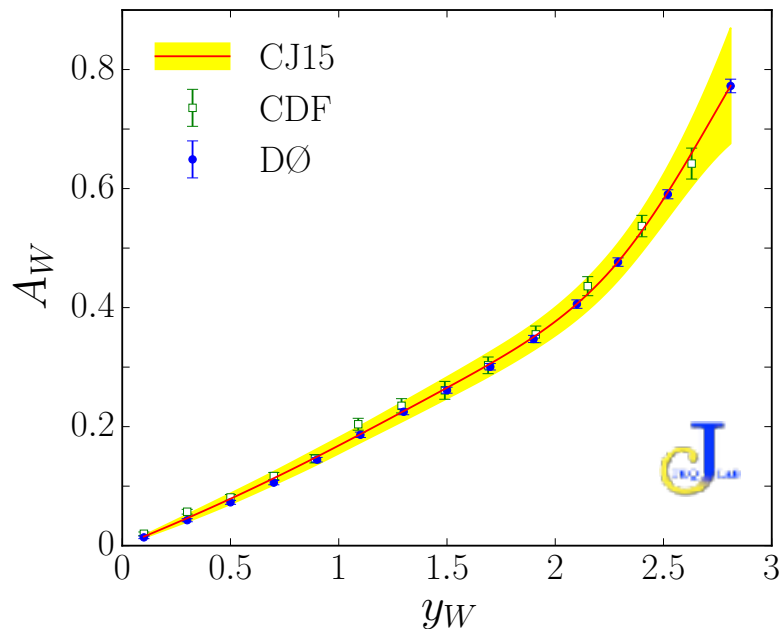
$$f(y, \gamma) = \int \frac{d^3p}{(2\pi)^3} |\psi_d(p)|^2 \delta\left(y - 1 - \frac{\varepsilon + \gamma p_z}{M}\right) \times \frac{1}{\gamma^2} \left[1 + \frac{\gamma^2 - 1}{y^2} \left(1 + \frac{2\varepsilon}{M} + \frac{\vec{p}^2}{2M^2} (1 - 3\hat{p}_z^2) \right) \right]$$

Unpolarized PDFs

Valence d/u ratio at high x of particular interest

→ significant reduction of PDF errors with new JLab tagged neutron & FNAL W -asymmetry data

$$d + \bar{u} \rightarrow W^- \rightarrow \ell^- + \bar{\nu}$$



→ extrapolated ratio at $x = 1$

$$d/u \rightarrow 0.09 \pm 0.03$$

does not match any model...

→ upcoming experiments at JLab (MARATHON, BONuS, SoLID) will determine d/u up to $x \sim 0.85$

Nucleon Helicity PDFs

Proton spin structure

- Question of how proton spin decomposed into its q & g constituents has engrossed community for > 30 years

- in nonrelativistic quark model, spin of proton is carried entirely by quarks $\Delta\Sigma = \Delta u^+ + \Delta d^+ + \Delta s^+ = 1$ while early data suggested that

$$\Delta q^+ \equiv \Delta q + \Delta \bar{q}$$

$$\Delta\Sigma \approx 0! \quad \Delta s^+ \approx -(0.1 - 0.2)$$

EMC (1988)

- proton spin sum requires

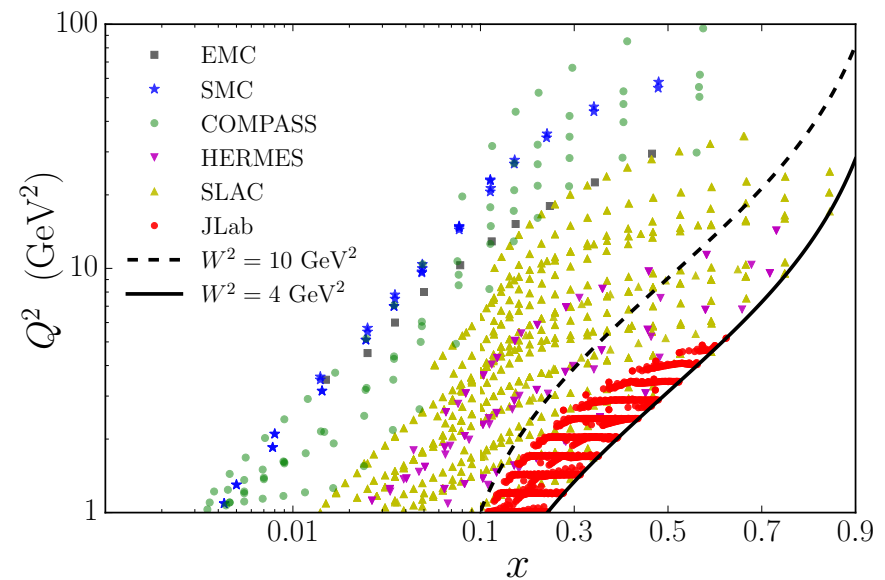
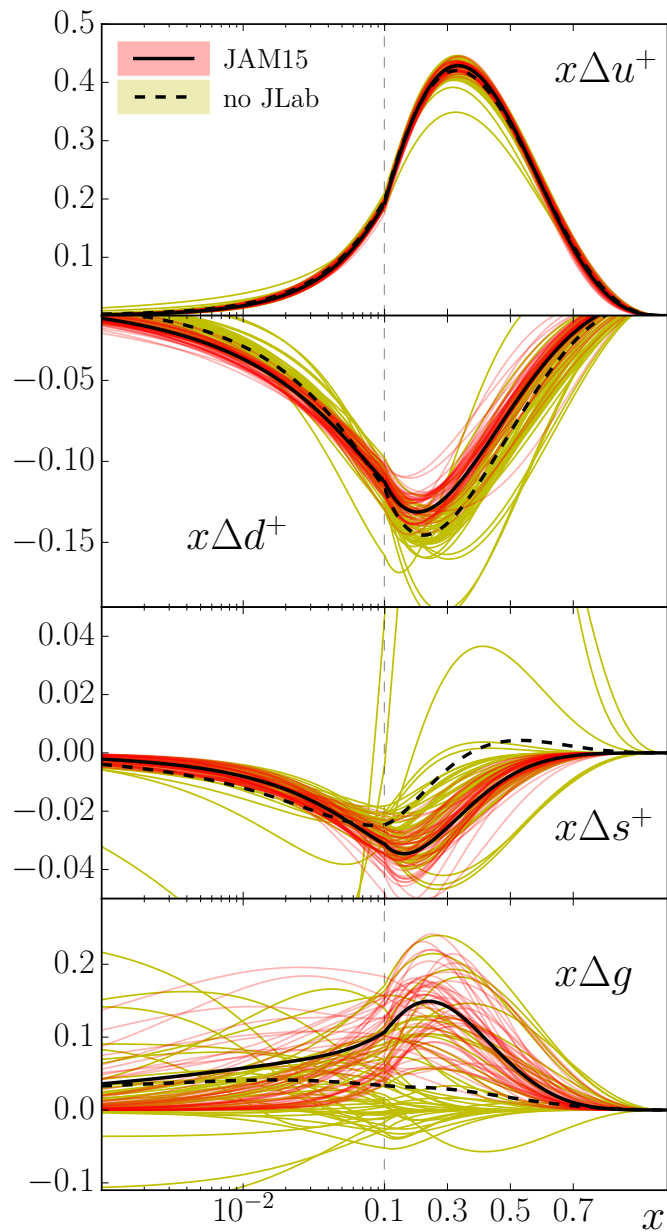
$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

... does remaining spin come from large *gluon* polarization or *orbital* angular momentum?

- stimulated many advances in theory, experiment & analysis
 - recent JAM global analyses, including JLab 6 GeV data

Proton spin structure

Impact of JLab data



- inclusion of JLab data increases # data points by factor ~ 2
- reduced uncertainty in Δs^+ , Δg through Q^2 evolution
- s -quark polarization *negative* from inclusive DIS data (assuming SU(3) symmetry)

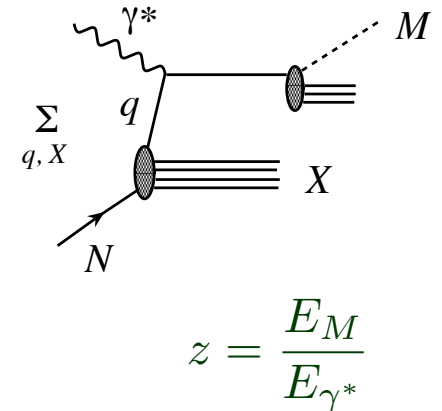
Polarization of quark sea?

- Inclusive DIS data cannot distinguish between q and \bar{q}

→ semi-inclusive DIS sensitive to Δq & $\Delta \bar{q}$

$$\sim \sum_q e_q^2 [\Delta q(x) D_q^h(z) + \Delta \bar{q}(x) D_{\bar{q}}^h(z)]$$

→ but need fragmentation functions!



- Global analysis of DIS + SIDIS data gives different *sign* for strange quark polarization for different fragmentation functions!

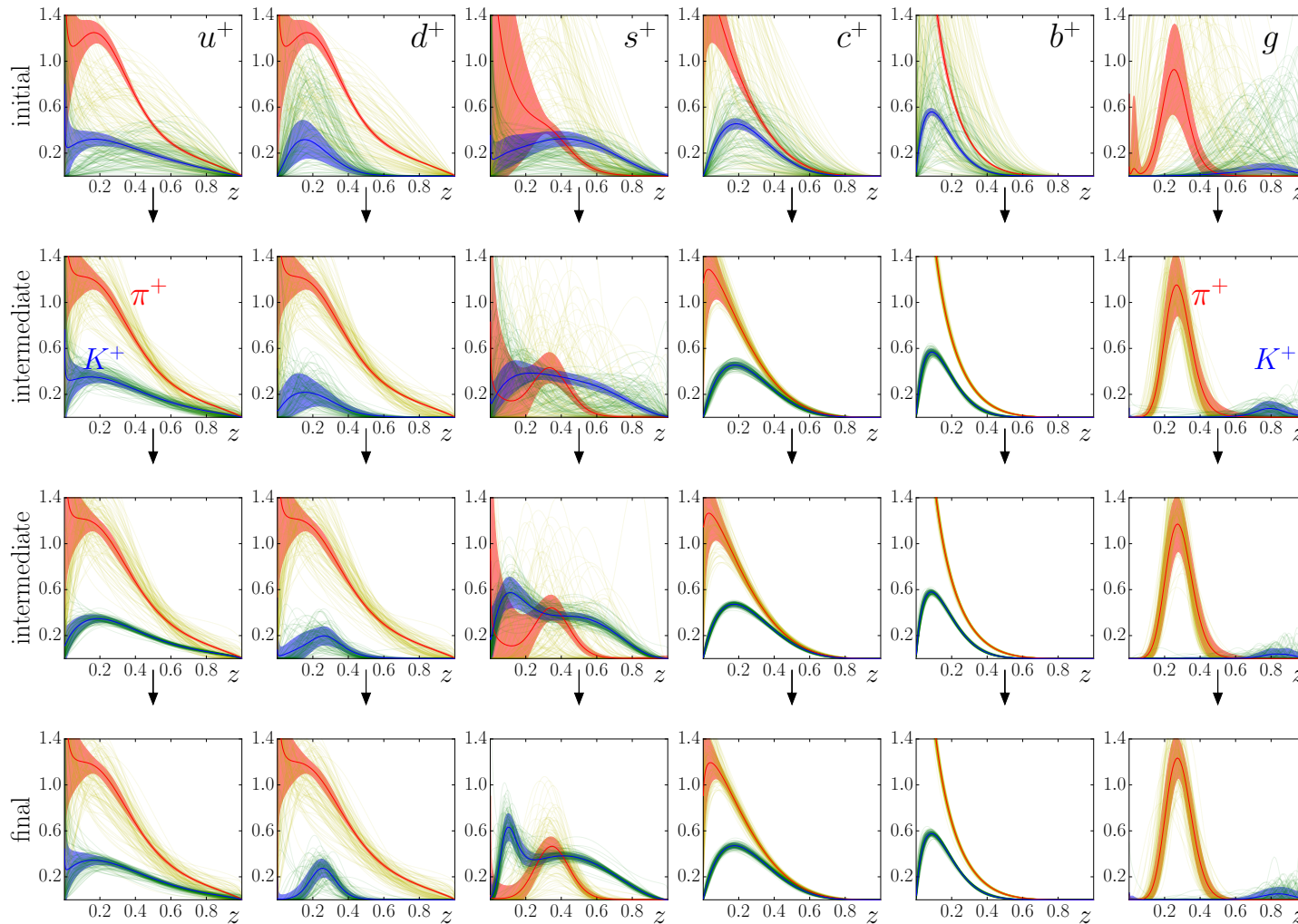
→ $\Delta s > 0$ for “DSS” FFs *de Florian et al. (2007)*

$\Delta s < 0$ for “HKNS” FFs *Hirai et al. (2007)*

→ need to understand origin of differences in fragmentation!

Polarization of quark sea?

● First MC analysis of fragmentation functions



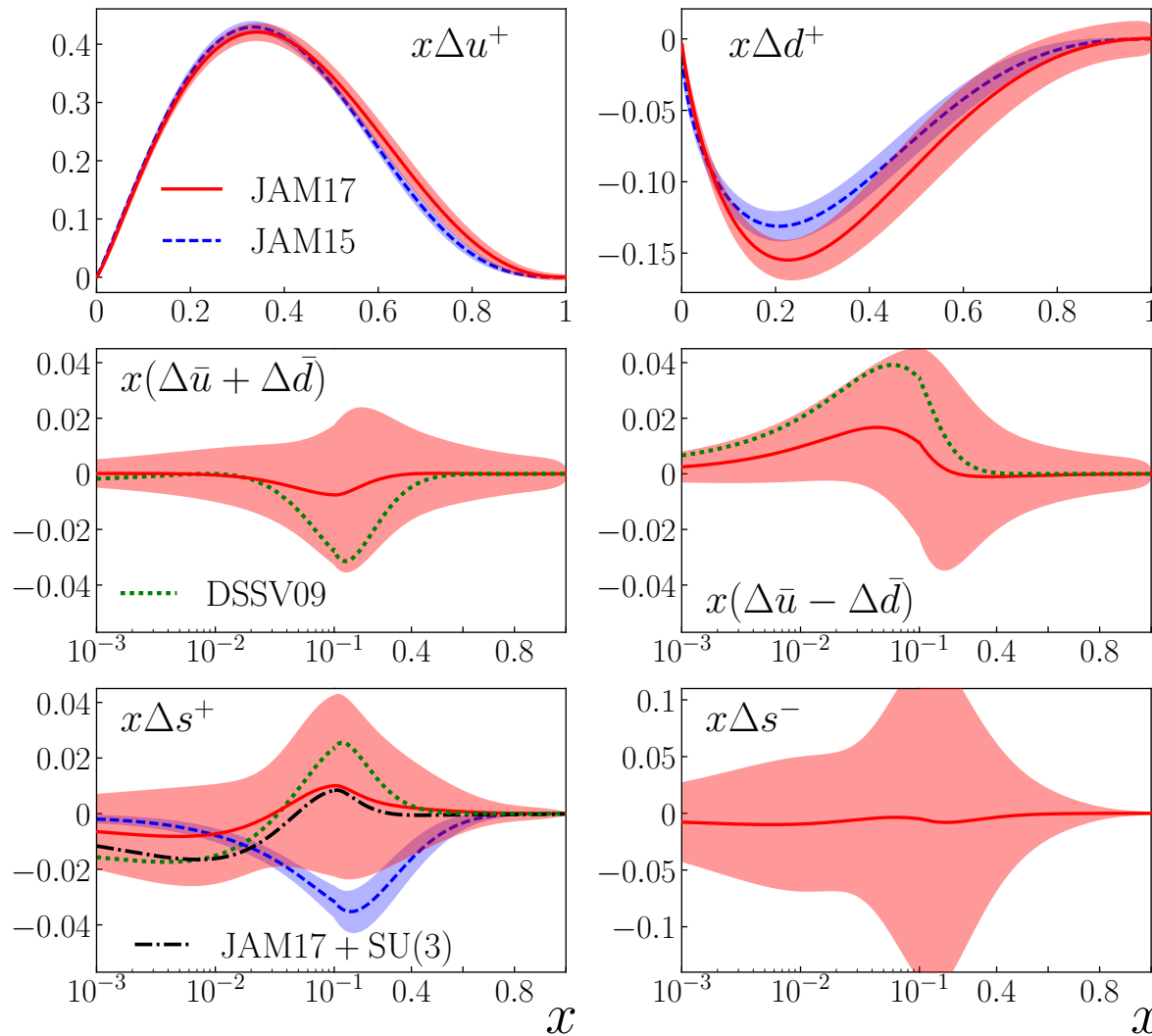
$e^+e^- \rightarrow h X$
single-inclusive
annihilation (SIA)

*Sato, Ethier, WM, Hirai,
Kumano, Accardi (2016)*

→ convergence after ~ 20 iterations

Polarization of quark sea?

- Simultaneous determination of spin PDFs and FFs, fitting to DIS, SIA and polarized SIDIS (HERMES, COMPASS) data



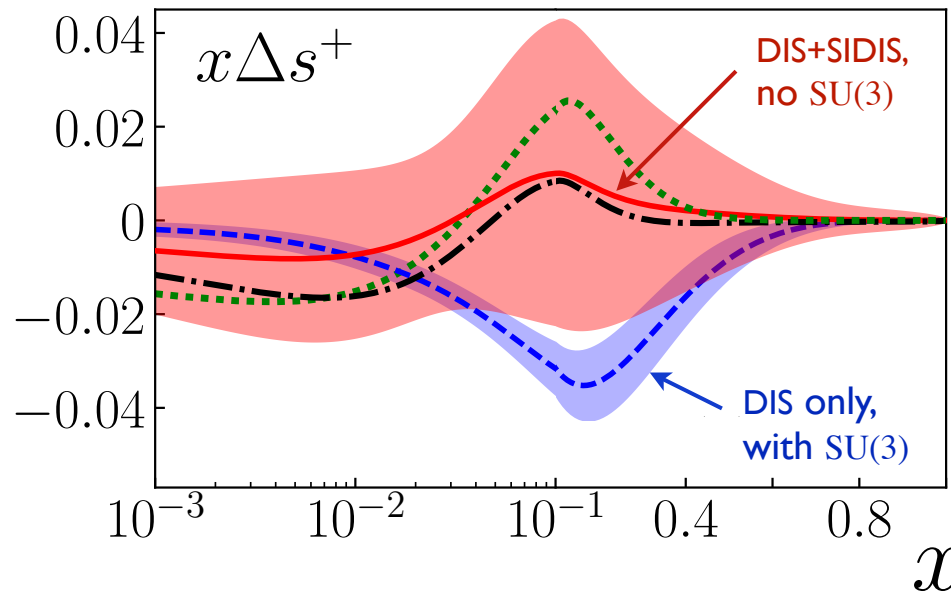
→ no assumption of SU(3) symmetry

→ Δ_s slightly > 0 at high x , consistent with zero

→ $\Delta_s - \Delta_{\bar{s}}$ & $\Delta_{\bar{u}} - \Delta_{\bar{d}}$ consistent with zero

Simultaneous analysis

- Polarized strangeness in previous, DIS-only analyses was negative at $x \sim 0.1$, induced by SU(3) and parametrization bias



Ethier, Sato, WM (2017)

- weak sensitivity to Δs^+ from DIS data & evolution
 - SU(3) pulls Δs^+ to generate moment ~ -0.1
 - negative peak at $x \sim 0.1$ induced by fixing $b \sim 6 - 8$
- less negative $\Delta s = -0.03(10)$ gives larger total helicity $\Delta \Sigma = 0.36(9)$

Nucleon Transversity PDFs

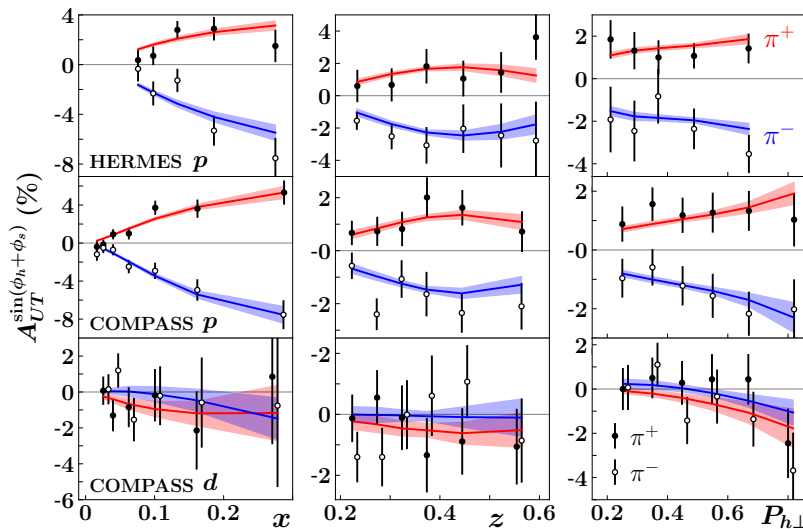
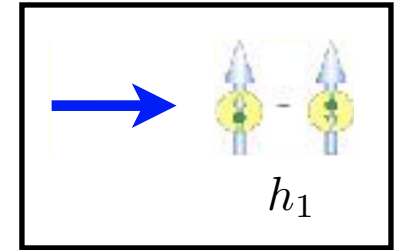
Transversity distributions



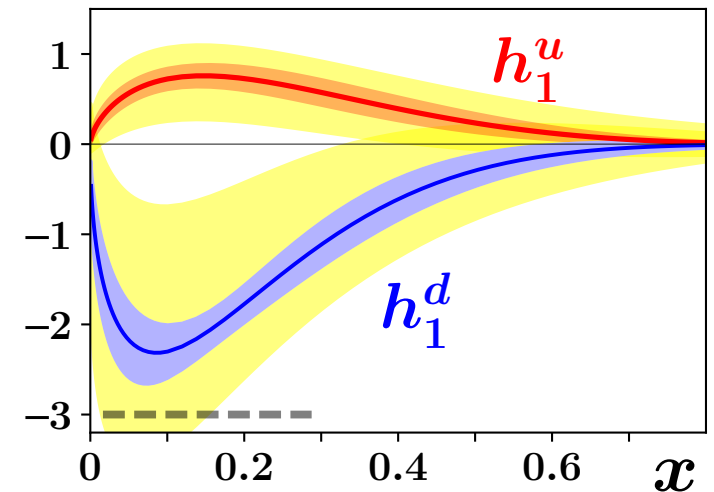
Extraction of transversity (TMD) PDF from SIDIS data
+ isovector moment $g_T = \int dx (h_1^u - h_1^d)$ from lattice QCD

→ Collins asymmetry

$$A_{UT}^{\sin(\phi_h + \phi_s)} \propto \frac{h_1 \otimes H_1^\perp}{f_1 \otimes D_1}$$



$$g_T^{\text{latt}} = 1.01(6)$$

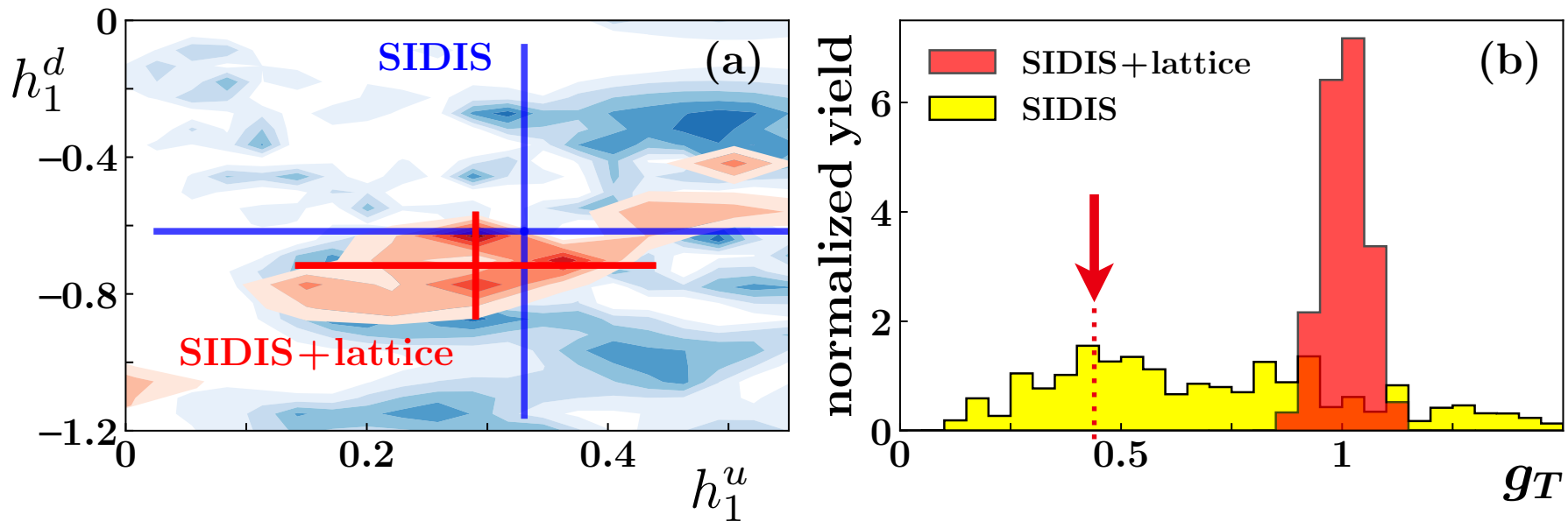


Lin, WM, Prokudin, Sato, Shows (2018)

→ significantly reduced uncertainties with lattice constraint

Transversity distributions

- Extraction of transversity (TMD) PDF from SIDIS data
+ isovector moment $g_T = \int dx (h_1^u - h_1^d)$ from lattice QCD



Lin, WM, Prokudin, Sato, Shows (PRL, 2018)

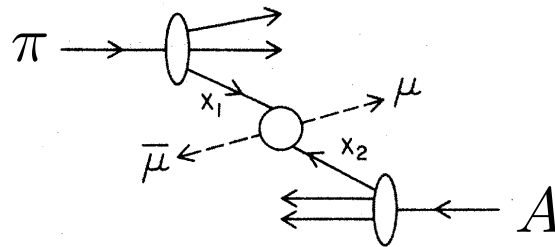
- distributions do not look very Gaussian!
- MC analysis gives $g_T = 1.0 \pm 0.1$
- maximum likelihood analysis would have given $g_T \approx 0.5$

Pion PDFs

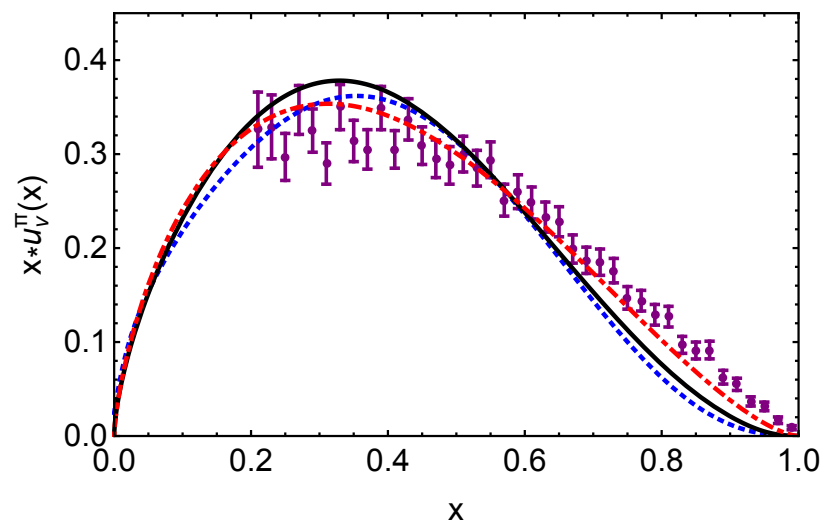
PDFs in the pion

- PDFs in the pion (in principle) simpler to compute than baryons, but are more difficult to study experimentally

→ most information has come from pion-nucleus (tungsten)
Drell-Yan data (CERN, Fermilab)



→ constrains valence PDFs at $x \gg 0$ (uncertainty from gluon resummation)



Shi, Mezrag, Zong (2018)

→ pion sea quark & gluon
PDFs at small x mostly
unconstrained

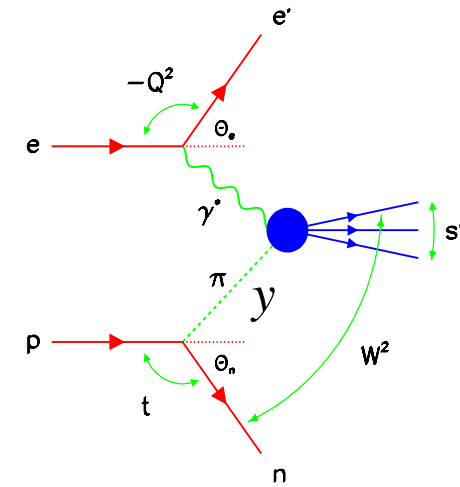
PDFs in the pion

- Recently a new (Monte Carlo-based) global analysis used chiral effective field theory to include also leading neutron electroproduction from HERA

$$\frac{d^3 \sigma^{\text{LN}}}{dx dQ^2 dy} \sim 2f_{\pi/N}(y) F_2^\pi(x_\pi, Q^2)$$

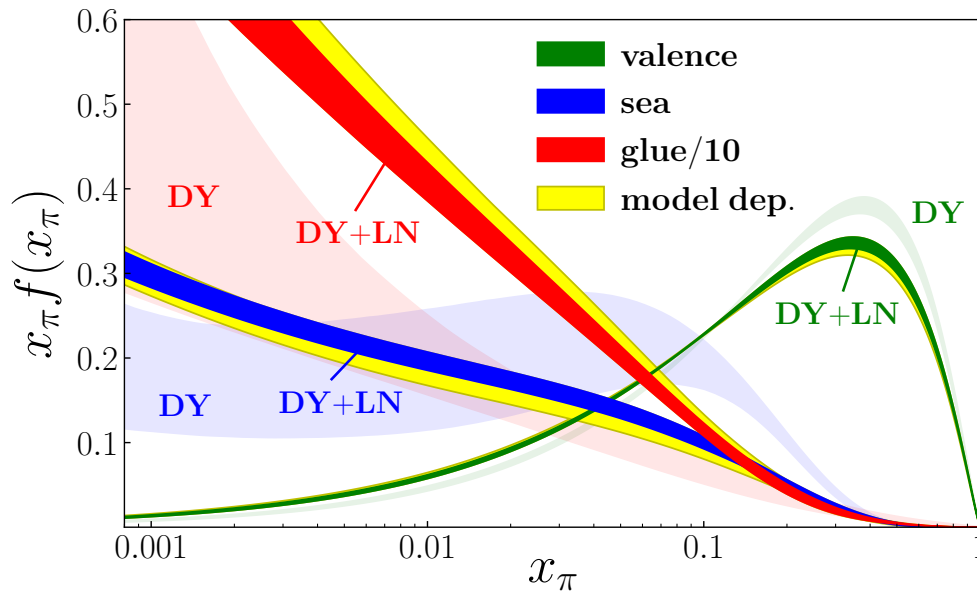
$N \rightarrow N + \pi$
splitting function
(computed from χEFT)

pion structure
function



$$x_\pi = x/y$$

McKenney, Sato, WM, C.-R. Ji (2018)

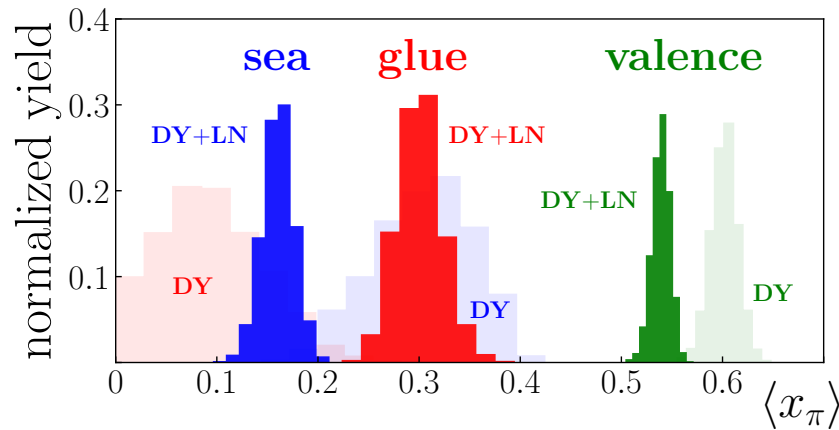


Barry, Sato, WM, C.-R. Ji (2018)

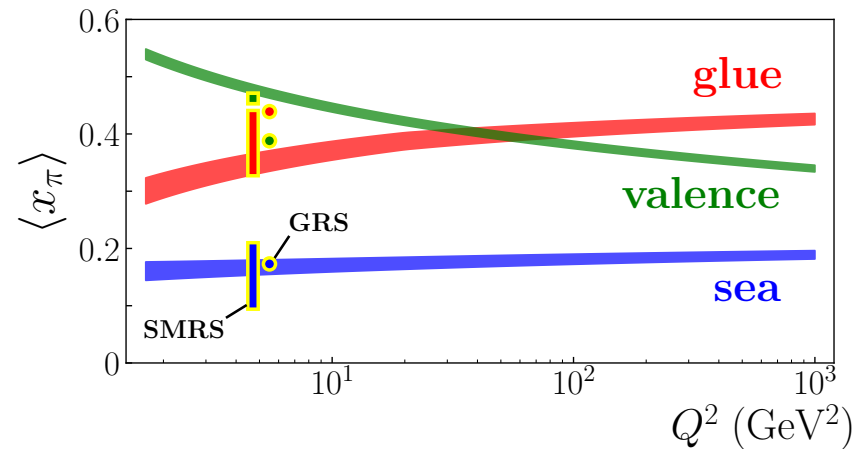
→ first constraints on
pion PDFs at low x

PDFs in the pion

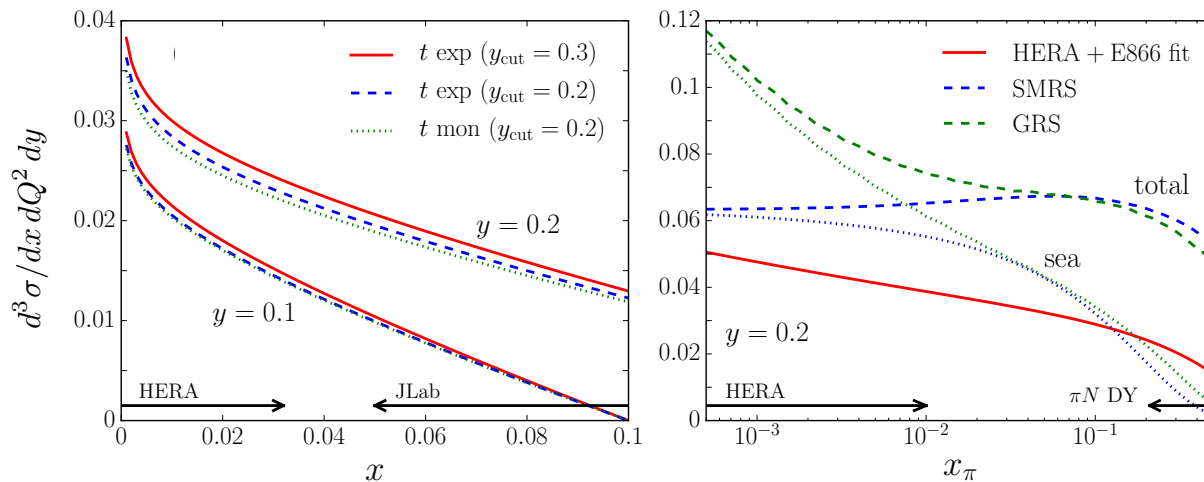
- Larger gluon fraction in the pion than without LN constraint



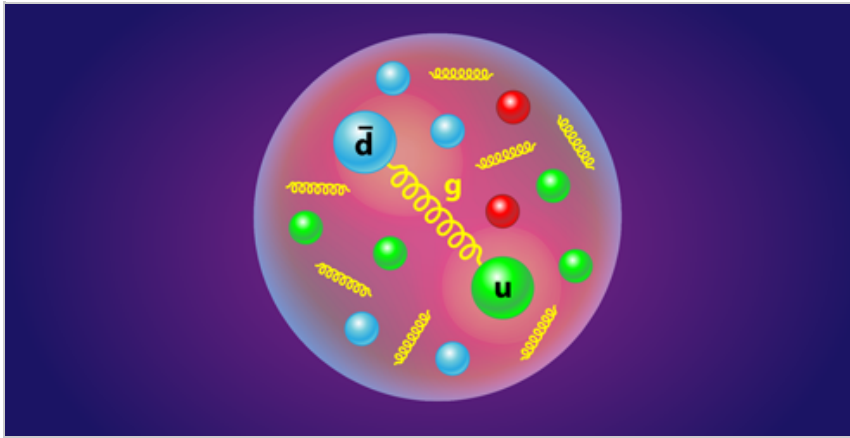
Barry, Sato, WM, C.-R. Ji (2018)



- Tagged DIS experiment at JLab ($e n \rightarrow e' p X$) will probe pion structure at intermediate x values (between DY and LN)



→ extension to hyperon final state will probe kaon structure



PARTICLES AND FIELDS

Synopsis: More Gluons in the Pion

October 10, 2018

A combined analysis of collider data finds that there are more gluons in the pion than earlier estimates. [Read More »](#)

physicsworld

PARTICLE AND NUCLEAR RESEARCH UPDATE

Gluons account for much more pion momentum than previously thought

19 Oct 2018



Pion exchange: the team used data from the HERA accelerator, which ran at DESY in Hamburg. (Courtesy: DESY)

Gluons contribute around 30% to the total momentum of energetic pions, which is about three times more than previously estimated. The research was done by a team led by [Chueng-Ryong Ji](#) at North Carolina State University in the US. They deduced the fraction by combining data gathered by two previous studies that took different approaches to exploring the interior structures of the particles.

Pions are the lightest members of the meson family. An individual pion comprises a quark and an antiquark, one of which has up flavour and the other down flavour. Yet this description is overly simplistic because the quark-antiquark pairs are embedded in a sea of “virtual” quarks and antiquarks which appear and disappear instantaneously. The quarks and antiquarks also interact

Outlook

- New paradigm in global analysis — *simultaneous* determination of collinear distributions using MC sampling of parameter space
→ providing new insights into quark/gluon structure of hadrons
- *Short-term*: “universal” QCD analysis of all observables sensitive to collinear (unpolarized & polarized) PDFs and FFs
- *Longer-term*: technology developed will be applied to global analysis of transverse momentum dependent (TMD) distributions to map out full 3-d image of hadrons
→ vital interplay between theory & experiment at JLab