,

Introduction to S-Matrix Bootstrap

Shaswat Subham Tiwari

June 2020

References

1. R.J Eden,P.V Landshoff,D.I Olive, J.C Polkinghorne ,"The Analytic S-Matrix"
2. M.F Paulos,J.Penedones,J.Toledo,B.C. van Rees, and P.Viera, "The S-matrix Bootstrap II:Two Dimensional Amplitudes,"arXiv:1607.06110[hep-th].
3. M. F. Paulos, J. Penedones, J. Toledo, B. C. van Rees and P. Vieira, "The S-matrix Bootstrap III: Higher Dimensional Amplitudes," arXiv:1708.06765 [hep-th].
4. L.Cordova,P.Vieira,"Adding flavour to S-matrix bootstrap,"arXiv:1805.111 43v3 [hep-th].
5. A.B Zamolodchikov,A.B Zamolodchikov,"Factorized S-Matrices in Two Dimensions as the exact Solutions of Certain Relativistic Quantum Field Theory Models".

Contents

Introduction

WHAT IS S-MATRIX BOOTSTRAP?

- S-matrix bootstrap is an attempt to determine the complete S-matrix of a theory by using analytic properties like:
  1. Unitarity: $S^\dagger S = 1$
  2. Crossing symmetry:$A_{a+b\to c+d}(s,t)$ gives the amplitude for the process $a+b\to c+d$ for $s\geq 4m^2$ and $t<0$. Crossing symmetry implies: $A_{a+\bar{c}\to\bar{b}+d}(t,s) = A_{a+b\to c+d}(s,t)$.
  3. Poles and branch cuts
- Completely Non-perturbative,it aimed to reformulate QFT.
- Was persued analytically in 1960s, abandoned after QCD

WHY NOW? WHAT HAPPENED?

- Advent of Conformal Bootstrap and it's methods breathed new life into S-matrix bootstrap
- The space of S-matrices was explored by extremizing the combinations of parameters under unitarity and crossing symmetry to find physical theories
- Pedro Viera, Juao Penedones et al published a series of papers employing bootstrap in 1+1 d and 3+1 d culminating in one which found theories close to pion pion S-matrix.
- Our work uses monotonicity of relative entropy to constrain the allowed S-matrix space

Introduction

WHY 1+1 Dimensions?

- The problem of maximizing coupling constants is analytically solved for 1+1 d
- This proves to be a fine check for our methods before we venture into 3+1 d, where no such solution is available.
- We will discuss the following papers:
  1. S-Matrix bootstrap II
  2. Adding Flavour to S-Matrix Bootstrap

General Setting

Both the papers have a recurring theme which can be summarized as:

- We consider 2-2 scattering of identical bosons in 1+1 d. The bosons are assumed to transform in the vector representation of O(N) in the second paper. Scattered particles are assumed to be the lightest in the theory.
- We make an ansatz for the S-matrix based on the:
  1. bound states : They are the intermediate particles in the theory. They occur as simple poles
  2. Branch cuts : Signify particle production. Disc(S) $\propto \sum_X \sigma_{2->x}$
- This dispersion relation is inherently crossing symmetric
- The coupling constant of the lightest bound state is maximized under unitarity and resultant theory is compared with a known model

S-Matrix Bootstrap II

setup

- $\langle p_3, p_4|\mathbf{S}|p_1, p_2\rangle = S(s)$, $\mathbf{S} = \mathbb{1} + \iota\mathbf{T}$
- Unitarity, $|S(s)|^2 \leq 1$ for $s > 4m^2$
- Crossing symmetry, $S(s) = S(4m^2 - s)$
- Poles are of the form:

$$S(s) \simeq -J_j \frac{g_j^2}{s - m_j^2} \text{ and } S(s) \simeq -J_j \frac{g_j^2}{4m^2 - s - m_j^2}$$

  g_j^2 is the residue of $\mathbf{T}$
- There is a branch cut from $s > 4m^2$ and from $s < 0$ by crossing symmetry.

S-Matrix Bootstrap II

Dispersion Relation

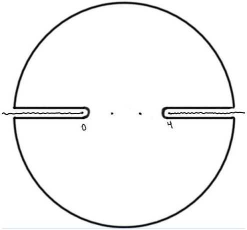

Figure: Contour of integration

- $S(s) - S_\infty = \oint_\gamma \frac{S(x)-S_\infty}{x-s}\frac{dx}{2\pi\iota}$
- $S(s) = S_\infty - \sum_j J_j(\frac{g_j^2}{s-m_j^2} + \frac{g_j^2}{4m^2-s-m_j^2}) + \int_{4m^2}^{\infty} dx\rho(x)(\frac{1}{x-s} + \frac{1}{x-4m^2+s})$
  where $2\pi\iota\,\rho(s) \equiv\ S(s+i0) - S(s-i0)$

S-Matrix Bootstrap II

- Approximating, $\rho(x) = \rho_i + \frac{\rho_{i+1}-\rho_i}{x_{i+1}-x_i}(x - x_i)$ for $x \in [x_i, x_{i+1}]$,$i < m$

$$\rho(x) = \rho_m x_m / x, \text{ for } x \in [x_m, \infty]$$

- $S(s) = S_\infty - \sum_j J_j(\frac{g_j^2}{s-m_j^2} + \frac{g_j^2}{4m^2-s-m_j^2}) + \sum_{i=0}^{n} \rho_i\, K_i(s)$
- We maximize g_1^2, under unitarity, in the space of (g_j^2, ρ_i) using FindMaximum function in Mathematica

We generate the solution for the case of single bound state at $m_1 = \sqrt{3}$ for n=20:

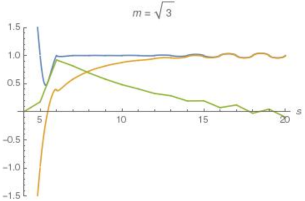

Figure: Graphs generated by us for n=20

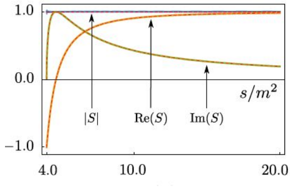

Figure: From [2]. This matches with sine gordon model

The result matches with the Sine-gordon S-matrix for the lightest breathers which is a theory saturating unitarity.

We next maximize g_1^2 for single bound-state case for various m_1 values and plot their graph:

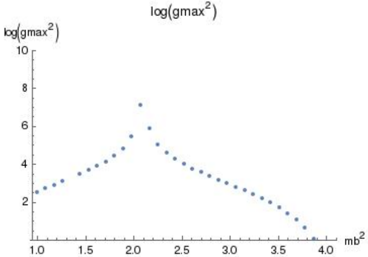

Figure: For n=20

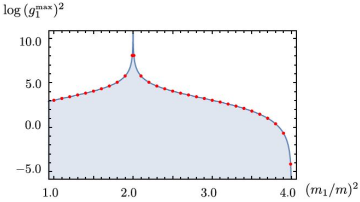

Figure: Taken from [2]. Plot for $Log(g_{max}^2)$ vs m_1^2

The peak at $m = \sqrt{2}$ occurs as s and t-channel poles cancel each other.

Analytic Comparison

- We write $p_1 = (m\cosh\theta_1, m\sinh\theta_1)$ and $p_2 = (m\cosh\theta_2.m\sinh\theta_2)$
- $s = 4m^2 Cosh^2(\frac{\theta}{2})$, Where $\theta = \theta_1 - \theta_2$
- Unitarity:$S(\theta + i0)S(-\theta + i0) = f(\theta)$ for $\theta \in R$
- Crossing symmetry:$S(\theta) = S(\iota\pi - \theta)$
- General Solution: $S(\theta) = S_{CDD}\, exp\left[\int_{-\infty}^{\infty} \frac{d\theta'}{2\pi} \frac{-i\sinh\theta\ \cosh(\theta')\log(f(\theta'))}{|\sinh(\theta-\theta')|^2}\right]$

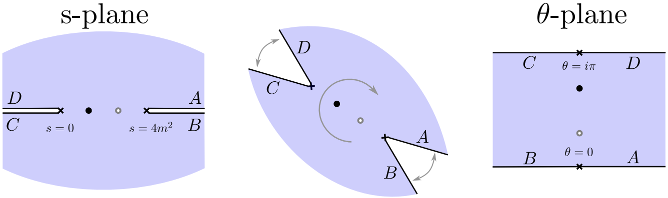

Figure: Coordinate transformation. From [2]

Analytic Comparison

The solution for analytic maximization is given by:

$$S(\theta) = \pm \prod_j [\alpha_j] \quad \text{where} \quad [\alpha] \equiv \frac{\sinh\theta + \iota \sin\alpha}{\sinh\theta - \iota\sin\alpha}$$

where $m_b = 2m\cos(\alpha/2)$ gives us the location of poles.These are the **CDD** factors. We need the above solution to match with $m = \sqrt{3}$ solution we found numerically and **it does!**

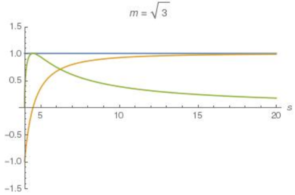

Figure: $[\alpha_1]$ for $\alpha_1 = 2\pi/3$