CTEQ Summer School University of Pittsburgh 7/16/19 – 7/26/19

Patrick Barry

Group Meeting

September 6th, 2019

Coordinated Theoretical-Experimental Project on QCD (CTEQ)

2019 CTEQ School Schedule

16 July 2019	17 Jul 2019	18 Jul 2019	19 Jul 2019	20 Jul 2019	21 Jul 2019	22 Jul 2019	23 Jul 2019	24 Jul 2019	25 Jul 2019	26 Jul 2019
Tuesday	**Wednesday**	**Thursday**	**Friday**	**Saturday**	**Sunday**	**Monday**	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
Arrive	**Day 1**	**Day 2**	**Day 3**	**Day 4**	**Free Day**	**Day 6**	**Day 7**	**Day 8**	**Day 9**	**Depart**
7:30 - 8:45	Breakfast	Breakfast	Breakfast	Breakfast	Breakfast	Breakfast	Breakfast	Breakfast	Breakfast	
9:00 - 10:00	**Intro 1 (Soper)**	**Intro 3 (Soper)**	**Intro 4 (Soper)**	**MC Intro 3 (Krauss)**		**Jets 1 (Sterman)**	**Jets 2 (Sterman)**	**Results: EW/Higgs (Mills)**	**Results: EW/Higgs (Mills)**	
10:00 - 10:30	Coffee	Coffee	Coffee	Coffee		Coffee	Coffee	Coffee	Coffee	
10:30 - 11:30	**DIS 1 (Aschenauer)**	**DIS 2 (Aschenauer)**	**PDF 1 (Guzzi)**	**PDF 2 (Guzzi)**		**Machine Learning 1 (Ostdiek)**	**Machine Learning 2 (Ostdiek)**	**Results QCD/Top (Nachman)**	**Heavy Quark 2 (Collins)**	
11:45 - 13:00	Lunch	Lunch	Lunch	Lunch		Lunch	Lunch	Lunch	Lunch	
13:00 - 14:00	**Intro 2 (Soper)**	**Higgs 2 (Dawson)**	**DY & Vec Boson Prod 1 (Stump)**	**DY & Vec Boson Prod 2 (Stump)**		**Neutrinos 1 (Betancourt)**	**Neutrinos 2 (Betancourt)**	**Results QCD/Top (Nachman)**	**Dark Matter (Batell)**	
14:00-14:30	Coffee	Coffee	Coffee	Coffee	**Sunday 21 July: 1:35pm Pittsburgh Pirates**	Coffee	Coffee	Coffee	Coffee	
14:30-15:30	**Higgs 1 (Dawson)**	**MC Intro 2 (Krauss)**	**Tutorial (Hoeche)**	**Tutorial (Hoeche)**		**Tutorial (Ostdiek)**	**Heavy Quark 1 (Sullivan/Nadolsky)**	**Tutorial (Ostdiek)**	**Beyond SM (Kilic)**	
15:30+	**MC Intro 1 (Krauss)**									
18:30 - 19:30	Dinner	Dinner	Dinner	Dinner		Dinner	Dinner	Dinner	Dinner	
19:30 - 21:00	**Recitation**	**Recitation**	**Recitation**	**Recitation**		**Recitation**	**Recitation**	**Recitation**	**Recitation**	
21:00 - 22:30	**NightCap**	**NightCap**	**NightCap**	**NightCap**		**NightCap**	**NightCap**	**NightCap**	**NightCap**	

Lecturers (Number of lectures) and Topics

Week 1

- Intro to QCD (4) – Dave Soper
- DIS (2) – Elke-Caroline Aschenauer
- Higgs (2) – Sally Dawson
- Monte Carlo (3) – Frank Krauss
- PDF (2) – Marco Guzzi
- Drell-Yan and Vector Boson Production (2) – Daniel Stump

Week 2

- Jets (2) – George Sterman
- Machine Learning (2) – Bryan Ostdiek
- Neutrinos (2) – Minerba Betancourt
- Heavy Quarks (2) – Paval Nadolsky/John Collins
- Results EW/Higgs (2) – Corrinne Mills
- Results QCD/Top Quark (2) – Benjamin Nachman
- Dark Matter (1) – Brian Batell
- Beyond Standard Model (1) -- Can Kiliç

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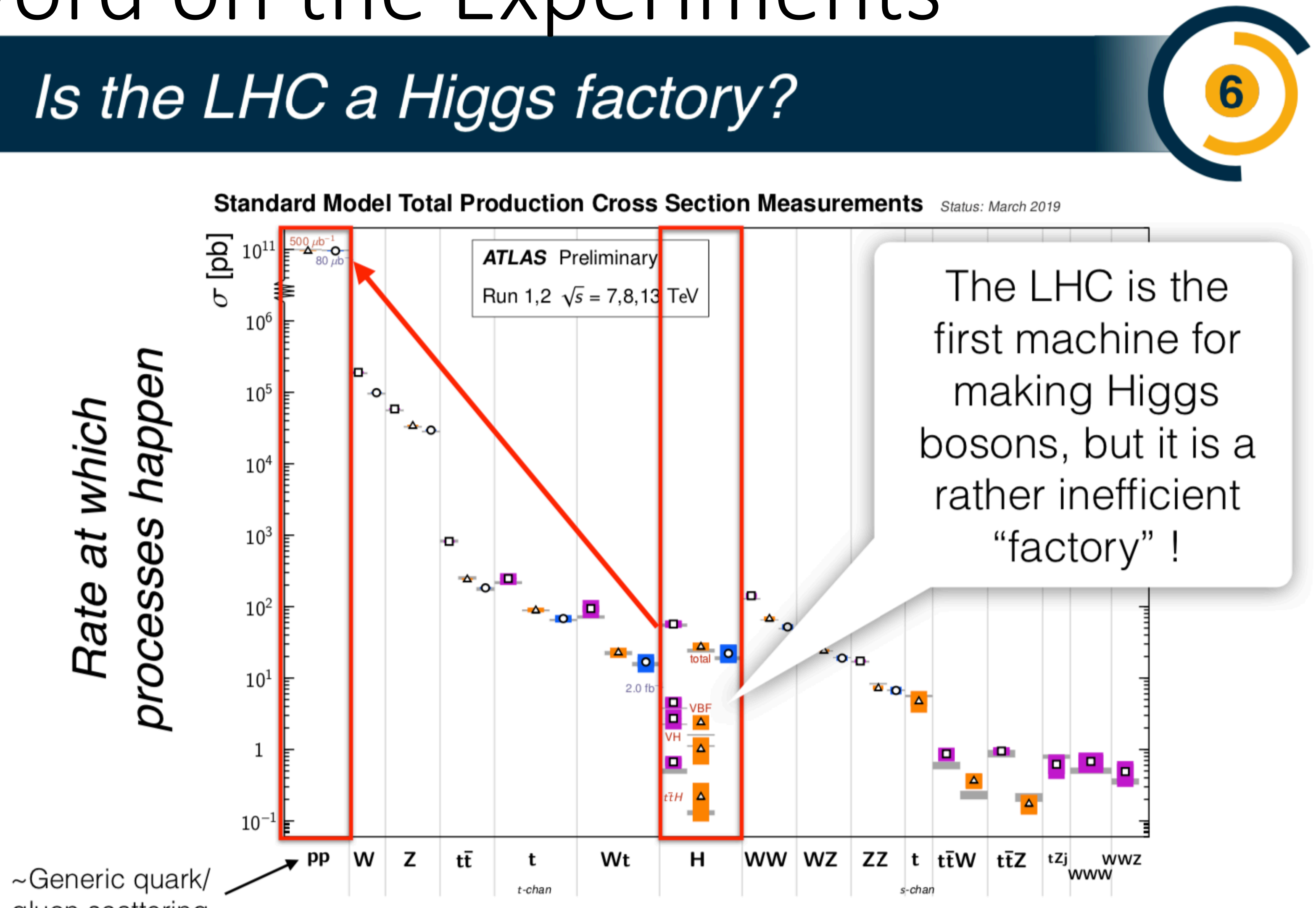
One Word on the Experiments

Is the LHC a Higgs factory?



Process name: proton + proton goes to X

Basics of QCD Perturbation Theory

Davison E. Soper
University of Oregon

CTEQ School, University of Pittsburgh
July 2019

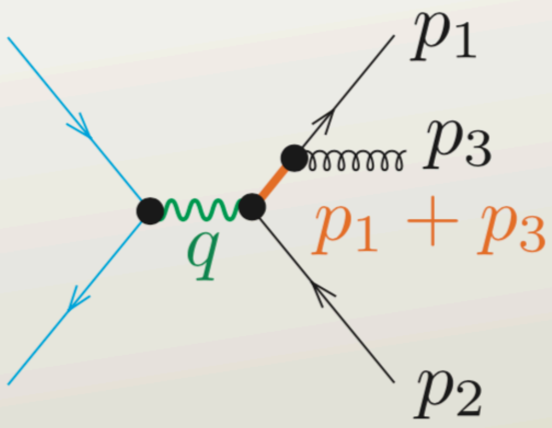
The Intro Lectures focused on the following topics

- Jet structure
- Renormalization group and running coupling
- Existence of Infrared safe observables
- Isolating soft initial state physics in PDFs

- Electron-positron annihilation (exploring final states)
- Deeply inelastic scattering
- Hard processes in hadron-hadron collisions

e^+e^- Scattering

General nature of the singularities



- $\mathcal{M}$ contains a factor $1/(p_1+p_3)^2$.

  $(p_1+p_3)^2 = 2E_1E_3(1-\cos\theta_{13})$
- This is singular for $\theta_{13} \to 0$ and for $E_3 \to 0$.
- The numerator has a factor θ_{13} for small θ_{13}.
- So

$$|\mathcal{M}|^2 \propto \frac{1}{E_3^2\theta_{13}^2}, \qquad \theta_{13} \to 0 \text{ or } E_3 \to 0$$

Taking a look at the Space-Time picture of interactions

See how long and where the emitted particles are traveling

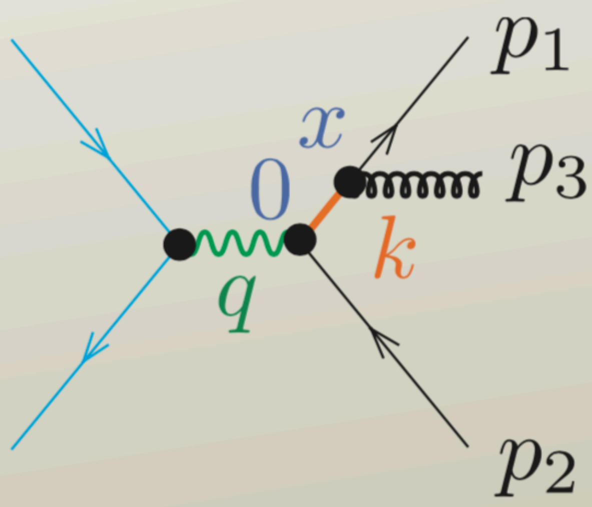
Use of light front coordinates (used in the talk as “null-plane” coordinates)

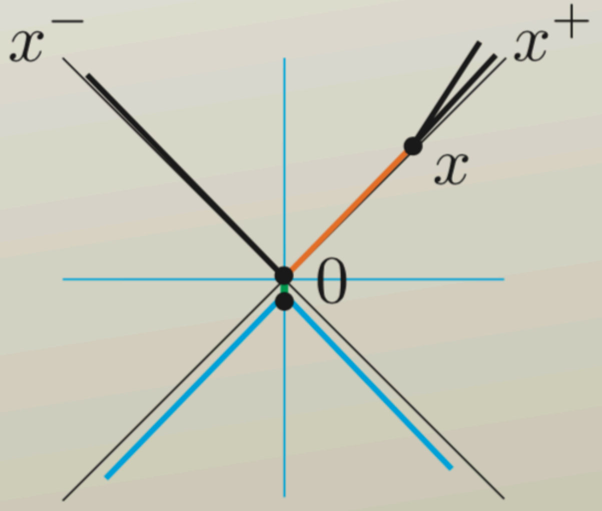
Consider the Fourier transform.

- The singularity corresponds to large k^+ and small k^-.

$$S_F(k) = \int dx^+ dx^- d\boldsymbol{x}\ \exp(i[k^+ x^- + k^- x^+ - \boldsymbol{k}\cdot\boldsymbol{x}])\ S_F(x).$$

- Contributing positions have large x^+ and small x^-.



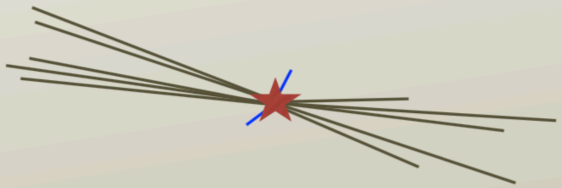


Infrared Safety

- The observable F is "infrared safe" if

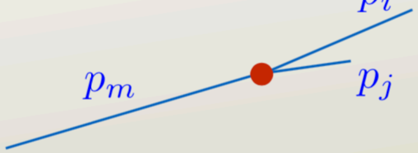
  for $z = 0$ or $0 < z < 1$

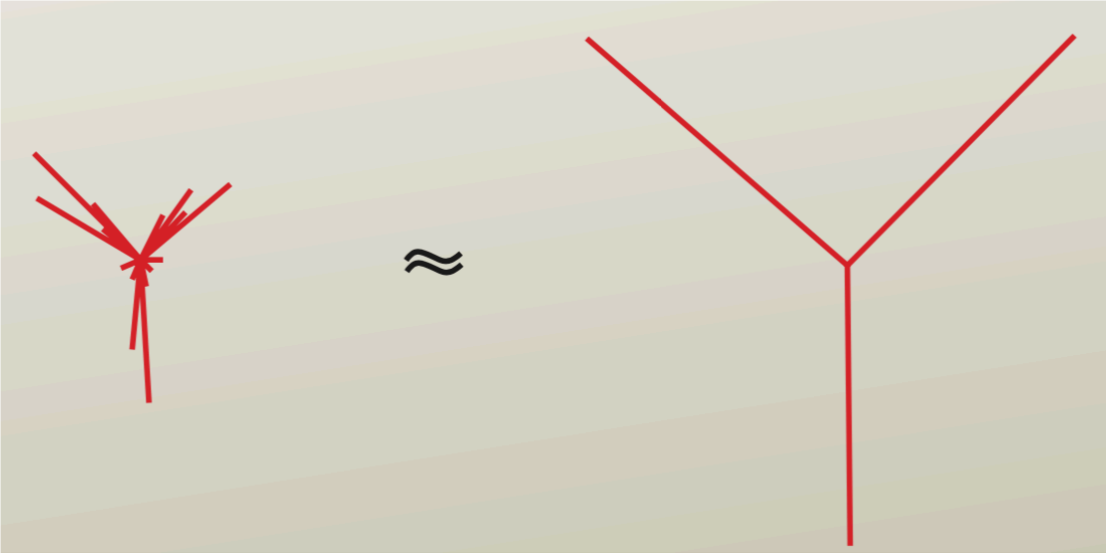
$$F_{m+1}(p_1^\mu, \ldots, (1-z)p_m^\mu, zp_m^\mu) = F_m(p_1^\mu, \ldots, p_m^\mu).$$



- For partons i and j becoming collinear or one becoming soft,

$$p_i \to z p_m$$
$$p_j \to (1-z) p_m$$





Perturbative QCD can't predict long time physics very well, and detectors are long distances away from reactions. Need to have IR safe observables.

If we can represent one or more outgoing particles' momenta in terms of other outgoing momenta (almost completely collinear outgoing particles), we can have infrared safety.

A more interesting example is the thrust distribution $d\sigma/dT$.

$$F_m(p_1^\mu, \ldots, p_m^\mu) = \delta\left(T - \mathcal{T}_m(p_1^\mu, \ldots, p_m^\mu)\right)$$

$$\mathcal{T}_m(p_1^\mu, \ldots, p_m^\mu) = \max_{\vec{u}} \frac{\sum_{i=1}^m |\vec{p}_i \cdot \vec{u}|}{\sum_{i=1}^m |\vec{p}_i|}$$

- Contribution from a particle with $\vec{p} = 0$ drops out.
- Replacing one parton by two collinear partons does not change T.

$$|(1-z)\,\vec{p}_m \cdot \vec{u}| + |z\,\vec{p}_m \cdot \vec{u}| = |\vec{p}_m \cdot \vec{u}|$$

$$|(1-z)\,\vec{p}_m| + |z\,\vec{p}_m| = |\vec{p}_m|$$

Since thrust distribution is infrared safe, we can calculate using perturbative QCD

Quiz Question (From Recitation)

- For e^+e^- annihilation, is the following quantity infrared (IR) safe?

$$F_N^{ij} = \sum_{a=1}^{N} p_a^i p_a^j$$

Where a is summed for all outgoing partons, and i, j are space-time coordinates, and N is the number of outgoing partons.

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Where a is summed for all outgoing partons, and i, j are space-time coordinates, and N is the number of outgoing partons.

- No! Looking at the definition on the previous slide, we cannot represent the quantity F in terms of $N-1$ momenta

Scales of IR safety

- Generally, IR safety is a ``yes-or-no” question
- IR safety can have a “degree of safety” per se for an observable, F, at a certain scale
- That is, we can say an observable is safe at a scale $Q^2(F)$

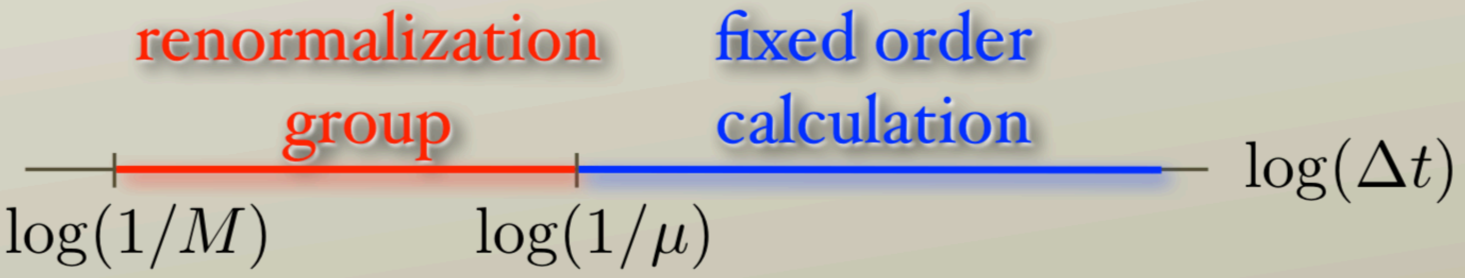
$$\frac{1}{Q^2(F)} = \max\left[\frac{\delta\sigma[F]/\sigma[F]}{\mu_{ij}^2}\right],$$

Which defines a new scale for IR safety

Running Coupling

- We account for time scales much smaller than $1/\mu$ (but bigger than a cutoff M at the "GUT scale") by using the running coupling.

- This sums the effects of short time fluctuations of the fields.



Calculating quantities with scale

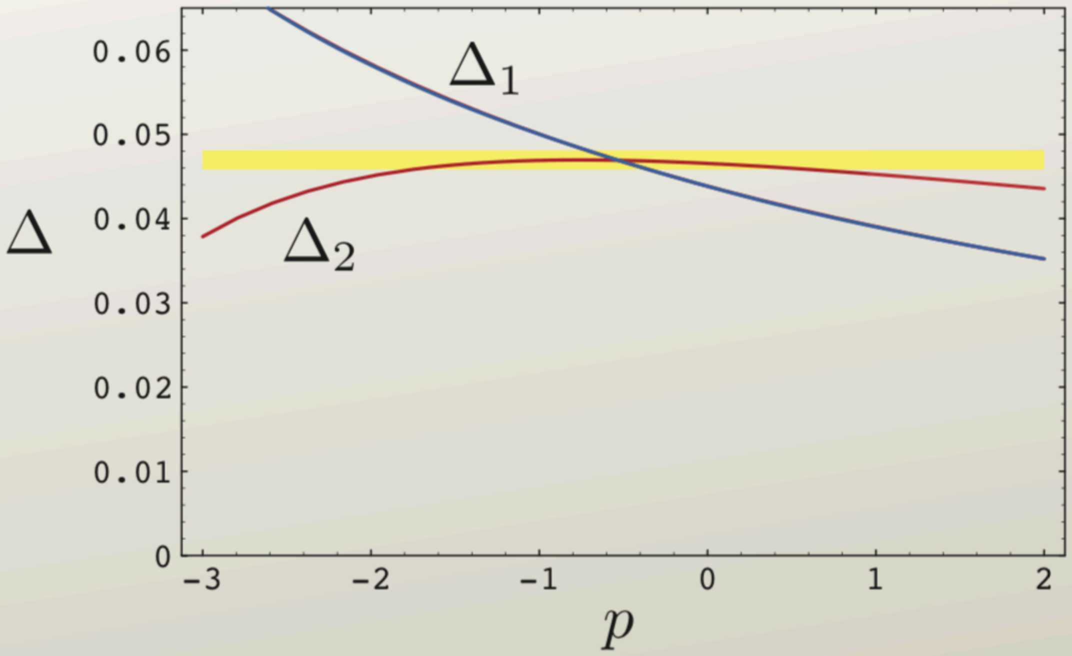
- Coefficients of perturbative calculations depend on μ
- α_S depends on μ
- However, the observable does not depend on μ

- How do we know which scale is appropriate?

- Consider some observable Δ
- Full Δ shouldn't depend on scale
- Higher orders of perturbation theory should get closer to the true Δ

I take $\alpha_s(M_Z) = 0.117$, $Q = 34$ GeV, 5 flavors of quarks.

I plot $\Delta(\mu)$ versus p defined by $\mu = 2^p Q$.



- Δ_1 includes one term.
- Δ_2 includes two terms.

- Possible choice: "principle of minimal sensitivity" point $\hat{\mu}$ where Δ_2 is flat.
- Error band estimated using $\mu = 2\hat{\mu}$ or $\mu = \hat{\mu}/2$.

Parton distribution functions

- They are defined as proton matrix elements of a certain operator.

- For quarks,

$$f_{i/h}(\xi,\mu_F) = \frac{1}{2}\int \frac{dy^-}{2\pi}\, e^{i\xi p^+ y^-} \langle p|\bar{\psi}_i(0,y^-,\mathbf{0})\gamma^+ F \psi_i(0)|p\rangle$$

$$F = \mathcal{P}\exp\left(ig \int_0^{y^-} dz^- A_a^+(0,z^-,\mathbf{0})\, t_a \right).$$

- For gluons, a similar definition.

- Renormalize with the so-called $\overline{\mathrm{MS}}$ prescription with scale μ_F.

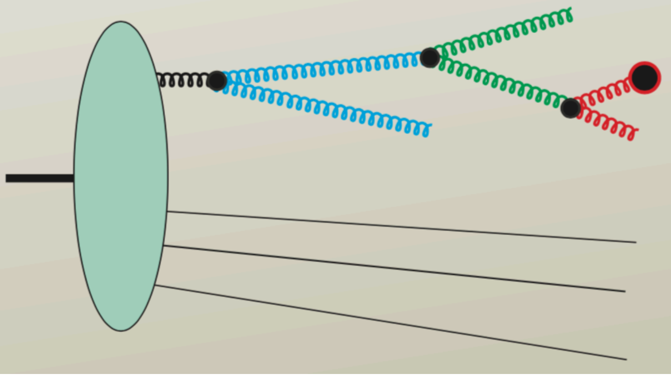
Evolution of the parton distribution functions

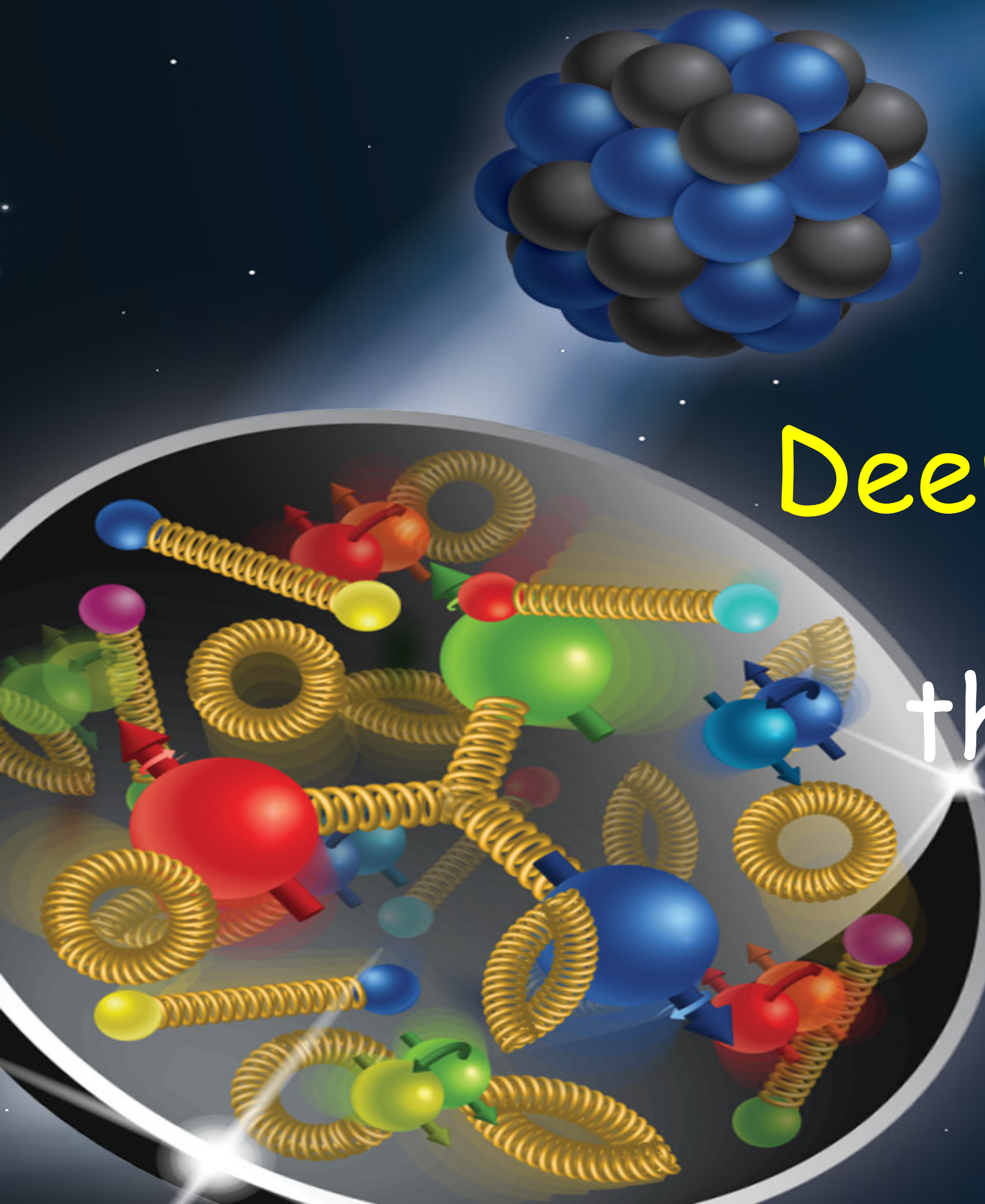
$$\frac{d}{d\log\mu_F} f_{a/h}(x,\mu_F) = \sum_b \int_x^1 \frac{d\xi}{\xi}\, P_{ab}(x/\xi, \alpha_s(\mu_F))\; f_{b/h}(\xi,\mu_F)$$

- This is called the Altarelli-Parisi equation or the DGLAP equation.

$$\begin{aligned} P_{ab}(x/\xi, \alpha_s(\mu_F)) \quad &= \quad P_{ab}^{(1)}(x/\xi)\, \frac{\alpha_s(\mu_F)}{\pi} \\ &\quad + P_{ab}^{(2)}(x/\xi)\, \left(\frac{\alpha_s(\mu_F)}{\pi}\right)^2 \\ &\quad + \cdots \end{aligned}$$

- The physical effect that we account for is fluctuations within fluctuations … as we look with a more powerful “microscope.”





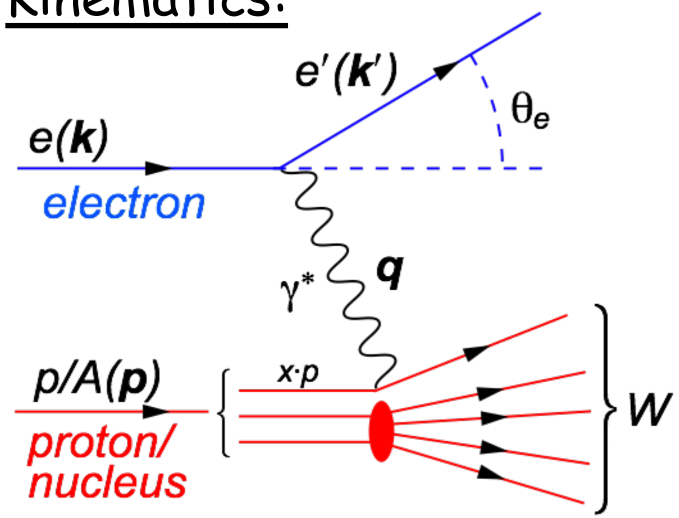
Deep Inelastic Scattering
The way to unravel the secrets of nucleons

E.C. Aschenauer
BNL

BROOKHAVEN NATIONAL LABORATORY
U.S. DEPARTMENT OF ENERGY | Office of Science

Deep Inelastic Scattering

Kinematics:



$$Q^2 = 2E_e E'_2(1-\cos\theta_{e'}) = -q^2$$

Measure of resolution power

$$y = 1 - \frac{E'_e}{E_e}\cos^2\left(\frac{\theta'_e}{2}\right)$$

Measure of inelasticity

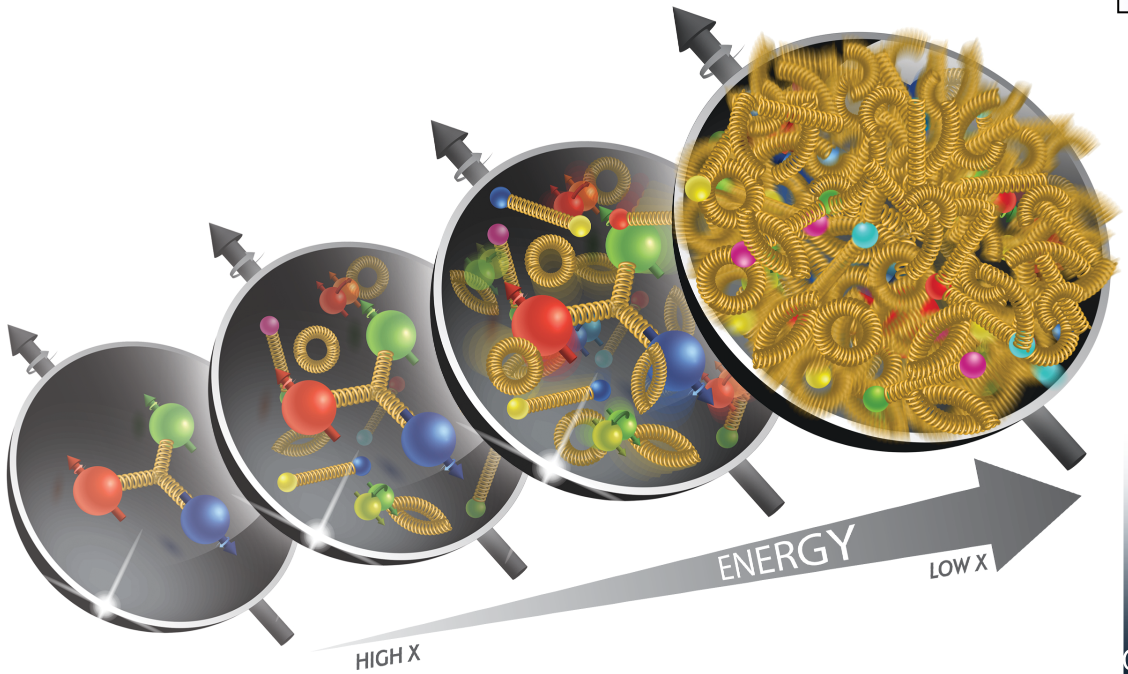
$$x = \frac{Q^2}{2pq}$$

Measure of momentum fraction of struck quark

$$\sqrt{s} = 2\sqrt{E_e E_p}$$

center-of-mass energy of electron-hadron system

$$\Rightarrow \quad Q^2 = s \bullet x \bullet y$$



DIS:

- As a probe, electron beams provide unmatched precision of the electromagnetic interaction
- Direct, model independent, determination of kinematics of physics processes

Why do we need different probes

Complementarity

QCD has two concepts which lay its foundation

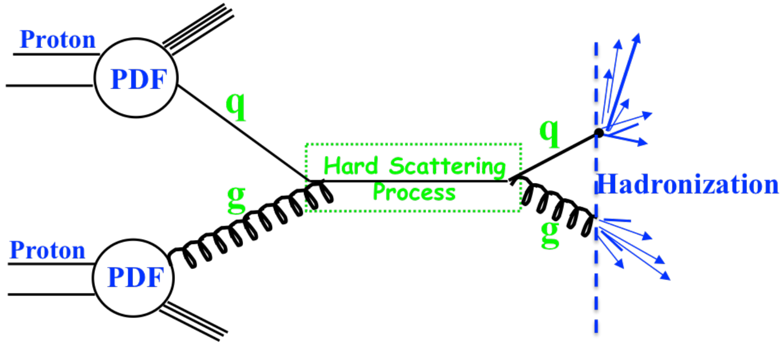
factorization and universality

To tests these concepts and separate interaction dependent phenomena from intrinsic nuclear properties

different complementary probes are critical

Probes: high precision data from ep, pp, e+e-

Factorization



(un)polarized cross section ~

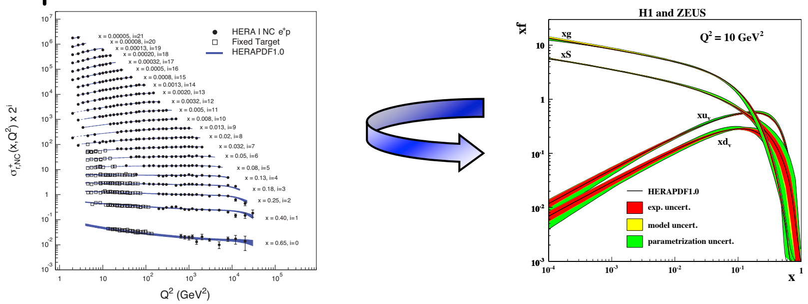
PDF ⊗ hard-scattering ⊗ Hadronization

hard-scattering : calculable in QCD

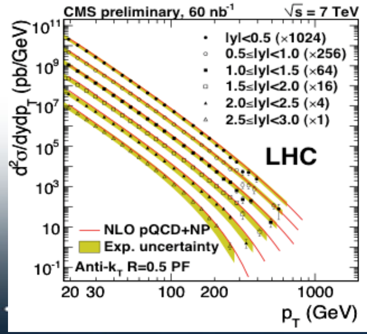
PDFs and Hadronization: need to be determined experimentaly

Universality

Example: Measure PDFs at HERA at √s=0.3 TeV:



Predict pp and $p\bar{p}$ measurements at √s=0.2, 1.96 & 7 TeV



How to access Gluons in DIS

Gluons manifest themselves through

1. the behavior of the cross section as function of x and Q^2

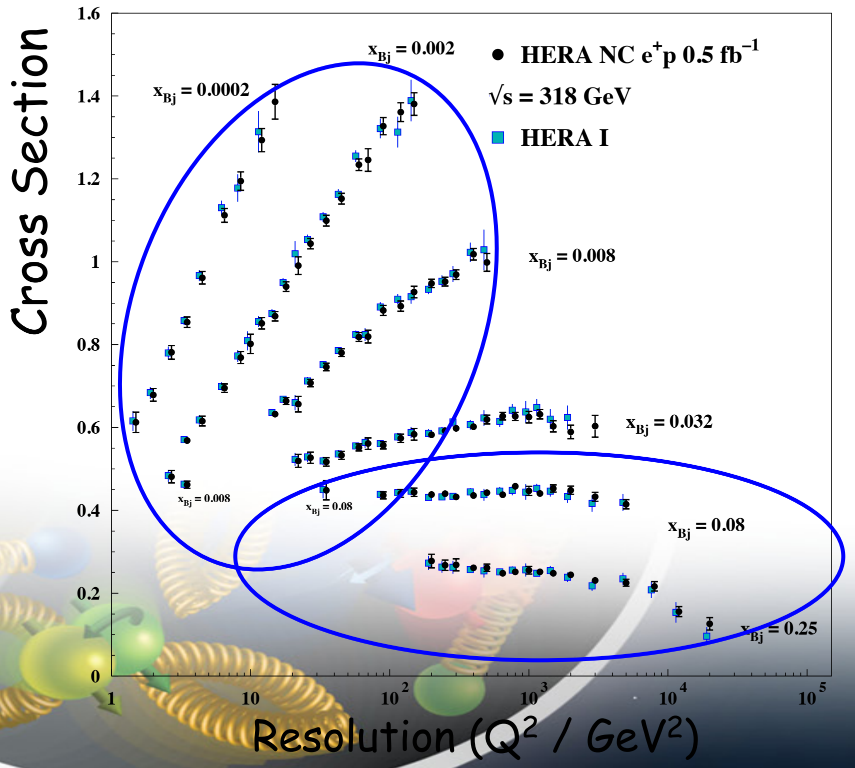
$$\frac{d^2\sigma^{ep\to eX}}{dxdQ^2} = \frac{4\pi\alpha_{e.m.}^2}{xQ^4}\left[\left(1-y+\frac{y^2}{2}\right)F_2(x,Q^2) - \frac{y^2}{2}F_L(x,Q^2)\right]$$

quark+anti-quark momentum distributions

gluon momentum distribution

without gluons the cross section depends only on x, no dependence on Q^2 → $F_2(x)$

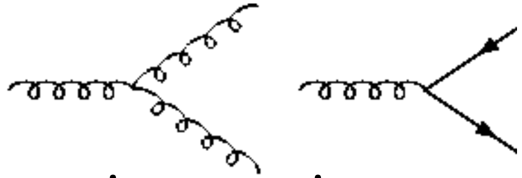
Bjorken scaling



BUT:

Observe strong rise of cross section with both x and Q^2

Because of gluon initiated processes



Scaling violation

→ Gluon Distribution: $d\sigma(x,Q^2)/d\ln Q^2$

SPIN: Fundamental Quantum Number

SPIN is one of the fundamental properties of matter
all elementary particles, but the Higgs carry spin
Studying Spin revealed many surprises in physics
→ proton anomalous magnetic moment → substructure
cannot be explained by a static picture of the proton

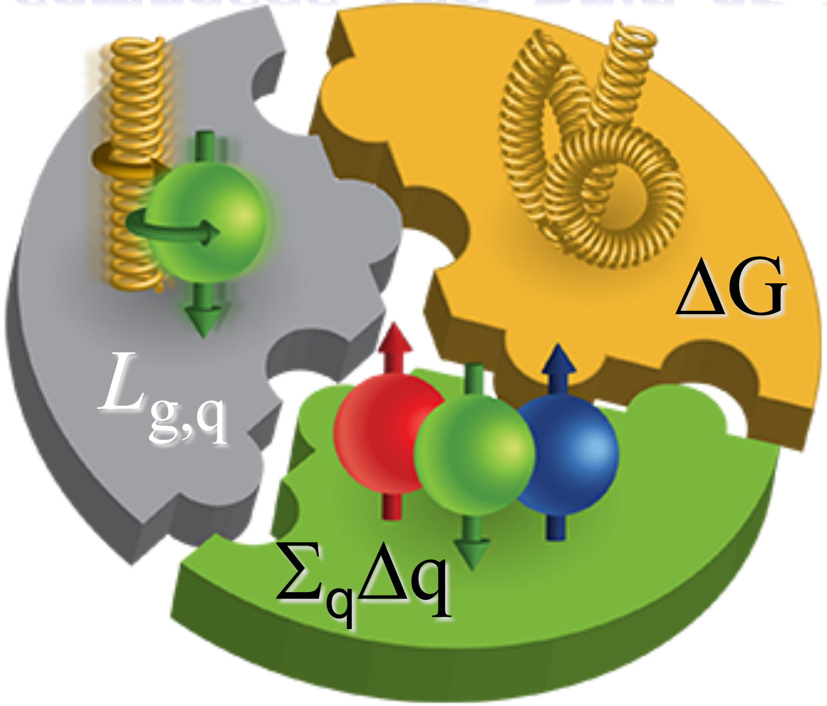
Proton Spin:

It is more than the number ½ ! It is the interplay between the intrinsic properties and interactions of quarks and gluons

If we do not understand the proton spin in QCD, we do not fully understand QCD !

need a polarized collider to have full access to the proton dynamics

What composes the Spin of the Proton



"Helicity sum rule"

$$\frac{1}{2}\hbar = \left\langle P, \frac{1}{2} | J^z_{QCD} | P, \frac{1}{2} \right\rangle = \overbrace{\sum_q \frac{1}{2} S^z_q}^{\text{total quark spin}} + \overbrace{S^z_g}^{\text{gluon spin}} + \overbrace{\sum_q L^z_q + L^z_g}^{\text{angular momentum}}$$

Can quarks and gluons explain all the spin?

→ what is the role of gluons?

→ what is the role of sea quarks?

→ How much orbital angular momentum is needed?

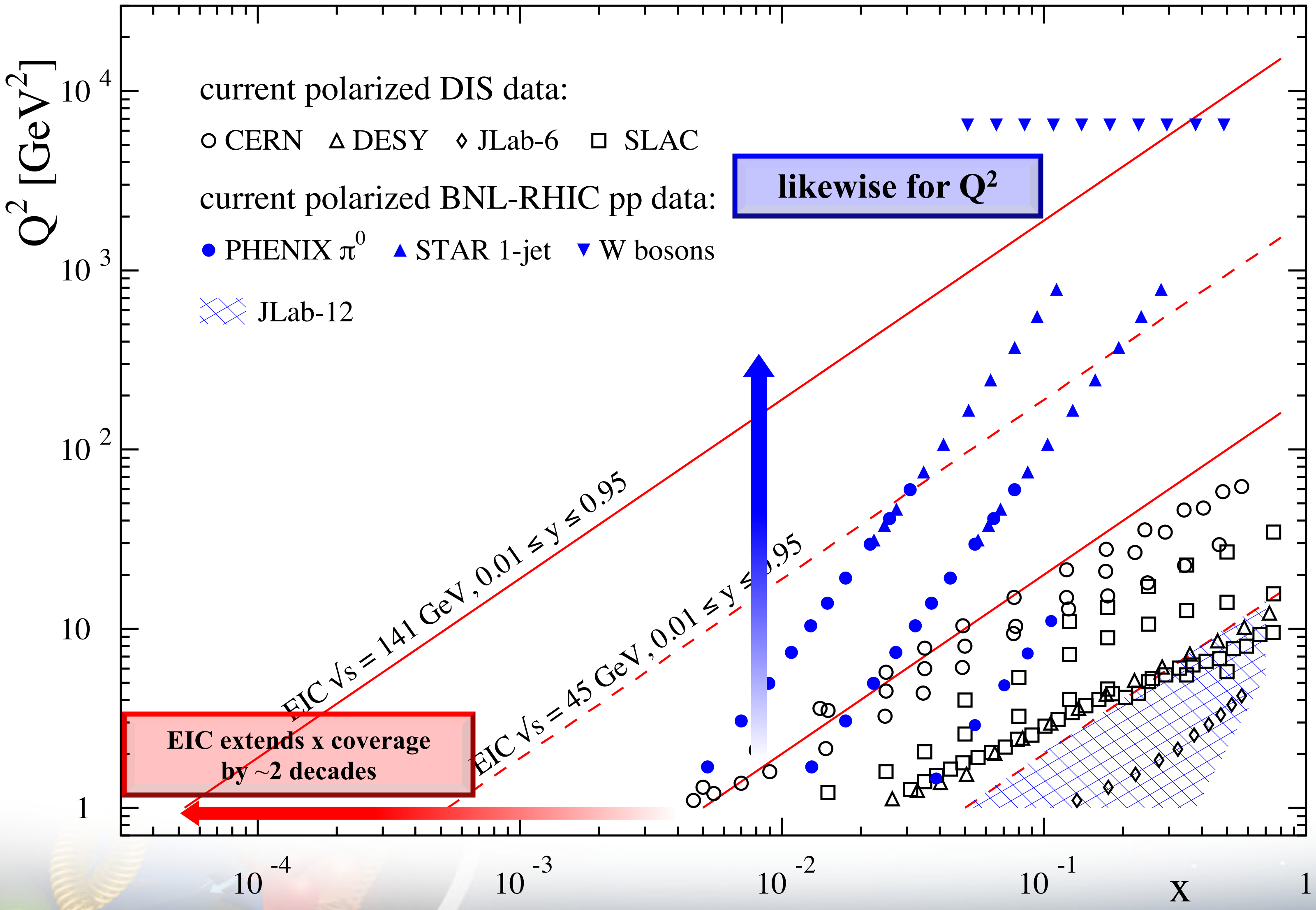
Why should we care?

Spin ideal tool to understand the dynamics of sea quarks and gluons inside the hadron

Despite decades of QCD – Spin one of the least understood quantities
→ Consequence very few models, but several physics pictures, which can be tested with high precision data

- ❑ the pion/kaon cloud model
  - → rooted in deeper concepts → chiral symmetry
  - → generated q-qbar pairs (sea quarks) at small(ish)-x are predicted to be unpolarized
  - → gluons if generated from sea quarks unpolarised → spatial imaging
  - → a high precision measurement of the flavor separated polarized quark and gluon distributions as fct. of x is a stringent way to test.
- ❑ the chiral quark-soliton model
  - → sea quarks are generated from a "Dirac sea" with a rich dynamical structure but excludes gluons at its starting scale
  - → sea quarks are polarized → asymmetry $\Delta\bar{u} \neq \Delta\bar{d}$
  - → a high precision measurement of the flavor separated polarized quark as fct. of x is a stringent way to test
- ❑ stringent test of lattice calculations
  - → the relative importance of lattice graphs
  - → probe quark is connected to the proton wave function or is created from the 'gluon soup' inside the proton

present vs EIC kinematic coverage



The Path to Imaging Quarks and Gluons

4+1-D

Wigner function

$\mathbf{W(x, k_T, b_T)}$

QCD genetic map of nucleons

$\int d^2b_T$ $\int d^2k_T$

2+1-D

$\mathbf{f(x, k_T)}$ — not related by Fourier transf. — $\mathbf{f(x, b_T)}$ $\xleftrightarrow{b \leftrightarrow t}$ $\mathbf{H(x, 0, t)}$ $\xleftarrow{\xi \to 0}$ $\mathbf{H(x, \xi, t)}$

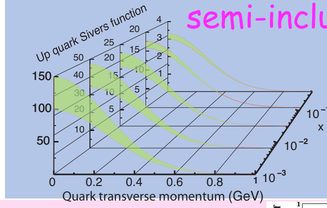
transv. mom. dep. PDF

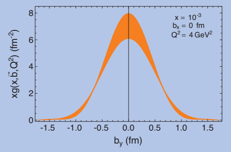
semi-inclusive DIS

impact par. dep. PDF

generalized PDF

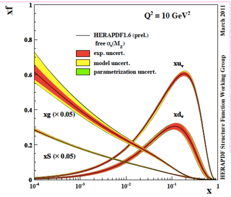
exclusive processes





$\int d^2k_T$ $\int d^2b_T$ $\int dx$

1-D

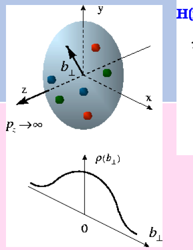


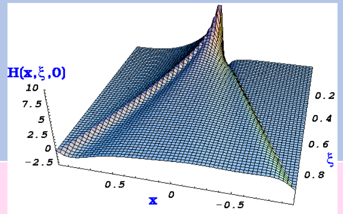
$\mathbf{f(x)}$

parton densities

$\mathbf{F(t)}$

form factor



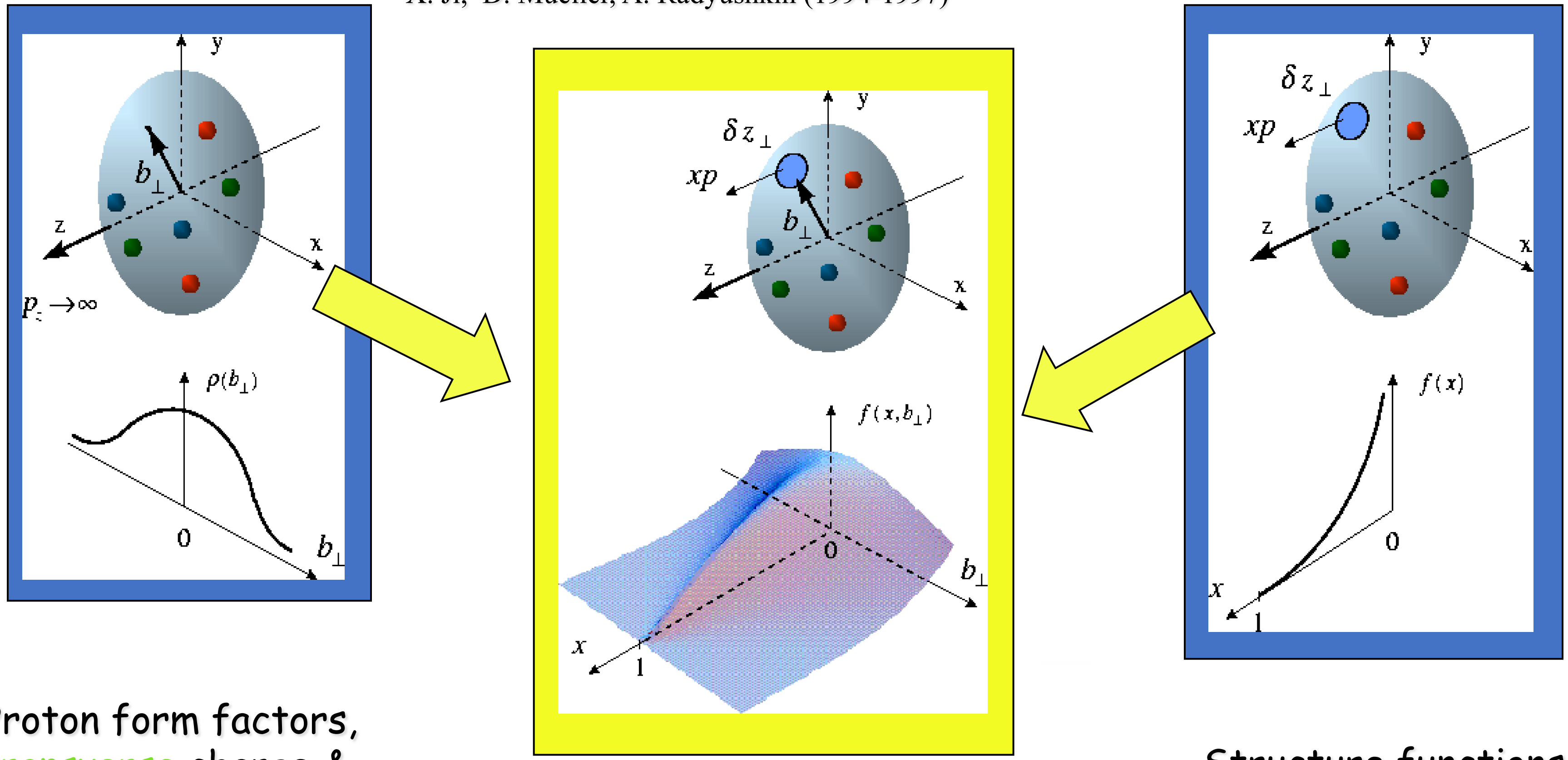


There are many reasons why one wants to have a 3d picture of nucleons and nuclei collective effects is one of them.

Getting the full picture is an other one

Generalized Parton Distributions

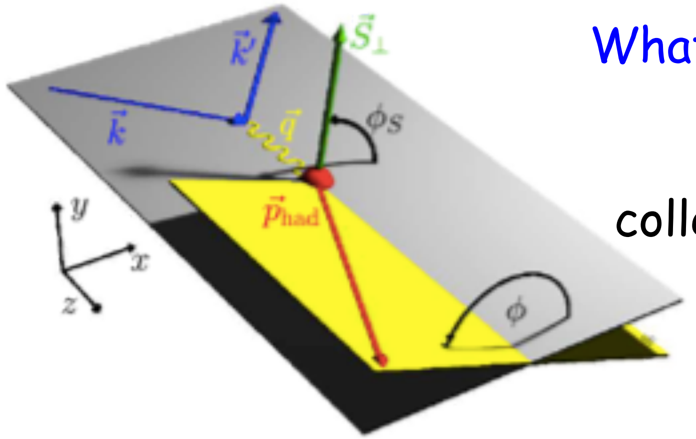
X. Ji, D. Mueller, A. Radyushkin (1994-1997)



Proton form factors, transverse charge & current densities

Correlated quark momentum and helicity distributions in transverse space - GPDs

Structure functions, quark longitudinal momentum & helicity distributions



What is the dynamic structure of the proton and nuclei
2D+1 picture in momentum space
Visualize color interactions in QCD
collective phenomena and correlations in fragmentation
New physics due to confined motion

Observable for TMDs:
azimuthal modulations of 6-fold differential cross section

$$\frac{d\sigma}{dx\,dQ^2\,dz\,d\phi_s\,d\phi_h\,dp_T^h}$$

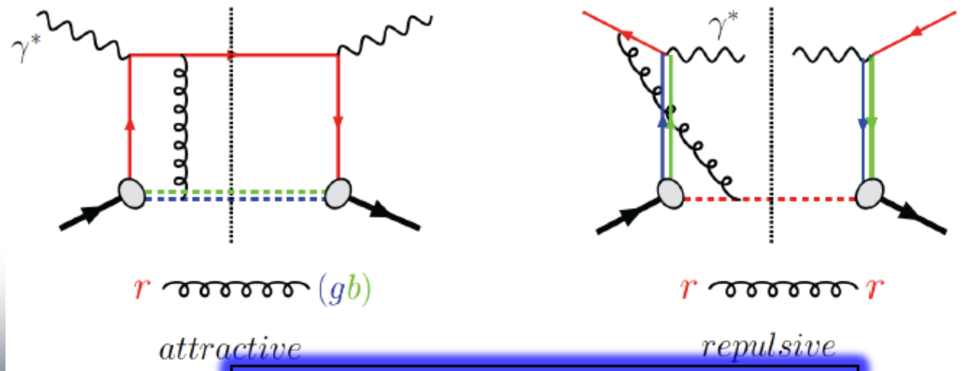
<u>Features:</u>

- very different evolution then collinear PDFs
  pertubative & non-pertubative contributions
- reveals non-trivial aspects of QCD color gauge invariance

QCD:

DIS:
γq-scattering
attractive FSI

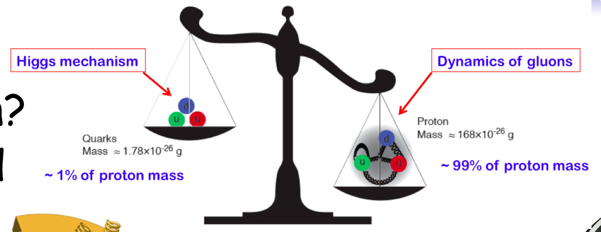
pp:
qqbar-anhilation
repulsive ISI

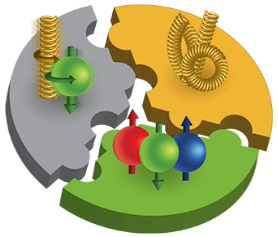


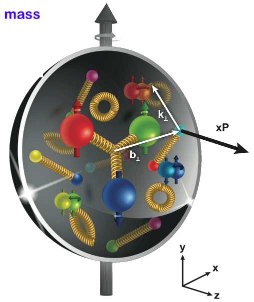
$TMD_{DIS} = - TMD_{DY/W/Z0}$

The EIC Physics

How are the sea quarks and gluons, and their spins, distributed in space and momentum inside the nucleon?
How do the nucleon properties emerge from them and their interactions?

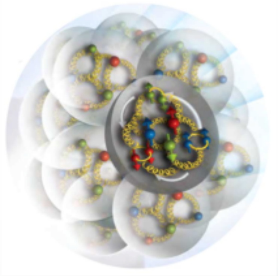




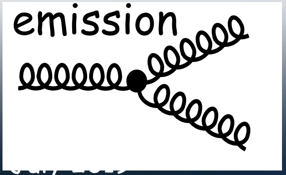


How do color-charged quarks and gluons, and colorless jets, interact with a nuclear medium?
How do the confined hadronic states emerge from these quarks and gluons?
How do the quark-gluon interactions create nuclear binding?

How does a dense nuclear environment affect the quarks and gluons, their correlations, and their interactions?
What happens to the gluon density in nuclei? Does it saturate at high energy, giving rise to a gluonic matter with universal properties in all nuclei, even the proton?



gluon emission



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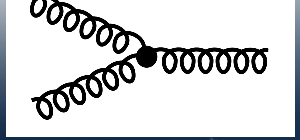
gluon recombination



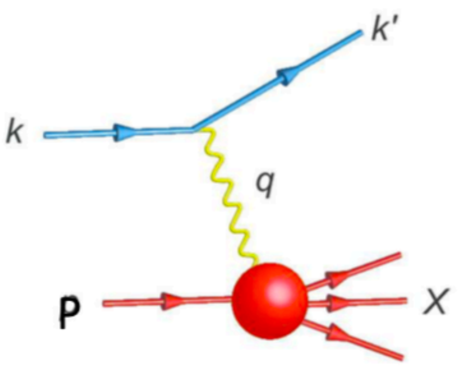
Parton Distribution Functions Introductory Lectures: PDF1

Marco Guzzi

CTEQ School - 19 July 2019

University of Pittsburgh

When the proton breaks apart (in DIS), the parametrization



$$\bar{u}(p')\left[F_1(Q^2)\gamma_\mu + F_2(Q^2) i\sigma_{\mu\nu}q^\nu\right]u(p)$$

is no longer good. Need to parametrize photon–proton–X interactions, where X is anything the proton can break up into.

Thus, it makes sense to parametrize the cross section (instead of the vertex) in terms of the momentum transfer q and the proton momentum P.

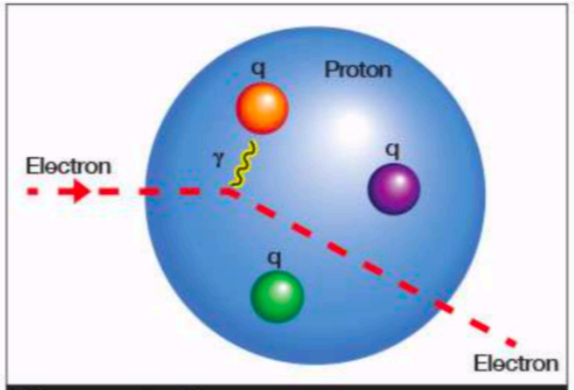
$$\left(\frac{d\sigma}{d\Omega\, dE'}\right)_{\text{lab}} = \frac{\alpha_e^2}{4\pi m_p q^4}\frac{E'}{E}L^{\mu\nu}W_{\mu\nu}$$

$$L_{\mu\nu} = \frac{1}{2}\mathrm{Tr}\left[\not{k}'\gamma^\mu \not{k}\gamma^\nu\right] = 2(k'^\mu k^\nu + k'^\nu k^\mu - k\cdot k' g^{\mu\nu})$$ Leptonic tensor

$$e^2\epsilon_\mu\epsilon_\nu^\star W^{\mu\nu} = \frac{1}{2}\sum_{X,\text{spins}}\int d\Pi_X (2\pi)^4\delta^4(q+P-p_X)\left|\mathcal{M}\left(\gamma^\star p^+ \to X\right)\right|^2$$

$$W^{\mu\nu} = W_1\left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) + W_2\left(P^\mu - \frac{P\cdot q}{q^2}q^\mu\right)\left(P^\nu - \frac{P\cdot q}{q^2}q^\nu\right)$$ Hadronic tensor

Unpolarized Parton Distribution functions of the proton



Electron scattering off a proton.

Parton Distribution Function of the Nucleon

$$\sigma\left(e^- P^+ \to e^- X\right) = \sum_i \int_0^1 d\xi\, f_i(\xi)\hat{\sigma}\left(e^- p_i \to e^- X\right)$$

$$\xi = x, \qquad \text{where} \quad x \equiv \frac{Q^2}{2P\cdot q}$$

PDF: probability that a parton i is emitted by the proton, which carries a longitudinal fraction ξ of proton's momentum.

Xsec in the parton model

$$\left(\frac{d\hat{\sigma}(e^- q \to e^- q)}{d\Omega\, dE'}\right)_{\text{lab}} = \frac{\alpha_e^2 Q_i^2}{4E^2\sin^4\frac{\theta}{2}}\left[\cos^2\frac{\theta}{2} + \frac{Q^2}{2m_q^2}\sin^2\frac{\theta}{2}\right]\delta\left(E - E' - \frac{Q^2}{2m_q}\right)$$

Qi = charge of the quark

$$\delta\left(E - E' - \frac{Q^2}{2m_q}\right) = \delta\left(\frac{Q^2}{2m_p x} - \frac{Q^2}{2m_p\xi}\right) = \frac{2m_p}{Q^2}x^2\delta(\xi - x)$$

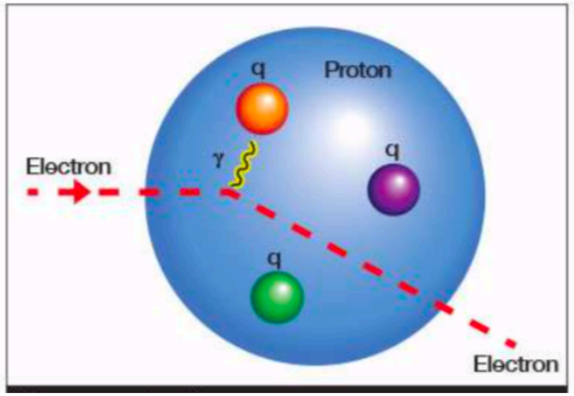
$$W_1(x,Q) = 2\pi\sum_i Q_i^2 f_i(x),$$

$$W_2(x,Q) = 8\pi\frac{x^2}{Q^2}\sum_i Q_i^2 f_i(x)$$

$$\left(\frac{d\sigma(e^- P \to e^- X)}{d\Omega\, dE'}\right)_{\text{lab}} = \sum_i f_i(x)\frac{\alpha_e^2 Q_i^2}{4E^2\sin^4\frac{\theta}{2}}\left[\frac{2m_p}{Q^2}x^2\cos^2\frac{\theta}{2} + \frac{1}{m_p}\sin^2\frac{\theta}{2}\right]$$

Physical justification of PDFs

- the momentum sloshes around among proton constituents at time scales $\approx \Lambda_{QCD}^{-1} \approx M_p^{-1}$
- these time scales are much slower than the time scales ~1/Q that the photon probes. The separation of scales $Q \gg \Lambda_{QCD}$ allows us to treat the parton wavefunctions within the proton as being decoherent, giving the **probabilistic interpretation**.
- to actually prove that this decoherence occurs, amounts to a proof of *factorization*.
- PDFs are non perturbative objects.



For a proton (uud) we have the valence quark sum rules

$$\int_0^1 dx\,[u(x) - \bar{u}(x)] = 2$$

$$\int_0^1 dx\,\left[d(x) - \bar{d}(x)\right] = 1$$

In principle we should expect $\sum_i \int dx\, x q_i(x) = 1$ Momentum sum rules

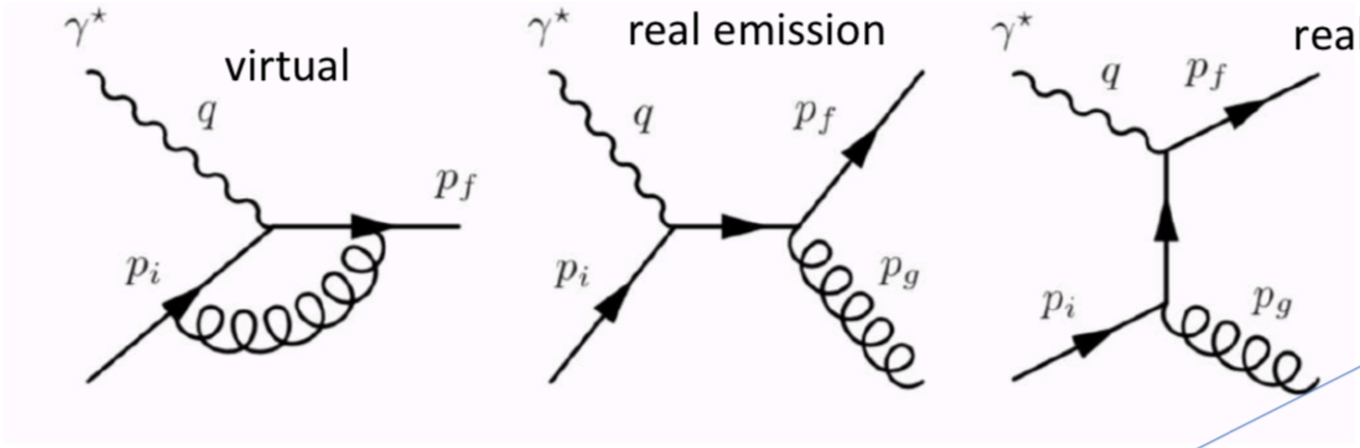
THE GLUON

$$\int_0^1 \mathrm{d}x\, x \left[(u(x) + \bar{u}(x) + d(x) + \bar{d}(x) + s(x) + \bar{s}(x) + \ldots \right] = 1$$

q_i	momentum
d_V	0.111
u_V	0.267
d_S	0.066
u_S	0.053
s_S	0.033
c_S	0.016
total	0.546

$$\int_0^1 dx\, x\,[f_u(x) + f_d(x) + f_{\bar{u}}(x) + f_{\bar{d}}(x) + f_g(x)] = 1$$

At NLO we have other Feynman diagrams contributing



IR Soft and collinear divergence.
UV divergences cancelled by renormalization

$$\hat{W}_0^V = 4\pi Q_i^2 \frac{\alpha_s}{2\pi} C_F \left(\frac{4\pi\mu^2}{Q^2}\right)^{\frac{\varepsilon}{2}} \frac{\Gamma\left(1-\frac{\varepsilon}{2}\right)}{\Gamma\left(1-\varepsilon\right)} \left(-\frac{8}{\varepsilon^2} - \frac{6}{\varepsilon} - 8 - \frac{\pi^2}{3}\right) \delta(1-z)$$

$\overline{MS}$: we need a prescription to deal with singularities.

In D dim, D = 4-ε

$$\hat{W}_0^R = 4\pi Q_i^2 \frac{\alpha_s}{2\pi} C_F \left(\frac{4\pi\mu^2}{Q^2}\right)^{\frac{\varepsilon}{2}} \frac{\Gamma(1-\frac{\varepsilon}{2})}{\Gamma(1-\varepsilon)} \times \left\{3 + 2z - \frac{1+z^2}{1-z}\ln z \right.$$
$$\left. + \left(\frac{8}{\varepsilon^2} + \frac{3}{\varepsilon} + \frac{7}{2}\right)\delta(1-z) - \left(2\frac{1+z^2}{\varepsilon} + \frac{3}{2}\right)\left[\frac{1}{1-z}\right]_+ + (1+z^2)\left[\frac{\ln(1-z)}{1-z}\right]_+ \right\}$$

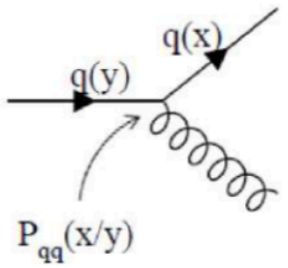
Soft and Collinear divergence

Putting all the contributions together the double $1/\varepsilon^2$ poles cancel out

$$\hat{W}_0 = \hat{W}_0^{\mathrm{LO}} + \hat{W}_0^{V} + \hat{W}_0^{R} = 4\pi Q_i^2 \Bigg\{ \left[\delta(1-z) - \frac{1}{\varepsilon}\frac{\alpha_s}{\pi} P_{qq}(z) \left(\frac{4\pi\mu^2}{Q^2} \right)^{\frac{\varepsilon}{2}} \frac{\Gamma(1-\frac{\varepsilon}{2})}{\Gamma(1-\varepsilon)} \right]$$

$$+ \frac{\alpha_s}{2\pi} C_F \left[(1+z^2) \left[\frac{\ln(1-z)}{1-z} \right]_+ - \frac{3}{2} \left[\frac{1}{1-z} \right]_+ - \frac{1+z^2}{1-z} \ln z + 3 + 2z - \left(\frac{9}{2} + \frac{1}{3}\pi^2 \right) \delta(1-z) \right] \Bigg\}$$

$$P_{qq}(z) = C_F \left[(1+z^2) \left[\frac{1}{1-z} \right]_+ + \frac{3}{2}\delta(1-z) \right]$$



DGLAP splitting function at LO.
(Dokshitzer, Gribov, Lipatov, Altarelli, Parisi)

Plus distribution

$$\int_0^1 dz \frac{f(z)}{[1-z]_+} \equiv \int_0^1 dz \frac{f(z)-f(1)}{1-z}$$

Inserting $\hat{W}_0$ back into $W^{\mu\nu}(x,Q)$ we obtain

$$W_0(x,Q) = 4\pi \sum_i Q_i^2 \int_x^1 \frac{d\xi}{\xi} f_i(\xi) \left[\delta\left(1-\frac{x}{\xi}\right) - \frac{\alpha_s}{2\pi} P_{qq}\left(\frac{x}{\xi}\right)\left(\frac{2}{\varepsilon} + \ln\frac{\tilde{\mu}^2}{Q^2}\right) + \text{finite}\right]$$

At fixed x the $1/\varepsilon$ pole does not cancel. We need to consider differences of cross sections to get a finite answer

$$W_0(x,Q) - W_0(x,Q_0) = 4\pi \sum_i Q_i^2 \int_x^1 \frac{d\xi}{\xi} f_i(\xi) \left[\frac{\alpha_s}{2\pi} P_{qq}\left(\frac{x}{\xi}\right) \ln\frac{Q^2}{Q_0^2}\right]$$

finite!!!

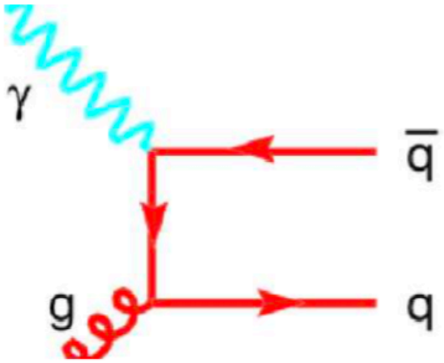
Q_0 is arbitrary. Renormalization Group Equation (RGE). Let's define, for every scale Q

$$W_0(x,Q) \equiv 4\pi \sum_i Q_i^2 f_i(x,\mu=Q) \quad \Rightarrow \quad f_i(x,\mu_1) = f_i(x,\mu) + \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi} f_i(\xi,\mu_1)\, P_{qq}\left(\frac{x}{\xi}\right) \ln\frac{\mu_1^2}{\mu^2}$$

$$\mu \frac{d}{d\mu} f_i(x,\mu) = \frac{\alpha_s}{\pi} \int_x^1 \frac{d\xi}{\xi} f_i(\xi,\mu)\, P_{qq}\left(\frac{x}{\xi}\right)$$

DGLAP equation

At NLO in QCD, we also have the following diagram contributing to F2(x, Q)



$$\frac{F_2(x,Q^2)}{x} = \int dz\, d\xi\, f_g(\xi) \left\{ \frac{\alpha_s}{2\pi} \left(\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon} \frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)} \left(-\frac{1}{\varepsilon}\right) (P_{qg}(z) + P_{\bar{q}g}(z)) + \frac{\alpha_s}{2\pi} \left[\left(z^2 + (1-z)^2\right) \ln \frac{1-z}{z} + 6z(1-z) \right] \right\} \delta(x - z\xi)$$

Therefore DGLAP equations are mixed together and we have 2 Nf + 1 coupled integro-differential equations

$$\mu \frac{d}{d\mu} \begin{pmatrix} f_i(x,\mu) \\ f_g(x,\mu) \end{pmatrix} = \sum_j \frac{\alpha_s}{\pi} \int_x^1 \frac{d\xi}{\xi} \begin{pmatrix} P_{q_i q_j}(\frac{x}{\xi}) & P_{q_i g}(\frac{x}{\xi}) \\ P_{g q_j}(\frac{x}{\xi}) & P_{gg}(\frac{x}{\xi}) \end{pmatrix} \begin{pmatrix} f_j(\xi,\mu) \\ f_g(\xi,\mu) \end{pmatrix}$$

$$P_{qq}(z) = C_F \left[\frac{1+z^2}{[1-z]_+} + \frac{3}{2}\delta(1-z) \right], \quad P_{gg}(z) = 2C_A \left[\frac{z}{[1-z]_+} + \frac{1-z}{z} + z(1-z) \right] + \frac{\beta_0}{2}\delta(1-z)$$ LO DGLAP Splitting functions

$$P_{qg}(z) = T_F \left[z^2 + (1-z)^2 \right],$$

$$P_{gq}(z) = C_F \left[\frac{1+(1-z)^2}{z} \right],$$

$$P_{ij}(x,\mu^2) = \frac{\alpha_s(\mu^2)}{4\pi} P_{ij}^{(0)}(x) + \left(\frac{\alpha_s(\mu^2)}{4\pi}\right)^2 P_{ij}^{(1)}(x) + \left(\frac{\alpha_s(\mu^2)}{4\pi}\right)^3 P_{ij}^{(2)}(x) + \ldots$$

PDFs universality

- Gluons, quarks and antiquarks are the known constituents of the proton.
- Their distributions as a function of x and generic scale μ, at which partons are probed, are universal quantities that do not depend on the specific hard process under consideration.

Differently from the hard-scattering cross section, the analytic structure of the PDFs cannot be fully predicted by perturbative QCD, but has to be determined by comparing standard sets of cross sections, to experimental measurements by using a variety of analytical/statistical methods.

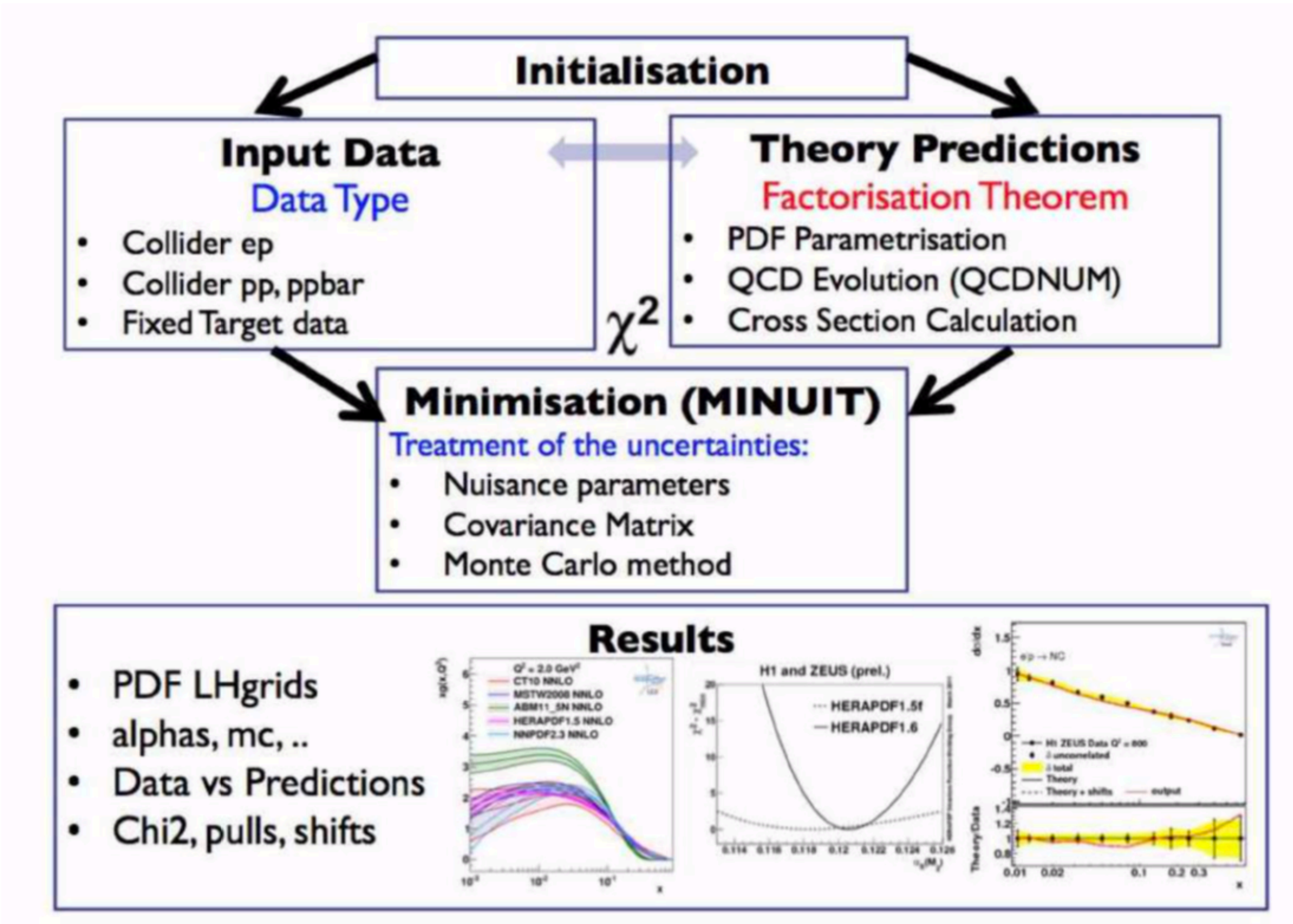
For this reason PDFs are “data-driven” quantities.

Start Again

September 27th, 2019

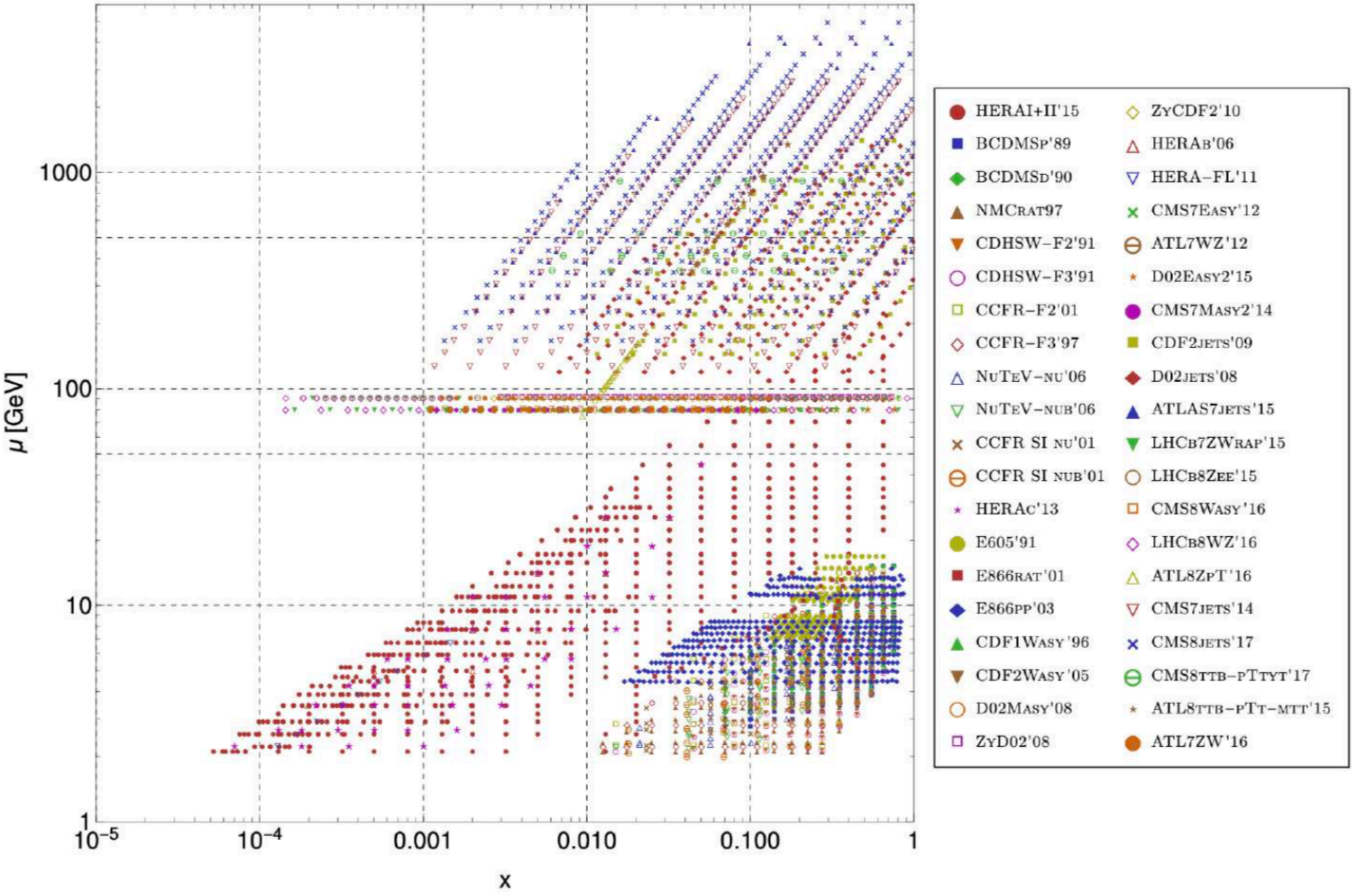
Global QCD analyses combine:

- Advanced QCD theory
- Precise measurements from HEP hadronic world data
- Sophisticated statistical methods



Welcome to the fitting machine!

Experimental data in CT18 PDF analysis



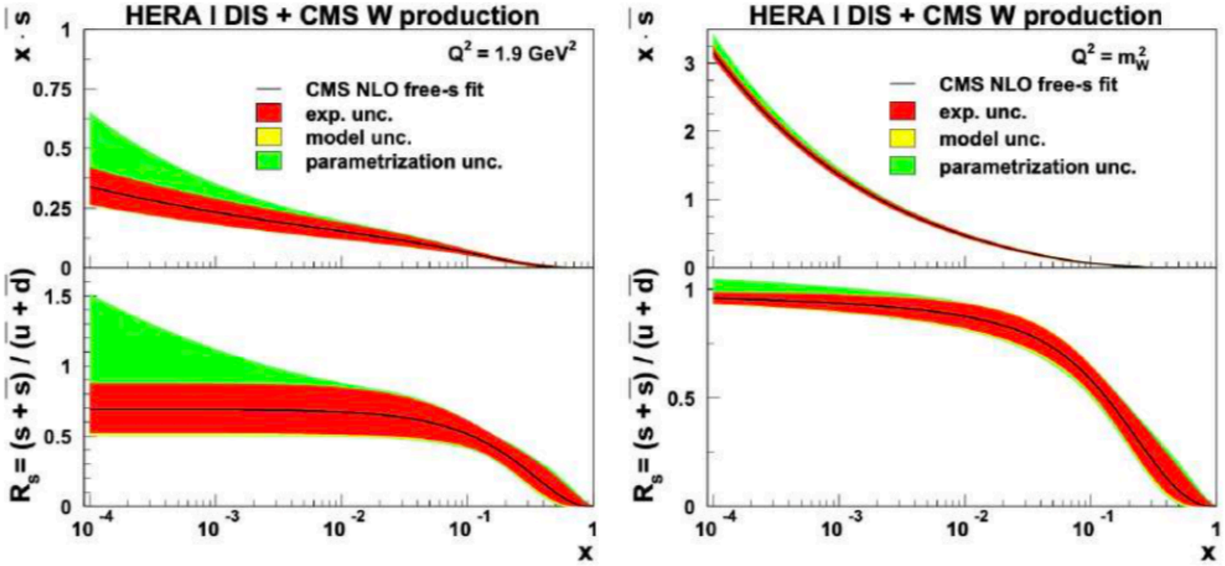
Constraints on PDFs: a few examples

Process	Sensitivity
Drell-Yan	Flavour decomposition of the sea, u_v, d_v, γ PDF
W+charm	Strange PDF
Jets	High-x gluon PDF
Photon	Medium-x gluon PDF
Top pair	Medium- and high-x gluon PDF

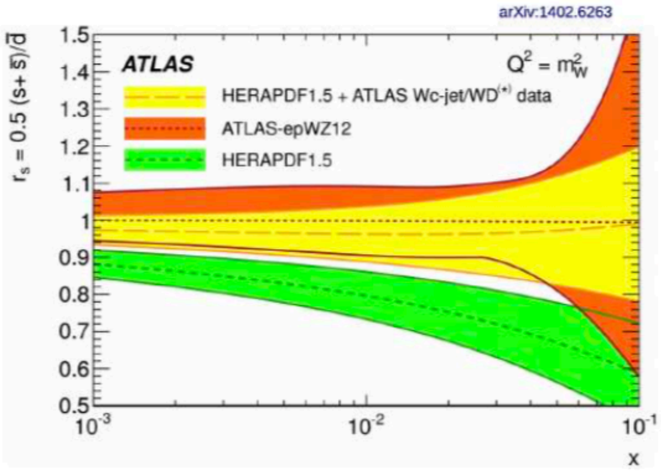
Of course, there are many other measurements which are not yet fully exploited

W + Charm production at the LHC

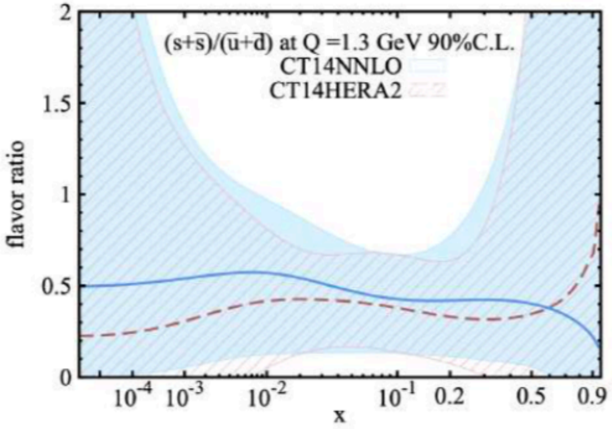
- W + single charm at LO is mainly produced by *gs* → *W+c*
- Sensitive to the strange PDF
- Exploit charge correlation to distinguish between *W + c* and *W + (g → c$\bar{c}$)*



The CMS coll. (PRD90 2014)



ATLAS Coll (2013)



CT14HERA2
PRD(2017)

Initial scale parametrization

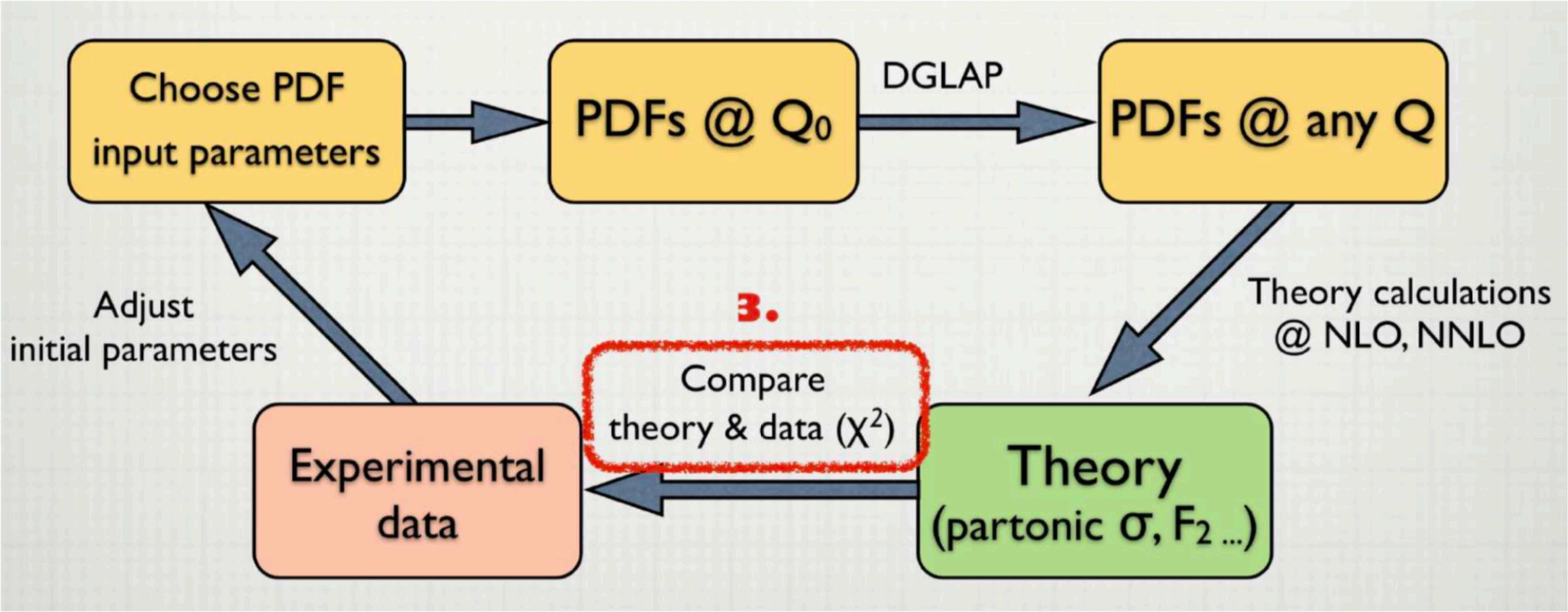
PDF parametrizations for $f_{a/p}(x, Q_0)$ must be "flexible just enough" to reach agreement with the data, without violating QCD constraints (sum rules, positivity, ...) or reproducing random fluctuations.

They are constructed by using an ansatz based on models for the two asymptotic behaviors $x \to 0$ and $x \to 1$

$$f_{i/p}(x, Q_0) = a_0 x^{a_1} (1-x)^{a_2} \times F(x; a_3, \ldots, a_n)$$

- Regge-like behavior $x \to 0 \Longrightarrow f \propto x^{a_1}$
- Quark counting rules $x \to 1 \Longrightarrow f \propto (1-x)^{a_2}$
- F(x, a3,…,an) interpolates between the small and large x regions

Flow of a PDF analysis



Statistics: Hessian method

- find N parameters {a$_i$} of the PDF (N=28 in CT14/CT18) ensemble from N_{exp} experiments with N_λ correlated systematic errors in each experiment
- Each systematic error is associated with a random parameter λ_α, assumed to be distributed as a Gaussian distribution with unit dispersion
- The most likely combination of a$_i$ and λ_α is found by minimizing the χ^2

$$\chi^2_E(\{a\},\{\lambda\}) = \chi^2_D + \chi^2_\lambda;$$

$$\chi^2_D \equiv \sum_{k=1}^{N_{pt}} \frac{1}{s_k^2}\left(D_k - T_k - \sum_{\alpha=1}^{N_\lambda} \beta_{k,\alpha}\lambda_\alpha\right)^2$$

Correlated systematic shifts

D_k and $T_k(ai)$ are the data and theory values at each point.

$$s_k = \sqrt{\sigma^2_{stat} + \sigma^2_{sys,uncor}}$$ is the total statistical + systematical **uncorrelated** error

$$\chi^2_\lambda \equiv \sum_{\alpha=1}^{N_\lambda} \lambda^2_\alpha$$ Penalty term for deviations of λ_α from their expected λ_α = 0 values

Statistics: Hessian method

Hessian matrix

After diagonalization

$$\chi^2(a) = \chi_0^2 + \frac{1}{2}\frac{\partial\chi^2}{\partial a_i \partial a_j}(a-a_0)_i(a-a_0)_j + \cdots \rightarrow \chi_0^2 + \sum_i z_i^2$$

$$y_i = a_i - a_i^0$$

$$\begin{aligned} \chi^2 &= \chi_0^2 + \sum_{i,j} H_{ij}\, y_i\, y_j\,, \\ H_{ij} &= \frac{1}{2}\left(\frac{\partial^2\chi^2}{\partial y_i\,\partial y_j}\right)_0\,, \end{aligned}$$

$$\sum_j H_{ij}\, v_{jk} = \epsilon_k\, v_{ik}$$ Eigenvalue equation

$$\sum_i v_{ij}\, v_{ik} = \delta_{jk}\,.$$ Orthonormality

$$z_i = \sqrt{\epsilon_i}\sum_j y_j\, v_{ji}$$ Change of basis in terms of the eigenvalues

$$\Delta\chi^2 = \chi^2 - \chi_0^2 = \sum_i z_i^2$$

the surfaces of constant χ2 are spheres in z_i space, with Δχ2 the squared distance from the minimum.

(1) The Drell-Yan Process and (2) Vector Boson Production in Hadron Collisions

Daniel Stump

Department of Physics and Astronomy
Michigan State University
East Lansing, Michigan

A comment on the importance of lepton-pair production processes in hadron collisions…

■ Search for resonances in the $l_1\,\bar{l}_2$ final state. Think of these discoveries from $\mu^+\,\mu^-$ production :

J/Ψ (+ other charmonium states ; p + Be)

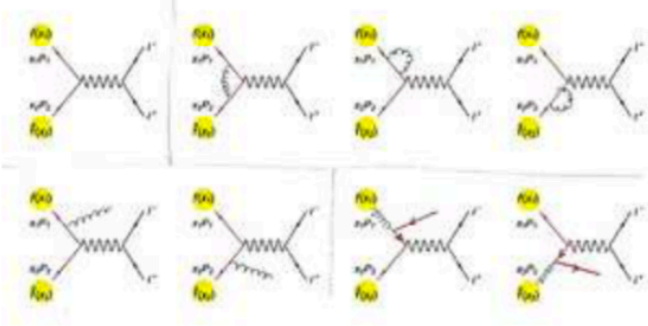
Υ (+ other upsilon states ; p + { Cu, Pt })

Z^0 (an intermediate vector boson ; p + $\overline{p}$) .

■ The cross sections depend on

Parton Distribution Functions

∴ The *measurements* of Drell-Yan cross sections and related processes provide quantitative information about PDFs. For example: the FNAL/E866 NuSea experiment (discuss later)



The 1-loop diagrams are familiar from QED, but with color factors. They have UV and IR divergences; ⇒ dimensional regularization, D = 4–2ϵ .

— The UV divergences { i.e., certain poles as $\epsilon \to 0$ } cancel with renormalization, which is familiar from QED.

— The IR divergences in $M_2^{(v)}$ { i.e., other poles as $\epsilon \to 0$ } cancel IR divergences in the real emission of soft gluons; familiar from the Bloch-Nordsieck cancellation in QED.

— But there remain some collinear divergences because the quark masses are "zero". These are not familiar from QED, because the electron mass is *not* zero. But they are familiar from ***the KLN theorem***: for massless particles the cross section is finite (or, IR safe) for ***inclusive*** initial and final states; the initial states must include all degenerate states.

Kinoshita (1962); Lee and Nauenberg (1964)

These remaining collinear divergences will be absorbed into the Parton Distribution Functions.

═ *f(x)* { the LO PDF} will be replaced by $f(x, \mu_F)$ { the NLO PDF}.

TRICKY QUESTION : where did the variable μ_F come from? (tomorrow)

Measuring the W-boson mass

Why is it important?

M_W is a fundamental parameter of the Standard Model, so its value is needed to test the Standard Model precisely.

For example, to compare the ***direct measurement*** to ***indirect measurements*** based on precision experiments.

The Z^0 mass is known very precisely because the decay $Z^0 \to l^+ + l^-$ has a resonant peak at invar.mass $m_{ll} = M_Z$; for example in the process $e^+ + e^- \to Z^0 \text{—} l^+ + l^-$ at $\sqrt{s} \sim 90$ GeV.

Measuring the W mass

But for $W^{\pm}$ the leptonic decay is $\mathrm{W} \to \mathrm{l}\ \nu$; the neutrino is "missing energy" so the detector cannot reconstruct the W mass peak.

V boson	Mass	Decay width	lBR
Z0	91.1876±0.0021 GeV/c	2.4952±0.0023 GeV/c	0.068
W±	80.379±0.012 GeV/c	2.085±0.042 GeV/c	0.210

However, there is a ***Jacobian peak*** in the distribution of the transverse momentum of the charged lepton, $p_T^{(\mathrm{lepton})}$. The shape of that peak depends on M_W.

Jet Cross Sections, Shapes and Substructure

July 22-23, 2019

PITT PAC

G. Sterman

C.N. Yang Institute for Theoretical Physics

Stony Brook Univ.

1. Why and where are there jets?
2. The measures and structures of jets.

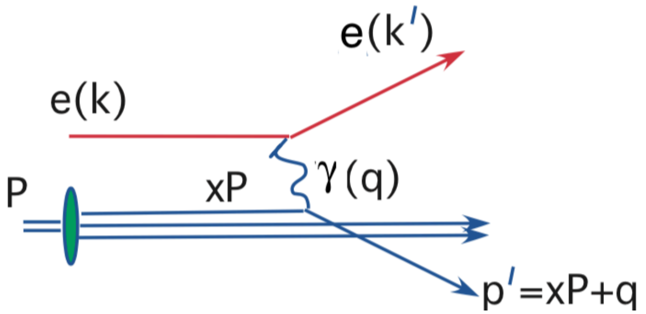
   We'll try and point out ways in which QCD jets are unique, yet part of a universal phenomenon in field theory.

- the question arose: what happens to partons in the final state?
  (Feynman, Bjorken & Paschos, Drell, Levy & Yan, 1969)
  Do "the hadrons 'remember' the directions along which the bare constituents were emitted? . . . "the observation of such 'jets' in colliding beam processes would be most spectacular." (Bjorken & Brodsky, 1969) Or does confinement forbid a it?

- To make this long story short: Quantum Chromodynamics (QCD) reconciled the irreconcilable. Here was the problem.

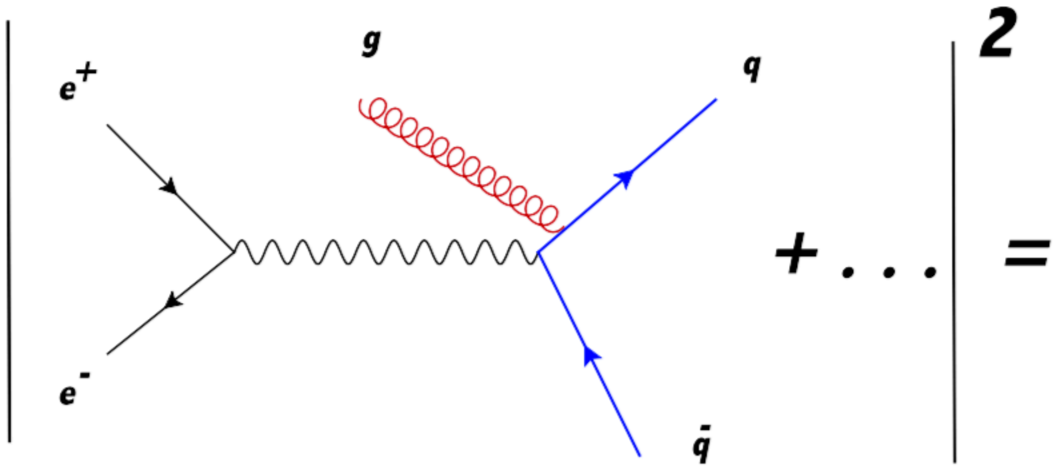
  1. Quarks and gluons explain spectroscopy, but aren't seen directly – confinement.

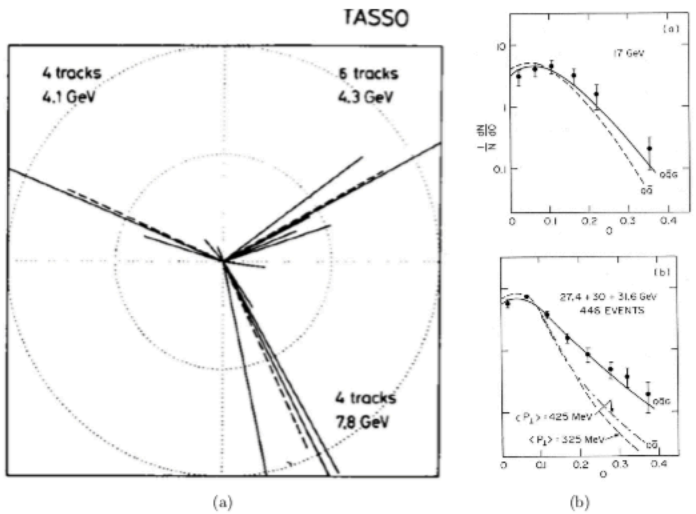
  2. In highly ("deep") inelastic, electron-proton scattering, the inclusive cross section was found to well-approximated by lowest-order elastic scattering of point-like (spin-1/2) particles (="partons" = quarks here) a result called "scaling":

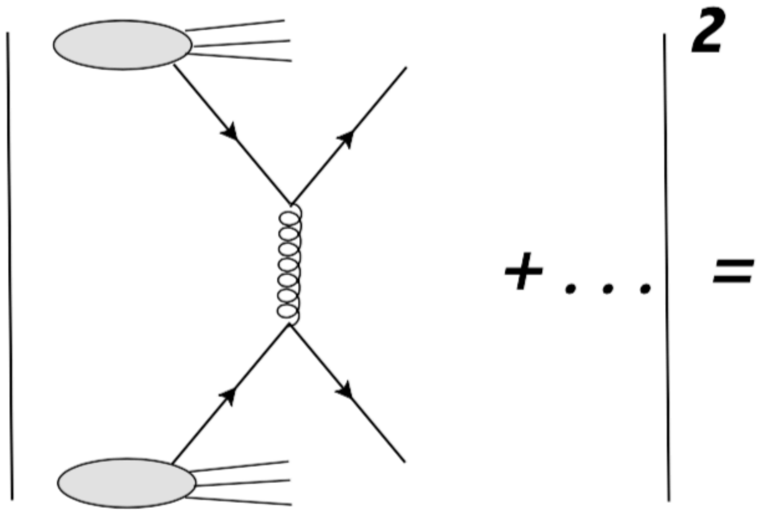


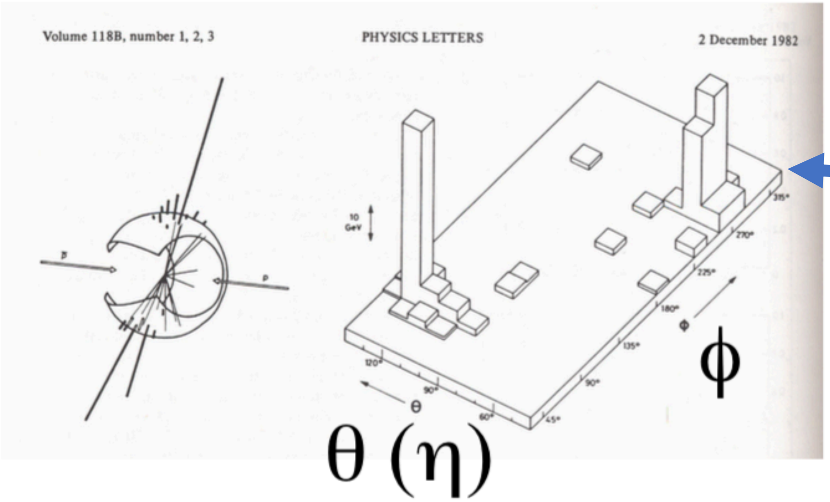
- The short explanation of DIS: Over the times $ct \leq \hbar/GeV$ it takes the electron to scatter from a quark-parton, the quark really does seem free. Later, the quark is eventually confined, but by then it's too late to change the probability for an event that has already happened.

- **Jets are “rare” because the high momentum transfer scattering of partons is rare (but calculable), but in e^+e^- annihilation to hadrons the “rarity” is in the likelihood of annihilation. Once that takes places, jets are nearly always produced.**









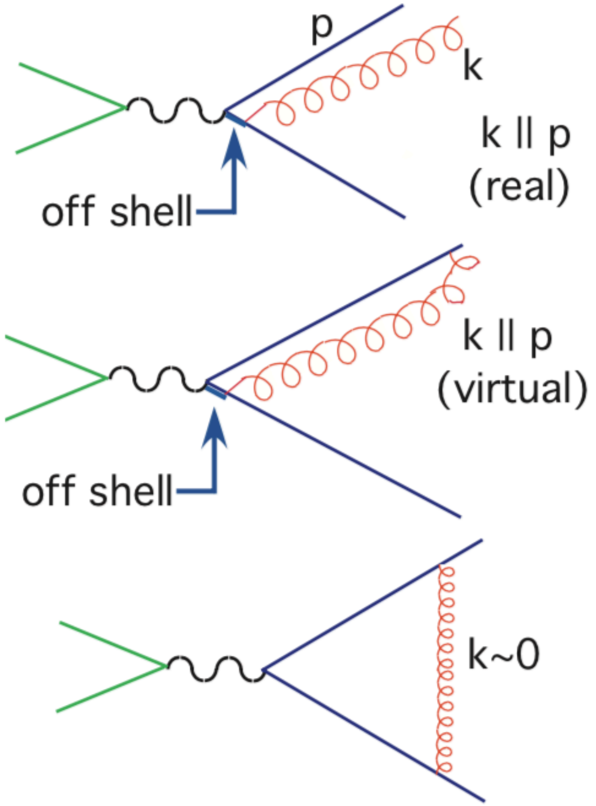
Lego Plot in terms of energy flow

- Whenever fast partons (quarks or gluons) emerge from the same point in space-time,

  they will rescatter for long times only with collinear partons.

When we get to cross sections, this is where the conditions for infrared safety will come from.

- RESULT: For particles emerging from a local scattering, (only) collinear or soft lines can give long-time behavior and enhancement. Example:



Finite-time cross sections and what they represent. Consider the probability for a sum over states f, each weighted by $S[f]$,

$$P[S] \;=\; \sum_f S[f] \sum_{n',n} \langle m_0|m_f\rangle^{(n')} \langle m_f|m_0\rangle^{(n)}$$

– Each matrix element and complex conjugate is a sum of ordered time integrals

– In any term of $P[S]$, there is a largest time.

– The largest time may be in the amplitude, or in the complex conjugate. We combine these two possibilities. Inside the sum over states, we find

$$\boxed{\Delta_n \;=\; E(\vec{k}_1 - \vec{k}_2) + E(\vec{k}_2) - \;E(\vec{k}_1)}$$

$$\ldots \times \int_{\tau'_{n-2}}^{\tau'_n} e^{i\Delta_{n-1}\tau_{n-1}}(-iV'_{f-2\to f-1})e^{-i\Delta_{n-1}\tau'_{n-1}} \quad \Leftarrow \text{in } \langle m_0|m_f\rangle$$

$$\times \int_{\tau_{f-1}}^{\infty} d\tau_n V_{f-1\to f} \left\{ ie^{i\Delta_n\tau_n} S[f] - ie^{-i(-\Delta_n)\tau_n} S[f-1] \right\}$$

$$\text{in } \langle m_f|m_0\rangle \Rightarrow \quad \times \int_{\tau_{n-2}}^{\tau_n} e^{i\Delta_{n-1}\tau_{n-1}} iV_{f-2\to f-1} e^{i\Delta_{n-1}\tau_{n-1}} \times \ldots$$

– When $S[f] = S[f-1]$ this vanishes! This is called the "largest time equation". It is an expression of unitarity – the sum of all probabilities has to be one.

– All that matters is the difference due to the last interaction: $V_{f-1\to f}$. When this produces a difference in $S[f]$, the result is nonzero.

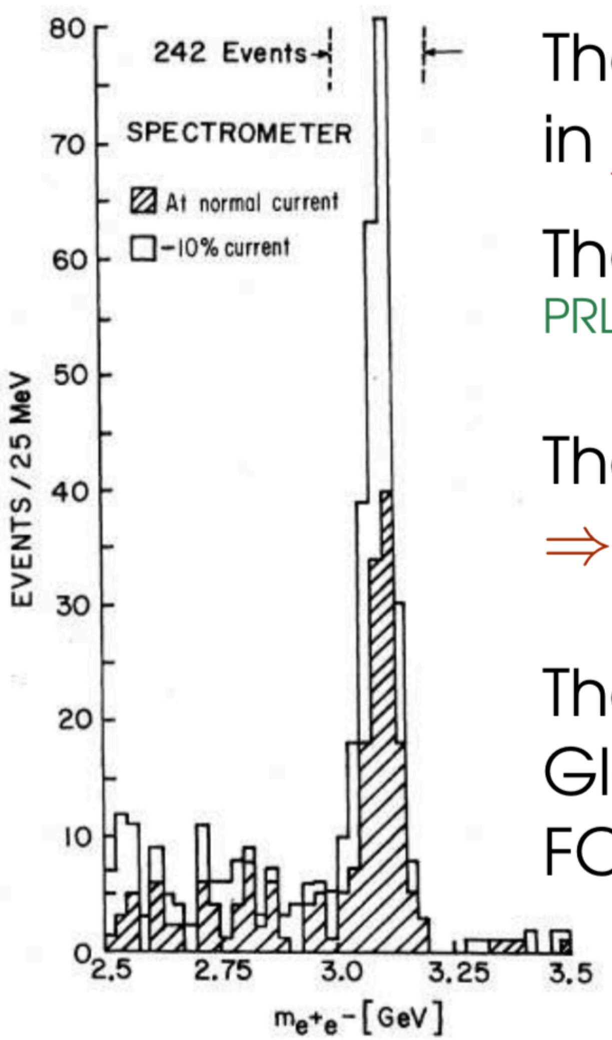
Charm, beauty, and truth at hadron colliders

Zack Sullivan[1], Pavel Nadolsky[2]

[1] Illinois Institute of Technology
[2] Southern Methodist University

Lecture 1
July 23, 2019

1974: Physicists discover charm



The first heavy quark, charm was discovered in in $p\bar{p}$ collisions at BNL and e^+e^- at SLAC

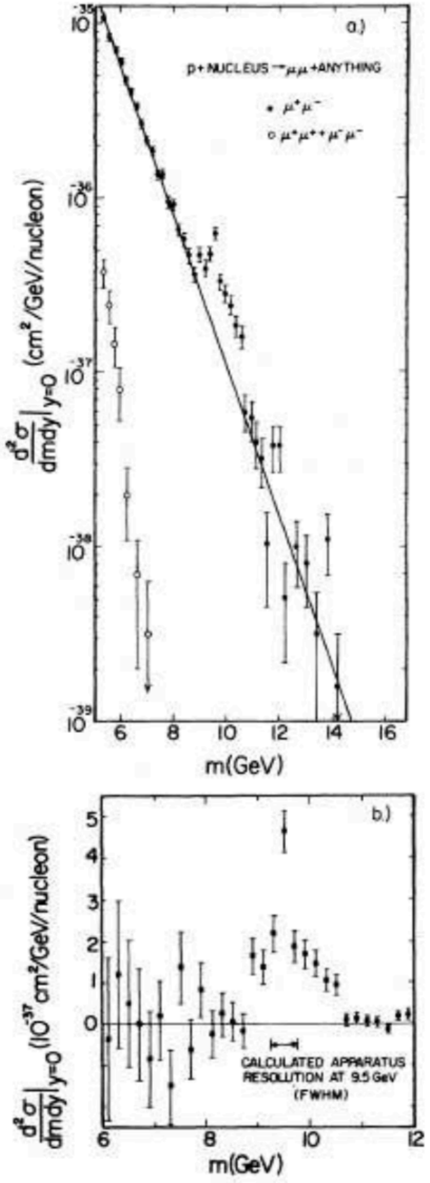
The observations were published together:
PRL 33, 1404 (1974); PRL 33, 1406 (1974)

The J/ψ was recognized as a $c\bar{c}$ bound state $\Rightarrow m_c \sim 1.5$ GeV

The existence of a 4th quark confirmed the Glashow-Iliopoulos-Maiani explanation for why FCNC decays ($s \to d\nu\bar{\nu}$) did not occur.

And it loosened the shackles of SU(3)$_{\mathrm{flavor}}$, Gell-Mann's "Eightfold way"

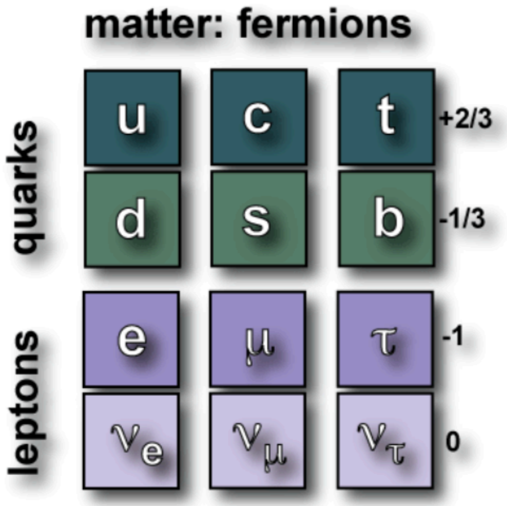
1977: Physicists discover beauty



In 1975 the τ was discovered and led to the search for other 3rd-generation particles.

In 1977 the Upsilon (a $b\bar{b}$ bound state) was observed at the Fermilab Tevatron.
(The Upsilon is also very narrow.) PRL 39, 252 (1977)

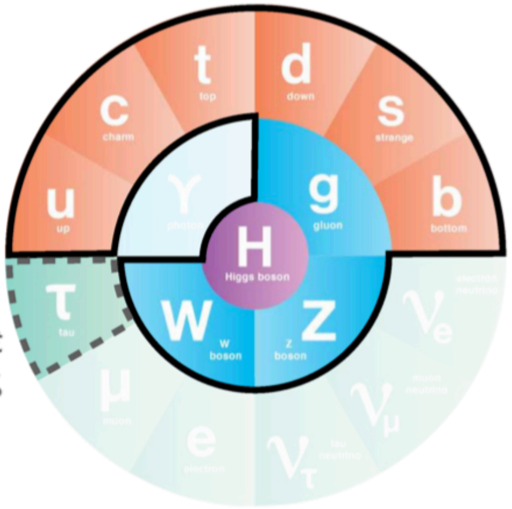
Once the bottom quark was found, it was clear that a sixth quark was needed to complete the family structure.

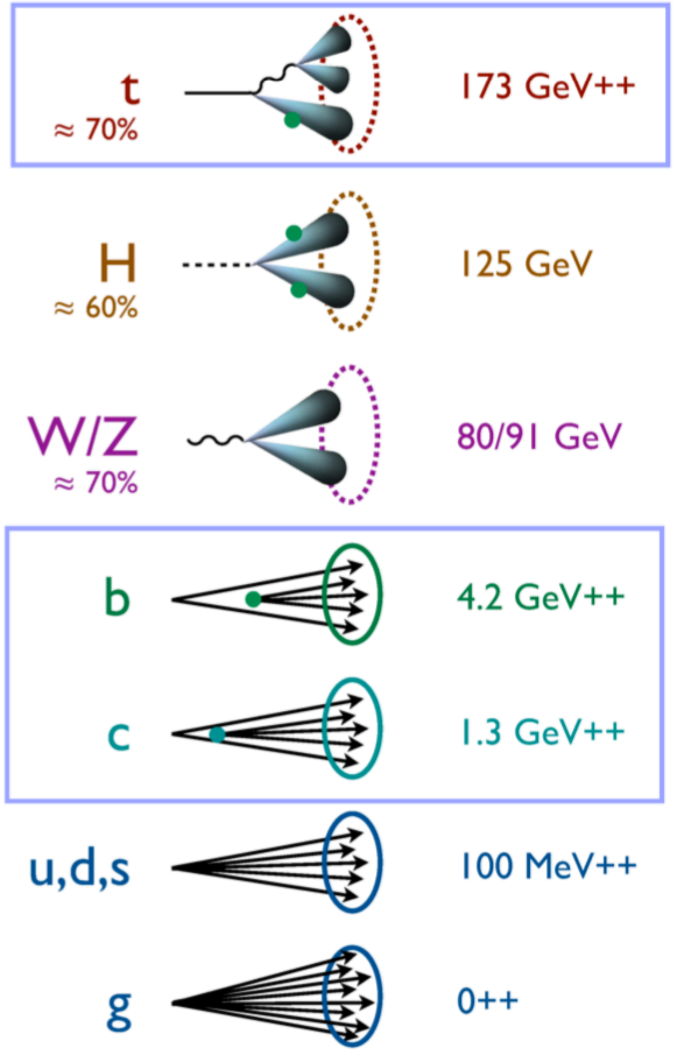


2009 - : The LHC era, hadronic decays

Jet Classification

Key supervised learning task at LHC





++ = Mass from QCD Radiation

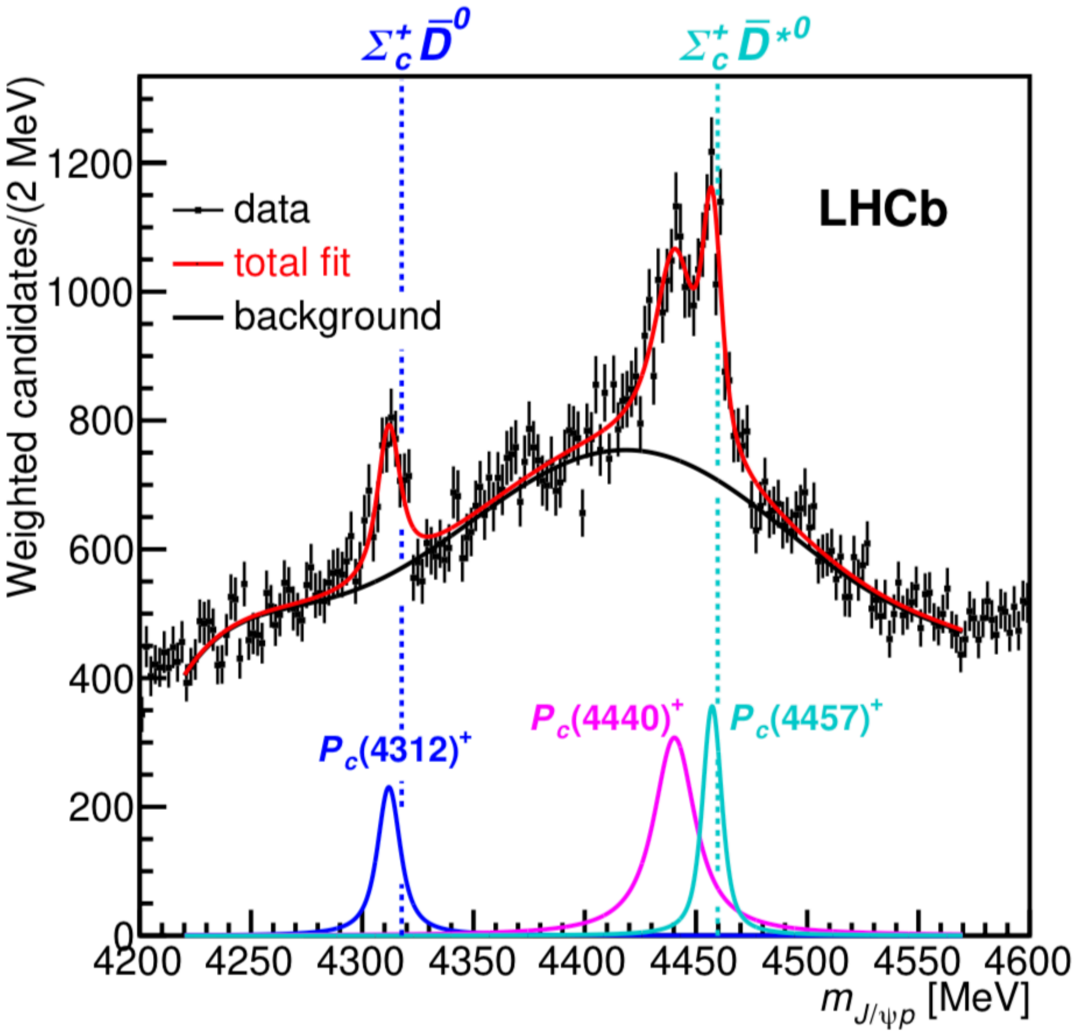
[see review in Larkoski, Moult, Nachman, 1709.04464]

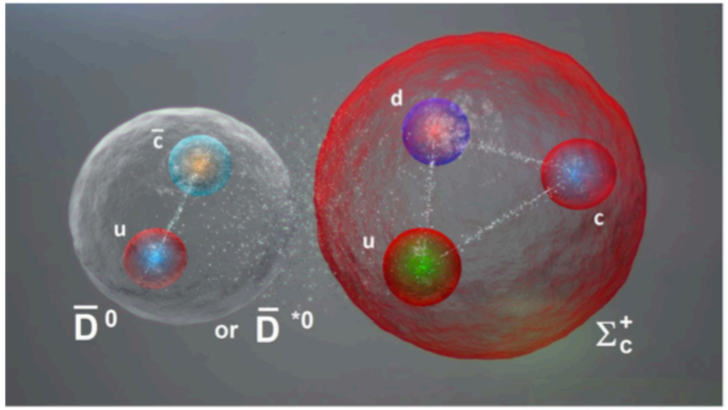
t: jet sub-structure (high $p_T^{t\bar{t}}$)

c, b: displaced decay vertices

2009 - : The LHC era, heavy-quark applications

Heavy-quark production is a means to many ends



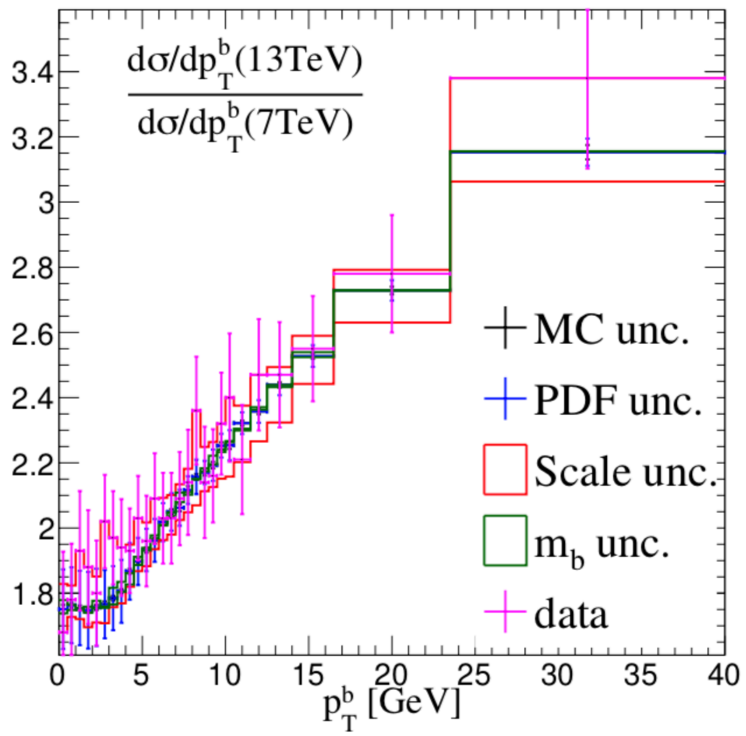


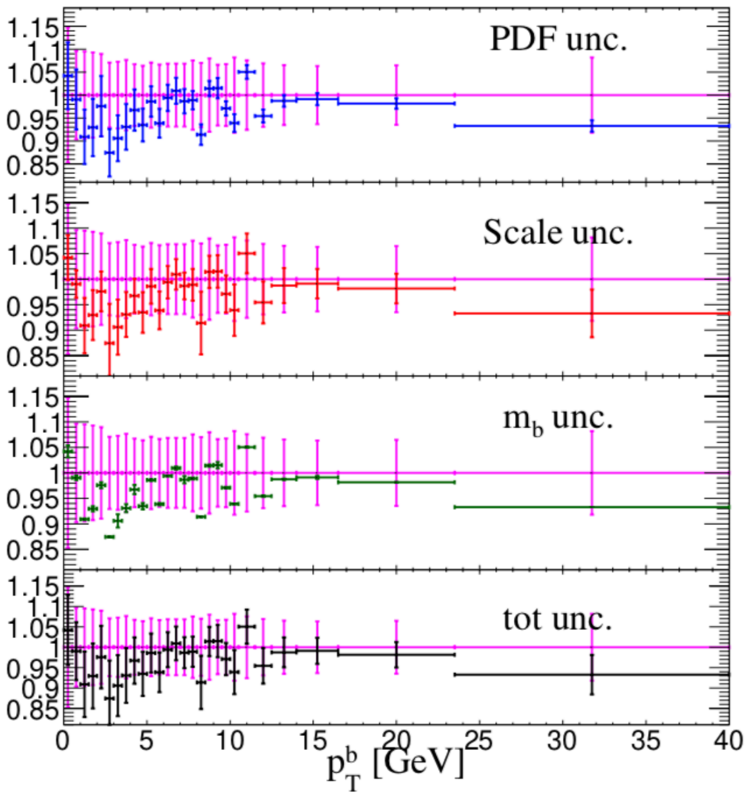
search for new QCD bound states, such as pentaquark "molecules"

2009 - : The LHC era, heavy-quark applications

Heavy-quark production is a means to many ends

The ratio $R(13\text{TeV}/7\text{TeV})$ of LHCb cross sections





Xie, Campbell, PN, 2019

LHCb c and b meson production can constrain the gluon PDF at $x \sim 10^{-5}$

Only one statement is correct. Which one? (1 minute)

1. The number N_f of active quark flavors is the number of quark masses satisfying $m_i \leq \mu_F$ at a given factorization scale μ_F.

2. In the $N_f = 4$ factorization scheme, no scattering contributions with b quarks are included.

3. The charm PDF $c(x, \mu_F)$ can be non-zero at $\mu_F < m_c$.

4. The $\overline{MS}$ PDFs are defined by setting heavy quark masses to zero to factorize collinear poles using dimensional regularization.

PDFs for heavy flavors

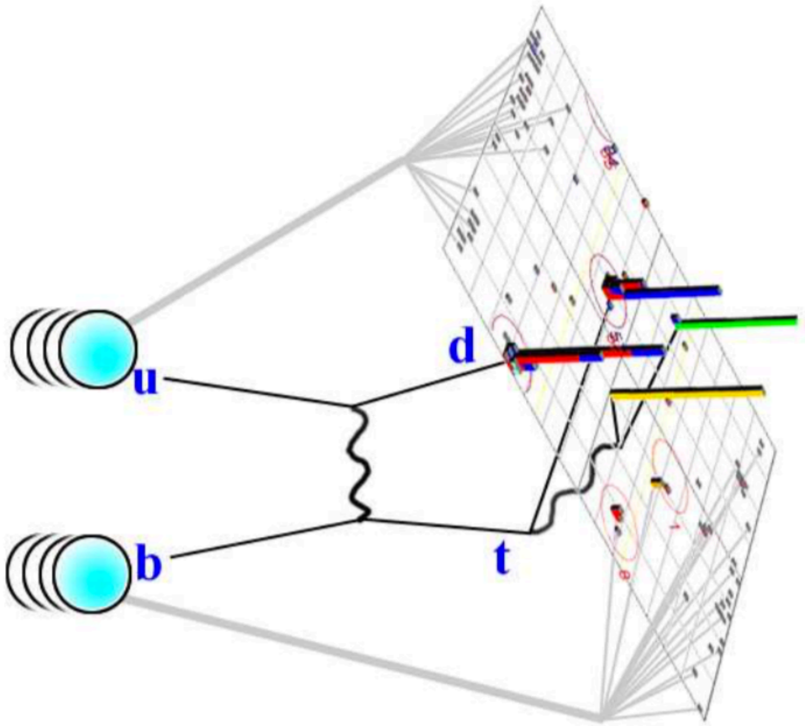
PDFs for heavy partons h can be generated via DGLAP evolution at $\mu \geq m$. At LO, a common boundary condition is $f_{h/p}(x,\mu) = 0$ at $\mu \leq m$.

In practice:

- PDFs are usually introduced for c and b quarks
  - starting from $\mathcal{O}(\alpha_s^2)$, an initial condition $f_{c/p}(x,\mu_0) \neq 0$ is generated at $\mu_0 = m_c$ by perturbative matching; also, one can obtain $f_{c/p}(x,\mu_0) \neq 0$ from twist-4 intrinsic charm DIS terms *(arXiv:1707.00657)*
- QCD coupling $\alpha_s(\mu)$ and PDFs are evaluated with 5 active flavors at all $\mu \geq m_b$
- Logarithmic enhancements may exist in collinear t, W, Z production at $\mu \gtrsim 1$ TeV; PDFs for t, W, Z "partons" may be introduced at such μ

A heavy quark testbed for QCD: single top

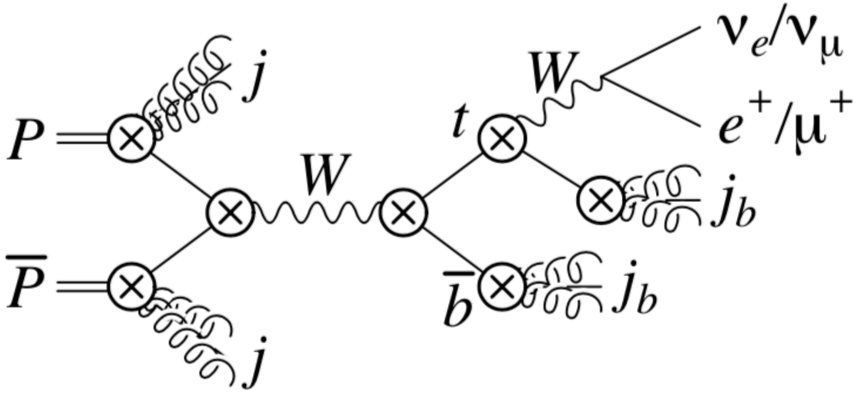
Experimentalist: Single top quark production is the observation of $b\,\ell^{\pm}\,\not{E}_T$ that reconstruct to a top quark mass, plus an extra jet (or two).



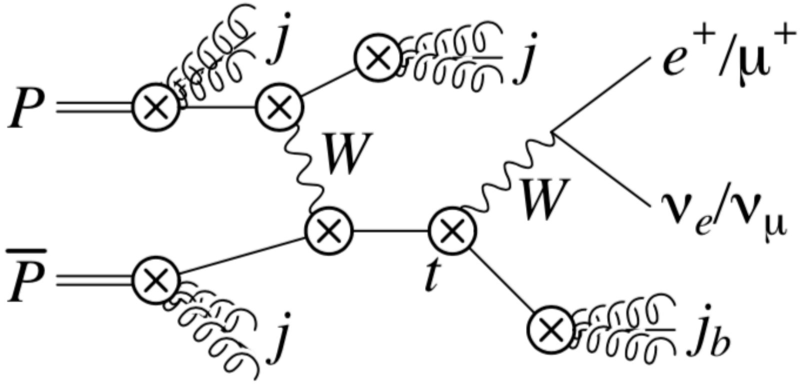
Theorist: Single top quark production is a playground in which we refine our understanding of perturbative QCD in the presence of heavy quarks.

s-/t-channel single-top-quark production (A generalized Drell-Yan and DIS)

A perfect factorization through next-to-leading order (NLO) makes single-top-quark production mathematically *identical*[†] *to DY and DIS!*



Generalized Drell-Yan. IS/FS radiation are independent.

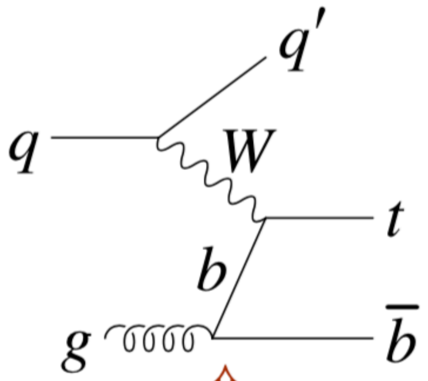


Double-DIS (DDIS) w/ 2 scales: $\mu_l = Q^2$, $\mu_h = Q^2 + m_t^2$

Color conservation forbids the exchange of just 1 gluon between the independent fermion lines.

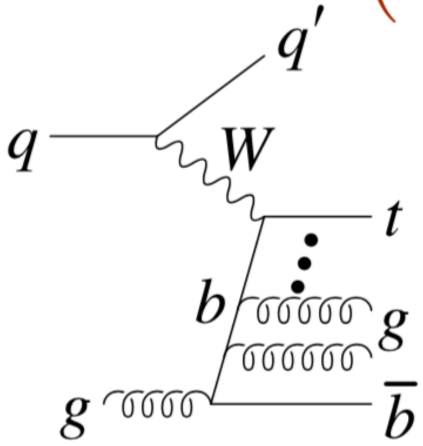
Rethinking the initial state: W-gluon fusion $\rightarrow$ t-channel single-top

W-gluon fusion (circa 1996)



$\Uparrow$

$$\sim \alpha_s \ln\left(\frac{Q^2+m_t^2}{m_b^2}\right) + \mathcal{O}(\alpha_s)$$



Each order adds $\frac{1}{n!}\left[\alpha_s \ln\left(\frac{Q^2+m_t^2}{m_b^2}\right)\right]^n$

Looks bad for perturbative expansion...

Look at the internal b.

The propagator is

$$\frac{1}{(P_g-P_{\bar{b}})^2-m_b^2} = \frac{1}{-2P_g\cdot P_{\bar{b}}}$$

$P_g = E_g(1,0,0,1)$, $P_{\bar{b}} = (E_b, \vec{p}_T, p_z)$

$$P_g \cdot P_{\bar{b}} = E_g\left(p_z\sqrt{1+\frac{p_T^2+m_b^2}{p_z^2}} - p_z\right)$$

$$\approx E_g p_z\left(\frac{p_T^2+m_b^2}{2p_z^2}\right) \sim (p_T^2+m_b^2)$$

$$\int_{p_{T\,\mathrm{cut}}} \frac{\mathrm{d}p_T^2}{p_T^2+m_b^2} \rightarrow \ln\left(\frac{1}{p_{T\,\mathrm{cut}}^2+m_b^2}\right)$$

We now have multiple scales entering the problem:

$Q, m_t, m_b, p_{T\,\mathrm{cut}}$.

$m_t \approx 35 m_b$! $\quad \alpha_s \ln \sim .7$-$.8$

Heavy Quark Theory

John Collins (Penn State)

- What is a heavy quark? Why study them especially?
- What theoretical methods are used?
- What is the meaning of 3-flavor, 4-flavor (. . .) coupling and parton densities? Why?
- What are they needed for?

Material today continues Zack Sullivan's lecture.

[*Warning:* I am only selectively examining the basics of a big subject.]

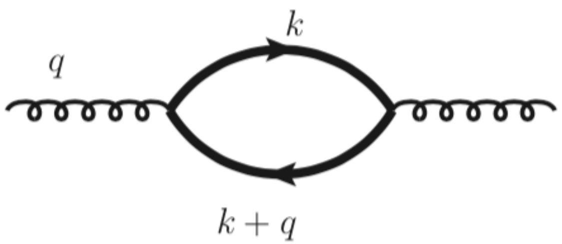
Is it really true . . .

- That effective field theory (EFT) $\widehat{\text{QCD}}_{n \text{ active flavors}}$ is obtained simply by dropping the $6 - n$ inactive flavors?
- That there is just one characteristic scale for a given process?
- That use of EFT is the best method?

Answers, with basic reasons:

- No: There are UV divergences in QFT: All scales to infinity matter. So something fancier is needed.
- No: See elastic scattering example.
- No: EFT alone is too limited . . . .

Unsuppressed effects when $M^2 \gg Q^2$ ($\overline{\mathrm{MS}}$ renormalization)



$$\propto g^2 \int \frac{\mathrm{tr}\, \gamma^\mu (\not{k} + M) \gamma^\nu (\not{q} + \not{k} + M)}{(k^2 - M^2)\ [(k+q)^2 - M^2]} \frac{\mathrm{d}^4 k}{(2\pi)^4} + \text{c.t.}$$

$$\propto (q^2 g^{\mu\nu} - q^\mu q^\nu) \frac{\alpha_s}{\pi} \int_0^1 x(1-x) \ln \frac{M^2 - q^2 x(1-x)}{\mu^2}\, \mathrm{d}x$$

$$= (q^2 g^{\mu\nu} - q^\mu q^\nu)\ \frac{\alpha_s}{6\pi} \ln \frac{M^2}{\mu^2} + \text{power-suppressed}$$

when $|q^2| \ll M^2$. This is *not* suppressed when $M^2 \gg |q^2|$.

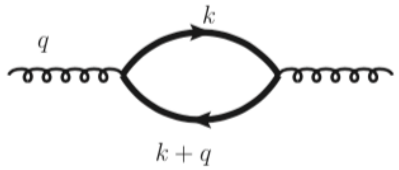
Add in light-quark graph. Mass m, with $m^2 \ll |q^2|$:

$$(q^2 g^{\mu\nu} - q^\mu q^\nu)\ \frac{\alpha_s}{6\pi} \left[\ln \frac{q^2}{\mu^2} + \text{constant} \right]$$

So no single choice of $\overline{\mathrm{MS}}$ μ eliminates large logarithms for sum of both heavy and light-quark graphs when $m^2 \ll |q^2| \ll M^2$.

First step: CWZ (Collins-Wilczek-Zee) for renormalization, coupling

Stay in full theory, but for "inactive" quarks, use zero-momentum subtraction:



$$+\ \text{c.t.} \ \propto \ (q^2 g^{\mu\nu} - q^\mu q^\nu) \frac{\alpha_s}{\pi} \int_0^1 x(1-x) \ln \frac{M^2 - q^2 x(1-x)}{M^2} \, \mathrm{d}x$$

$$= (q^2 g^{\mu\nu} - q^\mu q^\nu) \, \frac{\alpha_s}{6\pi} \, O\!\left(\frac{q^2}{M^2} \right) \qquad \text{when } |q^2| \ll M^2$$

Use $\overline{\mathrm{MS}}$ for everything else.

Key properties:

- Single theory QCD_6
- "Manifest decoupling"
- Automatically preserves gauge-invariance of QCD
- RG and DGLAP equations are same (mass-independent) as in the EFT approach.

Statement of CWZ

Technical definition:

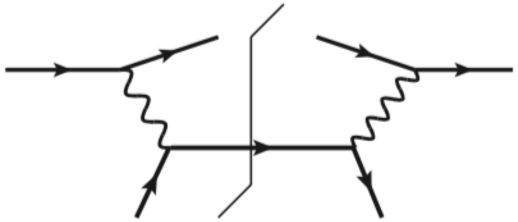
- Keep all (known or relevant) quarks in theory
- Define a sequence of subschemes with 3, 4, 5, etc "active" flavors. [(u, d, s), (u, d, s, c), etc]
- $\overline{\mathrm{MS}}$ for active flavors, zero-momentum subtraction for graphs with inactive flavors.
- Obtain relations of coupling, etc between subschemes by matching

Adjust choice of # of active flavors by the following principles:

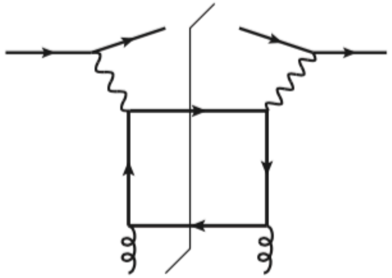
- At scale Q, quarks with $M \ll Q$ are active.
- Quarks with $M \gg Q$ are inactive.
- Overlapping ranges of usefulness for $m \sim Q$.

Overview of charm in DIS at $Q \gg$ few GeV, 4 active flavors

- Factorization, pdfs, etc:
  - Standard treatment of factorization says we need c quark as parton, since it can have collinear kinematics.
  - So we include c pdf term

$f_c \otimes$  $\Big|_{\text{on-shell quark}}$

  - Also have subtracted photon-gluon-fusion term, as usual:

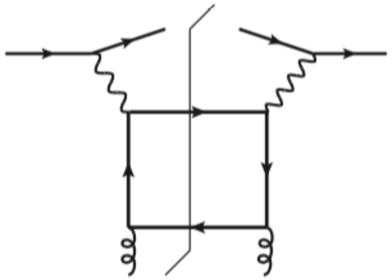
$f_g \otimes \Big[$  $+$ etc $-$ subtraction for c-in-g pdf $\Big]_{\text{gluon on-shell}}$

  - [Other subprocesses, NLO, NNLO, . . .]
  - Can keep m_c in hard scattering, for initial g, u, d, s.

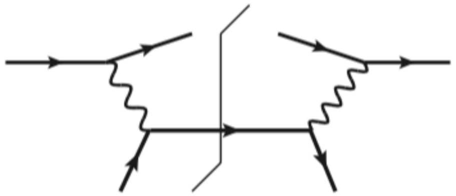
- Value of charm density: Perturbative estimate in terms of gluon density (etc).

4 active flavors: subtraction in γg hard scattering

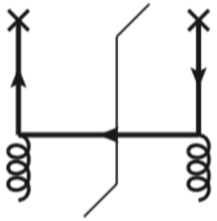
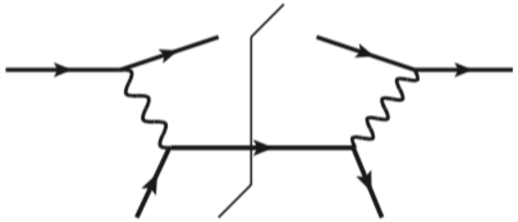
Subtraction in γg hard scattering,

$\Bigg[$  $+$ etc $-$ subtraction for c-in-g pdf $\Bigg]_{\text{gluon on-shell}}$

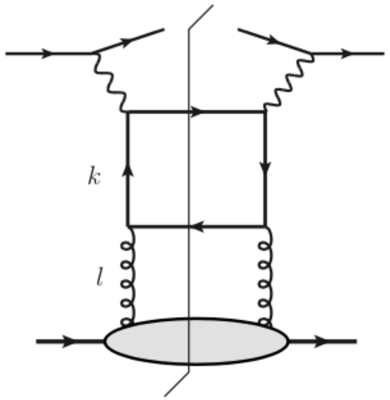
is to stop double counting of contribution included in LO term

$f_c \otimes$  $\Big|_{\text{on-shell quark}}$

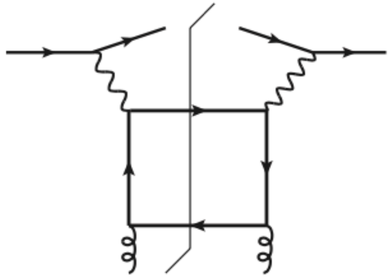
Subtraction term in order α_s photon-gluon hard scattering is

 $\Big|_{\text{gluon on-shell}}$ $\otimes$  $\Big|_{\text{on-shell quark}}$

Charm in DIS at $Q =$ few GeV: 3 active flavors



$+ \text{ etc } = f_g \otimes \Big[$



$+ \text{ etc} \Big]_{\text{gluon on-shell}}$

- Motivation for use of LO scattering on c quark lost. Therefore omit.
- Then charm generated dynamically in hard scattering only
- No gluon-to-$c\bar{c}$ collinear region nor divergence.
- So, there is no subtraction in hard scattering, unlike light-quark case

ACOT: To do this consistently, use 3-flavor CWZ including for pdfs.

In particular, would-be subtraction by c in gluon is zero. Generally $f^{(3)}_{c/p}$ is power suppressed by power of Λ/m_c.

ACOT implementation: Apply CWZ idea to pdfs and factorization, etc

3-flavor	Evolution: u, d, s only Usual 3-flavor DGLAP	c pdf suppressed by $(\Lambda/m_c)^p$, and not used
4-flavor	Evolution: u, d, s, c Usual 4-flavor DGLAP Start $f_{c/p}^{(4)}$ at $\mu \simeq m_c$ from calculated matching to $f_{c/p}^{(3)}$	Usually neglect $f_{c/p}^{(3)}$ in matching. (*Pace* Brodsky & intrinsic charm).

ETC

Summary

Basics:

- Heavy quarks, i.e., with masses in perturbative region, allow simplifications, and extra perturbative predictions c.w. light quarks.
- Simplest methods involve decoupling theorem and EFTs
- Fancier methods (CWZ/ACOT) allow keeping heavy quarks in the theory, without penalty of large logarithms in calculations
- Get concept of number of "active" partonic quarks
- See the vast literature for a range of views

But we need more work:

- Interesting processes have lots of different scales. E.g., $\sqrt{s}$, $P_{T,\text{jet}}$, jet width, relative momenta of components of events.
- Measurement of heavy hadrons (e.g., D-meson) in final state messes up rationale of ACOT, when heavy hadron is not strongly relativistic (i.e., not in a jet).

Thanks!