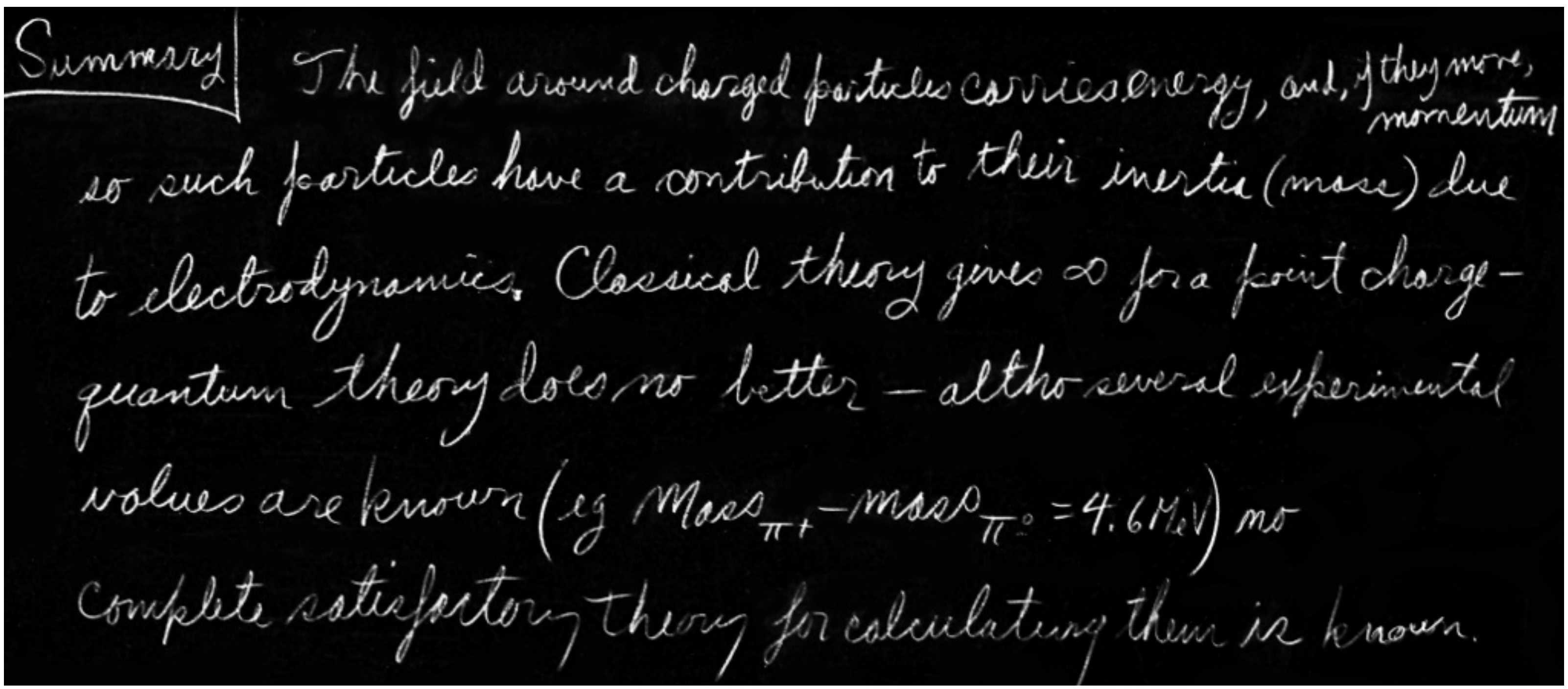
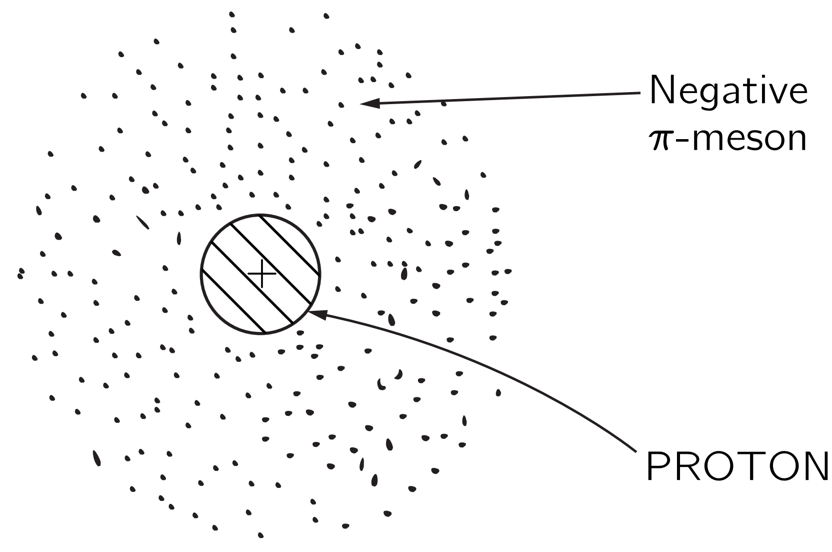
Relativistically Invariant Regularization and Renormalization of Point Particle Field Theories



Group Meeting April 17, 2020

Particle	Charge (electronic)	Mass (MeV)	Δm[1] (MeV)
n (neutron)	0	939.5	
p (proton)	+1	938.2	−1.3
π (π-meson)	0	135.0	
	±1	139.6	+4.6

[1] $\Delta m =$ (mass of charged) $-$ (mass of neutral).



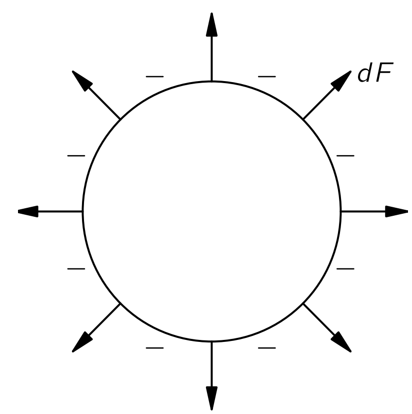
Outline

- Issue of Infinities in Point Particle Theories
- Proposition due to Bopp and Podolsky
- Interpretation as Pauli-Villars Regularization
- Renormalization Program in QFTs.

The field energy of a point charge

electric field is $q/4\pi\epsilon_0 r^2$

$$u = \frac{\epsilon_0}{2}E^2 = \frac{q^2}{32\pi^2\epsilon_0 r^4}$$



$$U_{\text{elec}} = \int \frac{q^2}{8\pi\epsilon_0 r^2}\,dr$$

$$U_{\text{elec}} = \frac{1}{2}\,\frac{q^2}{4\pi\epsilon_0}\,\frac{1}{a}$$

It is all fine until we set a equal to zero for a point charge.

Annalen der Physik. 5. Folge. Band 38. 1940

Eine lineare Theorie des Elektrons*)

Von Fritz Bopp

Inhalt: § 1. Feldgleichungen, Energie-Impuls-Tensor. — § 2. Punktladung und Bewegungsgleichungen. — § 3. Strahlungskraft. — § 4. Entwicklung der Hamiltonfunktion.

JULY 1 AND 15, 1942 PHYSICAL REVIEW VOLUME 62

A Generalized Electrodynamics

Part I—Non-Quantum

BORIS PODOLSKY
Department of Physics, University of Cincinnati, Cincinnati, Ohio

(Received March 23, 1942)

PHYSICAL REVIEW VOLUME 65, NUMBERS 7 AND 8 APRIL 1 AND 15, 1944

A Generalized Electrodynamics

Part II—Quantum

BORIS PODOLSKY AND CHIHIRO KIKUCHI*
Department of Physics, University of Cincinnati, Cincinnati, Ohio

(Received January 20, 1944)

$$S_0[A_\mu] = \int d^4x \left\{ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{a^2}{2} \partial_\nu F^{\mu\nu} \partial^\rho F_{\mu\rho} \right\}$$

$$S_{ext} = -\int d^4x \, j^\mu A_\mu$$

$$(1 + a^2 \Box) \partial_\mu F^{\mu\nu} = j^\nu$$

$$\partial_\nu j^\nu = (1 + a^2 \Box) \partial_\nu \partial_\mu F^{\mu\nu} \equiv 0$$

$$E^i = -F^{0i} \qquad B^i = -\frac{1}{2} \epsilon^{ijk} F^{jk}$$

Let's now focus on the static case

$$(1 - a^2 \nabla^2) \nabla^2 \phi = -4\pi \rho$$

$$\mathbb{P}_a \equiv (1 - a^2\nabla^2)\nabla^2 \qquad \mathbb{P}_a\phi = -4\pi\rho$$

a fixed unity point charge localized at $\mathbf{r}_0$

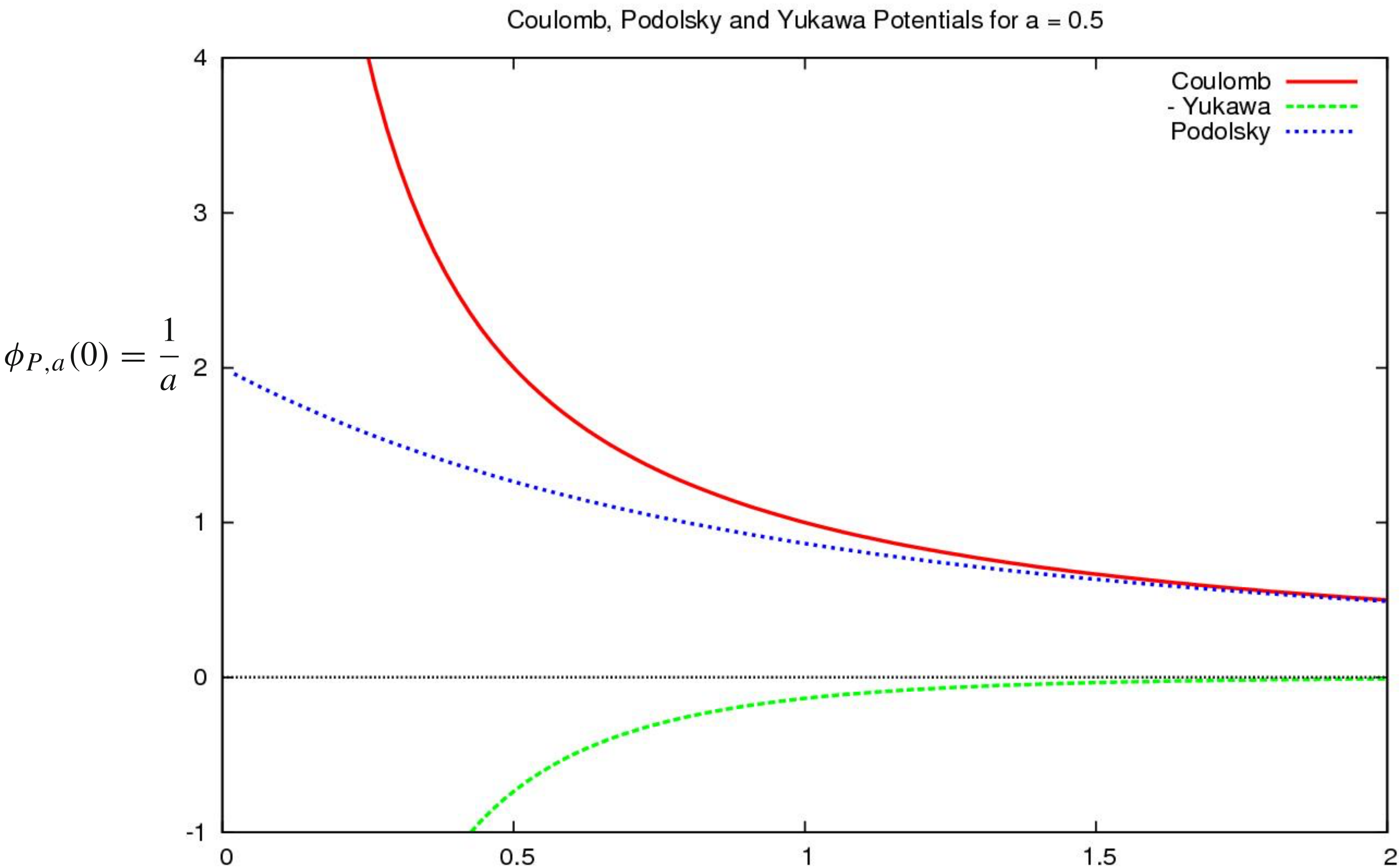
$$\rho_{\mathbf{r}_0}(\mathbf{r}) = \delta^{(3)}(\mathbf{r} - \mathbf{r}_0)$$

$$\phi_{P,a}(|\mathbf{r} - \mathbf{r}_0|) \equiv \phi_C(|\mathbf{r} - \mathbf{r}_0|) - \phi_{Y,a}(|\mathbf{r} - \mathbf{r}_0|)$$

$$= \frac{1 - e^{-|\mathbf{r}-\mathbf{r}_0|/a}}{|\mathbf{r} - \mathbf{r}_0|},$$

$$\phi_C(r) \equiv \frac{1}{r} \; ; \; \phi_{Y,a}(r) \equiv \frac{e^{-r/a}}{r}$$

$$\mathbb{P}_a\phi_C(\mathbf{r}) = -4\pi\left[\delta^{(3)}(\mathbf{r}) - a^2\nabla^2\delta^{(3)}(\mathbf{r})\right] \; ; \; \mathbb{P}_a\phi_{Y,a}(\mathbf{r}) = 4\pi a^2\nabla^2\delta^{(3)}(\mathbf{r})$$


Coulomb, Podolsky and Yukawa Potentials for a = 0.5
Coulomb
- Yukawa
Podolsky
$\phi_{P,a}(0) = \frac{1}{a}$
4
3
2
1
0
-1
0
0.5
1
1.5
2

The BP potential from ordinary electrostatics

$$\rho(b,r) = \frac{e^{-r/b}}{4\pi b^2 r} \; ; \; V(b,r) = \frac{1}{4\pi b^2} \int d\tau' \, \frac{e^{-r'/b}}{r' |\mathbf{r} - \mathbf{r}'|}$$

$$d\tau' = r'^2 \sin\theta \, d\phi \, d\theta \, dr'$$

$$V(b,r) = \frac{1}{2b^2} \int_0^\infty dr' \, r' e^{-r'/b} \int_0^\pi \frac{d\theta \sin\theta}{\sqrt{r^2 + r'^2 - 2rr' \cos\theta'}}$$

$$= \frac{1}{b^2} \int_0^\infty \frac{dr' \, e^{-r'/b}}{r} \left\{ |r + r'| - |r - r'| \right\}$$

$$= \frac{1}{b^2} \int_0^r dr' \, \frac{r' e^{-r'/b}}{r} + \frac{1}{b^2} \int_r^\infty dr' \, e^{-r'/b}$$

$$V(b,r) = \frac{1 - e^{-r/b}}{r} \equiv \phi_{P,b}(r)$$

$$\phi(\mathbf{r}) = \int \rho(\mathbf{r}') \frac{\left(1 - e^{-|\mathbf{r}-\mathbf{r}'|/a}\right)}{|\mathbf{r} - \mathbf{r}'|} d\tau'$$

$$\rho(b, r) = \frac{e^{-r/b}}{4\pi b^2 r}$$

$$\phi(\mathbf{r}) = \frac{1}{4\pi b^2} \int \frac{e^{-r'/b}\left(1 - e^{-|\mathbf{r}-\mathbf{r}'|/a}\right)}{r'\,|\mathbf{r} - \mathbf{r}'|} d\tau'$$

$$\psi(a, b, r) = \frac{1}{r} + \frac{a^2 e^{-r/a} - b^2 e^{-r/b}}{r(b^2 - a^2)}$$

$$\lim_{r \to 0} \psi(a, b, r) = \frac{1}{a + b}$$

Exchange of the role between the length scale parameter *a* introduced in the BP model for a point charge and the length scale parameter *b* for a specific charge distribution in the ordinary electrostatics.

Physical meaning of charge renormalization as the charge screening due to the Dirac vacuum in QED, Reinterpret the BP's generalized electrodynamic action as a natural way of providing Pauli–Villars regularization in ordinary QED.

REVIEWS OF MODERN PHYSICS VOLUME 21, NUMBER 3 JULY, 1949

On the Invariant Regularization in Relativistic Quantum Theory

W. PAULI AND F. VILLARS
Swiss Federal Institute of Technology, Zurich, Switzerland

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THE EUROPEAN PHYSICAL JOURNAL C

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Regular Article - Theoretical Physics

Pauli–Villars regularization elucidated in Bopp–Podolsky's generalized electrodynamics

Chueng-Ryong Ji[1], Alfredo Takashi Suzuki[2], Jorge Henrique Sales[3], Ronaldo Thibes[4,a]

$$S_{LGF} = \int d^4x \left\{ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{a^2}{2} \partial_\nu F^{\mu\nu} \partial^\rho F_{\mu\rho} - \frac{1}{2\xi} (\partial_\mu A^\mu)^2 + \frac{a^2}{2\xi} (\partial_\lambda \partial_\mu A^\mu)^2 \right\} .$$

$$\frac{\delta S_{LGF}}{\delta A_\mu(x)} = 0$$

$$(1 + a^2 \Box) \left(\Box \eta^{\mu\nu} - \partial^\mu \partial^\nu + \frac{1}{\xi} \partial^\mu \partial^\nu \right) A_\nu = 0$$

photon propagator

$$P_{\mu\nu}(k) = \frac{-1}{(1 - a^2 k^2) k^2} \left[\eta_{\mu\nu} + (\xi - 1) \frac{k_\mu k_\nu}{k^2} \right]$$

$$S_{LF} = \int d^4x \left\{ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{a^2}{2} \partial_\nu F^{\mu\nu} \partial^\rho F_{\mu\rho} \right.$$

$$\left. - \frac{1}{2\alpha} (n_\mu A^\mu)^2 + \frac{a^2}{2\alpha} (n_\mu \partial_\lambda A^\mu)^2 \right\} ,$$

$$P_{\mu\nu}(k) = \frac{-1}{k^2(1 - a^2k^2)}$$

$$\times \left[\eta_{\mu\nu} + \frac{(\alpha k^2 + n^2)}{(n \cdot k)^2} k_\mu k_\nu - \frac{1}{(n \cdot k)} (k_\mu n_\nu + k_\nu n_\mu) \right]$$

$$n^2 = 0 \qquad \alpha = 0$$

$$P_{\mu\nu} = \frac{-1}{k^2(1 - a^2k^2)} \left[\eta_{\mu\nu} - \frac{1}{(n \cdot k)} (k_\mu n_\nu + k_\nu n_\mu) \right]$$

axial Lorenz (AL) gauge-fixing

$$S_{AL} = \frac{1}{2} \int d^4x \, A_\mu$$

$$\times \left[(1 + a^2 \Box)(\Box \eta^{\mu\nu} - \partial^\mu \partial^\nu - \frac{1}{\beta}(n^\mu \partial^\nu - n^\nu \partial^\mu) \right] A_\nu$$

$$P_{\mu\nu}(k) = \frac{-1}{k^2(1 - a^2 k^2)}$$

$$\times \left[\eta_{\mu\nu} + \frac{\beta^2 k^2 + n^2}{(n \cdot k)^2 - n^2 k^2} k_\mu k_\nu - \frac{n \cdot k + i\beta k^2}{(n \cdot k)^2 - n^2 k^2} k_\mu n_\nu \right.$$

$$\left. + - \frac{n \cdot k + i\beta k^2}{(n \cdot k)^2 - n^2 k^2} k_\nu n_\mu + \frac{k^2}{(n \cdot k)^2 - n^2 k^2} n_\mu n_\nu \right].$$

$n^2 = 0$
$$P_{\mu\nu}(k) = \frac{-1}{k^2(1 - a^2 k^2)}$$

$\beta = 0$
$$\times \left[\eta_{\mu\nu} - \frac{1}{(n \cdot k)}(k_\mu n_\nu + k_\nu n_\mu) + \frac{k^2}{(n \cdot k)^2} n_\mu n_\nu \right]$$

$$A^{\hat{+}} = 0,\ \partial_{\hat{-}} A_{\hat{-}} + \boldsymbol{\partial}_{\perp} \mathbf{A}_{\perp} \mathbb{C} = 0 \quad (\mathbb{C} = \cos 2\delta)$$

$\delta \to 0$

$(\mathbb{C} \to 1)$

$\delta \to \pi/4$

$(\mathbb{C} \to 0)$

$$A^0 = 0,\ \nabla \cdot \mathbf{A} = 0$$

$$A^+ = 0$$

Coulomb Gauge

Light-front Gauge

C.Ji, Z. Li, and A. T. Suzuki, PRD91, 065020(2015)
C.Ji, Z. Li, B.Ma and A. T. Suzuki, PRD98, 036017(2018)

$$\sum_{\lambda=\pm} \epsilon^*_{\hat{\mu}}(\lambda)\epsilon_{\hat{\nu}}(\lambda) = -g_{\hat{\mu}\hat{\nu}} + \frac{(q\cdot n)(q_{\hat{\mu}} n_{\hat{\nu}} + q_{\hat{\nu}} n_{\hat{\mu}})}{\mathbf{q}_{\perp}^2 \mathbb{C} + q_{\hat{-}}^2} - \frac{\mathbb{C} q_{\hat{\mu}} q_{\hat{\nu}}}{\mathbf{q}_{\perp}^2 \mathbb{C} + q_{\hat{-}}^2} - \frac{q^2 n_{\hat{\mu}} n_{\hat{\nu}}}{\mathbf{q}_{\perp}^2 \mathbb{C} + q_{\hat{-}}^2}$$

LFD

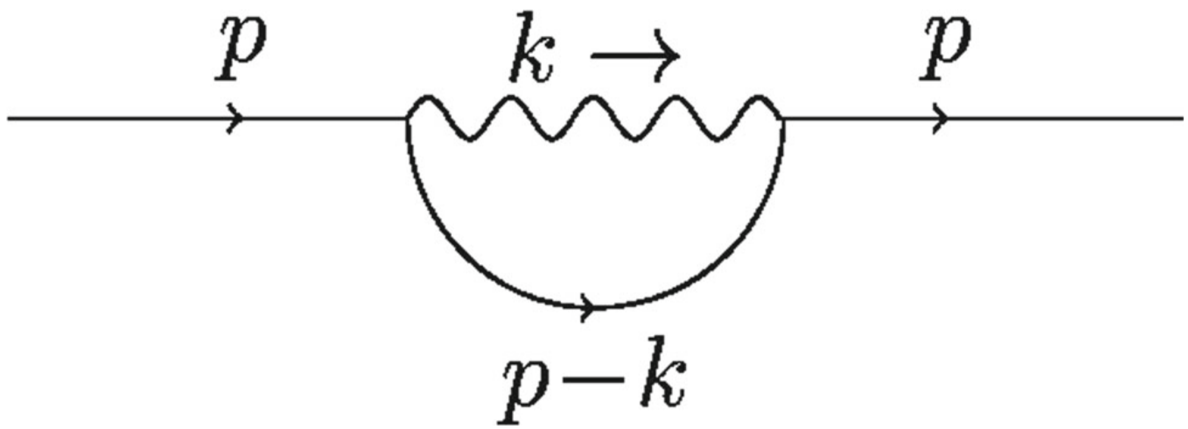
IFD

$$-g_{\mu\nu} + \frac{(q\cdot n)(q_\mu n_\nu + q_\nu n_\mu)}{(q\cdot n)^2} - \frac{q^2 n_\mu n_\nu}{(q\cdot n)^2}$$

P.Srivastava and S. Brodsky, PRD64,045006(2001)

$$-\eta_{\mu\nu} + \frac{(q\cdot n)(q_\mu n_\nu + q_\nu n_\mu)}{(q\cdot n)^2 - q^2} - \frac{q_\mu q_\nu}{(q\cdot n)^2 - q^2} - \frac{q^2 n_\mu n_\nu}{(q\cdot n)^2 - q^2}$$

Application – the electron self-energy



$$\mathcal{M} = \bar{u}(p)\hat{\Sigma}(p)u(p)$$

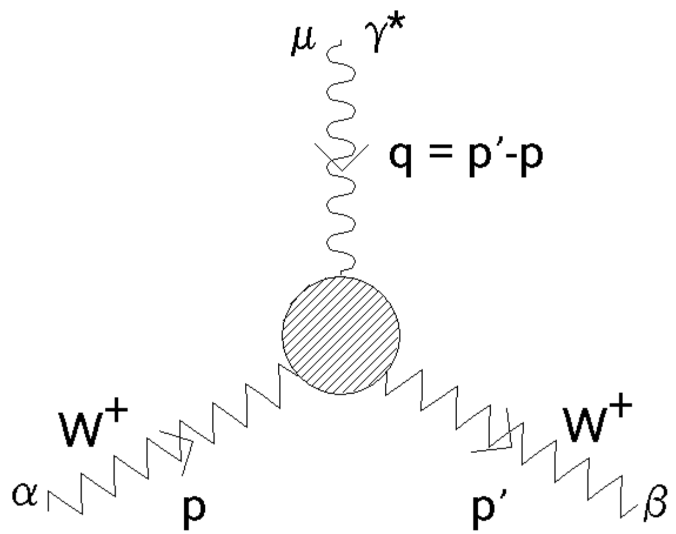
$$\hat{\Sigma}(p) = \int \frac{d^4k}{(2\pi)^4} \frac{\gamma^\mu(\not{p} - \not{k} - m)\gamma^\nu}{[(p-k)^2 - m^2 + i\epsilon]} P_{\mu\nu}(k)$$

$$\frac{1}{k^2(1-a^2k^2)} = \frac{1}{k^2} - \frac{1}{k^2 - (1/a)^2} \qquad a = 1/\Lambda$$

$$\hat{\Sigma}(p) = \frac{im}{8\pi^2} \int dx(1+x)\log\frac{x^2m^2}{x^2m^2 + (1-x)\Lambda^2}$$

CP-Even Electromagnetic Form Factors of $W^{\pm}$ Gauge Bosons

$$\Gamma^{\mu}_{\alpha\beta} = ie\left\{ A\left[(p+p')^{\mu} g_{\alpha\beta} + 2(q_{\beta} g^{\mu}_{\alpha} - q_{\alpha} g^{\mu}_{\beta})\right] + (\Delta\kappa)(g^{\mu}_{\alpha} q_{\beta} - g^{\mu}_{\beta} q_{\alpha}) + \frac{(\Delta Q)}{2M_W^2}(p+p')^{\mu} q_{\alpha} q_{\beta} \right\}$$



At tree level, for any q^2,

$$A = 1, \quad \Delta\kappa = 0, \quad \Delta Q = 0$$

Beyond tree level,

$$A = F_1(q^2),$$

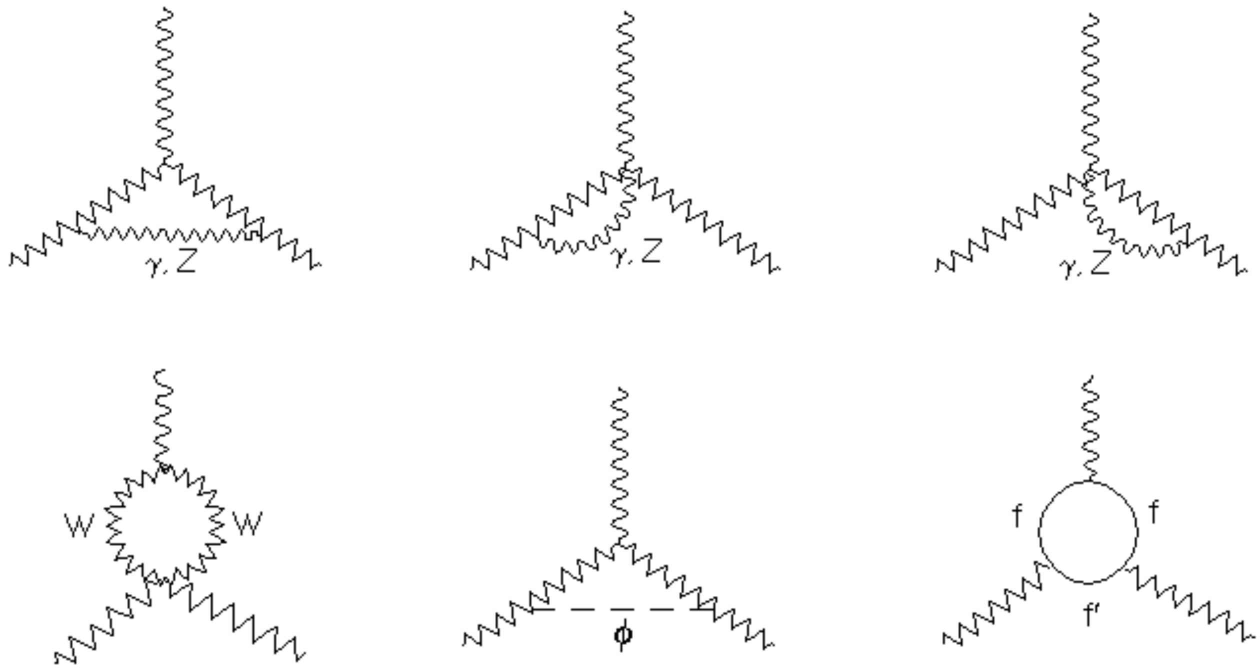
$$-(\Delta\kappa) = F_2(q^2) + 2F_1(q^2),$$

$$-(\Delta Q) = F_3(q^2),$$

$$\Gamma^{\mu}_{\alpha\beta} = -ie\, J^{\mu}_{\alpha\beta}$$

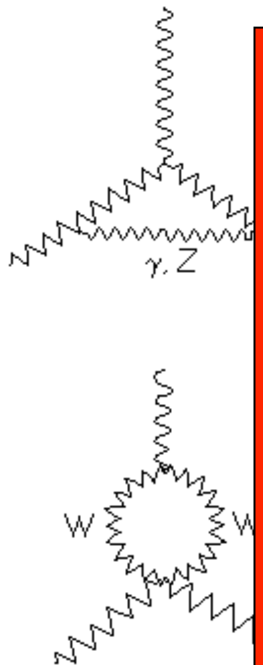
$$J^{\mu}_{\alpha\beta} = \left\{ -(p+p')^{\mu} g_{\alpha\beta} F_1(q^2) + (g^{\mu}_{\alpha} q_{\beta} - g^{\mu}_{\beta} q_{\alpha}) F_2(q^2) + \frac{q_{\alpha} q_{\beta}}{2M_W^2}(p+p')^{\mu} F_3(q^2) \right\}$$

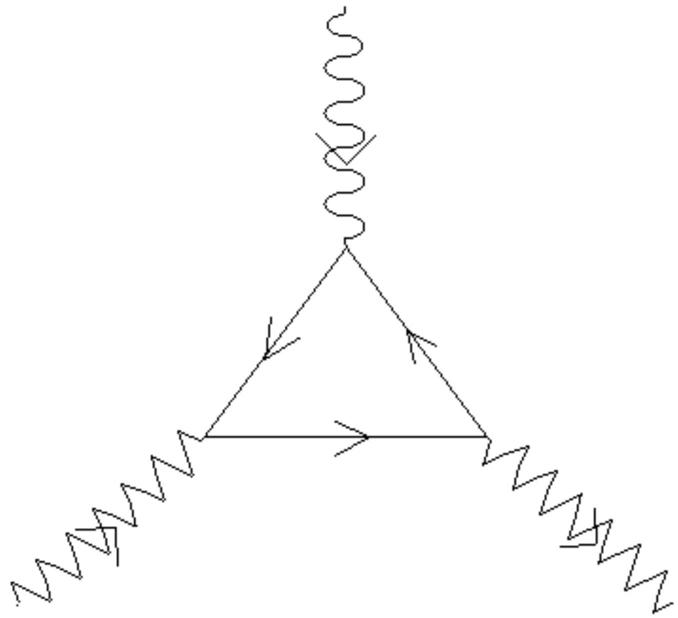
One-loop Contributions in



W.A.Bardeen,R.Gastmans and B.Lautrup, NPB46,319(1972)
G.Couture and J.N.Ng, Z.Phys.C35,65(1987)
E.N.Argyres et al.,NPB391,23(1993)
J.Papavassiliou and K.Philippidas,PRD48,4255(1993)

One-loop Contributions in

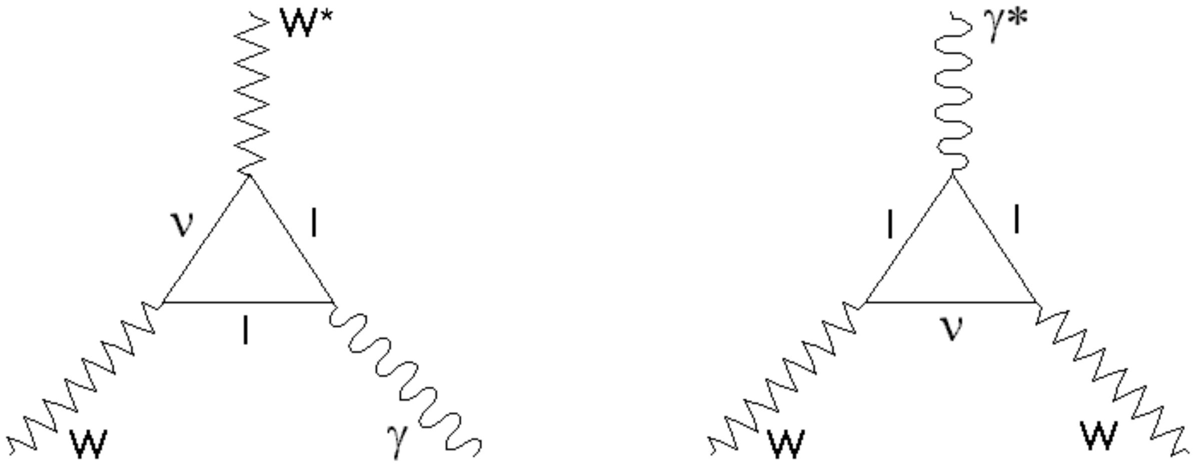




S.L.Adler,Phys.Rev.177,2426(1969)
J.S.Bell and R.Jackiw,Nuovo Cim.51,47(1969)
D.J.Gross and R.Jackiw,PRD6,477(1972)
C.Bouciat,J.I.Iliopoulos and Ph.Meyer,PLB38,519(1972)

W.A.Bardeen,R.Ga
G.Couture and J.N.
E.N.Argyres et al.,N
J.Papavassiliou and

Vector Anomaly in Fermion Triangle Loop



"Sidewise" channel

"Direct" channel

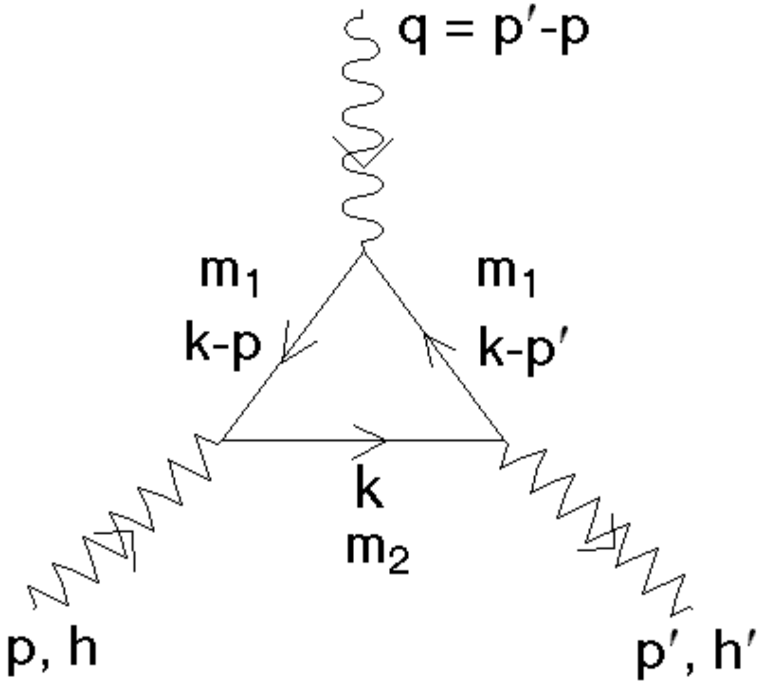
$$(\Delta\kappa)_{\text{"}Sidewise\text{"}} = (\Delta\kappa)_{\text{"}Direct\text{"}} + \frac{G_F M_W^2}{6\sqrt{2}\pi^2}$$

$$(\Delta Q)_{\text{"}Sidewise\text{"}} = (\Delta Q)_{\text{"}Direct\text{"}}$$

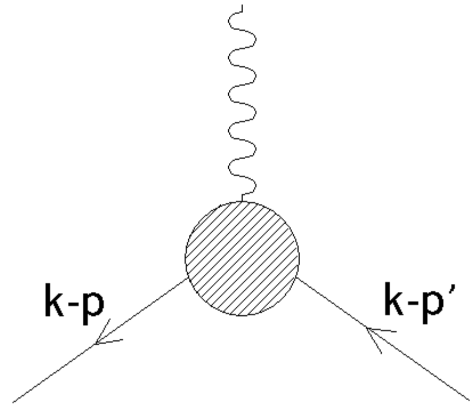
L.DeRaad, K.Milton and W.Tsai, PRD9, 2847(1974); PRD12, 3972(1975)

Vector Anomaly Revisited

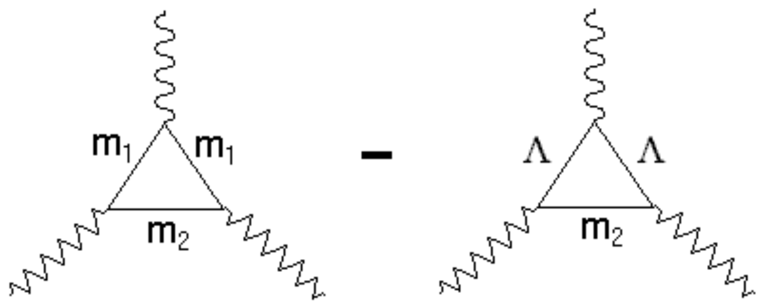
Dimensional Regularization (DR4,DR2)



Smearing of charge (SMR)



Pauli-Villars Regulation (PV1, PV2)



B.Bakker and C.Ji, PRD71,053005(2005)

Manifestly Covariant Results

$$(F_3)_{SMR} = (F_3)_{PV1} = (F_3)_{PV2} = (F_3)_{DR4}$$

$$(F_2 + 2F_1)_{SMR} = (F_2 + 2F_1)_{DR4} + \frac{g^2 Q_f}{4\pi^2}\left(\frac{1}{6}\right)$$

$$(F_2 + 2F_1)_{PV1} = (F_2 + 2F_1)_{DR4} + \frac{g^2 Q_f}{4\pi^2}\left(\frac{2}{3}\right)$$

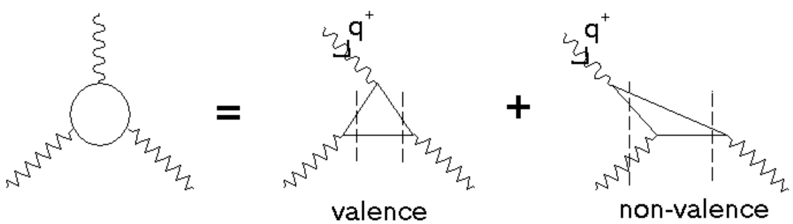
$$(F_2 + 2F_1)_{PV2} = (F_2 + 2F_1)_{DR4} + \frac{g^2 Q_f}{4\pi^2}\left(-\frac{1}{3}\right)$$

$$g^2 = \frac{G_F M_W^2}{\sqrt{2}}$$

LFD Results

$$G^{+}_{h'h} = \langle h', p' | J^{+} | h, p \rangle \quad in \quad q^{+} = 0 \quad frame \quad with \quad \eta = Q^2 / 4M_W^2 \quad (Q^2 = -q^2),$$

$$G^{+}_{++} = 2p^{+}(F_1 + \eta F_3), G^{+}_{+0} = p^{+}\sqrt{2\eta}(2F_1 + F_2 + 2\eta F_3), G^{+}_{+-} = -2p^{+}\eta F_3, G^{+}_{00} = 2p^{+}(F_1 - 2\eta F_2 - 2\eta^2 F_3)$$



$$\left(G^{+}_{00}\right)_{Z.M.} = \frac{g^2 Q_f p^{+}}{2\pi^3 M_W^2} \int_0^1 dx \int d^2 k_{\perp} \frac{k_{\perp}^2 + m_1^2 - x(1-x)Q^2}{k_{\perp}^2 + m_1^2 + x(1-x)Q^2} \neq 0$$

$$\left(F_2 + 2F_1\right)^{+0} = \frac{1}{p^{+}}\left[\frac{G^{+}_{+0}}{\sqrt{2\eta}} + G^{+}_{+-}\right], \quad \left(F_2 + 2F_1\right)^{00} = \frac{1}{4p^{+}\eta}\left[(1+2\eta)G^{+}_{++} - G^{+}_{00} + (1+4\eta)G^{+}_{+-}\right]$$

$$(F_2 + 2F_1)^{+0}_{DR2} = (F_2 + 2F_1)_{DR4} + \frac{g^2 Q_f}{4\pi^2}\left(\frac{1}{6}\right)$$

$$(F_2 + 2F_1)^{00}_{DR2} = (F_2 + 2F_1)_{DR4} - \frac{g^2 Q_f}{4\pi^2}\left(\frac{1}{2\eta}\right)\left(\frac{1}{3} + \frac{2\eta}{9}\right)$$

LFD Results for Other Regularizations

$$(F_2 + 2F_1)^{+0}_{SMR} = (F_2 + 2F_1)^{00}_{SMR} = (F_2 + 2F_1)^{\mathrm{cov}}_{SMR} = (F_2 + 2F_1)_{DR4} + \frac{g^2 Q_f}{4\pi^2}\left(\frac{1}{6}\right)$$

$$(F_2 + 2F_1)^{+0}_{PV1} = (F_2 + 2F_1)^{00}_{PV1} = (F_2 + 2F_1)^{\mathrm{cov}}_{PV1} = (F_2 + 2F_1)_{DR4} + \frac{g^2 Q_f}{4\pi^2}\left(\frac{2}{3}\right)$$

$$(F_2 + 2F_1)^{+0}_{PV2} \overset{?}{=} (F_2 + 2F_1)^{00}_{PV2} \quad (F_2 + 2F_1)^{\mathrm{cov}}_{PV2} = (F_2 + 2F_1)_{DR4} + \frac{g^2 Q_f}{4\pi^2}\left(-\frac{1}{3}\right)$$

Standard Model

$$\begin{array}{cccc} 2/3 & \begin{pmatrix} u \\ d \end{pmatrix} & \begin{pmatrix} c \\ s \end{pmatrix} & \begin{pmatrix} t \\ b \end{pmatrix} \\ -1/3 & & & \\ 0 & \begin{pmatrix} \nu_e \\ e \end{pmatrix} & \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} & \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \\ -1 & & & \end{array}$$

$$\sum_f Q_f = 0 \quad (Anomaly - Free \ \ Condition)$$

Conclusion

- This work demonstrated that the BP model solution for a point charge in classical electrodynamics corresponds to an ordinary classical electrodynamic solution for a specific charge distribution.
- We further elucidate the BP model as a natural way of providing Pauli–Villars regularization in ordinary QED.
- We note that the weakening of the divergence in QED as logarithmic, in contrast to the classical linear divergence, is a consequence of the non-trivial Dirac vacuum in cooperation with the UV cutoff parameter which is combined with another unmeasurable quantity, i.e., the bare mass, to yield the physically measurable renormalized mass of the electron.
- The BP parameter corresponding to the UV cutoff parameter is thus as unmeasurable as the bare mass but essential to regulate the loop divergence in QED and renormalize the mass of the electron as the physically measurable quantity.