Critical interpolation angles and boundaries

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HELICITY SPINOR FOR ANY INTERPOLATION ANGLE

Initial state at rest $|0; j, m>$

A spin projection along the z direction

$$J_3|0; j, m> = m|0; j, m>$$

Generalized helicity spinor state in each interpolation angle for a particle of spin *j* moving with momentum *p* and helicity *m*.

$$|p; j, m>_\delta = T|0; j, m>$$

Transformation matrix $T = e^{i\beta_1 \kappa^{\hat{1}} + i\beta_2 \kappa^{\hat{2}} - i\beta_3 K^{\hat{3}}}$

$$T = T_{12}T_3 = e^{i\beta_1 \mathcal{K}^{\hat{1}} + i\beta_2 \mathcal{K}^{\hat{2}}} e^{-i\beta_3 K^3}$$

$$P^{\hat{+}} = (\cos\delta \cosh\beta_3 + \sin\delta \sinh\beta_3)\, M,$$

$$P^{\hat{1}} = \beta_1 \frac{\sin\alpha}{\alpha} (\sin\delta \cosh\beta_3 + \cos\delta \sinh\beta_3)\, M,$$

$$P^{\hat{2}} = \beta_2 \frac{\sin\alpha}{\alpha} (\sin\delta \cosh\beta_3 + \cos\delta \sinh\beta_3)\, M,$$

$$P^{\hat{-}} = \frac{\mathbb{S} P^{\hat{+}} - P_{\hat{-}}}{\mathbb{C}},$$

$$\mathcal{K}^{\hat{1}} = -K^1 \sin\delta - J^2 \cos\delta,$$

$$\mathcal{K}^{\hat{2}} = J^1 \cos\delta - K^2 \sin\delta,$$

$$\cos\alpha = \frac{P_{\hat{-}}}{\mathbb{P}}$$

Interpolating Helicity Spinors Between the Instant Form and the Light-front Form

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New spin Operator for a moving particle- Generalized helicity operator

$$\mathfrak{J}_i = TJ_iT^{-1}$$

$$\mathfrak{J}_3|p;j,m>_{\delta} = TJ_iT^{-1}T|0;j,m> = m|p;j,m>_{\delta}$$

$\mathfrak{J}_3$ In terms of the particle's momentum

$$\mathfrak{J}_3 = \frac{1}{\mathbb{P}}(P_{\hat{-}}J_3 + P^1\kappa^{\hat{2}} - P^2\kappa^{\hat{1}})$$

$$\mathbb{P} = \sqrt{{P_{\hat{-}}}^2 + \mathbb{C}\mathbf{p}_{\perp}^2} = \sqrt{(P^{\hat{+}})^2 - \mathbb{C}M^2} \quad ; \mathbb{C} = Cos(2\delta)$$

Instant Form Limit

$$\kappa^{\hat{1}} \to -J^2, \kappa^{\hat{2}} \to J^1, P_{\hat{-}} \to P^3, \mathbb{P} \to \sqrt{(P^0)^2 - M^2} = |\mathbf{P}|$$

$$\mathfrak{J}_3 \to \frac{\mathbf{P}.\mathbf{J}}{|\mathbf{P}|}$$

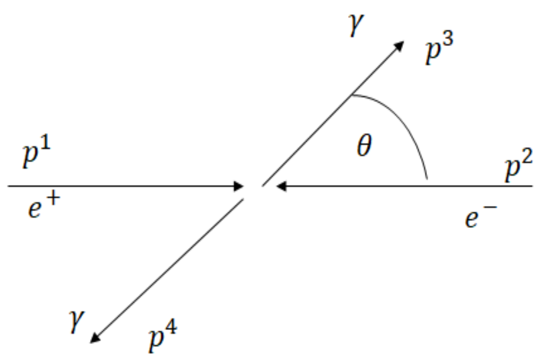
Light Front Limit

$$\kappa^{\hat{1}} \to -E_1, \kappa^{\hat{2}} \to -E_2, P_{\hat{-}} \to P^+, \mathbb{P} \to P^+$$

$$\mathfrak{J}_3 \to \frac{(P^2E_1 - P^1E_2)}{P^+}$$

If the particle is moving in the *+z* or *–z* direction ($P^1 = P^2 = 0$)

$$\mathfrak{J}_3 = \frac{1}{\mathbb{P}}(P_{\hat{-}} J_3)$$



Critical interpolation angle condition ; $P_{\hat{-}} = 0$

$$p^1 = \{E_0, 0, 0, P_e\}$$

$$p^2 = \{E_0, 0, 0, -P_e\}$$

$$p^3 = \{E_0, P_\gamma Sin(\theta), 0, P_\gamma Cos(\theta)\}$$

$$p^4 = \{E_0, -P_\gamma Sin(\theta), 0, -P_\gamma Cos(\theta)\}$$

$$\delta_p^{\pm} = -ArcTan\left[\frac{\left(E0 * pz + pe * \sqrt{4E0^2 + pz^2}\right)}{\left(pe * pz + E0 * \sqrt{4E0^2 + pz^2}\right)}\right]$$

$$\delta_e^{\pm} = -ArcTan\left[\frac{\left(E0 * pz - pe * \sqrt{4E0^2 + pz^2}\right)}{\left(-pe * pz + E0 * \sqrt{4E0^2 + pz^2}\right)}\right]$$

Interpolating quantum electrodynamics between instant and front forms

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Interpolating Helicity Spinors Between the Instant Form and the Light-front Form

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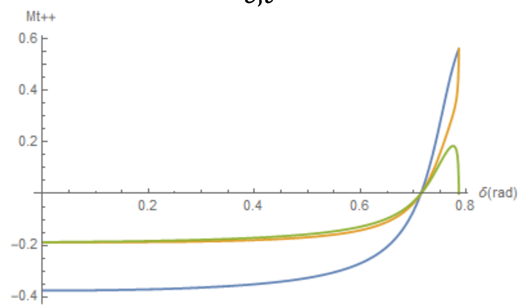
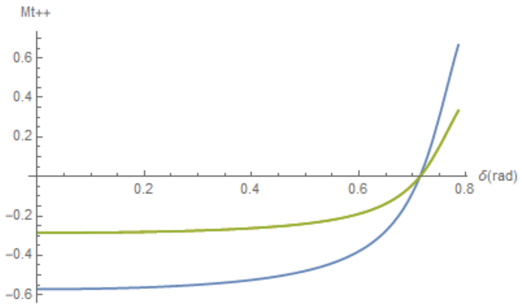
"$\pi^+\pi^-$ " → $\rho^+\rho^-$

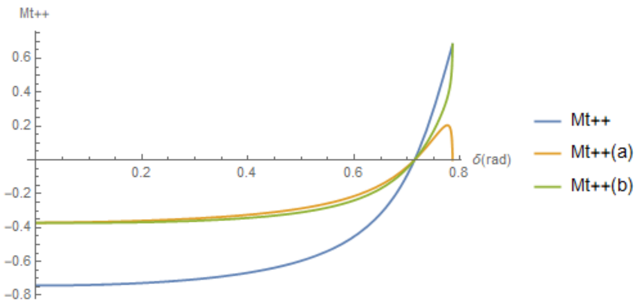
$\theta = \theta_{c,t} - 0.1$



$\theta = \theta_{c,t}$



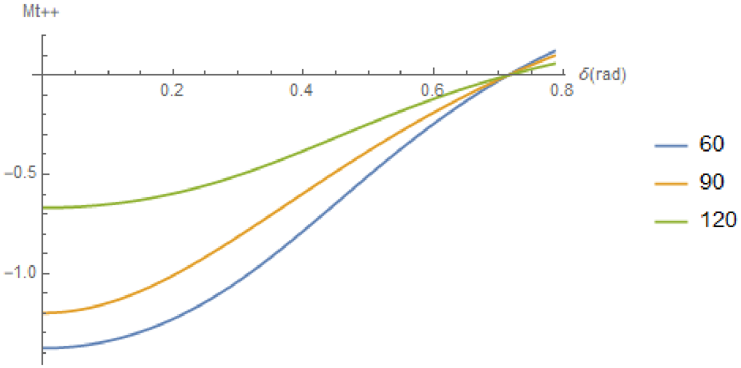
$\theta = \theta_{c,t} + 0.1$



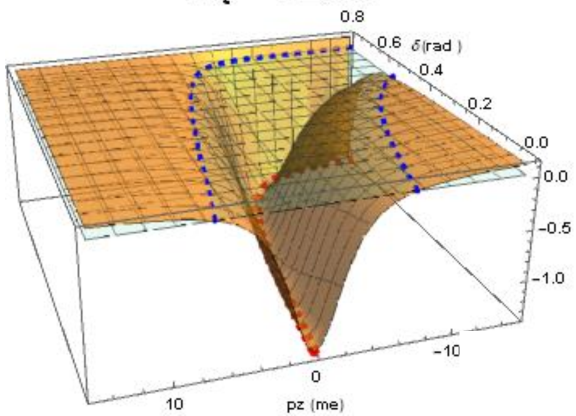
$$E_0 = 2m_e$$
$$P_\gamma = \sqrt{3.5}m_e$$
$$P_e = \sqrt{3}m_e$$

$\theta_{c,t}$ = critical annihilation angle

$P_\gamma = m_e$

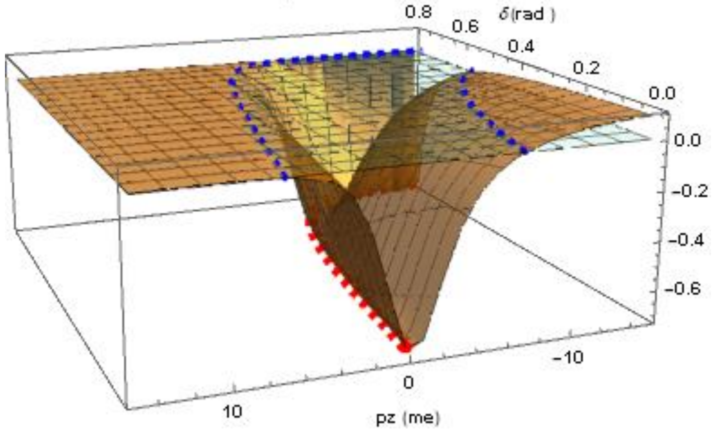


$^{\pi}M_t^{++}$ $\theta=\pi/3$

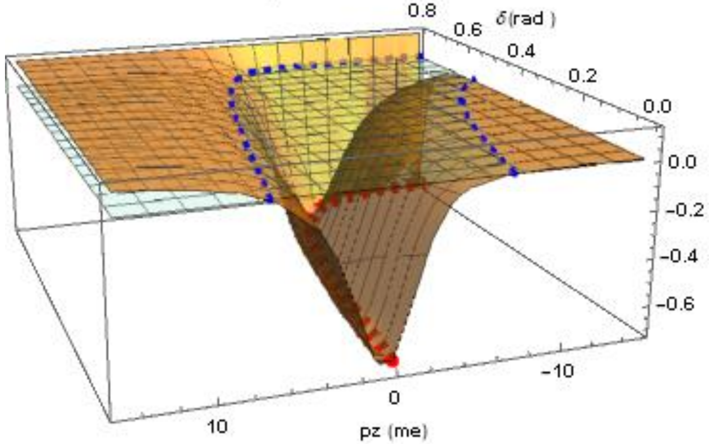


It seems there is a connection to the helicity.

$^{\pi}M_{t,a}^{++}$ $\theta=\pi/3$



$^{\pi}M_{t,b}^{++}$ $\theta=\pi/3$



$$\delta = ArcTan\left(\frac{P_e}{E_0}\right)$$
$$= 0.713724$$

$$M_t = (-p_1 \; + q_1)^{\hat{\mu}} \varepsilon^*_{\hat{\mu}}(p_3, \lambda_3) \frac{1}{q_1^2 - m^2} (p_2 \; + q_1)^{\hat{\nu}} \varepsilon^*_{\hat{\nu}}(p_4, \lambda_4)$$

$$(-p_1 \; + q_1)^{\hat{\mu}} \varepsilon^*_{\hat{\mu}}(p_3, \lambda_3) = -2 \, (p_1)^{\hat{\mu}} \varepsilon^*_{\hat{\mu}}(p_3, \lambda_3) = 0 \quad \rightarrow (\delta_p^{\pm}, \delta_{pt}^{00})$$

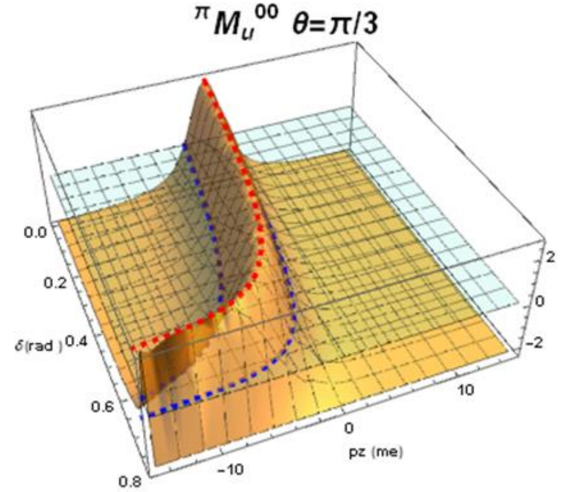
$$(p_2 \; + q_1)^{\hat{\nu}} \varepsilon^*_{\hat{\nu}}(p_4, \lambda_4) = 2(p_2)^{\hat{\nu}} \varepsilon^*_{\hat{\nu}}(p_4, \lambda_4) = 0 \quad \rightarrow (\delta_e^{\pm}, \delta_{et}^{00})$$

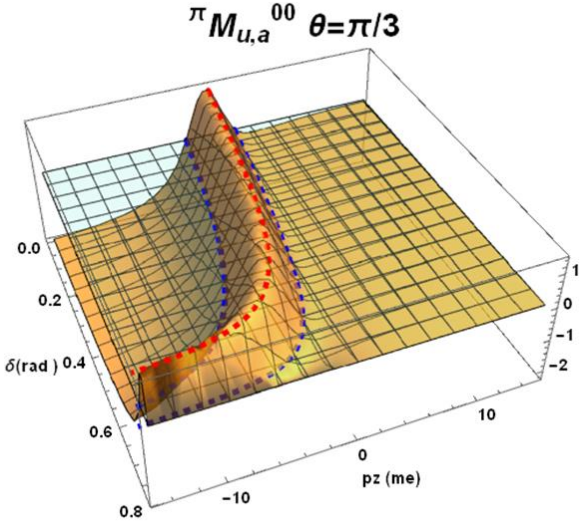
$$(p_1)^{\hat{\mu}} \varepsilon^*_{\hat{\mu}}(p_3, \pm) = (p_1)_{\hat{+}} \varepsilon^{*\hat{+}}(p_3, \pm) + (p_1)_{\hat{-}} \varepsilon^{*\hat{-}}(p_3, \pm) \qquad \rightarrow P_{\hat{-}} = 0$$

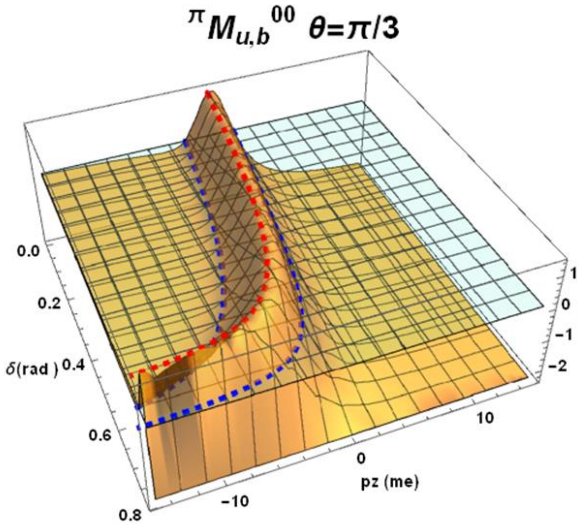
Transverse Interpolating polarization vector gauge conditions : $A^{\hat{+}} = 0$ and $\partial_{\hat{-}} A_{\hat{-}} + \; \partial_{\hat{\perp}} . A_{\hat{\perp}} \mathbb{C}$ =0

Light-front gauge : $A^{+} = 0$

Coulomb gauge in IFD : $\nabla . A = 0$







Does helicity 0 change ?

$$\delta_{et}^{00}(\theta) = \delta_{eu}^{00}(\pi - \theta)$$

$$\delta_{pt}^{00}(\theta) = \delta_{pu}^{00}(\pi - \theta)$$

$$Et = E0^2 p\gamma Cos[\theta] - pe(E0^2 - p\gamma^2 Sin[\theta]^2)$$

$$Pt = E0 p\gamma(p\gamma - peCos[\theta])$$

$$Eu = E0^2 p\gamma Cos[\theta] + pe(E0^2 - p\gamma^2 Sin[\theta]^2)$$

$$Pu = -E0 p\gamma(p\gamma + peCos[\theta])$$

$$\delta_{pt}^{00} = -ArcTan\left[\frac{\left(Et * pz + pt * \sqrt{4E0^2 + pz^2}\right)}{\left(pt * pz + Et * \sqrt{4E0^2 + pz^2}\right)}\right]$$

$$\delta_{et}^{00} = -ArcTan\left[\frac{\left(Et * pz - pt * \sqrt{4E0^2 + pz^2}\right)}{\left(-pt * pz + Et * \sqrt{4E0^2 + pz^2}\right)}\right]$$

$$\delta_{pu}^{00} = -ArcTan\left[\frac{\left(Eu * pz + pu * \sqrt{4E0^2 + pz^2}\right)}{\left(pu * pz + Eu * \sqrt{4E0^2 + pz^2}\right)}\right]$$

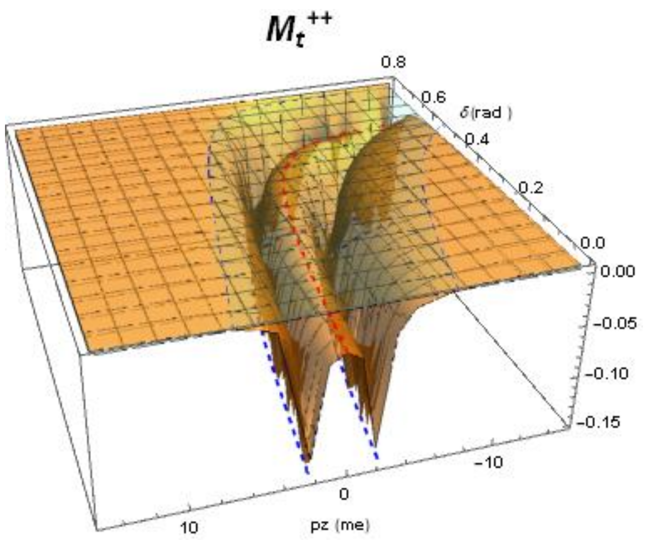
$$\delta_{eu}^{00} = -ArcTan\left[\frac{\left(Eu * pz - pu * \sqrt{4E0^2 + pz^2}\right)}{\left(-pu * pz + Eu * \sqrt{4E0^2 + pz^2}\right)}\right]$$

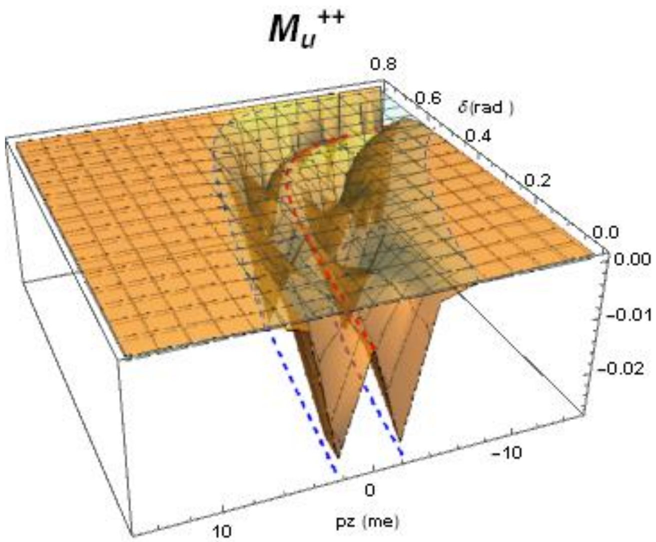
Annihilation angle $\theta \rightarrow 0$

$$\delta_{pt}^{00} = \delta_{pu}^{00} = -ArcTan\left[\frac{\left(E0 * pz + p\gamma * \sqrt{4E0^2 + pz^2}\right)}{\left(p\gamma * pz + E0 * \sqrt{4E0^2 + pz^2}\right)}\right]$$

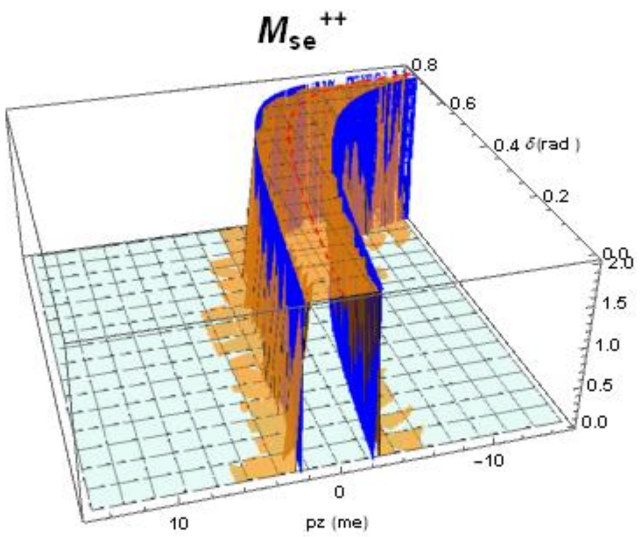
$$\delta_{et}^{00} = \delta_{eu}^{00} = -ArcTan\left[\frac{\left(E0 * pz - p\gamma * \sqrt{4E0^2 + pz^2}\right)}{\left(-p\gamma * pz + E0 * \sqrt{4E0^2 + pz^2}\right)}\right]$$

Critical interpolation angles from the helicity definition for rhp-mesons

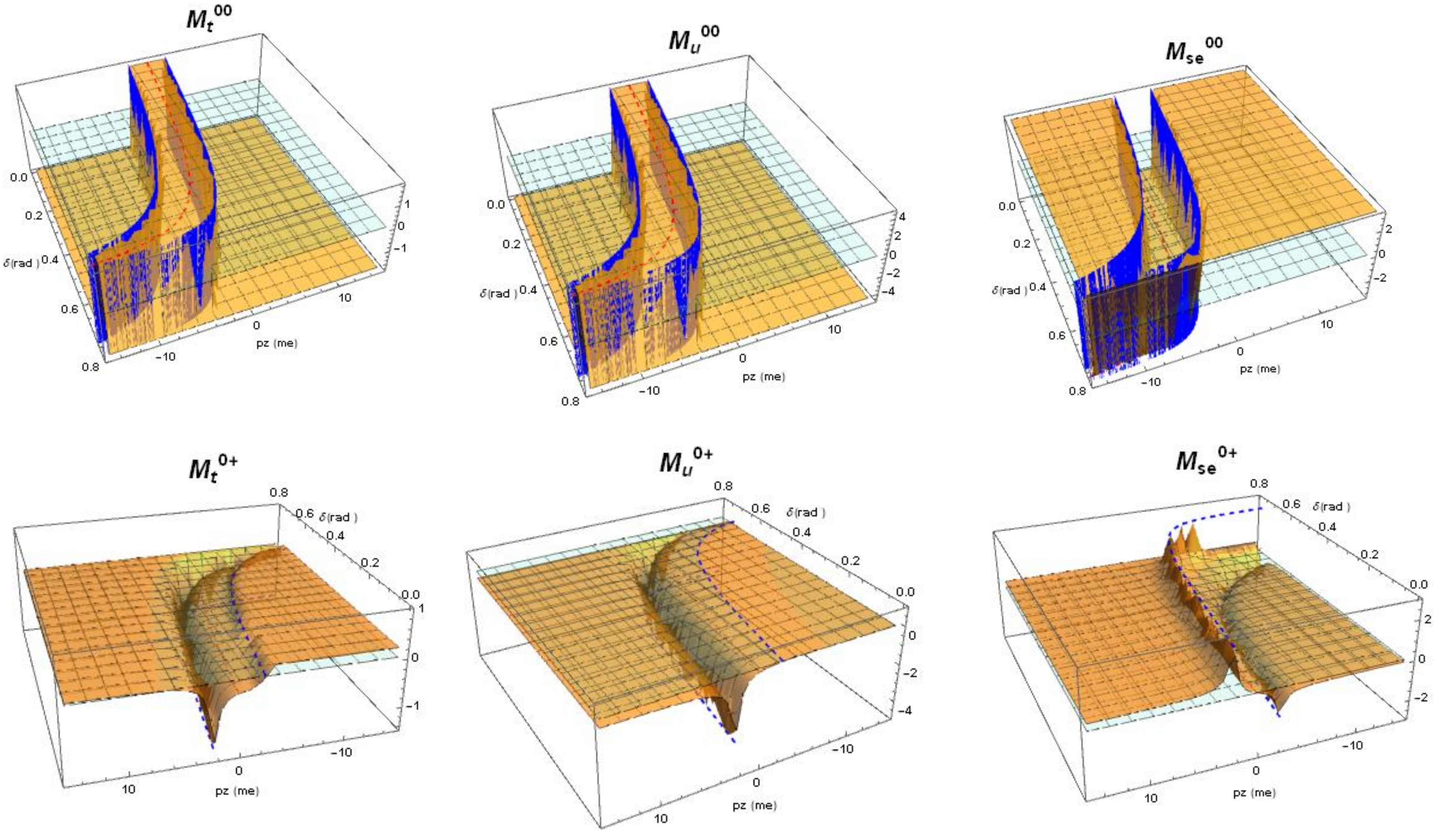




$\theta = 0.1$

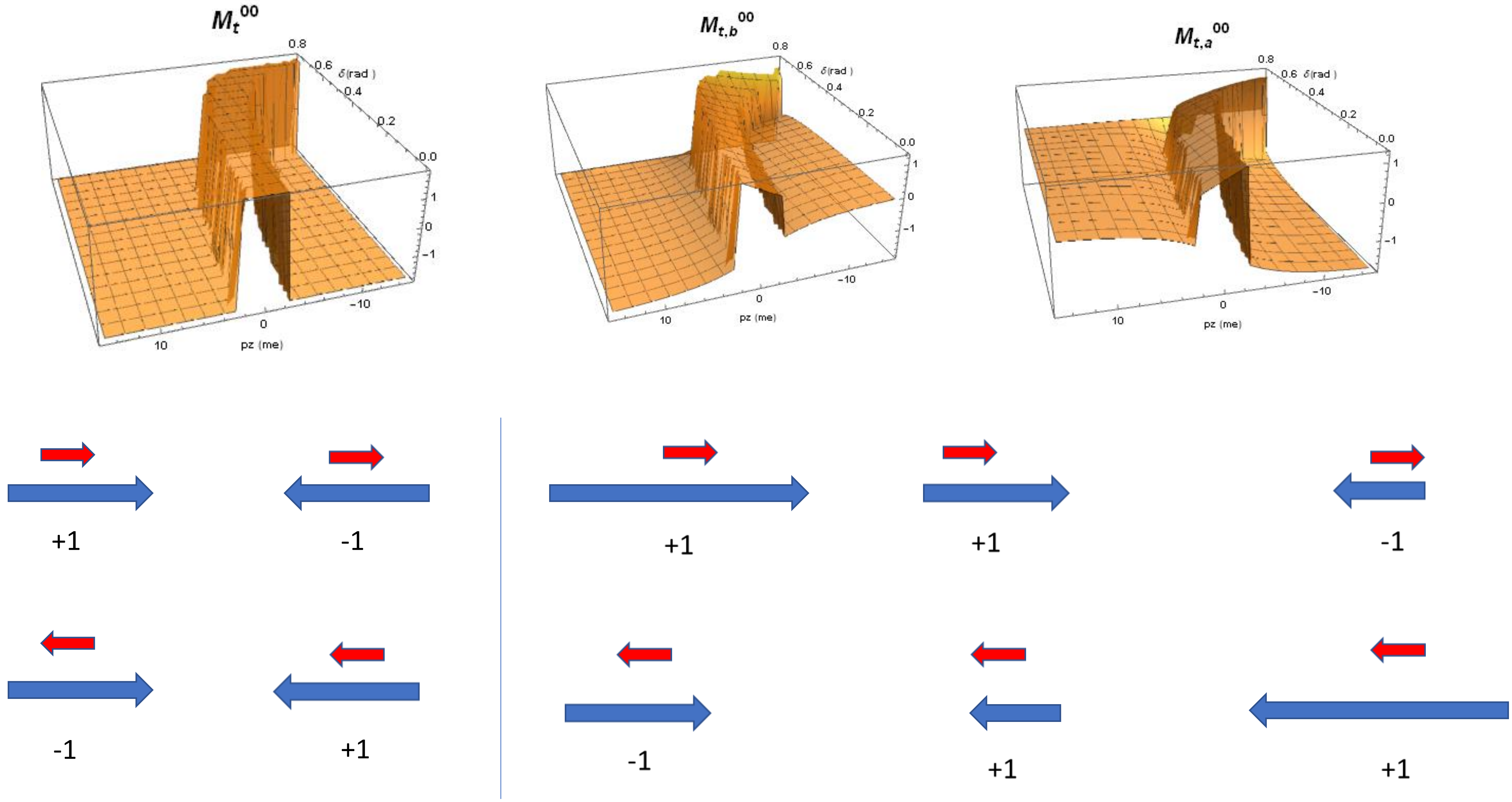


$\theta = 0$



$\theta = 0$

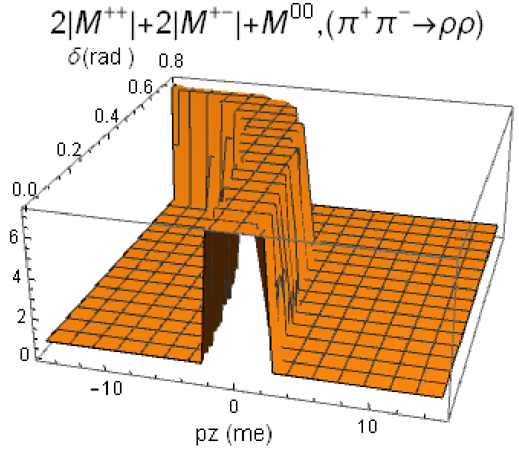
$\theta = 0.1$

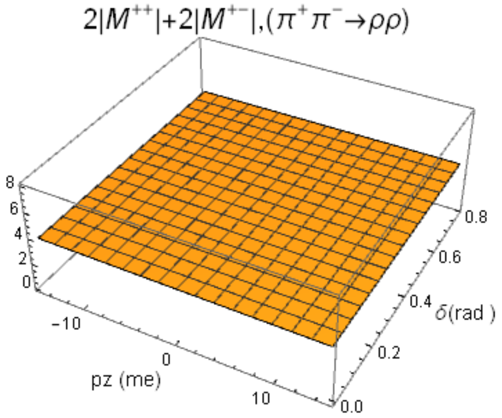


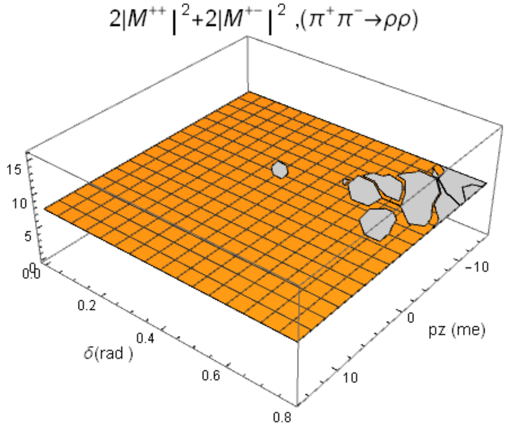
M_t^{00}
$M_{t,b}^{00}$
$M_{t,a}^{00}$
δ (rad)
pz (me)
0.8
0.6
0.4
0.2
0.0
1
0
-1
10
0
-10
+1
-1
-1
+1
+1
+1
-1
-1
+1
+1

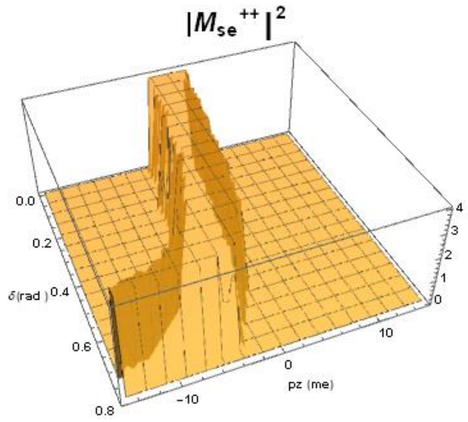
When $\theta = 0$ all the helicity amplitudes goes to zero except Mse++=Mse--, Mse+-=Mse-+, Mt00, Mu00, Mse00

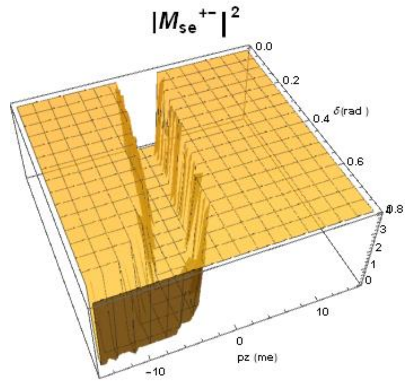
$$|M|^2 = \sum_{\lambda_1,\lambda_2} |M_t^{\lambda_1,\lambda_2} + M_u^{\lambda_1,\lambda_2} + M_{se}^{\lambda_1,\lambda_2}|^2$$

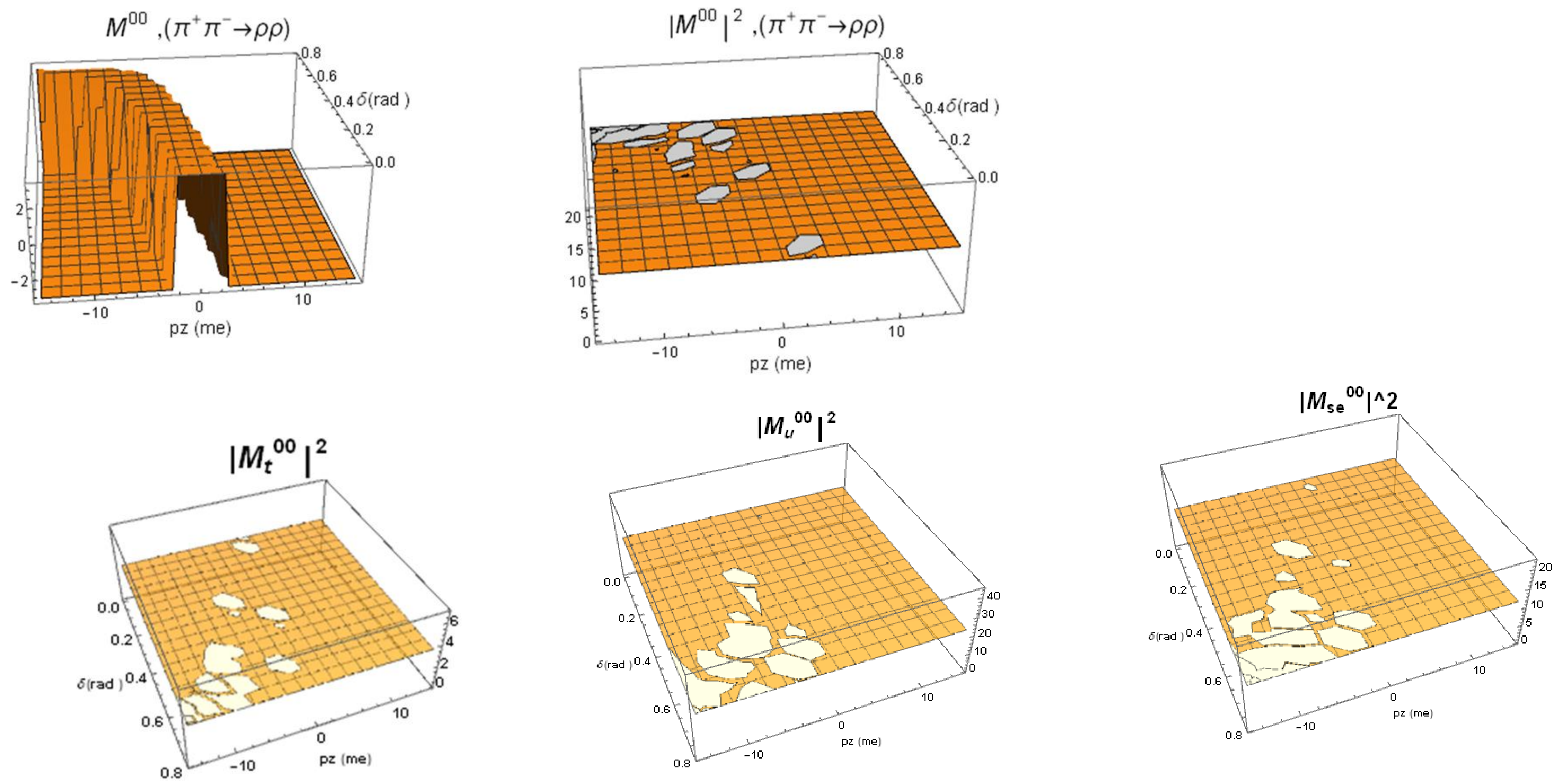






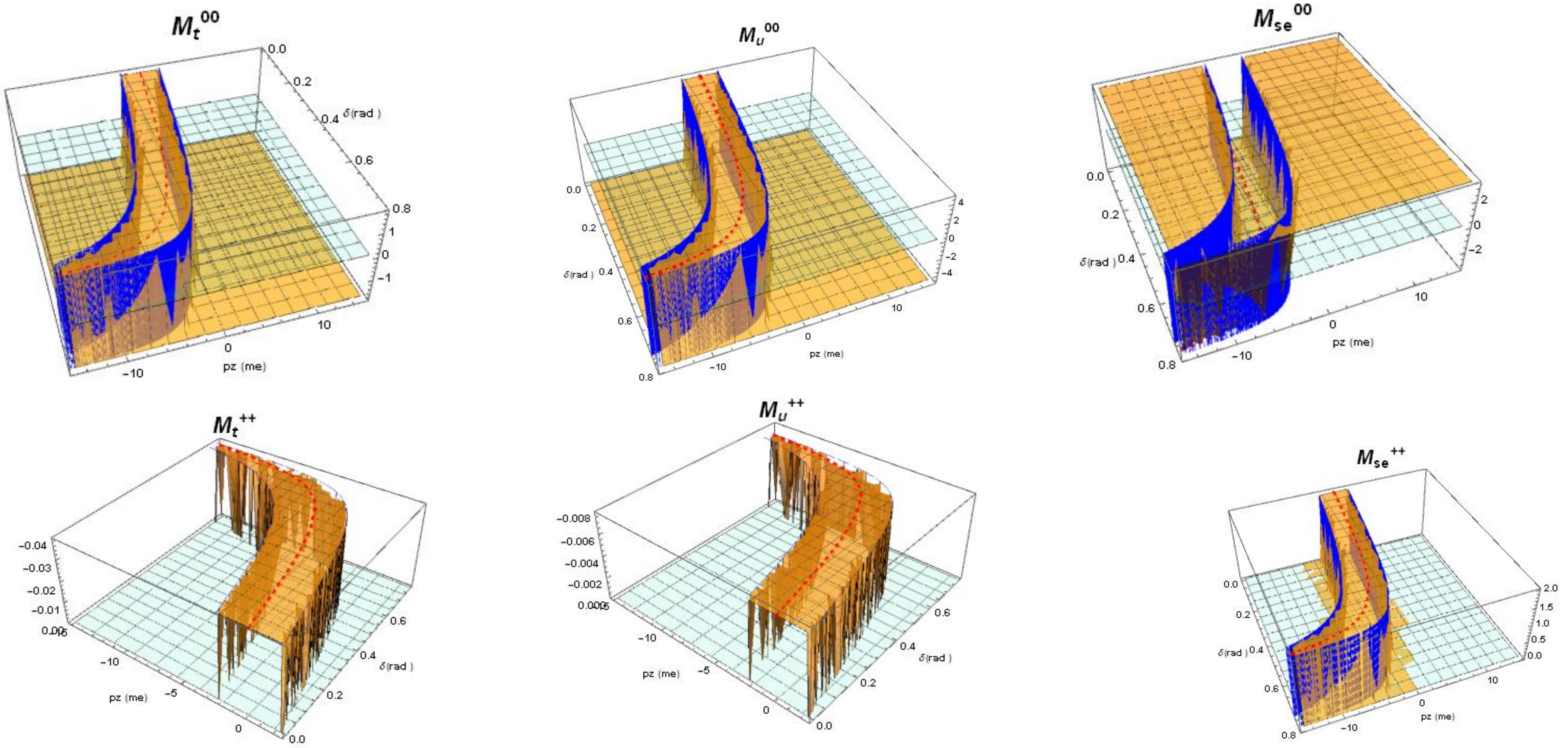





M^{00} ,$(\pi^+\pi^-\to\rho\rho)$
$|M^{00}|^2$,$(\pi^+\pi^-\to\rho\rho)$
$|M_t^{00}|^2$
$|M_u^{00}|^2$
$|M_{se}^{00}|$^2
pz (me)
δ(rad)
-10
0
10
0.0
0.2
0.4
0.6
0.8

$\rho^+\rho^- \to$ “$\pi^+\pi^-$ "

Don’t have stick with $\theta = 0$ to see clear boundaries as initial partials have spins.



Do I have to consider internal structure of the partials to have a proper understanding of the helicity zero ?