Soft Gluon Resummation Updates and Pion PDFs

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Group Meeting 7/31/2020

Outline

- Motivation
- How to compute resummation expressions
- Minimal Prescription
- Monte Carlo PDF results
- “Exact” Resummation with Double Mellin

Introduction/Motivation

Motivation

- QCD allows us to study the structure of protons in terms of partons (quarks, antiquarks, and gluons)
- Use factorization theorems to separate hard partonic physics out of soft, non-perturbative objects to quantify structure

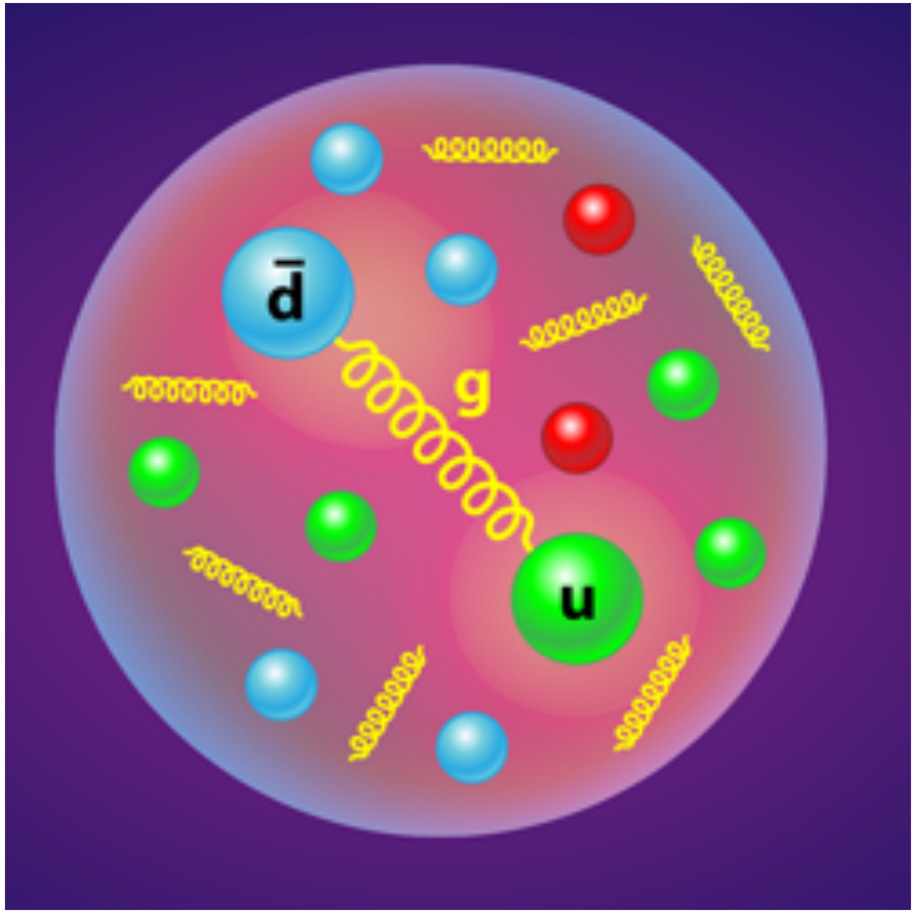
Motivation

What to do:

- Define a structure of nucleons in terms of quantum field theories
- Identify theoretical observables that factorize into non-perturbative objects and perturbatively calculable physics
- Perform global QCD analysis as structures are universal and are the same in all subprocesses

Pions

- Pion is the Goldstone boson associated with chiral symmetry breaking
- Lightest hadron as $\frac{m_\pi}{M_N} \ll 1$ and dictates the nature of hadronic interactions at low energies
- Simultaneously a $q\bar{q}$ bound state



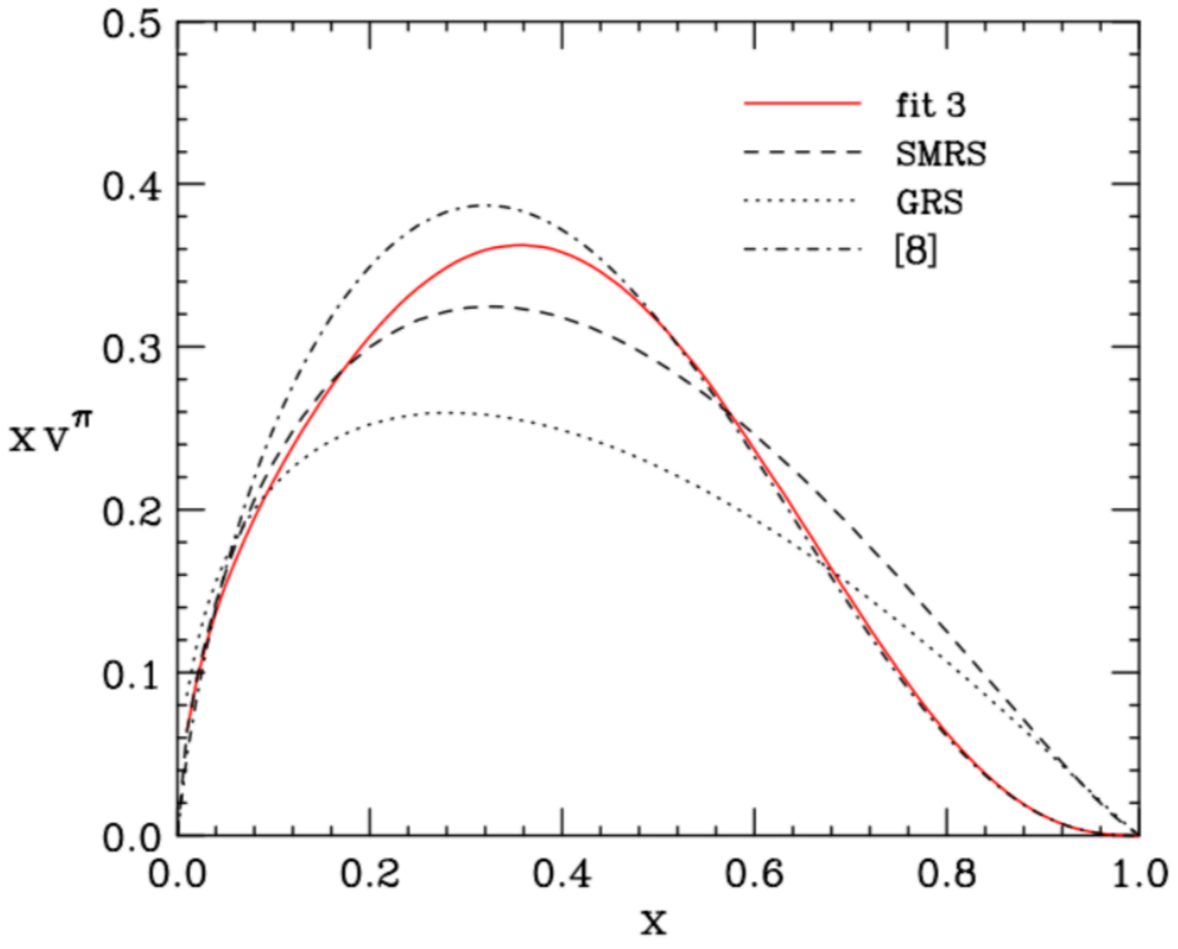
Theoretical Interest

- Behavior of PDF as $x_\pi \to 1$ ($\mathrm{v}_\pi \sim (1 - x_\pi)^{2\beta}$) can be related to momentum dependence of underlying interaction
- Perturbative QCD predicts that $\beta = 1$

Theoretical Interest

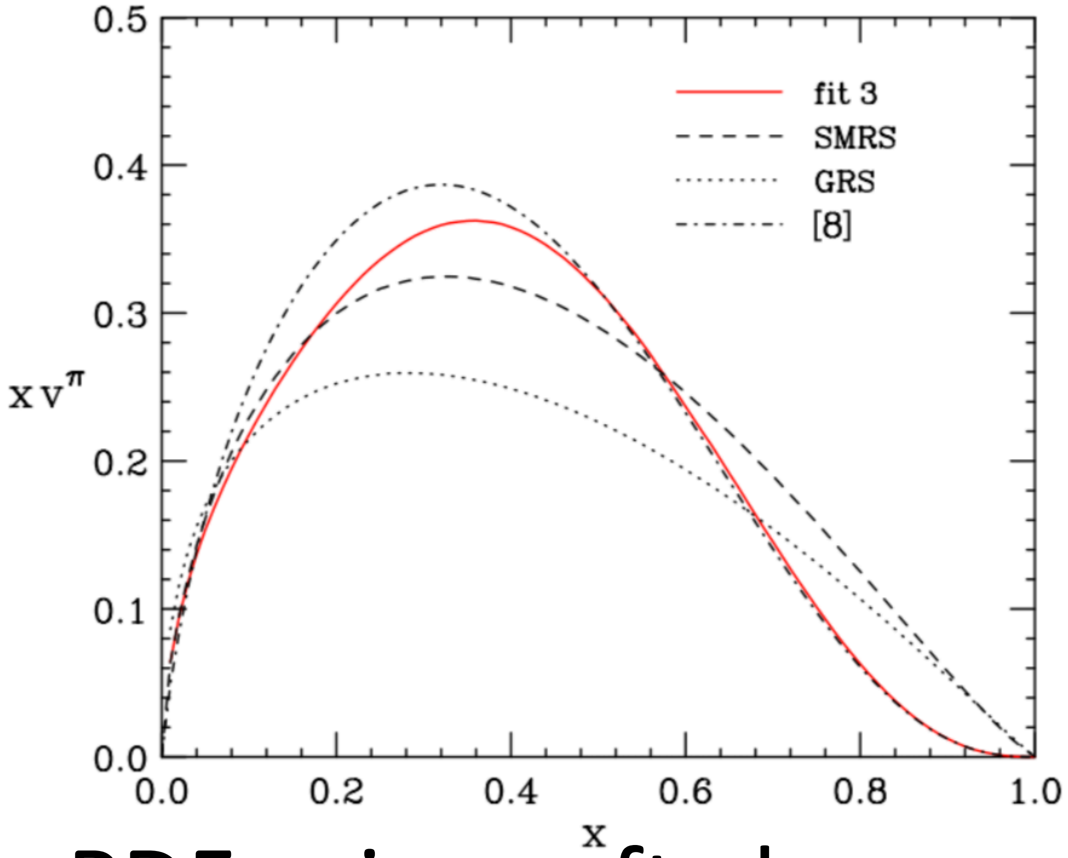
- Recent lattice calculations as well as phenomenologically determined valence quark PDFs using threshold resummation indicate $\beta = 1$ as opposed to fixed order ($\beta = 1/2$)
- Our analysis with threshold resummation will have impact on this question

Previous Pion Fits

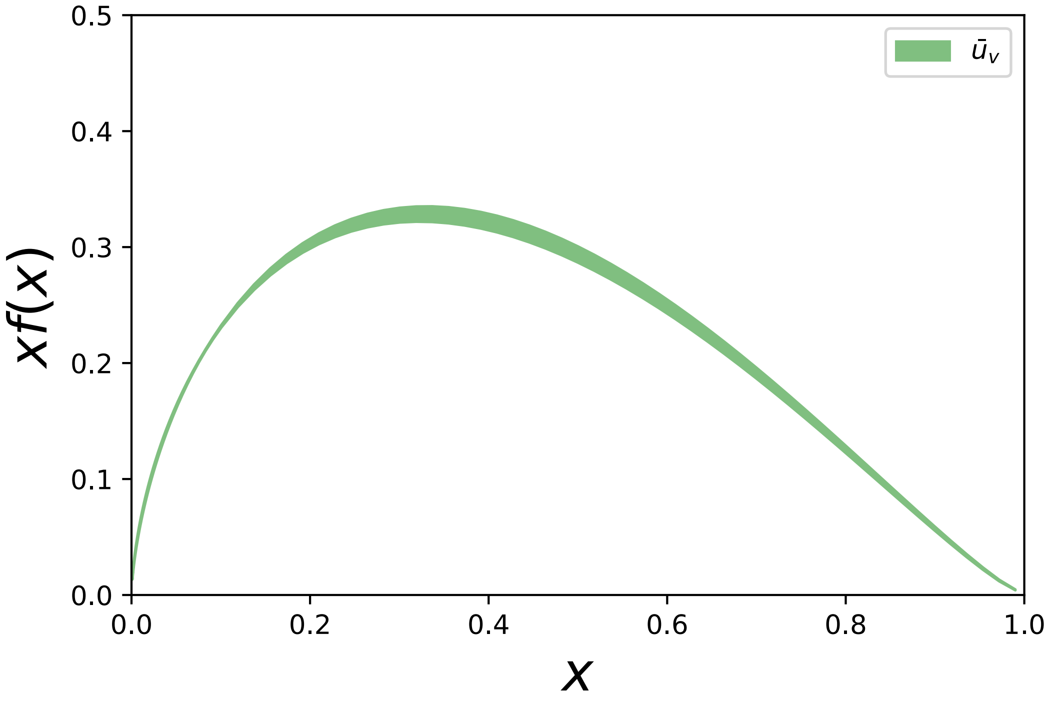


- Most recent (M. Aicher, et al, 2010) pion fit to DY data
- Fit uses soft gluon resummation

Comparison - Pion PDFs



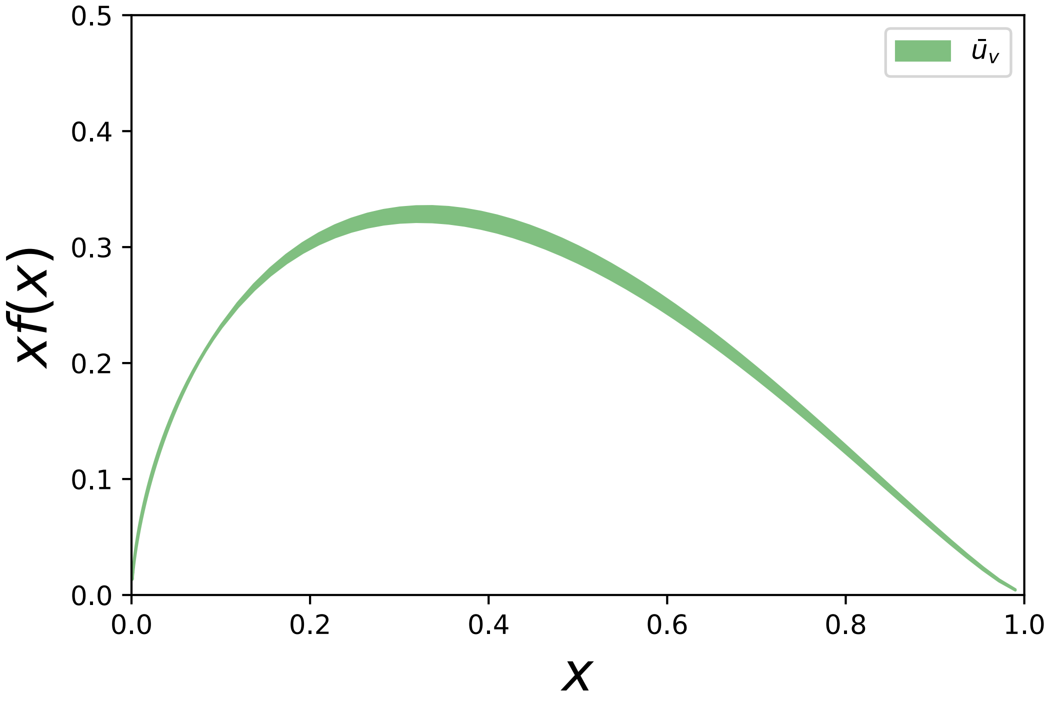
PDF using soft gluon resummation



PDF using only fixed-order pQCD (JAM18)

Uncertainty

- Note uncertainty band on PDFs are strictly from the data errors and parameterization bias
- No theoretical uncertainty shown (more on this later)



Drell-Yan (DY) Definitions

Hadronic variable

$$\tau = \frac{Q^2}{S}$$

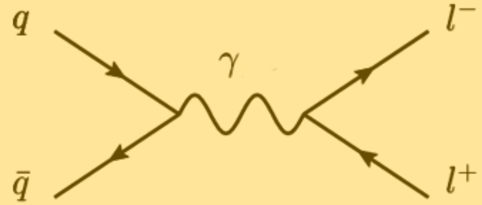
Partonic variable

$\hat{S}$ is the center of mass momentum squared of incoming partons

$$z \equiv \frac{Q^2}{\hat{S}} = \frac{\tau}{x_1 x_2}$$

Fixed Order Up to NLO

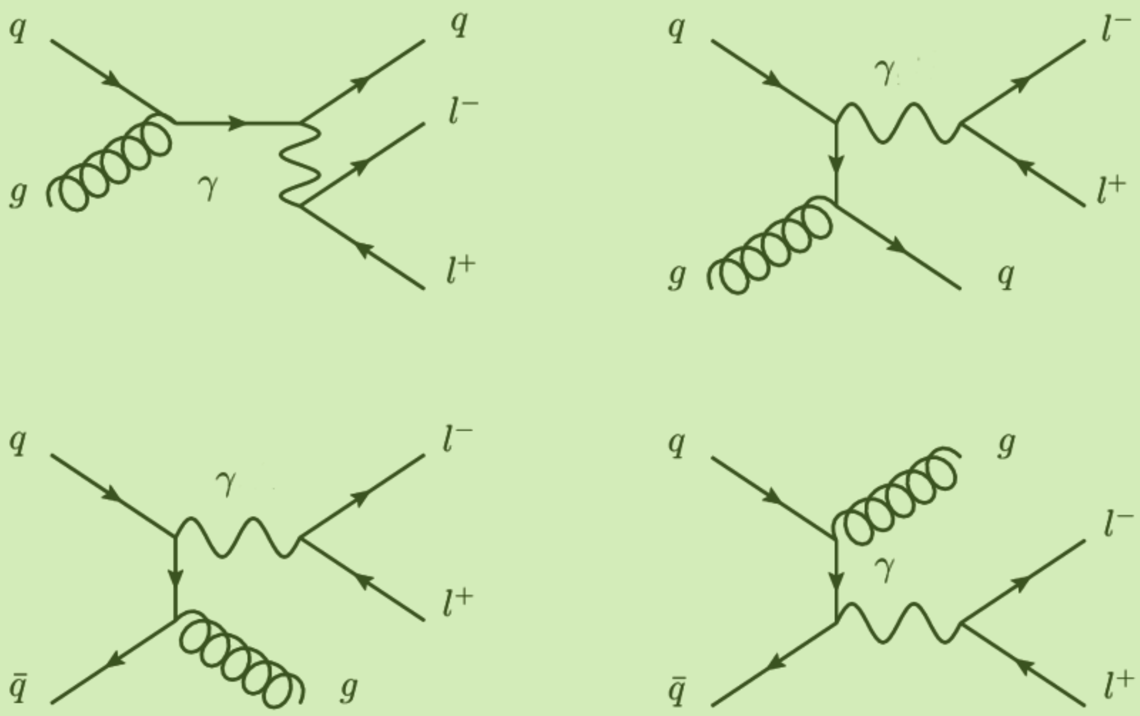
LO: $\mathcal{O}(1)$



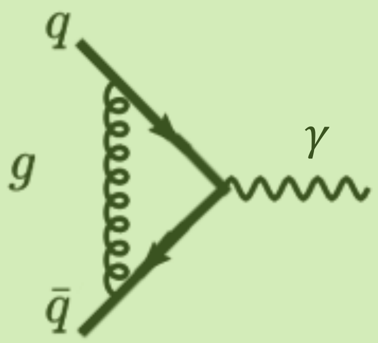
Feynman diagrams for DY amplitudes in collinear factorization

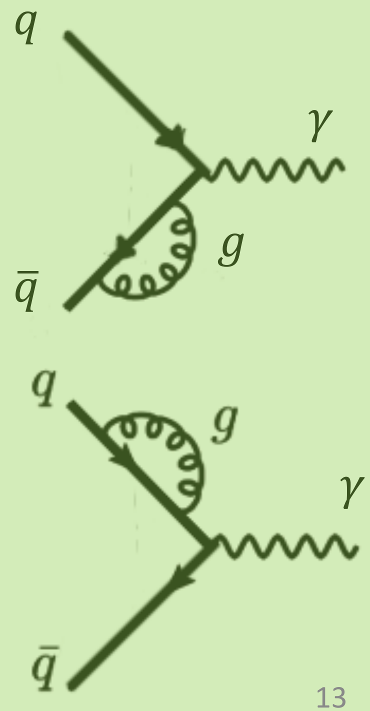
NLO: $\mathcal{O}(\alpha_S)$

Real emissions



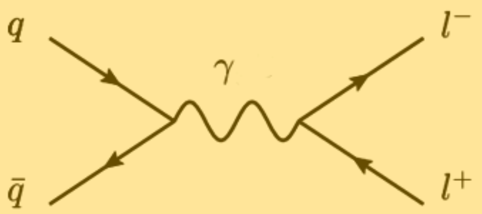
Virtual Corrections





LO

LO: $\mathcal{O}(1)$

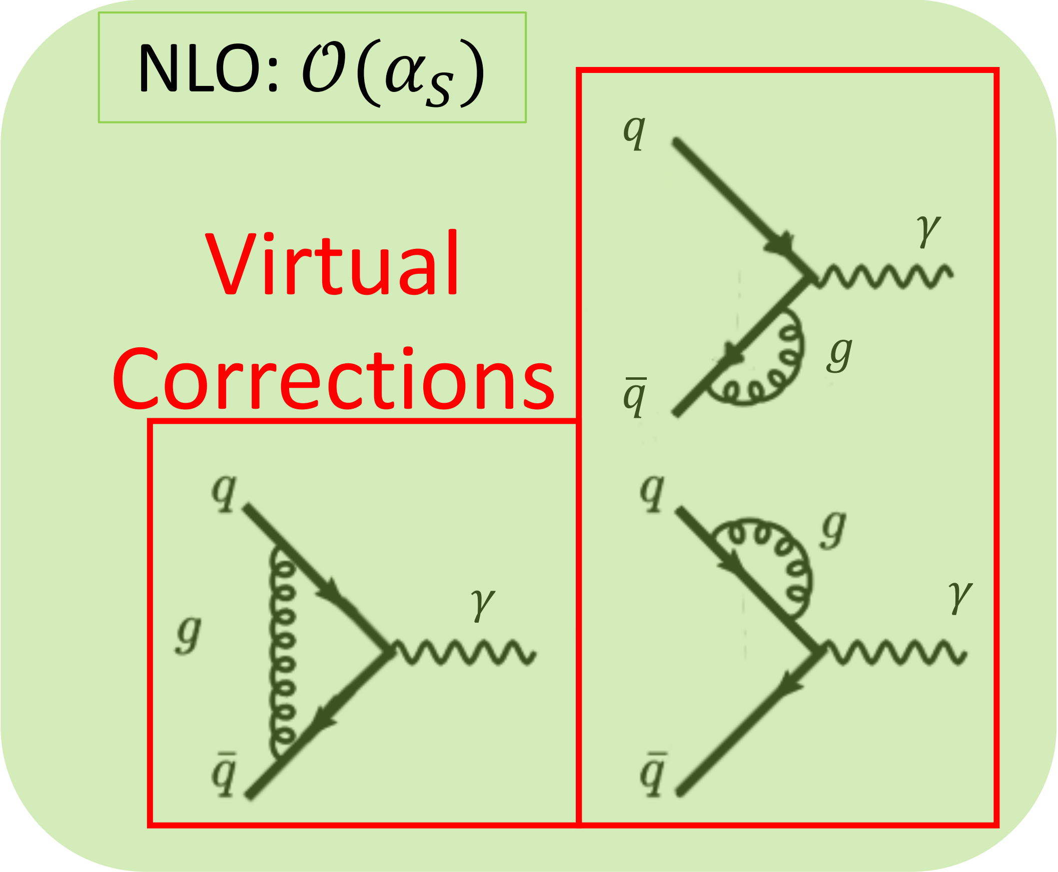


$$C_{q\bar{q}} = \delta(1-z)\frac{\delta(y)+\delta(1-y)}{2}$$

- $z = 1$ corresponds to partonic threshold
- All $\hat{S}$ is equal to Q^2
- All energy of hard partons turns into virtuality of photon

NLO Virtual

- Virtual corrections at NLO are proportional to $\delta(1-z)$
  - Exhibit Born kinematics



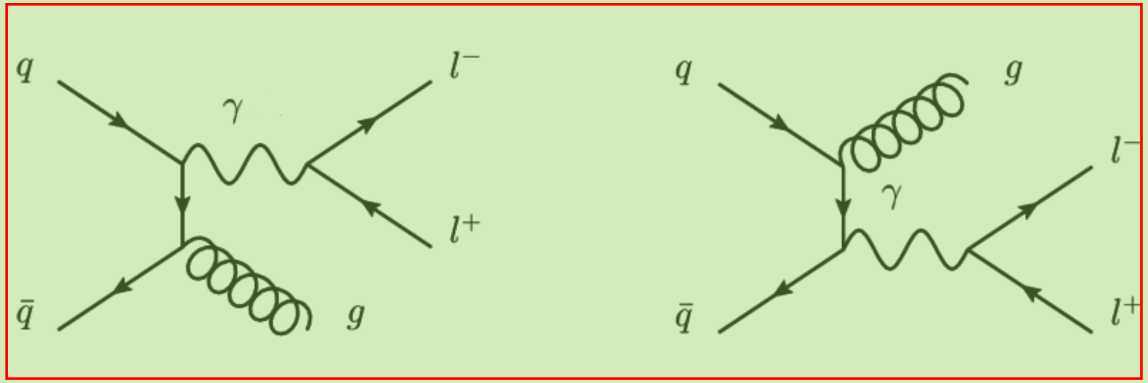
$$C_{q\bar{q}}^{\text{virtual}} = \delta(1-z)\frac{\delta(y)+\delta(1-y)}{2}\left[\frac{C_F\alpha_S}{\pi}\left(\frac{3}{2}\ln\frac{Q^2}{\mu^2}+\frac{2\pi^2}{3}-6\right)\right]$$

NLO Real Emission

- Next to leading order, real gluon emissions

$$C^{\text{real}}_{q\bar{q}} = \frac{C_F \alpha_S}{\pi}\left[\frac{\delta(y)+\delta(1-y)}{2}\left[(1+z^2)\left(\frac{1}{1-z}\ln\frac{Q^2(1-z)^2}{\mu^2 z}\right)_+ + 1 - z\right]\right.$$
$$\left. + \frac{1}{2}\left[\frac{(1-z)^2}{z}y(1-y)\right]\left[\frac{1+z^2}{1-z}\left(\left[\frac{1}{y}\right]_+ + \left[\frac{1}{1-y}\right]_+\right) - 2(1-z)\right]\right]$$

Real emissions

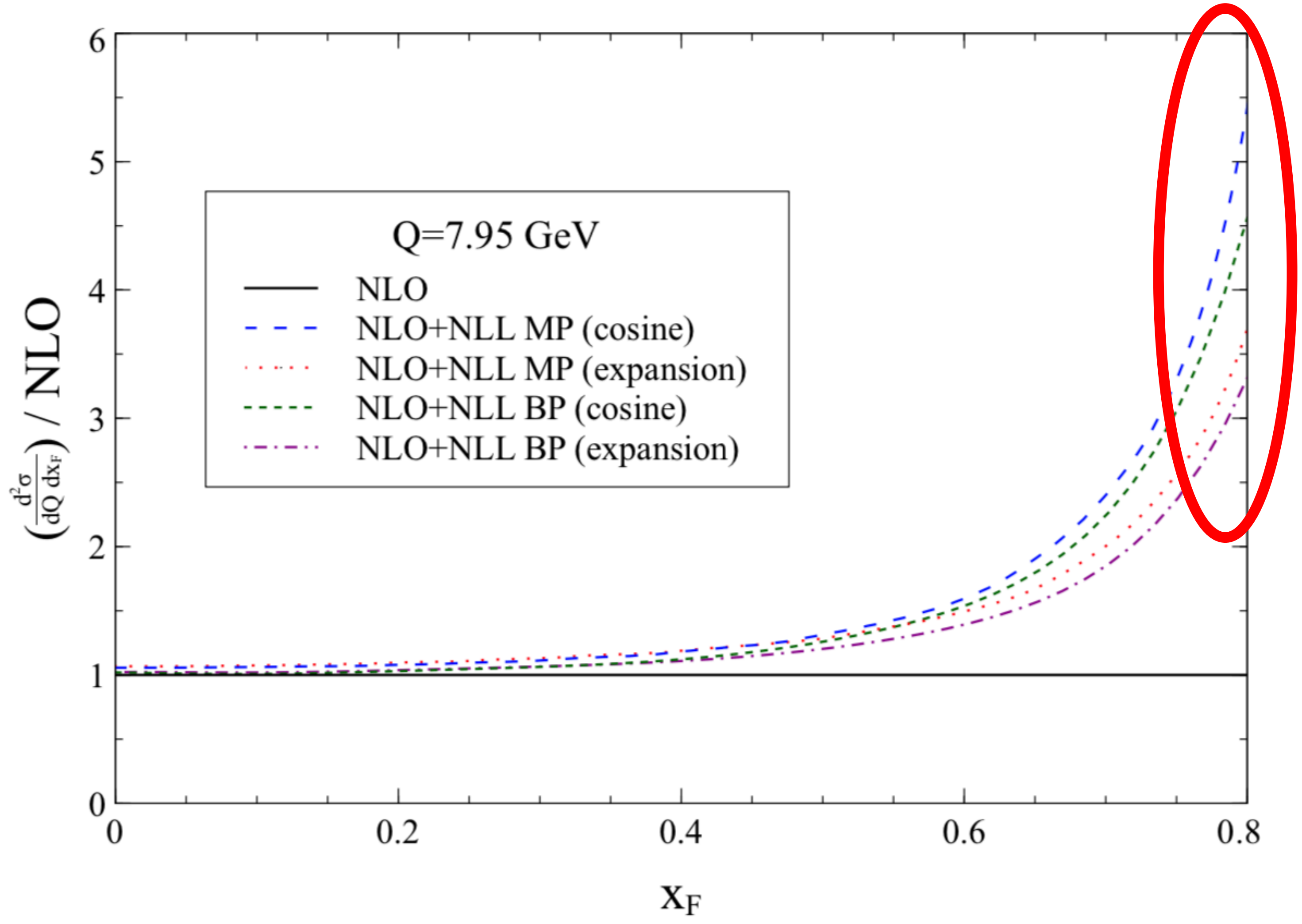


NLO Real Emission

- Plus distributions come from subtraction procedure of collinear singularities
- When $z \to 1, \log(1-z)$ can be large and potentially spoil perturbation
  - Appear in all orders in a predictable manner

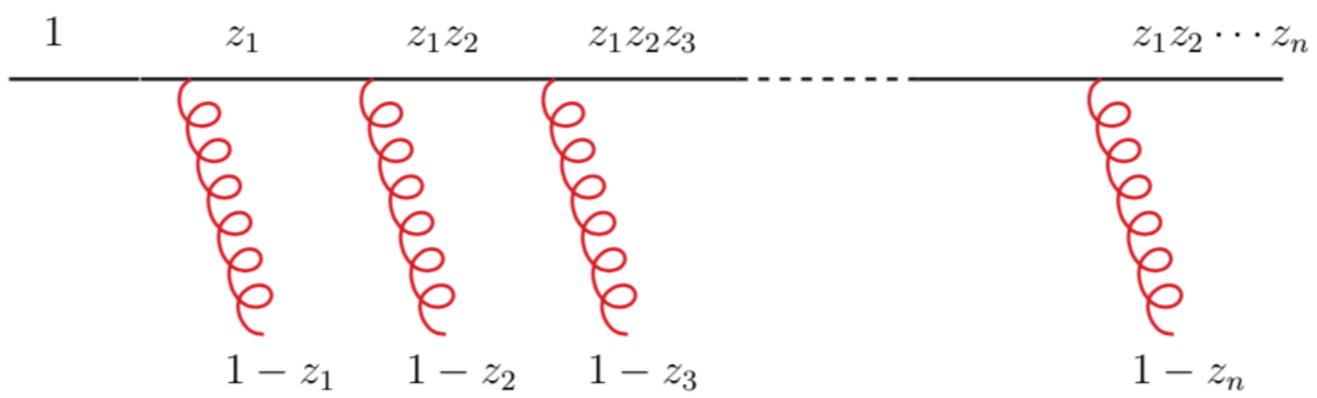
K-factor

- Different prescriptions show different results, especially at large x_F
- Could put a theoretical uncertainty on the pion PDFs



Computing Resummation Expressions

Soft Gluon Resummation



- These are Real Emitted Gluons from a quark line
- Can perturbatively calculate these emissions to all orders of α_S
- Here, z_i near 1

Setting it up

- Because of the Eikonal approximation, in the soft limit, matrix elements of large numbers of emitted gluons can be factorized as such:

$$\mathcal{M}_n(z_1, \ldots, z_n) \overset{\text{soft}}{\simeq} \frac{1}{n!} \prod_{i=1}^{n} \mathcal{M}_1(z_i)$$

- Even though the amplitudes factorize in z-space in that way, the phase space does not because of the presence of a delta function for conservation of momentum

$$\delta(z - z_1 z_2 \ldots z_n).$$

Setting it up

- In Mellin space, however, we do have factorization of the phase space,

$$\int_0^1 dz z^{N-1} \delta(z - z_1 z_2 ... z_n) = z_1^{N-1} z_2^{N-1} ... z_n^{N-1}$$

- So for hard kernels, for each order of α_S, we have:

$$C^{(n)}(N) \overset{\text{soft}}{\simeq} \frac{1}{n!} \left[C^{(1)}_{\text{soft}}(N) \right]^n$$

- Where $C^1_{soft}(N)$ is the hard kernel for one soft gluon emitted from the quark line

Exponentiation

- Thus, to sum over all the hard kernels is to exponentiate the emission of one soft gluon

$$
\begin{aligned}
\sum_{n=1}^{\infty} C^{(n)}(N) &= \sum_{n} \frac{1}{n!}[C^{(1)}_{\text{soft}}(N)]^n \\
&= \exp\left(C^{(1)}_{\text{soft}}(N)\right)
\end{aligned}
$$

- Exponentiation is a key concept in threshold resummation

Computing the Expressions

- Specifically for the DY case, we need to use the following for each initial state parton (2 for DY)

$$\log \Delta(N) = \int_0^1 dz \frac{z^{N-1}-1}{1-z} \int_{\mu^2}^{(1-z)^2 Q^2} \frac{dk_\perp^2}{k_\perp^2} A_q(\alpha_S(k_\perp))$$

where

$$A_q(\alpha_S) = \sum_{i=1}^{\infty} \alpha_S^i A_q^{(i)}$$

and

$$A_q^{(1)} = \frac{C_F}{\pi}, \qquad A_q^{(2)} = \frac{C_F}{2\pi^2}\left[C_A\left(\frac{67}{18} - \frac{\pi^2}{6}\right) - \frac{5}{9}N_f\right].$$

Computing the Expressions

- We also need a closed form for α_S, in which case, we use the two-loop (needed for up to NLL accuracy)

$$\alpha_S(k_T^2) = \frac{\alpha_S(\mu^2)}{1 + b_0\alpha_S(\mu^2)\log(\frac{k_T^2}{\mu^2})}\left[1 - \frac{b_1}{b_0}\frac{\alpha_S(\mu^2)\log\left(1 + b_0\alpha_S(\mu^2)\log\left(\frac{k_T^2}{\mu^2}\right)\right)}{1 + b_0\alpha_S(\mu^2)\log\left(\frac{k_T^2}{\mu^2}\right)}\right]$$

Computing the Expressions

- Plugging those in, we get the following

$$\log\Delta(N) = \int_0^1 dz \frac{z^{N-1}-1}{1-z}\int_{\mu^2}^{(1-z)^2Q^2}\frac{dk_\perp^2}{k_\perp^2}\Bigg\{A_q^{(1)}\frac{\alpha_S(\mu^2)}{1+b_0\alpha_S(\mu^2)\log\frac{k_\perp^2}{\mu^2}}$$

$$\times\Big[\frac{b_1}{b_0}\frac{\alpha_S(\mu^2)\log(1+b_0\alpha_S\log\frac{k_\perp^2}{\mu^2})}{1+b_0\alpha_S(\mu^2)\log\frac{k_\perp^2}{\mu^2}}\Big]$$

$$+A_q^{(2)}\Big(\frac{\alpha_S(\mu^2)}{1+b_0\alpha_S(\mu^2)\log\frac{k_\perp^2}{\mu^2}}\Big[\frac{b_1}{b_0}\frac{\alpha_S(\mu^2)\log(1+b_0\alpha_S\log\frac{k_\perp^2}{\mu^2})}{1+b_0\alpha_S(\mu^2)\log\frac{k_\perp^2}{\mu^2}}\Big]\Big)^2\Bigg\}.$$

Large N Approximation

- The z integral on the previous slide is difficult, but not impossible
- Recall, our aim is the soft limit, *i.e.* when $z \to 1$
- In Mellin space, soft limit is $N \to \infty$
- In the large N limit, we may use the approximation

$$z^{N-1} - 1 \approx -\Theta(1 - \frac{1}{\bar{N}} - z)$$

where

$$\bar{N} = N e^{\gamma_E}$$

Plugging it in

- We can use the large N approximation to compute the following

$$
\begin{aligned}
\log \Delta(N) = -\int_0^{1-\frac{1}{N}} \frac{dz}{1-z} \int_{\mu^2}^{(1-z)^2 Q^2} \frac{dk_\perp^2}{k_\perp^2} \Bigg\{ & A_q^{(1)} \frac{\alpha_S(\mu^2)}{1 + b_0 \alpha_S(\mu^2) \log \frac{k_\perp^2}{\mu^2}} \\
& \times \Big[\frac{b_1}{b_0} \frac{\alpha_S(\mu^2) \log(1 + b_0 \alpha_S \log \frac{k_\perp^2}{\mu^2})}{1 + b_0 \alpha_S(\mu^2) \log \frac{k_\perp^2}{\mu^2}} \Big] \\
+ A_q^{(2)} \Big(\frac{\alpha_S(\mu^2)}{1 + b_0 \alpha_S(\mu^2) \log \frac{k_\perp^2}{\mu^2}} & \Big[\frac{b_1}{b_0} \frac{\alpha_S(\mu^2) \log(1 + b_0 \alpha_S \log \frac{k_\perp^2}{\mu^2})}{1 + b_0 \alpha_S(\mu^2) \log \frac{k_\perp^2}{\mu^2}} \Big] \Big)^2 \Bigg\}.
\end{aligned}
$$

Form for exponent

- Performing for the case of DY, we have

$$\log \Delta(N) = 2h^{(1)} \log(\bar{N}) + 2h^{(2)}(\lambda, Q^2/\mu^2)$$

$$\lambda = b_0 \alpha_s(\mu^2) \ln \bar{N}.$$

$$h^{(1)} = \frac{A_q^{(1)}}{2b_0\lambda}[2\lambda + (1-2\lambda)\log(1-2\lambda)]$$

$$h^{(2)} = (A_q^{(1)} b_1 - b_0 A_q^{(2)}) \frac{2\lambda + \log(1-2\lambda)}{2b_0^3} + \frac{A_q^{(1)} b_1}{4b_0^3} \log^2(1-2\lambda) + \frac{A_q^{(1)}}{2b_0} \log(1-2\lambda) \log\frac{Q^2}{\mu^2}$$

Full Hard Kernel to Calculate

$$C^{\mathrm{N}^p\mathrm{LO}}_{\mathrm{N}^k\mathrm{LL}}(N,\alpha_s) = \sum_{j=0}^{p} \alpha_s^j C^{(j)}(N) + C^{\mathrm{res}}_{\mathrm{N}^k\mathrm{LL}}(N,\alpha_s) - \sum_{j=0}^{p} \frac{\alpha_s^j}{j!}\left[\frac{d^j C^{\mathrm{res}}_{\mathrm{N}^k\mathrm{LL}}(N,\alpha_s)}{d\alpha_s^j}\right]_{\alpha_s=0}$$

Fixed order Kernel
Already have calculated this at NLO!

Resummation Kernel.
Calculate such as Leading Log,
or Next-to-Leading Log

Matching coefficients
Need to subtract in
order to avoid double
counting

Next-to-Leading + Next-to-Leading Logarithm Order Calculation

Make sure only counted once!
- Subtract the matching

	LL	NLL	...	N^pLL
LO	1	--	...	--
NLO	$\alpha_s \log(N)^2$	$\alpha_s \log(N)$	...	--
NNLO	$\alpha_S^2 \log(N)^4$	$\alpha_S^2 (\log(N)^2, \log(N)^3)$	...	--
...	...	...	...	...
N^kLO	$\alpha_S^k \log(N)^{2k}$	$\alpha_S^k \left(\log(N)^{2k-1}, \log(N)^{2k-2}\right)$	...	$\alpha_S^k \log(N)^{2k-2p} + \cdots$

The need for prescriptions

- To compare with data, one must Mellin invert so that the formulas are in momentum-fraction space and not moment space
- The Mellin inversion of the hard kernel appears order-by-order, but it is divergent because of the divergence of α_S
- One can locate the divergences and avoid them as in the Minimal Prescription (main focus)
- Or one can manipulate the summation to make it convergent as in the Borel prescription (out of the scope of this talk)

Minimal Prescription

Cosine vs Expansion

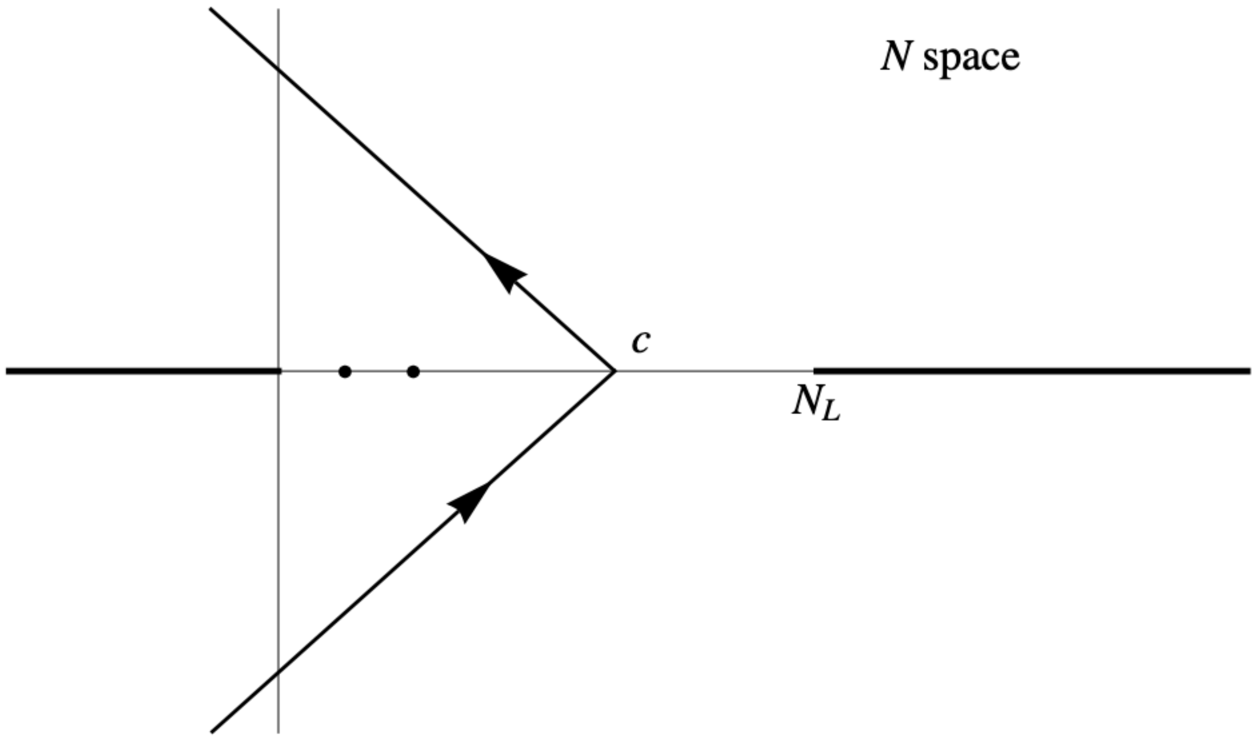
Minimal Prescription

- In principle, one can just do the Mellin inversion exactly
- However, the ambiguity appears in the Landau pole
- We can locate the Landau and avoid it
- By looking at *e.g.* the $h^1(\lambda)$ term, we can see where the arguments of the logarithms go to 0 and become negative
- This location is the Landau pole

$$1 - 2\lambda > 0 \implies \bar{N} < \exp\left(1/2\alpha_S b_0\right)$$

Avoiding the Landau Pole

- The minimal prescription attempts to avoid the Landau pole by making its Mellin inversion contour away the left of the pole and to the right of the other poles



Rapidity Distribution

- In order to compare with data, we need to compare with the rapidity dependent resummed formulas
- Instead of a single Mellin, a Mellin-Fourier transform must be taken

$$\sigma(N, M) \equiv \int_0^1 \mathrm{d}\tau \tau^{N-1} \int_{-\ln \frac{1}{\sqrt{\tau}}}^{\ln \frac{1}{\sqrt{\tau}}} \mathrm{d}\eta e^{iM\eta} \frac{\mathrm{d}\sigma}{\mathrm{d}Q^2 \mathrm{d}\eta},$$

Rapidity Distribution

- Substituting the hadronic rapidity with a partonic rapidity

$$\hat{\eta} = \eta - \frac{1}{2}\log\left(x_1/x_2\right)$$

- We get

$$C^{\mathrm{res}}(N,M) = \int_0^1 dz z^{N-1} \int_{-\log 1/\sqrt{z}}^{\log 1/\sqrt{z}} d\hat{\eta} e^{iM\hat{\eta}} C^{\mathrm{res}}(z)$$

- Since $C^{res}(z)$ is even under $\hat{\eta}$, the exponent can be converted to a cosine

$$C^{\mathrm{res}}(N,M) = \int_0^1 dz z^{N-1} \int_{-\log 1/\sqrt{z}}^{\log 1/\sqrt{z}} d\hat{\eta} \cos(M\hat{\eta}) C^{\mathrm{res}}(z)$$

Rapidity Distribution

- Because in the threshold limit, the hard part has delta functions, the $\hat{\eta}$ integration can be completed, namely

$$C^{\mathrm{res}}(N,M) = \int_0^1 dz z^{N-1} \cos(\frac{M}{2}\log z) C^{\mathrm{res}}(z)$$

Cosine vs Expansion

- Since we focus on the threshold region, that is when $z \rightarrow 1$, the log of z will be close to 0, meaning the argument of the cosine will be close to 0
- One can expand to the cosine term such that

$$\cos(\frac{M}{2}\log z) \approx 1$$

- Or, one can take the cosine exactly how it is

$$\cos(\frac{M}{2}\log z) = \frac{1}{2}(e^{i\frac{M}{2}\log z} + e^{-i\frac{M}{2}\log z})$$

Expansion

- If we have the expansion, then

$$C^{\mathrm{res}}(N,M)=\int_0^1 dz z^{N-1}\cos(\frac{M}{2}\log z)C^{\mathrm{res}}(z)$$

- Goes to

$$C^{\mathrm{res}}(N,M)=\int_0^1 dz z^{N-1}C^{\mathrm{res}}(z)=C^{\mathrm{res}}(N)$$

- Note the independence of *C* on *M*

Cosine

- If we have the cosine, then

$$C^{\text{res}}(N,M) = \int_0^1 dz z^{N-1} \cos(\frac{M}{2}\log z) C^{\text{res}}(z)$$

- Goes to

$$\begin{aligned} C^{\text{res}}(N,M) &= \int_0^1 dz z^{N-1} \left[\frac{1}{2}\left(z^{iM/2} + z^{-iM/2}\right)\right] C^{\text{res}}(z) \\ &= \int_0^1 dz \frac{1}{2}\left(z^{(N+iM/2)-1} + z^{(N-iM/2)-1}\right) C^{\text{res}}(z) \end{aligned}$$

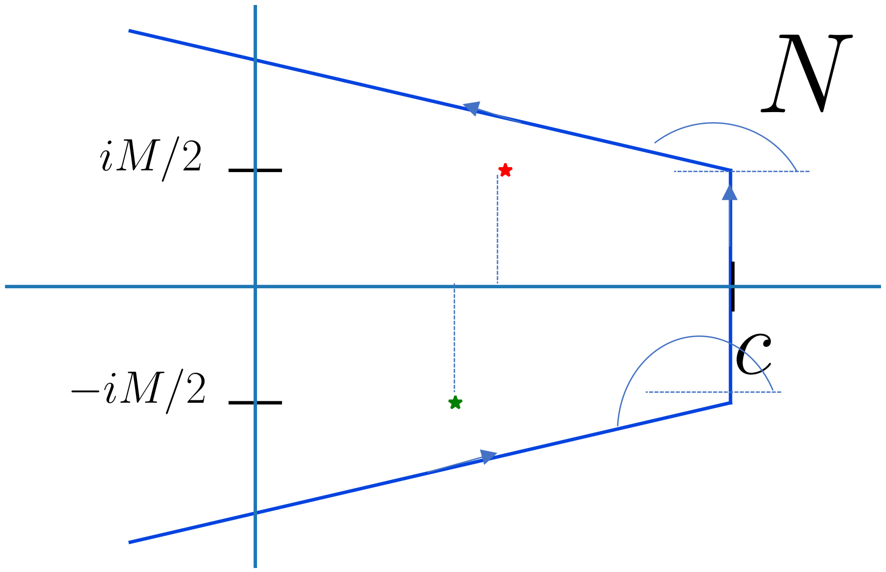
PDFs

- Because of the change of $\eta \to \hat{\eta}$, the PDFs gather a $\pm i\,\frac{M}{2}$ in their Mellin moments

$$\sigma(N, M) = \sigma_0 \sum_{q\bar{q}} f_A^{\pi}(N + iM/2) f_B^{W}(N - iM/2) C^{\mathrm{res}}(N, M)$$

- Whether *C* is dependent on *M* or not

MELLIN CONTOUR



- Here, c is to the right of the PDFs' rightmost poles
- Because the PDF moments are evaluated at $N \pm i\frac{M}{2}$ instead of the usual N, the poles are also located $\pm i\frac{M}{2}$ from the real axis (red and green stars)
- Contour is misshapen to ensure poles are encapsulated

$$N_1 = c - i\frac{M}{2} + z_1\, e^{\phi_1} \qquad 0 < z_1 < \infty$$

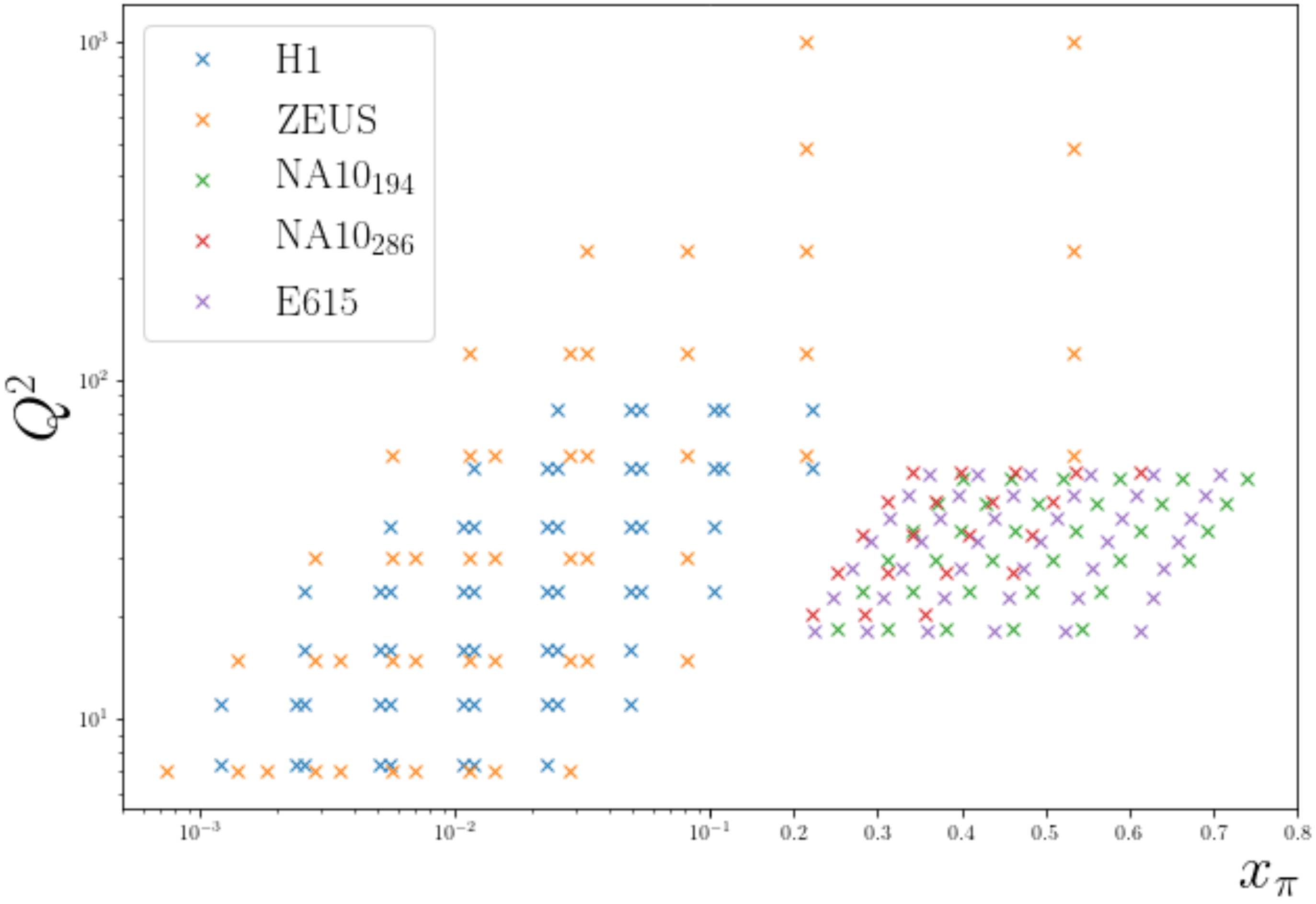
$$N_2 = c - i\frac{M}{2} + z_2\, i\, M \qquad 0 < z_2 < 1$$

$$N_3 = c + i\frac{M}{2} + z_3\, e^{\phi_3} \qquad 0 < z_3 < \infty$$

Monte Carlo Results

Kinematic Coverage

- We want to be able to fit simultaneously the Drell-Yan and Leading Neutron data
- We can shape the pion PDFs at both high- and low-x_π with both datasets
- E615, NA10 – DY
- H1, ZEUS – LN



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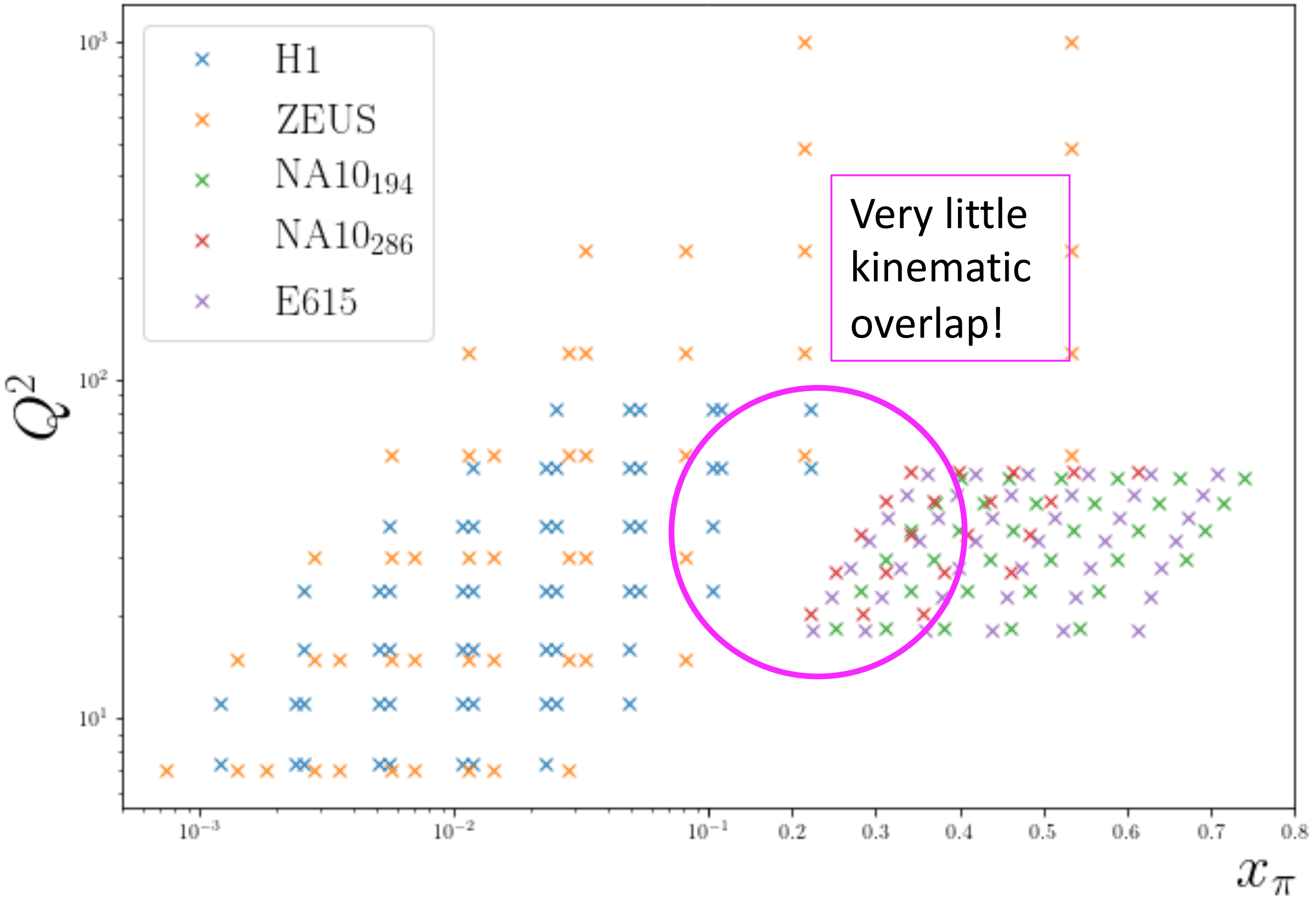


Parametrization of the PDF

- We open the shape up a little for the valence (important for resummation in DY)

$$q_v(x_\pi, Q_0^2, \mathbf{a}) = \frac{N}{N_v'} x_\pi^\alpha (1 - x_\pi)^\beta (1 + \gamma x^2)$$

As was done in Aicher et al.

where

$$N_v' = B(2 + \alpha, \beta + 1) + \gamma B(4 + \alpha, \beta + 1)$$

- And for the sea and the gluon, we parametrize by

$$f(x_\pi, Q_0^2, \mathbf{a}) = \frac{N}{N'} x_\pi^\alpha (1 - x_\pi)^\beta$$

where

$$N' = B(2 + \alpha, \beta + 1)$$

Parameterization of the PDF (in terms of π^-)

- We equate the valence distributions: $\bar{u}_v^{\pi^-} = d_v^{\pi^-}$
- We equate the light sea distributions: $u^{\pi^-} = \bar{d}^{\pi^-} = u_s^{\pi^-} = d_s^{\pi^-} = s = \bar{s}$
- Normalizations of the valence and sea PDFs are fixed by the sum rules

Quark sum rule $$\int_0^1 dx_\pi q_v^\pi = 1$$

Momentum Sum Rule $$\int_0^1 dx_\pi x_\pi (2q_v^\pi + 6q_s^\pi + g^\pi) = 1$$

Monte Carlo

- Using Bayesian statistics, we describe the probability

$$\mathcal{P}(\mathbf{a}|\mathrm{data}) \propto \mathcal{L}(\mathrm{data}|\mathbf{a})\pi(\mathbf{a})$$

- We quantify the expectation value and variance of our observable $\mathcal{O}$ as a function of the parameter set $\boldsymbol{a}_i$

$$E[\mathcal{O}] = \frac{1}{N}\sum_i \mathcal{O}(\mathbf{a}_i)$$

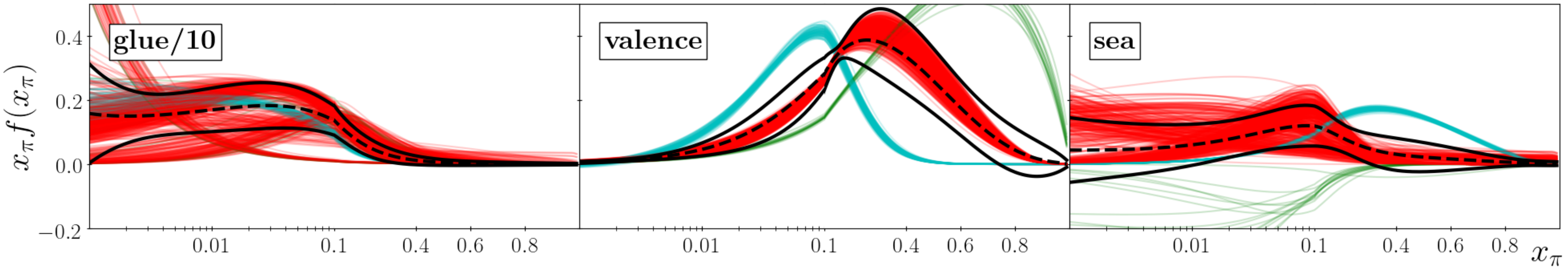
$$V[\mathcal{O}] = \frac{1}{N}\sum_i \left[\mathcal{O}(\mathbf{a}_i) - E[\mathcal{O}]\right]^2$$

Multi-Step Strategy

- Fitting PDFs to many types of observables all at once is time consuming and slows the fit
- We start with many replicas with flat priors to fit to one observable, the $\pi^{-}W$ DY data
- The posteriors from that fit are used as the priors for the next fit, which includes the LN data

PDF Results – Cosine

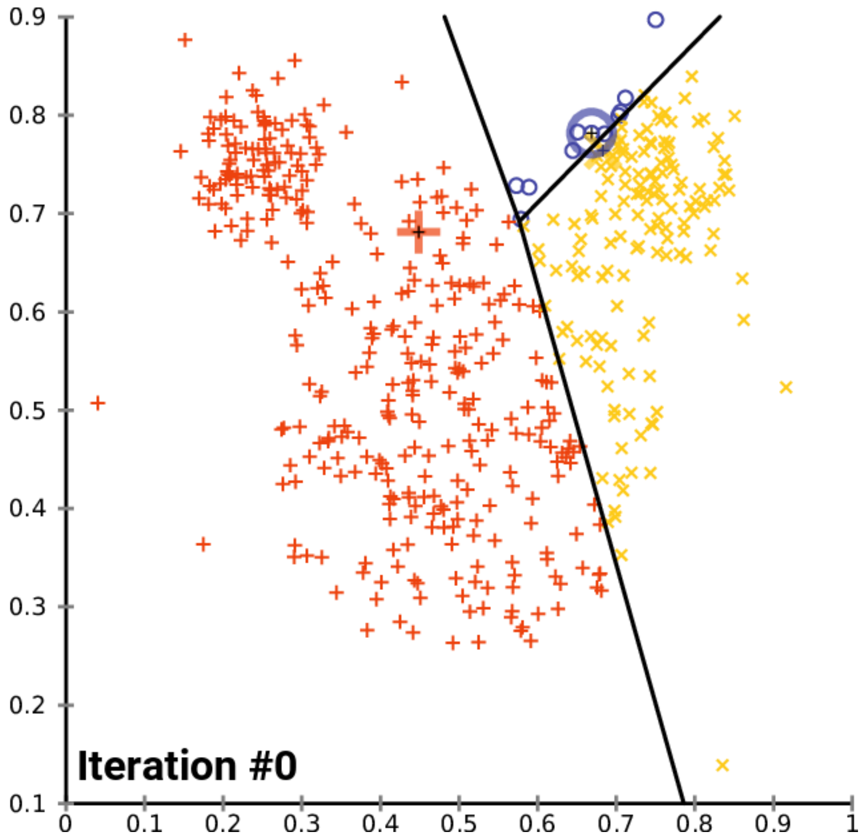
- Fitting to both DY and LN data using the cosine approximation in the minimal prescription



- Clearly there are multiple solutions (evident in the valence)

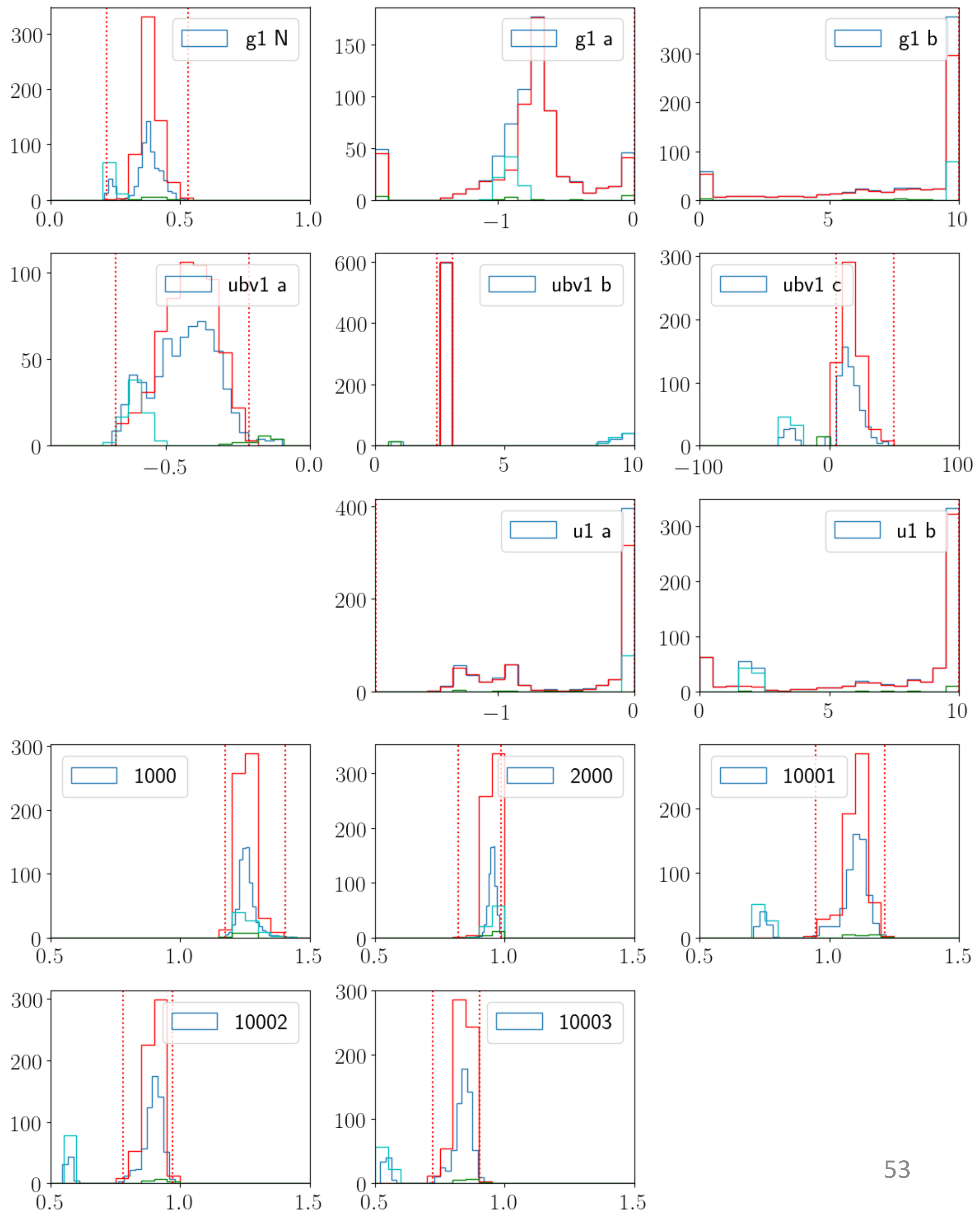
k-means clustering

- The different colors represent different clusters of parameters



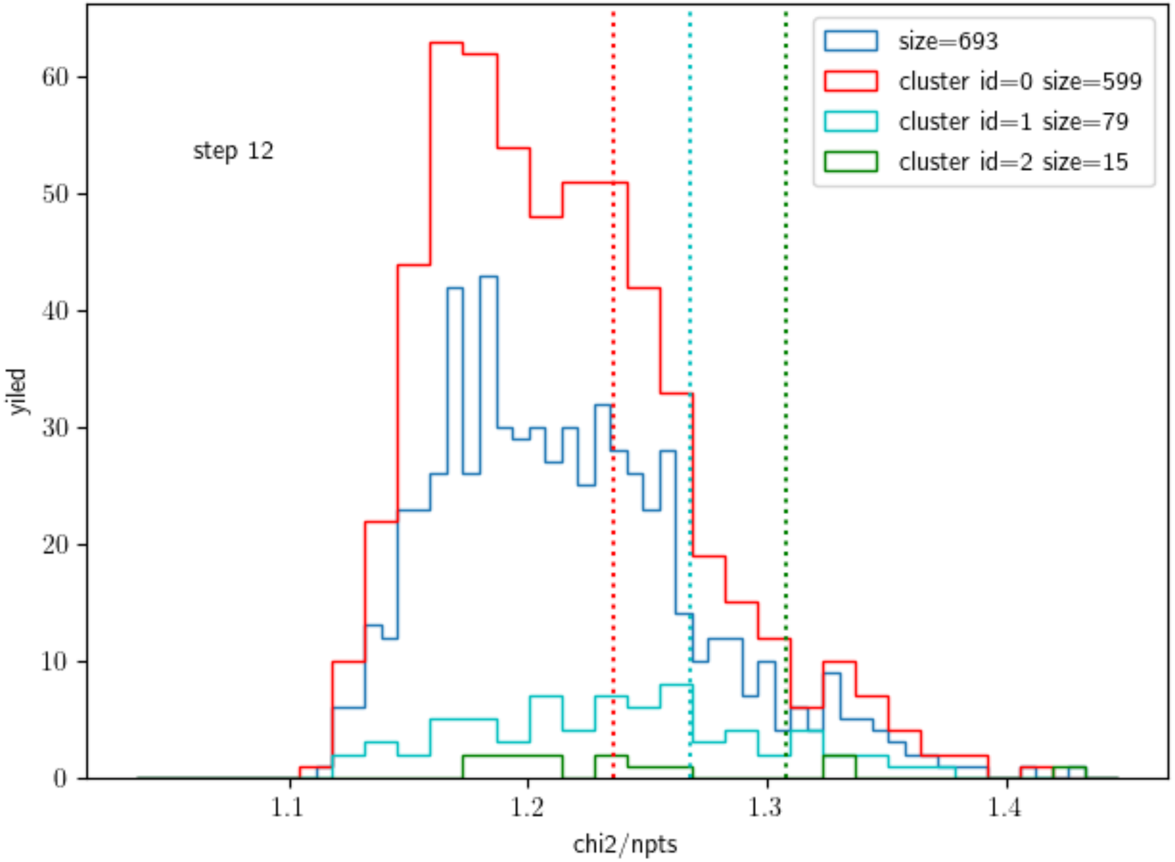
PDF parameter results – Cosine

- Fit parameter histograms
- Perform k-means clustering on the valence b parameter

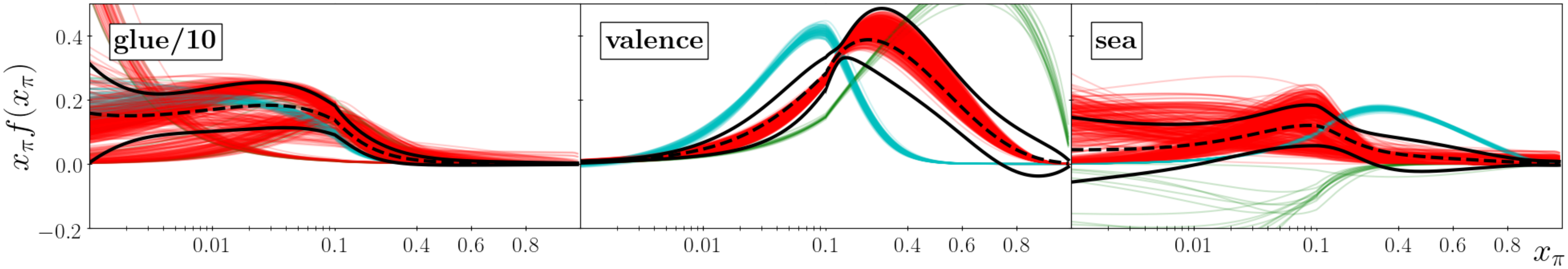


χ^2 profile of clusters

- Showing histogram of χ^2 values for the different clusters
- Red is best, but not by much!
- Can look to physics for justification of throwing out cyan and green solutions



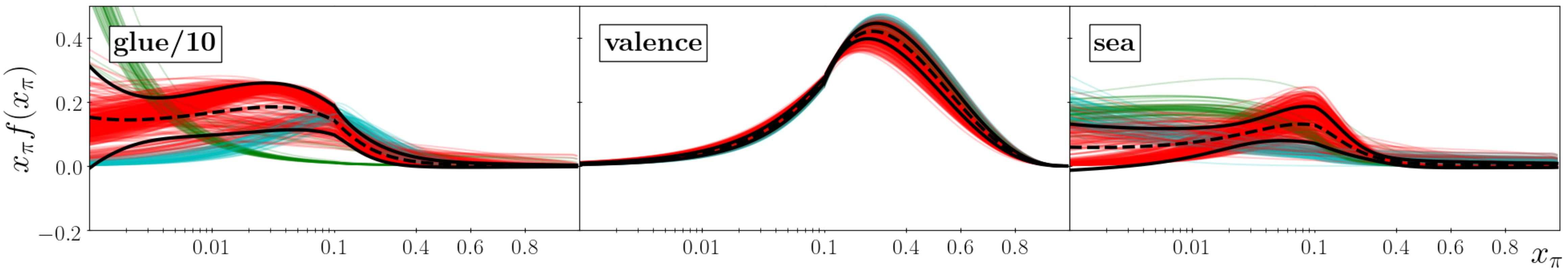
PDF Results – Cosine



- The cyan and green solutions are not favorable
- Cyan: the sea is larger than the valence at large x_π; those distributions belong in the valence
- Green: The sea is negative, and the valence is much to large at high x_π

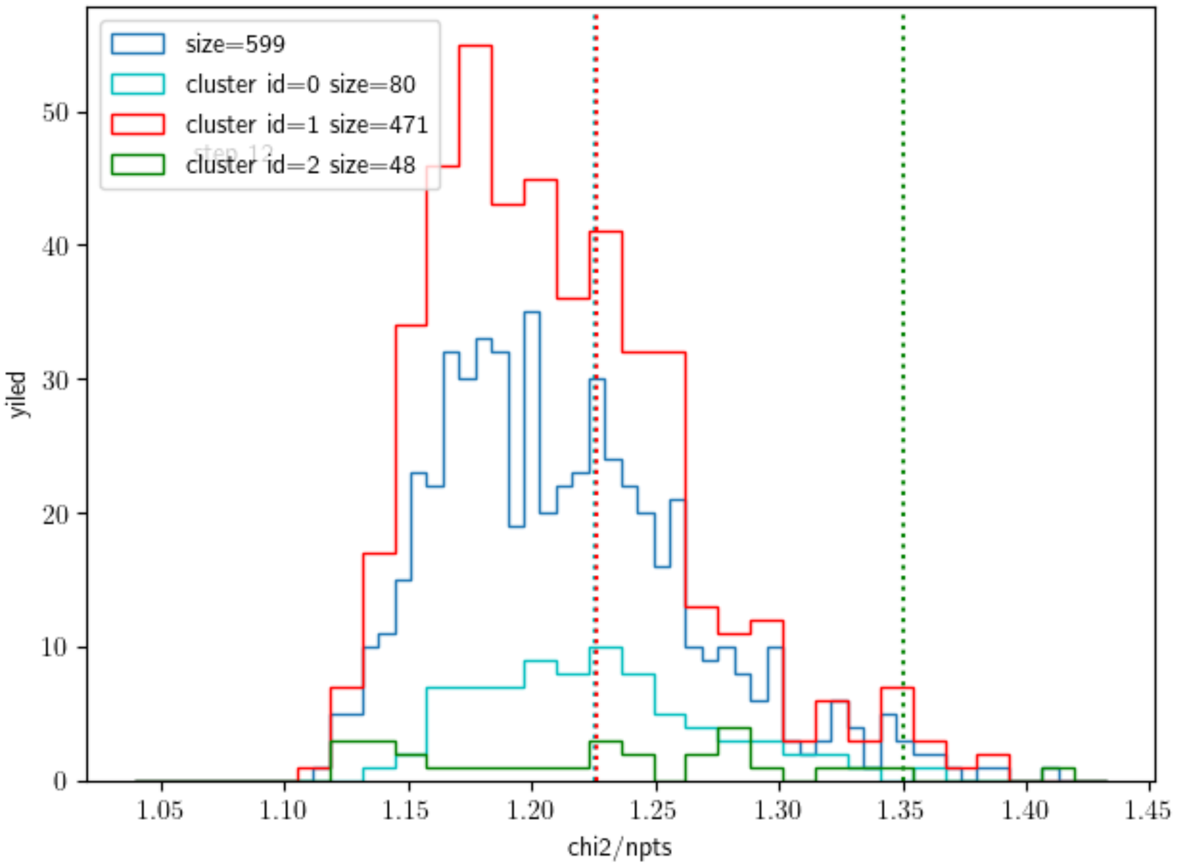
PDF Results – Cosine

- Remove the improper solutions
- Perform a k-means clustering on the gluon's a parameter as there are multiple solutions in the gluon PDF



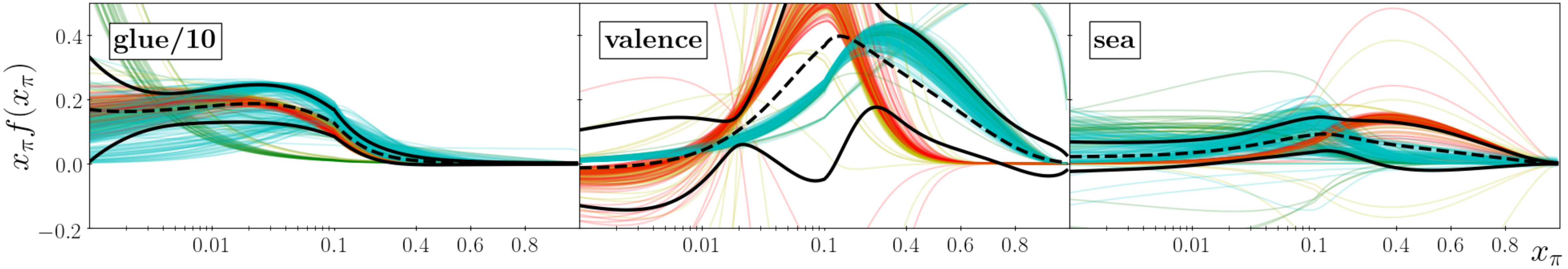
χ^2 profile

- May not be able to distinguish results with χ^2 so close…
- Perhaps green may be thrown away



PDF Results – Expansion

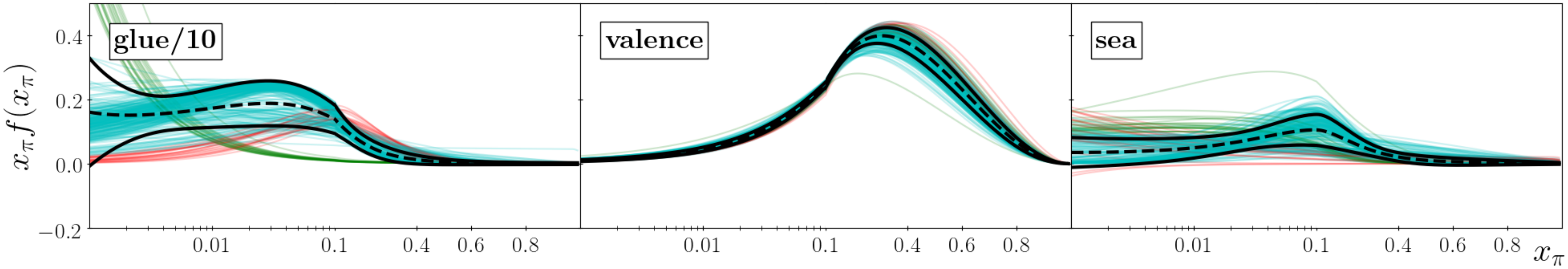
- Replicas are a little more chaotic



- k-means clustering here is not so trustworthy!
- Use physics to eliminate some solutions, *i.e.* limit the valence parameter $1 < b < 3$

PDF results – Expansion

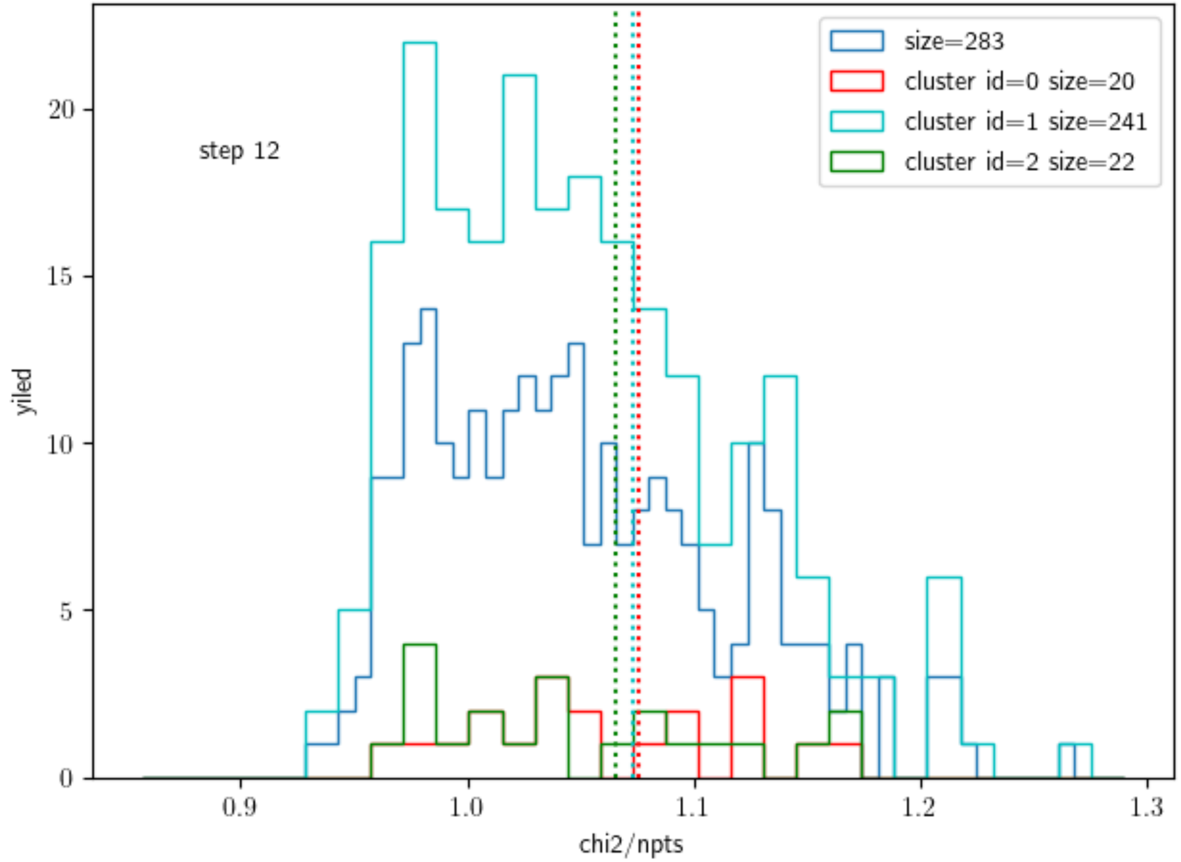
- Elimination of unphysical solutions



- Here, k-means clustering shows red as having the best χ^2, but cyan is most populated and probably the “best” solution
  - More work needs to be done here to justify certain solutions

χ^2 profile – Expansion

- Probably need to keep all solutions as χ^2 is not a good metric to distinguish among clusters



Comparisons – χ^2

Cosine method

number of replicas = 599

summary of testcluster

prediction	reaction	idx	col	npts	chi2	chi2/npts	norm
False	ln	1000	H1	58	22.07	0.38	1.26
False	ln	2000	ZEUS	50	72.75	1.46	0.95
False	dy-pion	10001	E615	55	72.19	1.31	1.10
False	dy-pion	10002	NA10	36	48.97	1.36	0.90
False	dy-pion	10003	NA10	20	27.85	1.39	0.84
				219	243.82	1.11	

Expansion method

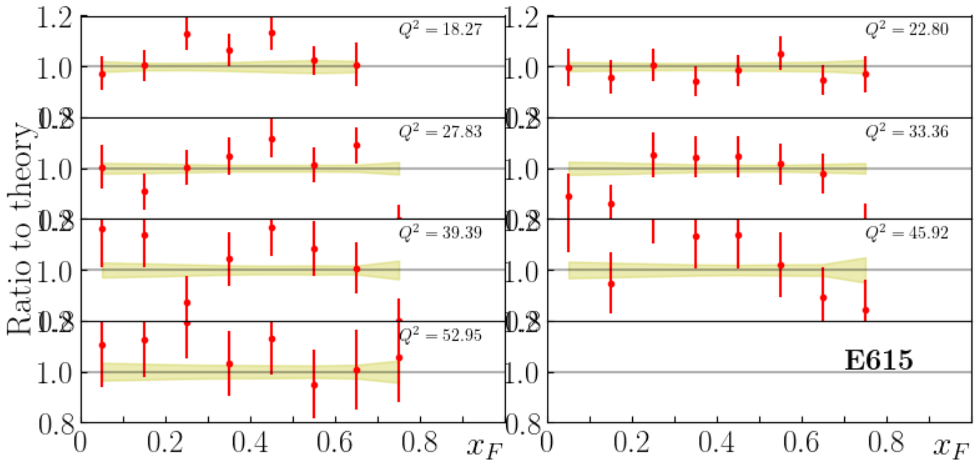
number of replicas = 283

summary of testcluster2

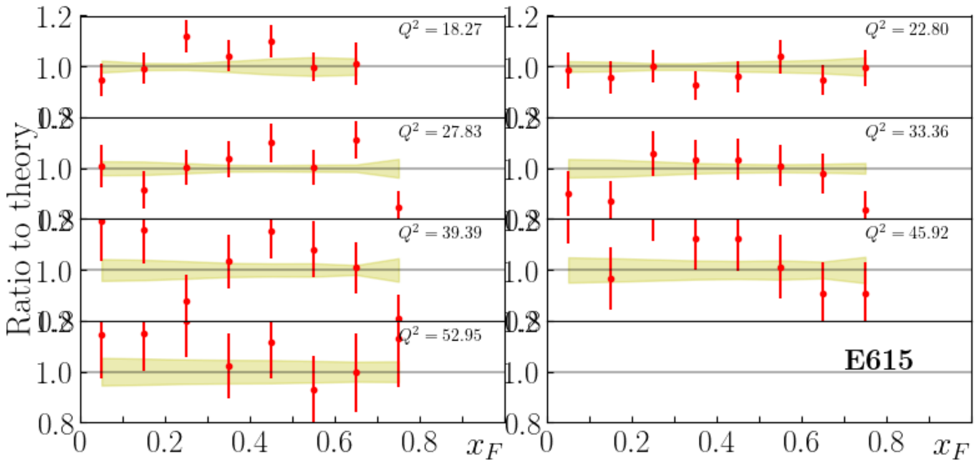
prediction	reaction	idx	col	npts	chi2	chi2/npts	norm
False	ln	1000	H1	58	21.93	0.38	1.25
False	ln	2000	ZEUS	50	74.39	1.49	0.95
False	dy-pion	10001	E615	55	57.77	1.05	1.08
False	dy-pion	10002	NA10	36	26.58	0.74	0.90
False	dy-pion	10003	NA10	20	15.29	0.76	0.83
				219	195.96	0.89	

Comparisons – data over theory DY E615

- Most notably the resummation affects the E615 dataset as it contains higher x_F values than NA10

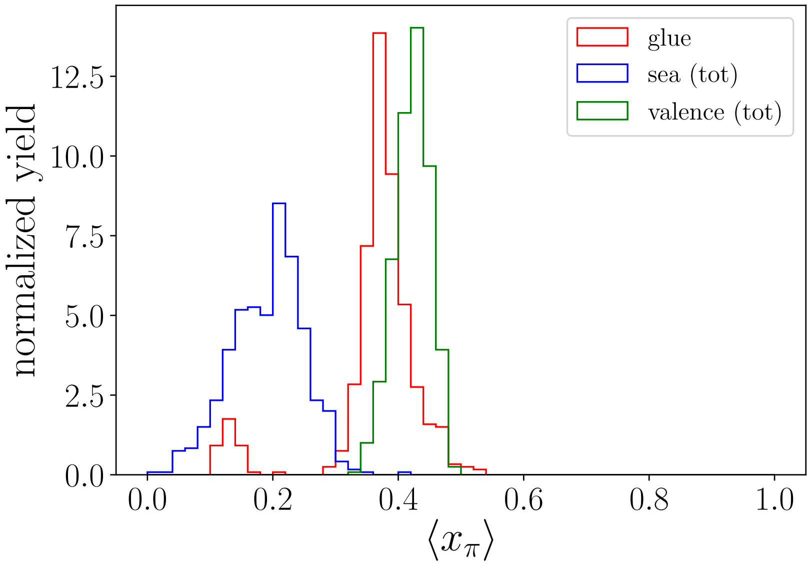


Cosine method



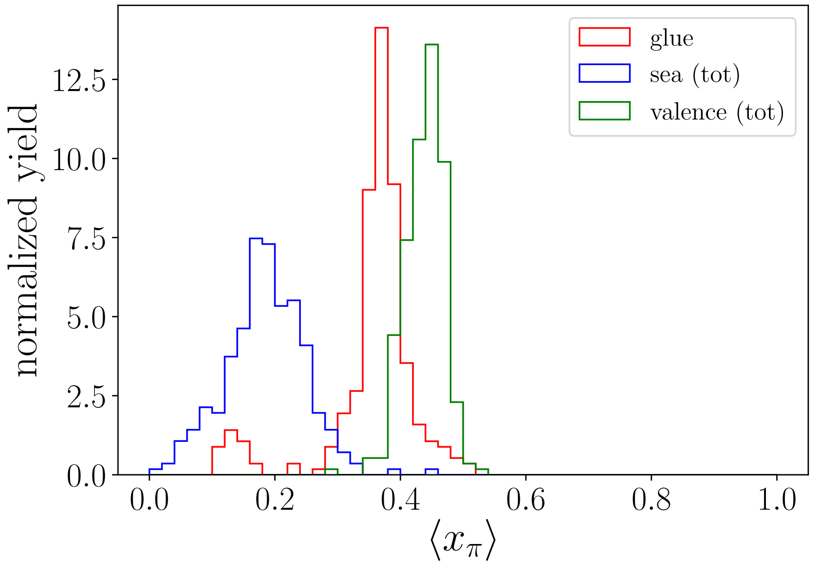
Expansion method

Comparisons – Momentum Fractions

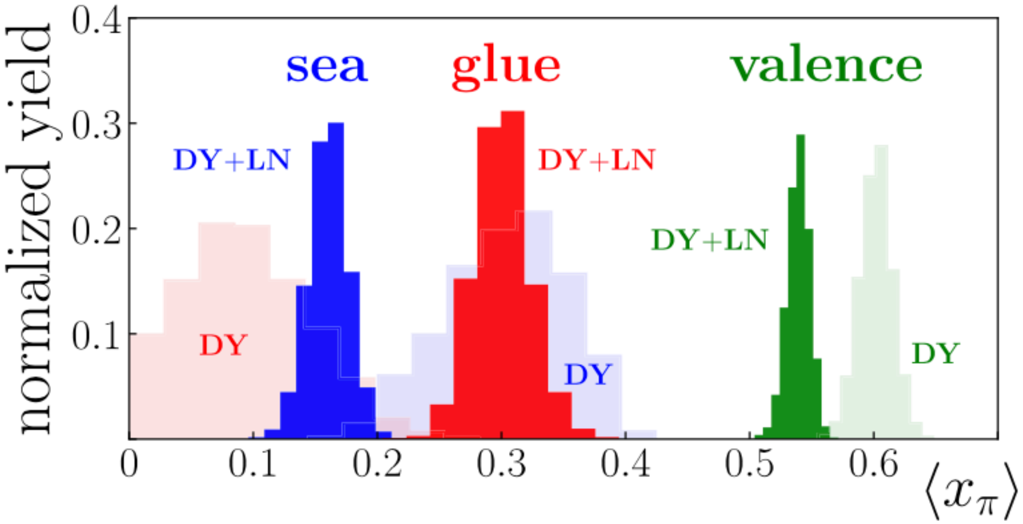


Cosine method

Expansion method

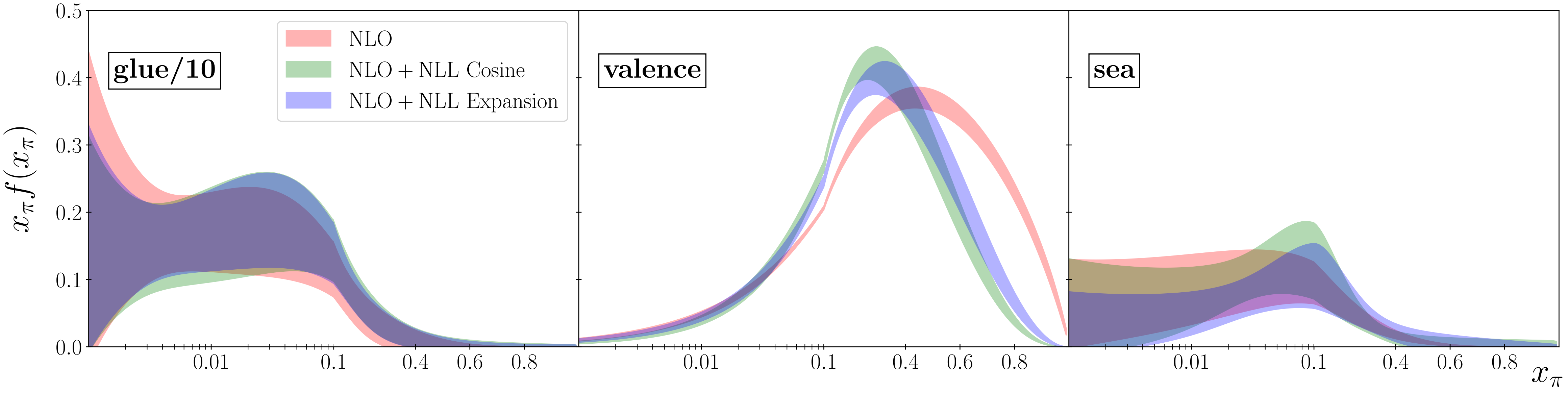


Fixed order result from 2018 PRL



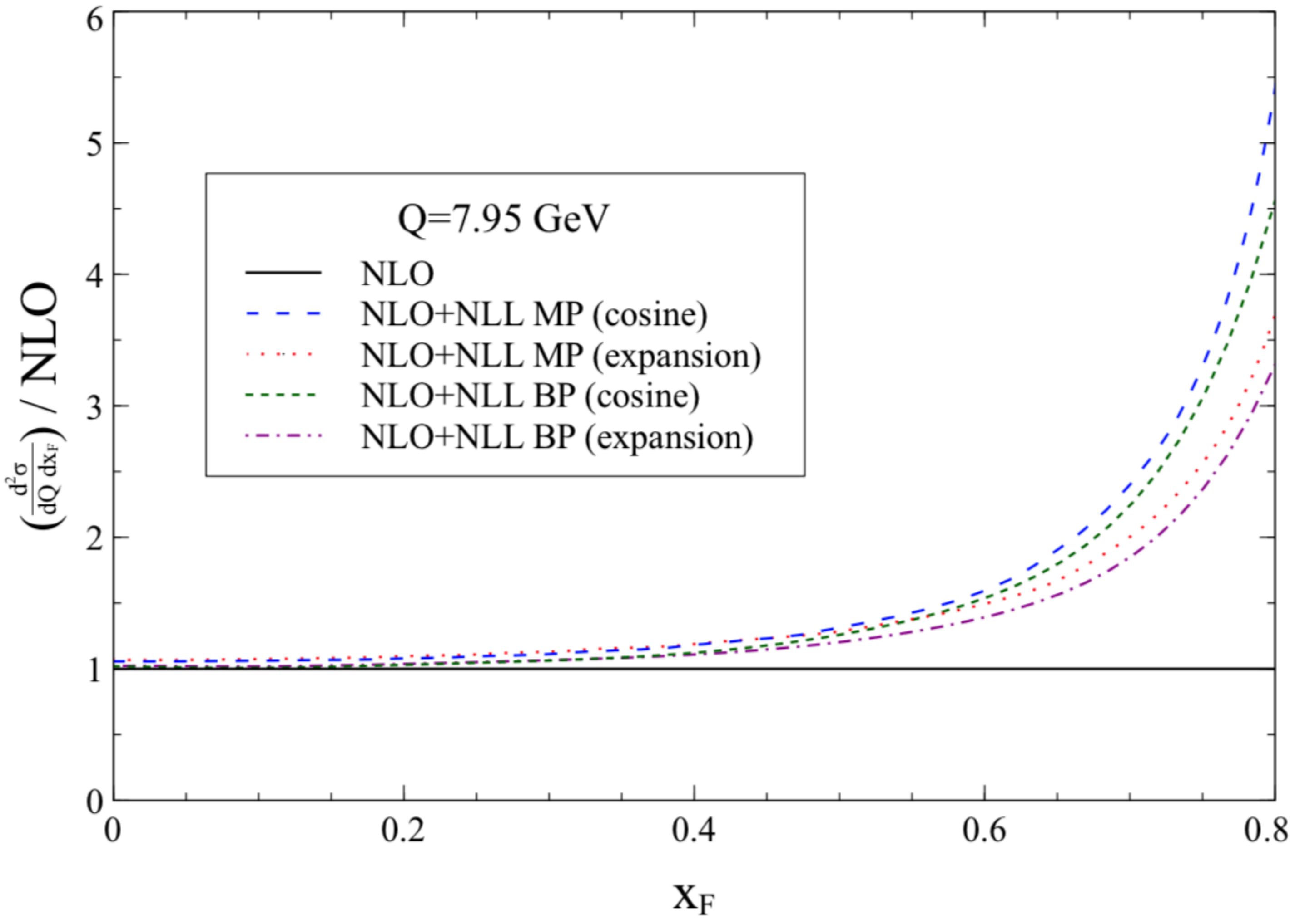
Comparisons – PDFs

- Comparison of the PDFs at the initial scale with fixed order



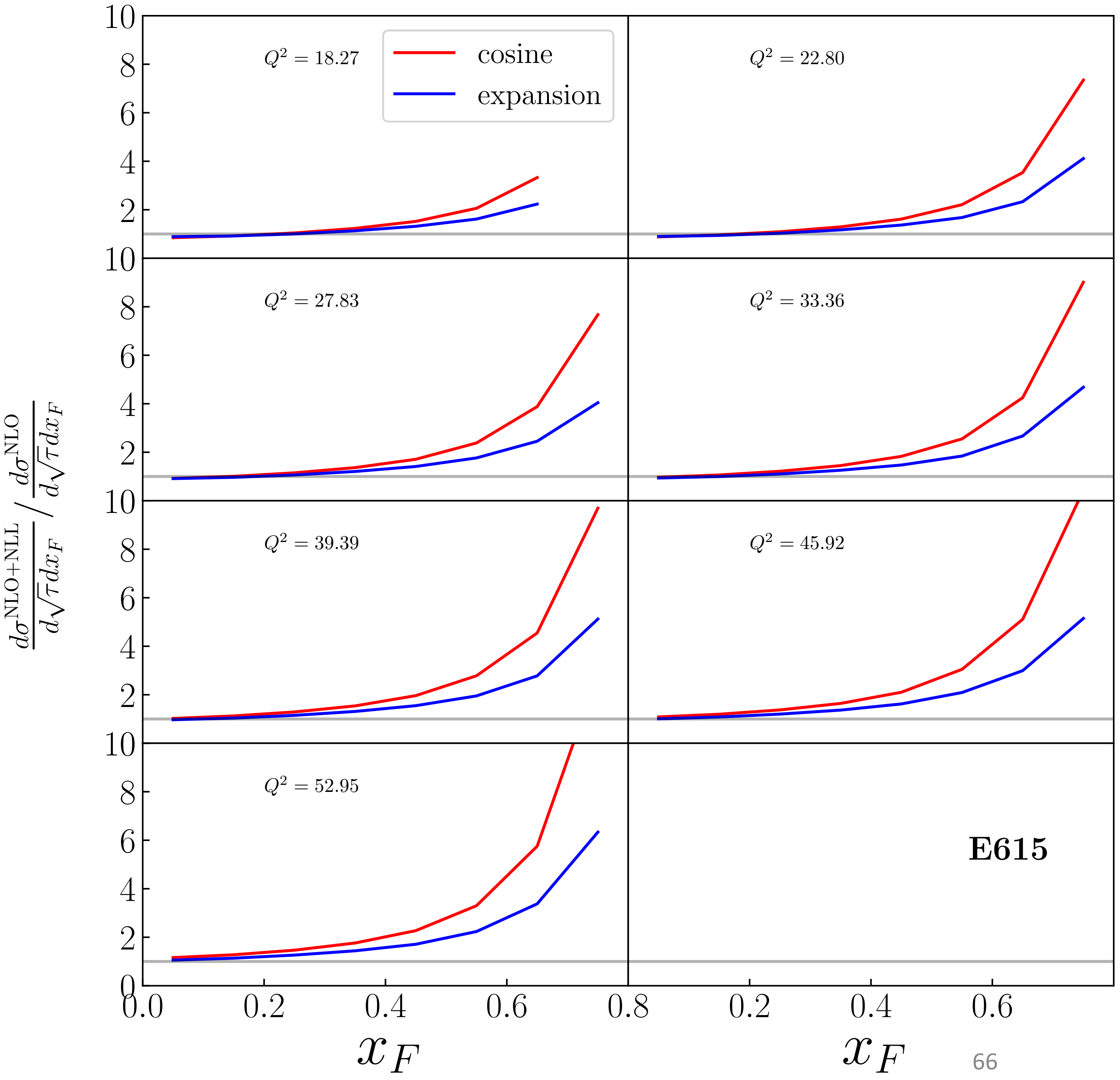
K Factor

- We can study the impact of each resummation method by comparing cross sections divided by the NLO piece
- Shown to the right is the result by Westmark/Owens



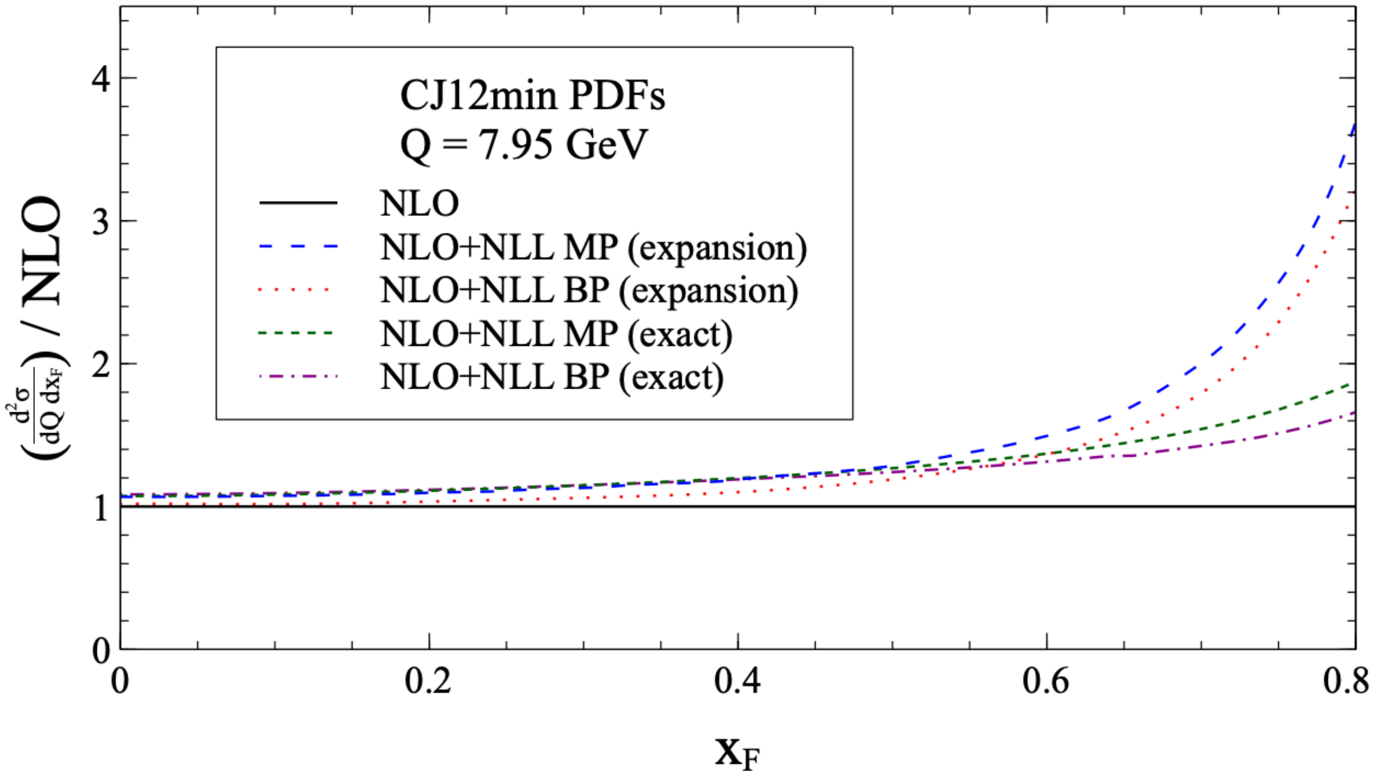
K Factor

- K facto for kinematics associated with the E615 dataset
- PDFs are consistent with each curve
- Cosine seems to gather more terms at higher orders than expansion



K Factor

- Additional K Factor provided by Westmark/Owens
- Invokes an "exact" resummation prescription
- Lowers the K factor from expansion to near 1 for all x_F
- Exact will probably be closest to NLO



Double Mellin Resummation
“Exact”

Exact Method

- Instead of a Mellin-Fourier transform as was the case for the cosine and expansion methods, use double Mellin
- The task is to calculate

$$
\begin{aligned}
\log \Delta(N_1, N_2) = &\int_0^1 dz_1 \int_0^1 dz_2 \frac{(z_1^{N_1-1} - 1)(z_2^{N_2-1} - 1)}{(1-z_1)(1-z_2)} A_q\Big(\alpha_S\big((1-z_1)(1-z_2)Q^2\big)\Big) \\
&+ \int_0^1 dz_1 \frac{z_1^{N_1-1} - 1}{1-z_1} \int_{\mu_F^2}^{(1-z_1)Q^2} \frac{dk_\perp^2}{k_\perp^2} A_q\big(\alpha_S(k_\perp^2)\big) \\
&+ \int_0^1 dz_2 \frac{z_2^{N_2-1} - 1}{1-z_2} \int_{\mu_F^2}^{(1-z_2)Q^2} \frac{dk_\perp^2}{k_\perp^2} A_q\big(\alpha_S(k_\perp^2)\big)
\end{aligned}
$$

Westmark/Owens Expressions

- Westmark and Owens have already derived their expressions
- These are to be compared with $h^{(1)}$ and $h^{(2)}$ as shown previously

$$\begin{aligned}
g_1(\lambda_1,\lambda_2) &= \frac{A_1}{b_0(\lambda_1+\lambda_2)}\Big(\left(1-(\lambda_1+\lambda_2)\right)\ln\left(1-(\lambda_1+\lambda_2)\right)+\lambda_1+\lambda_2\Big),\\
g_2(\lambda_1,\lambda_2) &= \frac{A_1 b_1}{b_0^2}\left(\ln\left(1-(\lambda_1+\lambda_2)\right)+\frac{1}{2}\ln^2\left(1-(\lambda_1+\lambda_2)\right)-(\lambda_1+\lambda_2)\right)\\
&\quad-\frac{2A_1\gamma_E}{b_0}\ln\left(1-(\lambda_1+\lambda_2)\right)+\frac{A_1}{b_0}\ln\left(1-(\lambda_1+\lambda_2)\right)\ln\frac{Q^2}{\mu_R^2}\\
&\quad-\frac{A_2}{b_0^2}\Big(\lambda_1+\lambda_2+\ln\left(1-(\lambda_1+\lambda_2)\right)\Big)+\frac{A_1}{b_0}(\lambda_1+\lambda_2)\ln\frac{\mu_F^2}{\mu_R^2}.
\end{aligned}$$

- Here, $\lambda_1 = b_0\alpha_S\log(N_1)$ and $\lambda_2 = b_0\alpha_S\log(N_2)$

Westmark/Owens Expressions

- However, we note that $\bar{N}$ is not used whereas I use $\bar{N}$ in the cosine and expansion methods

$$
\begin{aligned}
g_1(\lambda_1,\lambda_2) &= \frac{A_1}{b_0(\lambda_1+\lambda_2)}\Big((1-(\lambda_1+\lambda_2))\ln\left(1-(\lambda_1+\lambda_2)\right)+\lambda_1+\lambda_2\Big),\\
g_2(\lambda_1,\lambda_2) &= \frac{A_1 b_1}{b_0^2}\left(\ln\left(1-(\lambda_1+\lambda_2)\right)+\frac{1}{2}\ln^2\left(1-(\lambda_1+\lambda_2)\right)-(\lambda_1+\lambda_2)\right)\\
&\quad -\frac{2A_1\gamma_E}{b_0}\ln\left(1-(\lambda_1+\lambda_2)\right)+\frac{A_1}{b_0}\ln\left(1-(\lambda_1+\lambda_2)\right)\ln\frac{Q^2}{\mu_R^2}\\
&\quad -\frac{A_2}{b_0^2}\Big(\lambda_1+\lambda_2+\ln\left(1-(\lambda_1+\lambda_2)\right)\Big)+\frac{A_1}{b_0}(\lambda_1+\lambda_2)\ln\frac{\mu_F^2}{\mu_R^2}.
\end{aligned}
$$

- Additionally, a term $\propto \gamma_E$ appears that is not consistent with our expressions

Strategy

- Evaluate the integrations using the large N approximation as before

$$z^{N-1} - 1 \approx -\Theta(1 - \frac{1}{\bar{N}} - z)$$

- Obtain our own terms to use in the PDF extractions

Warning – Landau Pole

- In the case of the cosine and expansion methods, the Landau pole appeared on the real axis at one point
- However, since we are doing a Double Mellin calculation, the Landau pole moves in Mellin space

Warning – Landau Pole

- We want to calculate the Double Mellin inversion to compare with data

$$\frac{1}{\sigma_0}\frac{d^2\sigma}{dQ^2 dx_F} = \sum_{q,\bar{q}} e_q^2 \frac{1}{(2\pi i)^2}\int_{C_1-i\infty}^{C_1+i\infty} dN_1 (x_1^0)^{-N_1}\int_{C_2-i\infty}^{C_2+i\infty} dN_2 (x_2^0)^{-N_2} f_\pi(N_1) f_W(N_2)\sigma(N_1,N_2)$$

- For each N_1, we need to perform a contour integration over N_2
- In this case, the Landau pole appears at

$$N_L = \frac{1}{N_1} e^{\frac{1}{\alpha_S b_0}}$$

Warning – Landau Pole

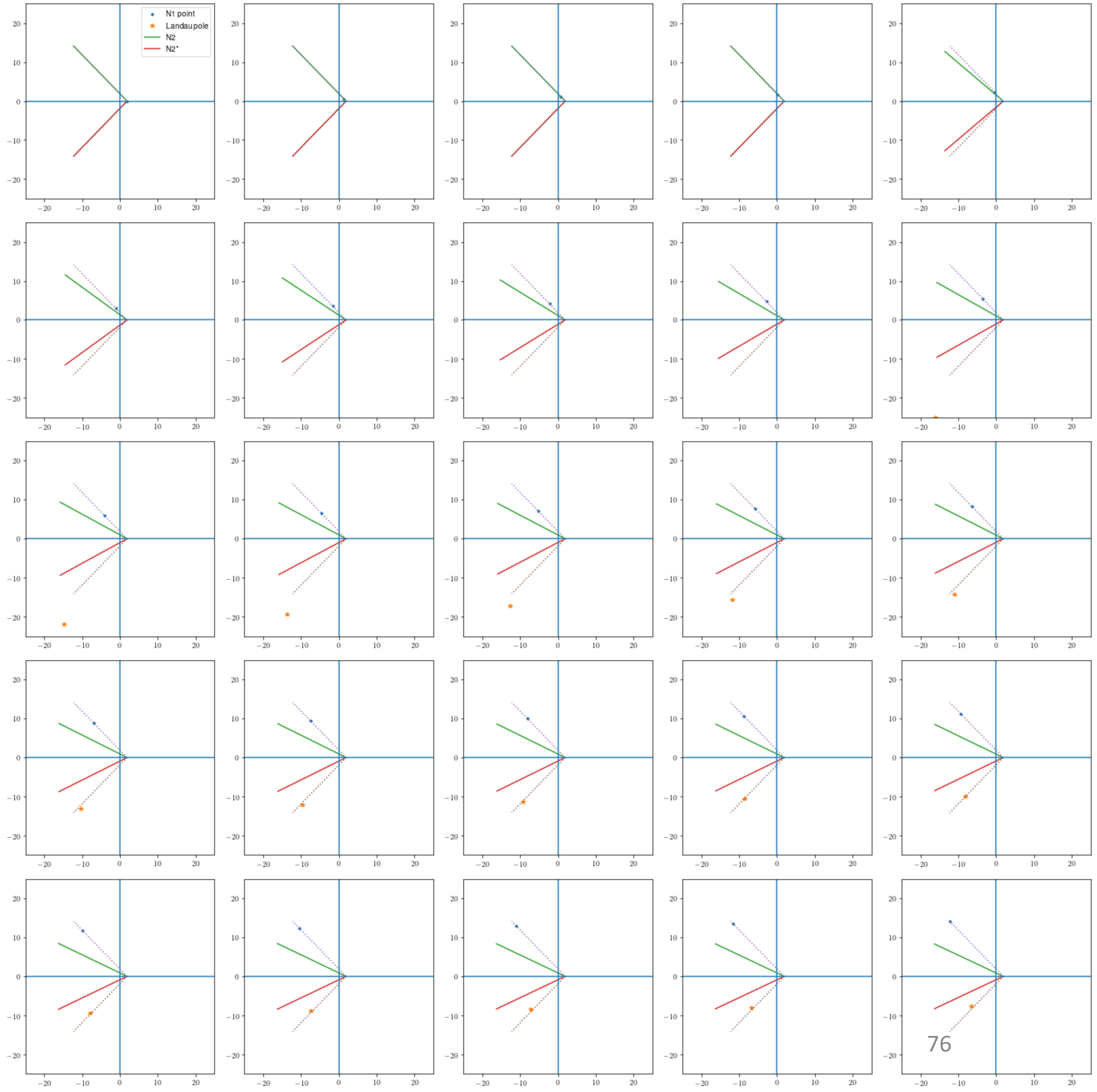
$$\frac{1}{\sigma_0}\frac{d^2\sigma}{dQ^2dx_F} = \sum_{q,\bar{q}} e_q^2 \frac{1}{(2\pi i)^2}\int_{C_1-i\infty}^{C_1+i\infty} dN_1 (x_1^0)^{-N_1}\int_{C_2-i\infty}^{C_2+i\infty} dN_2 (x_2^0)^{-N_2} f_\pi(N_1) f_W(N_2)\sigma(N_1,N_2)$$

- We can freely choose N_1, but we have to be careful with our choice of N_2 in order to keep the Landau pole to the right of our contour while making sure to enclose the poles associated with the PDFs!
- <u>Suggestions</u>
  - Where the Landau pole does not encounter the nominal choice of $N_2 = c_2 + z_2 e^{i\phi_2}$ where $\phi_2 = 3\pi/4$ use it
  - When the Landau pole comes close, we need to shrink the angle to be closer to the real axis

$$\frac{1}{\sigma_0}\frac{d^2\sigma}{dQ^2dx_F} = \sum_{q,\bar{q}} \frac{e_q^2}{2\pi^2}\int_0^\infty dz_1 \int_0^\infty dz_2 \mathrm{Re}\left[e^{i(\phi_1-\phi_2)}\sigma(N,M^*) - e^{i(\phi_1+\phi_2)}\sigma(N,M)\right]$$

Warning – Landau Pole

- Following points in N_1 (the blue dots), we have to draw an N_2 contour (green) and conjugate (red)
- The Landau pole appears as the orange star
- The dotted lines are the "nominal" choice, where $\phi_2 = 3\pi/4$



Choice of Angle

- The Choice of the angle is the following

$$\phi_2 = \max\left(\frac{3\pi}{4}, \frac{\theta_c + \pi}{2}\right)$$

where

$$\theta_c = -\arg\left(\frac{1}{N_1} e^{\frac{1}{\alpha_S b_0}}\right) = \arctan\frac{\mathrm{Im}(N_L)}{\mathrm{Re}(N_L)}$$

Conclusions

Conclusions

- The analysis of the resummation will focus on the Minimal Prescription
- Cosine and expansion methods need a little cleaning up, but results are almost complete
- Derivation of the Double Mellin “exact” expressions is needed
- Fits done with “exact” method need to done and compared with cosine and expansion
- Expectation is that the K factor for the “exact” method will be closer to 1 and the PDFs will be closer to the NLO calculation