Impact of q_T-dependent Drell-Yan data on pion PDFs

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Group Meeting 3/27/2020

Outline

1. Motivation
2. Theory
    a. DY & LN theory
    b. q_T-dependent DY theory
3. Fitting Methodology
4. Results
5. Impact of q_T data
6. Scale Dependence
7. Future works

1. Motivation

Motivation

- QCD allows us to study the structure of protons in terms of partons (quarks, antiquarks, and gluons)
- Use factorization theorems to separate hard partonic physics out of soft, non-perturbative objects to quantify structure

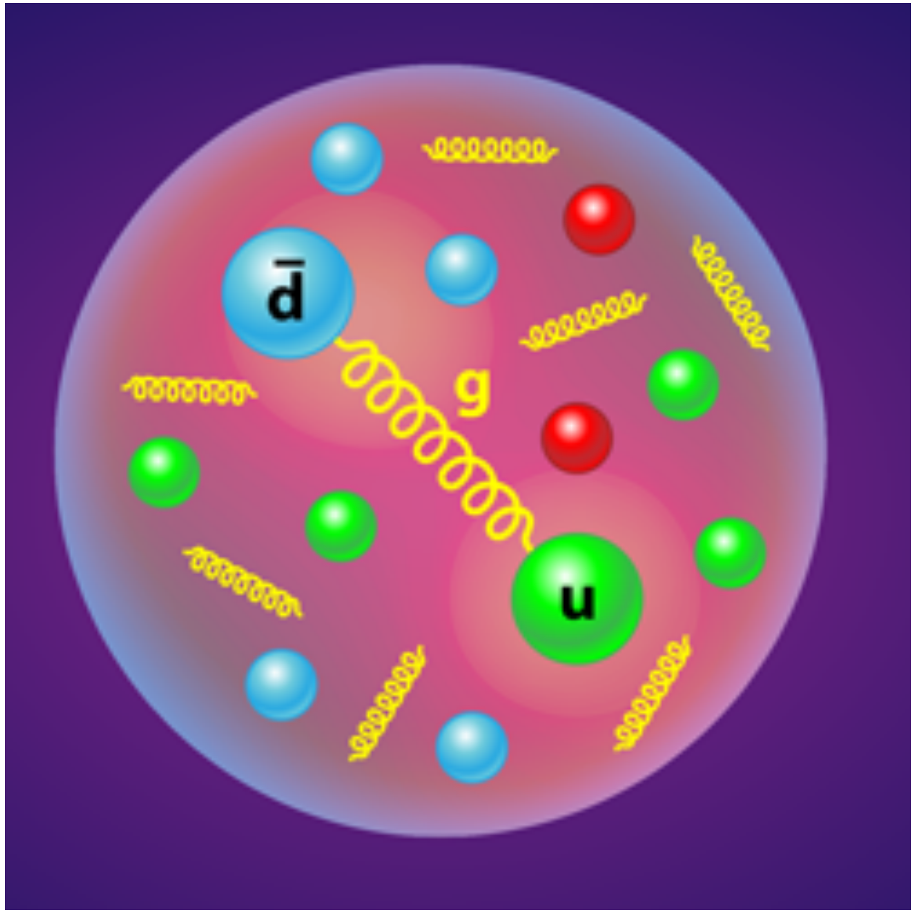
Motivation

What to do:

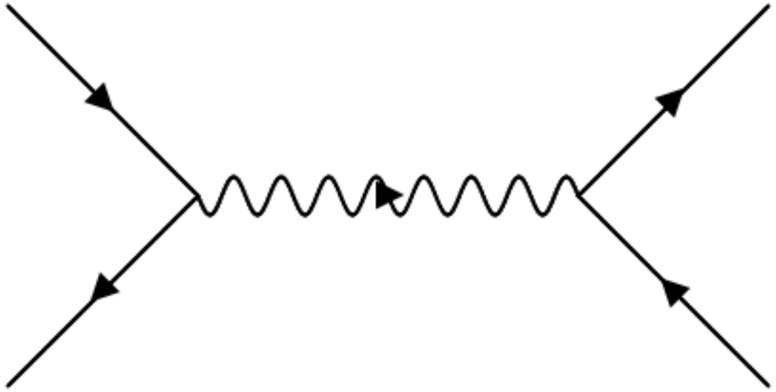
- Define a structure of nucleons in terms of quantum field theories
- Identify theoretical observables that factorize into non-perturbative objects and perturbatively calculable physics
- Perform global QCD analysis as structures are universal and are the same in all subprocesses

Pions

- Pion is the Goldstone boson associated with chiral symmetry breaking
- Lightest hadron as $\frac{m_\pi}{M_N} \ll 1$ and dictates the nature of hadronic interactions at low energies
- Simultaneously a $q\bar{q}$ bound state

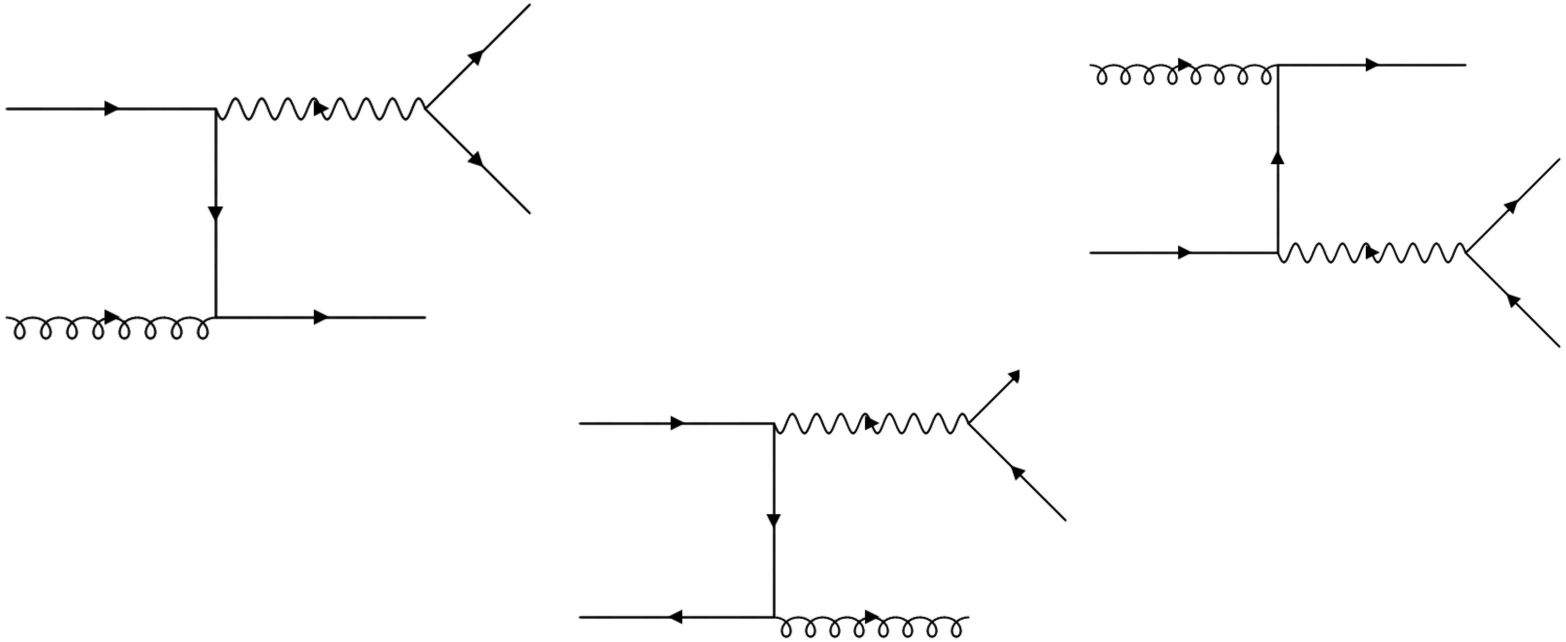


Why q_T-dependent Drell-Yan?



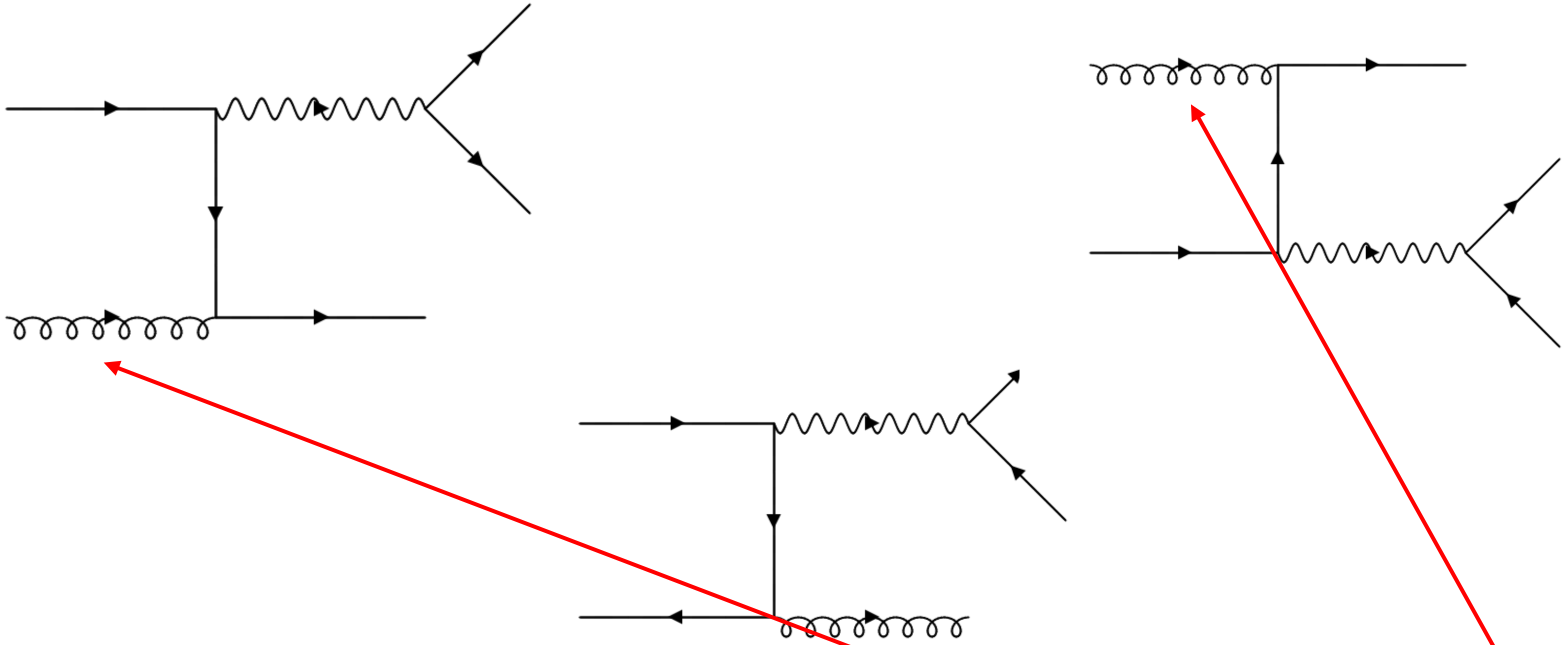
- At Leading order in q_T-integrated Drell-Yan, all of the initial quark/antiquark momentum goes directly into producing a virtual photon
- Leading order contributions dominate cross section
- No recoil, no q_T component

Why q_T-dependent Drell-Yan?



- Any recoil must involve a gluon, meaning the leading order diagrams in q_T dependence are of order $\mathcal{O}(\alpha_S)$

Why q_T-dependent Drell-Yan?



- Any recoil must involve a gluon, meaning the leading order diagrams in q_T dependence are of order $\mathcal{O}(\alpha_s)$
- In principle, there should be a better constraint on the gluon PDF

Motivation

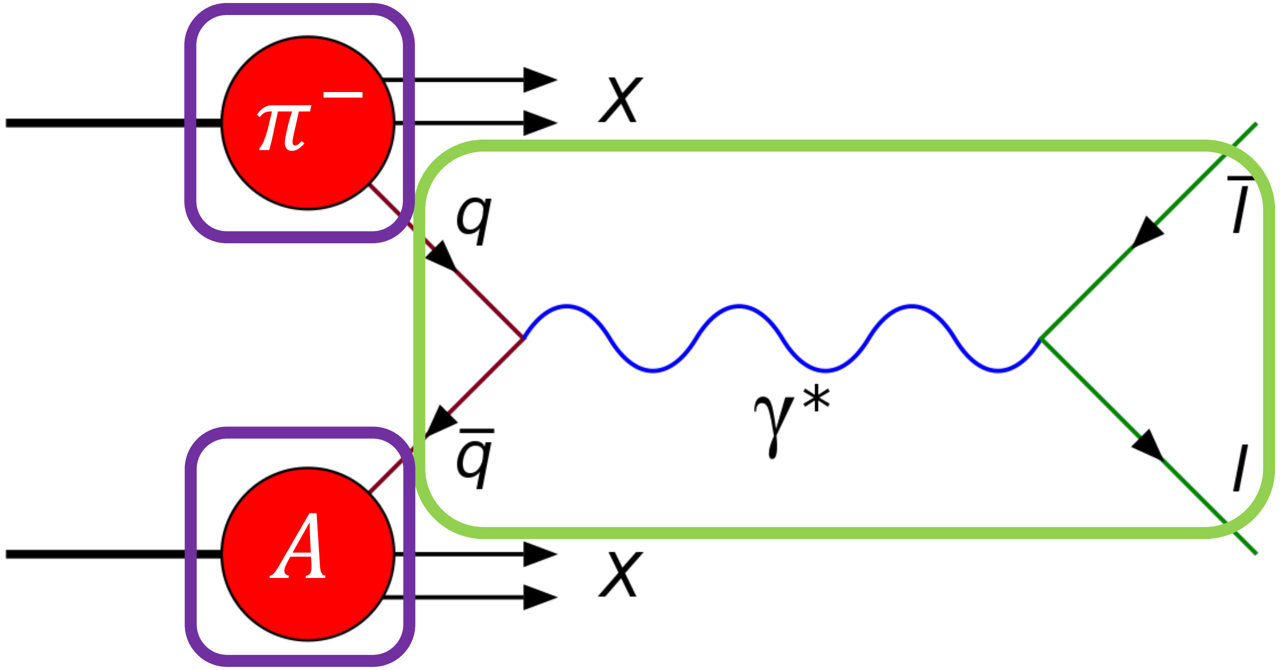
- There also has not been a successful analysis of collinear PDFs fit to q_T-dependent data
- Studying the fixed-order perturbative calculation in q_T has connections with transverse momentum dependent PDFs (TMDPDFs)
- The aim is to fit the q_T-dependent Drell-Yan data along with q_T-integrated Drell-Yan and the Leading Neutron data

2. Theory

2a. Drell-Yan & Leading Neutron Theory

P. C. Barry, N. Sato, W. Melnitchouk, and C. -R. Ji, Phys. Rev. Lett. **121**, 152001 (2018).

Drell-Yan (DY)



$$\sigma \propto \sum_{i,j} f_i^{\pi}(x_\pi, \mu) \otimes f_j^{A}(x_A, \mu) \otimes \hat{\sigma}_{i,j}(x_\pi, x_A, Q/\mu)$$

Drell-Yan (DY)

- p_T integrated DY

$$\frac{d\sigma}{dQ^2 dY} = \frac{4\pi\alpha^2}{3N_C Q^2 S} \sum_{i,j} e_q^2 \int_{x_\pi^0}^1 dx_\pi \int_{x_A^0}^1 dx_A {\color{red} f_i^\pi(x_\pi, \mu)} f_j^A(x_A, \mu) \times \frac{d\hat{\sigma}_{i,j}}{dQ^2 dY}(x_\pi, x_A, Q/\mu)$$

Drell-Yan (DY) Definitions

Hadronic variable

$$\tau = \frac{Q^2}{S}$$

Partonic variable

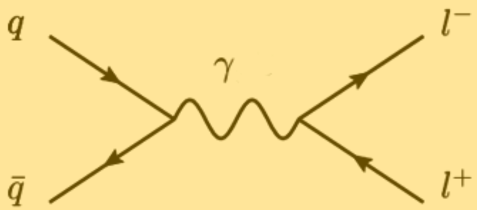
$\hat{S}$ is the center of mass momentum squared of incoming partons

$$z \equiv \frac{Q^2}{\hat{S}} = \frac{\tau}{x_1 x_2}$$

Fixed Order Up to NLO

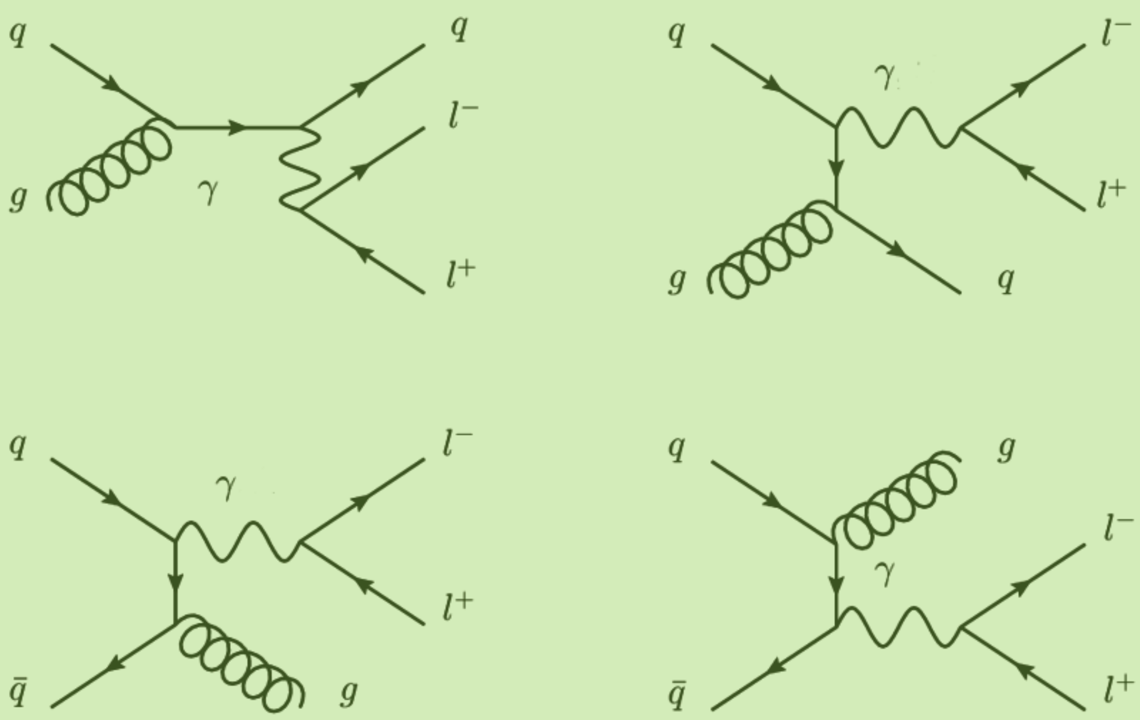
Feynman diagrams for DY amplitudes in collinear factorization

LO: $\mathcal{O}(1)$

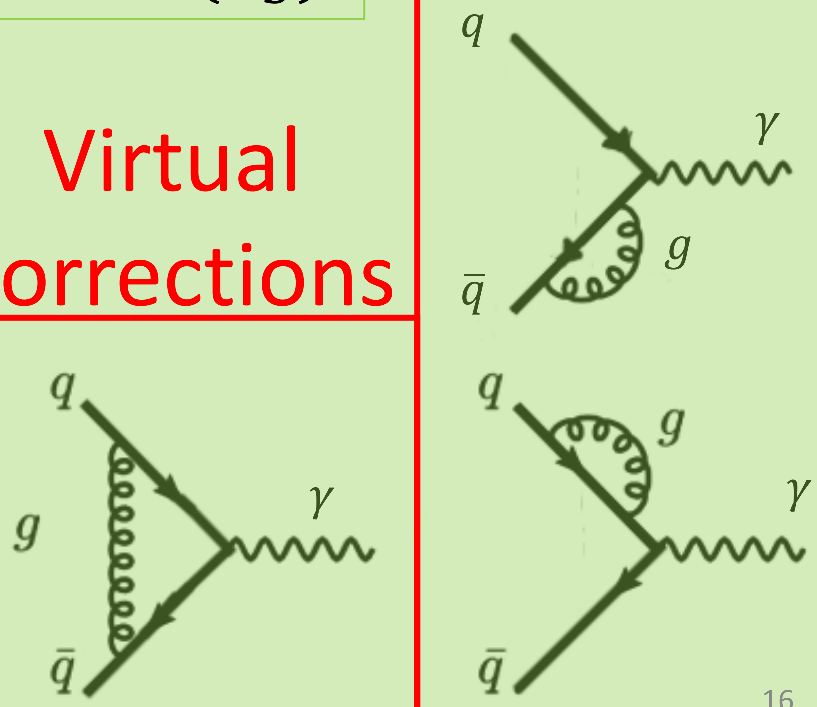


NLO: $\mathcal{O}(\alpha_S)$

Real emissions

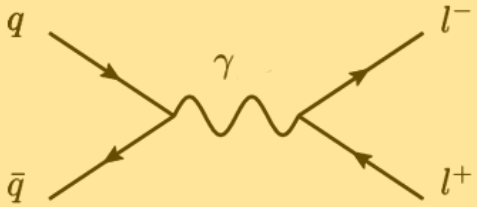


Virtual Corrections



LO

LO: $\mathcal{O}(1)$

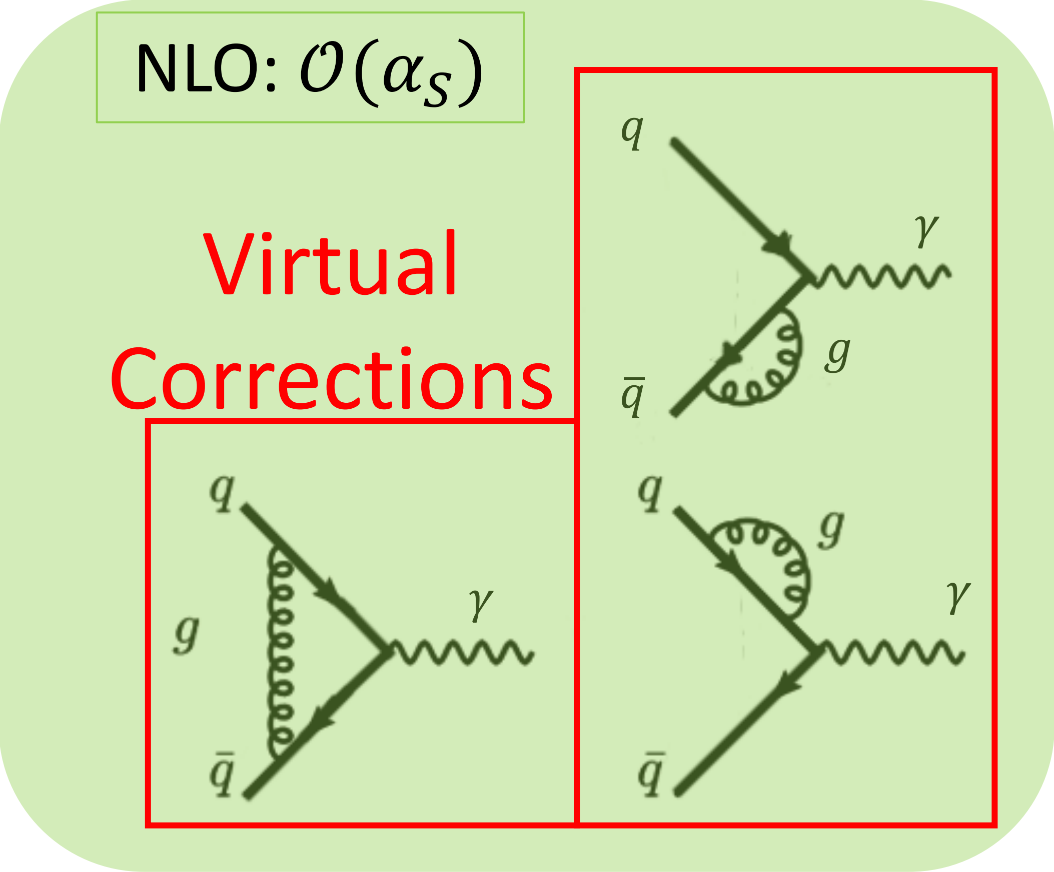


$$C_{q\bar{q}} = \delta(1-z)\frac{\delta(y)+\delta(1-y)}{2}$$

- $z = 1$ corresponds to partonic threshold
- All $\hat{S}$ is equal to Q^2
- All energy of hard partons turns into virtuality of photon

NLO Virtual

- Virtual corrections at NLO are proportional to $\delta(1-z)$
  - Exhibit Born kinematics



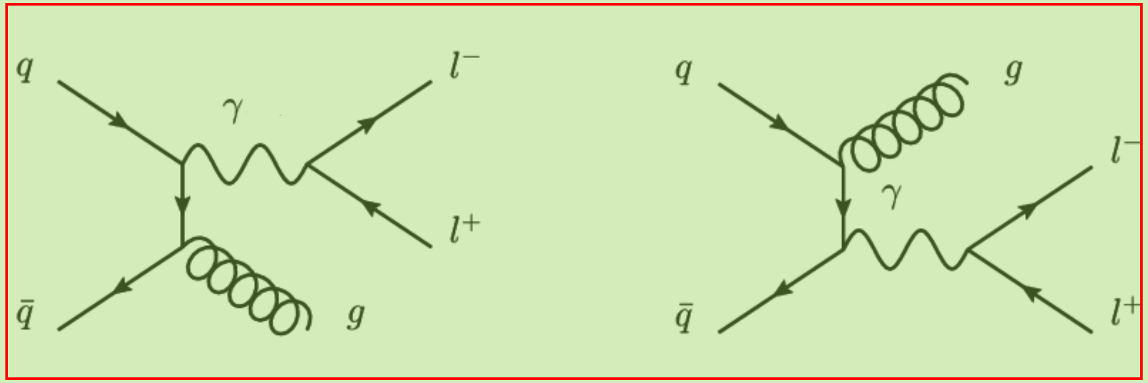
$$C_{q\bar{q}}^{\text{virtual}} = \delta(1-z)\frac{\delta(y)+\delta(1-y)}{2}\left[\frac{C_F\alpha_S}{\pi}\left(\frac{3}{2}\ln\frac{Q^2}{\mu^2}+\frac{2\pi^2}{3}-6\right)\right]$$

NLO Real Emission

- Next to leading order, real gluon emissions

$$C_{q\bar{q}}^{\mathrm{real}} = \frac{C_F \alpha_S}{\pi}\left[\frac{\delta(y)+\delta(1-y)}{2}\left[(1+z^2)\left(\frac{1}{1-z}\ln\frac{Q^2(1-z)^2}{\mu^2 z}\right)_+ + 1 - z\right]\right.$$
$$\left. + \frac{1}{2}\left[\frac{(1-z)^2}{z}y(1-y)\right]\left[\frac{1+z^2}{1-z}\left(\left[\frac{1}{y}\right]_+ + \left[\frac{1}{1-y}\right]_+\right) - 2(1-z)\right]\right]$$

Real emissions

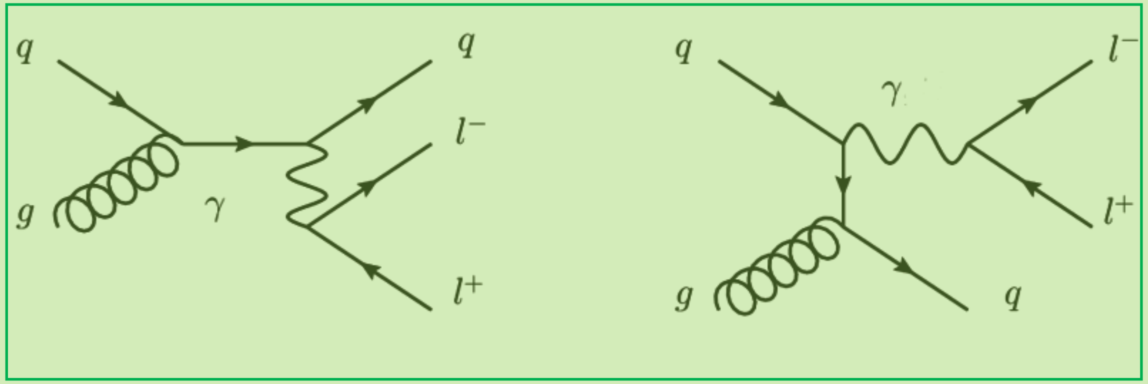


NLO Real Emission

- Plus distributions come from subtraction procedure of collinear singularities
- When $z \to 1, \log(1-z)$ can be large and potentially spoil perturbation
  - Appear in all orders in a predictable manner

NLO Real Emission

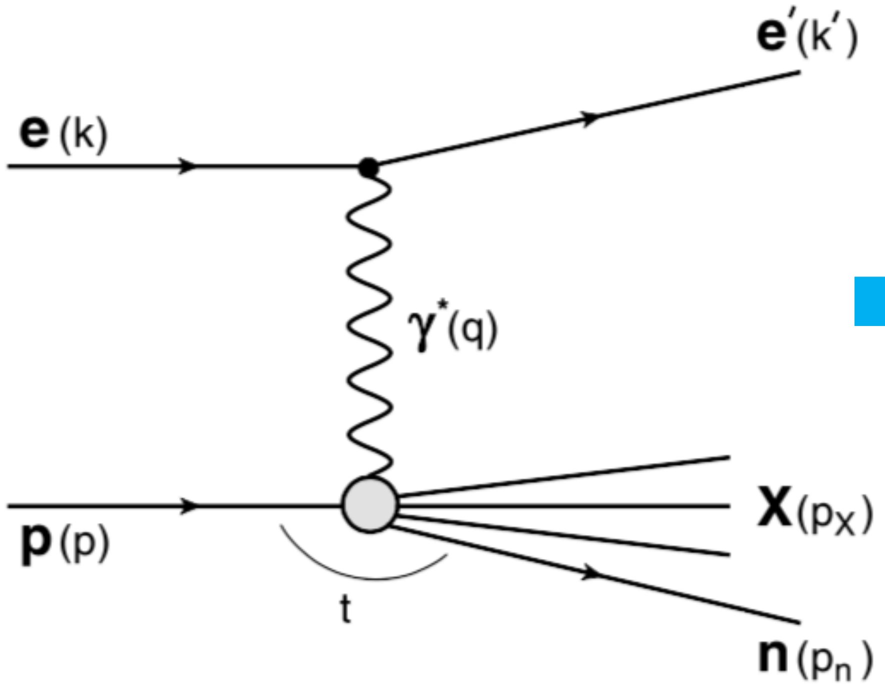
Real emissions

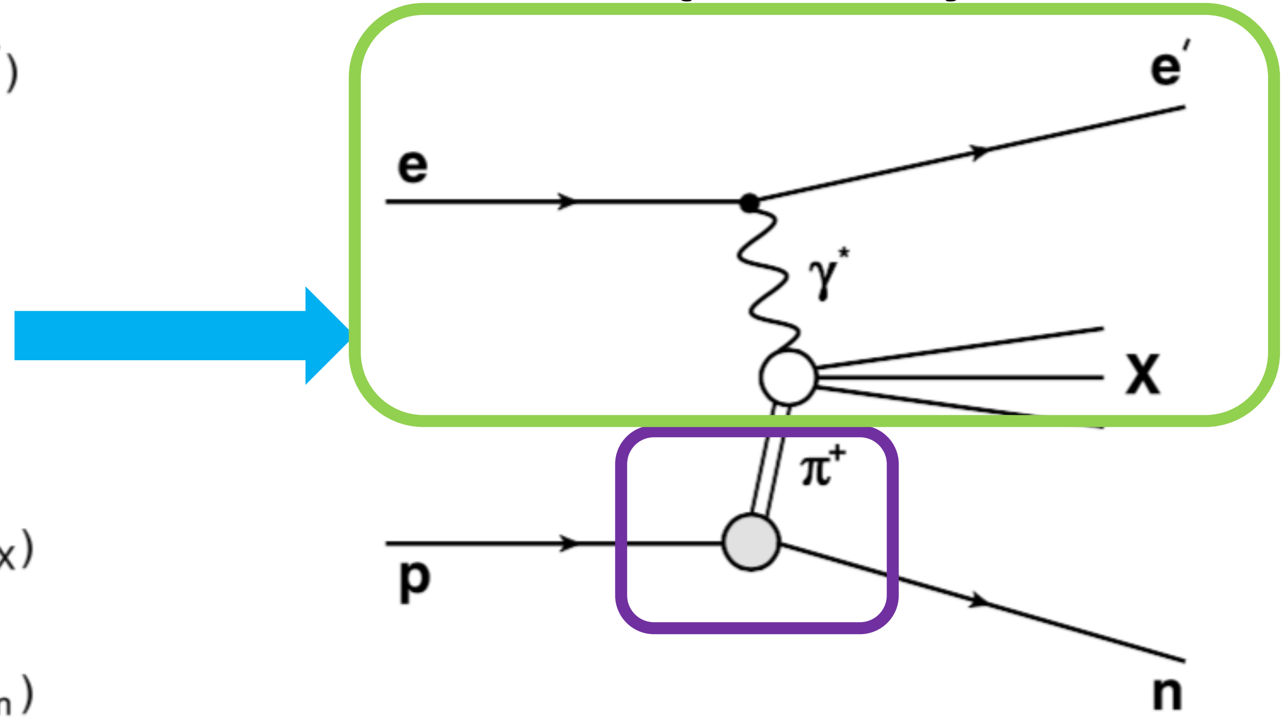


- Real quark emissions
- $C_{qg} = C_{gq}|_{y \to 1-y}$

$$C_{qg}^{\text{real}} = \frac{T_F \alpha_S}{2\pi} \Bigg[\delta(y) \left[(z^2 + (1-z)^2) \ln \frac{Q^2(1-z)^2}{\mu^2 z} + 2z(1-z) \right] + \left[1 + \frac{(1-z)^2}{z} y(1-y) \right] \left[(z^2 + (1-z)^2) \left(\frac{1}{y} \right)_+ + 2z(1-z) + (1-z)^2 y \right] \Bigg]$$

Leading Neutron (LN)





$$\frac{d\sigma}{dx\,dQ^2\,dy} \sim f_{p\to\pi^+n}(y) \times \sum_q \int_{x/y}^{1} \frac{d\xi}{\xi} C(\xi)\, q\left(\frac{x/y}{\xi}, \mu^2\right)$$

LN

- These data provide indirect measure of pion PDFs, since it is virtual
- Need to have as small of $|t|$ as possible
- Assumed dominance of t-channel exchange process by quantum numbers
- Introduce some model dependence

Leading Neutron (LN)

Splitting Function:

$$f_{\pi N}(y) = \frac{g_A^2 M^2}{(4\pi f_\pi)^2} \int dk_\perp^2 \frac{y[k_\perp^2 + y^2 M^2]}{x_L^2 D_{\pi N}^2} |\mathcal{F}|^2$$

Where $y = k^+/p^+ = x/x_\pi$,
$g_A = 1.267$, $f_\pi = 93\text{MeV}$

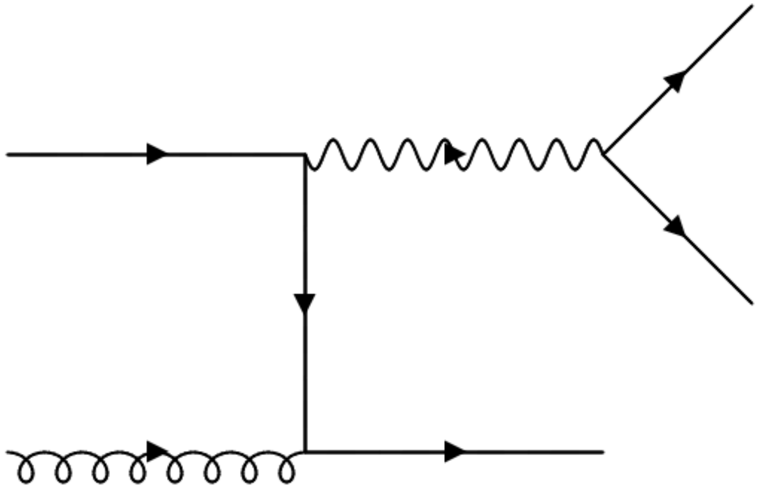
$$D_{\pi N} \equiv t - m_\pi^2 = -\frac{1}{1-y}[k_\perp^2 + y^2 M^2 + (1-y) m_\pi^2]$$

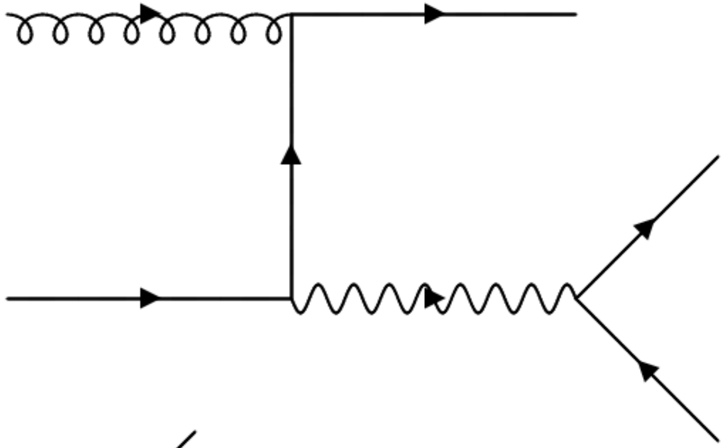
$$\mathcal{F} = \begin{cases} \text{(i)} & \exp\left((M^2 - s)/\Lambda^2\right) & s\text{-dep. exponential} \\ \text{(ii)} & \exp\left(D_{\pi N}/\Lambda^2\right) & t\text{-dep. exponential} \\ \text{(iii)} & (\Lambda^2 - m_\pi^2)/(\Lambda^2 - t) & t\text{-dep. monopole} \\ \text{(iv)} & \bar{x}_L^{-\alpha_\pi(t)} \exp\left(D_{\pi N}/\Lambda^2\right) & \text{Regge} \\ \text{(v)} & \left[1 - D_{\pi N}^2/(\Lambda^2 - t)^2\right]^{1/2} & \text{Pauli-Villars} \end{cases}$$

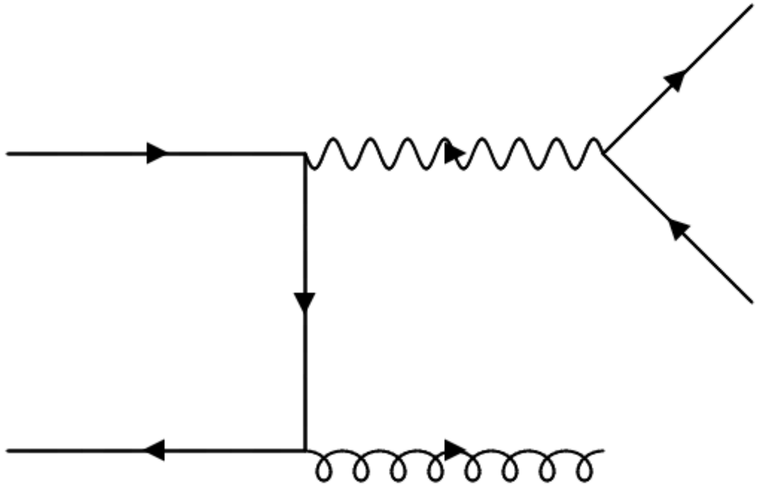
UV regulators used in the literature

2a. DY-q_T Dependent Theory

Leading Order Diagrams







Drell-Yan (DY)

- p_T dependent DY

$$\frac{d\sigma}{dQ^2 dy dq_T^2} = \sum_{ab} \int_{x_{a,\min}}^{1} dx_a \frac{x_a x_b}{x_a - 1} f_{a/A}(x_a, \mu^2) f_{b/B}(x_b, \mu^2) \frac{d\hat{\sigma}_{ab}}{dQ^2 d\hat{t}}$$

Here, y is the rapidity, Q^2 is the invariant mass squared of the virtual photon, q_T is the transverse momentum of the virtual photon

q_T-dependent DY definitions

- Effectively a $2 \to 2$ process, where we have
  - $q\bar{q} \to \gamma^* g$,
  - $qg \to \gamma^* q$,
  - and $gq \to \gamma^* q$

$$p_a = x_a P_A \qquad p_b = x_b P_B$$

$$s = (P_A + P_B)^2 \qquad t = \left(P_{\gamma^*} - P_A\right)^2 \qquad u = \left(P_{\gamma^*} - P_B\right)^2$$

$$\hat{s} = x_a x_b s \qquad \hat{t} - M^2 = x_a(t - Q^2) \qquad \hat{u} - Q^2 = x_b(u - Q^2)$$

$$\hat{s} + \hat{t} + \hat{u} = Q^2$$

q_T-dependent DY definitions

$$x_a = \frac{x_b x_1 - \tau}{x_b - x_2}$$

$$x_b = \frac{x_a x_2 - \tau}{x_a - x_1}$$

$$x_1 = -\frac{u - Q^2}{s} = \frac{1}{2}(x_T^2 + 4\tau)^{1/2} e^{Y}$$

$$x_2 = -\frac{t - Q^2}{s} = \frac{1}{2}(x_T^2 + 4\tau)^{1/2} e^{-Y}$$

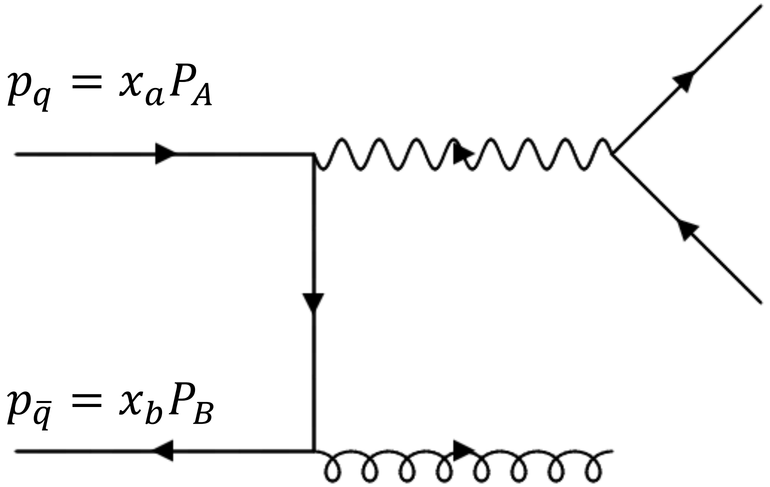
$$x_T = 2p_T/\sqrt{s}$$

$$\hat{t} = -x_a \hat{s} x_2 + Q^2$$

$$\hat{u} = -x_b \hat{s} x_1 + Q^2$$

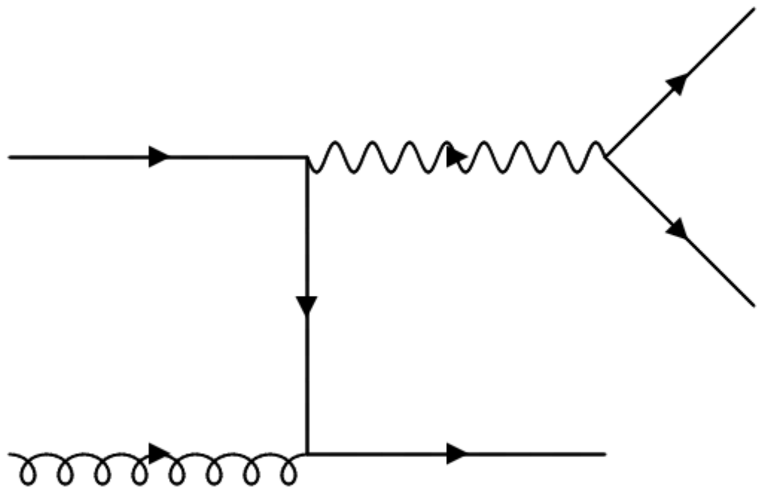
$$x_a^{\min} = \frac{x_1 - \tau}{1 - x_2}$$

Annihilation Term



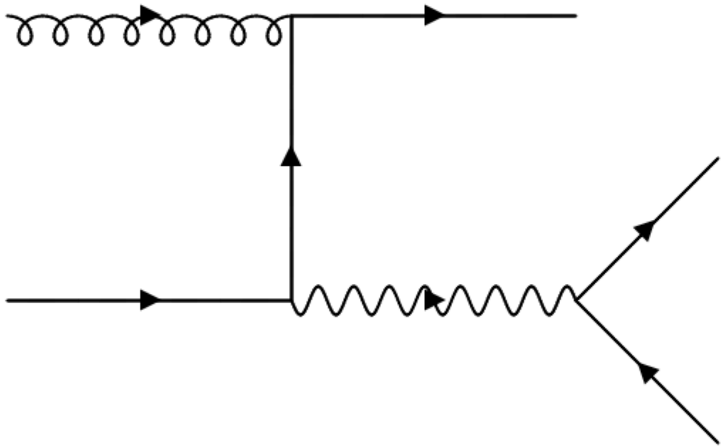
$$\frac{d\hat{\sigma}_A}{dQ^2 d\hat{t}}(\hat{s},\hat{t}) = \frac{8}{27}\frac{\alpha^2 \alpha_S e_q^2}{Q^2 \hat{s}^2}\left(\frac{2Q^2\hat{s} + \hat{u}^2 + \hat{t}^2}{\hat{t}\hat{u}}\right)$$

Compton term



$$\frac{d\hat{\sigma}_C}{dQ^2 d\hat{t}}(\hat{s},\hat{t}) = -\frac{1}{9}\frac{\alpha^2\alpha_S e_q^2}{Q^2\hat{s}^2}\left(\frac{2Q^2\hat{u}+\hat{s}^2+\hat{t}^2}{\hat{s}\hat{t}}\right)$$

Compton term



$$\frac{d\hat{\sigma}_C}{dQ^2 d\hat{t}}(\hat{s},\hat{t}) = -\frac{1}{9}\frac{\alpha^2\alpha_S e_q^2}{Q^2\hat{s}^2}\left(\frac{2Q^2\hat{t}+\hat{s}^2+\hat{u}^2}{\hat{s}\hat{u}}\right)$$

Observables

Same E615 $\pi^- W \rightarrow \mu^+ \mu^- X$ experiment as in the q_T-integrated case

$$\frac{d\sigma}{dQdp_T} = 4Qq_T \int_{y_{\min}}^{y_{\max}} dy \sum_{a,b} \int_{x_{a,\min}}^{1} dx_a \frac{x_a x_b}{x_a - 1} f_{a/\pi}(x_a, \mu^2) f_{b/W}(x_b, \mu^2) \frac{d\hat{\sigma}_{ab}}{dQ^2 d\hat{t}}$$

$$\frac{d\sigma}{dx_F dp_T} = 4q_T m_T \cosh y \int_{Q_{\min}}^{Q_{\max}} QdQ \sum_{a,b} \int_{x_{a,\min}}^{1} dx_a \frac{x_a x_b}{x_a - 1} f_{a/\pi}(x_a, \mu^2) f_{b/W}(x_b, \mu^2) \frac{d\hat{\sigma}_{ab}}{dQ^2 d\hat{t}}$$

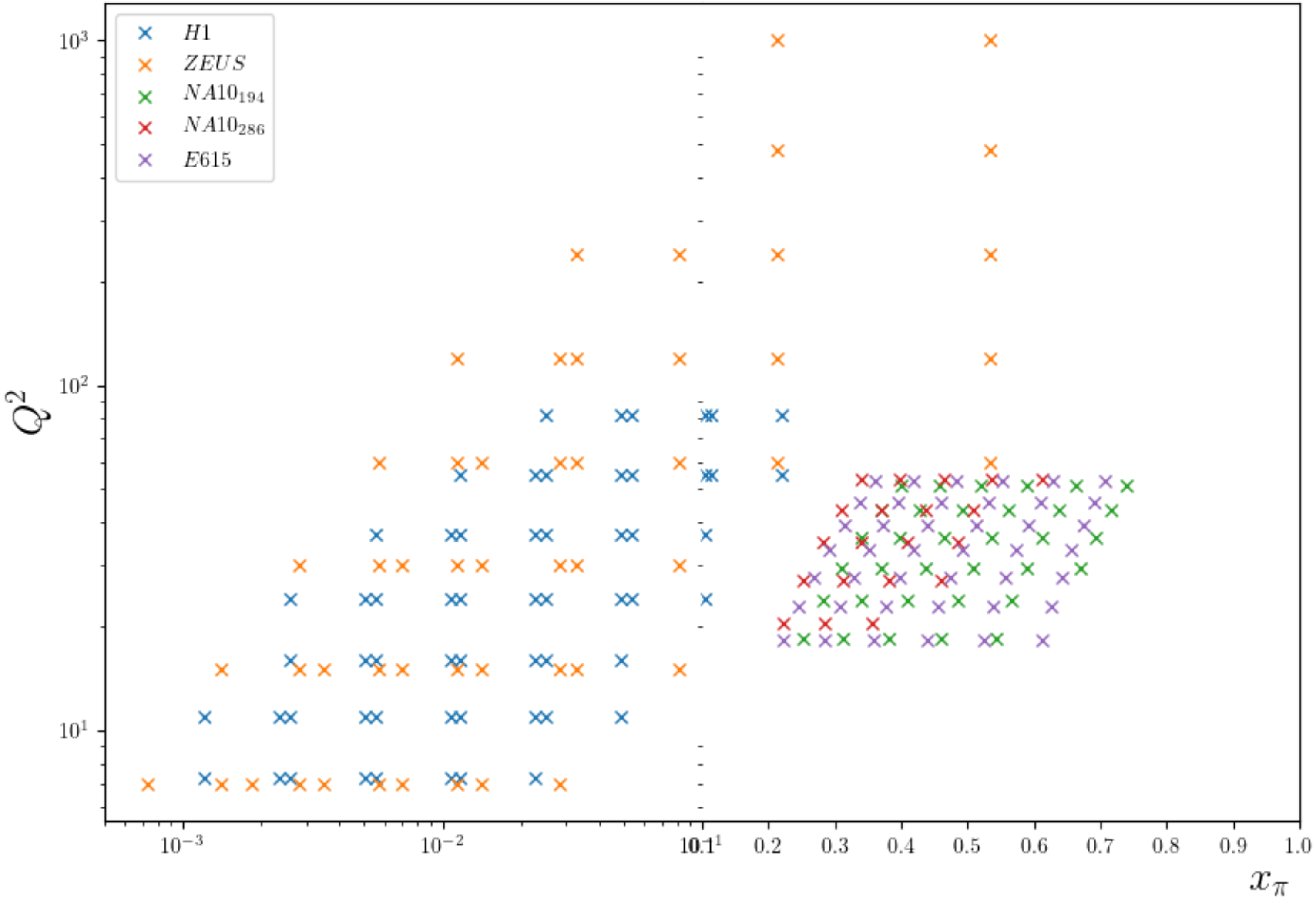
$$m_T = \sqrt{Q^2 + q_T^2}$$

3. Fitting Methodology

Datasets

- We fit the PDFs to the following datasets
- DY:
  - E615 – FNAL
  - NA10 – CERN
- LN:
  - H1 – HERA at DESY
  - ZEUS – HERA at DESY
- q_T-dependent DY
  - E615 – FNAL (Q-dependent and x_F-dependent)

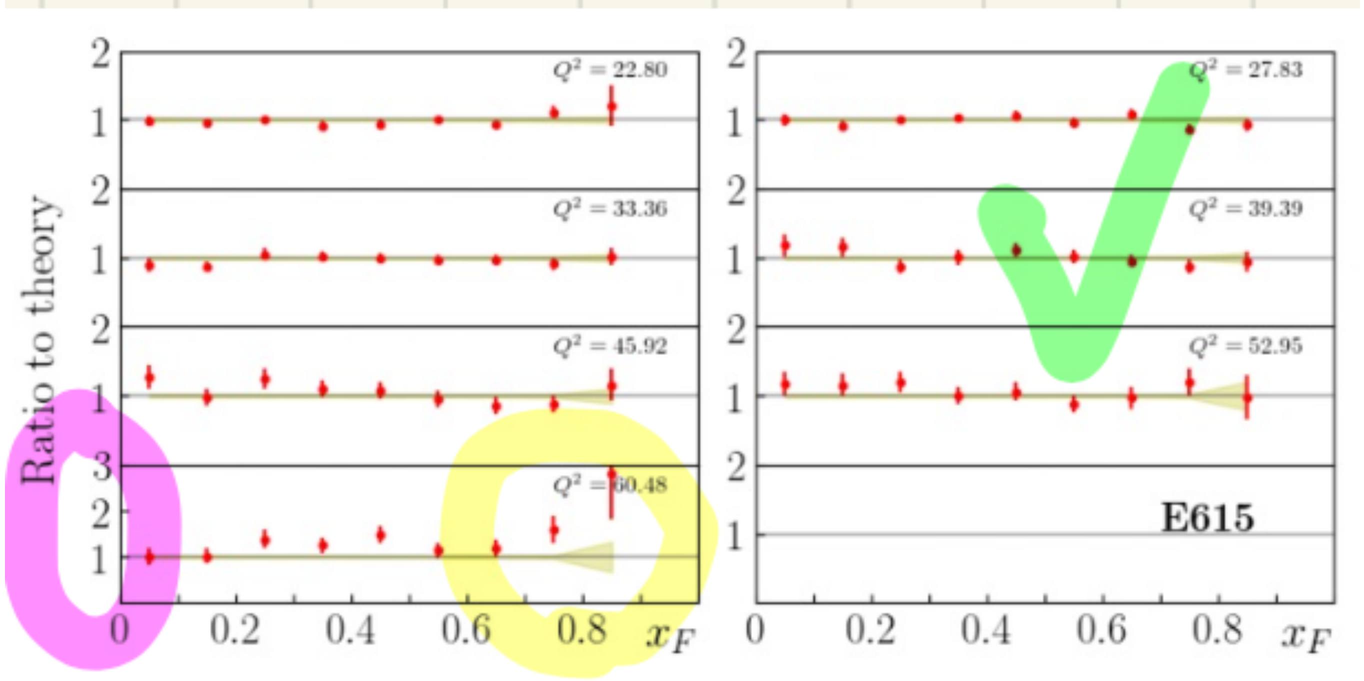
Kinematics



H1
ZEUS
NA10194
NA10286
E615
Q^2
10^3
10^2
10^1
10^{-3}
10^{-2}
10^{-1}
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
x_π

Kinematic Cuts

- DY
  - We first make a cut on $4.16^2 < Q^2 < 8.34^2$ in GeV² to avoid the J/ψ and Υ resonances
  - However, we find that the highest Q^2 bin in the E615 dataset is difficult to fit



The theory is undershooting the data in the yellow circle.

At such a high Q^2 and x_F, the data may not be well described by NLO DY theory.

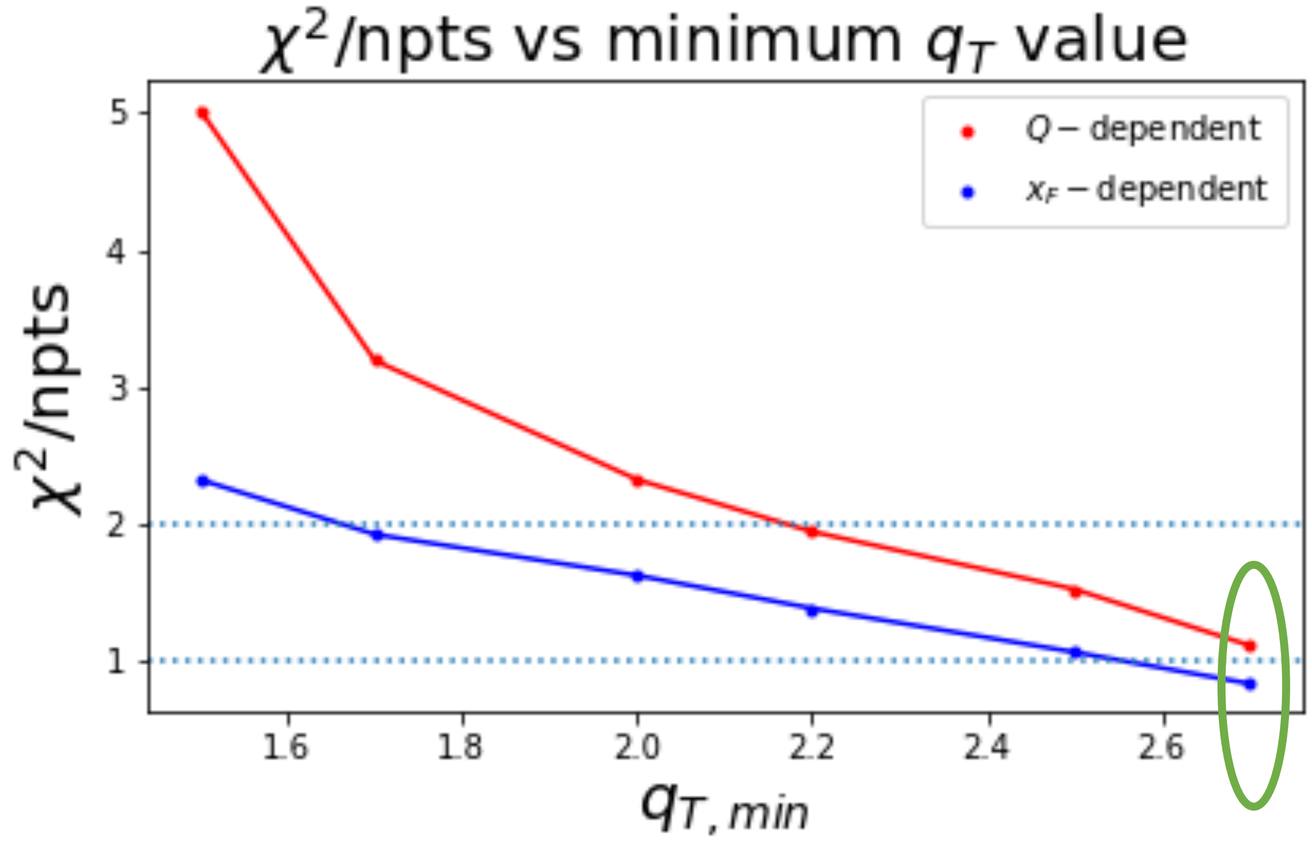
Perhaps threshold resummation is needed

Kinematic Cuts

- DY
  - We limit $4.16^2 < Q^2 < 7.68^2$ in GeV^2
  - To also ensure that factorization theorems are holding, we limit the Feynman x to be $0 < x_F < 0.6$.
    - At such a large x_π, we are nearing the threshold of the phase space, and fixed order calculations may be dangerous
- LN
  - We fit data only where $x_L > 0.8$
- q_T-dependent DY
  - Because the falloff of the cross-sections at large Q^2 and large x_F, the integration of the dQ or dx_F observable will not draw much from the high Q^2or x_F region and we do not make kinematic cuts for those variables.
  - We do limit Q^2 by the resonances, however, as in the inclusive case

q_T cuts for DY

- The fixed-order q_T-dependent DY is only applicable at large q_T
- TMD physics comes into play at small-q_T
- We need to determine what is a good cutoff for q_T
- Systematically include more q_T points and determine the goodness of fit by looking at χ^2



Good agreement at $q_T = 2.7$ GeV!

PDF Parametrization

- We parametrize the PDF by the following functional form

$$f_i^{\pi}(x_\pi, \mu^2) = \frac{N_i x_\pi^{a_i}(1-x_\pi)^{b_i}}{\int_0^1 dx_\pi x_\pi^{a_i+1}(1-x_\pi)^{b_i}}$$

- We've tried Two shapes for more flexibility:

$$f_i^{\pi}(x_\pi, \mu^2) = \frac{N_{1,i} x_\pi^{a_{1,i}}(1-x_\pi)^{b_{1,i}}}{\int_0^1 dx_\pi x_\pi^{a_{1,i}+1}(1-x_\pi)^{b_{1,i}}} + \frac{N_{2,i} x_\pi^{a_{2,i}}(1-x_\pi)^{b_{2,i}}}{\int_0^1 dx_\pi x_\pi^{a_{2,i}+1}(1-x_\pi)^{b_{2,i}}}$$

- But one shape always seems to have just as good of χ^2, and it does the job

PDF Parametrization

- We parametrize in the same way the
  - valence quark distribution, $v^{\pi} \equiv \bar{u}_v^{\pi^-} = \bar{u}^{\pi^-} - u^{\pi^-} = d_v^{\pi^-} = u_v^{\pi^+} = \bar{d}_v^{\pi^+}$,
  - the sea quark distribution, $s^{\pi} \equiv u^{\pi^-} = \bar{u}_s^{\pi^-} = d_s^{\pi^-} = \bar{d}^{\pi^-} = s^{\pi} = \bar{s}^{\pi}$,
  - and the gluon distribution, g
- Two sum rules exist to constrain normalizations:
  - The quark number rule:

$$\int_0^1 dx_\pi v^\pi(x_\pi, \mu^2) = 1$$

Constrains normalization of the valence PDF

  - And the momentum sum rule:

$$\int_0^1 dx_\pi x_\pi (2v^\pi(x_\pi, \mu^2) + 6s^\pi(x_\pi, \mu^2) + g(x_\pi, \mu^2)) = 1$$

Constrains the normalization of the *sea*

Why Fix Normalization of the Sea?

- Previously, we fit the normalization of the gluon by the momentum sum rule
- We tend to see negative gluon distributions if

$$N_g = 1 - \int_0^1 dx_\pi x_\pi (2v^\pi(x_\pi, \mu^2) + 6s^\pi(x_\pi, \mu^2))$$

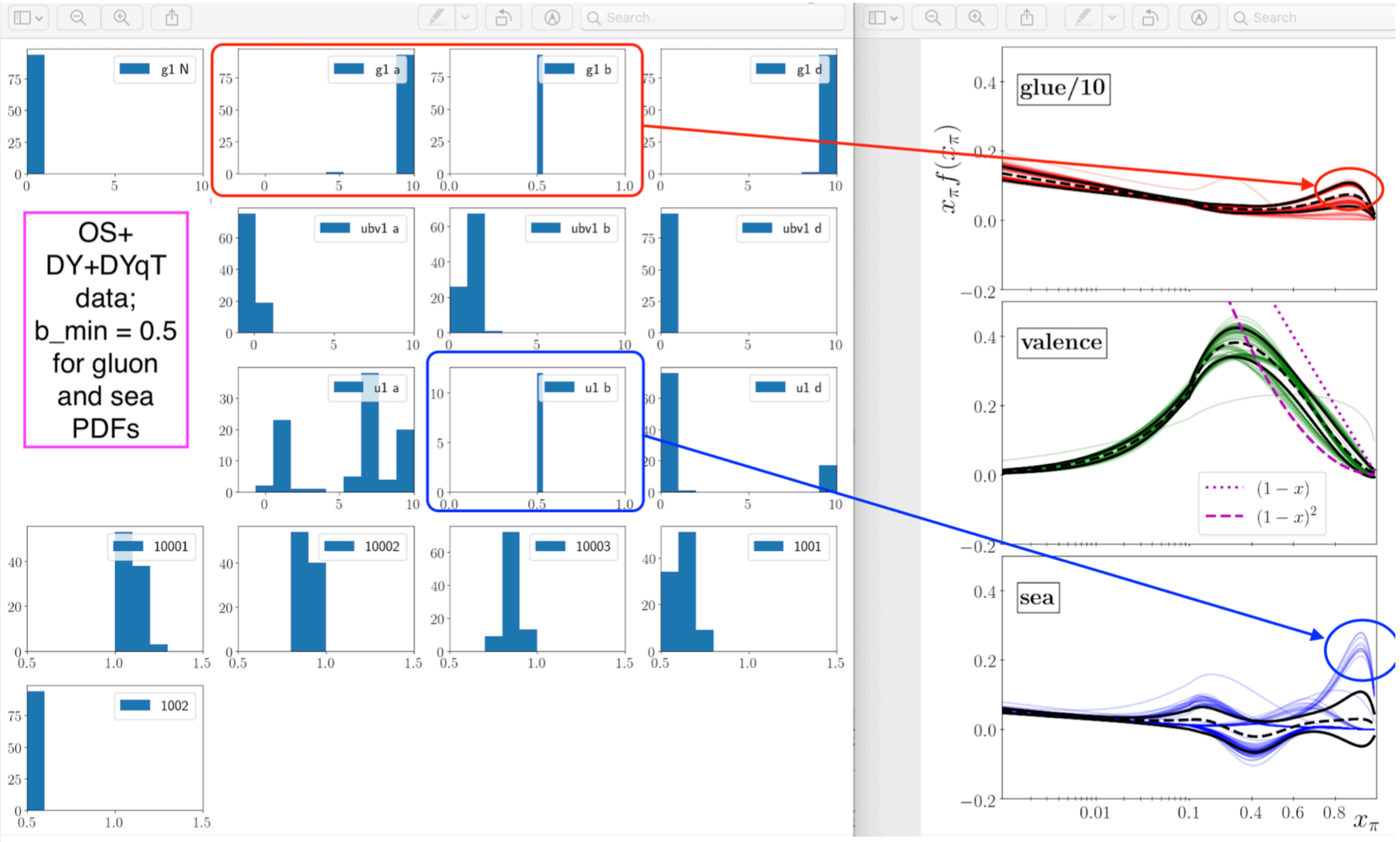
- We previously had this problem too! But we included a large penalty on the χ^2 when the gluon's normalization was negative
  - The penalty is not ideal when fitting with steepest gradient descent!

Fixing Normalization of the Sea

- We can more easily fix the normalization of the sea and hard-code the normalization of the gluon to be a free parameter, but strictly positive
- The sea never really becomes negative even though it is allowed to

Additional parameter constraints

- We also constrained the b parameter of the sea and gluon to be greater than 5
- Tried first a minimum of 0.5:
- In the probabilistic interpretation of PDFs: doesn't make sense to have such large sea and gluon distributions at x_π near 1



Bayesian Statistics

- The probability of the parameter set $\vec{a}$ given the data is

$$\mathcal{P}(\vec{a}|\text{data}) = \frac{1}{Z}\mathcal{L}(\text{data}|\vec{a})\pi(\vec{a})$$

- Where Z is the Bayesian evidence, π is the Bayesian priors, and the likelihood function is

$$\mathcal{L}(\text{data}|\vec{a}) = \exp\Big(-\frac{1}{2}\chi^2(\vec{a})\Big)$$

Bayesian Statistics

- The χ^2 function is

$$\chi^2_{\text{expt}}(\vec{a}) = \sum_i \frac{(D_i + S_i - T_i(\vec{a})/N)^2}{\sum_j (\alpha_{i,j})^2}$$

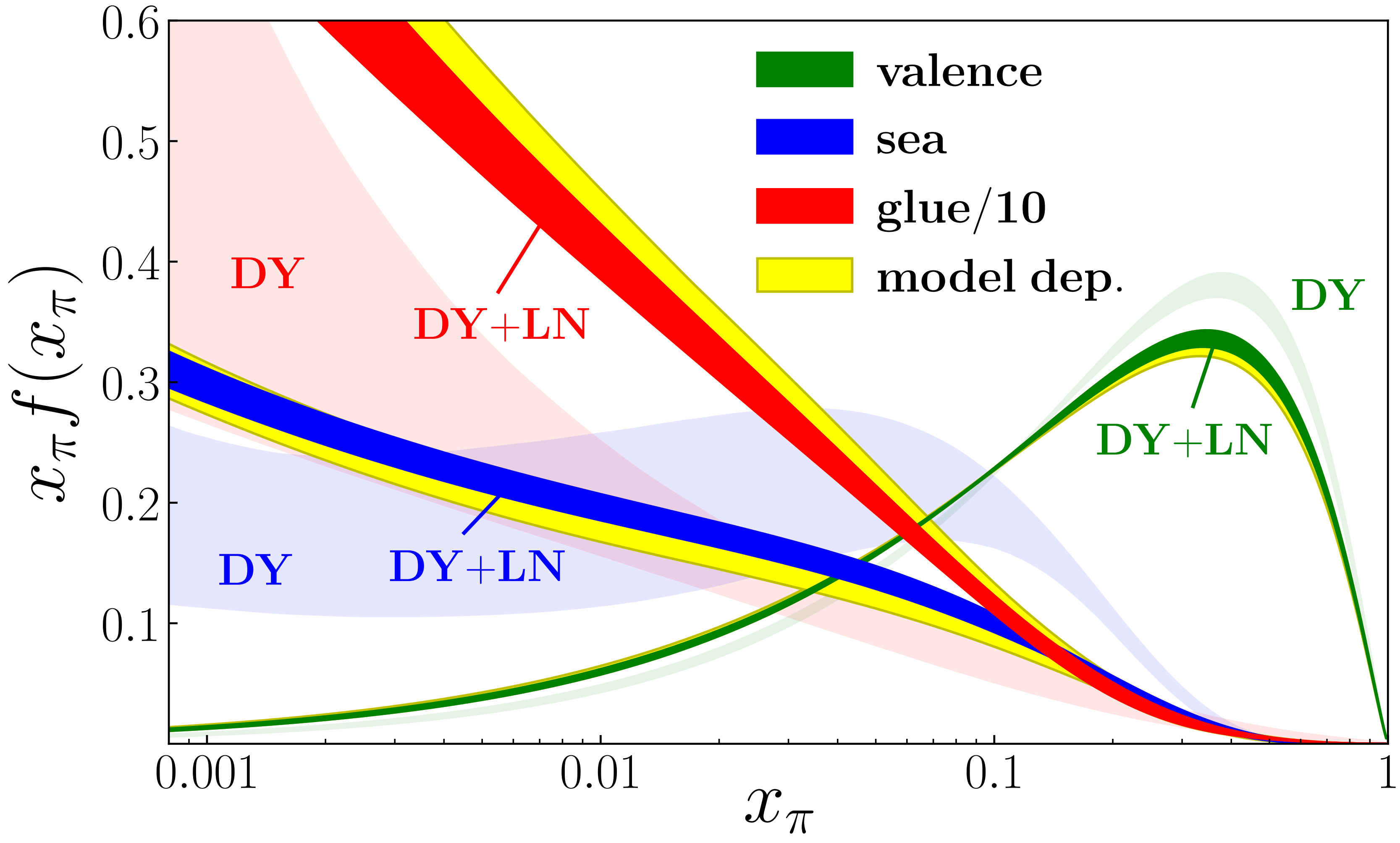
- D is each data point, S is the systematic shift associated with correlated uncertainties, $T(\vec{a})$ is the theory calculation, based on the parameter set, N is the overall normalization for the experiment, and α are the uncorrelated statistical uncertainties

The strategy

- We start with a flat Bayesian prior, π, where the parameter set is randomly chosen
- We bootstrap the data, meaning the central values of the data are allowed to fluctuate randomly within the uncertainties
  - This ensures that we fit a slightly different dataset each time, reflecting the statistical uncertainties in the data into the fit
- We fit first only the q_T-integrated DY data
- Next, the priors are the $\vec{a}$ that we found from the DY only fit. We add the LN data to the DY data and fit.
- Using the priors from DY+LN, we fit the full datasets, DY+LN+q_T-dependent DY

4. Results

DY & LN study



valence
sea
glue/10
model dep.
DY
DY+LN
DY
DY+LN
DY
DY+LN
$x_\pi f(x_\pi)$
x_π
0.6
0.5
0.4
0.3
0.2
0.1
0.001
0.01
0.1
1

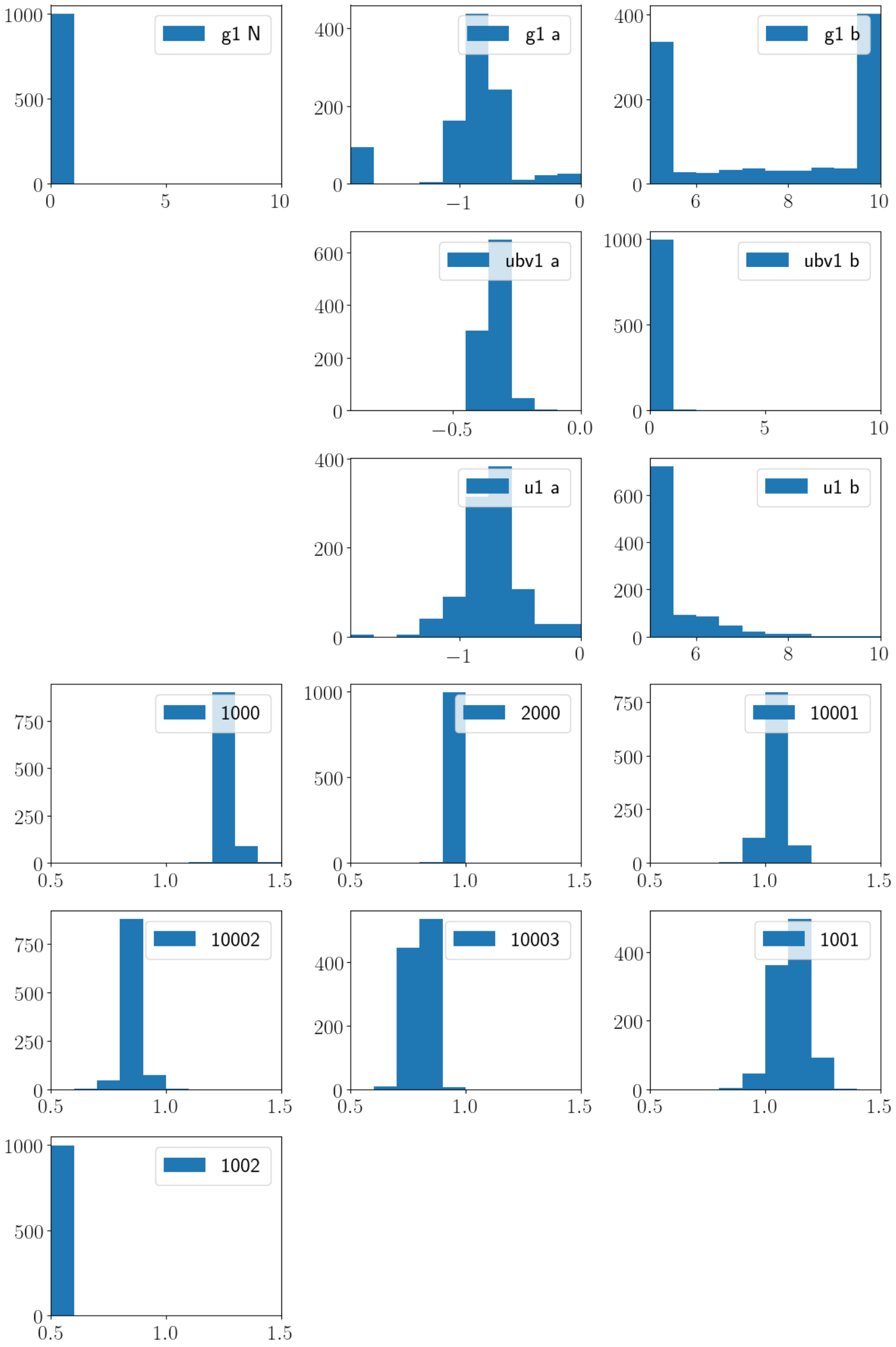
Including q_T-data

Process	Experiment	# of points	χ^2	χ^2 /npts	norm
DY	E615	42	37.03	0.88	1.05
	$NA10_{194}$	34	18.53	0.54	0.86
	$NA10_{286}$	20	14.11	0.71	0.80
LN	H1	58	22.52	0.39	1.26
	ZEUS	50	73.22	1.46	0.95
DY-q_T	E615 x_F-integrated	34	37.86	1.11	1.11
	E615 Q-integrated	49	40.49	0.83	0.51
	Total	287	243.74	0.85	

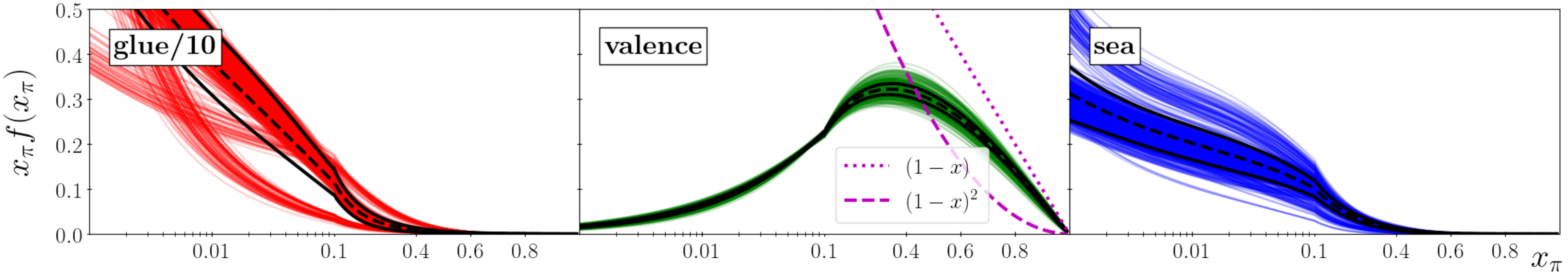
Parameters

- Shrank the range of certain parameters (b parameters for the sea and gluon)

$$f_i^{\pi}(x_\pi, \mu^2) = \frac{N_i x_\pi^{a_i}(1-x_\pi)^{b_i}}{\int_0^1 dx_\pi x_\pi^{a_i+1}(1-x_\pi)^{b_i}}$$



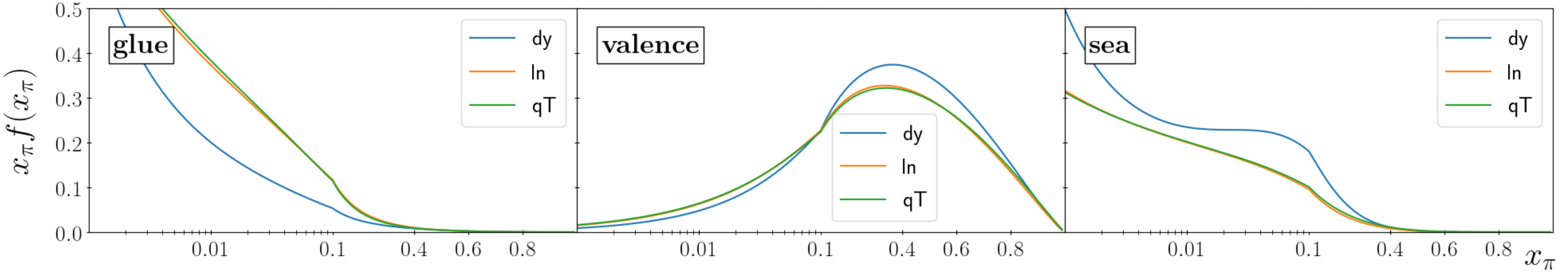
PDFs



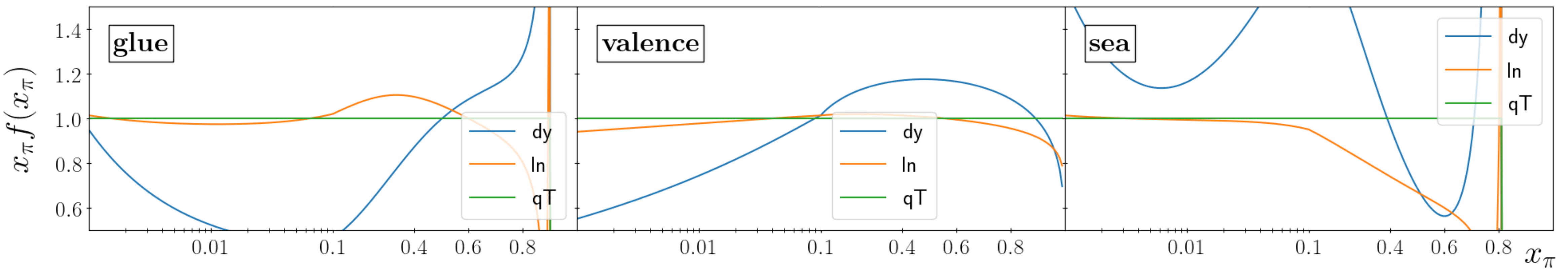
- Shown are the PDFs fit to all the datasets, and the scale is $\mu = q_T/2$ for the q_T-dependent DY data

PDFs

- Central Values

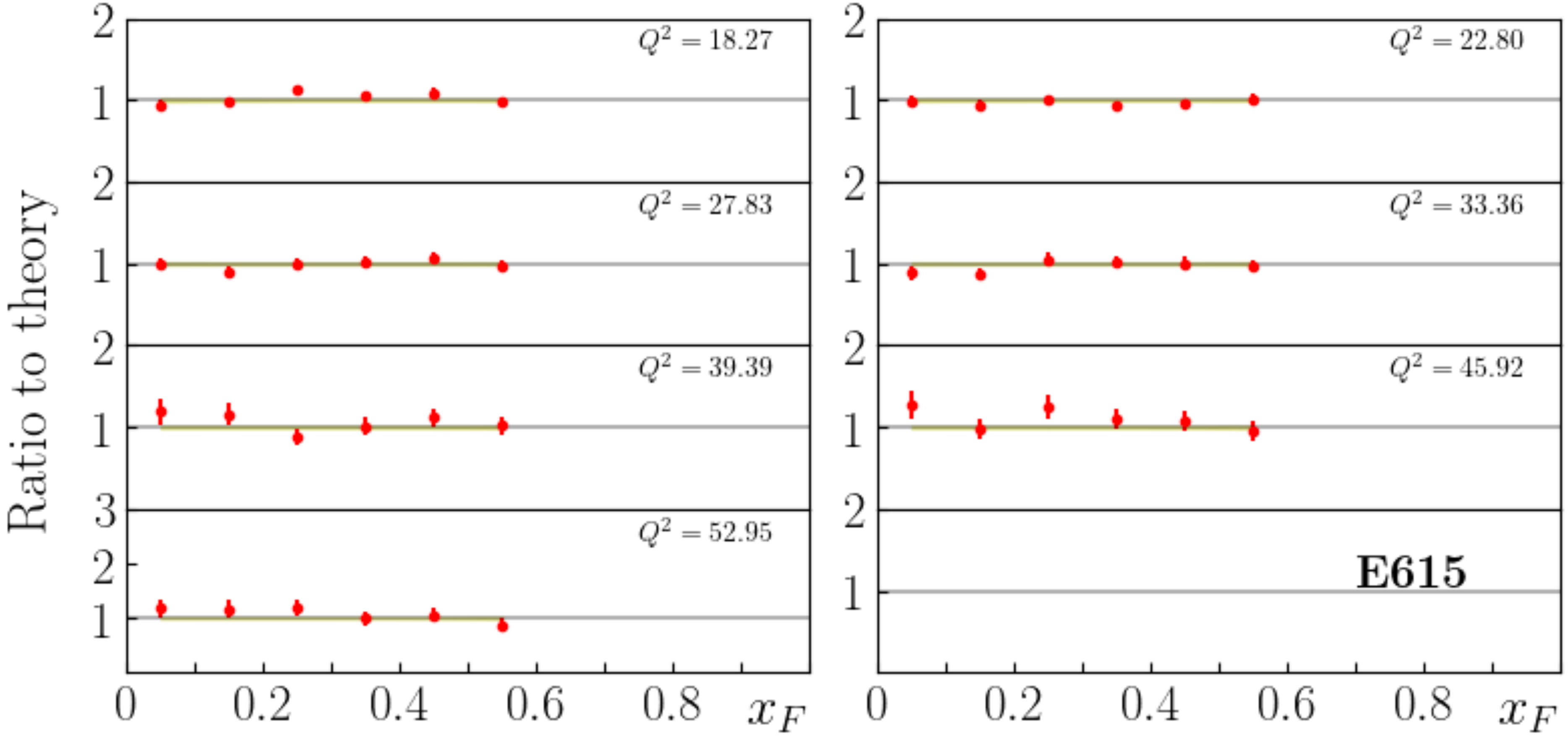


- Central Values relative to the full fit



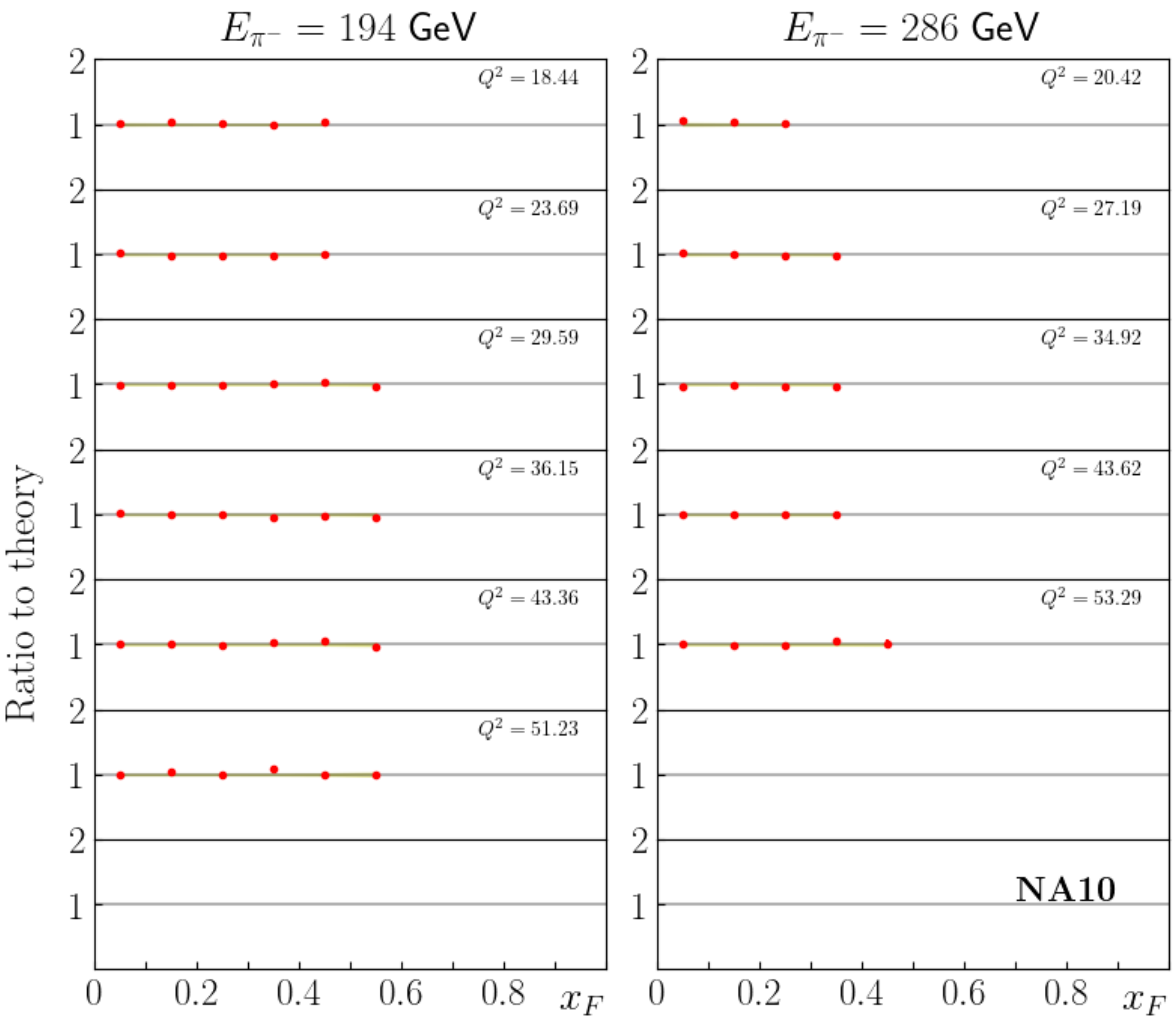
DY data divided by theory

- E615



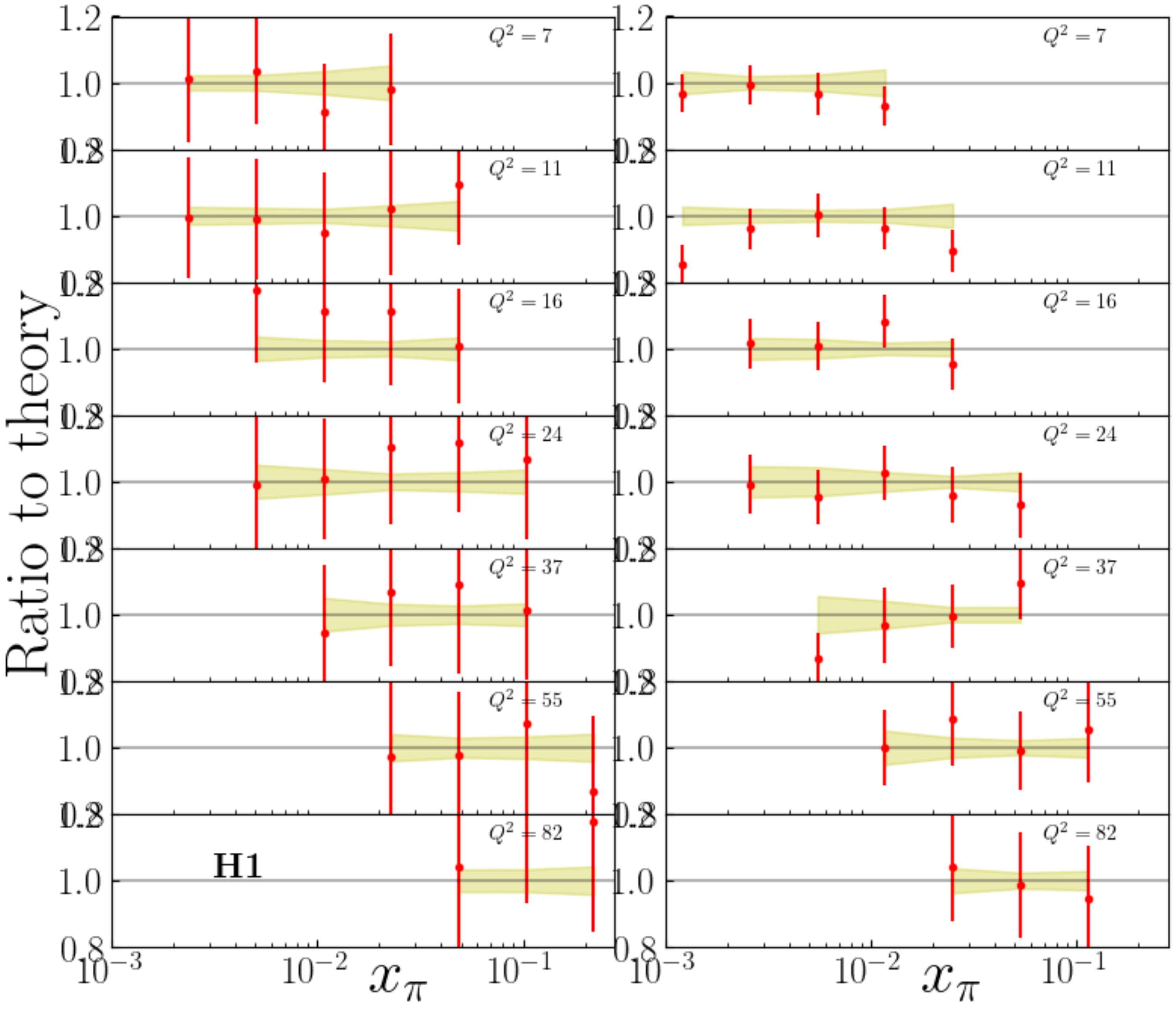
DY data divided by theory

- NA10



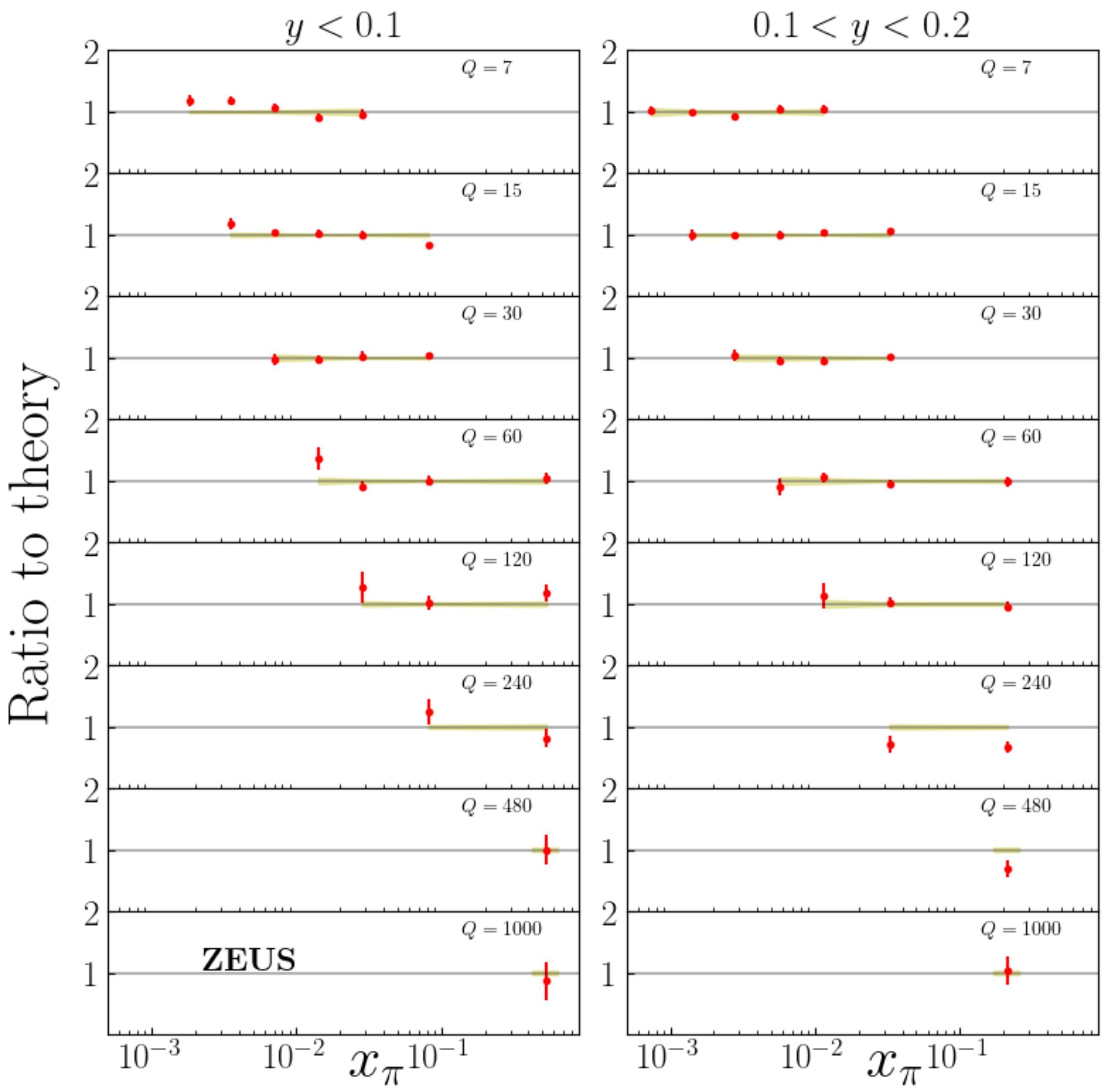
LN data divided by theory

- H1



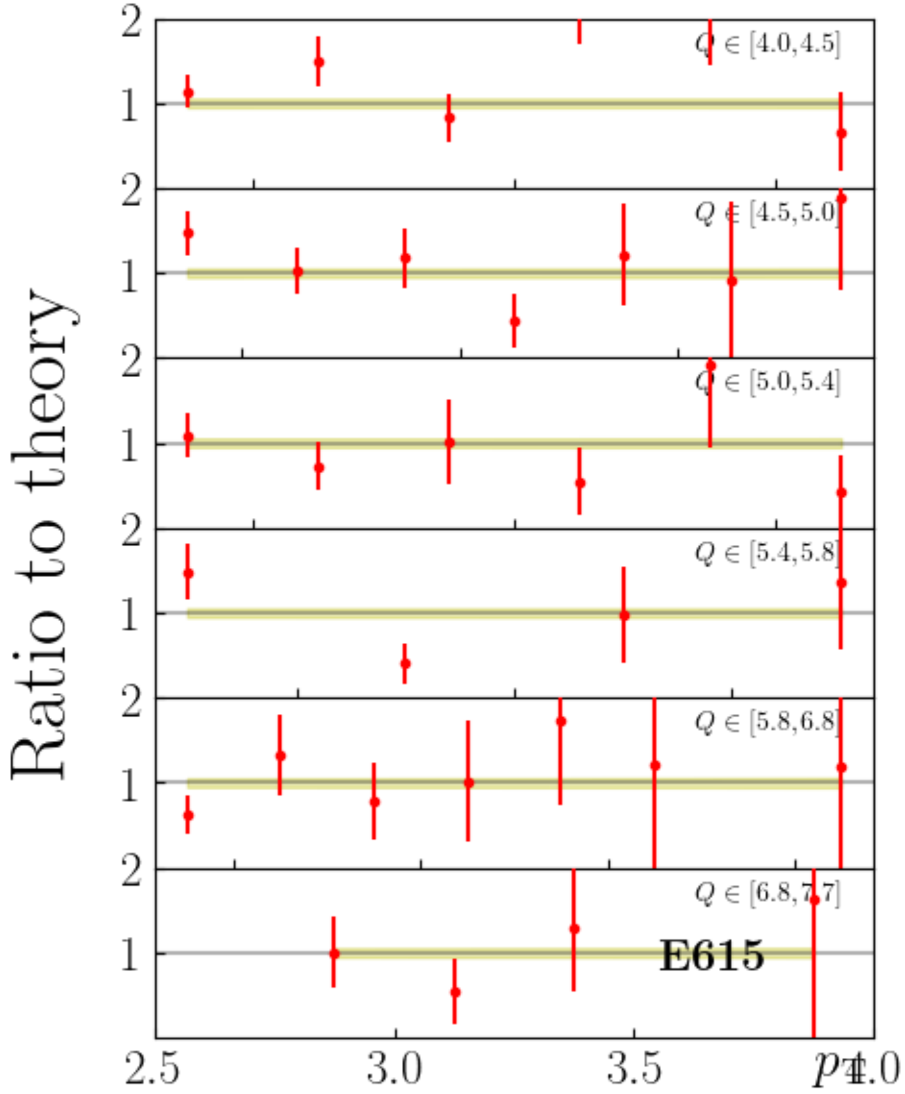
LN data divided by theory

- ZEUS



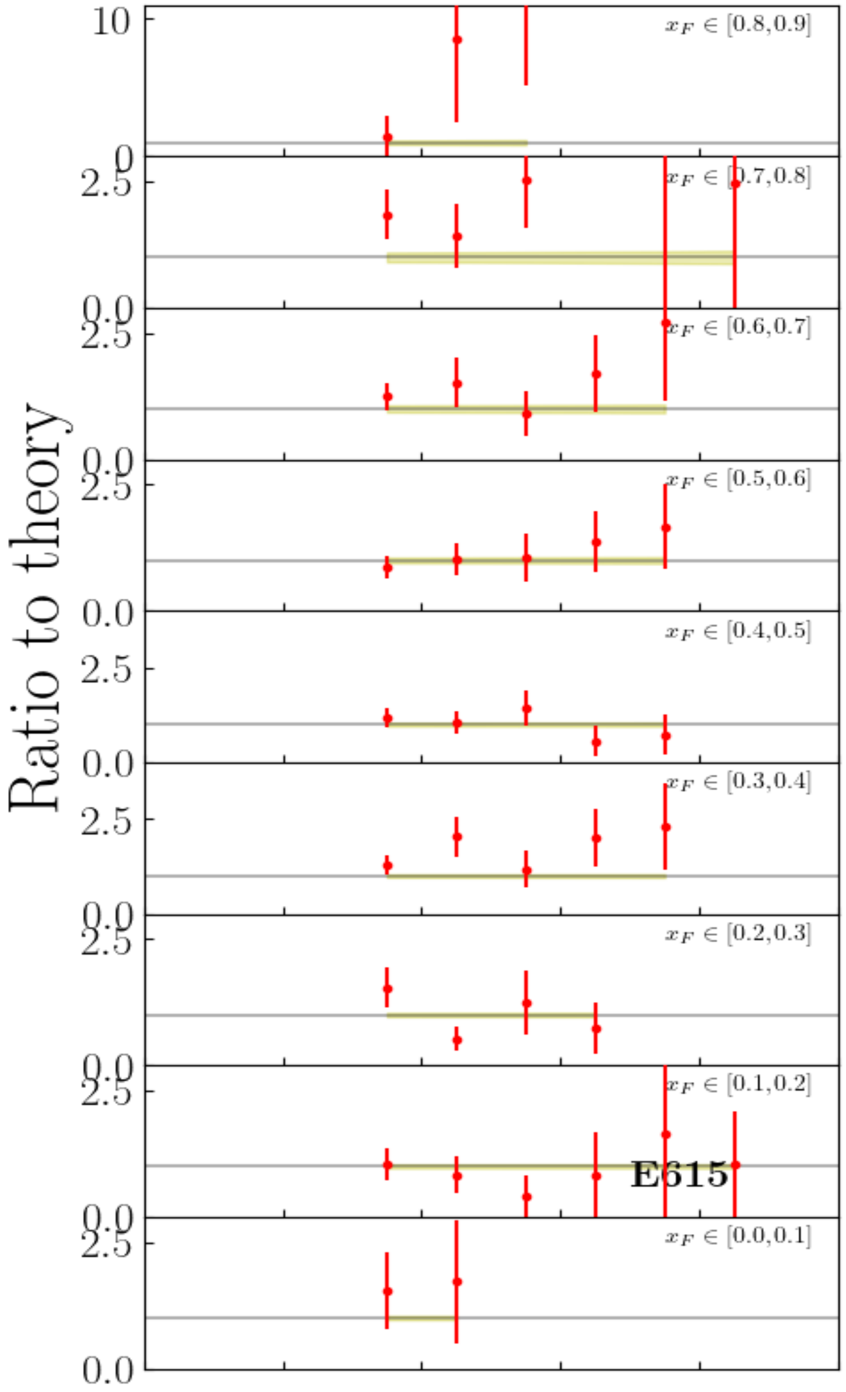
DY q_T-dependent data divided by theory

- E615 $\frac{d\sigma}{dQdqT}$
- Scale is $\mu = \frac{q_T}{2}$



DY q_T-dependent data divided by theory

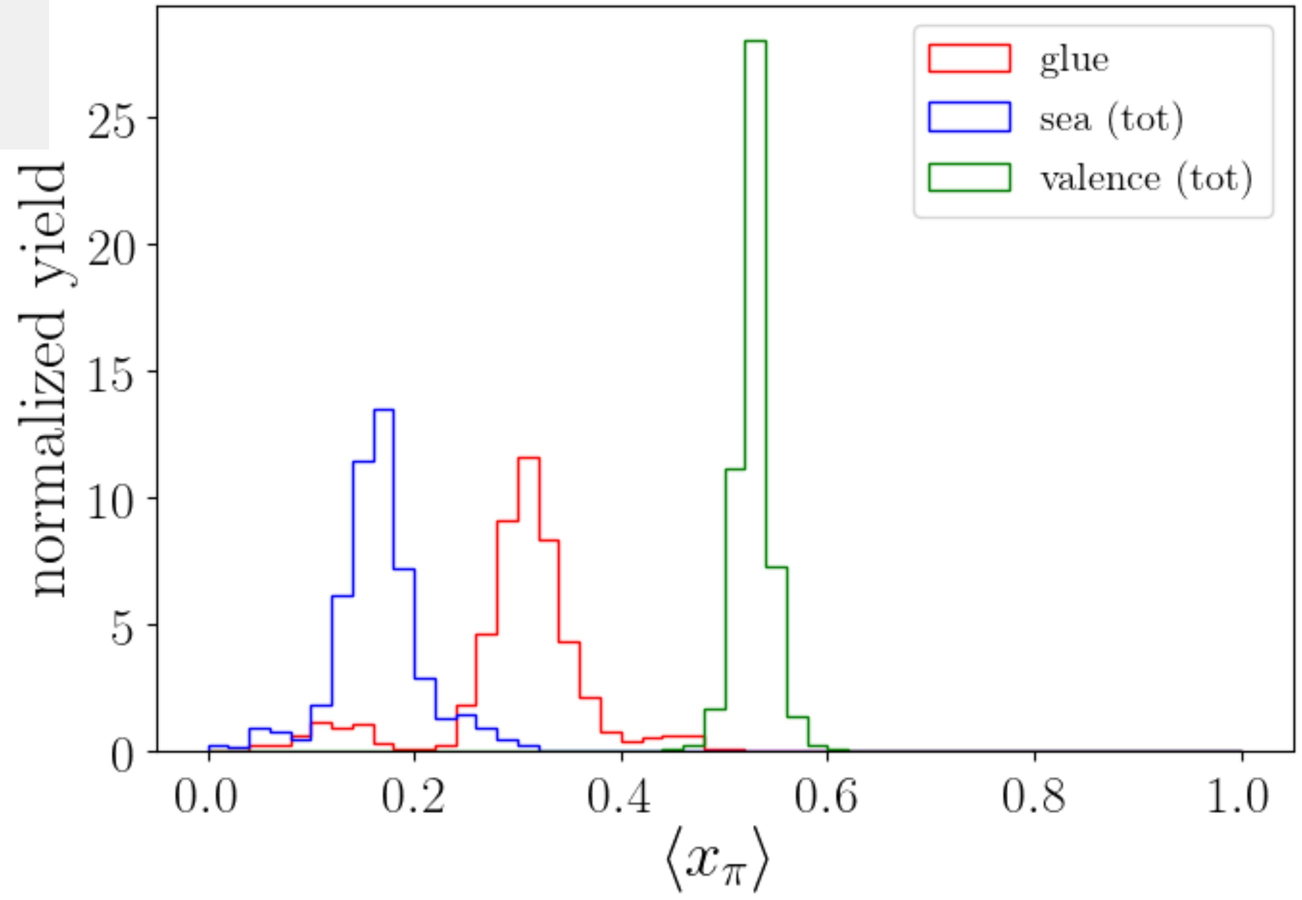
- E615 $\frac{d\sigma}{dx_F dqT}$
- Scale is $\mu = \frac{q_T}{2}$



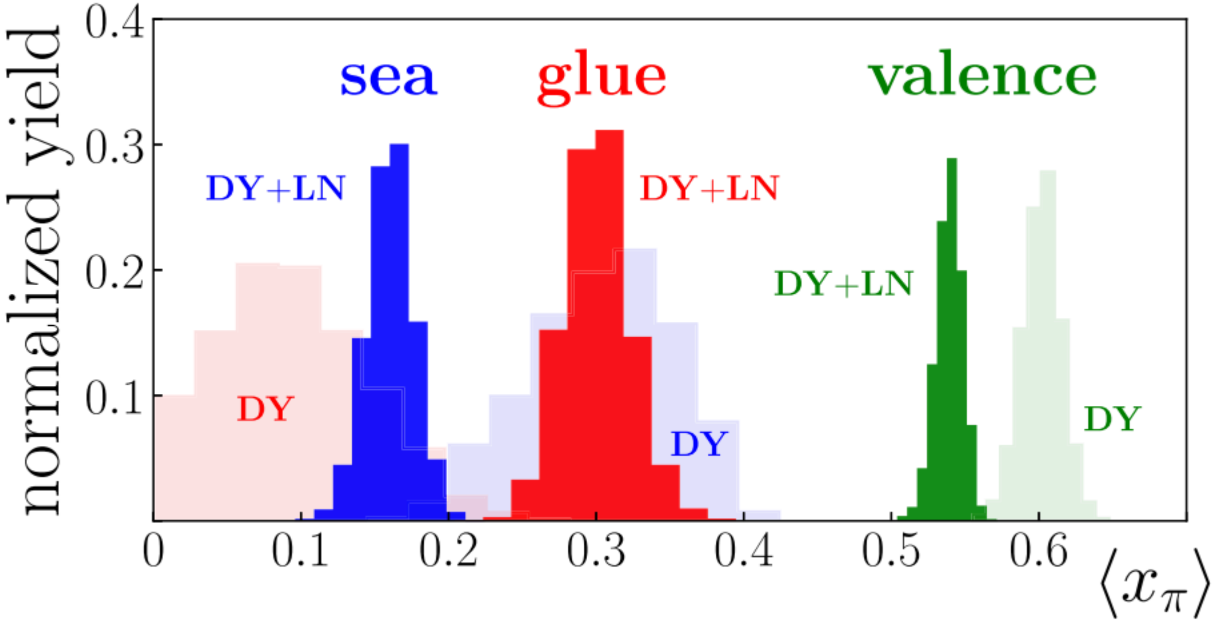
Momentum Fractions

$$\langle x_\pi \rangle_i = \int_0^1 dx_\pi x_\pi f_i^\pi(x_\pi)$$

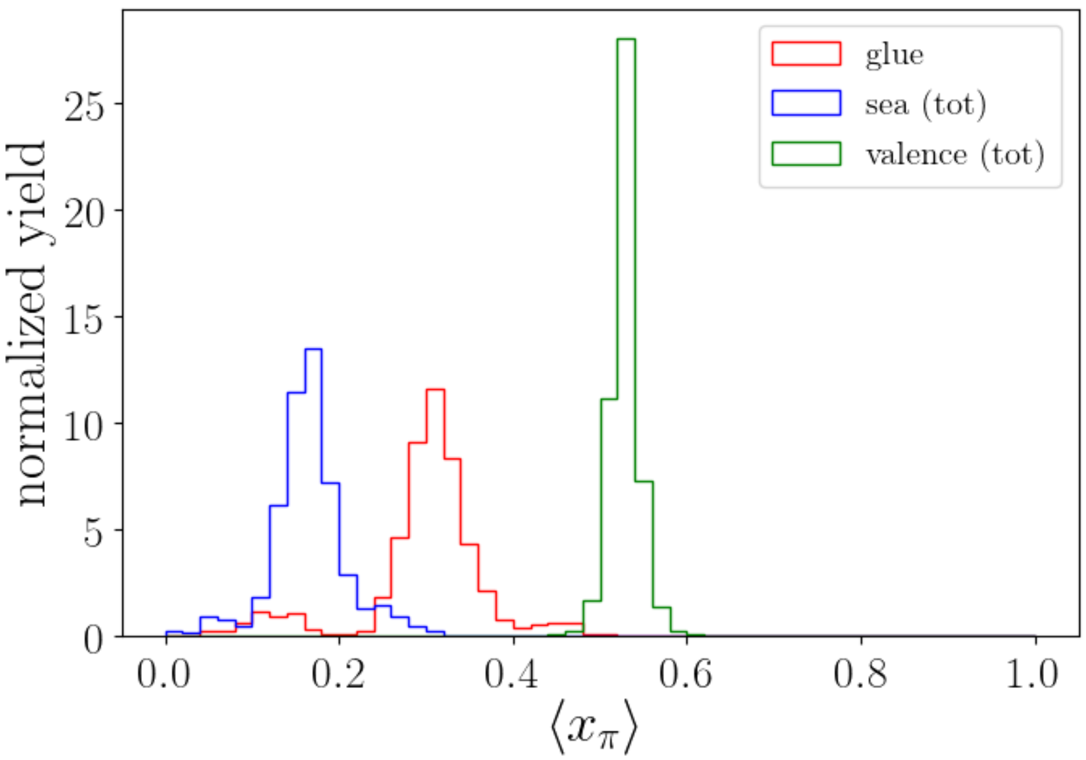
```
moments from resultsFinal/step13b

        g =    2.99e-01 +/-    7.12e-02
      sea =    1.65e-01 +/-    4.29e-02
  valence =    5.28e-01 +/-    1.61e-02
```



Momentum Fractions



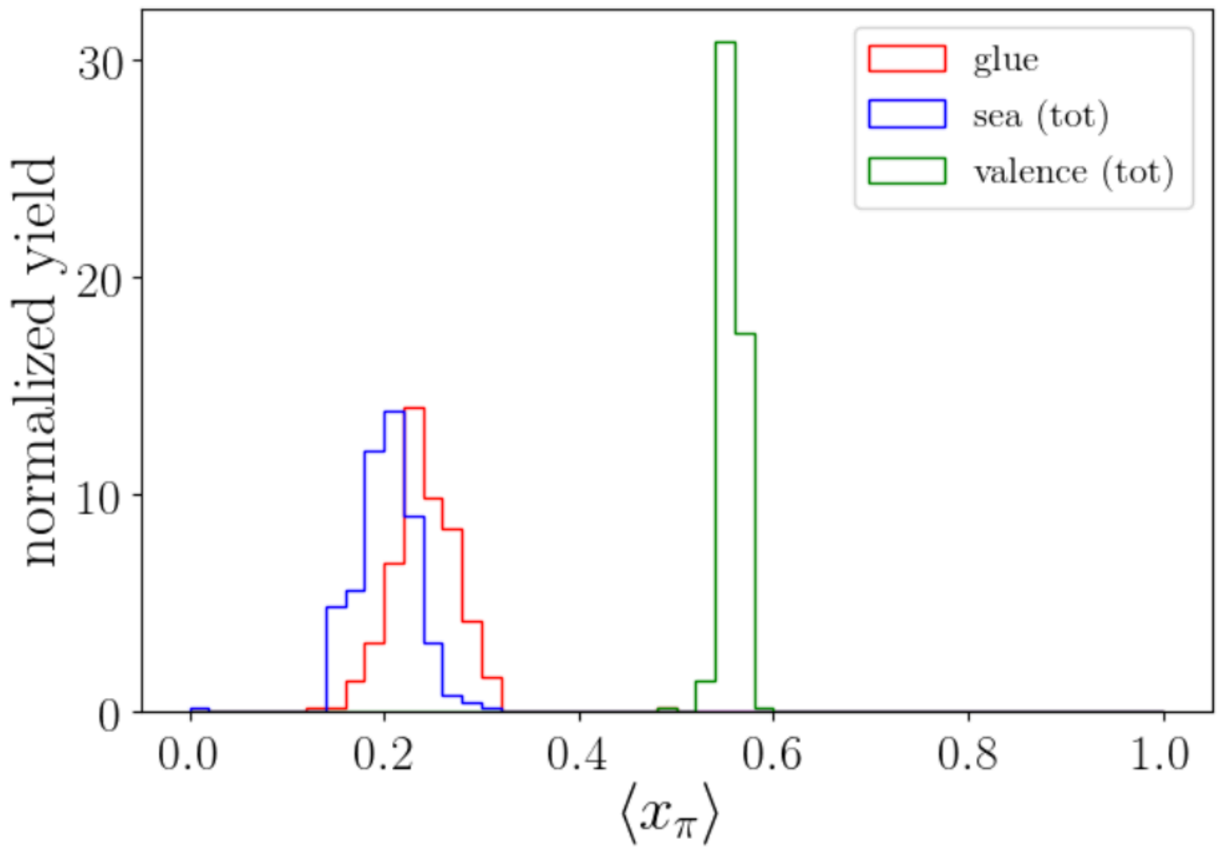
PRL: DY+LN



New: DY+LN+q_T-dependent DY

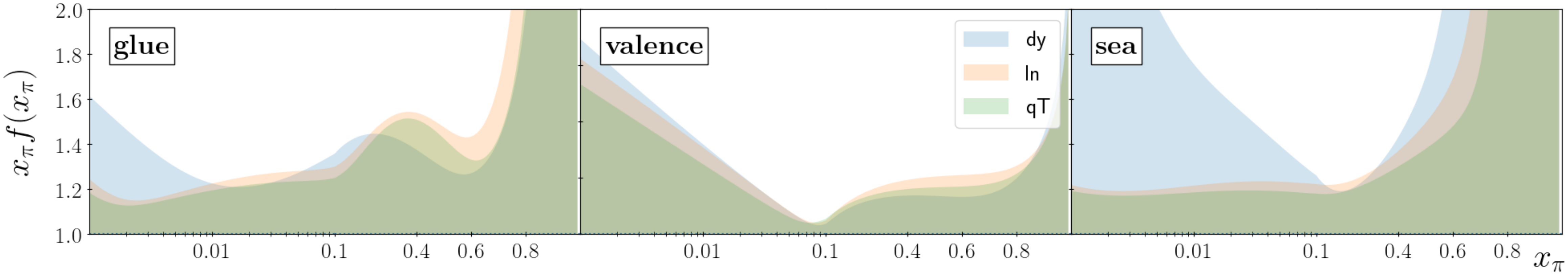
Zeus data has huge effect

- When using only the H1 data to fit for the LN observables, the momentum fractions are different than using H1 and ZEUS
- Recall, in
  - H1: $7 < Q^2 < 82$ GeV^2
  - ZEUS: $7 < Q^2 < 1000$ GeV^2



5. Impact of q_T-dependent DY

Reducing PDF uncertainties



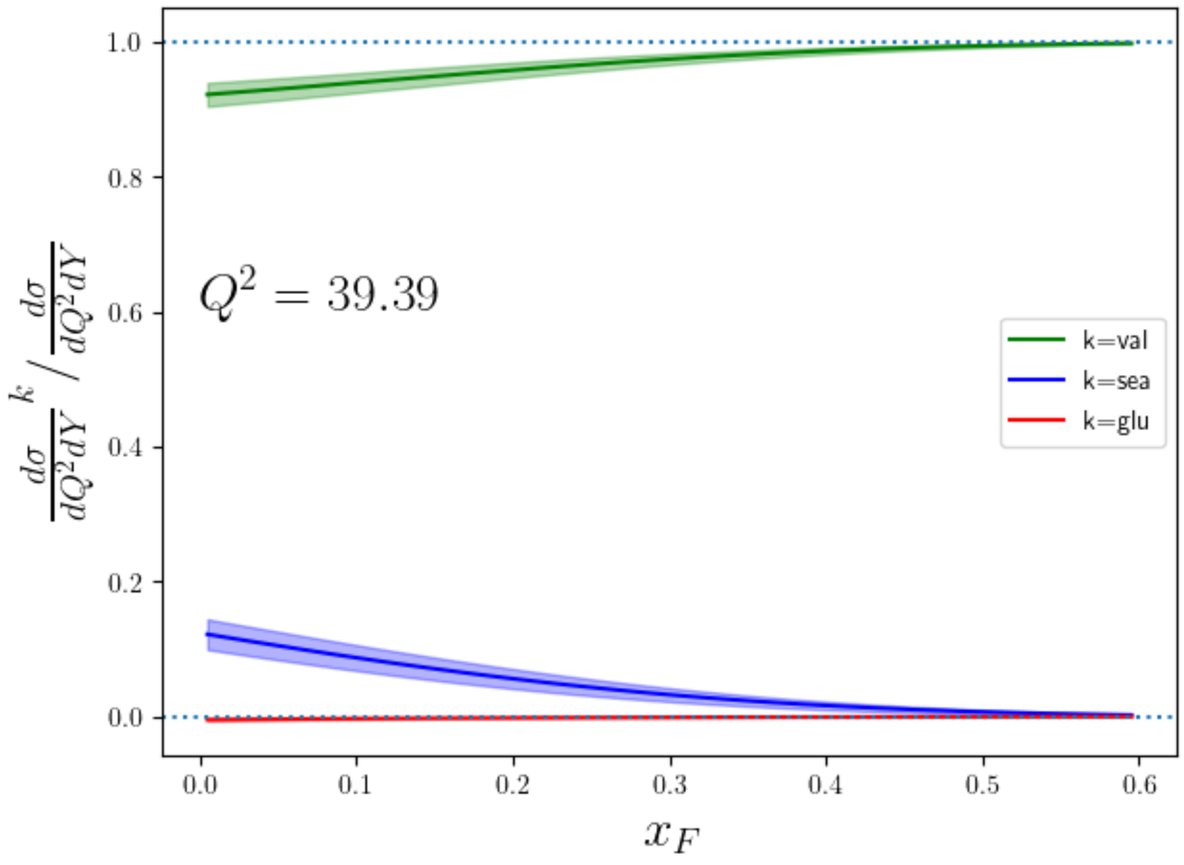
- Adding more and more datasets reduces the uncertainties on the PDFs
- Shown is the uncertainties relative to each fit's central values
- Blue is DY only fit, orange is DY+LN fit, Green is DY+LN+q_T fit

Channel-by-channel contribution

- We can see the observables of each experiment in terms of the degrees of freedom that we fit, *i.e.* valence, sea, and gluon PDFs
- The larger the contribution to the overall cross-section means the more constraints come from that observable

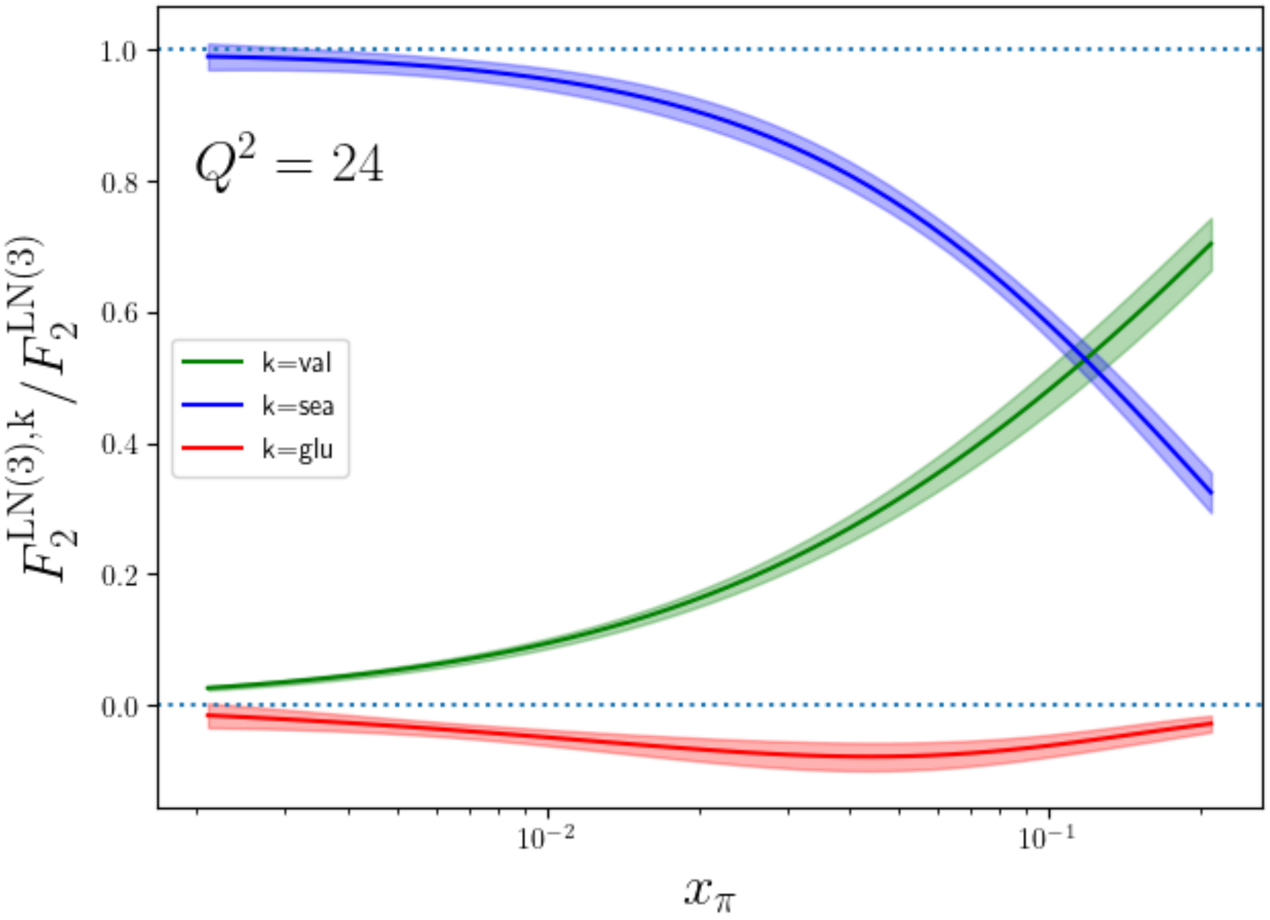
DY Channel-by-channel

- The valence dominates the DY cross section, especially as x_F grows
- The gluon contribution is negligible!
  - DY can't tell us much of anything about gluons at these kinematics!



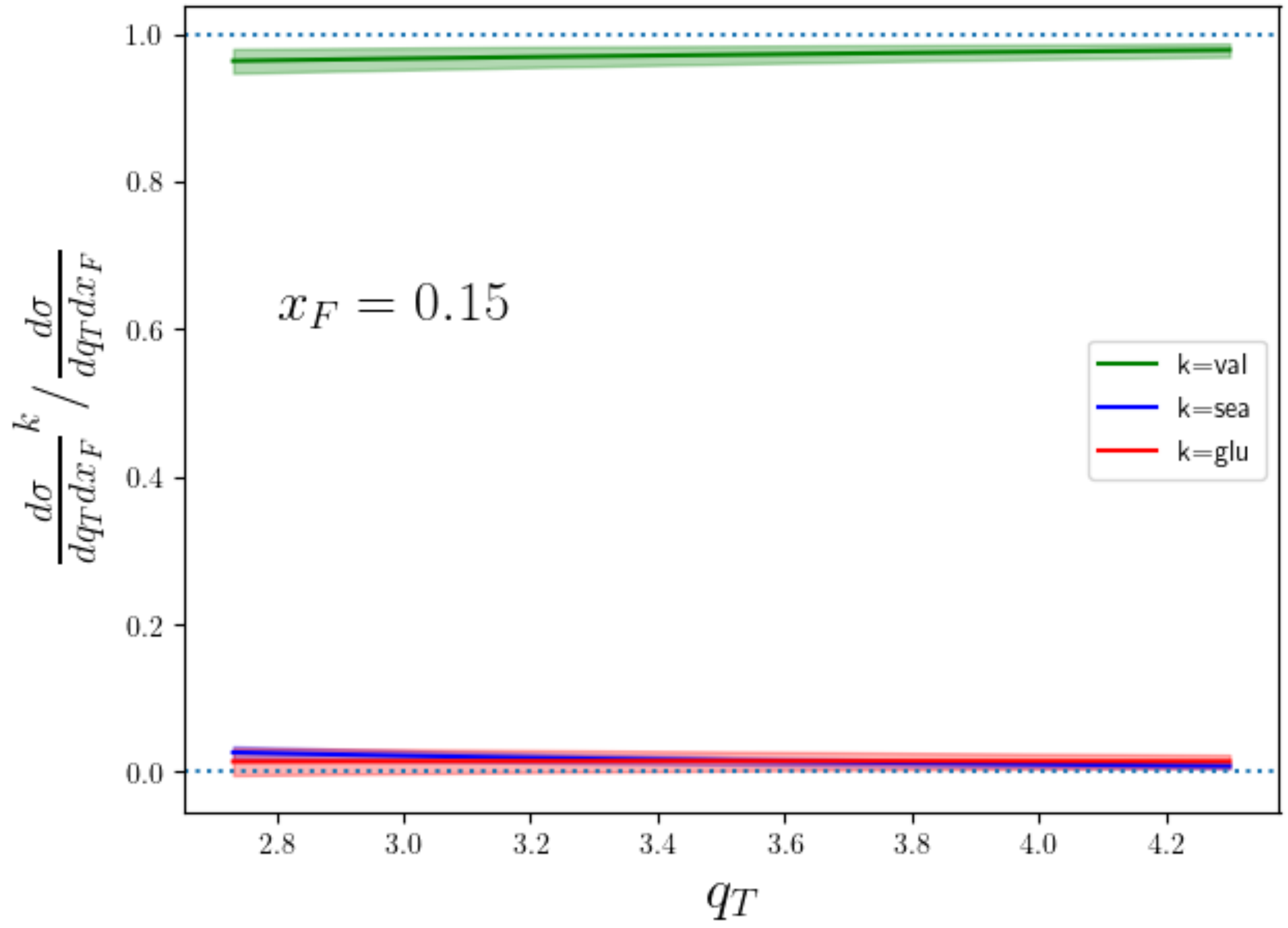
LN channel-by-channel

- For the LN observables, we see the sea and gluon contribute *much* more!
- Better constraints on sea and gluon
  - Especially at low-x_π
- Valence becomes more important at high-x_π region, where DY already lives



q_T-dependent DY Channel-by-Channel

- The gluon is NOT well constrained by the q_T-dependent DY as hypothesized!
- The gluon PDF is so tiny at these kinematics, it is hard to have much of any contribution!



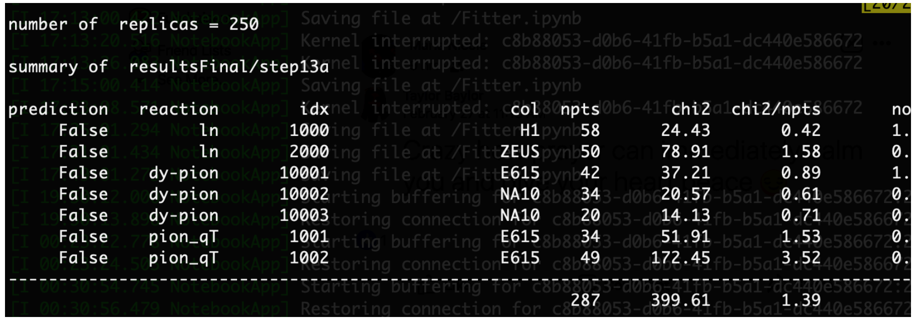
6. Scale Dependence

Ambiguity of Scale

- In Collinear Factorization, one needs a hard scale that is $\mu \gg \Lambda$, where μ is a hard, partonic scale, and Λ is a scale associated with soft, non-perturbative physics
- In DIS, for instance, one hard scale exists, Q^2, which is the invariant mass of the virtual photon
- In DY, again, only one hard scale exists, Q^2
- However, in the q_T-dependent DY, two scales exist
  - The invariant mass of the dilepton pair, Q^2 is measured, but also the transverse momentum of the dilepton pair, p_T
  - Which scale is appropriate?

Exploration of Scale

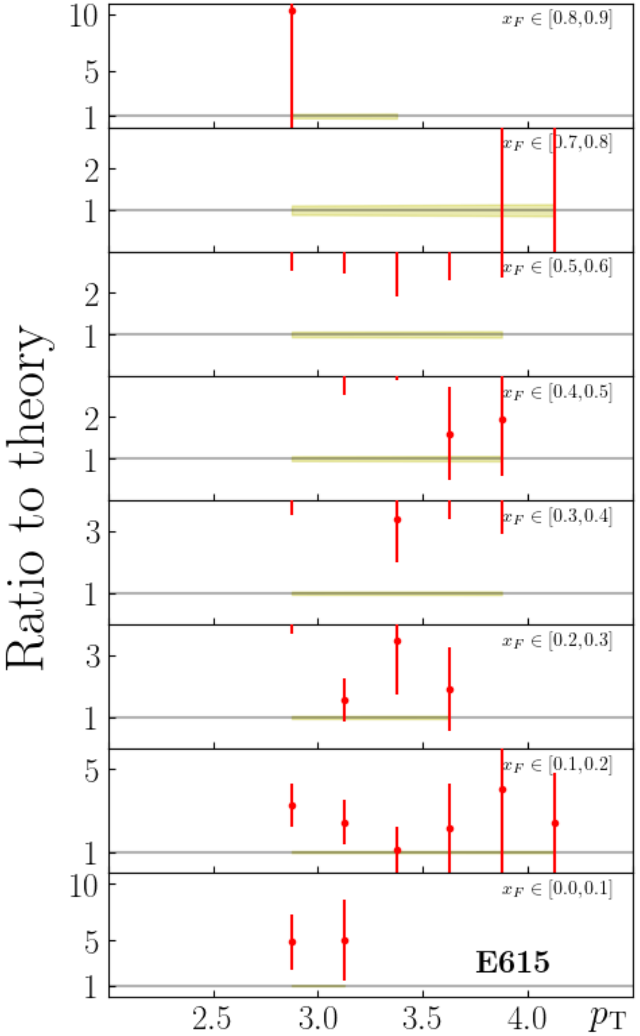
- We performed fits with $\mu = Q$, and had trouble fitting the q_T-dependent data

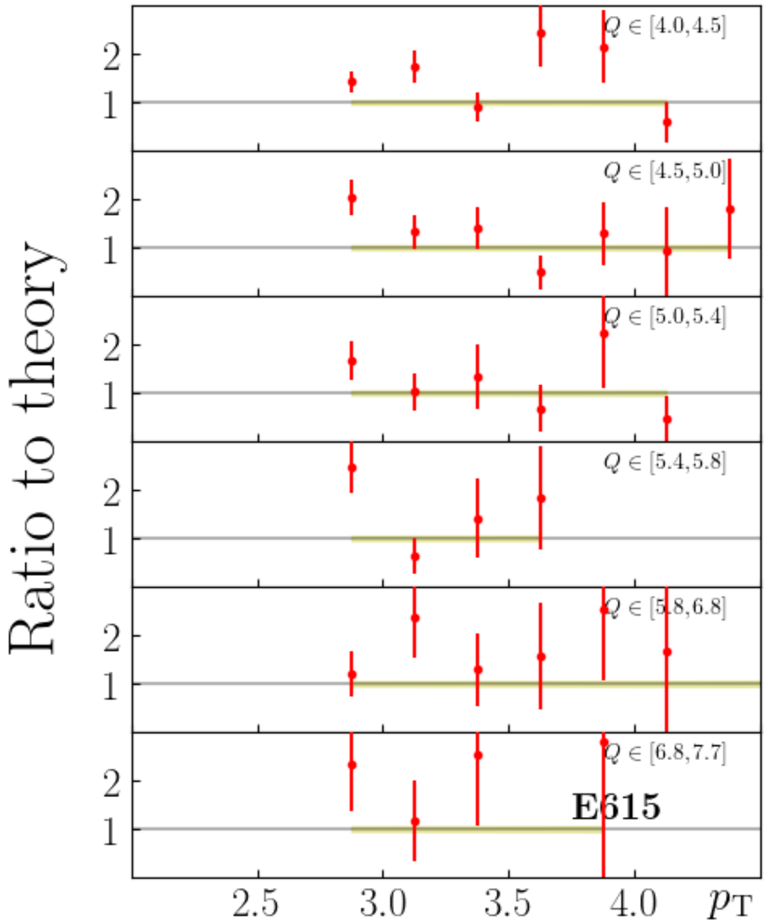


number of replicas = 250

summary of resultsFinal/step13a

prediction	reaction	idx	col	npts	chi2	chi2/npts	no
False	ln	1000	H1	58	24.43	0.42	1.
False	ln	2000	ZEUS	50	78.91	1.58	0.
False	dy-pion	10001	E615	42	37.21	0.89	1.
False	dy-pion	10002	NA10	34	20.57	0.61	0.
False	dy-pion	10003	NA10	20	14.13	0.71	0.
False	pion_qT	1001	E615	34	51.91	1.53	0.
False	pion_qT	1002	E615	49	172.45	3.52	0.
				287	399.61	1.39	





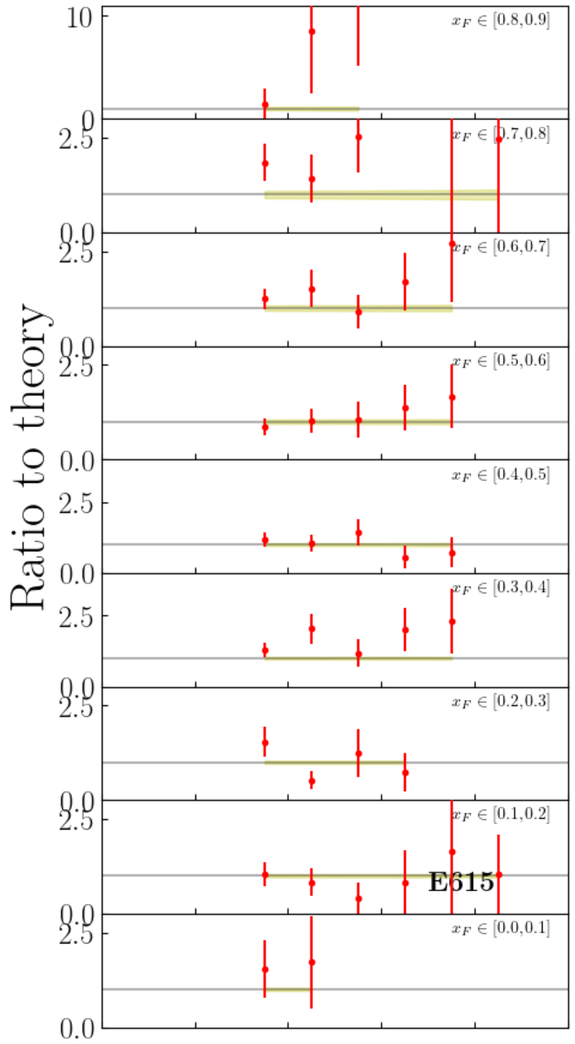
Exploration of Scale

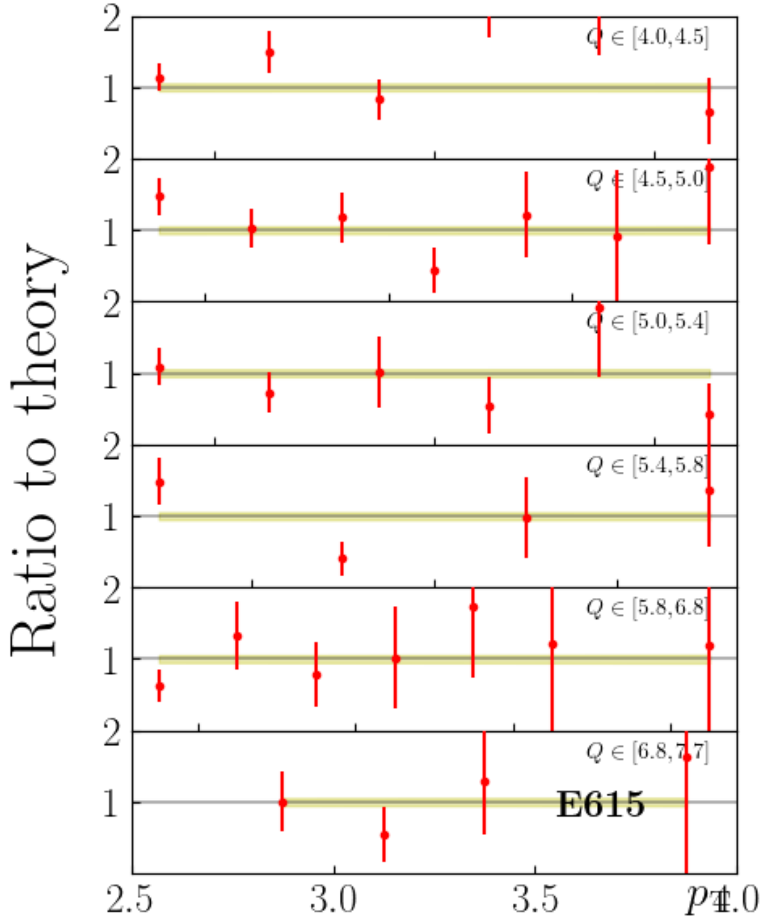
- We performed fits with $\mu = q_T/2$, and had much better success

number of replicas = 999

summary of resultsFinal/step13b

prediction	reaction	idx	col	npts	chi2	chi2/npts	norm
False	ln	1000	H1	58	22.52	0.39	1.26
False	ln	2000	ZEUS	50	73.22	1.46	0.95
False	dy-pion	10001	E615	42	37.03	0.88	1.05
False	dy-pion	10002	NA10	34	18.53	0.54	0.86
False	dy-pion	10003	NA10	20	14.11	0.71	0.80
False	pion_qT	1001	E615	34	37.86	1.11	1.11
False	pion_qT	1002	E615	49	40.49	0.83	0.51
				287	243.74	0.85	





χ^2 for Scale Variation

number of replicas = 250

summary of resultsFinal/step13a

prediction	reaction	idx	col	npts	chi2	chi2/npts	norm
False	ln	1000	H1	58	24.43	0.42	1.27
False	ln	2000	ZEUS	50	78.91	1.58	0.94
False	dy-pion	10001	E615	42	37.21	0.89	1.11
False	dy-pion	10002	NA10	34	20.57	0.61	0.91
False	dy-pion	10003	NA10	20	14.13	0.71	0.85
False	pion_qT	1001	E615	34	51.91	1.53	0.50
False	pion_qT	1002	E615	49	172.45	3.52	0.50
				287	399.61	1.39	

$\mu = Q$

number of replicas = 999

summary of resultsFinal/step13b

prediction	reaction	idx	col	npts	chi2	chi2/npts	norm
False	ln	1000	H1	58	22.52	0.39	1.26
False	ln	2000	ZEUS	50	73.22	1.46	0.95
False	dy-pion	10001	E615	42	37.03	0.88	1.05
False	dy-pion	10002	NA10	34	18.53	0.54	0.86
False	dy-pion	10003	NA10	20	14.11	0.71	0.80
False	pion_qT	1001	E615	34	37.86	1.11	1.11
False	pion_qT	1002	E615	49	40.49	0.83	0.51
				287	243.74	0.85	

$\mu = q_T/2$

number of replicas = 243

summary of resultsFinal/step13c

prediction	reaction	idx	col	npts	chi2	chi2/npts	norm
False	ln	1000	H1	58	23.53	0.41	1.26
False	ln	2000	ZEUS	50	76.08	1.52	0.94
False	dy-pion	10001	E615	42	37.48	0.89	1.09
False	dy-pion	10002	NA10	34	20.33	0.60	0.89
False	dy-pion	10003	NA10	20	13.44	0.67	0.83
False	pion_qT	1001	E615	34	36.89	1.08	0.58
False	pion_qT	1002	E615	49	123.45	2.52	0.50
				287	331.19	1.15	

$\mu = q_T$

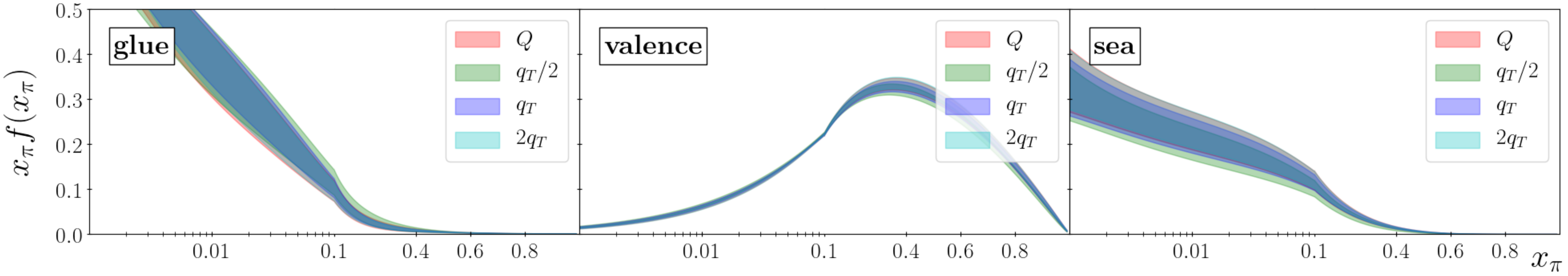
number of replicas = 247

summary of resultsFinal/step13d

prediction	reaction	idx	col	npts	chi2	chi2/npts	norm
False	ln	1000	H1	58	24.56	0.42	1.27
False	ln	2000	ZEUS	50	78.50	1.57	0.94
False	dy-pion	10001	E615	42	37.13	0.88	1.12
False	dy-pion	10002	NA10	34	21.08	0.62	0.92
False	dy-pion	10003	NA10	20	14.49	0.72	0.85
False	pion_qT	1001	E615	34	67.19	1.98	0.50
False	pion_qT	1002	E615	49	188.10	3.84	0.50
				287	431.05	1.50	

$\mu = 2q_T$

PDFs at different scales



- The good news is that the PDFs change
- However, since we can better describe the data with the $\mu = q_T/2$ scale, that is the preferred choice

7. Future Work

This Analysis

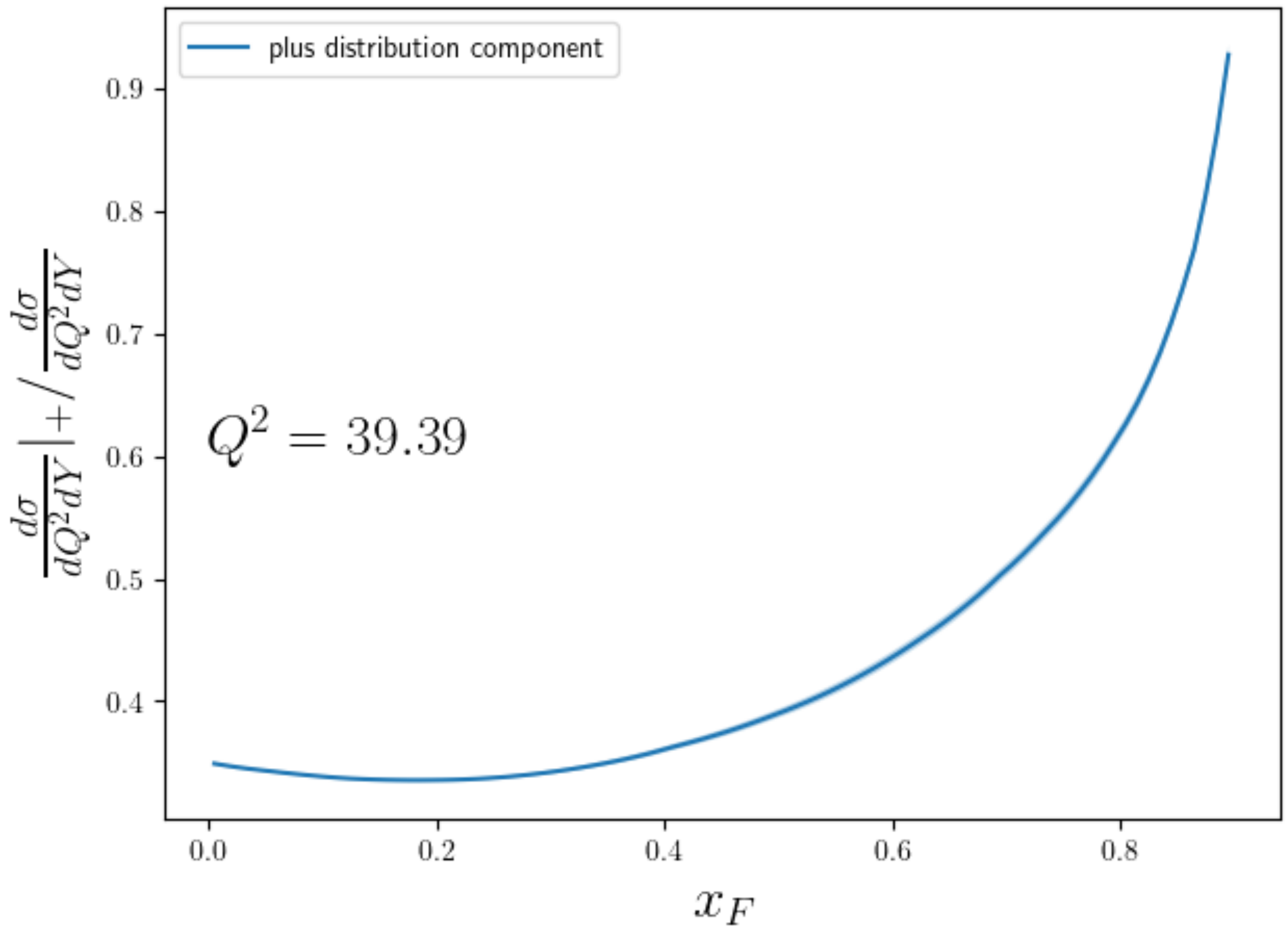
- Put PDFs on LHAPDF for community use
- Compare the proton vs pion
- Make projections for EIC kinematics

Projections for EIC Kinematics

- We can use the EIC kinematics to predict the $F_2^{LN(3)}$ for tagged DIS experiments as “pseudodata”
- Construct number of events from $F_2^{LN(3)}$ and calculate a statistical uncertainty
- At small t and large x_L in LN, we can get the one pion exchange and get new LN data

Threshold Resummation

- Indicates a need for resummation since the terms contribute a lot at high x_π



Pion TMDs

Toy example: how we expect the W+FO-ASY to work