Some applications of nonlocal chiral effective Lagrangian

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Perturbative chiral quark model

$$\mathcal{L}_{\rm inv}(x) = \bar\psi(x)[i\,\partial\!\!\!/ - \gamma^0 V(r)]\psi(x) + \frac{1}{2}[D_\mu \Phi_i(x)]^2 - S(r)\bar\psi(x)\exp\left[i\gamma^5 \frac{\hat\Phi(x)}{F}\right]\psi(x),$$

$$\mathcal{L}_{\chi SB}(x) = -\bar\psi(x)\mathcal{M}\psi(x) - \frac{B}{2}{\rm Tr}\Big[\hat\Phi^2(x)\,\mathcal{M}\Big],$$

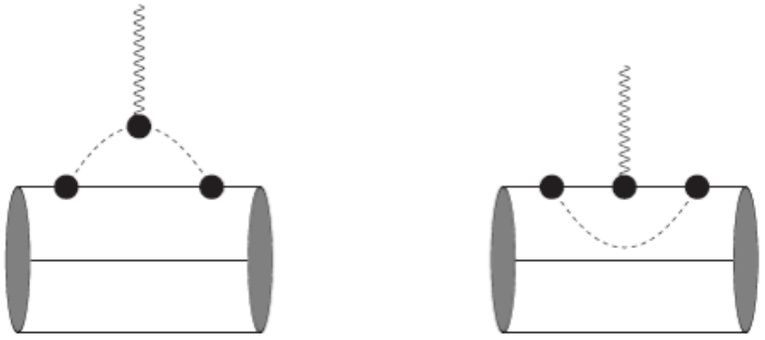
$$\left[-i\gamma^0\vec\gamma\cdot\vec\nabla + \gamma^0 S(r) + V(r) - \mathcal{E}_\alpha\right] u_\alpha(\vec x) = 0$$

$$u_0(\vec x) \;=\; N\,\exp\Big[-\frac{\vec x^{\,2}}{2R^2}\Big]\begin{pmatrix} 1 \\ i\rho\,\vec\sigma\vec x/R \end{pmatrix}$$

$$\psi(x) = \sum_\alpha b_\alpha u_\alpha(\vec x)\exp(-i\mathcal{E}_\alpha t) + \sum_\beta d^\dagger_\beta v_\beta(\vec x)\exp(i\mathcal{E}_\beta t)$$

$$iG_\psi(x,y) \to iG_0(x,y) \doteq u_0(\vec x)\,\bar u_0(\vec y)\,e^{-i\mathcal{E}_\alpha(x_0-y_0)}\,\theta(x_0-y_0),$$

Nucleon electromagnetic form factors



$$G_E^N(Q^2)\Big|_{MC} = \frac{9}{400}\left(\frac{g_A}{\pi F}\right)^2 \int_0^\infty dp\, p^2 \int_{-1}^{1} dx\,(p^2 + p\sqrt{Q^2}x)$$

$$\times \mathcal{F}_{\pi NN}(p^2,Q^2,x)\, t_E^N(p^2,Q^2,x)\Big|_{MC},$$

$$t_E^p(p^2)|_{VC} = \frac{1}{2}W_\pi(p^2) - W_K(p^2) + \frac{1}{6}W_\eta(p^2),$$

$$t_E^n(p^2)|_{VC} = W_\pi(p^2) - W_K(p^2),$$

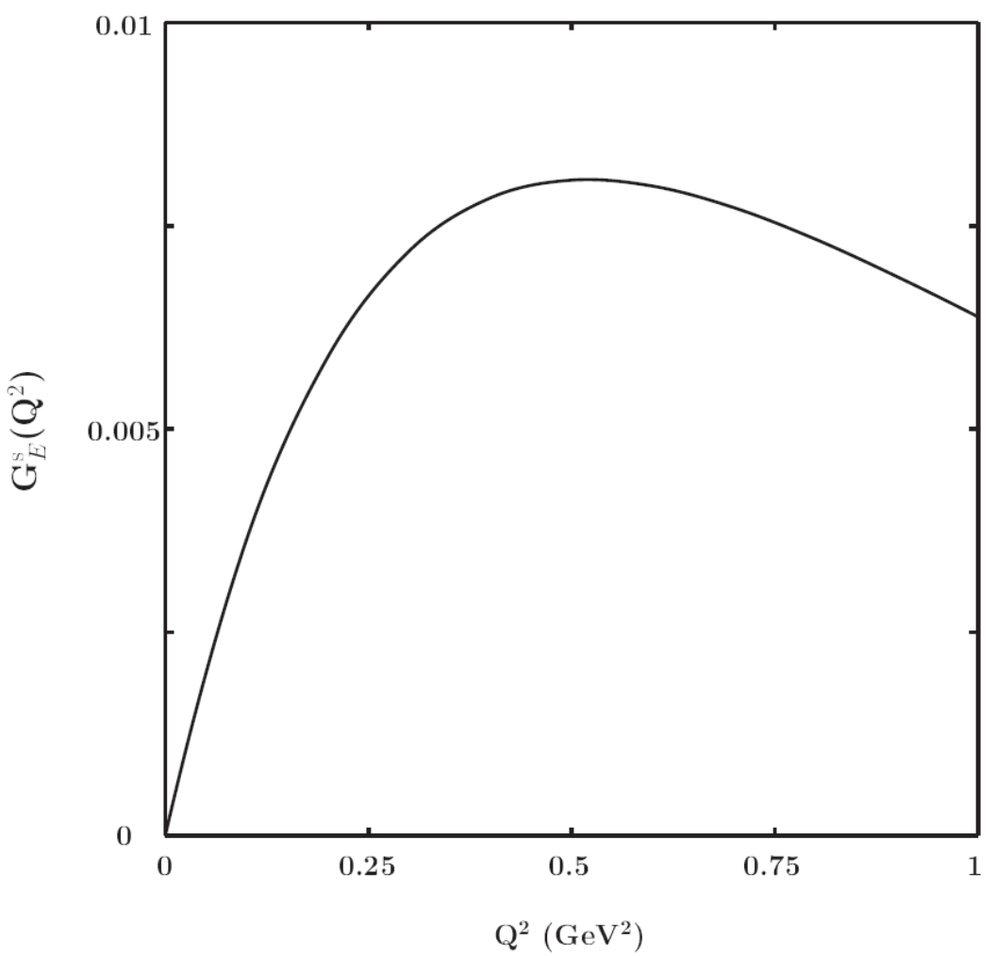
$$W_\Phi(p^2) = \frac{1}{w_\Phi^3(p^2)}$$

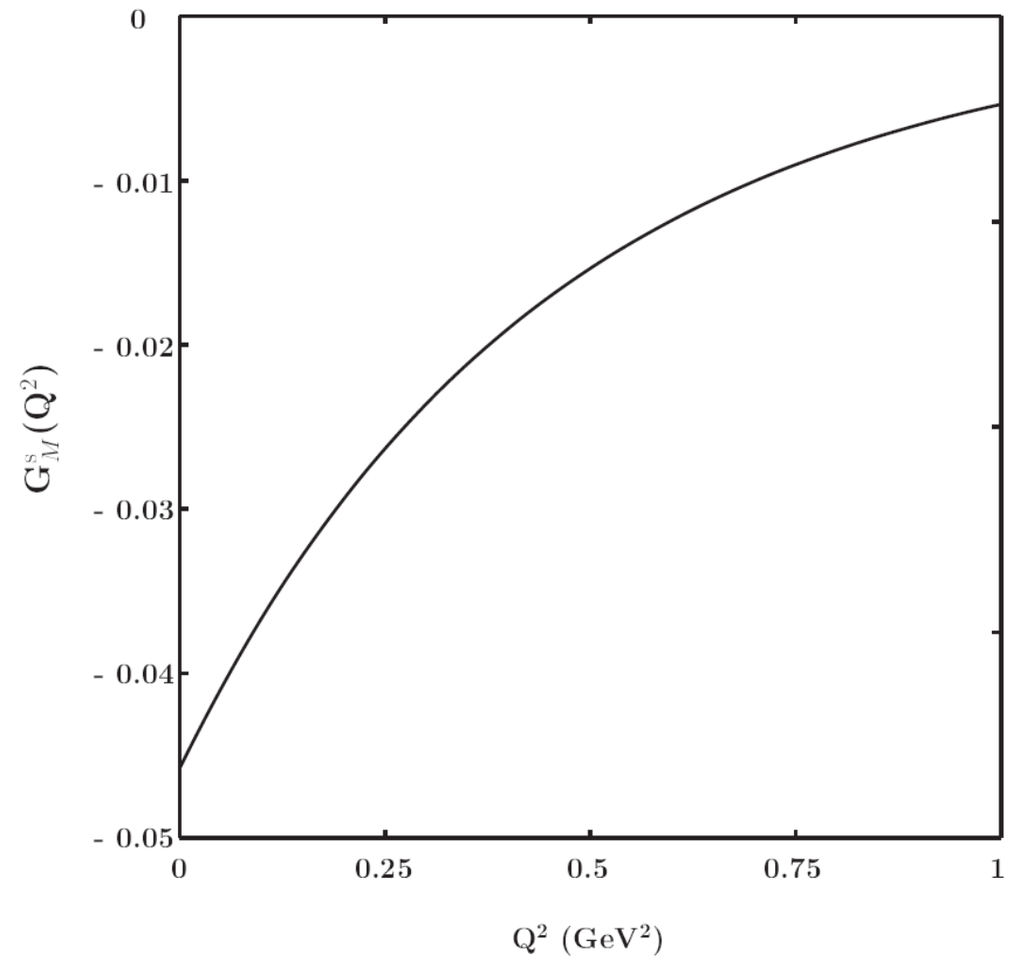
$$\mathcal{F}_{\pi NN}(p^2,Q^2,x) = F_{\pi NN}(p^2)F_{\pi NN}(p^2 + Q^2 + 2p\sqrt{Q^2}x),$$

$$F_{\pi NN}(p^2) = \exp\left(-\frac{p^2R^2}{4}\right)\left\{1 + \frac{p^2R^2}{8}\left(1 - \frac{5}{3g_A}\right)\right\}$$

Strange quark contribution to the form factor

$$< B(p')|J_\mu|B(p) >= \bar{u}(p') \left\{ v_\mu G_E(Q^2) + \frac{i\epsilon_{\mu\nu\alpha\beta} v^\alpha S_v^\beta q^\nu}{m_N} G_M(Q^2) \right\} u(p)$$



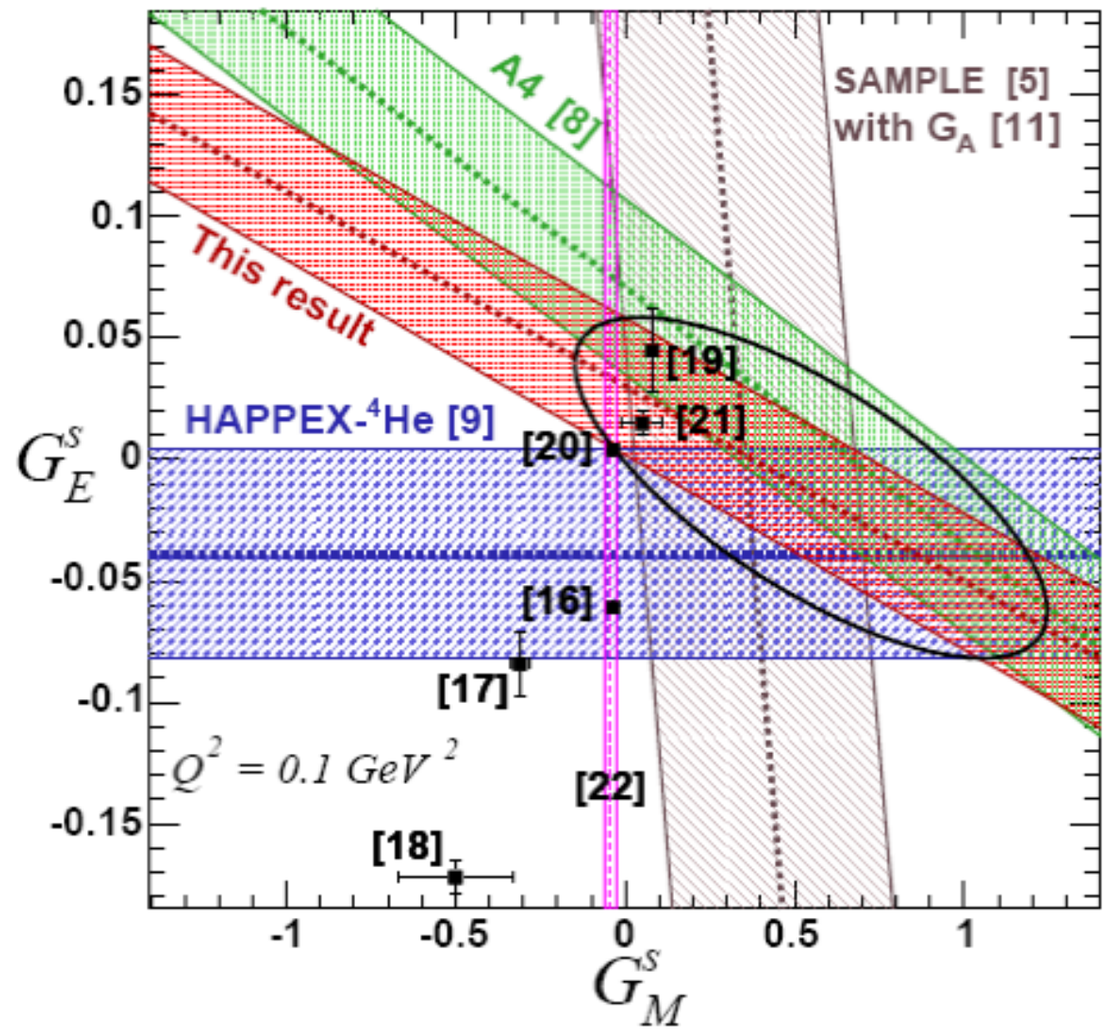


V. Lyubovitskij, P. Wang, T. Gutsche, A. Faessler, Phys. Rev. C 66 (2002) 055204

Approach	μ^s (n.m.)	$< r^2 >_E^s$ (fm^2)	$< r^2 >_M^s$ (fm^2)	ρ^s	$\rho^s + \mu_p \mu^s$
QCD equalities [7]	-0.75 ± 0.30				
Lattice QCD [8]	-0.16 ± 0.18				
Lattice QCD [9]	-0.36 ± 0.20	-0.16 ± 0.06		2.02 ± 0.75	1 ± 0.75
Lattice QCD [10]	-0.28 ± 0.10				
HBChPT [12]	0.18 ± 0.34	0.05 ± 0.09	-0.14		
Poles [13]	-0.31 ± 0.09	0.14 ± 0.07		-2.1	-2.97
Poles [14]	-0.185 ± 0.075	0.14 ± 0.06		-2.93	-3.60
Poles [15]	-0.24 ± 0.03				
Kaon loop [16]	-0.355 ± 0.045	-0.0297 ± 0.0026			
Kaon loop + VMD [17]	-0.28 ± 0.04	-0.0425 ± 0.0026			
Skyrme model [19]	-0.13	-0.11		1.64	1.27
NJL soliton model [20]	0.10 ± 0.15	-0.15 ± 0.05		3.06	2.92
χQSM [21]	0.115	-0.095	0.073		
CBM [22]	0.37				
CQM [23]	~ -0.05				
CQM [24]	-0.046		~ 0.02		
PCQM	-0.048 ± 0.012	-0.011 ± 0.003	-0.024 ± 0.003	0.17 ± 0.04	0.05

u/d quark in Σ /Σ

HAPPEX Collaboration
Phys. Lett. B635 (2006) 275



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[8] F. E. Mass *et al*, Phys. Rev. Lett. 94 (2005) 152001
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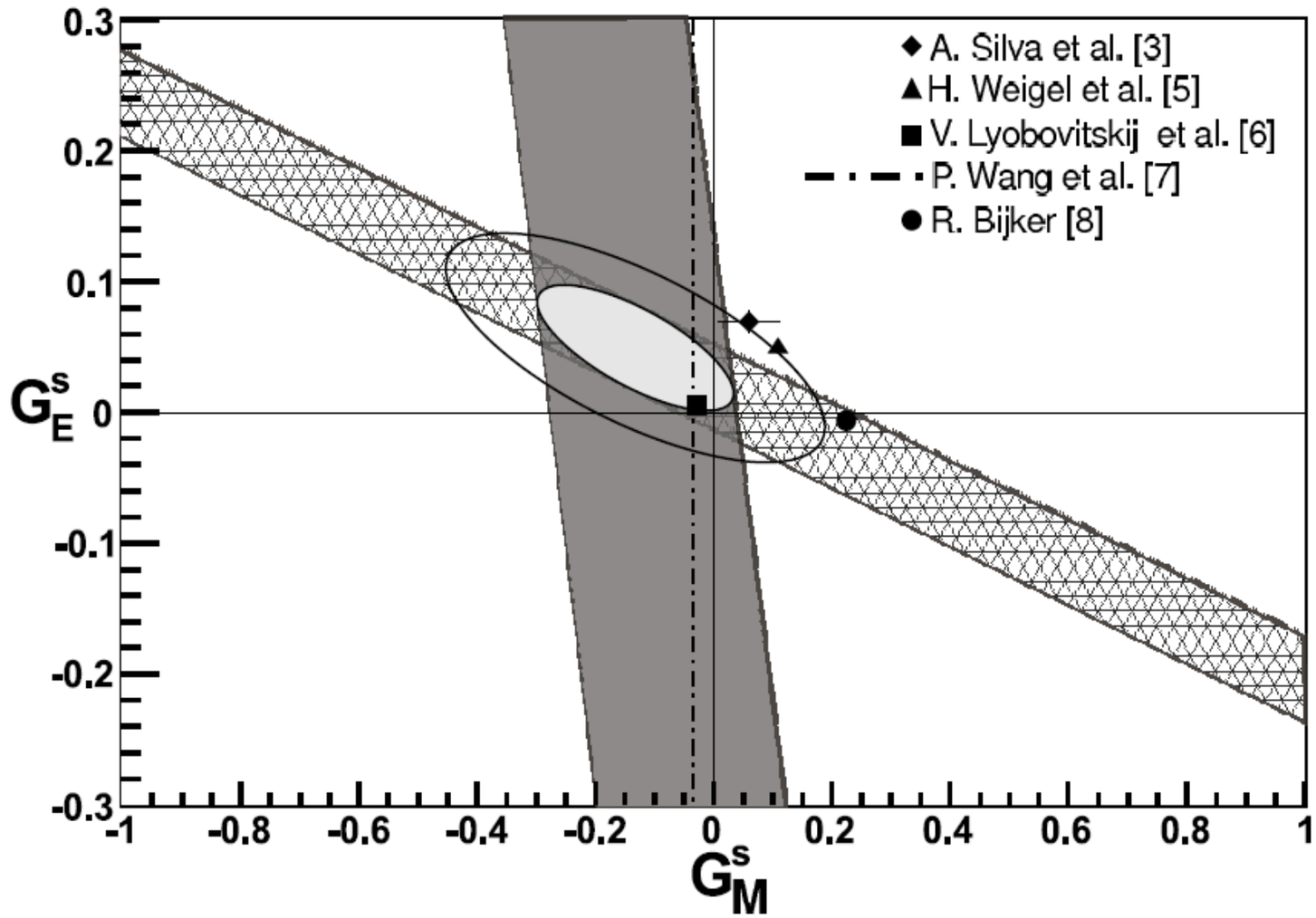

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Non-local quark-meson coupling model

$$\mathcal{L}_{\rm int}^{\rm str}(x) = g_H H(x) \int dx_1 \int dx_2 F_H(x, x_1, x_2) \bar{q}_2(x_2) \Gamma_H \lambda_H q_1(x_1)$$

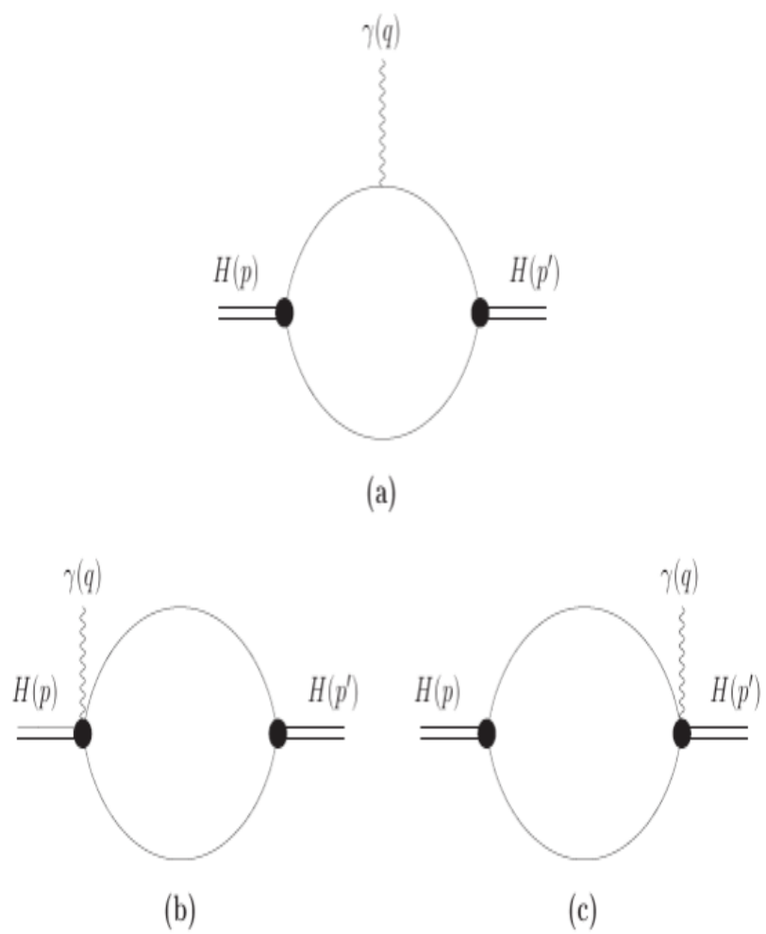
$$F_H(x, x_1, x_2) = \delta(x - w_{21} x_1 - w_{12} x_2) \Phi_H((x_1 - x_2)^2)$$

$$\begin{aligned}\mathcal{L}_{\rm int}^{\rm em(1)}(x) &= e\bar{q}(x) \not{A} Q\, q(x) \\ &+ ieA_\mu(x) \left(H^-(x) \partial^\mu H^+(x) - H^+(x) \partial^\mu H^-(x) \right) + e^2 A_\mu^2(x) H^-(x) H^+(x)\end{aligned}$$

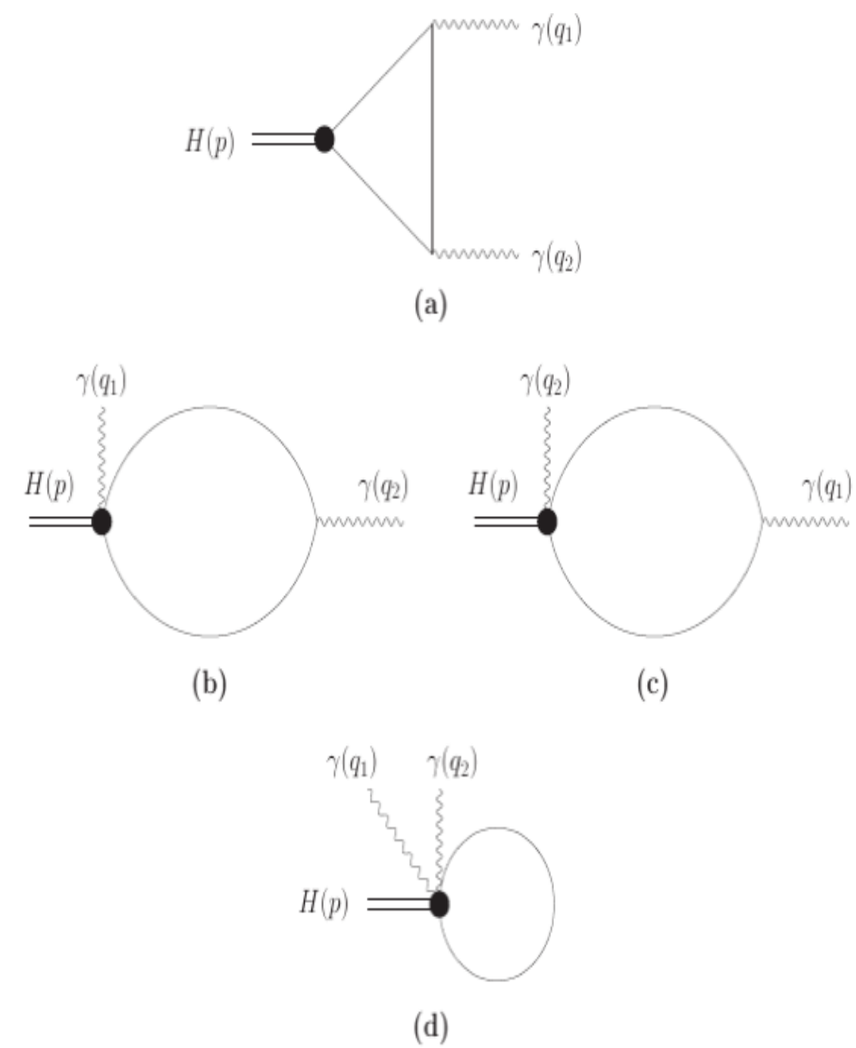
$$\begin{aligned}\mathcal{L}_{\rm int}^{\rm str+em(2)}(x) = g_H H(x) &\int dx_1 \int dx_2 F_H(x, x_1, x_2) \bar{q}_2(x_2)\, e^{ie_{q_2} I(x_2, x, P)} \\ &\times \Gamma_H \lambda_H\, e^{-ie_{q_1} I(x_1, x, P)}\, q_1(x_1),\end{aligned}$$

$$I(x_i, x, P) = \int_x^{x_i} dz_\mu A^\mu(z)$$

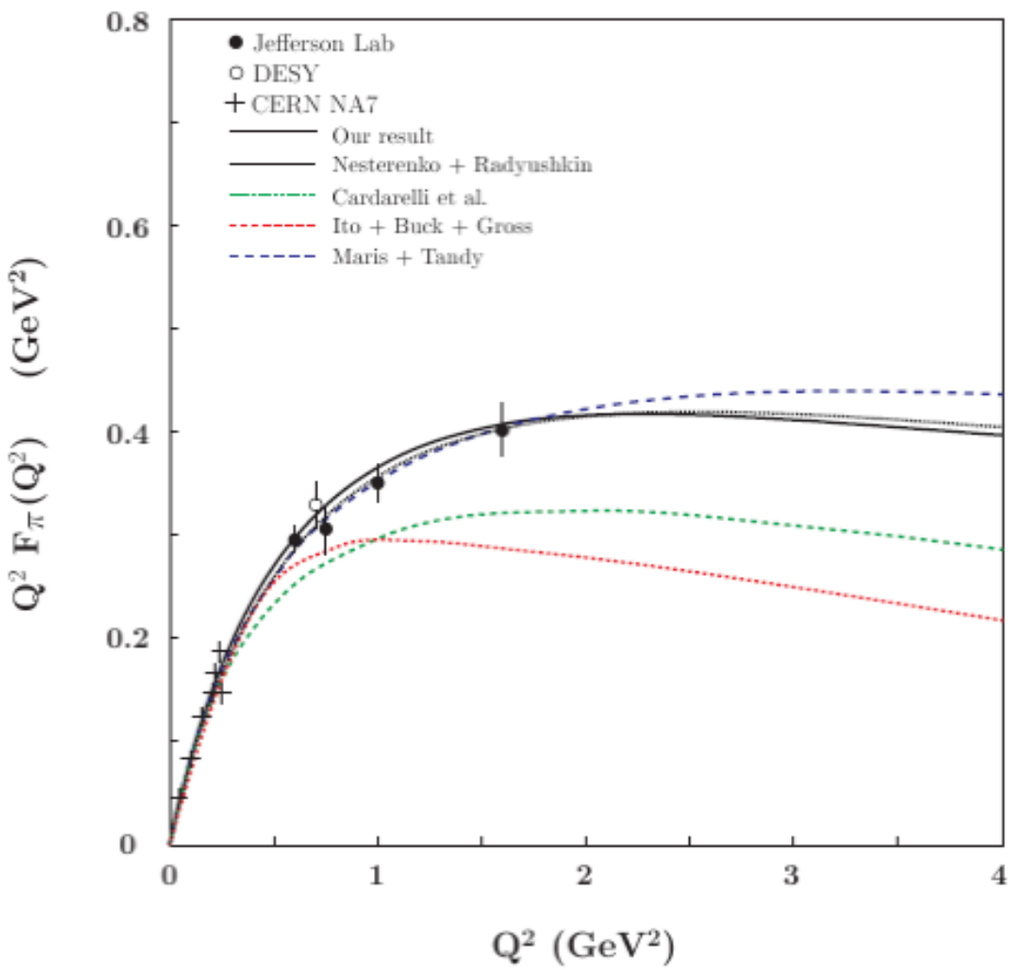
pion form factor

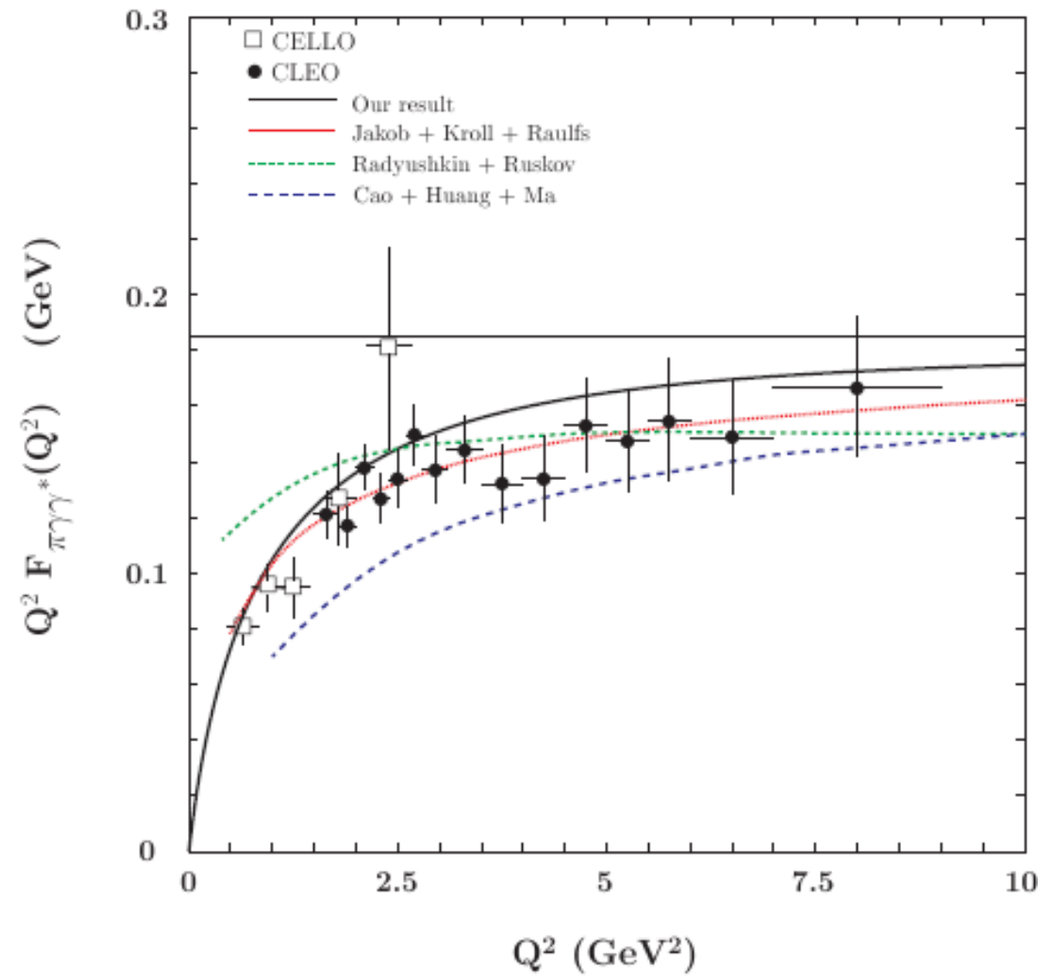


pion —> 2 gama



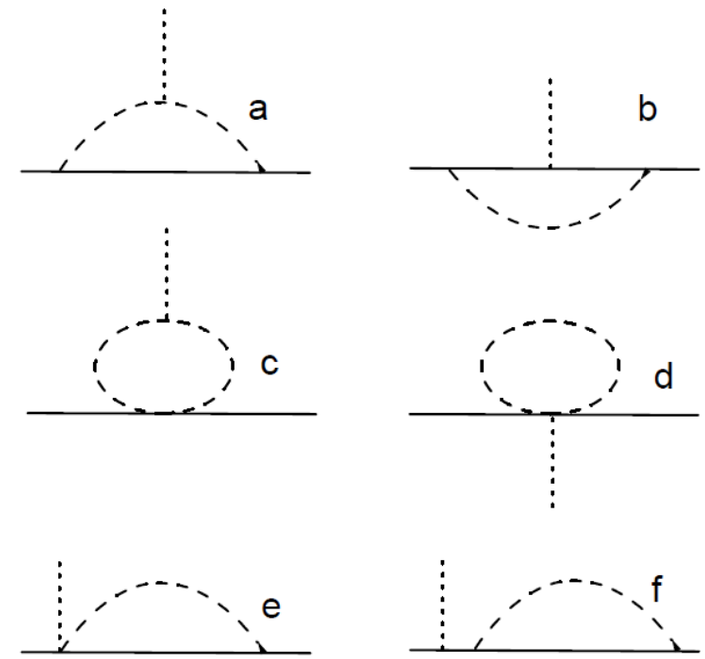
$$I^{\mu}_{\triangle_\perp}(p,p') = \int \frac{d^4k}{4\pi^2 i}\,\tilde{\Phi}\Big(-\Big[k+\frac{p}{2}\Big]^2\Big)\,\tilde{\Phi}\Big(-\Big[k+\frac{p'}{2}\Big]^2\Big)\,\mathrm{tr}[\gamma^5 S(k+p')\gamma^{\mu}_{\perp;q} S(k+p)\gamma^5 S(k)]$$





A. Faessler, T. Gutsche, M.A. Ivanov, Valery E. Lyubovitskij and P. Wang, Phys. Rev. D 68 (2003) 014011

Finite-Range-Regularization



$$\mathcal{G}^v_{2,0}(Q^2, m_\pi^2) = Z(g^v_{2,0} + g^v_\pi m_\pi^2 + (g^v_t + g^v_{\pi,t} m_\pi^2)Q^2) + \sum_{i=a}^{f} \mathcal{G}^{v,i}_{2,0}$$

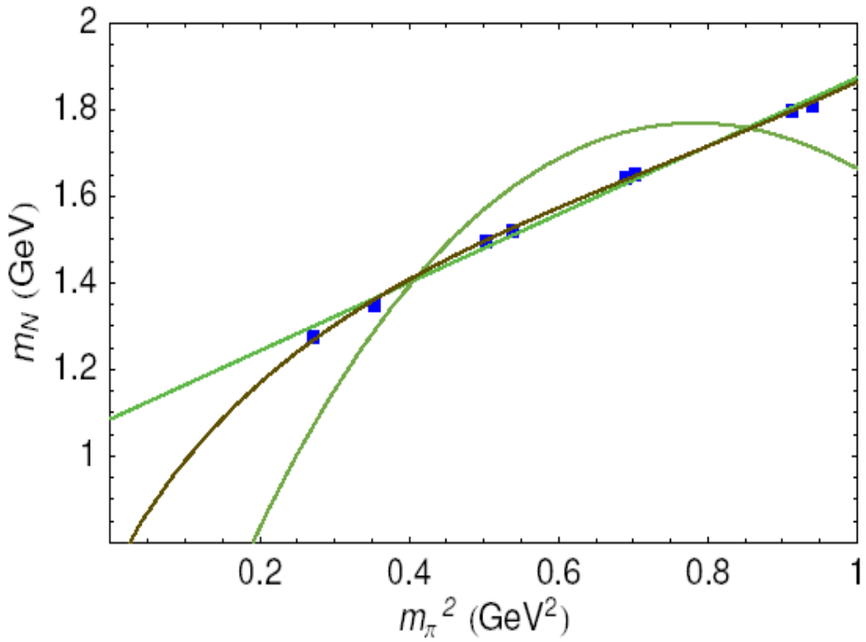
$$\mathcal{A}^{v,b+f}_{2,0} = Z a^v_{2,0} - \frac{g_A^2 a^v_{2,0}}{64\pi^3 F_\pi^2} \int d\vec{k} \frac{\vec{k}^2 u^2(\vec{k})}{\omega^3(\vec{k})} - \frac{5\mathcal{C}^2 a^v_{2,0}}{72\pi^3 F_\pi^2} \int d\vec{k} \frac{\vec{k}^2 u^2(\vec{k})}{\omega(\vec{k})(\omega(\vec{k}) + \Delta)^2}$$

$$Z = 1 - \frac{3g_A^2}{64\pi^3 F_\pi^2} \int d\vec{k} \frac{\vec{k}^2 \vec{k}^2 u^2(\vec{k})}{\omega^3(\vec{k})} - \frac{g_A^2}{24\pi^3 F_\pi^2} \int d\vec{k} \frac{\vec{k}^2 \vec{k}^2 u^2(\vec{k})}{\omega^3(\vec{k})}$$

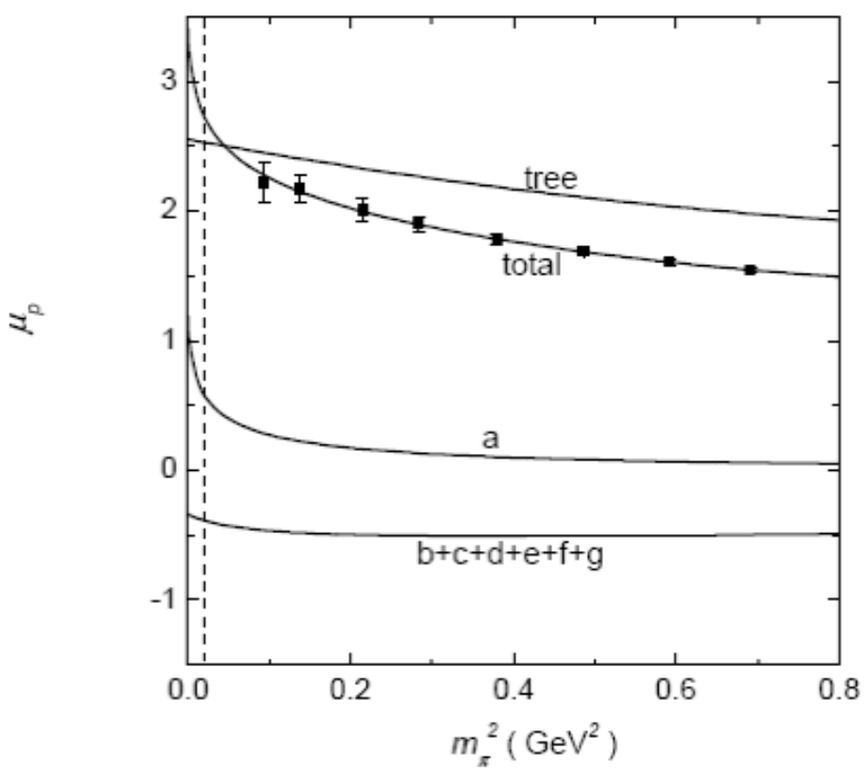
$$u(k) = \frac{1}{(1 + k^2/\Lambda^2)^2}$$

Λ = 0.8 GeV

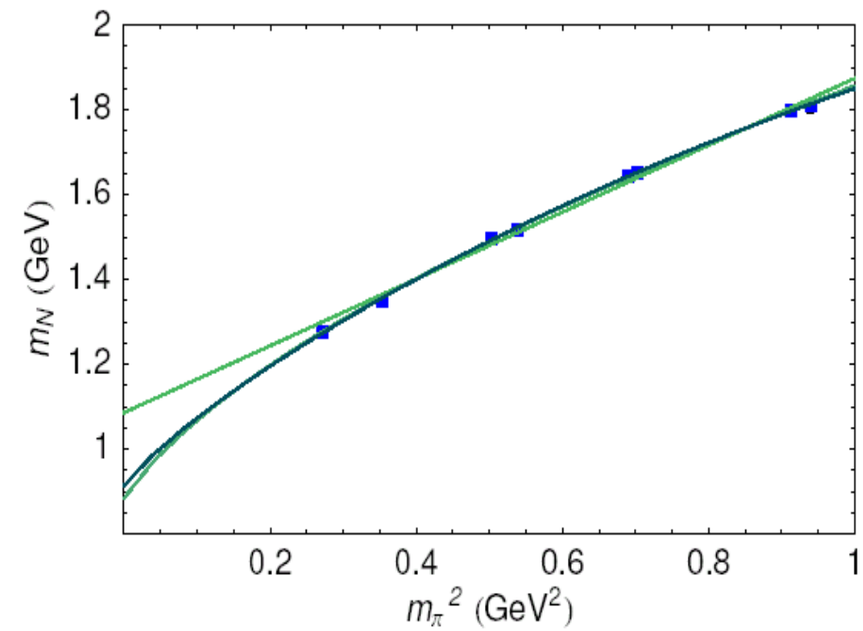
- CP-PACS collaboration results Phys. Rev. D65 (2002) 054505



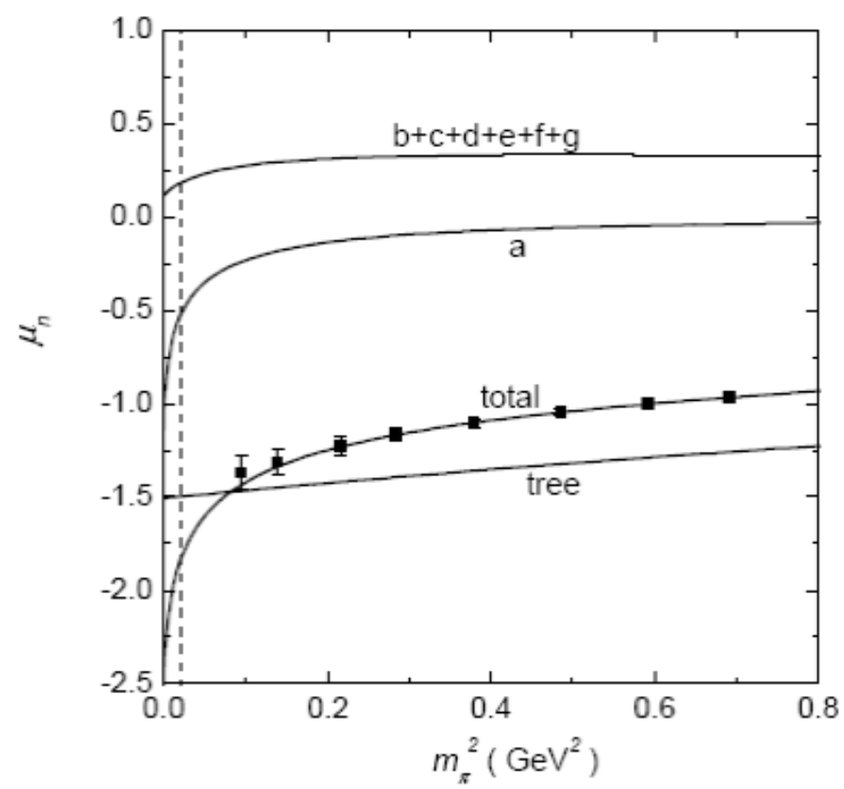
- C: $a_0 + a_2 m_\pi^2 + \chi_\pi I_\pi + a_4 m_\pi^4$



- Dipole Regularisation



- C: $a_0 + a_2 m_\pi^2 + \chi_\pi I_\pi + a_4 m_\pi^4$



Second quantization, Canonical quantization

$$
\begin{aligned}
[\phi(\vec{x},t),\phi(\vec{y},t)] &= [\pi(\vec{x},t),\pi(\vec{y},t)] = 0, \\
[\phi(\vec{x},t),\pi(\vec{y},t)] &= i\delta^{(3)}\left(\vec{x}-\vec{y}\right).
\end{aligned}
$$

$$
\phi(\vec{x},t) = \int \widetilde{dp}\left[a(\vec{p})e^{i\vec{p}\cdot\vec{x}-i\omega_p t} + a^\dagger(\vec{p})e^{-i\vec{p}\cdot\vec{x}+i\omega_p t}\right]
$$

$$
\pi(\vec{x},t) = \int \widetilde{dp}(-i)\omega_p\left[a(\vec{p})e^{i\vec{p}\cdot\vec{x}-i\omega_p t} - a^\dagger(\vec{p})e^{-i\vec{p}\cdot\vec{x}+i\omega_p t}\right]
$$

$$
\begin{aligned}
[a(\vec{p}),a(\vec{q})] &= \left[a^\dagger(\vec{p}),a^\dagger(\vec{q})\right] = 0, \\
\left[a(\vec{p}),a^\dagger(\vec{q})\right] &= (2\pi)^3 2\omega_p \delta^{(3)}(\vec{p}-\vec{q})
\end{aligned}
$$

$$
\Delta_F(x'-x) = \langle 0|T\phi(x')\phi(x)|0\rangle = \int \frac{d^4k}{(2\pi)^4}\frac{i}{k^2-m^2+i\epsilon}e^{-ik\cdot(x'-x)}
$$

Path integral method

$$Z_0(J) = \int \mathcal{D}\phi e^{i\int d^4x[\mathcal{L}_0 + J\phi]}$$

$$\mathcal{L}_0 = -\frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{2}m^2\phi^2$$

$$\langle 0|T\{\phi(x_1)\phi(x_2)\}|0\rangle = \frac{1}{Z_0}\left(-i\frac{\delta}{\delta J(x_1)}\right)\left(-i\frac{\delta}{\delta J(x_2)}\right)Z[J]\Big|_{J=0}$$

o

Ultraviolet Divergence, Dimensional Regularization

$$\begin{aligned}
\int \frac{d^dp}{(2\pi)^d}\frac{1}{(p^2+m^2)^n} &= i\frac{1}{(4\pi)^{d/2}}\frac{\Gamma(n-d/2)}{\Gamma(n)}\left(\frac{1}{m^2}\right)^{n-d/2} \\
\int \frac{d^dp}{(2\pi)^d}\frac{(p^2)^k}{(p^2+m^2)^n} &= i\frac{1}{(4\pi)^{d/2}}\frac{\Gamma(n-d/2-k)\Gamma(d/2+k)}{\Gamma(d/2)\Gamma(n)}\left(\frac{1}{m^2}\right)^{n-d/2-k} \\
\int \frac{d^dp}{(2\pi)^d}\frac{p^\mu p^\nu}{(p^2+m^2)^n} &= i\frac{1}{(4\pi)^{d/2}}\frac{g^{\mu\nu}}{2}\frac{\Gamma(n-d/2-1)}{\Gamma(n)}\left(\frac{1}{m^2}\right)^{n-d/2-1} \\
\int \frac{d^dp}{(2\pi)^d}\frac{p^\mu p^\nu p^\rho p^\sigma}{(p^2+m^2)^n} &= i\frac{1}{(4\pi)^{d/2}}\frac{\Gamma(n-d/2-2)}{\Gamma(n)}\left(\frac{1}{m^2}\right)^{n-d/2-2} \\
&\quad \cdot\frac{1}{4}\left(g^{\mu\nu}g^{\rho\sigma} + g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho}\right)
\end{aligned}$$

Solid quantization

$$
\begin{aligned}
[\phi(\vec{x},t),\phi(\vec{y},t)] &= [\pi(\vec{x},t),\pi(\vec{y},t)] = 0, \\
[\phi(\vec{x},t),\pi(\vec{y},t)] &= i\Phi\left(\vec{x}-\vec{y}\right).
\end{aligned}
$$

$$
\phi(\vec{x},t) = \int \widetilde{dp}\left[A(\vec{p})e^{i\vec{p}\cdot\vec{x}-i\omega_p t} + A^\dagger(\vec{p})e^{-i\vec{p}\cdot\vec{x}+i\omega_p t}\right]
$$

$$
\begin{aligned}
[A(\vec{p}),A(\vec{q})] &= \left[A^\dagger(\vec{p}),A^\dagger(\vec{q})\right] = 0, \\
\left[A(\vec{p}),A^\dagger(\vec{q})\right] &= (2\pi)^3\delta^{(3)}(\vec{p}-\vec{q})\Psi(\vec{p})
\end{aligned}
$$

$$
\Phi\left(\vec{x}\right) = \int \frac{d^3p}{(2\pi)^3}\frac{\Psi(\vec{p})}{2}(e^{i\vec{p}\cdot\vec{x}} + e^{-i\vec{p}\cdot\vec{x}})
$$

$$
\Psi\left(\vec{p}\right) = \int d^3x\frac{\Phi(\vec{x})}{2}(e^{i\vec{p}\cdot\vec{x}} + e^{-i\vec{p}\cdot\vec{x}})
$$

$$
\phi(\vec{x},t) = \int \widetilde{dp}\sqrt{\Psi(\vec{p})}\left[a(\vec{p})e^{i\vec{p}\cdot\vec{x}-i\omega_p t} + a^\dagger(\vec{p})e^{-i\vec{p}\cdot\vec{x}+i\omega_p t}\right]
$$

$$
\Delta_F(x'-x) = \int \frac{d^4k}{(2\pi)^4}\frac{i\Psi(\vec{k})e^{-ik\cdot(x'-x)}}{k^2-m^2+i\epsilon}
$$

Nonlocal Lagrangian

$$\mathcal{L}(x) = \int d^3x_1 d^3x_2 \bar{\psi}(t,\vec{x}_1) e^{iI(\vec{x}_1,\vec{x})} F(\vec{x}_1 - \vec{x}_2)\gamma^\mu i D_\mu$$
$$e^{-iI(\vec{x}_2,\vec{x})}\psi(t,\vec{x}_2),$$

$$D_\mu = \partial_\mu \delta(\vec{x} - \tfrac{\vec{x}_1+\vec{x}_2}{2}) - igA_\mu(x)G(\vec{x} - \tfrac{\vec{x}_1+\vec{x}_2}{2})$$

$$I(y,x) = g\int_x^y dz_\mu A^\mu(z)G(\vec{z} - \tfrac{\vec{x}_1+\vec{x}_2}{2})$$

$$\psi(t,\vec{x}_2) \to e^{ig\theta(t,\vec{x}_2)}\psi(t,\vec{x}_2),$$
$$\bar{\psi}(t,\vec{x}_1) \to \bar{\psi}(t,\vec{x}_1)e^{-ig\theta(t,\vec{x}_1)},$$
$$A_\mu(t,\vec{x}) \to A_\mu(t,\vec{x}) + G(\vec{x} - \frac{\vec{x}_1+\vec{x}_2}{2})^{-1}\partial_\mu\theta(t,\vec{x})$$

$$\mathcal{L}_0(x) = \int d^3a \bar{\psi}(t,\vec{x} + \frac{\vec{a}}{2})\gamma^\mu i\partial_\mu \psi(t,\vec{x} - \frac{\vec{a}}{2})F(\vec{a})$$

$$\mathcal{L}_0(x) = \bar{\psi}(x)\gamma^\mu i\partial_\mu \tilde{F}(i\vec{\partial})\psi(x)$$

$$\tilde{F}(i\vec{\partial}) = \int d^3a e^{i\vec{a}\cdot i\vec{\partial}}F(\vec{a})$$

Relativistic case

$$\phi(x) = \int \frac{d^4p}{(2\pi)^4} H(p^2) \left[\alpha_p e^{-ip\cdot x} + \alpha_p^\dagger e^{ip\cdot x}\right]$$

$$\begin{aligned} [\alpha_p, \alpha_q] &= [\alpha_p^\dagger, \alpha_q^\dagger] = 0, \\ [\alpha_p, \alpha_q^\dagger] &= (2\pi)^4 \delta^{(4)}(p-q) \end{aligned}$$

$$\begin{aligned} [\phi(\vec{x},t), \pi(\vec{y},t)] &= \int \frac{d^4p}{(2\pi)^4} H^2(p^2) i p_0 (e^{i\vec{p}\cdot\vec{x}} + e^{-i\vec{p}\cdot\vec{x}}) \\ &= \int \frac{d^3p}{(2\pi)^3} \frac{i\Psi(\vec{p})}{2} (e^{i\vec{p}\cdot\vec{x}} + e^{-i\vec{p}\cdot\vec{x}}) \\ &\equiv i\Phi(\vec{x}-\vec{y}), \end{aligned} \tag{44}$$

$$\Psi(\vec{p}) = \int \frac{dp_0}{\pi} H^2(p^2) p_0$$

$$H^2(p^2) = 2\pi\delta(p^2 - m^2)$$

$$\begin{aligned} \phi(x) &= \int \frac{d^4p}{(2\pi)^4} dM^2 H(M^2) \delta(p^2 - M^2) \\ &\quad \left[\alpha_p e^{-ip\cdot x} + \alpha_p^\dagger e^{ip\cdot x}\right] \\ &= \int \frac{d^3p}{(2\pi)^4 2\omega_M} dM^2 H(M^2) \\ &\quad \left[\alpha_{\vec{p},\omega_M} e^{i\vec{p}\cdot\vec{x} - i\omega_M t} + \alpha_{\vec{p},\omega_M}^\dagger e^{-i\vec{p}\cdot\vec{x} + i\omega_M t}\right] \end{aligned}$$

$$\Delta_F(x'-x) = \int \frac{d^3k}{4(2\pi)^4 \omega_M \omega_{M'}} dM^2 dM'^2 H(M^2) H(M'^2)$$

$$\delta(\omega_{M'} - \omega_M) \left[\theta(t'-t) e^{ik\cdot(x'-x)} + \theta(t-t') e^{ik\cdot(x-x')}\right] = \int \frac{d^4k}{(2\pi)^4} \frac{dM^2}{2\pi} \frac{iH^2(M^2)}{k^2 - M^2 + i\epsilon} e^{-ik\cdot(x'-x)}$$

$$H^2(M^2) = \begin{cases} 2\pi\delta(M^2 - m^2) & \text{Point particle} \\ 2\pi[\delta(M^2 - m^2) - \delta(M^2 - \Lambda^2)] & \text{Pauli-Villars} \end{cases}$$

P. Wang, Chin. Phys. C 35 (2011) 223
P. Wang, Eur. Phys. J. A 50 (2014) 172

Nonlocal effective Lagrangian

Motivation:

Finite-range-regularisation → Relativistic regulator

Heavy baryon EFT → Covariant EFT

Gauge symmetry distroyed → Gauge symmetry restored

Ward identity, Charge conservation, Renormalised nucleon charge

Solution:

Gauge invariant nonlocal chiral effective Lagrangian

Regulator derived from the Lagrangian

Make it possible to study hadron structure at relatively large Q^2

FFs, PDFs, GPDs, TMDs, … …

Simplified version

$$\mathcal{L} = i\,Tr\,\bar{B}\gamma_\mu \not{\partial} B - m_B\,Tr\,\bar{B}B + \bar{T}^{abc}_\mu(i\gamma^{\mu\nu\alpha}D_\alpha - m_T\gamma^{\mu\nu})T^{abc}_\nu + \frac{f^2}{4}Tr\,\partial_\mu\Sigma\partial^\mu\Sigma^+ + D\,Tr\,\bar{B}\gamma_\mu\gamma_5\{A_\mu, B\}$$
$$+ F\,Tr\,\bar{B}\gamma_\mu\gamma_5[A_\mu, B] + \left[\frac{\mathcal{C}}{f}\epsilon^{abc}\bar{T}^{ade}_\mu(g^{\mu\nu} + z\gamma_\mu\gamma_\nu)B^e_c\partial_\nu\phi^d_b + H.C\right] + H\bar{T}^\mu\gamma_\nu\gamma^5 A^\nu T_\mu,$$

$$V_\mu = \frac{1}{2}(\zeta\partial_\mu\zeta^\dagger + \zeta^\dagger\partial_\mu\zeta), \qquad A_\mu = \frac{1}{2}(\zeta\partial_\mu\zeta^\dagger - \zeta^\dagger\partial_\mu\zeta)$$

$$\Sigma = \zeta^2 = e^{2i\phi/f}, \qquad f = 93\ \text{MeV}$$

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}$$

$$B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}$$

$$T_{111} = \Delta^{++}, \quad T_{112} = \frac{1}{\sqrt{3}}\Delta^+, \quad T_{122} = \frac{1}{\sqrt{3}}\Delta^0,$$
$$T_{222} = \Delta^-, \quad T_{113} = \frac{1}{\sqrt{3}}\Sigma^{*,+}, \quad T_{123} = \frac{1}{\sqrt{6}}\Sigma^{*,0},$$
$$T_{223} = \frac{1}{\sqrt{3}}\Sigma^{*,-}, \quad T_{133} = \frac{1}{\sqrt{3}}\Xi^{*,0}, \quad T_{233} = \frac{1}{\sqrt{3}}\Xi^{*,-}, \quad T_{333} = \Omega^-.$$

Local interaction including pi meson

$$\mathcal{L}_\pi{}^{local} = \int dx \frac{D+F}{\sqrt{2}f} \bar{p}(x)\gamma^\mu\gamma_5\, n(x)(\partial_\mu + ie\,\mathscr{A}_\mu(x))\pi^+(x)$$

Corresponding nonlocal Lagrangian

$$\mathcal{L}_\pi{}^{nl} = \int dx \int dy \frac{D+F}{\sqrt{2}f} \bar{p}(x)\gamma^\mu\gamma_5 n(x) F(x-y) \exp[ie \int_x^y dz_\nu \int da\, \mathscr{A}^\nu(z-a)F(a)]$$
$$\times (\partial_\mu + ie \int da\, \mathscr{A}_\mu(y-a)F(a))\pi^+(y),$$

Local EM interaction

$$\mathcal{L}_{EM}^{local} = -\, e\bar{p}(x)\gamma^\mu p(x)\mathscr{A}_\mu(x) + \frac{(c_1-1)e}{4m_N}\bar{p}(x)\sigma^{\mu\nu}p(x)F_{\mu\nu}(x)$$

Corresponding nonlocal EM interaction

$$\mathcal{L}_{EM}^{nl} = -e \int da \bar{p}(x)\gamma^\mu p(x)\mathscr{A}_\mu(x-a)F_1(a) + \frac{(c_1-1)e}{4m_N}\int da \bar{p}(x)\sigma^{\mu\nu}p(x)F_{\mu\nu}(x-a)F_2(a)$$

EM currents with pi meson:

$$\mathcal{L}^{nor} = ie \int dx \int dy \frac{D+F}{\sqrt{2}f}\bar{p}(x)\gamma^\mu\gamma_5 n(x)F(x-y)\pi^+(y) \int da\, \mathscr{A}_\mu(y-a)F(a)$$

$$\mathcal{L}^{add} = ie \int dx \int dy \frac{D+F}{\sqrt{2}f}\bar{p}(x)\gamma^\mu\gamma_5 n(x)F(x-y) \int_x^y dz_\nu \int da\, \mathscr{A}^\nu(z-a)F(a)\partial_\mu\pi^+(y)$$

The Lagrangian is gauge invariant under the following transformation:

$$\pi^+(y) \to e^{i\alpha(y)}\pi^+(y), \quad p(x) \to e^{i\alpha(x)}p(x), \quad \mathscr{A}_\mu(x) \to \mathscr{A}_\mu(x) - \frac{1}{e}\partial_\mu\alpha'(x)$$

$$\alpha(x) = \int da\alpha'(x-a)F(a)$$

Parameters:

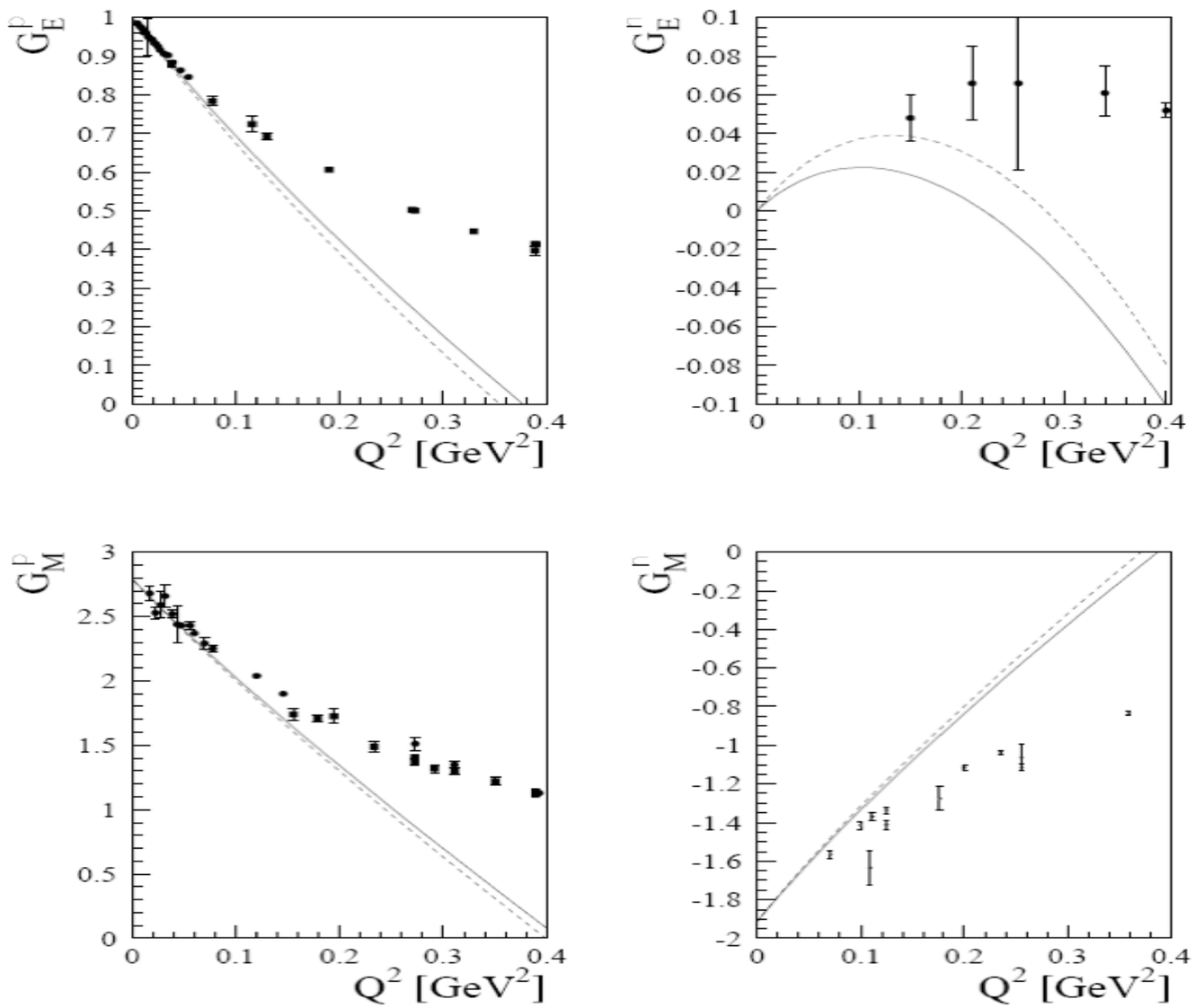
$D = 0.76$ and $F = 0.50$ ($g_A = D + F = 1.26$) $\mathcal{C} = 1$

Correlation function: $\tilde{F}(k) = \frac{1}{(1-k^2/\Lambda^2)^2}$ $\Lambda = 0.85$ GeV

c_1 , c_2 are determined by the proton and neutron magnetic moments 2.79 and -1.91.

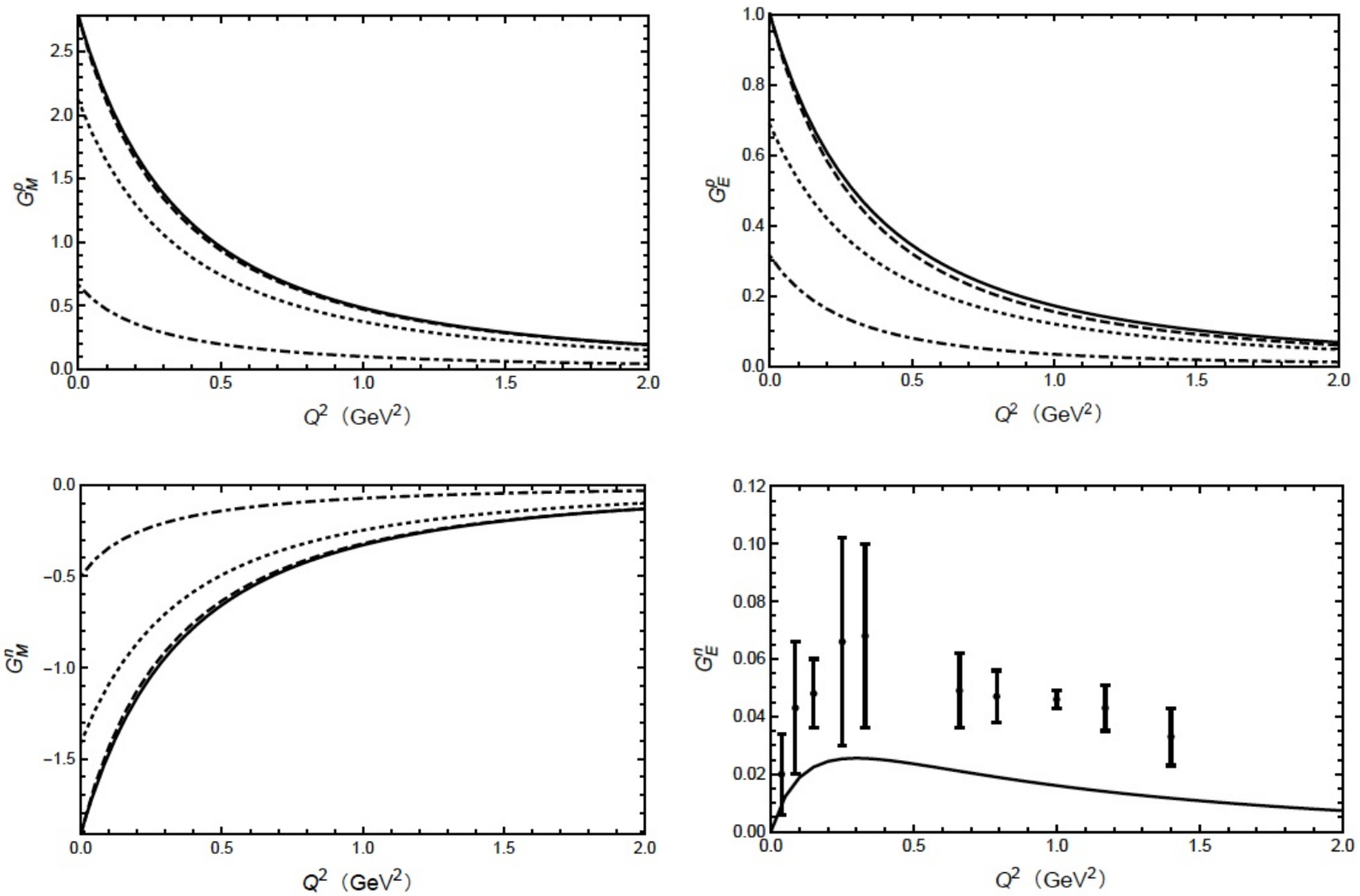
Parameters in traditional ChPT: c_1, c_2, c_3, c_4, c_5, c_6

Nucleon Form Factors



T. Fuchs, J. Gegelia, S. Scherer, J. Phys. G30 (2004) 1407

Nucleon Form Factors



$\langle r_M \rangle p$ = 0.848 (0.836) fm

$\langle r_E \rangle p$ = 0.857 (0.847) fm

$\langle r_M \rangle n$ = 0.867 (0.889) fm

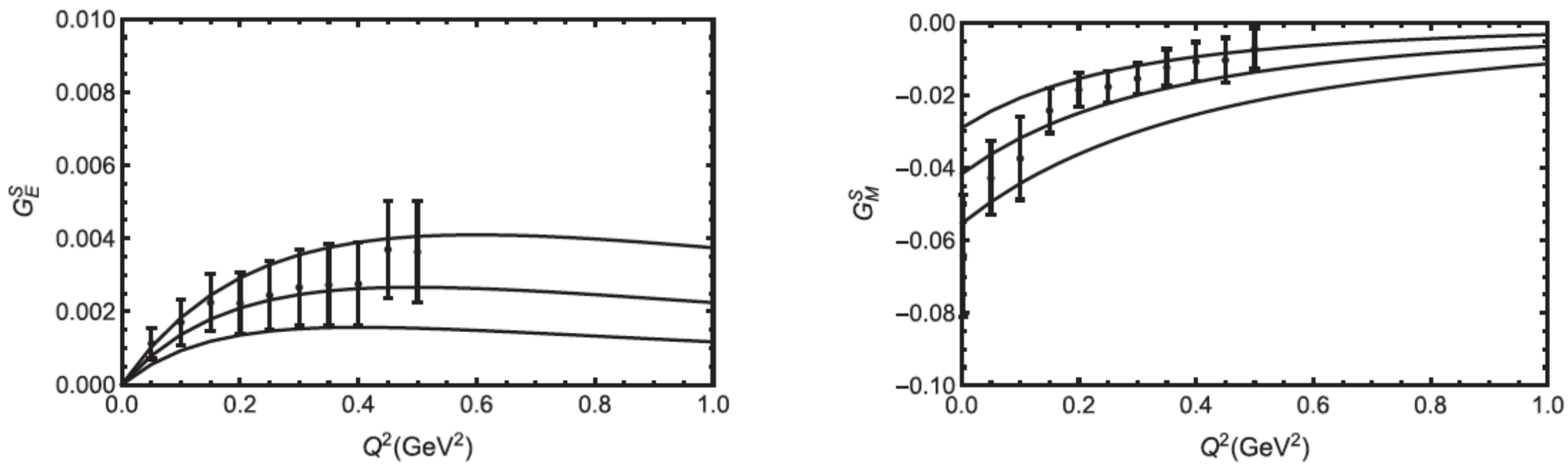
$\langle r^2_E \rangle n$ = -0.077 (-0.113) fm^2

F. C. He and P. Wang, Phys. Rev. D97 (2018) 036007

Strange form factors

$$< N(p')|J^s_\mu|N(p) >= \bar{u}(p')\left\{\gamma^\mu F^s_1(Q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m_N}F^s_2(Q^2)\right\}u(p)$$

$$G^s_E(Q^2) = F^s_1(Q^2) - \frac{Q^2}{4m_N^2}F^s_2(Q^2), \qquad G^s_M(Q^2) = F^s_1(Q^2) + F^s_2(Q^2)$$



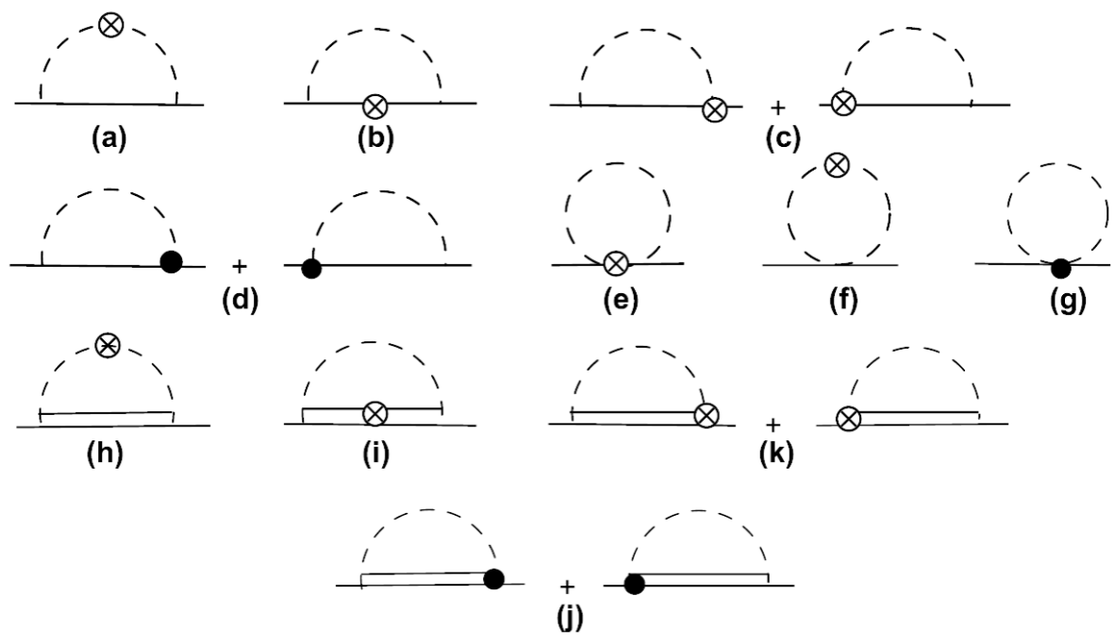
$$< (r^s_E)^2 >= -6\frac{dG^s_E(Q^2)}{dQ^2}|_{Q^2=0}, \qquad < (r^s_M)^2 >= -6\frac{dG^s_M(Q^2)}{dQ^2}|_{Q^2=0}$$

$$< (r^s_E)^2 >= -0.004 \pm 0.001 \text{ fm}^2 \text{ and } < (r^s_M)^2 >= -0.028 \pm 0.003 \text{ fm}^2$$

F. C. He and P. Wang, Phys. Rev. D98 (2018) 036007

Parton distribution functions
Splitting functions

One loop diagrams:



$$\Gamma^{\mu}_{\phi B}\,(2\pi)^4\delta^{(4)}(p-p) = \langle p|i^2\int d^4x\,d^4y\,d^4z\,\mathcal{L}^{(\mathrm{nonloc})}_{\mathrm{had}(B)}(x)\,J^{\mu}_{q,\mathrm{em}}(y)\,\mathcal{L}^{(\mathrm{nonloc})}_{\mathrm{had}(B)}(z)|p\rangle$$

$$= \frac{i^2C^2_{B\phi}}{f^2}\langle p|\int d^4x\,d^4y\,d^4z\int d^4a\,F(a)\int d^4b\,F(b)\,\bar{p}(x)\,\gamma^{\nu}\gamma^5B(x)\,\partial_{\nu}\phi(x+a)$$

$$\times\,\big(-i[\phi(y)\partial^{\mu}\phi^{\dagger}(y)-\phi^{\dagger}(y)\partial^{\mu}\phi(y)]\big)\,\bar{B}(z)\gamma^{\rho}\gamma^5p(z)\,\partial_{\rho}\phi^{\dagger}(z+b)\,|p\rangle,$$

Splitting functions

$$f_{\phi B}^{(\mathrm{rbw})}(y) = \frac{M}{p^+} \int \frac{d^4k}{(2\pi)^4} \widetilde{\mathbf{\Gamma}}_{\phi B}^{+} \, \delta\left(y - \frac{k^+}{p^+}\right)$$

Dipole form factor:

$$\widetilde{F}(k) = \left(\frac{\overline{\Lambda}}{D_\Lambda}\right)^2 \qquad D_\Lambda = k^2 - \Lambda^2 + i\varepsilon \qquad \overline{\Lambda}^2 \equiv \Lambda^2 - m_\phi^2$$

(a)

$$f_{\phi B}^{(\mathrm{rbw})}(y) = \frac{iC_{B\phi}^2 \overline{\Lambda}^8}{f^2} \int \frac{d^4k}{(2\pi)^4} \left[\frac{y\overline{M}^2(\Delta^2 - m_\phi^2)}{D_\phi^2 \, D_B \, D_\Lambda^4} - \frac{y\overline{M}^2}{D_\phi \, D_B \, D_\Lambda^4} + \frac{y(\overline{M}\Delta - 2p\cdot k)}{D_\phi^2 \, D_\Lambda^4} \right] \times \delta\left(y - \frac{k^+}{p^+}\right),$$

$$f_{\phi B}^{(\mathrm{rbw})}(y) = \frac{C_{B\phi}^2 \overline{M}^2}{(4\pi f)^2} \left[f_B^{(\mathrm{on})}(y) + f_B^{(\delta)}(y) - \delta f_B^{(\delta)}(y) \right]$$

On-shell contribution:

$$f_B^{(\mathrm{on})}(y) = \overline{\Lambda}^8 \int dk_\perp^2 \, \frac{y\left[k_\perp^2 + (yM + \Delta)^2\right]}{\bar{y}^2 \, D_{\phi B}^2 \, D_{\Lambda B}^4}$$

$$D_{\phi B} = -\frac{k_\perp^2 + yM_B^2 - y\,\bar{y}\,M^2 + \bar{y}\,m_\phi^2}{\bar{y}}$$

$$D_{\Lambda B} = -\frac{k_\perp^2 + yM_B^2 - y\,\bar{y}\,M^2 + \bar{y}\,\Lambda^2}{\bar{y}}.$$

$$\text{when } \Lambda \to \infty, \qquad f^{(\mathrm{on})}_{\pi+n}(y) \longrightarrow \int dk_\perp^2 \frac{y\left(k_\perp^2 + y^2 M^2\right)}{\left[k_\perp^2 + y^2 M^2 + \bar{y}\, m_\pi^2\right]^2}$$

Delta function terms:

$$\begin{aligned} f_B^{(\delta)}(y) &= -\frac{\overline{\Lambda}^8}{\overline{M}^2} \int dk_\perp^2 \int_0^1 dz\, \frac{z^3}{(k_\perp^2 + \Omega)^4}\, \delta(y) \\ &= \frac{1}{\overline{M}^2} \int dk_\perp^2 \left[\log \frac{\Omega_\phi}{\Omega_\Lambda} + \frac{\overline{\Lambda}^2 (11\,\Omega_\Lambda^2 - 7\,\Omega_\Lambda \Omega_\phi + 2\,\Omega_\phi^2)}{6\Omega_\Lambda^3} \right] \delta(y) \end{aligned}$$

$$\begin{aligned} \delta f_B^{(\delta)}(y) &= -\frac{\overline{\Lambda}^8}{\overline{M}^2} \int dk_\perp^2 \int_0^1 dz\, \frac{z^4}{(k_\perp^2 + \Omega)^4}\, \delta(y) \\ &= \frac{1}{\overline{M}^2} \int dk_\perp^2 \left[-4 \frac{\Omega_\phi}{\overline{\Lambda}^2} \log \frac{\Omega_\phi}{\Omega_\Lambda} - \frac{3\Omega_\Lambda^3 + 13\Omega_\Lambda^2 \Omega_\phi - 5\Omega_\Lambda \Omega_\phi^2 + \Omega_\phi^3}{3\Omega_\Lambda^3} \right] \end{aligned}$$

$$\Omega_\phi \;=\; k_\perp^2 + m_\phi^2\,, \qquad \Omega_\Lambda \;=\; k_\perp^2 + \Lambda^2$$

$\text{when } \Lambda \to \infty,$ the delta function terms are the same as our previous point particle result.

General regulator:

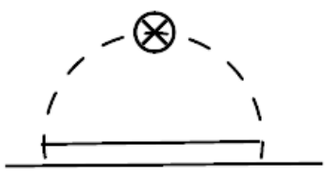
$$F(t,\ u) \equiv F[k^2,\ (p-k)^2] = \left(\frac{\Lambda_t - m^2}{\Lambda_t - t}\right)\cdot\left(\frac{\Lambda_u - M^2}{\Lambda_u - u}\right) = \frac{a}{d_N}\frac{b}{d_\pi}$$

$$f_{n\pi+}(y) = \frac{-i\cdot(F+\mathrm{D})^2}{4f^2}\cdot\int\frac{\mathrm{d}^4k}{(2\pi)^4}\left(\frac{4\ (p.\ k)}{\mathrm{D}_\pi^2} + \frac{8\ M^2}{\mathrm{D}_\pi\mathrm{D}_N} + \frac{8\ M^2\ m^2}{\mathrm{D}_\pi^2\mathrm{D}_N}\right)\cdot F^2(t,\ u)\cdot y\cdot\delta\left(y - \frac{k^+}{p^+}\right)$$

$$\left(\left(-\frac{\pi\ i}{\Lambda_t} - \frac{9\ \pi\ i\ M^2}{a^2} - \frac{3\ \pi\ i}{a}\right)\ \ln(m^2)\right)\ m^4 + \left(-\pi\ i\ \ln(m^2)\right)\ m^2$$

$$\left(\left(\frac{3}{2}\ \frac{\pi\ i\ M^2}{a^2} - \frac{1}{4}\ \frac{\pi\ i}{M^2}\right)\ \ln(m^2)\right)\ m^4 - \frac{1}{2}\ \frac{\pi^2\ i\ m^3}{M} + \left(\frac{1}{2}\ \pi\ i\ \ln(m^2)\right)\ m^2$$

$$\left(\left(\frac{3\ \pi\ i\ M^2}{a^2} + \frac{\pi\ i}{\Lambda_t} - \frac{1}{2}\ \frac{\pi\ i}{M^2}\right)\ \ln(m^2)\right)\ m^4 - \frac{3}{4}\ \frac{\pi^2\ i\ m^3}{M} + \left(\frac{1}{2}\ \pi\ i\ \ln(m^2)\right)\ m^2$$



(h)

$$f_{\phi T}^{(\mathrm{rbw})}(y) = \frac{iC_{T\phi}^2\,\overline{\Lambda}^8}{6M_T^2 f^2}\int\frac{d^4k}{(2\pi)^4}\Bigg[\frac{y(\overline{M}_T^2-m_\phi^2)^2(\Delta_T^2-m_\phi^2)}{D_\phi^2\,D_T\,D_\Lambda^4} - \frac{y(\overline{M}_T^2-m_\phi^2)(\overline{M}_T^2+2\Delta_T^2-3m_\phi^2)}{D_\phi\,D_T\,D_\Lambda^4} + \frac{y(2\overline{M}_T^2+\Delta_T^2-k^2-2m_\phi^2)}{D_T\,D_\Lambda^4} + \frac{y}{D_\phi^2\,D_\Lambda^4}\Big(4(p\cdot k)^2-2(\overline{M}_T^2-k^2)\,p\cdot k+(M_T^2-k^2)^2 + M(2M_T^3-M^3-2M^2M_T)-2Mk^2(2M+M_T)\Big)\Bigg]\delta\left(y-\frac{k^+}{p^+}\right)$$

$$f_{\phi T}^{(\mathrm{rbw})}(y) = \frac{C_{T\phi}^2\overline{M}_T^2}{(4\pi f)^2}\left[f_T^{(\mathrm{on})}(y)+f_T^{(\mathrm{on\,end\,pt})}(y)-\frac{1}{18}f_T^{(\delta)}(y)+\frac{\overline{M}_T^2-m_\phi^2}{6M_T^2}\Big(f_\phi^{(\delta)}(y)-\delta f_\phi^{(\delta)}(y)\Big)\right]$$

On-shell contribution:

$$f_T^{(\mathrm{on})}(y) = \frac{\overline{\Lambda}^8}{6M_T^2\,\overline{M}_T^2}\int dk_\perp^2\,\frac{y\,(\overline{M}_T^2-m_\phi^2)}{\bar{y}}\left[\frac{(\overline{M}_T^2-m_\phi^2)(\Delta_T^2-m_\phi^2)}{D_{\phi T}^2\,D_{\Lambda T}^4}-\frac{3(\Delta_T^2-m_\phi^2)+4MM_T}{D_{\phi T}\,D_{\Lambda T}^4}\right]$$

when $\Lambda\to\infty$,

$$f_\Delta^{(\mathrm{on})}(y) = C_\Delta\int dk_\perp^2\,\frac{y\,(\overline{M}^2-m_\pi^2)}{1-y}\times\left[\frac{(\overline{M}^2-m_\pi^2)(\Delta^2-m_\pi^2)}{D_{\pi\Delta}^2}-\frac{3(\Delta^2-m_\pi^2)+4MM_\Delta}{D_{\pi\Delta}}\right]$$

Y. Salamu, C. R. Ji, W. Melnitchouk and P. Wang, Phys. Rev. Lett. 114 (2015) 122001.

End-point term:

$$f_T^{(\text{on end pt})}(y) = \frac{\overline{\Lambda}^8}{6M_T^2\,\overline{M}_T^2}\int dk_\perp^2\,\frac{y}{\bar{y}^2\,D_{\Lambda T}^4} \times\left[k_\perp^2 + y^2M^2 + 3\overline{M}_T^2 - 4MM_T - 2y(\overline{M}_T^2 - M\Delta_T) - 2\bar{y}\,m_\phi^2\right]$$

Local limit:

$$f_T^{(\text{on end pt})}(y) \xrightarrow[\Lambda\to\infty]{} \frac{1}{6M_T^2\,\overline{M}_T^2}\int dk_\perp^2\,\delta(\bar{y})\left\{\left[\Omega_T - 2(\Delta_T^2 - m_\phi^2) - 6MM_T\right]\log\frac{\Omega_T}{\mu^2} - \Omega_T + 2(\Delta_T^2 - m_\phi^2) + 6MM_T\right\}.$$

Delta term:

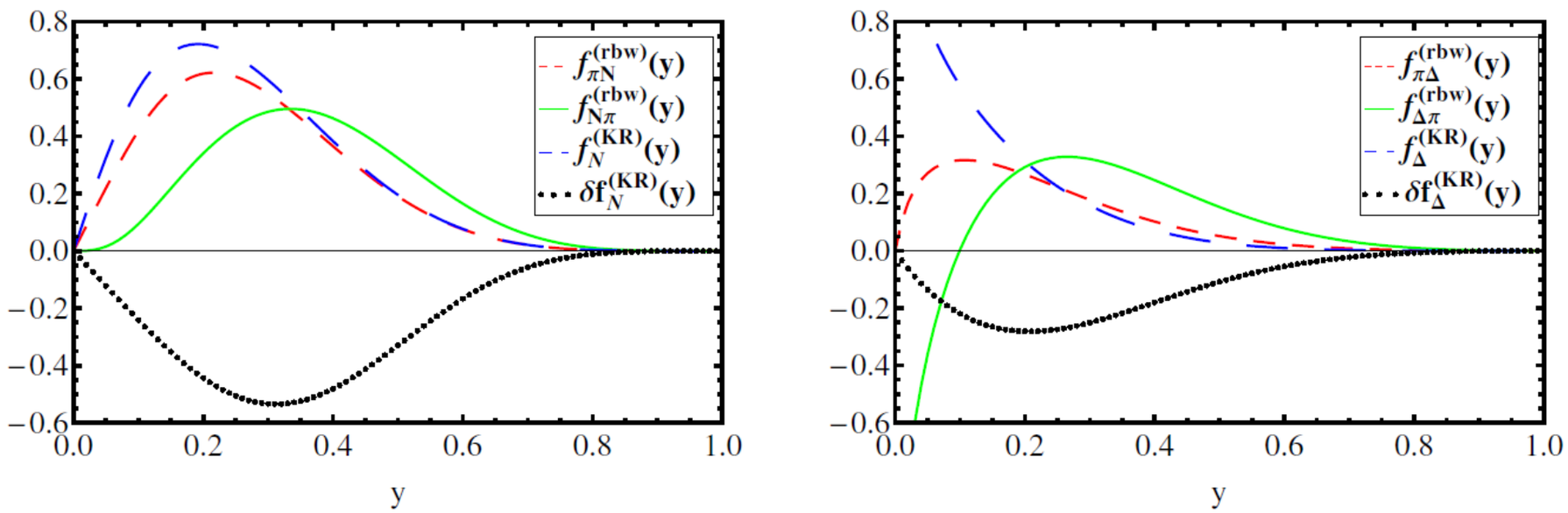
$$f_T^{(\delta)}(y) = \frac{\overline{\Lambda}^8}{M_T^2\overline{M}_T^2}\int dk_\perp^2\int_0^1 dz\,\frac{z^3}{(k_\perp^2+\Omega)^3}\,\delta(y)$$
$$= \frac{1}{M_T^2\overline{M}_T^2}\int dk_\perp^2\frac{1}{2\Omega_\Lambda^2}\left[6\Omega_\Lambda^2\Omega_\phi\log\frac{\Omega_\phi}{\Omega_\Lambda} + (\Omega_\phi - \Omega_\Lambda)(\Omega_\phi^2 - 5\Omega_\phi\Omega_\Lambda - 2\Omega_\Lambda^2)\right]\delta(y)$$

Relationships between splitting functions:

$$f_{\phi B}^{(\mathrm{rbw})}(y) = f_{B\phi}^{(\mathrm{rbw})}(y) + f_{B}^{(\mathrm{KR})}(y) + \delta f_{B}^{(\mathrm{KR})}(y)$$

$$f_{\phi}^{(\mathrm{bub})}(y) = f_{\phi}^{(\mathrm{tad})}(y) + \delta f_{\phi}^{(\mathrm{tad})}(y)$$

Gauge invariance of nonlocal Lagrangian, net strangeness zero.



Y. Salamu, C. R. Ji, W. Melnitchouk, A. W. Thomas and P. Wang, Phys. Rev. D 99 (2019) 014041.

Numerical results of PDFs

dbar-ubar asymmetry:

$$\bar{d}(x) - \bar{u}(x) = [f_{\pi^+ n}(y) + f_{bub,\pi^+}(y) + f_{\pi^+\Delta^0}(y) - f_{\pi^-\Delta^{++}}(y)] \otimes \bar{q}_\pi(x)$$

strange-antistrange quark asymmetry:

$$\begin{aligned} s(x) - \bar{s}(x) = \sum_{B,T,\phi} \Big[& f^{(\mathrm{rbw})}_{B\phi}(y) \otimes s_B(x) + f^{(\mathrm{KR})}_B(y) \otimes s^{(KR)}_B(x) + \delta f^{(\mathrm{KR})}_B(y) \otimes s^{\delta}_B(x) \\ & - f^{(\mathrm{rbw})}_{\phi B}(y) \otimes \bar{s}_\phi(x) + f^{(\mathrm{rbw})}_{T\phi}(y) \otimes s_T(x) + f^{(\mathrm{KR})}_T(y) \otimes s^{(KR)}_T(x) \\ & + \delta f^{(\mathrm{KR})}_T(y) \otimes s^{\delta}_T(x) - f^{(\mathrm{rbw})}_{\phi T}(y) \otimes \bar{s}_\phi(x) + f^{(\mathrm{tad})}_\phi \otimes s^{(tad)}_\phi(x) \\ & + \delta f^{(\mathrm{tad})}_\phi(y) \otimes s^{\delta}_\phi(x) - f^{(\mathrm{bub})}_\phi(y) \otimes \bar{s}_\phi(x) \Big], \end{aligned}$$

$$q(x) = \sum_j (f_j \otimes q^v_j)(x) \equiv \sum_j \int_0^1 dy \int_0^1 dz\, \delta(x - yz)\, f_j(y)\, q^v_j(z)$$

Input PDFs of valence at tree level:

valence quark in mesons:

$$u_{\pi^+}(x) = \bar{d}_{\pi^+}(x) = \bar{u}_{\pi^-}(x) = d_{\pi^-}(x) = \bar{s}_{K^+}(x) = \bar{s}_{K^0}(x) = \bar{q}_\pi(x)$$

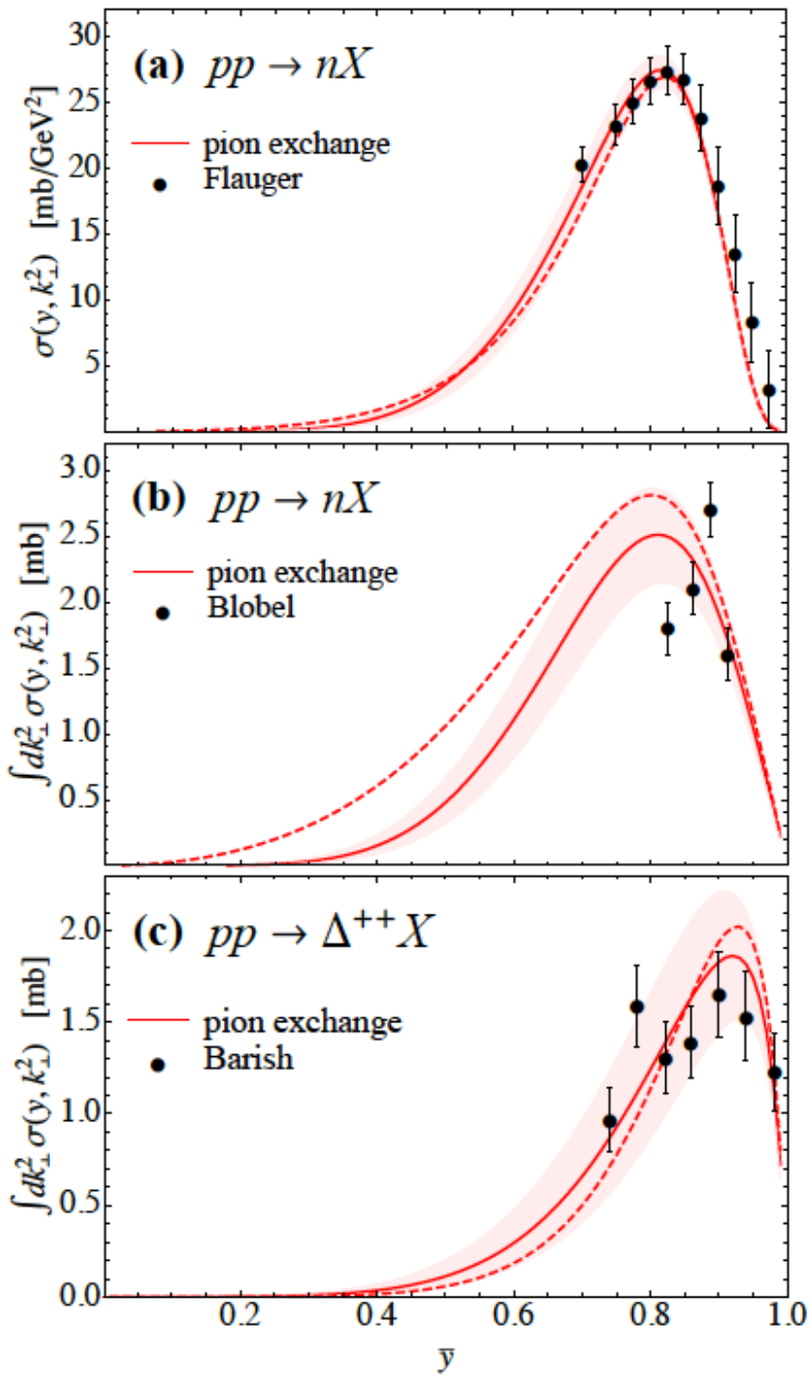
valence quark in baryons:

$$u_n(x) = d(x), u_{\Sigma^+}(x) = 2u_{\Sigma^0} = u(x), u_\Lambda(x) = \frac{4\,d(x) + u(x)}{6}, u_{\Delta^0}(x) = \frac{1}{2}u_{\Delta^+}(x) = \frac{1}{3}\,u_{\Delta^{++}}(x)$$
$$= u_{\Sigma^{*0}}(x) = \frac{1}{2}u_{\Sigma^{*+}}(x) = d(x), d_n(x) = u(x), 2\,d_{\Sigma^0}(x) = d_{\Sigma^-}(x) = u(x), d_\Lambda(x) = \frac{4d(x) + u(x)}{6}$$
$$\frac{1}{3}d_{\Delta^-}(x) = \frac{1}{2}\,d_{\Delta^0}(x) = d_{\Delta^+}(x) = \frac{1}{2}d_{\Sigma^{*-}}(x) = d_{\Sigma^{*0}}(x) = d(x), s_\Lambda(x) = \frac{2\,u(x) - d(x) + 2s(x)}{3},$$
$$s_{\Sigma^+}(x) = s_{\Sigma^0}(x) = d(x), s_{\Sigma^{+*}}(x) = s_{\Sigma^{0*}}(x) = d(x),$$

quark distributions For KR diagrams:

$$u_n^{(KR)}(x) = \frac{\Delta u(x) - \Delta d(x)}{F + D}, u_{\Sigma^0}^{(KR)}(x) = \frac{\Delta d(x)}{F - D}, u_\Lambda^{(KR)} = \frac{2\Delta u(x) - \Delta d(x)}{3F + D}$$
$$u_{\Delta^0}^{(KR)}(x) = u_{\Delta^{++}}^{(KR)}(x) = u_{\Sigma^{*0}}^{(KR)}(x) = \frac{2\Delta d(x) - \Delta u(x)}{-2D},$$
$$d_n^{(KR)}(x) = \frac{\Delta u(x) - \Delta d(x)}{F + D}, d_{\Sigma^+}^{(KR)}(x) = \frac{\Delta d(x)}{F - D},$$
$$d_{\Delta^0}^{(KR)}(x) = d_{\Delta^{++}}^{(KR)}(x) = d_{\Sigma^{*+}}^{(KR)}(x) = \frac{2\Delta d(x) - \Delta u(x)}{-2D},$$
$$s_{\Sigma^+}^{(KR)}(x) = s_{\Sigma^0}^{(KR)}(x) = \frac{\Delta d(x)}{F - D}, s_\Lambda^{(KR)}(x) = \frac{2\Delta u(x) - \Delta d(x)}{3F + D},$$
$$s_{\Sigma^{*+}}^{(KR)}(x) = s_{\Sigma^{*0}}^{(KR)}(x) = \frac{2\Delta d(x) - \Delta u(x)}{-2D},$$

Lambda determination



$$\sigma(pp \to nX) = \frac{2(D+F)^2 M^2}{(4\pi f)^2} \frac{\bar{y}}{\pi}\, \hat{f}_N^{(\mathrm{on})}(y, k_\perp^2)\, \sigma_{\mathrm{tot}}^{\pi^+ p}(ys)$$

$$\sigma(pp \to \Delta^{++}X) = \frac{\mathcal{C}^2 \overline{M}^2}{2(4\pi f)^2} \frac{\bar{y}}{\pi} \left[\hat{f}_\Delta^{(\mathrm{on})}(y, k_\perp^2) + \hat{f}_\Delta^{(\mathrm{on\,end})}(y, k_\perp^2) \right] \sigma_{\mathrm{tot}}^{\pi^- p}(ys)$$

$$f_N^{(\mathrm{on})}(y) \equiv \int dk_\perp^2\, \hat{f}_N^{(\mathrm{on})}(y, k_\perp^2)$$

$$f_\Delta^{(\mathrm{on})}(y) \equiv \int dk_\perp^2\, \hat{f}_\Delta^{(\mathrm{on})}(y, k_\perp^2),$$

$$f_\Delta^{(\mathrm{on\,end})}(y) \equiv \int dk_\perp^2\, \hat{f}_\Delta^{(\mathrm{on\,end})}(y, k_\perp^2),$$

dbar - ubar asymmetry

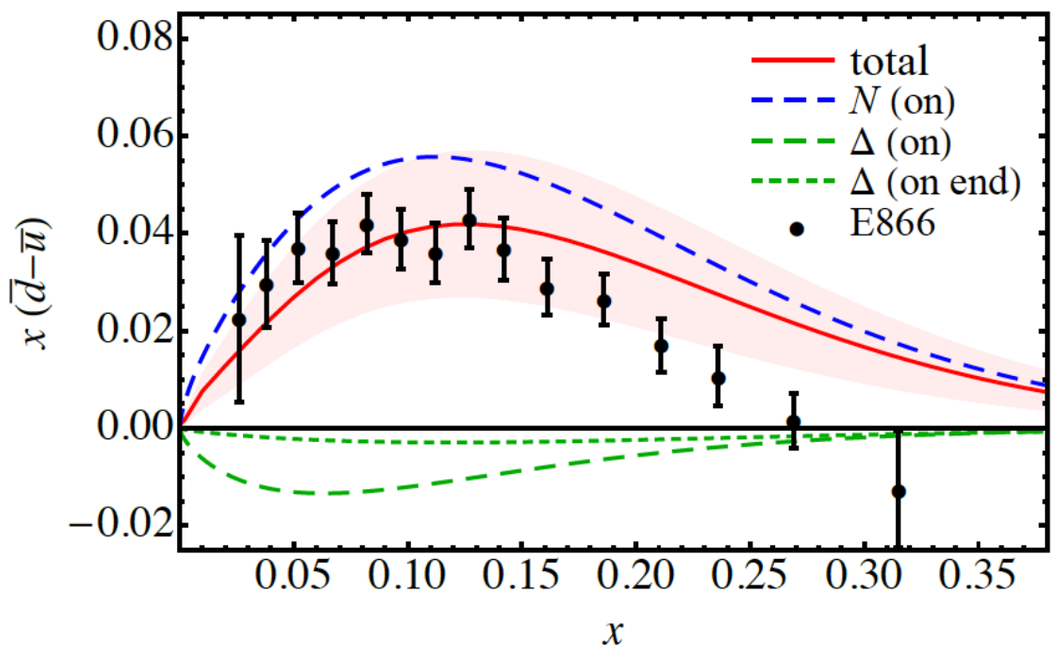
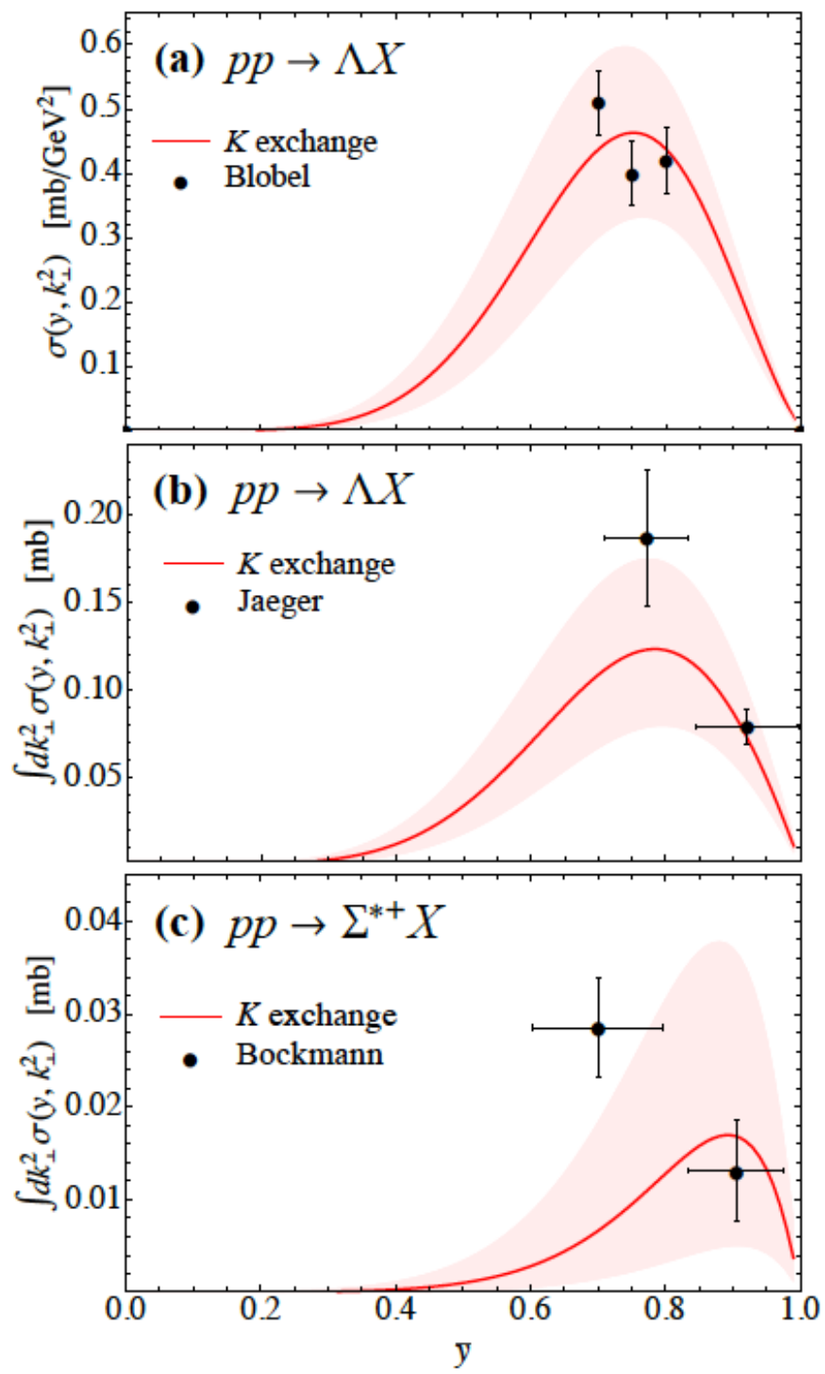


diagram		$\langle \bar{d} - \bar{u} \rangle$
πN (rbw)	$f_N^{(\mathrm{on})}$	$0.152^{+0.032}_{-0.030}$
	$f_\pi^{(\delta)}$	$-(0.079^{+0.020}_{-0.018})$
	$\delta f_\pi^{(\delta)}$	$0.044^{+0.010}_{-0.009}$
total πN		$0.116^{+0.022}_{-0.022}$
$\pi\Delta$ (rbw)	$f_\Delta^{(\mathrm{on})}$	$-(0.044^{+0.012}_{-0.012})$
	$f_\Delta^{(\mathrm{on\,end})}$	$-(0.009^{+0.004}_{-0.003})$
	$f_\Delta^{(\delta)}$	$0.002^{+0.001}_{-0.001}$
	$f_\pi^{(\delta)}$	$0.039^{+0.010}_{-0.010}$
	$\delta f_\pi^{(\delta)}$	$-(0.022^{+0.005}_{-0.005})$
total $\pi\Delta$		$-(0.033^{+0.010}_{-0.010})$
π (bub)	$f_\pi^{(\delta)}$	$0.099^{+0.025}_{-0.022}$
	$\delta f_\pi^{(\delta)}$	$-(0.054^{+0.013}_{-0.012})$
total π bubble		$0.044^{+0.012}_{-0.010}$
total		$\mathbf{0.127^{+0.044}_{-0.042}}$
$x>0$		$0.099^{+0.047}_{-0.046}$
$x=0$		$0.028^{+0.008}_{-0.007}$
local		$0.159^{+0.041}_{-0.039}$
nonlocal		$-(0.032^{+0.008}_{-0.008})$

Y. Salamu, C. R. Ji, W. Melnitchouk, A. W. Thomas, P. Wang and X. G. Wang, arXiv:1907.08551.

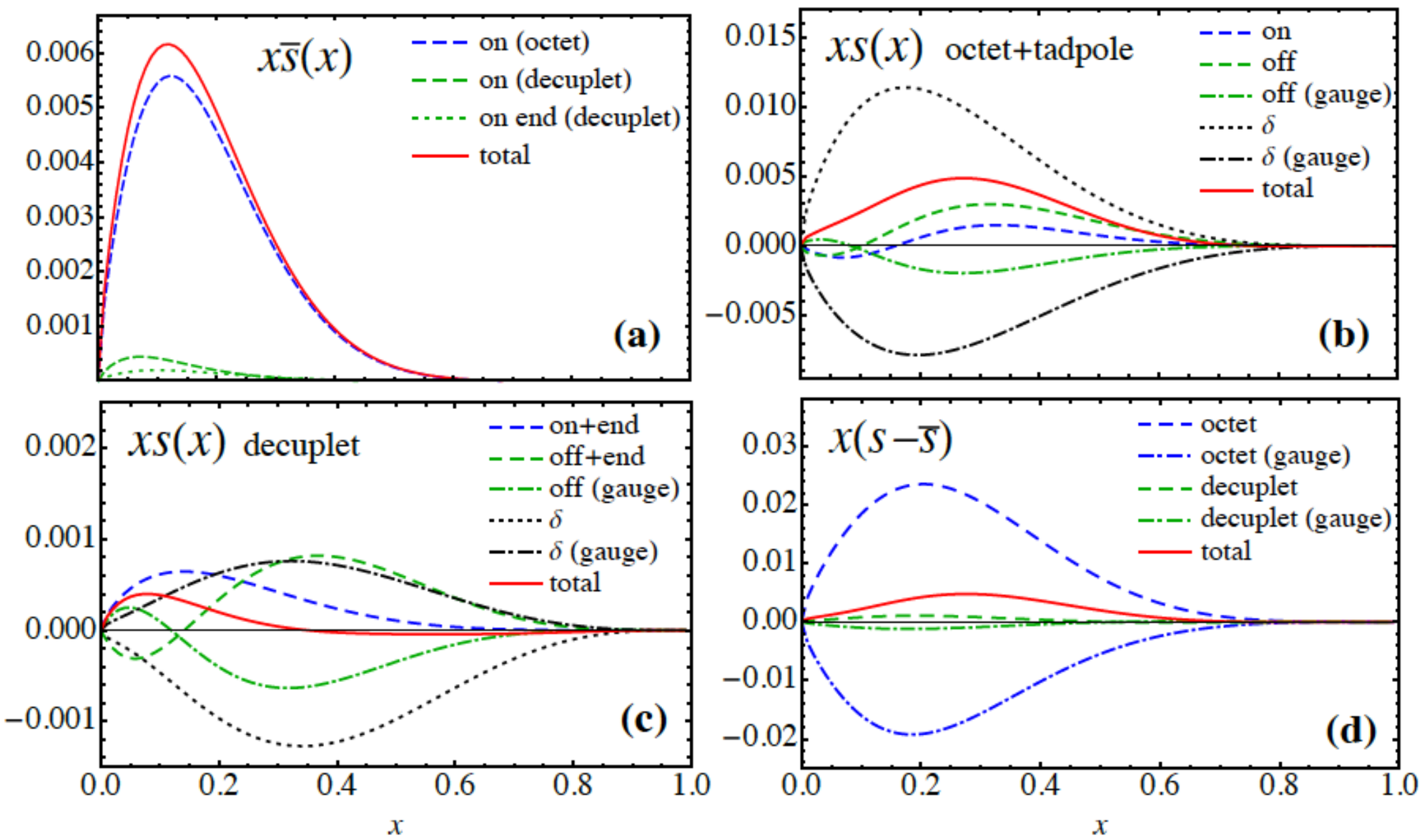
Lambda determination



$$\sigma(pp \to \Lambda X) = \frac{(D+3F)^2 (M+M_\Lambda)^2}{12(4\pi f)^2}\, \frac{\bar{y}}{\pi}\, \hat{f}_\Lambda^{(\mathrm{on})}(y, k_\perp^2)\, \sigma_{\mathrm{tot}}^{K^+p}(ys),$$

$$\sigma(pp \to \Sigma^{*+} X) = \frac{\mathcal{C}^2\, (M+M_{\Sigma^*})^2}{6(4\pi f)^2}\, \frac{\bar{y}}{\pi} \left[\hat{f}_{\Sigma^{*+}}^{(\mathrm{on})}(y, k_\perp^2) + \hat{f}_{\Sigma^{*+}}^{(\mathrm{on\,end})}(y, k_\perp^2) \right] \sigma_{\mathrm{tot}}^{K^0p}(sy)$$

s-sbar asymmetry



Y. Salamu, C. R. Ji, W. Melnitchouk, A. W. Thomas, P. Wang and X. G. Wang, arXiv:1907.08551.

		$\langle\bar{s}\rangle$ $(\times 10^{-2})$	$\langle x\bar{s}\rangle$ $(\times 10^{-3})$			$\langle s\rangle$ $(\times 10^{-2})$	$\langle xs\rangle$ $(\times 10^{-3})$
KY(rbw)	$f_Y^{(\mathrm{on})}$	$1.39^{+0.69}_{-0.54}$	$1.33^{+0.74}_{-0.56}$	YK(rbw)	$f_Y^{(\mathrm{on})}$	$1.39^{+0.69}_{-0.54}$	$1.67^{+0.78}_{-0.63}$
	$f_K^{(\delta)}$	$-(1.66^{+0.79}_{-0.63})$	0		$f_Y^{(\mathrm{off})}$	$-(4.01^{+1.68}_{-1.42})$	$-(5.35^{+2.12}_{-1.83})$
	$\delta f_K^{(\delta)}$	$1.12^{+0.50}_{-0.41}$	0		$\delta f_Y^{(\mathrm{off})}$	$2.70^{+1.07}_{-0.92}$	$3.12^{+1.13}_{-1.02}$
					$f_K^{(\delta)}$	$1.66^{+0.79}_{-0.63}$	$2.82^{+1.35}_{-1.07}$
				YK(KR)	$f_Y^{(\mathrm{off})}$	$4.01^{+1.68}_{-1.42}$	$6.29^{+2.50}_{-2.15}$
					$f_K^{(\delta)}$	$-(3.31^{+1.58}_{-1.26})$	$-(6.66^{+3.18}_{-2.53})$
				$YK(\delta$KR)	$\delta f_Y^{(\mathrm{off})}$	$-(2.70^{+1.07}_{-0.92})$	$-(3.68^{+1.33}_{-1.20})$
					$\delta f_K^{(\delta)}$	$1.12^{+0.50}_{-0.41}$	$2.24^{+1.01}_{-0.82}$
total octet		$0.85^{+0.40}_{-0.32}$	$1.33^{+0.74}_{-0.56}$	total octet		$0.85^{+0.40}_{-0.32}$	$0.46^{+0.14}_{-0.14}$
K(bub)	$f_K^{(\delta)}$	$4.85^{+2.32}_{-1.84}$	0	K(tad)	$f_K^{(\delta)}$	$4.85^{+2.32}_{-1.84}$	$7.87^{+3.76}_{-2.98}$
	$\delta f_K^{(\delta)}$	$-(3.27^{+1.47}_{-1.20})$	0	$K(\delta$tad)	$\delta f_K^{(\delta)}$	$-(3.27^{+1.47}_{-1.20})$	$-(5.30^{+2.38}_{-1.94})$
total bubble		$1.59^{+0.85}_{-0.64}$	0	total tadpole		$1.59^{+0.85}_{-0.64}$	$2.57^{+1.38}_{-1.04}$
KY^*(rbw)	$f_{Y^*}^{(\mathrm{on})}$	$0.09^{+0.13}_{-0.07}$	$0.06^{+0.09}_{-0.04}$	Y^*K(rbw)	$f_{Y^*}^{(\mathrm{on})}$	$0.09^{+0.13}_{-0.07}$	$0.10^{+0.14}_{-0.08}$
	$f_{Y^*}^{(\mathrm{on\,end})}$	$0.04^{+0.07}_{-0.03}$	$0.03^{+0.06}_{-0.03}$		$f_{Y^*}^{(\mathrm{on\,end})}$	$0.04^{+0.07}_{-0.03}$	$0.04^{+0.07}_{-0.03}$
	$f_{Y^*}^{(\delta)}$	$-(0.01^{+0.01}_{-0.01})$	0		$f_{Y^*}^{(\mathrm{off})}$	$-(0.59^{+0.72}_{-0.42})$	$-(0.75^{+0.89}_{-0.52})$
	$f_K^{(\delta)}$	$-(0.15^{+0.20}_{-0.11})$	0		$f_{Y^*}^{(\mathrm{off\,end})}$	$0.17^{+0.23}_{-0.12}$	$0.21^{+0.29}_{-0.15}$
	$\delta f_K^{(\delta)}$	$0.11^{+0.14}_{-0.08}$	0		$\delta f_{Y^*}^{(\mathrm{off})}$	$0.34^{+0.45}_{-0.24}$	$0.38^{+0.47}_{-0.27}$
					$f_K^{(\delta)}$	$0.18^{+0.24}_{-0.13}$	$0.26^{+0.34}_{-0.19}$
					$f_{Y^*}^{(\delta)}$	$0.01^{+0.01}_{-0.01}$	$0.01^{+0.02}_{-0.01}$
					$\delta f_{Y^*}^{(\delta)}$	$-(0.07^{+0.11}_{-0.05})$	$-(0.10^{+0.16}_{-0.07})$
				Y^*K(KR)	$f_{Y^*}^{(\mathrm{off})}$	$0.59^{+0.72}_{-0.42}$	$1.02^{+1.21}_{-0.71}$
					$f_{Y^*}^{(\mathrm{off\,end})}$	$-(0.17^{+0.23}_{-0.12})$	$-(0.29^{+0.39}_{-0.21})$
					$f_K^{(\delta)}$	$-(0.34^{+0.44}_{-0.24})$	$-(0.65^{+0.85}_{-0.47})$
					$f_{Y^*}^{(\delta)}$	$-(0.02^{+0.03}_{-0.01})$	$-(0.03^{+0.05}_{-0.02})$
					$\delta f_{Y^*}^{(\delta)}$	$0.05^{+0.08}_{-0.03}$	$0.09^{+0.15}_{-0.07}$
				$Y^*K(\delta$KR)	$\delta f_{Y^*}^{(\mathrm{off})}$	$-(0.34^{+0.45}_{-0.24})$	$-(0.51^{+0.63}_{-0.36})$
					$\delta f_K^{(\delta)}$	$0.11^{+0.14}_{-0.08}$	$0.22^{+0.27}_{-0.15}$
					$\delta f_{Y^*}^{(\delta)}$	$0.02^{+0.04}_{-0.02}$	$0.05^{+0.07}_{-0.03}$
total decuplet		$0.08^{+0.12}_{-0.06}$	$0.09^{+0.15}_{-0.07}$	total decuplet		$0.08^{+0.12}_{-0.06}$	$0.04^{+0.04}_{-0.03}$
total		$\mathbf{2.51^{+1.36}_{-1.02}}$	$\mathbf{1.42^{+0.89}_{-0.62}}$	**total**		$\mathbf{2.51^{+1.36}_{-1.02}}$	$\mathbf{3.08^{+1.55}_{-1.20}}$
non δ-function		$1.52^{+0.88}_{-0.64}$	$1.42^{+0.89}_{-0.62}$	non δ-function		$1.52^{+0.88}_{-0.64}$	$2.25^{+1.21}_{-0.92}$
δ-function		$0.99^{+0.60}_{-0.50}$	0	δ-function		$0.99^{+0.60}_{-0.50}$	$0.82^{+0.67}_{-0.63}$
local		$4.55^{+2.23}_{-1.77}$	$1.42^{+0.89}_{-0.62}$	local		$4.55^{+2.23}_{-1.77}$	$6.58^{+3.14}_{-2.60}$
nonlocal		$-(2.04^{+1.04}_{-0.92})$	0	nonlocal		$-(2.04^{+1.04}_{-0.92})$	$-(3.50^{+1.66}_{-1.46})$

Sivers function

$$f_1^q(x,\vec{k}_\perp) \; + \; \frac{\epsilon^{ji}k_\perp^i S_\perp^j}{m_p} f_{1T}^q(x,\vec{k}_\perp) = \frac{1}{2}\int \frac{d\xi^- d^2\vec{\xi}_\perp}{(2\pi)^3}$$
$$\times \; e^{-ixP^+\xi^- + i\vec{k}_\perp\cdot\vec{\xi}_\perp} \langle P,\vec{S}_\perp \mid \mathcal{O}^q \mid P,\vec{S}_\perp\rangle$$

$$\mathcal{O}^q = \bar{q}(\xi^-,\vec{\xi}_\perp)\mathcal{L}^\dagger_{\xi_\perp}(\infty,\xi^-)\gamma^+\mathcal{L}_0(\infty,0)q(0,0)$$

$$\mathcal{L}^\dagger_{\xi_\perp}(\infty,\xi^-) = P\, e^{-ig_c\int_{\xi^-}^{\infty} A^+(z^-,\vec{\xi}_\perp)dz^-}$$

$$\mathcal{O}^{\pi^+} = i\big[\pi^-(y^-,\vec{y}_\perp)\partial^+\pi^+(0) - \partial^+\pi^-(y^-,\vec{y}_\perp)\pi^+(0)\big]$$

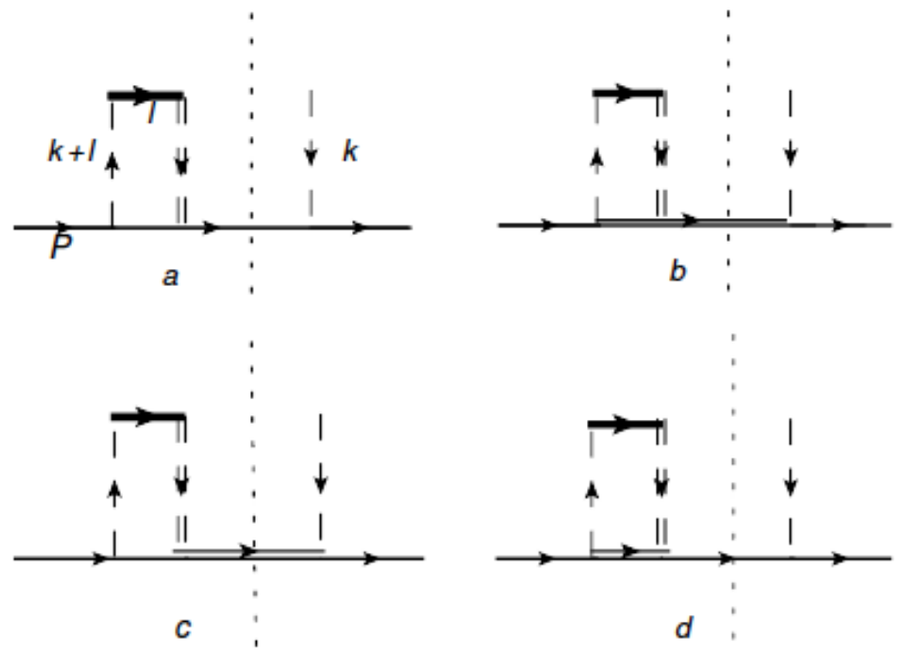
$$\mathcal{S} = \int dx\, dy f_\pi^2 Tr[\alpha_\mu(y)W(y,x)\alpha^\mu(x)W^\dagger(y,x)]$$

$$W(y,x) = Pe^{I(x,y)} = Pe^{-ig\int_x^y dz_\nu V^\nu(z)}$$

$$\mathcal{J}^\mu_{\bar{d}/\pi^+} \;=\; i[\pi^-(y)\partial^\mu\pi^+(x) - \partial^\mu\pi^-(y)\pi^+(x)]$$
$$\times \; \big[1 - i\sqrt{2}g\int_x^y dz_\nu \rho^{0\nu}(z)\big].$$

Pion Sivers function in proton:

$$\frac{\epsilon^{ji} k^i_{\perp\pi} S^j_\perp}{m_p} f_{1T}^{\pi/p}(z, \vec{k}_{\perp\pi}) = \frac{1}{2} \int \frac{dy^- d^2\vec{y}_\perp}{(2\pi)^3}$$
$$\times \; e^{-i(zP^+y^- - \vec{y}_\perp \cdot \vec{k}_{\perp\pi})} \langle P, \vec{S}_\perp \mid \mathcal{O}^\pi_{Sivers} \mid P, \vec{S}_\perp \rangle$$



$$\frac{\epsilon^{ji} k^i_{\perp\pi} S^j_\perp}{m_p} f_{1T}^{\pi/p}(z, \vec{k}_{\perp\pi}) = \frac{i g^2 g_A^2}{4 f_\pi^2} \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4l}{(2\pi)^4} \bar{U}(P, \vec{S}_\perp)$$
$$\not{k} \gamma^5 S_{\rm on}(P-k) V_\mu(l) S(P-k-l) \gamma^5 (\not{k} + \not{l}) U(P, \vec{S}_\perp)$$
$$\times \; \frac{(2k^+ + l^+)}{(l^+ + i\epsilon)} S_\pi(k) S_\rho^{\mu+}(l) S_\pi(k+l)$$
$$\times \; \delta(k^+ - zP^+) \delta^2(\vec{k}_\perp - \vec{k}_{\perp\pi}) + h.c., \qquad (17)$$

Convolution form:

$$k_\perp^i f_{1T}^{\bar{q}/p}(x,\vec{k}_\perp) = \int d^2\vec{k}_{\perp\pi} k_{\perp\pi}^i \int_x^1 \frac{dz}{z} f_{1v}^{\bar{q}/\pi}(\frac{x}{z},\vec{k}_\perp - \frac{x}{z}\vec{k}_{\perp\pi}) f_{1T}^{\pi/p}(z,\vec{k}_{\perp\pi})$$

The first moment of Sivers function:

$$\Delta^N f_{\bar{q}}^{(1)}(x) = \int d^2\vec{k}_\perp \frac{-k_\perp^2}{2m_p^2} f_{1T}^{\bar{q}/p}(x,\vec{k}_\perp)$$
$$= \frac{1}{2m_p^2}\int_1^x d(\frac{x}{z}) f_{1v}^{\bar{q}/\pi}(\frac{x}{z}) \int d^2\vec{k}_{\perp\pi}\vec{k}_{\perp\pi}^2 f_{1T}^{\pi/p}(z,\vec{k}_{\perp\pi})$$

$$\tilde{F}_j(k) = \left(\frac{M_j^2 - \Lambda_j^2}{k^2 - \Lambda_j^2}\right)^2$$

$$0.8\ \text{GeV} \leq \Lambda_\pi \leq 1.2\ \text{GeV and } 1.6\ \text{GeV} \leq \Lambda_\rho \leq 2.0\ \text{GeV}$$

Numerical results:

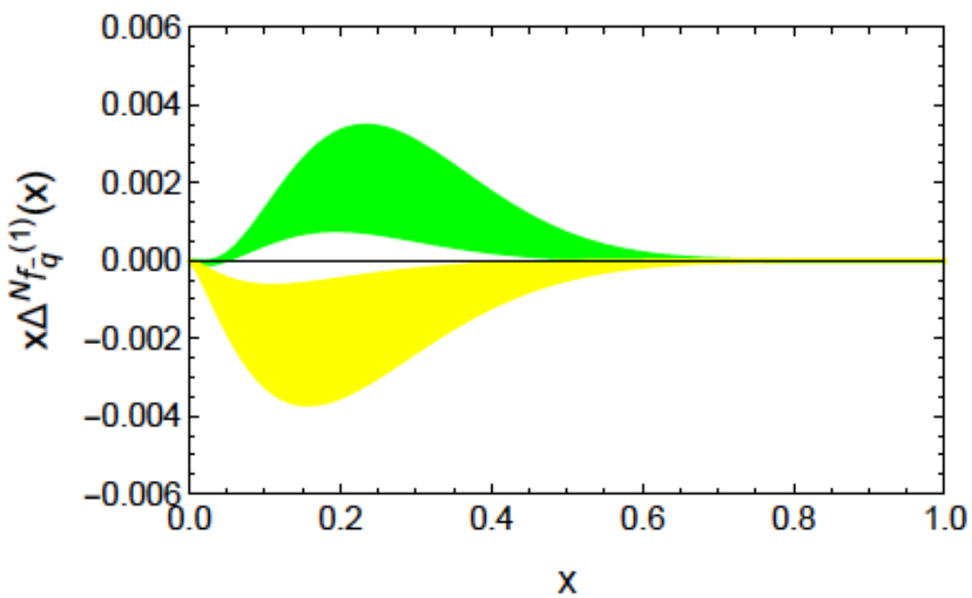


FIG. 3: The first momentum of sea quark Sivers distribution functions versus x at $Q = 0.63$ GeV. The green and yellow bands are the results of $x\Delta^N f_{\bar{d}}^{(1)}(x)$ and $x\Delta^N f_{\bar{u}}^{(1)}(x)$ with 0.8 GeV $\leq \Lambda_\pi \leq$ 1.2 GeV and 1.6 GeV $\leq \Lambda_\rho \leq$ 2.0 GeV.

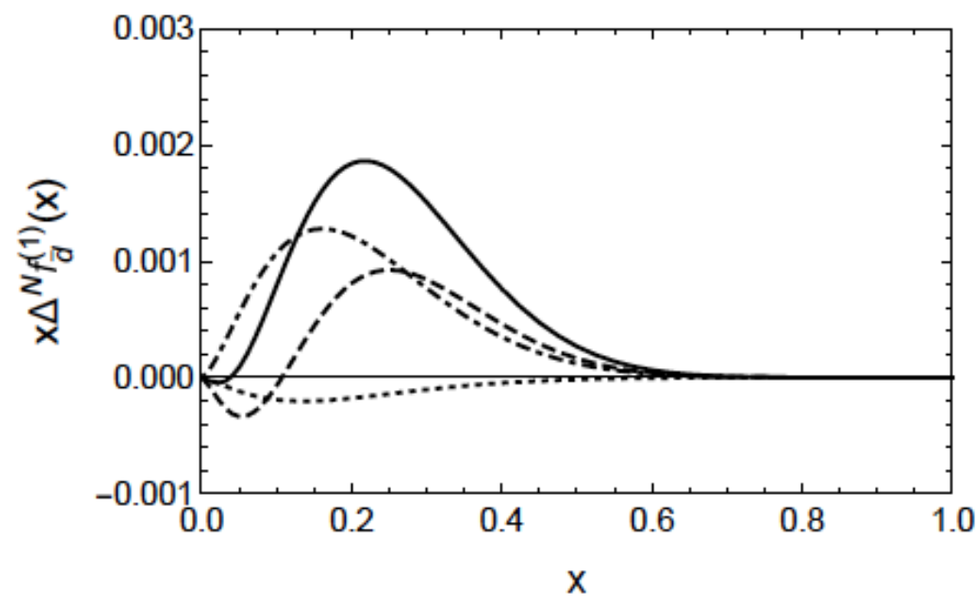
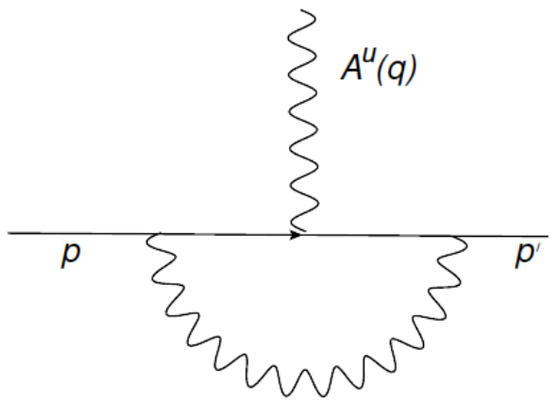


FIG. 4: Contributions to $x\Delta^N f_{\bar{d}}^{(1)}(x)$ from different intermediate states with $\Lambda_\pi = 1$ GeV, $\Lambda_\rho = 1.85$ GeV and $\kappa_N = 6.1$. The dashed, dotted and dot-dashed lines are for the contributions from intermediate octet, decuplet and octet-decuplet transition, respectively. The solid line is for the total result.

Fangcheng He and P. Wang, arXiv:1904.06815

Nonlocal behaviour is general for all the interactions?

Nonlocal QED?



$$\Delta a_e = (-87 \pm 36) \times 10^{-14} \qquad \Delta a_\mu = (2.74 \pm 0.73) \times 10^{-9}$$

$$\mathcal{L}^{nl}_{QED} = \bar{\psi}(x)(i\partial\!\!\!/ - m)\psi(x) - \frac{1}{4}F_{\mu\nu}(x)^2 - e\int d^4a\bar{\psi}(x)A\!\!\!/_\mu(x+a)\psi(x)F(a)$$

$$\psi(x) \to e^{i\alpha(x)}\psi(x), \qquad A_\mu \to A_\mu - \frac{1}{e}\partial_\mu\,\alpha'(x)$$

$$\alpha(x) = \int da\alpha'(x+a)F(a)$$

Ward identity:

$$-iq_\mu \Gamma^\mu(p+q,p) = \tilde{F}(q)(S^{-1}(p+q) - S^{-1}(p))$$

$$S(p+q) = \frac{iZ_2}{\not{p}+\not{q}-m}, \qquad S(p) = \frac{iZ_2}{\not{p}-m}$$

$$Z_2\Gamma^\mu(p,p) = \gamma^\mu \tilde{F}(0)$$

$$Z_2\Gamma^\mu(p+q,p) = \gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2)$$

$$F_1(0) = \tilde{F}(0)$$

$$Z_2 - 1 \;=\; \left.\frac{d\Sigma(\not{p})}{d\not{p}}\right|_{\not{p}=m}$$

$$\Sigma(\not{p}) = -\int \frac{d^4k}{(2\pi)^4}\tilde{F}^2(k)(ie\gamma^\nu)\frac{1}{\not{p}-\not{k}-m}(-ie\gamma^\rho)\frac{-ig_{\nu\rho}}{k^2}$$

$$\frac{d\Sigma(\not{p})}{d\not{p}} = -F_1^{loop}(0) \qquad F_1(0) = Z_2 + F_1^{loop}(0) = 1$$

Pauli form factor:

$$F_2^{nl}(Q^2) = \frac{\alpha}{2\pi}\tilde{F}(-Q^2)\int_0^1 dx\int_0^{1-x} dy\frac{2\Lambda^8 m^2(x+y)(1-x-y)^5}{(Q^2xy+m^2(x+y)^2)(m^2(x+y)^2+Q^2xy+(1-x-y)\Lambda^2)^4}.$$

When Λ goes to infinity, the local result will be recovered as

$$F_2^{lo}(Q^2) = \frac{\alpha}{2\pi}\int_0^1 dx\int_0^{1-x} dy\frac{2m^2(1-x-y)(x+y)}{m^2(x+y)^2+Q^2xy},$$

Q, Lambda >> m:

$$F_2^{nl}(Q^2) = \alpha m^2\left[\frac{\Lambda^6Q^2-\Lambda^4Q^4}{\pi Q^4\left(Q^2-\Lambda^2\right)\left(\Lambda^2+Q^2\right)^2}\log\left(\frac{m^2}{Q^2}\right)+C(Q^2,\Lambda^2)\right]$$

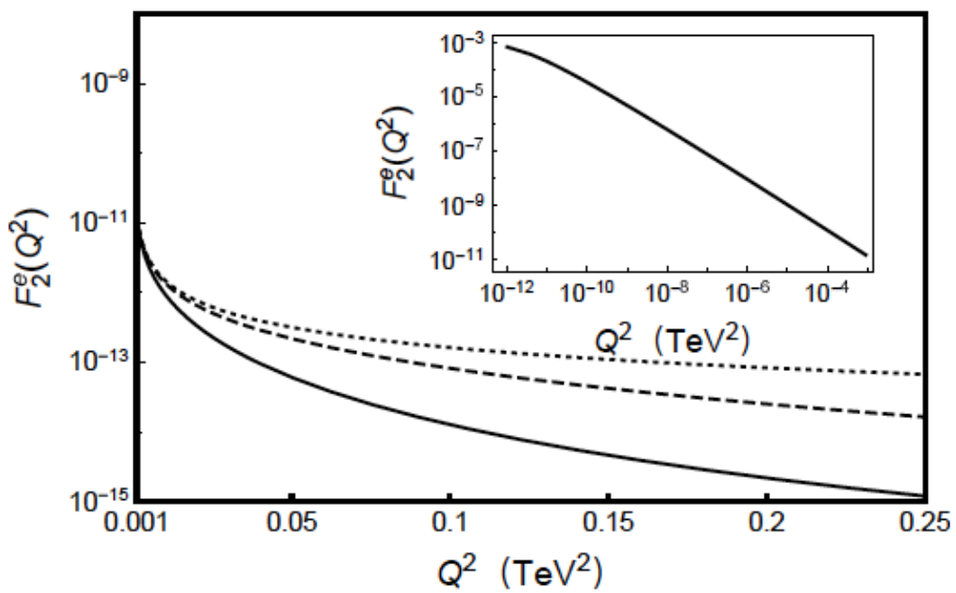
where $C(Q^2,\Lambda^2)$ is lepton mass independent. It is written as

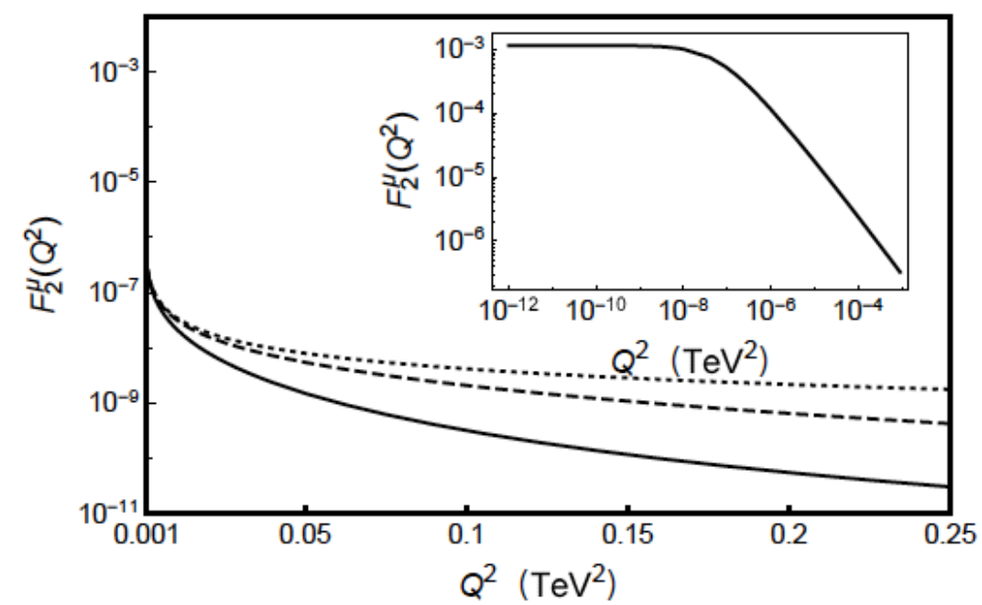
$$\begin{aligned}C(Q^2,\Lambda^2) &= \frac{\Lambda^4\left(-6\Lambda^4+Q^4+3\Lambda^2Q^2\right)\log\left(\frac{\Lambda^2}{Q^2}\right)}{\pi Q^4\left(Q^2-\Lambda^2\right)\left(\Lambda^2+Q^2\right)^2}-\frac{3\Lambda^8\log^2\left(\frac{\Lambda^2}{Q^2}\right)}{\pi Q^6\left(\Lambda^2+Q^2\right)^2}\\ &- \frac{6\Lambda^8\mathrm{Li}_2\left(-\frac{\Lambda^2-Q^2}{Q^2}\right)}{\pi Q^6\left(\Lambda^2+Q^2\right)^2}-\frac{\pi\Lambda^8}{Q^6\left(\Lambda^2+Q^2\right)^2}+\frac{3\Lambda^4\left(4\Lambda^2+Q^2\right)}{2\pi Q^4\left(\Lambda^2+Q^2\right)^2}\end{aligned}$$

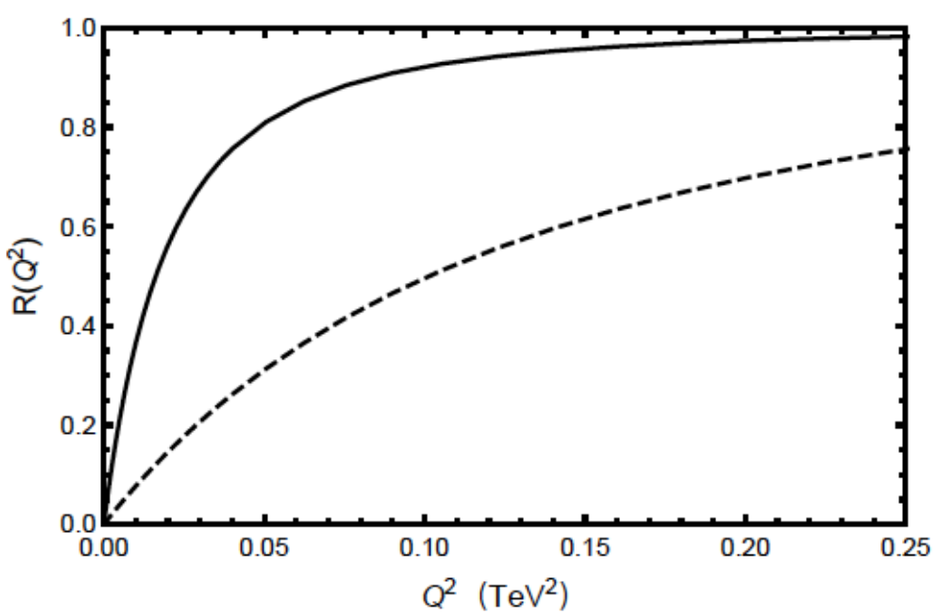
$C(Q^2,\infty)=0$ and $F_2^{lo}(Q^2)$ at leading order is expressed as

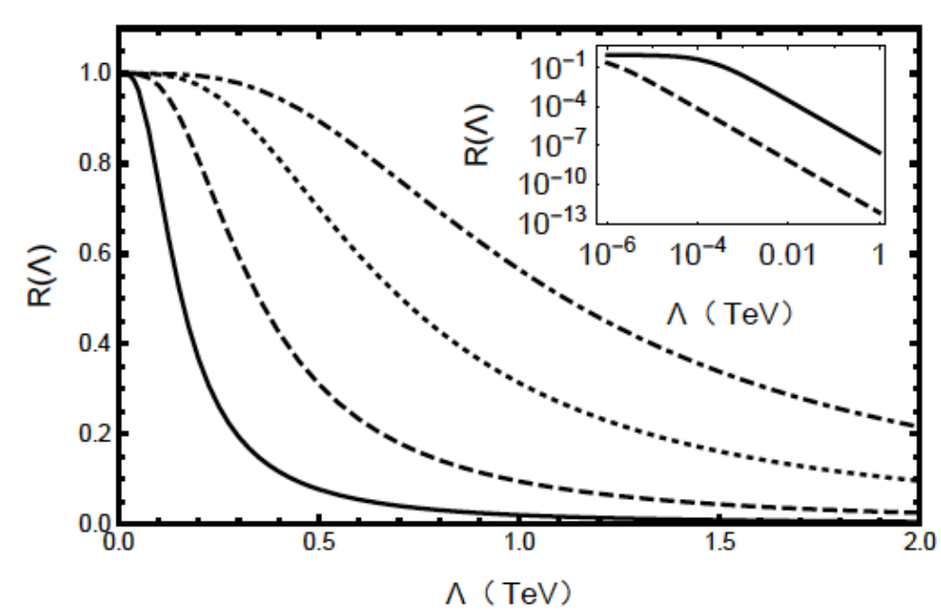
$$F_2^{lo}(Q^2) = -\frac{\alpha\, m^2}{\pi Q^2}\log\left(\frac{m^2}{Q^2}\right)$$

Numerical results:









$$\Delta F_2 = F_2^{lo} - F_2^{nl} \qquad R = \Delta F_2 / F_2^{lo}$$

TABLE I: The discrepancy between nonlocal QED and SM and the relative deviation for electron and muon.

Q^2 (TeV^2)	0	0.001	0.01	0.05	0.1	0.2
ΔF_2^e ($\Lambda = 0.2$ TeV)	2.03×10^{-14}	6.63×10^{-13}	5.41×10^{-13}	2.55×10^{-13}	1.49×10^{-13}	8.08×10^{-14}
ΔF_2^e ($\Lambda = 0.5$ TeV)	3.27×10^{-15}	1.10×10^{-13}	1.14×10^{-13}	9.79×10^{-14}	8.03×10^{-14}	5.79×10^{-14}
R^e ($\Lambda = 0.2$ TeV)	1.74×10^{-11}	0.050	0.370	0.808	0.922	0.973
R^e ($\Lambda = 0.5$ TeV)	2.81×10^{-12}	0.008	0.077	0.311	0.496	0.697
ΔF_2^μ ($\Lambda = 0.2$ TeV)	8.65×10^{-10}	1.50×10^{-8}	1.32×10^{-8}	6.45×10^{-9}	3.84×10^{-9}	2.11×10^{-9}
ΔF_2^μ ($\Lambda = 0.5$ TeV)	1.38×10^{-10}	2.49×10^{-9}	2.80×10^{-9}	2.50×10^{-9}	2.08×10^{-9}	1.52×10^{-9}
R^μ ($\Lambda = 0.2$ TeV)	7.44×10^{-7}	0.051	0.371	0.812	0.924	0.975
R^μ ($\Lambda = 0.5$ TeV)	1.19×10^{-7}	0.008	0.079	0.314	0.500	0.701

Fangcheng He and P. Wang, arXiv:1901.00271

Summary

- Nonlocal Lagrangian is constructed by introducing the correlation function and the gauge link.
- Correlation function generates the relativistic regulator and the loop integral is convergent.
- Gauge link guarantees the locally gauge invariance and generate additional diagrams.
- When Lambda goes to infinity, all the formulas are the same as those in the local case.
- At finite Lambda, the LNAs are the same as those in the local case.
- The nonlocal Lagrangian can be applied to study hadron properties at relatively at large Q^2.
- Without fine-tuning, the agreement between the calculation and experiment is very good.
- The nonlocal behaviour could be the general property for all the interactions.
- Pauli form factor of lepton is the unique quantity to test the point particle assumption.
- For what ever Lambda, the relative deviation R is always significant as long as Q^2 is large enough.
- … …

The End !