Off-Shell Pion Issue

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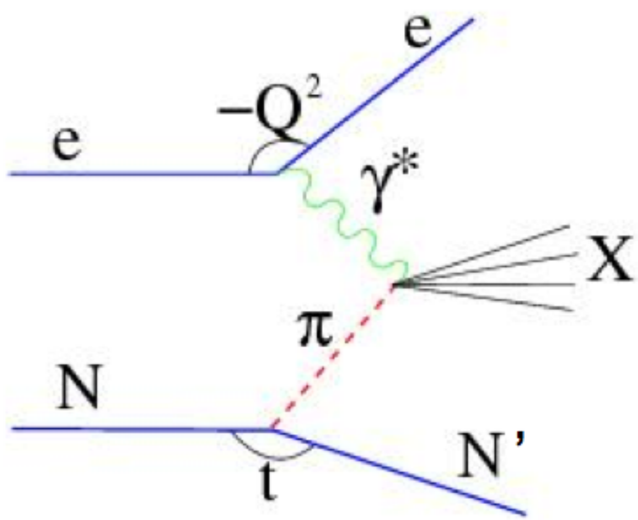
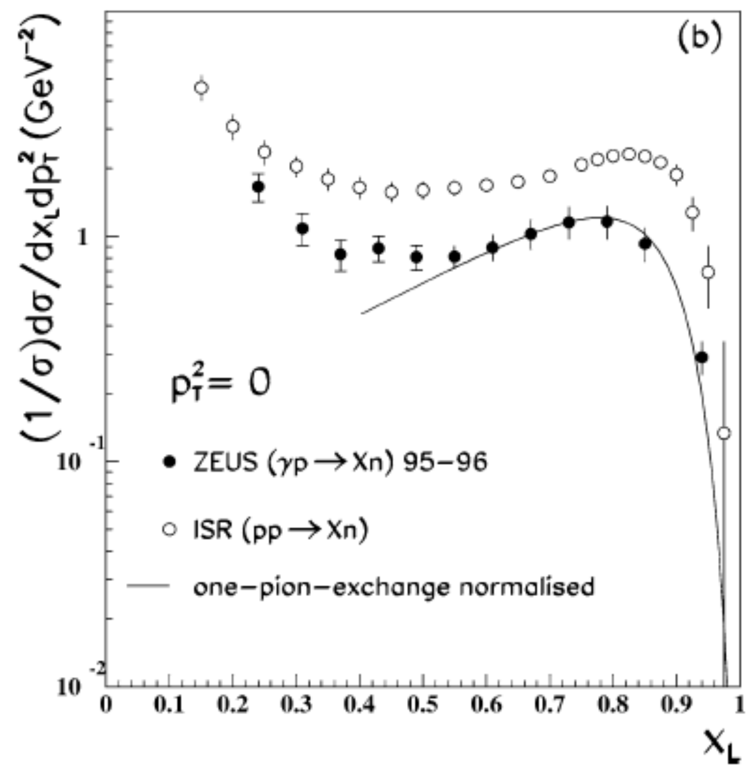


$$e + p(or\ n) \to e' + p + X$$
$$e + D \to e' + p + p + X$$



Leading neutron production in e^+p collisions at HERA

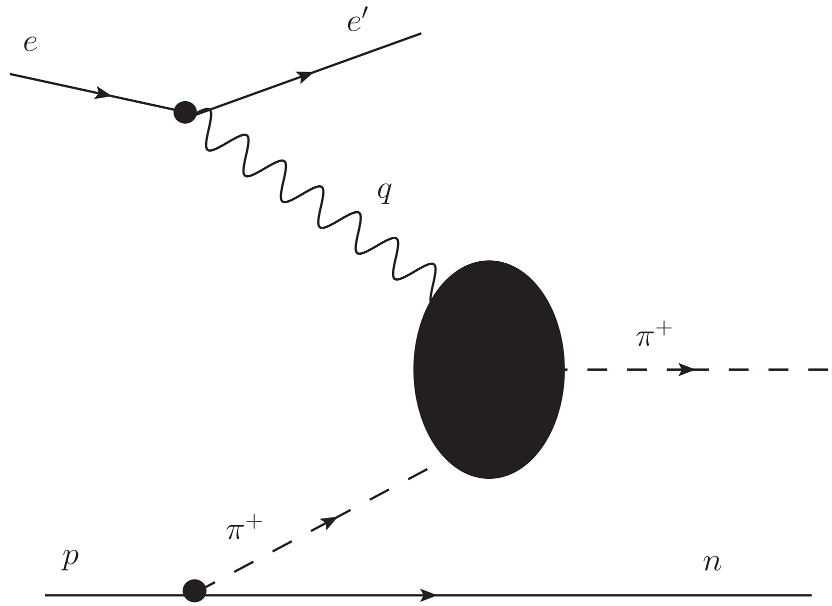
Group Meeting, April 2nd, 2021

Motivation

- What is “Pion Target”?
- Pion lifetime is too short:

  ~10^{-8} sec($\Pi^{+/-}$), ~10^{-17} sec(Π^0)
- The exact pion pole is not accessible

  in electroproduction processes ($t<0<m_{\Pi}^2$).
- Validity of the extrapolation from the off-shell results to the on-shell limit is questionable/ debated.
- EM structure of the off-shell hadron is more complicate involving more unknown functions with more dynamical variables.



Pion off-shell electromagnetic form factors: Data extraction and model analysis

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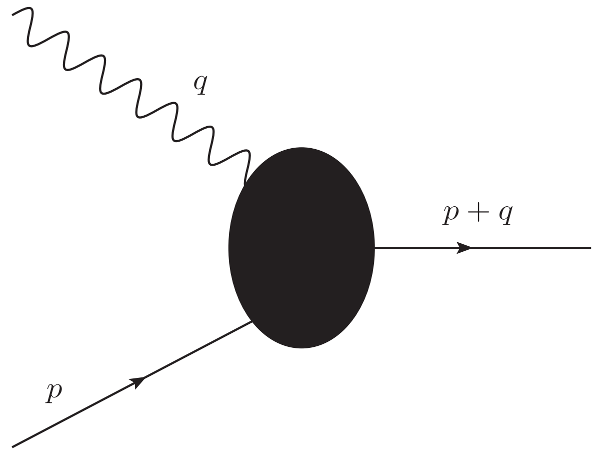
(Received 6 August 2019; published 23 December 2019)

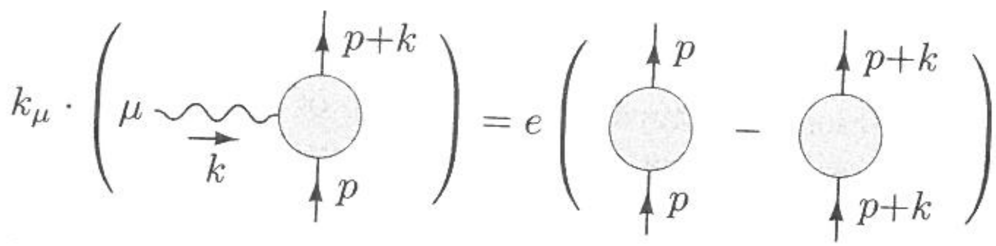
Outline

- Off-shell Pion EM Form Factors
- Off-shell Form Factor Sum Rule
- Manifestly Covariant Model Calculation
- Extraction of Off-shell FFs from Data
- Comparison of Extracted vs. Model FFs
- Conclusion and Outlook

OFF-SHELL PION ELECTROMAGNETIC FORM FACTORS



$$\Gamma_\mu(p, p') = (p' + p)_\mu G_1(q^2, p^2, p'^2) + q_\mu G_2(q^2, p^2, p'^2)$$



$$q^\mu \Gamma_\mu(p, p') = \Delta^{-1}(p') - \Delta^{-1}(p),$$

where

$$\Delta(p) = \frac{1}{p^2 - m_\pi^2 - \Pi(p^2) + \iota\varepsilon} \qquad \Pi(m_\pi^2) = 0.$$

$$(p'^2 - p^2)G_1(q^2, p^2, p'^2) + q^2 G_2(q^2, p^2, p'^2)$$
$$= \Delta^{-1}(p') - \Delta^{-1}(p).$$

In particular, for the case of real photons (i.e., $q^2 = 0$) and $p'^2 = m_\pi^2$ with $\Delta^{-1}(p') = 0$,

$$\Delta^{-1}(p) = (p^2 - m_\pi^2)G_1(0, p^2, m_\pi^2)$$
$$= (p^2 - m_\pi^2)G_1(0, m_\pi^2, p^2).$$

Thus, the form factor normalization $G_1(0, m_\pi^2, m_\pi^2) = 1$ since $\lim_{p^2 \to m_\pi^2}[(p^2 - m_\pi^2)\Delta(p)]^{-1} = 1.$

$$G_1(q^2, p^2, p'^2) = G_1(q^2, p'^2, p^2) \;\; ; \; G_2(q^2, p^2, p'^2) = -G_2(q^2, p'^2, p^2)$$

$$G_1(0, p^2, p'^2) = \frac{\Delta^{-1}(p') - \Delta^{-1}(p)}{p'^2 - p^2}$$

$$G_2(q^2, p^2, p'^2) = \frac{(p'^2 - p^2)[G_1(0, p^2, p'^2) - G_1(q^2, p^2, p'^2)]}{q^2}$$

Half off-mass-shell form factors : $p^2 = t$ and $p'^2 = m_\pi^2$,

$$F_2(Q^2, t) = \frac{t - m_\pi^2}{Q^2}[F_1(0, t) - F_1(Q^2, t)],$$

where $F_i(Q^2, t) \equiv G_i(q^2, t, m_\pi^2)$ $(i = 1, 2)$ and $Q^2 = -q^2$.

$$F_2(Q^2, t) = 0 \text{ if } p^2 = p'^2 = m_\pi^2$$

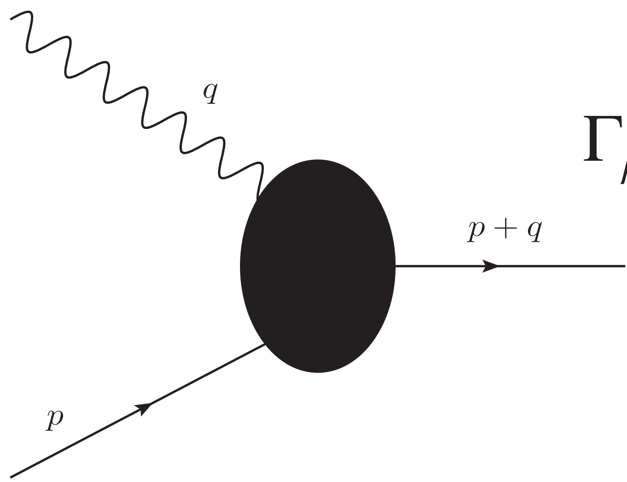
consistent with $G_2(Q^2, p^2, p'^2) = -G_2(Q^2, p'^2, p^2)$

normalization of F_1; $F_1(Q^2 = 0, t = m_\pi^2) = 1$

The renormalized pion self-energy $\Pi(t)$ is related to the off-shell pion form factor $F_1(Q^2 = 0, t)$

$$\Pi(t) = (t - m_\pi^2)[1 - F_1(0, t)]$$

assuring the on-mass-shell condition $\Pi(t = m_\pi^2) = 0$



$$\Gamma_\mu = (p' + p)_\mu F_1(Q^2, t) + q_\mu \frac{(t - m_\pi^2)}{Q^2}[F_1(0, t) - F_1(Q^2, t)]$$

$F_2(Q^2, t)$ to $t - m_\pi^2$ is nonzero in the limit of $t \to m_\pi^2$

$$g(Q^2, t) \equiv \frac{F_2(Q^2, t)}{t - m_\pi^2}$$

off-shell form factor sum rule

$$F_1(Q^2,t) - F_1(0,t) + Q^2 g(Q^2,t) = 0$$

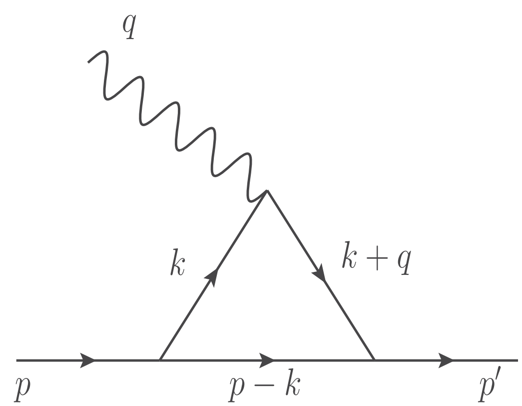
$$\frac{\partial}{\partial Q^2} F_1(Q^2,t) + g(Q^2,t) + Q^2 \frac{\partial g(Q^2,t)}{\partial Q^2} = 0$$

$$g(Q^2=0,m_\pi^2) = -\frac{\partial}{\partial Q^2} F_1(Q^2=0,m_\pi^2) = \frac{1}{6}\langle r_\pi^2\rangle$$

$$\frac{\partial}{\partial t} F_1(Q^2,t) - \frac{\partial F_1(0,t)}{\partial t} + Q^2 \frac{\partial g(Q^2,t)}{\partial t} = 0$$

$$\frac{\partial^2}{\partial t \partial Q^2} F_1(Q^2,t) + \frac{\partial g(Q^2,t)}{\partial t} + Q^2 \frac{\partial^2 g(Q^2,t)}{\partial t \partial Q^2} = 0$$

MANIFESTLY COVARIANT MODEL CALCULATION



$$\Gamma^\mu = iN_c g^2_{\pi q\bar{q}} \int \frac{d^4k}{(2\pi)^4} \frac{S^\mu}{N_k N_{k+q} N_{p-k}}$$

$$N_k = k^2 - m_q^2 + i\varepsilon,\ N_{k+q} = (k+q)^2 - m_q^2 + i\varepsilon,$$

$$N_{p-k} = (p-k)^2 - m_q^2 + i\varepsilon$$

$$S^\mu = \mathrm{Tr}[\gamma_5(\not{k} + \not{q} + m_q)\gamma^\mu(\not{k} + m_q)\gamma_5(\not{k} - \not{p} + m_q)]$$

$$\Gamma^\mu = (p' + p)^\mu F_1(Q^2, t) + q^\mu F_2(Q^2, t)$$

$$F_1(Q^2,t) = -\frac{N_c g^2_{\pi q\bar{q}}}{8\pi^2}\int_0^1 dx \int_0^x dy \times \left[(1+3y)\left(\gamma - \frac{1}{\varepsilon} + \frac{1}{2} + \mathrm{Log}C\right) + \frac{\alpha}{C}\right],$$

and

$$F_2(Q^2,t) = -\frac{N_c g^2_{\pi q\bar{q}}}{8\pi^2}\int_0^1 dx \int_0^x dy \times \left[3(1-2x+y)\mathrm{Log}C + \frac{2\beta-\alpha}{C}\right],$$

where $\gamma \simeq 0.577$ is the Euler-Mascheroni constant

$$\alpha = (1+y)(E^2 - m_q^2) - q\cdot E + 2yp\cdot E - yq\cdot p,$$

$$\beta = (1-x+y)(E^2 - m_q^2) + (1-2x+2y)p\cdot E + (x-y)q\cdot p$$

where $E=(x-y)q-yp$, $C=(x-y)(x-y-1)q^2-y(1-y)t-2y(x-y)q\cdot p+m_q^2$, and $q\cdot p = (m_\pi^2 + Q^2 - t)/2$.

$$F_1^{\rm ren}(Q^2,t) = 1 + [F_1(Q^2,t) - F_1(0,m_\pi^2)],$$

$$F_1(0,m_\pi^2) = -\frac{N_c g_{\pi q\bar{q}}^2}{8\pi^2}\left[\mathrm{Log}(m_q^2) + \gamma - \frac{1}{\varepsilon} - \frac{7}{6}\right.$$

$$\left. - \frac{2(m_\pi^2 - 2m_q^2)}{m_\pi\sqrt{4m_q^2 - m_\pi^2}}\tan^{-1}\left(\frac{m_\pi}{\sqrt{4m_q^2 - m_\pi^2}}\right)\right]$$

on-shell pion decay constant f_π

$$\langle 0|\bar{q}\gamma^\mu\gamma_5 q|\pi(p)\rangle = i f_\pi p^\mu,$$

$$f_\pi = -\frac{N_c g_{\pi q\bar{q}}}{4\pi^2} m_q \left[\gamma - \frac{1}{\varepsilon} - \frac{3}{2} + \mathrm{Log}(m_q^2)\right.$$

$$\left. + \frac{2}{m_q}\sqrt{4m_q^2 - m_\pi^2}\tan^{-1}\left(\frac{m_\pi}{\sqrt{4m_q^2 - m_\pi^2}}\right)\right].$$

$$\frac{g_{\pi q\bar{q}}}{2m_q} = \frac{F_1(0, m_\pi^2)}{f_\pi} + \mathcal{O}(\varepsilon)$$

$$F_1^{\text{ren}}(0, m_\pi^2)/f_\pi^{\text{Exp}} \text{ with } F_1^{\text{ren}}(0, m_\pi^2) = 1$$

$$g_{\pi q\bar{q}} \quad \sim \quad 2m_q/f_\pi^{\text{Exp}}$$

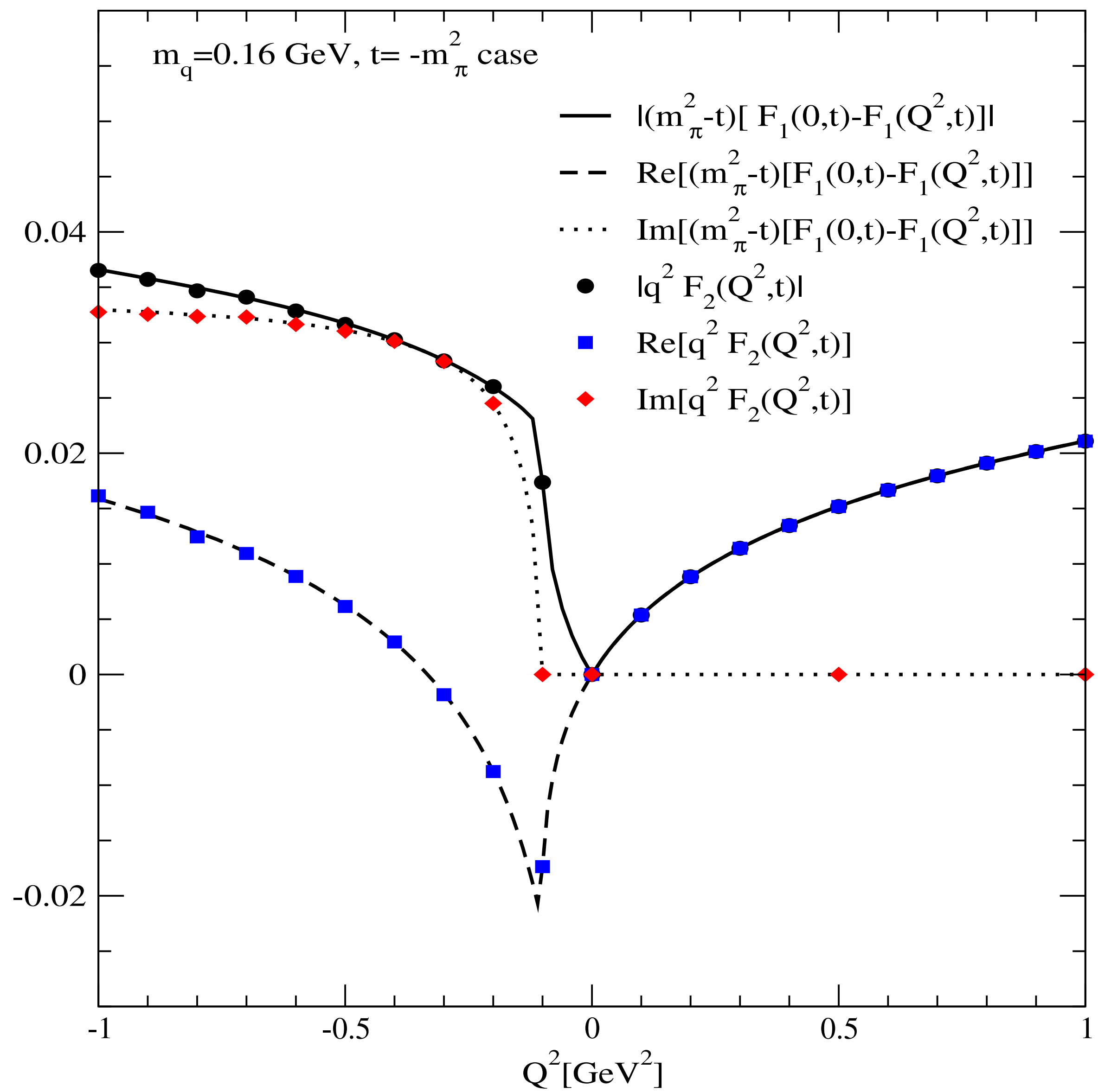
$$f_\pi^{\text{Exp}} = 130 \text{ MeV}$$

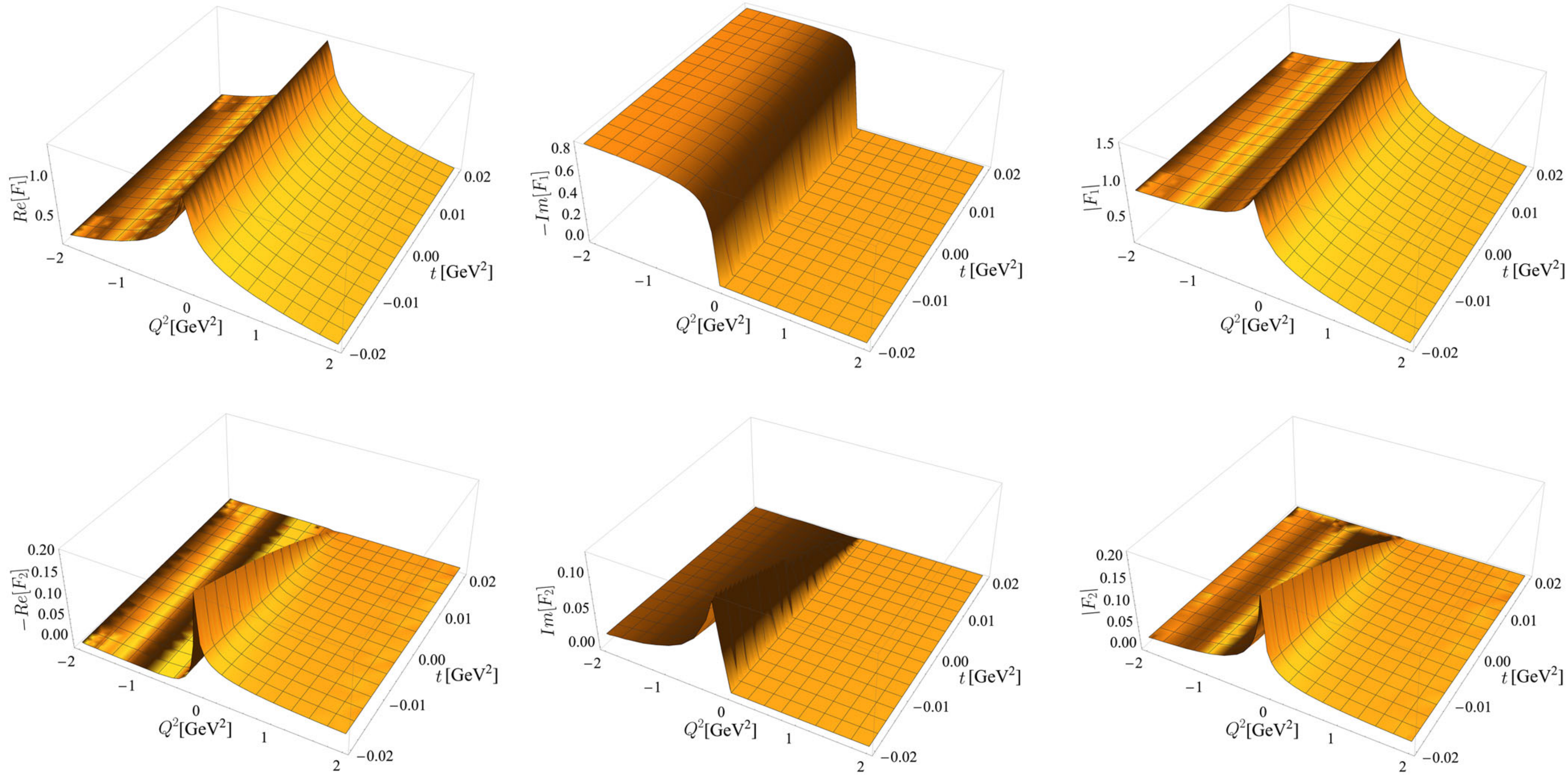
$$g_{\pi q\bar{q}} = (1.32, 1.20, 1.11)(2m_q/f_\pi^{\text{Exp}})$$

$$\text{for } m_q = (0.12, 0.14, 0.16) \text{ GeV}$$

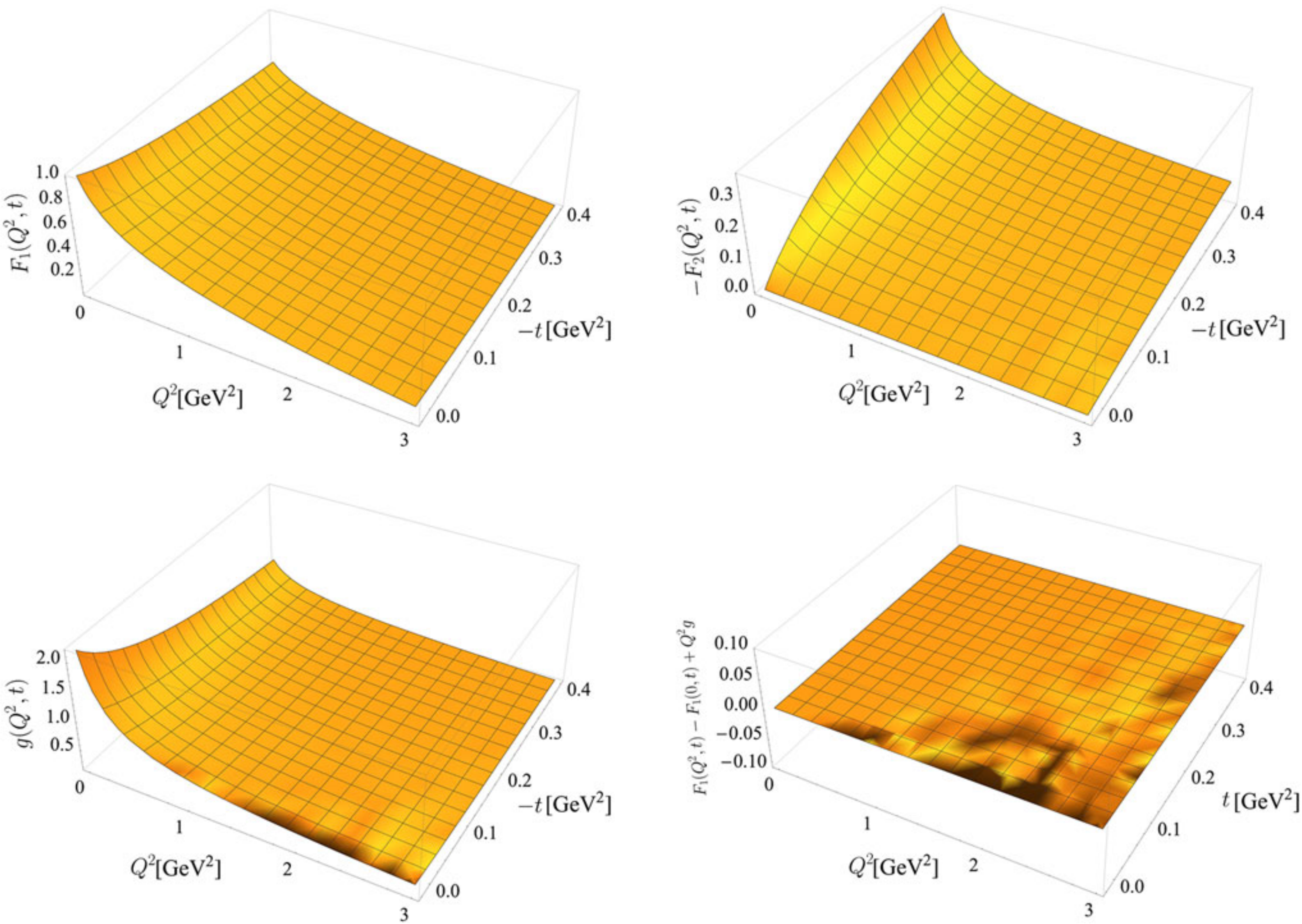
explicit proof of the WTI

$$F_2(Q^2,t) = \frac{t - m_\pi^2}{Q^2}[F_1(0,t) - F_1(Q^2,t)],$$



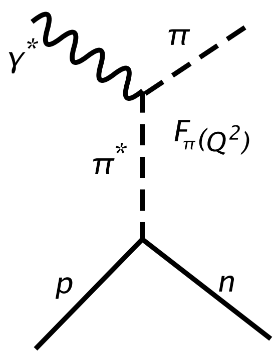


Re[F1]
Q^2[GeV^2]
t [GeV^2]
−Im[F1]
|F1|
−Re[F2]
Im[F2]
|F2|



$F_1(Q^2,t)$
1.0
0.8
0.6
0.4
0.2
0
1
2
3
Q^2[GeV^2]
0.0
0.1
0.2
0.3
0.4
$-t$ [GeV^2]
$-F_2(Q^2,t)$
0.3
0.2
0.1
0.0
0
1
2
3
Q^2[GeV^2]
0.0
0.1
0.2
0.3
0.4
$-t$ [GeV^2]
$g(Q^2,t)$
2.0
1.5
1.0
0.5
0
1
2
3
Q^2[GeV^2]
0.0
0.1
0.2
0.3
0.4
$-t$ [GeV^2]
$F_1(Q^2,t) - F_1(0,t) + Q^2 g$
0.10
0.05
0.00
−0.05
−0.10
0
1
2
3
Q^2[GeV^2]
0.0
0.1
0.2
0.3
0.4
t [GeV^2]

EXTRACTION OF THE OFF-SHELL FORM FACTORS FROM THE EXPERIMENTAL CROSS SECTION



$$N\frac{d\sigma_L}{dt} = 4\hbar c(eG_{\pi NN})^2 \frac{-tQ^2}{(t-m_\pi^2)^2} F_\pi^2(Q^2)$$

$$e^2/(4\pi\hbar c) = 1/137$$

$$N = 32\pi(W^2-m_p^2)\sqrt{(W^2-m_p^2)^2+Q^4+2Q^2(W^2+m_p^2)}$$

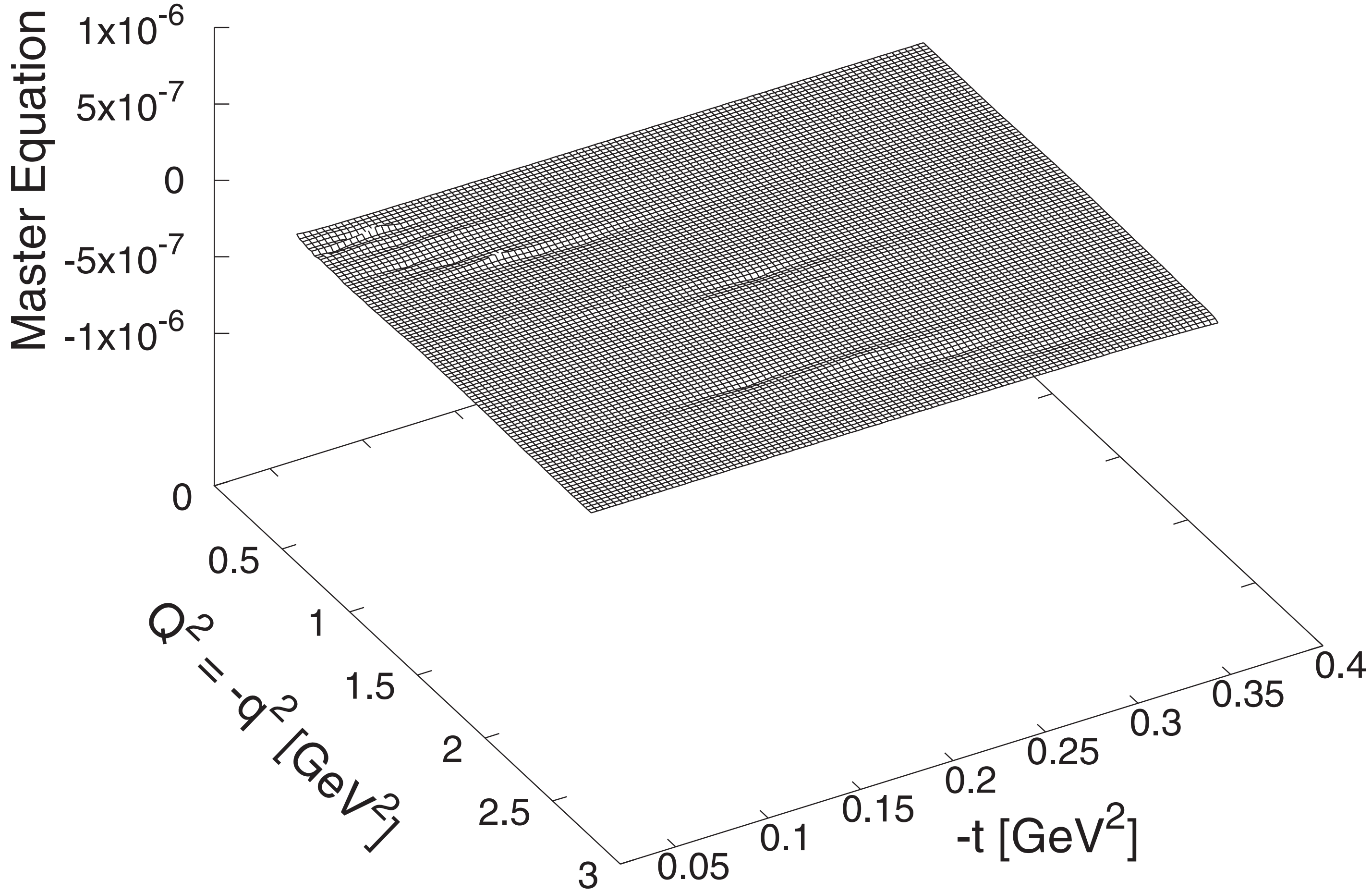
$$G_{\pi NN}(t) = G_{\pi NN}(m_\pi^2)\left(\frac{\Lambda_\pi^2-m_\pi^2}{\Lambda_\pi^2-t}\right)$$

where $G_{\pi NN}(m_\pi^2) = 13.4$ and $\Lambda_\pi = 0.80$ GeV have been taken in the extraction of F_π from the Jefferson Lab (JLAB)

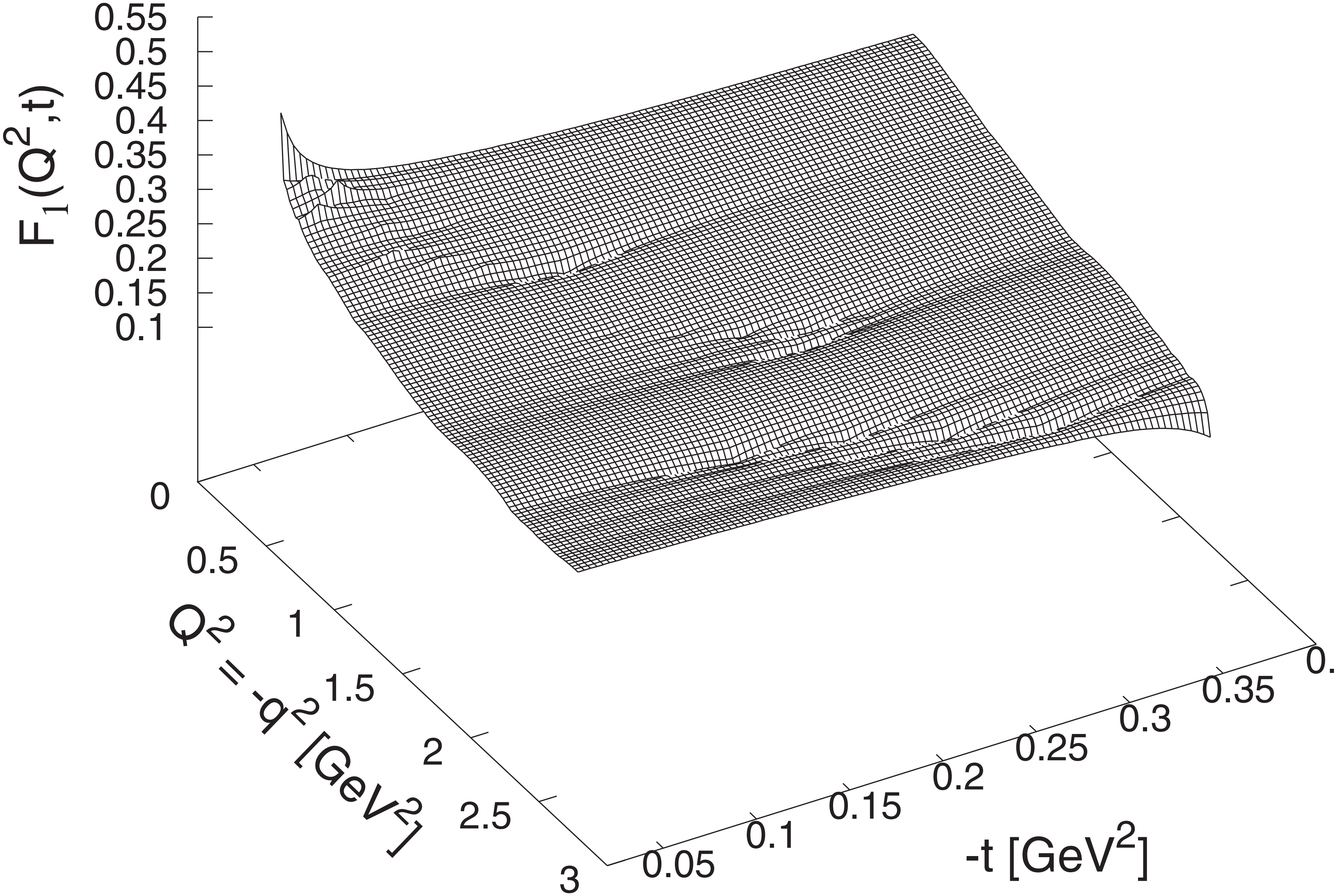
TABLE I. Pion form factors extracted from experimental cross section for $d\sigma_{\rm L}/dt$ given in Table VII of Ref. [7] vs solvable model with $m_q = 0.14 \pm 0.02$ GeV. The coupling constants, $g_{\pi q\bar{q}} = (1.32, 1.20, 1.11)(2m_q/f_\pi^{\rm Exp})$, are used for $m_q = (0.12, 0.14, 0.16)$ GeV, respectively. (Q^2, t) are in units of $\rm GeV^2$, and $g(Q^2, t)$ is in units of $\rm GeV^{-2}$.

Q^2	$-t$	$F_1^{\rm Exp}(Q^2,t)$	$F_1^{\rm Cov}(Q^2,t)$	$F_1^{\rm Cov}(0,t)$	$g^{\rm Exp}(Q^2,t)$	$g^{\rm Cov}(Q^2,t)$
$\langle Q^2\rangle = 0.60$ GeV^2, $W = 1.95$ GeV						
0.526	0.026	0.502 ± 0.013	$0.487^{+0.032}_{-0.039}$	$0.891^{+0.019}_{-0.030}$	$0.740^{+0.060}_{-0.082}$	$0.768^{-0.024}_{+0.018}$
0.576	0.038	0.440 ± 0.010	$0.462^{+0.032}_{-0.039}$	$0.869^{+0.022}_{-0.033}$	$0.745^{+0.055}_{-0.075}$	$0.708^{-0.016}_{+0.008}$
0.612	0.050	0.413 ± 0.011	$0.443^{+0.030}_{-0.038}$	$0.849^{+0.024}_{-0.036}$	$0.712^{+0.058}_{-0.076}$	$0.664^{-0.010}_{+0.003}$
0.631	0.062	0.371 ± 0.014	$0.430^{+0.030}_{-0.036}$	$0.831^{+0.026}_{-0.038}$	$0.729^{+0.063}_{-0.082}$	$0.635^{-0.007}_{-0.002}$
0.646	0.074	0.340 ± 0.022	$0.419^{+0.030}_{-0.036}$	$0.814^{+0.027}_{-0.039}$	$0.734^{+0.076}_{-0.095}$	$0.611^{-0.004}_{-0.005}$
$\langle Q^2\rangle = 0.75$ GeV^2, $W = 1.95$ GeV						
0.660	0.037	0.397 ± 0.019	$0.435^{+0.030}_{-0.036}$	$0.870^{+0.023}_{-0.032}$	$0.717^{+0.063}_{-0.078}$	$0.660^{-0.012}_{+0.005}$
0.707	0.051	0.360 ± 0.017	$0.414^{+0.030}_{-0.035}$	$0.848^{+0.024}_{-0.036}$	$0.690^{+0.058}_{-0.075}$	$0.613^{-0.006}_{-0.001}$
0.753	0.065	0.358 ± 0.015	$0.394^{+0.029}_{-0.034}$	$0.827^{+0.026}_{-0.039}$	$0.623^{+0.054}_{-0.072}$	$0.574^{-0.003}_{-0.006}$
0.781	0.079	0.324 ± 0.018	$0.381^{+0.027}_{-0.033}$	$0.807^{+0.028}_{-0.040}$	$0.618^{+0.059}_{-0.074}$	$0.546^{-0.001}_{-0.009}$
0.794	0.093	0.325 ± 0.022	$0.371^{+0.028}_{-0.032}$	$0.789^{+0.029}_{-0.041}$	$0.584^{+0.065}_{-0.079}$	$0.526^{+0.003}_{-0.011}$
$\langle Q^2\rangle = 1.00$ GeV^2, $W = 1.95$ GeV						
0.877	0.060	0.342 ± 0.014	$0.366^{+0.027}_{-0.031}$	$0.834^{+0.026}_{-0.038}$	$0.561^{+0.046}_{-0.059}$	$0.533^{-0.001}_{-0.006}$
0.945	0.080	0.327 ± 0.012	$0.343^{+0.025}_{-0.030}$	$0.806^{+0.028}_{-0.040}$	$0.507^{+0.042}_{-0.055}$	$0.490^{+0.003}_{-0.010}$
1.010	0.100	0.311 ± 0.012	$0.322^{+0.024}_{-0.029}$	$0.781^{+0.030}_{-0.042}$	$0.465^{+0.042}_{-0.053}$	$0.454^{+0.006}_{-0.013}$
1.050	0.120	0.282 ± 0.016	$0.307^{+0.023}_{-0.027}$	$0.758^{+0.031}_{-0.043}$	$0.453^{+0.045}_{-0.056}$	$0.430^{+0.007}_{-0.015}$
1.067	0.140	0.233 ± 0.028	$0.297^{+0.023}_{-0.026}$	$0.737^{+0.032}_{-0.043}$	$0.472^{+0.057}_{-0.066}$	$0.412^{+0.009}_{-0.015}$
$\langle Q^2\rangle = 1.60$ GeV^2, $W = 1.95$ GeV						
1.455	0.135	0.258 ± 0.010	$0.237^{+0.018}_{-0.021}$	$0.742^{+0.032}_{-0.043}$	$0.332^{+0.029}_{-0.037}$	$0.347^{+0.010}_{-0.015}$
1.532	0.165	0.245 ± 0.010	$0.219^{+0.016}_{-0.020}$	$0.714^{+0.032}_{-0.044}$	$0.306^{+0.028}_{-0.035}$	$0.323^{+0.011}_{-0.016}$
1.610	0.195	0.222 ± 0.012	$0.201^{+0.015}_{-0.018}$	$0.688^{+0.033}_{-0.044}$	$0.289^{+0.028}_{-0.034}$	$0.302^{+0.012}_{-0.016}$
1.664	0.225	0.203 ± 0.013	$0.188^{+0.014}_{-0.017}$	$0.665^{+0.034}_{-0.045}$	$0.278^{+0.028}_{-0.035}$	$0.286^{+0.012}_{-0.016}$
1.702	0.255	0.227 ± 0.016	$0.177^{+0.014}_{-0.015}$	$0.644^{+0.034}_{-0.044}$	$0.245^{+0.029}_{-0.035}$	$0.274^{+0.012}_{-0.017}$
$\langle Q^2\rangle = 1.60$ GeV^2, $W = 2.22$ GeV						
1.416	0.079	0.270 ± 0.010	$0.259^{+0.019}_{-0.022}$	$0.807^{+0.028}_{-0.040}$	$0.379^{+0.027}_{-0.035}$	$0.387^{+0.006}_{-0.012}$
1.513	0.112	0.258 ± 0.010	$0.235^{+0.018}_{-0.021}$	$0.767^{+0.030}_{-0.043}$	$0.336^{+0.027}_{-0.035}$	$0.351^{+0.009}_{-0.014}$
1.593	0.139	0.251 ± 0.010	$0.217^{+0.016}_{-0.019}$	$0.738^{+0.032}_{-0.043}$	$0.306^{+0.026}_{-0.034}$	$0.327^{+0.010}_{-0.015}$
1.667	0.166	0.241 ± 0.012	$0.201^{+0.015}_{-0.018}$	$0.713^{+0.033}_{-0.044}$	$0.283^{+0.027}_{-0.033}$	$0.307^{+0.011}_{-0.016}$
1.763	0.215	0.200 ± 0.018	$0.179^{+0.013}_{-0.017}$	$0.672^{+0.034}_{-0.044}$	$0.268^{+0.029}_{-0.035}$	$0.280^{+0.011}_{-0.017}$
$\langle Q^2\rangle = 2.45$ GeV^2, $W = 2.22$ GeV						
2.215	0.145	0.188 ± 0.008	$0.146^{+0.010}_{-0.012}$	$0.732^{+0.033}_{-0.043}$	$0.246^{+0.018}_{-0.023}$	$0.265^{+0.010}_{-0.014}$
2.279	0.202	0.178 ± 0.008	$0.129^{+0.009}_{-0.011}$	$0.682^{+0.034}_{-0.044}$	$0.221^{+0.019}_{-0.023}$	$0.243^{+0.011}_{-0.015}$
2.411	0.245	0.163 ± 0.009	$0.109^{+0.008}_{-0.009}$	$0.650^{+0.037}_{-0.044}$	$0.202^{+0.019}_{-0.022}$	$0.224^{+0.011}_{-0.014}$
2.539	0.288	0.156 ± 0.011	$0.092^{+0.006}_{-0.007}$	$0.622^{+0.034}_{-0.043}$	$0.184^{+0.017}_{-0.022}$	$0.209^{+0.011}_{-0.014}$
2.703	0.365	0.150 ± 0.016	$0.068^{+0.004}_{-0.005}$	$0.579^{+0.033}_{-0.043}$	$0.159^{+0.018}_{-0.022}$	$0.189^{+0.011}_{-0.014}$

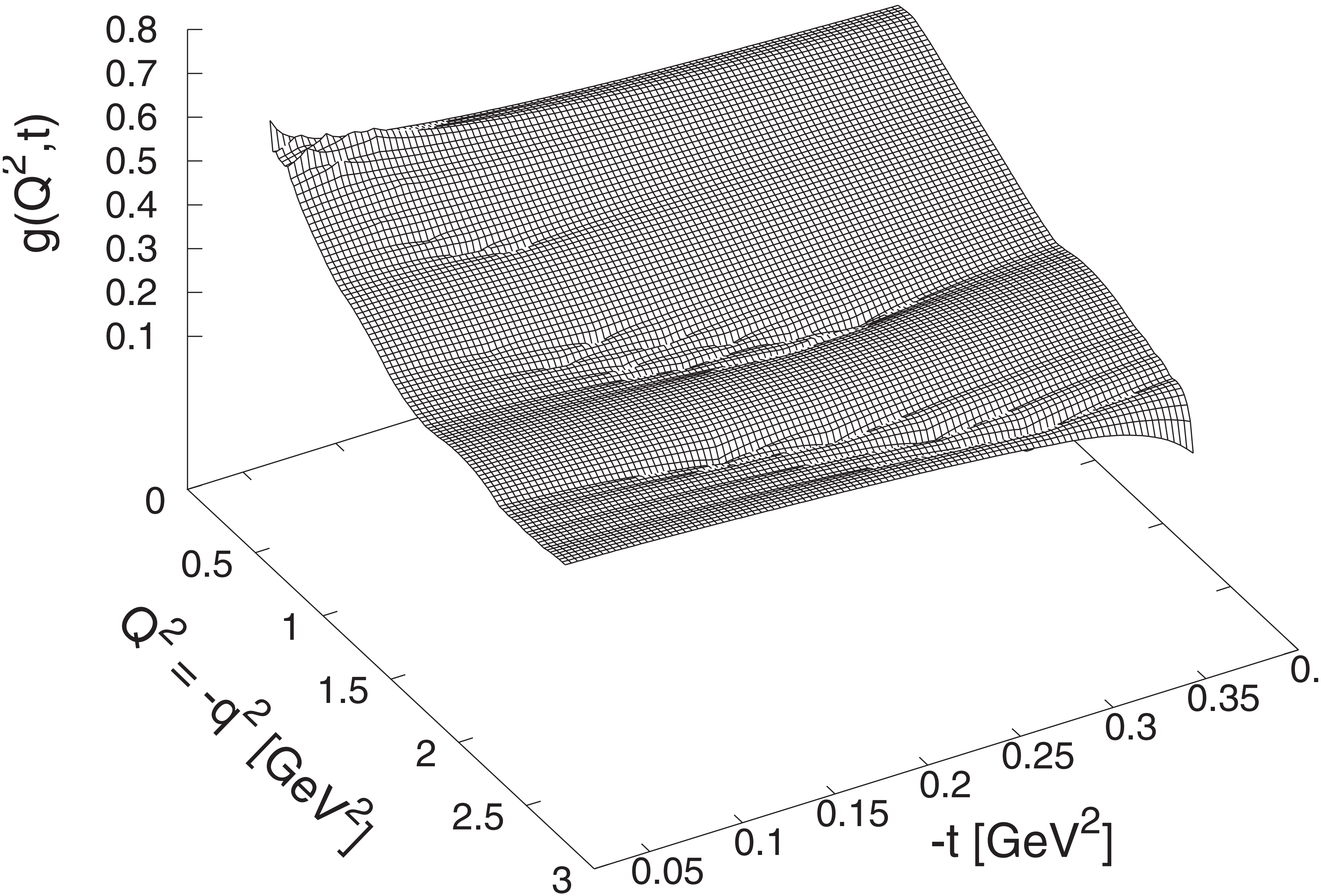
$$g^{\mathrm{Exp}}(Q^2,t) = [F_1^{\mathrm{Cov}}(0,t) - F_1^{\mathrm{Exp}}(Q^2,t)]/Q^2$$



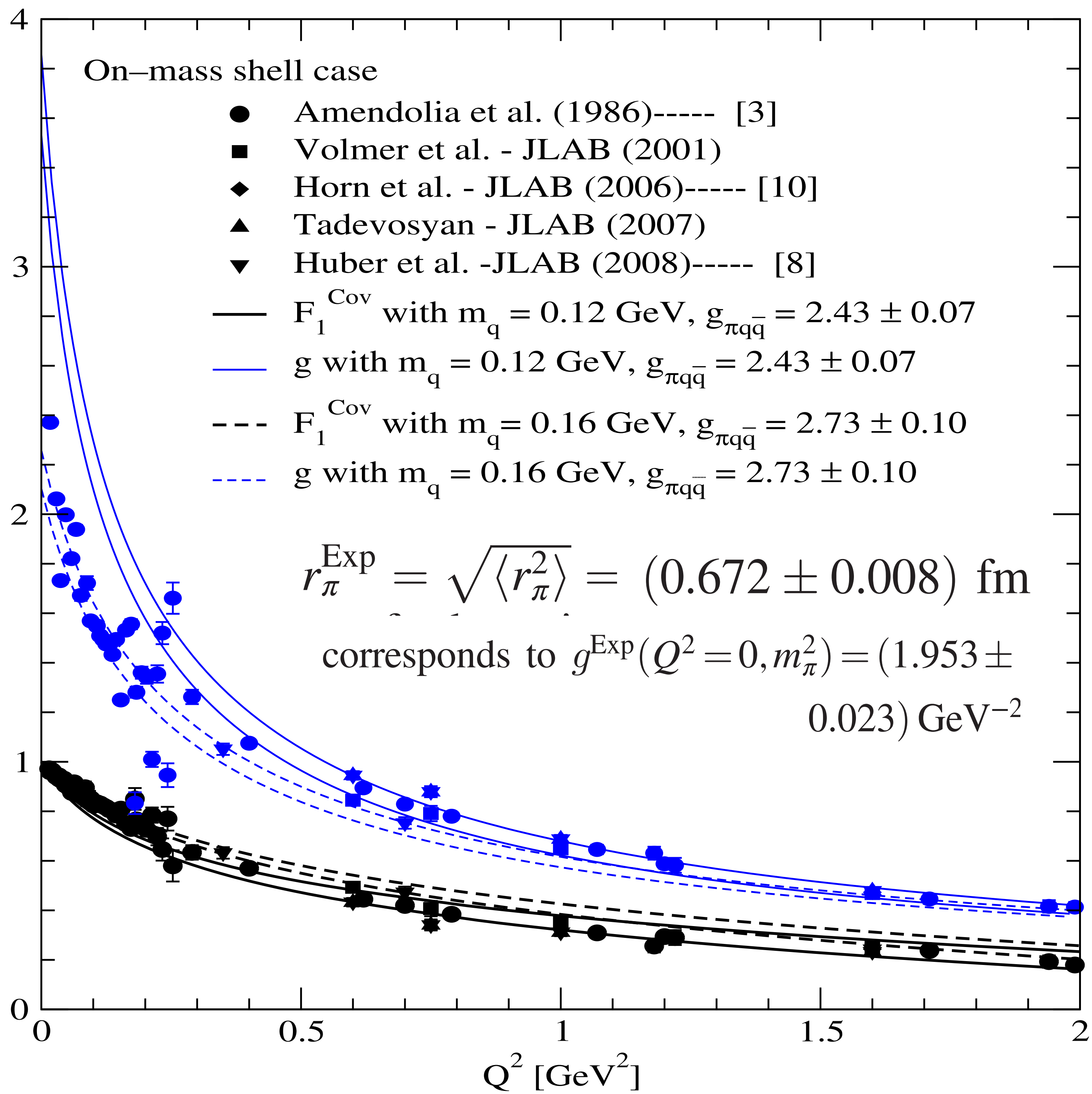
$$F_1(Q^2,t) - F_1(0,t) + Q^2 g(Q^2,t) = 0$$



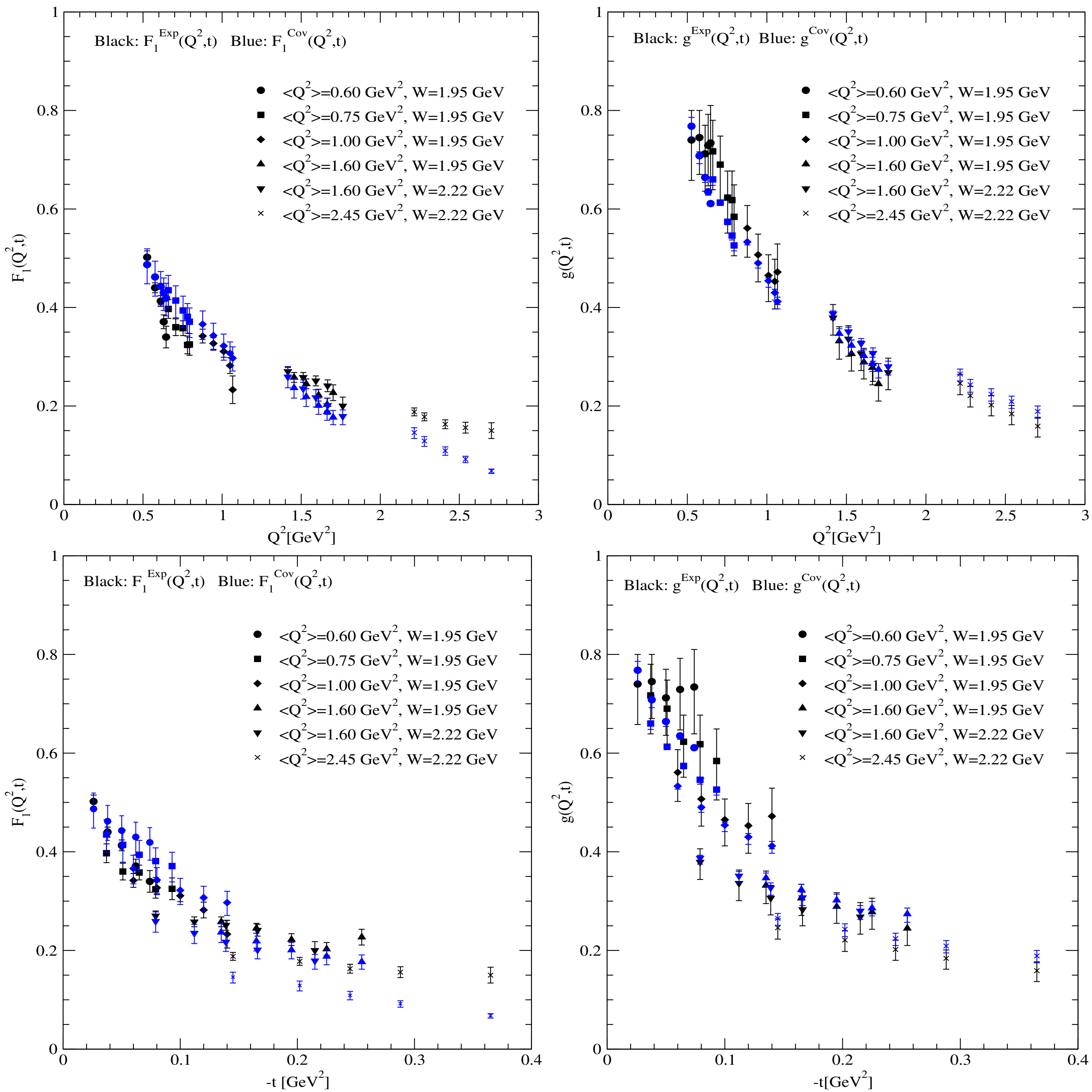
0.55
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
$F_1(Q^2,t)$
0
0.5
1
1.5
2
2.5
3
$Q^2 = -q^2$ [GeV^2]
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
$-t$ [GeV^2]



g(Q^2,t)
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.5
1
1.5
2
2.5
3
Q^2 = -q^2 [GeV^2]
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
-t [GeV^2]



On–mass shell case
Amendolia et al. (1986)----- [3]
Volmer et al. - JLAB (2001)
Horn et al. - JLAB (2006)----- [10]
Tadevosyan - JLAB (2007)
Huber et al. -JLAB (2008)----- [8]
F_1^{Cov} with $m_q = 0.12$ GeV, $g_{\pi q\bar{q}} = 2.43 \pm 0.07$
g with $m_q = 0.12$ GeV, $g_{\pi q\bar{q}} = 2.43 \pm 0.07$
F_1^{Cov} with $m_q = 0.16$ GeV, $g_{\pi q\bar{q}} = 2.73 \pm 0.10$
g with $m_q = 0.16$ GeV, $g_{\pi q\bar{q}} = 2.73 \pm 0.10$
$r_\pi^{Exp} = \sqrt{\langle r_\pi^2 \rangle} = (0.672 \pm 0.008)$ fm
corresponds to $g^{Exp}(Q^2=0, m_\pi^2) = (1.953 \pm 0.023)$ GeV^{-2}
0
1
2
3
4
0
0.5
1
1.5
2
Q^2 [GeV^2]



Black: F1^Exp(Q^2,t) Blue: F1^Cov(Q^2,t)
<Q^2>=0.60 GeV^2, W=1.95 GeV
<Q^2>=0.75 GeV^2, W=1.95 GeV
<Q^2>=1.00 GeV^2, W=1.95 GeV
<Q^2>=1.60 GeV^2, W=1.95 GeV
<Q^2>=1.60 GeV^2, W=2.22 GeV
<Q^2>=2.45 GeV^2, W=2.22 GeV
F1(Q^2,t)
Q^2[GeV^2]
Black: g^Exp(Q^2,t) Blue: g^Cov(Q^2,t)
g(Q^2,t)
-t [GeV^2]

Conclusion and Outlook

- New form factor $g(Q^2,t) = F_2(Q^2,t)/(t - m_\pi^2)$ appears measurable even in $t \to m_\pi^2$.
- The value of $g(Q^2 = 0, t = m_\pi^2)$ corresponds to the charge radius of pion.
- One needs $F_1(0,t)$ to determine $g(Q^2,t)$.
- Main features appear consistent between the model calculation and the data extraction although the evolution in Q^2 and/or t is not in full agreement between them as expected.
- QCD effects deserve further study including the extension to inclusive processes.