SPINORS AND PHOTON POLARIZATION VECTORS

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Deepasika Dayananda

OUTLINE

- Representations
  Standard and Chiral
- Helicity and Dirac Spinors
- Electric and magnetic filed tensor and Gauge field
- Results and Difficulties
- Future

Standard representation (S) and Chiral representation (C)

The transformation between two representation S and C can be made by the transformation matrix S

$$S = S^\dagger = \frac{1}{\sqrt{2}}\begin{pmatrix} I & I \\ I & -I \end{pmatrix}$$

Where ***I*** is the 2×2 identity matrix for spin (1/2) representation and 3×3 identity matrix for spin 1 representation

Gamma matrices are related by

$$\gamma^\mu_S = S\gamma^\mu_C S^\dagger,$$
$$\gamma^\mu_C = S^\dagger\gamma^\mu_S S,$$

Spinors are related by

$$u_S(p) = S u_C(p),$$
$$u_C(p) = S^\dagger u_S(p),$$

Pauli matrices for spin 1

$$\sigma_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\sigma_y = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

$$\sigma_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Pauli matrices for spin (1/2)

$$\sigma_1 = \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$
$$\sigma_2 = \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$
$$\sigma_3 = \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Gamma matrices in chiral basis

$$\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & -\sigma^i \\ \sigma^i & 0 \end{pmatrix}$$

The six Lorentz group generators of rotation (J) and boost (K)

$$\mathbf{J}_S = \mathbf{J}_C = \frac{1}{2}\begin{pmatrix} \boldsymbol{\sigma} & 0 \\ 0 & \boldsymbol{\sigma} \end{pmatrix}, \qquad \mathbf{K}_S = \frac{i}{2}\begin{pmatrix} 0 & \boldsymbol{\sigma} \\ \boldsymbol{\sigma} & 0 \end{pmatrix}, \mathbf{K}_C = \frac{i}{2}\begin{pmatrix} \boldsymbol{\sigma} & 0 \\ 0 & -\boldsymbol{\sigma} \end{pmatrix},$$

Spin (1/2)

The combination between rotation and boost suggest the two decoupled helical motion of the particle that can be denotes as the right -handed and left- handed chirality

$$\mathbf{A} = \frac{1}{2}(\mathbf{J} + i\mathbf{K}),$$
$$\mathbf{B} = \frac{1}{2}(\mathbf{J} - i\mathbf{K}),$$

Helicity Spinor and Dirac Spinor

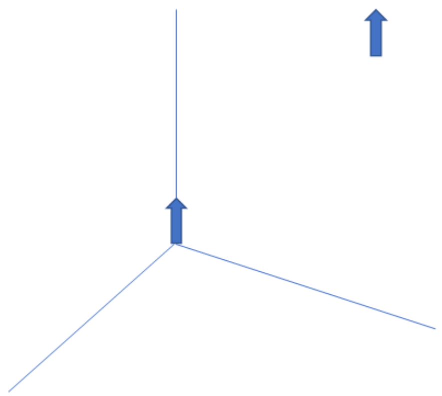
Helicity Spinor : Starting with the state at rest having a spin projection along the z-direction equal to the desired helicity , then boosting it in the z-direction to get the desired magnitude of momentum $|\vec{P}|$, and then rotating it subsequently to get the momentum and spin projection in the desired direction.

→Direction of the helicity spinor always align to the momentum direction

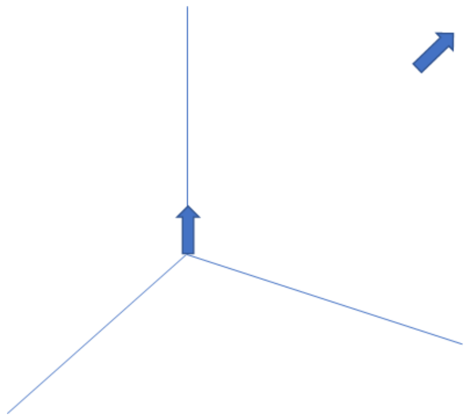
$$T = T_{12}T_3 = e^{i\beta_1 \mathcal{K}^{\hat{1}} + i\beta_2 \mathcal{K}^{\hat{2}}} e^{-i\beta_3 K^3}$$

Dirac Spinor : Boosting the initial state at rest that has a spin projection along the z-direction to the state with the desired momentum $\vec{P}$

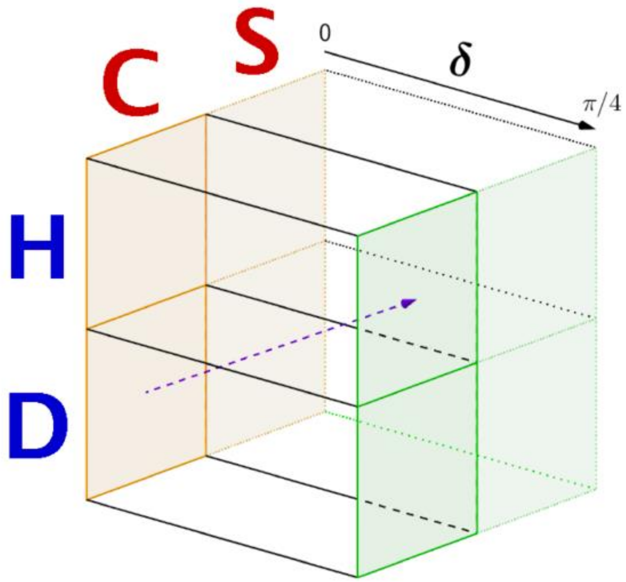
→ The spin direction of the Dirac spinor is in general not align to the moving direction.



Dirac



Helicity



Dashed Purple line -Melosh transformation between instant form Dirac spinor connect to the Light front form helicity spinor

Electric and magnetic field tensor and gauge field .

Electric and magnetic field tensor

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}$$

We can see correspondence of boost operators and electric fields $K \leftrightarrow E$ and rotation operators and magnetic fields $\mathrm{J} \leftrightarrow \mathrm{B}$

Poincare Matrix

$$M^{\mu\nu} = \begin{pmatrix} 0 & K^1 & K^2 & K^3 \\ -K^1 & 0 & J^3 & -J^2 \\ -K^2 & -J^3 & 0 & J^1 \\ -K^3 & J^2 & -J^1 & 0 \end{pmatrix}$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

Gauge fields

$$A^\mu(x) = \int \frac{d^3k}{\sqrt{2(2\pi)^3\omega_k}} \varepsilon^\mu (k) e^{-ik.x}$$

We can write down a matrix which connects a 4 degrees of freedom polarization vectors in (1/2,1/2) Lorentz group and a six-component spin -1 spinor (1,0)+(0,1)

$$\begin{pmatrix} E_x \\ E_y \\ E_z \\ B_x \\ B_y \\ B_z \end{pmatrix} = -i \begin{pmatrix} p^1 & -p^0 & 0 & 0 \\ p^2 & 0 & -p^0 & 0 \\ p^3 & 0 & 0 & -p^0 \\ 0 & 0 & -p^3 & p^2 \\ 0 & p^3 & 0 & -p^1 \\ 0 & -p^2 & p^1 & 0 \end{pmatrix} \begin{pmatrix} \phi \\ A_x \\ A_y \\ A_z \end{pmatrix}$$

Similar to the spin (1/2) **A** and **B** notation

$F^{\mu\nu}$ can be separated in to right-handed and left-handed as $E + iB \epsilon\ (1,0)$ and $E - iB \epsilon\ (0,1)$

We can find the component of the u- spinor by expressing the electric field and magnetic field in terms of the spherical harmonics +, - ,0

$$u(p) = \frac{i}{\sqrt{2}m}\begin{bmatrix} E^- - iB^- \\ -(E^3 - iB^3) \\ -(E^+ - iB^+) \\ E^- + iB^- \\ -(E^3 + iB^3) \\ -(E^+ + iB^+) \end{bmatrix}$$

Where $E^{\pm} = (E^1 \pm iE^2)/\sqrt{2}$

$$B^{\pm} = (B^1 \pm iB^2)/\sqrt{2}$$

$$\ell = 1 \quad \begin{cases} Y_{\ell=1}^{m=-1}(\theta,\phi) = \sqrt{\frac{3}{8\pi}}\sin\theta e^{-i\phi} & \sqrt{\frac{3}{8\pi}}\frac{x-iy}{r} \\ Y_1^0(\theta,\phi) = \sqrt{\frac{3}{4\pi}}\cos\theta & \sqrt{\frac{3}{4\pi}}\frac{z}{r} \\ Y_1^{+1}(\theta,\phi) = -\sqrt{\frac{3}{8\pi}}\sin\theta e^{i\phi} & -\sqrt{\frac{3}{8\pi}}\frac{x+iy}{r} \end{cases}$$

$$\mathrm{u1} = \frac{-(\mathrm{P0}+\mathrm{P3})\epsilon 1 + \mathrm{i}(\mathrm{P0}+\mathrm{P3})\epsilon 2 + (\mathrm{P1}-\mathrm{iP2})(\epsilon 0+\epsilon 3)}{2m}$$

$$\mathrm{u2} = \frac{-\mathrm{P3}\epsilon 0 - \mathrm{iP2}\epsilon 1 + \mathrm{iP1}\epsilon 2 + \mathrm{P0}\epsilon 3}{\sqrt{2}m}$$

$$\mathrm{u3} = \frac{(\mathrm{P0}-\mathrm{P3})\epsilon 1 + \mathrm{i}(\mathrm{P0}-\mathrm{P3})\epsilon 2 + (\mathrm{P1}+\mathrm{iP2})(-\epsilon 0+\epsilon 3)}{2m}$$

$$\mathrm{u4} = \frac{(-\mathrm{P0}+\mathrm{P3})\epsilon 1 + \mathrm{i}(\mathrm{P0}-\mathrm{P3})\epsilon 2 + (\mathrm{P1}-\mathrm{iP2})(\epsilon 0-\epsilon 3)}{2m}$$

$$\mathrm{u5} = \frac{-\mathrm{P3}\epsilon 0 + \mathrm{iP2}\epsilon 1 - \mathrm{iP1}\epsilon 2 + \mathrm{P0}\epsilon 3}{\sqrt{2}m}$$

$$\mathrm{u6} = \frac{(\mathrm{P0}+\mathrm{P3})\epsilon 1 + \mathrm{i}(\mathrm{P0}+\mathrm{P3})\epsilon 2 + (\mathrm{P1}+\mathrm{iP2})(-\epsilon 0-\epsilon 3)}{2m}$$

$$\begin{bmatrix} U^1 \\ U^2 \\ U^3 \\ U^4 \\ U^5 \\ U^6 \end{bmatrix} = \frac{1}{2m} \begin{pmatrix} \mathrm{p1} - I\mathrm{p2} & -\mathrm{p0} - \mathrm{p3} & I(\mathrm{p0} + \mathrm{p3}) & \mathrm{p1} - I\mathrm{p2} \\ -\mathrm{p3}\sqrt{2} & -I\mathrm{p2}\sqrt{2} & I\mathrm{p1}\sqrt{2} & \mathrm{p0}\sqrt{2} \\ -\mathrm{p1} - I\mathrm{p2} & \mathrm{p0} - \mathrm{p3} & I(\mathrm{p0} - \mathrm{p3}) & \mathrm{p1} + I\mathrm{p2} \\ \mathrm{p1} - I\mathrm{p2} & -\mathrm{p0} + \mathrm{p3} & I(\mathrm{p0} - \mathrm{p3}) & -\mathrm{p1} + I\mathrm{p2} \\ -\mathrm{p3}\sqrt{2} & I\mathrm{p2}\sqrt{2} & -I\mathrm{p1}\sqrt{2} & \mathrm{p0}\sqrt{2} \\ -\mathrm{p1} - I\mathrm{p2} & \mathrm{p0} + \mathrm{p3} & I(\mathrm{p0} + \mathrm{p3}) & -\mathrm{p1} - I\mathrm{p2} \end{pmatrix} \begin{bmatrix} \in^0 \\ \in^1 \\ \in^2 \\ \in^3 \end{bmatrix}$$

U^{μ}=C $\in^{\mu}$

Dirac Photon polarization vectors in chiral basis

$$\epsilon^{\mu}(\mathrm{P},+) = \frac{-1}{m(\mathrm{p0}+m)} \begin{bmatrix} (\mathrm{p0}+m)\,(\mathrm{p1}+i\mathrm{p2})/\sqrt{2} \\ (\mathrm{p0}+m)\,m/\sqrt{2} + \mathrm{p1}\,(\mathrm{p1}+i\mathrm{p2})/\sqrt{2} \\ i(\mathrm{p0}+m)\,m/\sqrt{2} + \mathrm{p2}\,(\mathrm{p1}+i\mathrm{p2})/\sqrt{2} \\ \mathrm{p3}\,(\mathrm{p1}+i\mathrm{p2})/\sqrt{2} \end{bmatrix}$$

$$\epsilon^{\mu}(\mathrm{P},0) = \frac{1}{m(\mathrm{p0}+m)} \begin{bmatrix} (\mathrm{p0}+m)\mathrm{p3} \\ \mathrm{p1p3} \\ \mathrm{p2p3} \\ (\mathrm{p0}+m)m + \mathrm{p3}^2 \end{bmatrix}$$

$$\epsilon^{\mu}(\mathrm{P},-) = \frac{1}{m(\mathrm{p0}+m)} \begin{bmatrix} (\mathrm{p0}+m)\,(\mathrm{p1}-i\mathrm{p2})/\sqrt{2} \\ (\mathrm{p0}+m)\,m/\sqrt{2} + \mathrm{p1}\,(\mathrm{p1}-i\mathrm{p2})/\sqrt{2} \\ -i(\mathrm{p0}+m)\,m/\sqrt{2} + \mathrm{p2}\,(\mathrm{p1}-i\mathrm{p2})/\sqrt{2} \\ \mathrm{p3}\,(\mathrm{p1}-i\mathrm{p2})/\sqrt{2} \end{bmatrix}$$

Lorenz Gauge condition satisfied -> $\partial_{\mu} A^{\mu} = 0$

<u>Dirac spin-1 spinors in chiral basis</u>

$$U^{\mu}(P,+) = \left\{\begin{matrix} \frac{(m+\mathrm{p0}+\mathrm{p3})^2}{2\sqrt{2}m(m+\mathrm{p0})} \\ \frac{(\mathrm{p1}+\mathrm{ip2})(m+\mathrm{p0}+\mathrm{p3})}{2m(m+\mathrm{p0})} \\ \frac{(\mathrm{p1}+\mathrm{ip2})^2}{2\sqrt{2}m(m+\mathrm{p0})} \\ \frac{(m+\mathrm{p0}-\mathrm{p3})^2}{2\sqrt{2}m(m+\mathrm{p0})} \\ -\frac{(\mathrm{p1}+\mathrm{ip2})(m+\mathrm{p0}-\mathrm{p3})}{2m(m+\mathrm{p0})} \\ \frac{(\mathrm{p1}+\mathrm{ip2})^2}{2\sqrt{2}m(m+\mathrm{p0})} \end{matrix}\right\}$$

$$U^{\mu}(P,0) = \left\{\begin{matrix} \frac{(\mathrm{p1}-i\mathrm{p2})(m+\mathrm{p0}+\mathrm{p3})}{2m(m+\mathrm{p0})} \\ \frac{m\mathrm{p0}+\mathrm{p0}^2-\mathrm{p3}^2}{\sqrt{2}m(m+\mathrm{p0})} \\ \frac{(\mathrm{p1}+i\mathrm{p2})(m+\mathrm{p0}-\mathrm{p3})}{2m(m+\mathrm{p0})} \\ -\frac{(\mathrm{p1}-i\mathrm{p2})(m+\mathrm{p0}-\mathrm{p3})}{2m(m+\mathrm{p0})} \\ \frac{m\mathrm{p0}+\mathrm{p0}^2-\mathrm{p3}^2}{\sqrt{2}m(m+\mathrm{p0})} \\ -\frac{(\mathrm{p1}+i\mathrm{p2})(m+\mathrm{p0}+\mathrm{p3})}{2m(m+\mathrm{p0})} \end{matrix}\right\}$$

$$U^{\mu}(P,-) = \left\{\begin{matrix} \frac{(\mathrm{p1}-\mathrm{ip2})^2}{2\sqrt{2}m(m+\mathrm{p0})} \\ \frac{(\mathrm{p1}-\mathrm{ip2})(m+\mathrm{p0}-\mathrm{p3})}{2m(m+\mathrm{p0})} \\ \frac{(m+\mathrm{p0}-\mathrm{p3})^2}{2\sqrt{2}m(m+\mathrm{p0})} \\ \frac{(\mathrm{p1}-\mathrm{ip2})^2}{2\sqrt{2}m(m+\mathrm{p0})} \\ -\frac{(\mathrm{p1}-\mathrm{ip2})(m+\mathrm{p0}+\mathrm{p3})}{2m(m+\mathrm{p0})} \\ \frac{(m+\mathrm{p0}+\mathrm{p3})^2}{2\sqrt{2}m(m+\mathrm{p0})} \end{matrix}\right\}$$

$$C = \sum_{\lambda} -(U^{\mu}(P,\lambda) \otimes \epsilon_{\mu}^{*}(P,\lambda)$$

Properties of photon polarization vector

$$\epsilon^{*}(P,\lambda) \cdot \epsilon(P,\lambda') = -\delta_{\lambda\lambda'}$$

$$C = -(U^{\mu}(P,+) \otimes \epsilon_{\mu}^{*}(P,+) - (U^{\mu}(P,-) \otimes \epsilon_{\mu}^{*}(P,-) - (U^{\mu}(P,0) \otimes \epsilon_{\mu}^{*}(P,0)$$

If I multiply C by ϵ^{μ}(P,+)

$U^{\mu}(P,+)$=C $\in^{\mu}$ $(P,+)$

If the two vectors have dimensions *n* and *m*, then their outer product is an *n* × *m* matrix.

$$\mathbf{u} \otimes \mathbf{v} = \mathbf{A} = \begin{bmatrix} u_1 v_1 & u_1 v_2 & \dots & u_1 v_n \\ u_2 v_1 & u_2 v_2 & \dots & u_2 v_n \\ \vdots & \vdots & \ddots & \vdots \\ u_m v_1 & u_m v_2 & \dots & u_m v_n \end{bmatrix}$$

$$\bar{C} = \sum_{\lambda} (\epsilon^{\mu}(P,\lambda) \otimes \bar{U}\,(P,\lambda))$$

$$\bar{U}\,(P,\lambda) = \gamma_0 U^{*}(P,\lambda)$$

$$\gamma_0 = \begin{pmatrix} 0_3 & I_3 \\ I_3 & 0_3 \end{pmatrix}$$

$$\bar{C} = \begin{pmatrix} -\frac{\mathrm{p1}+\mathrm{ip2}}{2m} & \frac{\mathrm{p3}}{\sqrt{2}m} & \frac{\mathrm{p1}-\mathrm{ip2}}{2m} & -\frac{\mathrm{p1}+\mathrm{ip2}}{2m} & \frac{\mathrm{p3}}{\sqrt{2}m} & \frac{\mathrm{p1}-\mathrm{ip2}}{2m} \\ \frac{-\mathrm{p0}+\mathrm{p3}}{2m} & -\frac{\mathrm{ip2}}{\sqrt{2}m} & \frac{\mathrm{p0}+\mathrm{p3}}{2m} & -\frac{\mathrm{p0}+\mathrm{p3}}{2m} & \frac{\mathrm{ip2}}{\sqrt{2}m} & \frac{\mathrm{p0}-\mathrm{p3}}{2m} \\ -\frac{\mathrm{i(p0-p3)}}{2m} & \frac{\mathrm{ip1}}{\sqrt{2}m} & -\frac{\mathrm{i(p0+p3)}}{2m} & -\frac{\mathrm{i(p0+p3)}}{2m} & -\frac{\mathrm{ip1}}{\sqrt{2}m} & -\frac{\mathrm{i(p0-p3)}}{2m} \\ -\frac{\mathrm{p1}+\mathrm{ip2}}{2m} & \frac{\mathrm{p0}}{\sqrt{2}m} & -\frac{\mathrm{p1}-\mathrm{ip2}}{2m} & \frac{\mathrm{p1}+\mathrm{ip2}}{2m} & \frac{\mathrm{p0}}{\sqrt{2}m} & \frac{\mathrm{p1}-\mathrm{ip2}}{2m} \end{pmatrix}$$

$$\in^{\mu}\,(P,\lambda) = \bar{C}U^{\mu}(P,\lambda)$$

$$C\bar{C} = \sum_{\lambda} -(U^{\mu}(P,\lambda) \otimes \epsilon^{*}_{\mu}(P,\lambda).\,(\epsilon^{\mu}(P,\lambda) \otimes \bar{U}\,(P,\lambda)) = \sum_{\lambda} (U^{\mu}(P,\lambda) \otimes \bar{U}\,(P,\lambda))$$

$$C\bar{C} = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{(\mathrm{p0}+\mathrm{p3})^2}{2m^2} & \frac{(\mathrm{p1}-i\mathrm{p2})(\mathrm{p0}+\mathrm{p3})}{\sqrt{2}m^2} & \frac{(\mathrm{p1}-i\mathrm{p2})^2}{2m^2} \\ 0 & \frac{1}{2} & 0 & \frac{(\mathrm{p1}+i\mathrm{p2})(\mathrm{p0}+\mathrm{p3})}{\sqrt{2}m^2} & -\frac{1}{2}+\frac{\mathrm{p0}^2-\mathrm{p3}^2}{m^2} & \frac{(\mathrm{p1}-i\mathrm{p2})(\mathrm{p0}-\mathrm{p3})}{\sqrt{2}m^2} \\ 0 & 0 & \frac{1}{2} & \frac{(\mathrm{p1}+i\mathrm{p2})^2}{2m^2} & \frac{(\mathrm{p1}+i\mathrm{p2})(\mathrm{p0}-\mathrm{p3})}{\sqrt{2}m^2} & \frac{(\mathrm{p0}-\mathrm{p3})^2}{2m^2} \\ \frac{(\mathrm{p0}-\mathrm{p3})^2}{2m^2} & -\frac{(\mathrm{p1}-i\mathrm{p2})(\mathrm{p0}-\mathrm{p3})}{\sqrt{2}m^2} & \frac{(\mathrm{p1}-i\mathrm{p2})^2}{2m^2} & \frac{1}{2} & 0 & 0 \\ -\frac{(\mathrm{p1}+i\mathrm{p2})(\mathrm{p0}-\mathrm{p3})}{\sqrt{2}m^2} & -\frac{1}{2}+\frac{\mathrm{p0}^2-\mathrm{p3}^2}{m^2} & -\frac{(\mathrm{p1}-i\mathrm{p2})(\mathrm{p0}+\mathrm{p3})}{\sqrt{2}m^2} & 0 & \frac{1}{2} & 0 \\ \frac{(\mathrm{p1}+i\mathrm{p2})^2}{2m^2} & -\frac{(\mathrm{p1}+i\mathrm{p2})(\mathrm{p0}+\mathrm{p3})}{\sqrt{2}m^2} & \frac{(\mathrm{p0}+\mathrm{p3})^2}{2m^2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

$$\sum_{\lambda}(U^{\mu}(0,\lambda)\otimes\bar{U}\,(0,\lambda)) = \frac{(I+\gamma_0)}{2}$$

Spin (1/2)

$$\sum_{\lambda} u(p,\lambda)\bar{u}(p,\lambda) = (\not{p}+m)/2m$$

Spin one

$$(\not{p}+m)/2m = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{\mathrm{p0}+\mathrm{p3}}{2m} & \frac{\mathrm{p1}-i\mathrm{p2}}{2\sqrt{2}m} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{\mathrm{p1}+i\mathrm{p2}}{2\sqrt{2}m} & \frac{\mathrm{p0}}{2m} & \frac{\mathrm{p1}-i\mathrm{p2}}{2\sqrt{2}m} \\ 0 & 0 & \frac{1}{2} & 0 & \frac{\mathrm{p1}+i\mathrm{p2}}{2\sqrt{2}m} & \frac{\mathrm{p0}-\mathrm{p3}}{2m} \\ \frac{\mathrm{p0}-\mathrm{p3}}{2m} & -\frac{\mathrm{p1}-i\mathrm{p2}}{2\sqrt{2}m} & 0 & \frac{1}{2} & 0 & 0 \\ -\frac{\mathrm{p1}+i\mathrm{p2}}{2\sqrt{2}m} & \frac{\mathrm{p0}}{2m} & -\frac{\mathrm{p1}-i\mathrm{p2}}{2\sqrt{2}m} & 0 & \frac{1}{2} & 0 \\ 0 & -\frac{\mathrm{p1}+i\mathrm{p2}}{2\sqrt{2}m} & \frac{\mathrm{p0}+\mathrm{p3}}{2m} & 0 & 0 & \frac{1}{2} \end{pmatrix} \neq \sum_{\lambda}(U^{\mu}(P,\lambda)\otimes\bar{U}\,(P,\lambda))$$

Rest Frame

- Spin one spinors

$U^{+} = \{\frac{1}{\sqrt{2}}, 0,0, \frac{1}{\sqrt{2}}, 0,0\}$

$U^{-} = \{0,0, \frac{1}{\sqrt{2}}, 0,0, \frac{1}{\sqrt{2}}\}$

$U^{0} = \{0, \frac{1}{\sqrt{2}}, 0,0, \frac{1}{\sqrt{2}}, 0\}$

- Photon polarization vectors

$\in^{+} = \{0, -\frac{1}{\sqrt{2}}, -\frac{i}{\sqrt{2}}, 0\}$

$\in^{-} = \{0, \frac{1}{\sqrt{2}}, -\frac{i}{\sqrt{2}}, 0\}$

$\in^{0} = \{0,0,0,1\}$

$$C = \begin{bmatrix} 0 & -\frac{1}{2} & \frac{i}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{2} & \frac{i}{2} & 0 \\ 0 & -\frac{1}{2} & \frac{i}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{2} & \frac{i}{2} & 0 \end{bmatrix}$$

Interpolating Helicity Spinors and Photon polarization vectors in chiral basis

$$\epsilon^{+}_{\hat{\mu}}(P) = -\frac{1}{\sqrt{2}\mathbb{P}} \times \left(\mathbb{S}|\mathbf{P}_{\perp}|, \frac{P_1 P_{\hat{-}} - iP_2\mathbb{P}}{|\mathbf{P}_{\perp}|}, \frac{P_2 P_{\hat{-}} + iP_1\mathbb{P}}{|\mathbf{P}_{\perp}|}, -\mathbb{C}|\mathbf{P}_{\perp}| \right),$$

$$\epsilon^{-}_{\hat{\mu}}(P) = \frac{1}{\sqrt{2}\mathbb{P}} \times \left(\mathbb{S}|\mathbf{P}_{\perp}|, \frac{P_1 P_{\hat{-}} + iP_2\mathbb{P}}{|\mathbf{P}_{\perp}|}, \frac{P_2 P_{\hat{-}} - iP_1\mathbb{P}}{|\mathbf{P}_{\perp}|}, -\mathbb{C}|\mathbf{P}_{\perp}| \right),$$

$$\epsilon^{0}_{\hat{\mu}}(P) = \frac{P^{\hat{+}}}{M\mathbb{P}} \left(P_{\hat{+}} - \frac{M^2}{P^{\hat{+}}}, P_1, P_2, P_{\hat{-}} \right).$$

$$u_H^{(+1)} = \frac{1}{2\sqrt{M\mathbb{P}^2}} \begin{pmatrix} \frac{(P_{\hat{-}} + \mathbb{P})(P^{\hat{+}} + \mathbb{P})}{(A-B)} \\ \sqrt{2}P^R(P^{\hat{+}} + \mathbb{P}) \\ \frac{(A-B)(P^R)^2(P^{\hat{+}} + \mathbb{P})}{(P_{\hat{-}} + \mathbb{P})} \\ (A-B)(P_{\hat{-}} + \mathbb{P})\mathbb{X} \\ \sqrt{2}P^R(P^{\hat{+}} - \mathbb{P}) \\ \frac{(A+B)(P^R)^2(P^{\hat{+}} - \mathbb{P})}{(P_{\hat{-}} + \mathbb{P})} \end{pmatrix},$$

$$u_H^{(0)} = \sqrt{\frac{M}{2\mathbb{P}^2}} \begin{pmatrix} -(A+B)P^L \\ \sqrt{2}P_{\hat{-}} \\ (A-B)P^R \\ (-A+B)P^L \\ \sqrt{2}P_{\hat{-}} \\ (A+B)P^R \end{pmatrix}.$$

$$u_H^{(-1)} = \frac{1}{2\sqrt{M\mathbb{P}^2}} \begin{pmatrix} \frac{(A+B)(P^L)^2(P^{\hat{+}} - \mathbb{P})}{(P_{\hat{-}} + \mathbb{P})} \\ -\sqrt{2}P^L(P^{\hat{+}} - \mathbb{P}) \\ (A-B)(P_{\hat{-}} + \mathbb{P})\mathbb{X} \\ \frac{(A-B)(P^L)^2(P^{\hat{+}} + \mathbb{P})}{(P_{\hat{-}} + \mathbb{P})} \\ -\sqrt{2}P^L(P^{\hat{+}} + \mathbb{P}) \\ \frac{(P_{\hat{-}} + \mathbb{P})(P^{\hat{+}} + \mathbb{P})}{(A-B)} \end{pmatrix},$$

$$A = \cos\delta,\ B = -\sin\delta$$

$$\mathbb{X} \equiv \frac{P^{\hat{+}} - \mathbb{P}}{\mathbb{C}} = \frac{P^{\hat{+}} - \sqrt{(P^{\hat{+}})^2 - M^2\mathbb{C}}}{\mathbb{C}}$$

Conditions

$$\epsilon^{\hat{+}} = \mathbb{C}\epsilon_{\hat{+}} + \mathbb{S}\epsilon_{\hat{-}} = 0,$$

$$\epsilon_{\hat{-}}(\lambda)P_{\hat{-}} + \epsilon_{\perp}(\lambda)\mathbf{P}_{\perp}\mathbb{C} = 0,$$

$$A^{\hat{+}} = 0 \text{ and } \partial_{\hat{-}}A_{\hat{-}} + \partial_{\perp} \cdot \mathbf{A}_{\perp}\mathbb{C} = 0.$$

$$P^R = P^1 + iP^2$$

$$P^L = P^1 - iP^2$$

$$P^+ = (P^0 + P^3)/\sqrt{2}$$

$$U_H = C_H \in_H$$

Spinors are all normalized , $\bar{U}U = 2M$

$$C = \sum_{\lambda} -(U^{\mu}(P,\lambda) \otimes \epsilon^{*}_{\mu}(P,\lambda))$$

Light front spin-1 spinors in Standard representation

Spin-1 hadronic spinors with even spinor parity:

$$\mathcal{U}_{+1}(p^\mu)=\frac{1}{2}\begin{bmatrix} p^+ + (m^2/p^+) \\ \sqrt{2}p_r \\ p_r^2/p^+ \\ p^+ - (m^2/p^+) \\ \sqrt{2}p_r \\ p_r^2/p^+ \end{bmatrix}, \quad \mathcal{U}_0(p^\mu)=m\sqrt{1/2}\begin{bmatrix} -p_l/p^+ \\ \sqrt{2} \\ p_r/p^+ \\ p_l/p^+ \\ 0 \\ p_r/p^+ \end{bmatrix}, \quad \mathcal{U}_{-1}(p^\mu)=\frac{1}{2}\begin{bmatrix} p_l^2/p^+ \\ -\sqrt{2}p_l \\ p^+ + (m^2/p^+) \\ -p_l^2/p^+ \\ \sqrt{2}p_l \\ -p^+ + (m^2/p^+) \end{bmatrix}. \tag{B2}$$

$$P^+ = (P^0 + P^3)$$

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Front-form spinors in the Weinberg-Soper formalism and generalized Melosh transformations for any spin

D. V. Ahluwalia
Medium Energy Physics Division, MS H-844, Los Alamos National Laboratory, Los Alamos, New Mexico 87545

Mikolaj Sawicki
Department of Physics, Iowa State University, Ames, Iowa 50011

(Received 25 September 1992)

Generalized Melosh Transformation for spinors

Connects Dirac spinors with generalized helicity spinors

$$\Omega^{[u_D u_H]}_{\lambda\rho} = \frac{1}{2M}\bar{u}^{(\rho)}_H(P)u^{(\lambda)}_D(P) = \mathcal{D}(\hat{\mathbf{m}}, -\theta_s)_{\rho\lambda}$$

→ Transpose of the Wigner rotation matrix

$$\Omega(i) = \begin{pmatrix} \omega(i) & 0 \\ 0 & \omega(i) \end{pmatrix}.$$

$$\cos\frac{\theta_s}{2} = \frac{1}{2}\sqrt{\frac{2M}{E+M}}\sqrt{\frac{P_{\hat{-}} + \mathbb{P}}{2\mathbb{P}}}\left(\sqrt{\frac{P^{\hat{+}} + \mathbb{P}}{M(\sin\delta + \cos\delta)}} + \sqrt{\frac{M(\sin\delta + \cos\delta)}{P^{\hat{+}} + \mathbb{P}}}\right)$$

Spin 1

$$\cos\phi_s = \frac{P^1}{\sqrt{\mathbf{P}^2_\perp}} \qquad \sin\phi_s = \frac{P^2}{\sqrt{\mathbf{P}^2_\perp}}$$

$$\omega(1) = \begin{pmatrix} \cos^2\frac{\theta_s}{2} & -\frac{e^{i\phi_s}\sin\theta_s}{\sqrt{2}} & e^{2i\phi_s}\sin^2\frac{\theta_s}{2} \\ \frac{e^{-i\phi_s}\sin\theta_s}{\sqrt{2}} & \cos\theta_s & -\frac{e^{i\phi_s}\sin\theta_s}{\sqrt{2}} \\ e^{-2i\phi_s}\sin^2\frac{\theta_s}{2} & \frac{e^{-i\phi_s}\sin\theta_s}{\sqrt{2}} & \cos^2\frac{\theta_s}{2} \end{pmatrix}.$$

$$U_D = \Omega U_H$$

$$\in_D = \overline{C_D} U_D$$

$$\in_D = \overline{C_D} \Omega U_H$$

$$\in_D = \overline{C_{DH}} U_H$$

$\Omega \rightarrow 6 \times 6$ matrix

$\overline{C_D} \rightarrow 4 \times 6$ matrix

$\overline{C_{DH}} \rightarrow 4 \times 6$ matrix

Possibility

If I can calculate generalized transformation matrix that connect $\in_D$ and $\in_H$

$$\in_D = \Omega_\epsilon \in_H$$

$\Omega_\epsilon \rightarrow 4 \times 4$ matrix

$$\in_H = {\Omega_\epsilon}^{-1} \overline{C_{DH}} U_H$$

THANK YOU