GPDs in DGLAP & ERBL Regions

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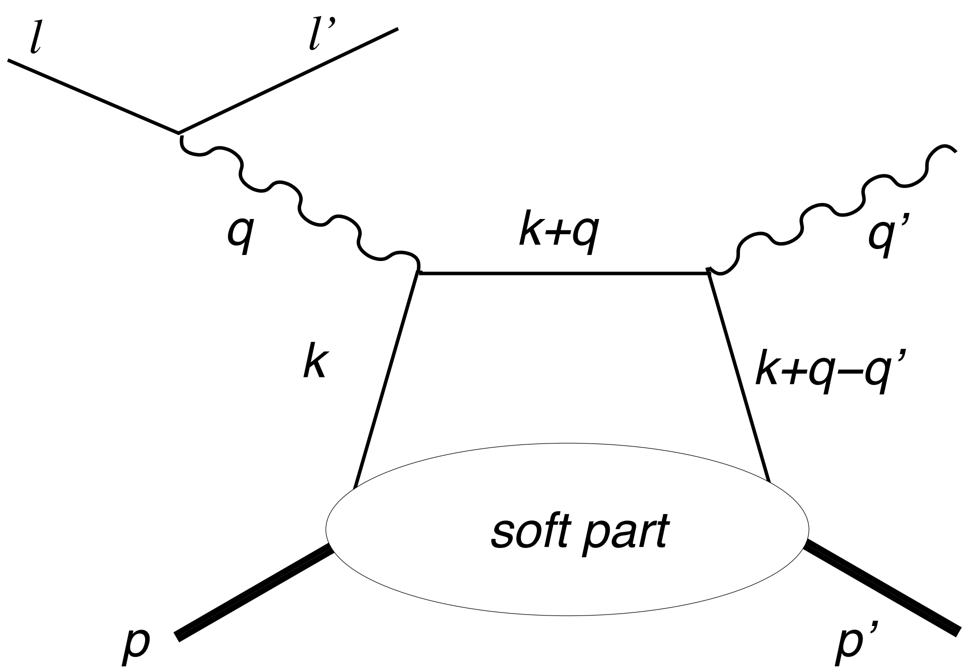


Group Meeting, July 9th, 2021

Outline

- EMT and GPDs
- D-term Interpretation and Access from GPDs
- BSA and Imaginary Part of GPDs
- Computation of GPDs in LFQM
- Crossover between DGLAP and ERBL Regions
- Continuity at Crossover Point
- Zero or Nonzero GPDs at Crossover Point
- Conclusion and Outlook

Mechanical properties of particles

M.V. Polyakov

EMT form factors for the nucleon

$$T_q^{\mu\nu} = \frac{1}{4}\bar{\psi}_q\left(-i\overleftarrow{\mathcal{D}}^{\mu}\gamma^{\nu} - i\overleftarrow{\mathcal{D}}^{\nu}\gamma^{\mu} + i\overrightarrow{\mathcal{D}}^{\mu}\gamma^{\nu} + i\overrightarrow{\mathcal{D}}^{\nu}\gamma^{\mu}\right)\psi_q - g^{\mu\nu}\bar{\psi}_q\left(-\frac{i}{2}\overleftarrow{\not{\mathcal{D}}} + \frac{i}{2}\overrightarrow{\not{\mathcal{D}}} - m_q\right)\psi_q,$$

$$T_g^{\mu\nu} = F^{a,\mu\eta}\, F^{a,}{}_{\eta}{}^{\nu} + \frac{1}{4} g^{\mu\nu} F^{a,\kappa\eta}\, F^{a,}{}_{\kappa\eta}.$$

$$\langle p'|T^a_{\mu\nu}(0)|p\rangle = \bar{u}'\left[A^a(t)\,\frac{P_\mu P_\nu}{M_N} + J^a(t)\,\frac{i\,P_{\{\mu}\sigma_{\nu\}\rho}\Delta^\rho}{2M_N} + D^a(t)\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4M_N} + M_N\,\bar{c}^a(t)g_{\mu\nu}\right]u$$

Kobzarev, Okun '1962 , Pagels '1966

$P = (p' + p)/2,\ \Delta = p' - p.$

The name **"D-term"** is rather technical, it can be traced back to more or less accidental notations chosen in /Weiss, MVP '99/. Nowadays, given more clear physics meaning of this quantity, we might call this term as ***"Druck-term"*** derived from German word for pressure

Interaction of the nucleon with gravity

$$\langle p'|T^a_{\mu\nu}(0)|p\rangle = \bar{u}'\left[A^a(t)\,\frac{P_\mu P_\nu}{M_N} + J^a(t)\,\frac{i\,P_{\{\mu}\sigma_{\nu\}\rho}\Delta^\rho}{2M_N} + D^a(t)\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4M_N} + M_N\,\bar{c}^a(t)g_{\mu\nu}\right]u$$

$a = g, Q$ (gluon or quark parts)

δg^{00} δg^{0i} δg^{ij} $\text{non}-\text{conservation of EMT pieces}$

Mass

$$\sum_a A^a(0) = 1$$

Spin

$$\sum_a J^a(0) = \frac{1}{2}$$

deformation of space = elastic properties of N

$$\sum_a \bar{c}^a(t) = 0$$

Elasticity stress tensor (Landau & Lifshitz vol. 7)

Shear forces distribution (pressure anisotropy)

Pressure distribution

MVP ‘2003

$$T^a_{ij}(\vec{r}) = \left(\frac{r_i r_j}{r^2} - \frac{1}{3}\,\delta_{ij}\right)s^a(r) + \delta_{ij}\,p^a(r)$$

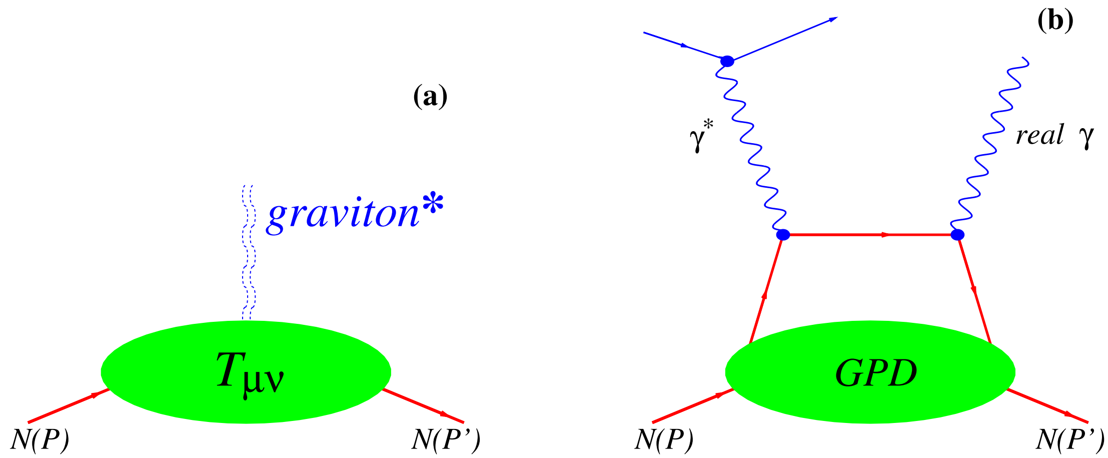
$$s^a(r) = -\frac{1}{4M_N}\;r\frac{d}{dr}\frac{1}{r}\frac{d}{dr}\widetilde{D}^a(r),\quad p^a(r) = \frac{1}{6M_N}\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr}\widetilde{D^a}(r) - M_N\int\frac{d^3\Delta}{(2\pi)^3}\;e^{-i\vec{\Delta}\vec{r}}\;\bar{c}^a(-\vec{\Delta}^2).$$

D-term is a global and fundamental quantity related to the distribution of strong forces (pressure and shear) inside a hadron

$$D = M \int d^3r\ r^2\ p(r) = -\frac{4M}{15} \int d^3r\ r^2\ s(r)$$

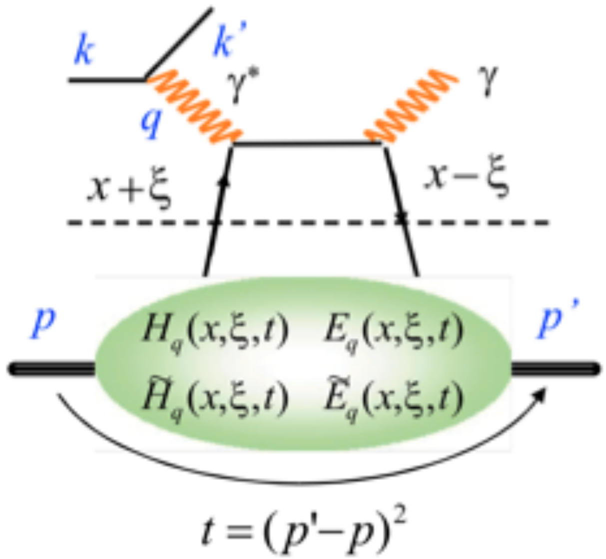
- gravitational D-form factor is related to "elastic properties" of the nucleon, and gives access to details of strong forces inside the nucleon.
- D(0) (the D-term) is the last unexplored global (in the same sense as mass and spin) property of the nucleon

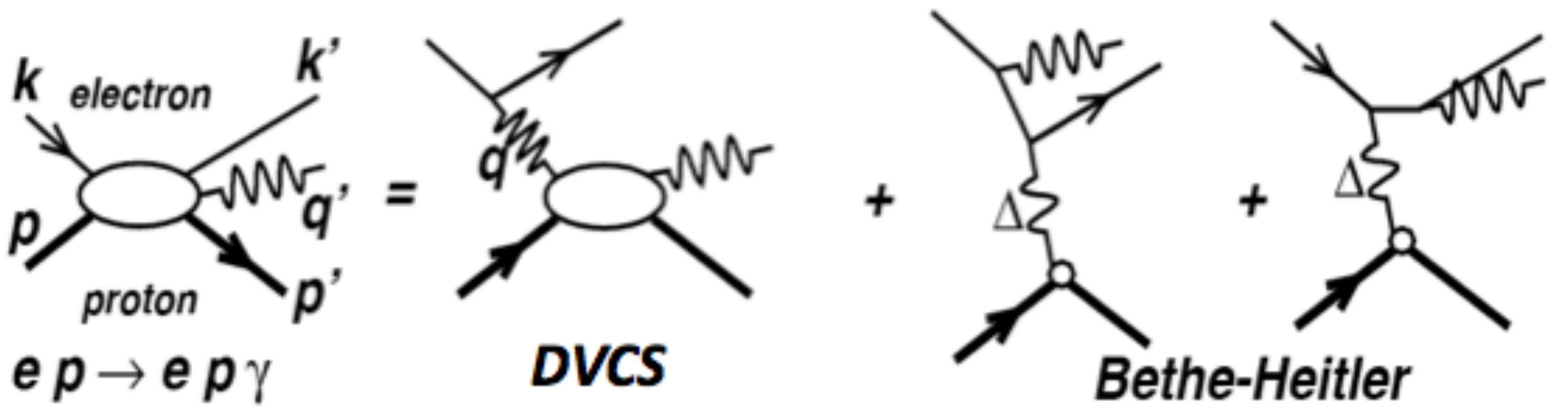
Accessing D(t) in hard exclusive processes



$$\mathcal{H}(\xi,t) = \int_{-1}^{1} dx \left(\frac{1}{\xi - x - i0} - \frac{1}{\xi + x - i0} \right) H(x,\xi,t)$$ DVCS amplitude at LO

$$\int_{-1}^{1} \mathrm{d}x\ x\, H^a(x,\xi,t) = A^a(t) + \xi^2 D^a(t)\,,$$





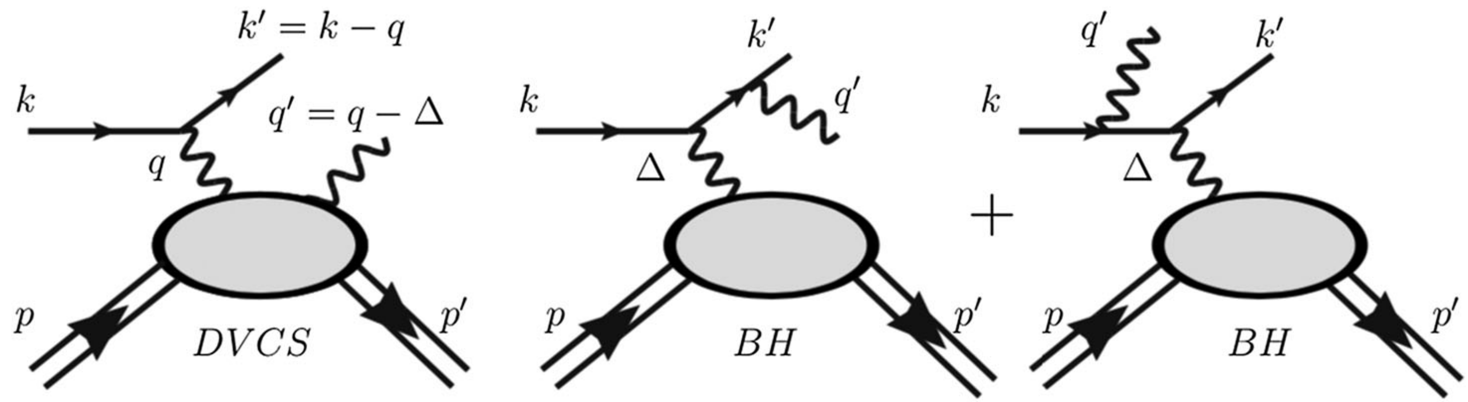
$$d^5\sigma \propto |T_{BH}|^2 + \mathrm{Re}(T_{BH}\cdot T_{VCS}^{\dagger}) + |T_{VCS}|^2$$

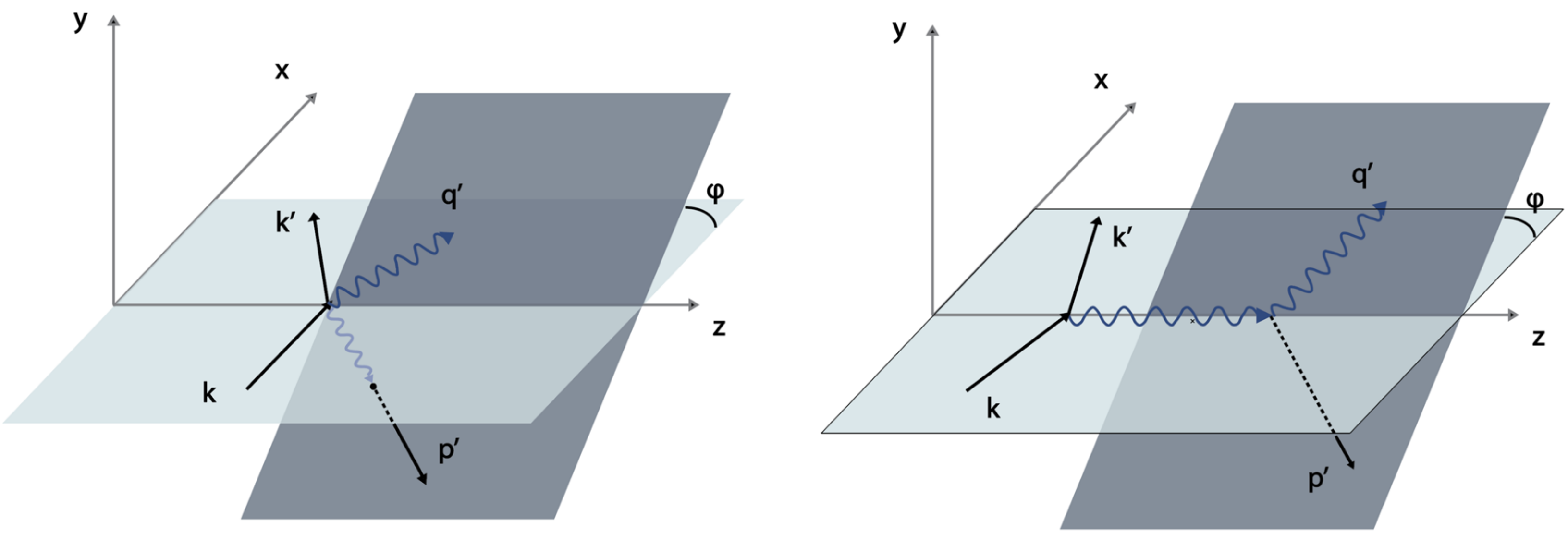
$$d^5\sigma_L - d^5\sigma_R \propto \mathrm{Im}(T_{BH}\cdot T_{VCS}^{\dagger})$$

PHYSICAL REVIEW D **101,** 054021 (2020)

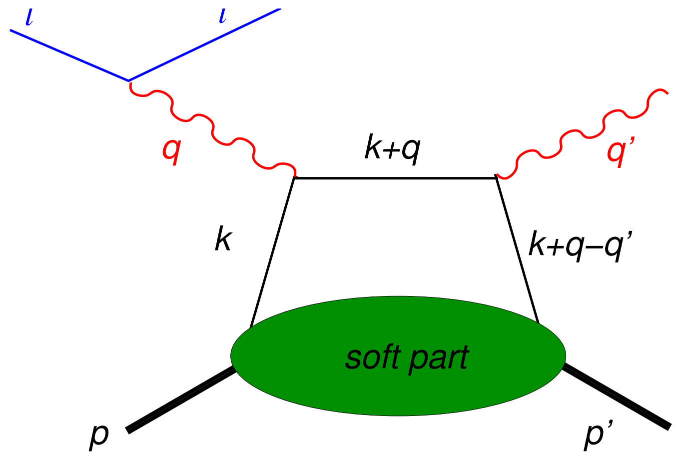
Extraction of generalized parton distribution observables from deeply virtual electron proton scattering experiments

Brandon Kriesten,[*] Andrew Meyer,[†] Simonetta Liuti,[‡] Liliet Calero Diaz,[§] and Dustin Keller[‖] Gary R. Goldstein[¶] J. Osvaldo Gonzalez-Hernandez[**]





Beam-spin asymmetry



Handbag diagram for VCS, including the leptonic part

$$\langle|\mathcal{M}|^2\rangle = \left(\frac{e^2}{q^2}\right)^2 \mathcal{L}^{\mu\nu}\mathcal{H}_{\mu\nu} \qquad \begin{aligned} \mathcal{L}^{\mu\nu} &= q^2\Lambda^{\mu\nu} + 2i\lambda\epsilon^{\mu\nu\alpha\beta}k_\alpha k'_\beta \\ \Lambda^{\mu\nu} &= g^{\mu\nu} + \tfrac{2}{q^2}(k^\mu k'^\nu + k'^\mu k^\nu) \end{aligned}$$

Whenever there is an interference term, it can used to determine the CFFs. This term will show up in the single-spin asymmetry A_{LU}, which is defined as

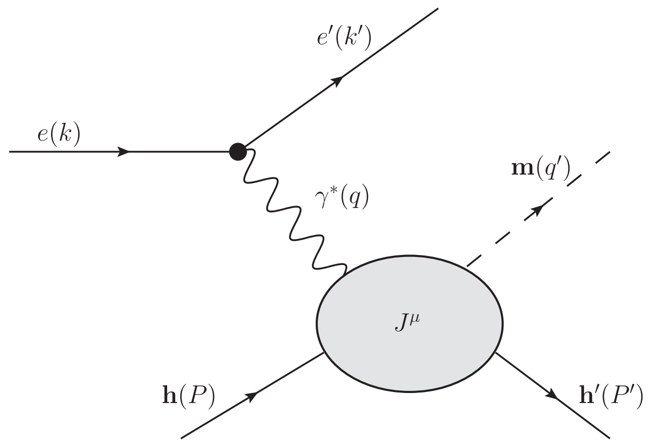
$$A_{\mathrm{LU}} = \frac{d\sigma^+ - d\sigma^-}{d\sigma^+ + d\sigma^-}.$$

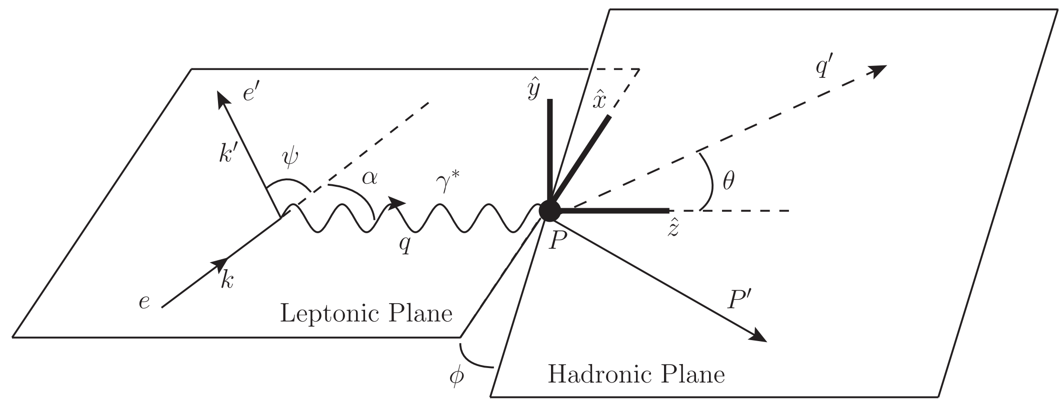
It is well known that in case the CFFs are real, this observable vanishes.

PHYSICAL REVIEW D **99**, 116008 (2019)

Beam spin asymmetry in the electroproduction of a pseudoscalar meson or a scalar meson off the scalar target

Chueng-Ryong Ji,[1] Ho-Meoyng Choi,[2] Andrew Lundeen,[1] and Bernard L. G. Bakker[3]





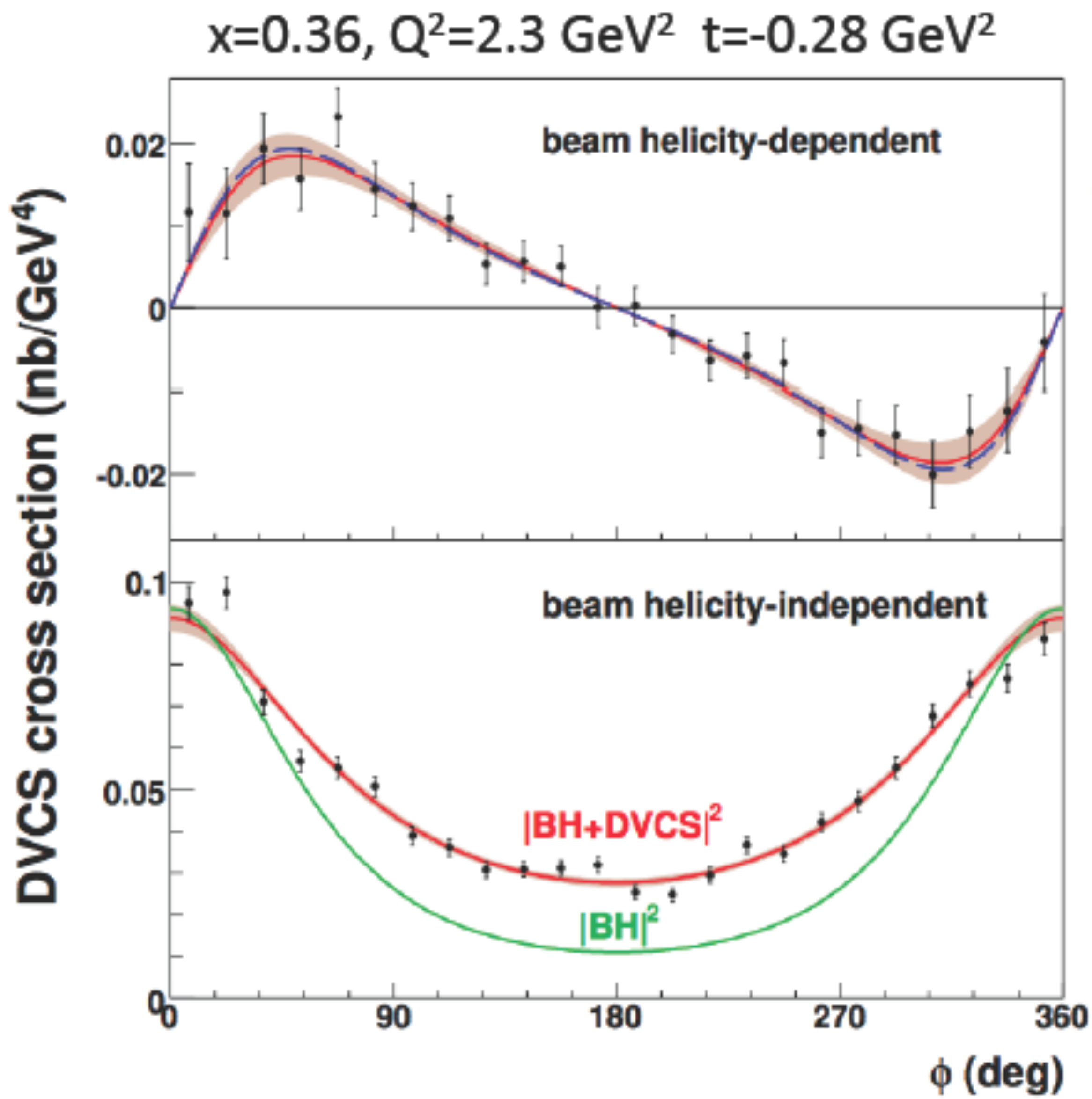
$$d\sigma_\lambda^S = d\sigma_U^S + d\sigma_P^S \epsilon \cos(2\phi) + d\sigma_L^S \epsilon_L + d\sigma_I^S \cos\phi \sqrt{\epsilon_L(1+\epsilon)} + \lambda\, d\sigma_{\mathrm{SSA}}^S$$

$$\begin{bmatrix} d\sigma_U^S \\ d\sigma_L^S \\ d\sigma_I^S \\ d\sigma_{\mathrm{SSA}}^S \end{bmatrix} = \begin{bmatrix} U_1 & U_2 & U_3 & 0 \\ L_1 & L_2 & L_3 & 0 \\ I_1 & I_2 & I_3 & 0 \\ 0 & 0 & 0 & S_A \end{bmatrix} \begin{bmatrix} |F_1|^2 \\ |F_2|^2 \\ F_{12}^+ \\ F_{12}^- \end{bmatrix}$$

$$F_{12}^\pm = F_1 F_2^* \pm F_2 F_1^*$$

$$S_A = -\kappa e^4 \frac{\sin\theta \sin\phi}{2Mx^2 y}(m^2 + Q^2 - t)\sqrt{Q^2(y-1) + M^2x^2y^2}\sqrt{x^2(t^2 - 4m^2M^2) + Q^4 + 2Q^2 tx},$$

Typical results from the first generation proton experiment E00-110



C. M. Camacho *et al.* [Jefferson Lab Hall A and Hall A DVCS Collaborations], Phys. Rev. Lett. **97**, 262002 (2006)

International Journal of Modern Physics E
Vol. 22, No. 2 (2013) 1330002 (98 pages)

Review

CONCEPTUAL ISSUES CONCERNING GENERALIZED PARTON DISTRIBUTIONS

CHUENG-RYONG JI BERNARD L. G. BAKKER

PHYSICAL REVIEW D **73,** 114013 (2006)

Higher Fock-state contributions to the generalized parton distribution of pion

Chueng-Ryong Ji,[1] Yuriy Mishchenko,[1,2] and Anatoly Radyushkin[2,3,*]

PHYSICAL REVIEW D **66**, 053011 (2002)

Continuity of generalized parton distributions for the pion virtual Compton scattering

Ho-Meoyng Choi Chueng-Ryong Ji L. S. Kisslinger

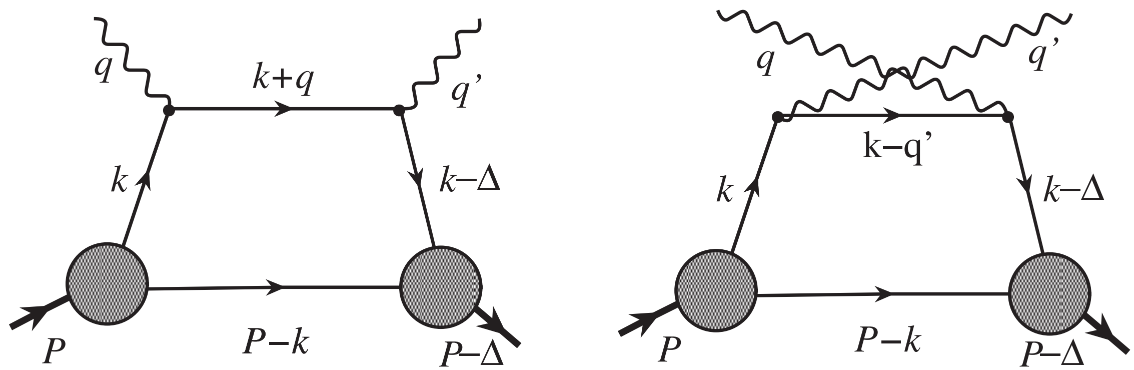
PHYSICAL REVIEW D, VOLUME 64, 093006

Skewed quark distribution of the pion in the light-front quark model

Ho-Meoyng Choi Chueng-Ryong Ji L. S. Kisslinger

(Received 11 April 2001; published 4 October 2001)

$$\gamma^*(q) + \pi(P) \longrightarrow \gamma(q') + \pi(P')$$



$$P = (P^+, M^2/P^+, 0_\perp),$$

$$P' = ((1-\zeta)P^+, (M^2 + \mathbf{\Delta}_\perp^2)/(1-\zeta)P^+, -\mathbf{\Delta}_\perp),$$

and

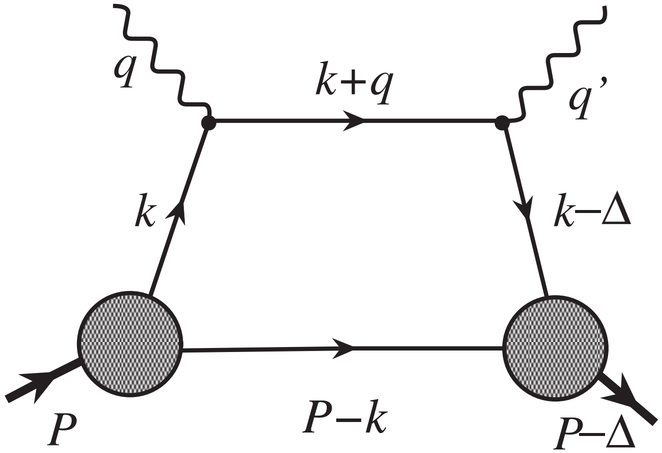
$$\Delta = P - P' = (\zeta P^+, (\Delta^2 + \mathbf{\Delta}_\perp^2)/\zeta P^+, \mathbf{\Delta}_\perp),$$

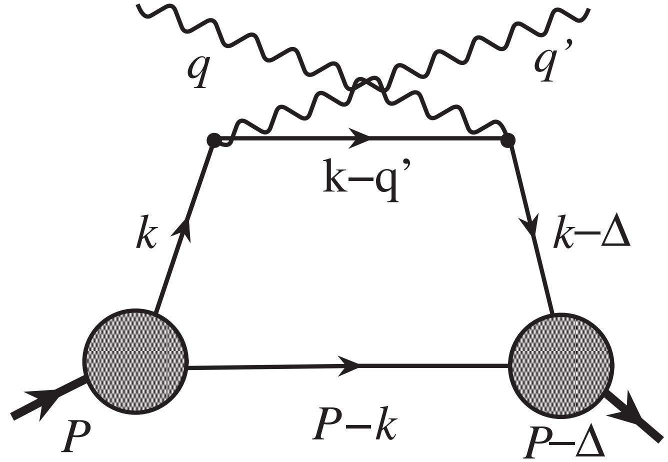
$$q = (0, (\mathbf{q}_\perp + \mathbf{\Delta}_\perp)^2/\zeta P^+ + (\zeta M^2 + \mathbf{\Delta}_\perp^2)/(1-\zeta)P^+, \mathbf{q}_\perp),$$

$$q' = (\zeta P^+, (\mathbf{q}_\perp + \mathbf{\Delta}_\perp)^2/\zeta P^+, \mathbf{q}_\perp + \mathbf{\Delta}_\perp).$$

$$t = \Delta^2 = 2P\cdot\Delta = -\frac{\zeta^2 M^2 + \mathbf{\Delta}_\perp^2}{1-\zeta}.$$

$$M^{\mu\nu} = M_s^{\mu\nu} + M_u^{\mu\nu},$$

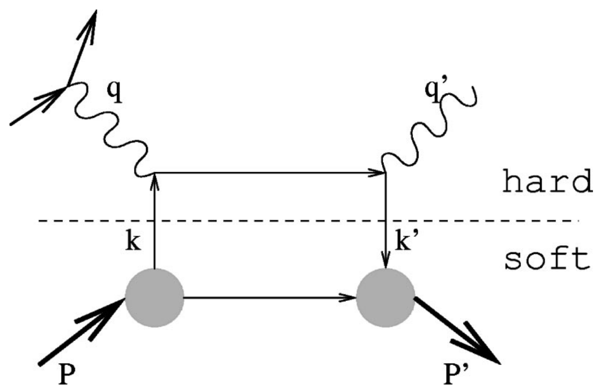


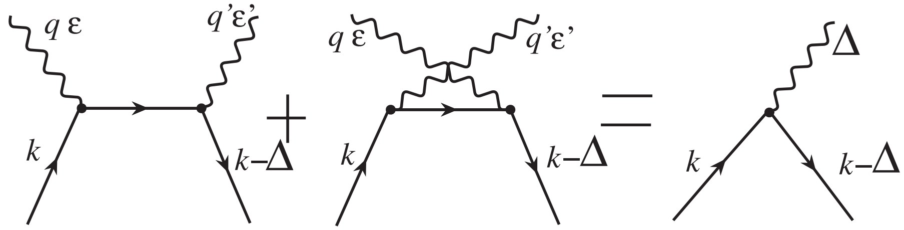


$$M_u^{\mu\nu} = M_s^{\mu\nu}(q \to -q')$$

$$M_s^{\mu\nu} = -iN_c e_q^2 \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}[\gamma_5(\not{k} + m)\gamma^\mu(\not{k} + \not{q} + m)\gamma^\nu(\not{k} - \not{\Delta} + m)\gamma_5(-\not{P} + \not{k} + m)]$$

$$\times \frac{H_{\mathrm{cov}}(k, P)H'_{\mathrm{cov}}(k - \Delta, P - \Delta)}{[k^2 - m^2 + i\varepsilon][(k + q)^2 - m^2 + i\varepsilon][(k - \Delta)^2 - m^2 + i\varepsilon][(P - k)^2 - m^2 + i\varepsilon]},$$





$$q^2, q \cdot a \gg a \cdot b, m^2, \epsilon \cdot q, \epsilon' \cdot q.$$

$$(-ie_q)^2\left(\frac{\not{\epsilon}\, i(\not{q}+\not{k}+m)\not{\epsilon}'}{(q+k)^2-m^2+i\varepsilon}+\frac{\not{\epsilon}'\, i(-\not{q}'+\not{k}+m)\not{\epsilon}}{(k-q')^2-m^2+i\varepsilon}\right).$$

$$-ie_q^2\left(\frac{-\not{q}\not{\epsilon}\not{\epsilon}'}{(q+k)^2-m^2+i\varepsilon}+\frac{\not{q}'\not{\epsilon}'\not{\epsilon}}{(k-q')^2-m^2+i\varepsilon}\right)$$

$\not{\epsilon}\not{\epsilon}'$ can be written as $\epsilon \cdot \epsilon' + i\sigma^{\mu\nu}\epsilon_\mu \epsilon'_\nu$, where $\sigma^{\mu\nu} = \frac{1}{2i}[\gamma^\mu, \gamma^\nu]$.

$$\left(\frac{1}{(q+k)^2-m^2+i\varepsilon}-\frac{1}{(k-q')^2-m^2+i\varepsilon}\right)\sim\frac{1}{P^+q^-}\left(\frac{1}{x-\zeta+i\varepsilon}+\frac{1}{x-i\varepsilon}\right)$$

$$M^{IJ} = \epsilon^I_\mu \epsilon^{*J}_\nu M^{\mu\nu}$$

$$= -\frac{iN_c}{2P^+}e_q^2\int\frac{d^4k}{(2\pi)^4}\left(\frac{1}{x-\zeta+i\varepsilon}+\frac{1}{x-i\varepsilon}\right)H_{\mathrm{cov}}(k,P)H'_{\mathrm{cov}}(k-\Delta,P-\Delta)$$

$$\times\frac{\mathrm{Tr}[\gamma_5(\not{k}+m)\not{\epsilon}^I\gamma^+\not{\epsilon}^{*J}(\not{k}-\not{\Delta}+m)\gamma_5(-\not{P}+\not{k}+m)]}{[k^2-m^2+i\varepsilon][(k-\Delta)^2-m^2+i\varepsilon][(P-k)^2-m^2+i\varepsilon]}.$$

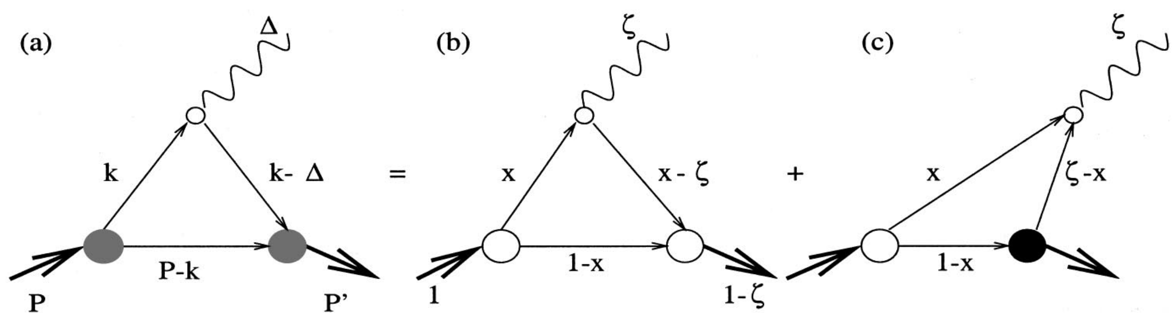
$$\not{\epsilon}^I\gamma^+\not{\epsilon}^{*J} = (\epsilon^I_\perp \cdot \epsilon^{*J}_\perp)\gamma^+ + i(\epsilon^I_\perp \times \epsilon^{*J}_\perp)_3\gamma^+\gamma_5$$

$$M^{\uparrow\uparrow}(P,q,P') = M^{\downarrow\downarrow}(P,q,P')$$
$$= -e_q^2 \int dx \left(\frac{1}{x-\zeta+i\varepsilon} + \frac{1}{x-i\varepsilon} \right) \mathcal{F}_\pi(\zeta,x,t),$$

$$\mathcal{F}_\pi(\zeta,x,t) = \frac{iN_c}{2} \int \frac{dk^- d^2\mathbf{k}_\perp}{2(2\pi)^4} H_{\rm cov}(k,P) H'_{\rm cov}(k-\Delta, P-\Delta)$$
$$\times \frac{\mathrm{Tr}[\gamma_5(\not{k}+m)\gamma^+(\not{k}-\not{\Delta}+m)\gamma_5(-\not{P}+\not{k}+m)]}{[k^2-m^2+i\varepsilon][(k-\Delta)^2-m^2+i\varepsilon][(P-k)^2-m^2+i\varepsilon]}.$$

$$\mathcal{J}^\mu = iN_c \int \frac{d^4k}{(2\pi)^4} \frac{\delta(x-k^+/P^+) H_{\rm cov} H'_{\rm cov} S^\mu}{[k^2-m^2+i\epsilon][(k-\Delta)^2-m^2+i\epsilon][(P-k)^2-m^2+i\epsilon]}$$

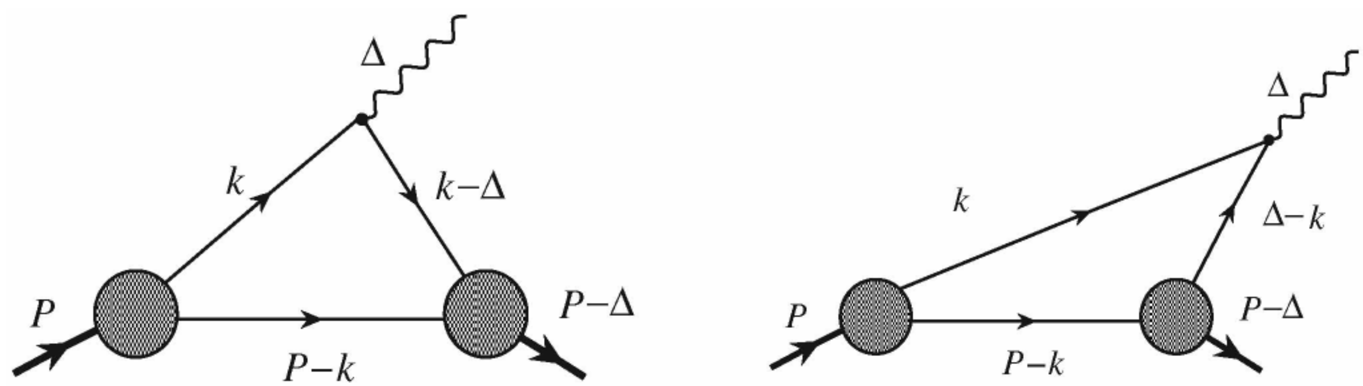
$$\mathcal{J}^+ \equiv \int \frac{dz^-}{4\pi} e^{ixP^+z^-/2} \langle P' | \bar{\psi}(0)\gamma^+\psi(z) | P \rangle \Big|_{z^+=\mathbf{z}_\perp=0}$$

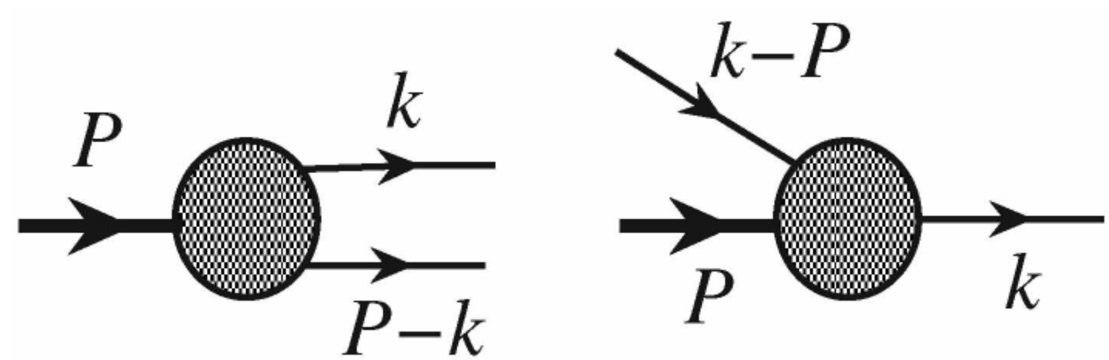


$$J^+(0) \equiv \langle P' | \bar{\psi}(0)\gamma^+\psi(0) | P \rangle = F_\pi(t)(P+P')^+$$

$$\int_0^1 dx \mathcal{F}_\pi(\zeta, x, t) = (1 - \zeta/2) F_\pi(t).$$

$$\int_0^1 dx x^{n-1} \mathcal{F}_\pi(\zeta, x, t) = (1 - \zeta/2) F_n(\zeta, t)$$





$$\mathcal{F}_{\pi}^{\text{val}}(\zeta, x, t) = \frac{N_c}{2P^+} \frac{1}{16\pi^3} \frac{1}{(1-x)xx'}$$

$$\times \int d^2\mathbf{k}_\perp \chi_{(2\to2)}(x, \mathbf{k}_\perp) S^+_{\text{val}} \chi'_{(2\to2)}(x', \mathbf{k}'_\perp),$$

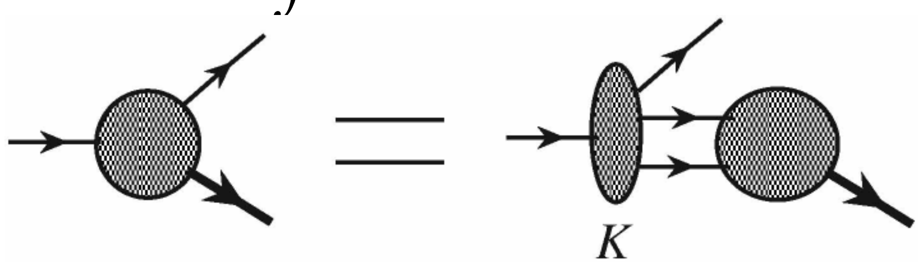
$$\mathcal{F}_{\pi}^{\text{nonval}}(\zeta, x, t) = \frac{N_c}{2P^+} \frac{1}{16\pi^3} \frac{1}{x(1-x)(1-x'')}$$

$$\times \int d^2\mathbf{k}_\perp \chi_{(2\to2)}(x, \mathbf{k}_\perp) S^+_{\text{nonval}} \chi^g(x, \mathbf{k}''_\perp) h'_{\text{LF}},$$

$$(M^2 - M_0^2)\chi(x_i, \mathbf{k}_{i\perp}) = \int [dy][d^2\mathbf{l}_\perp] \mathcal{K}(x_i, \mathbf{k}_{i\perp}; y_j, \mathbf{l}_{j\perp}) \chi(y_j, \mathbf{l}_{j\perp})$$

$$\chi_{(2\to2)} = \chi^{\text{val}}, \qquad \chi_{(1\to3)} = \chi^{\text{nonval}}.$$

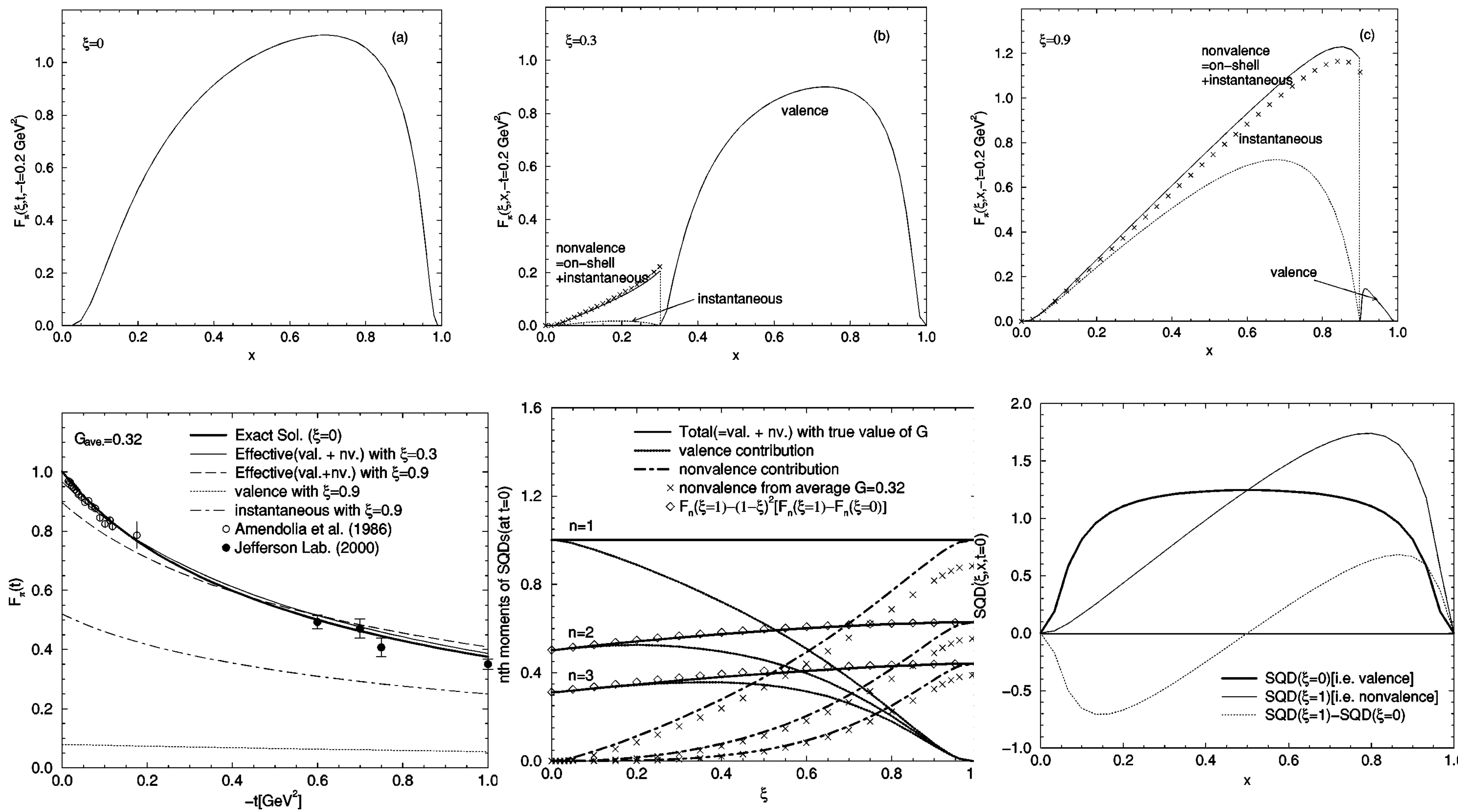
$$(M^2 - M_0^2)\chi_{(1\to3)}(x_i, \mathbf{k}_{i\perp}) = \int [dy][d^2\mathbf{l}_\perp] \mathcal{K}(x_i, \mathbf{k}_{i\perp}; y_j, \mathbf{l}_{j\perp})\, \chi_{(2\to2)}(y_j, \mathbf{l}_{j\perp})$$



$$\mathcal{F}_{\pi}^{\text{nonval}}(\zeta, x, t) = \frac{N_c}{2P^+}\frac{1}{16\pi^3}\frac{1}{x(1-x)(1-x'')} \times \int d^2\mathbf{k}_\perp \chi_{(2\to2)}(x, \mathbf{k}_\perp) S^+_{\text{nonval}} \chi^g(x, \mathbf{k}''_\perp) \times \int \frac{dy}{y(1-y)} \times \int d^2\mathbf{l}_\perp \mathcal{K}(x, \mathbf{k}_\perp; y, \mathbf{l}_\perp) \chi_{(2\to2)}(y, \mathbf{l}_\perp).$$

$$\chi^g(x,\mathbf{k}''_\perp) = \frac{1}{(1-\xi)\left[\frac{\Delta^2}{\xi} - \left(\frac{\mathbf{k}''^2_\perp + m^2}{x} + \frac{\mathbf{k}''^2_\perp + m^2}{\xi - x}\right)\right]}$$

$$\chi_{(2\to2)}(x,\mathbf{k}_\perp) = \left(\frac{8\pi^3}{N_c}\right)^{1/2}\left(\frac{\partial k_z}{\partial x}\right)^{1/2}\frac{[x(1-x)]^{1/2}}{M_0}\phi(x,\mathbf{k}_\perp)$$



The *D* term generates the highest power in the polynomial, and *D* term is effectively included in the ERBL region.

$$\mathcal{M}_{\gamma^*\pi\to\gamma\pi} \sim \int_0^1 dx \, \frac{F_\pi(x,\xi,t)}{x-\xi+i\epsilon}$$

$$= \underbrace{\mathcal{P}\int_0^1 dx \, \frac{F_\pi(x,\xi,t)}{x-\xi}}_{\shortparallel} + i\pi F_\pi(\xi,\xi,t)$$

$$\lim_{\epsilon\to 0}\left[\int_0^{\xi-\epsilon} dx \, \frac{F_\pi(x,\xi,t)}{x-\xi} + \int_{\xi+\epsilon}^1 dx \, \frac{F_\pi(x,\xi,t)}{x-\xi}\right]$$

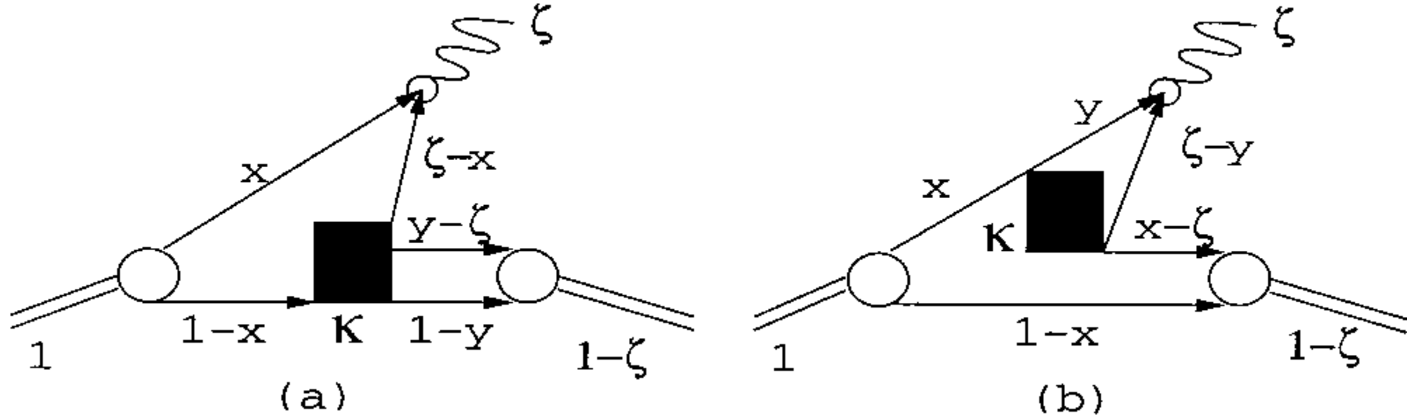
$$\int$$

$$\log\frac{1}{\epsilon}\cdot\left[F_\pi(\xi+\epsilon,\xi,t) - F_\pi(\xi-\epsilon,\xi,t)\right]$$

+ finite result

- If $F_\pi(\xi_+) \neq F_\pi(\xi_-)$, then the real part blows up logarithmically as $\epsilon \to 0$.
- The imaginary part is given by the value of $F_\pi(\xi,\xi,t)$. $\longrightarrow$ SSA (Hermes, CLAS)

Consistent treatment of gauge-boson and meson vertex functions in LFQM: Update



$$\int_0^\zeta dx \mathcal{F}^{\rm nv}_{\pi(a)}(\zeta,x,t) = \int_0^\zeta dx \int d^2\mathbf{k}_\perp \int_\zeta^1 dy \int d^2\mathbf{l}_\perp \chi^{\rm f}(y,\mathbf{l}_\perp) \times \mathcal{K}_a(x,\mathbf{k}_\perp;y,\mathbf{l}_\perp)\chi^g(x,\mathbf{k}_\perp)S^+_{nv}(x,\mathbf{k}_\perp)\widetilde{\chi}^{\rm i}_a(x,\mathbf{k}_\perp),$$

$$\int_0^\zeta dy \mathcal{F}^{\rm nv}_{\pi(b)}(\zeta,y,t) = \int_0^\zeta dy \int d^2\mathbf{l}_\perp \int_\zeta^1 dx \int d^2\mathbf{k}_\perp \chi^{\rm f}(x,\mathbf{k}_\perp) \times \mathcal{K}_b(y,\mathbf{l}_\perp;x,\mathbf{k}_\perp)\chi^g(y,\mathbf{l}_\perp)S^+_{nv}(x,\mathbf{k}_\perp)\widetilde{\chi}^{\rm i}_b(x,\mathbf{k}_\perp),$$

$$\mathcal{F}_{\pi}^{nv}(\zeta,x,t)=\frac{-N_c}{2x(1-x)(1-x'')}$$

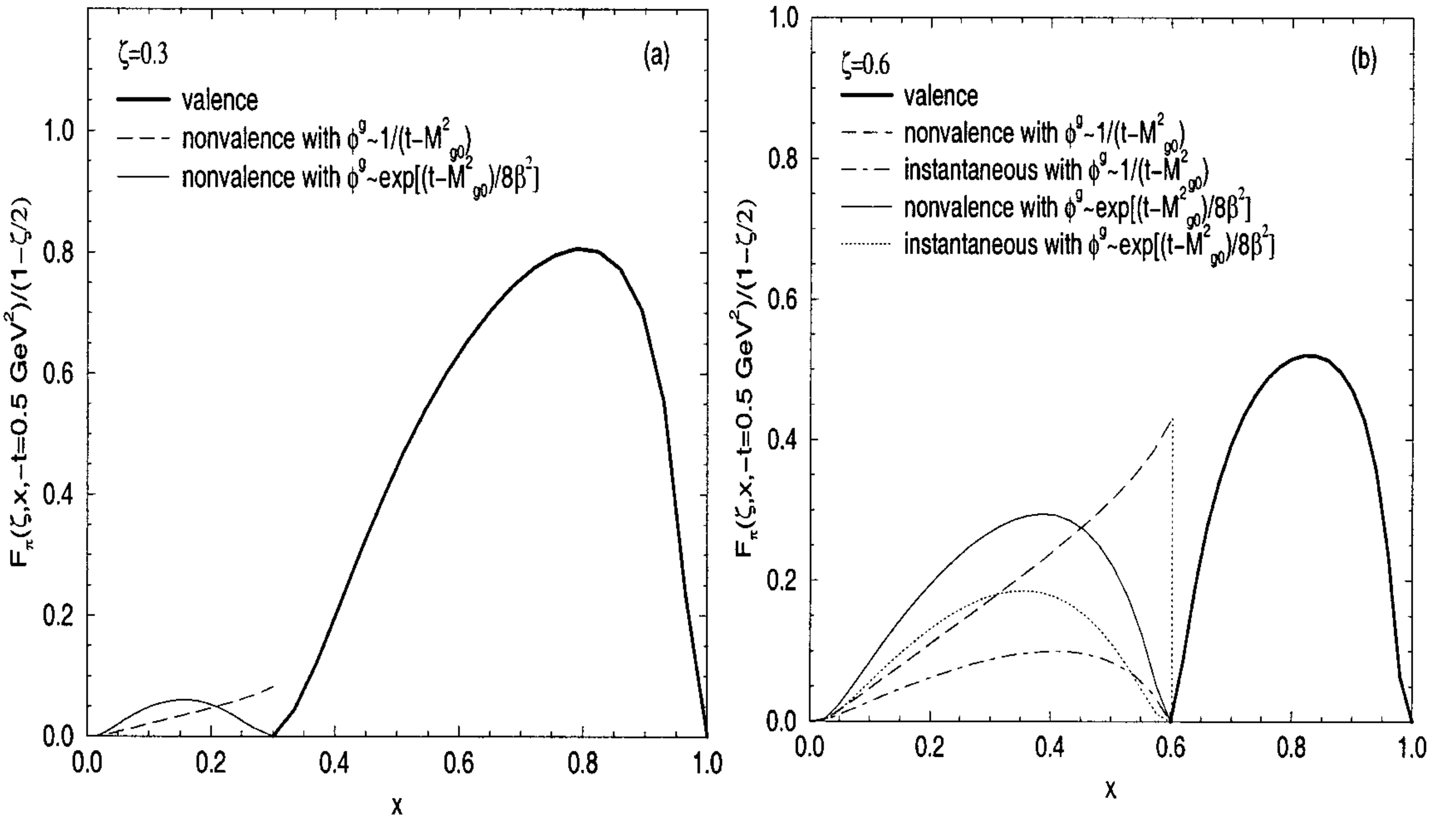
$$\times\int\frac{d^2\mathbf{k}_{\perp}}{16\pi^3}\chi_{(2\to 2)}(x,\mathbf{k}_{\perp})S^{+}_{nv}(x,\mathbf{k}_{\perp})\chi^{g}(x'',\mathbf{k}''_{\perp})$$

$$\times\int_0^1\frac{dy'}{y'(1-y')}\int d^2\mathbf{l}_{\perp}\tilde{\mathcal{K}}(x',\mathbf{k}'_{\perp};y',\mathbf{l}'_{\perp})\,\chi'_{(2\to 2)}(y',\mathbf{l}'_{\perp})$$

$$\tilde{\mathcal{K}}(x',\mathbf{k}'_{\perp};y',\mathbf{l}'_{\perp})\equiv\mathcal{K}(x',\mathbf{k}'_{\perp};y',\mathbf{l}'_{\perp})\left[1-\frac{S^{+}_{\mathrm{nonval}}(y,\mathbf{l}_{\perp})\tilde{\chi}^{\mathrm{i}}_{b}(y,\mathbf{l}_{\perp})}{S^{+}_{\mathrm{nonval}}(x,\mathbf{k}_{\perp})\tilde{\chi}^{\mathrm{i}}_{a}(x,\mathbf{k}_{\perp})}\right],$$

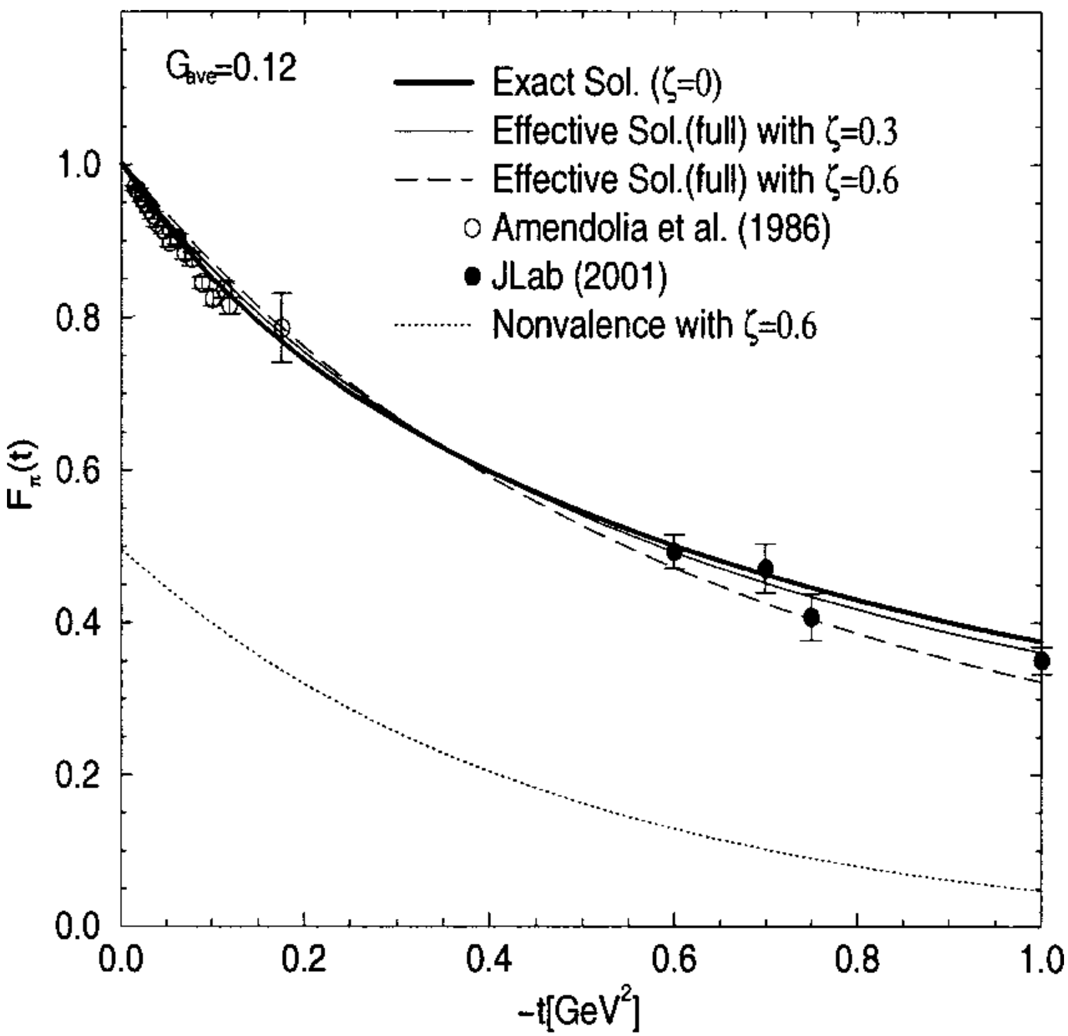
$$\chi_{(2\to 2)}(x,\mathbf{k}_{\perp})=\sqrt{\frac{8\pi^3}{N_c}}\sqrt{\frac{\partial k_z}{\partial x}}\frac{[x(1-x)]^{1/2}}{M_0}\phi(x,\mathbf{k}_{\perp}),$$

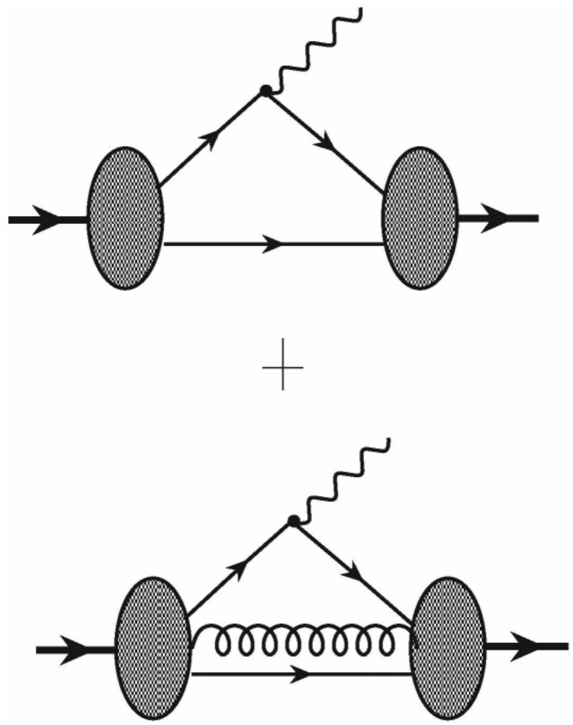
$$\chi^{g}(x'',\mathbf{k}''_{\perp})=\sqrt{\frac{8\pi^3}{N_c}}\sqrt{\frac{\partial k''_z}{\partial x''}}\frac{[x''(1-x'')]^{1/2}}{M''_0}\phi^{g}(x'',\mathbf{k}''_{\perp}),$$

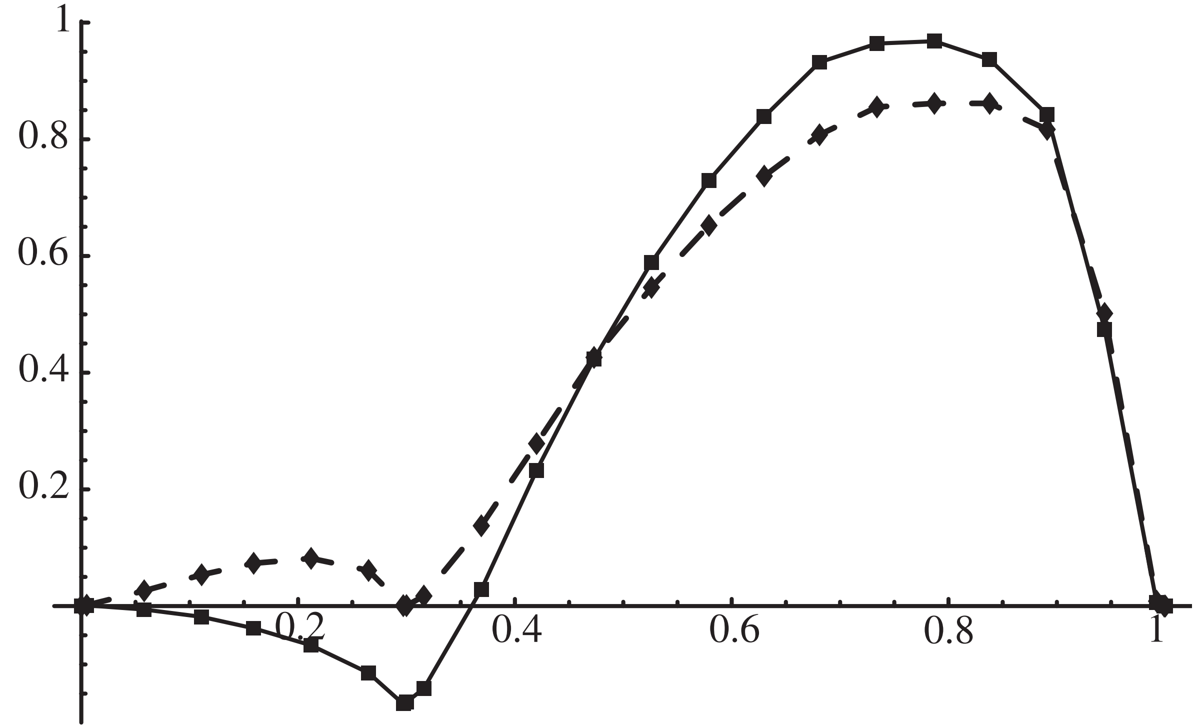


ζ=0.3
(a)
valence
nonvalence with φ^g~1/(t−M^2_g0)
nonvalence with φ^g~exp[(t−M^2_g0)/8β^2]
F_π(ζ,x,−t=0.5 GeV^2)/(1−ζ/2)
x
ζ=0.6
(b)
valence
nonvalence with φ^g~1/(t−M^2_g0)
instantaneous with φ^g~1/(t−M^2_g0)
nonvalence with φ^g~exp[(t−M^2_g0)/8β^2]
instantaneous with φ^g~exp[(t−M^2_g0)/8β^2]
F_π(ζ,x,−t=0.5 GeV^2)/(1−ζ/2)
x

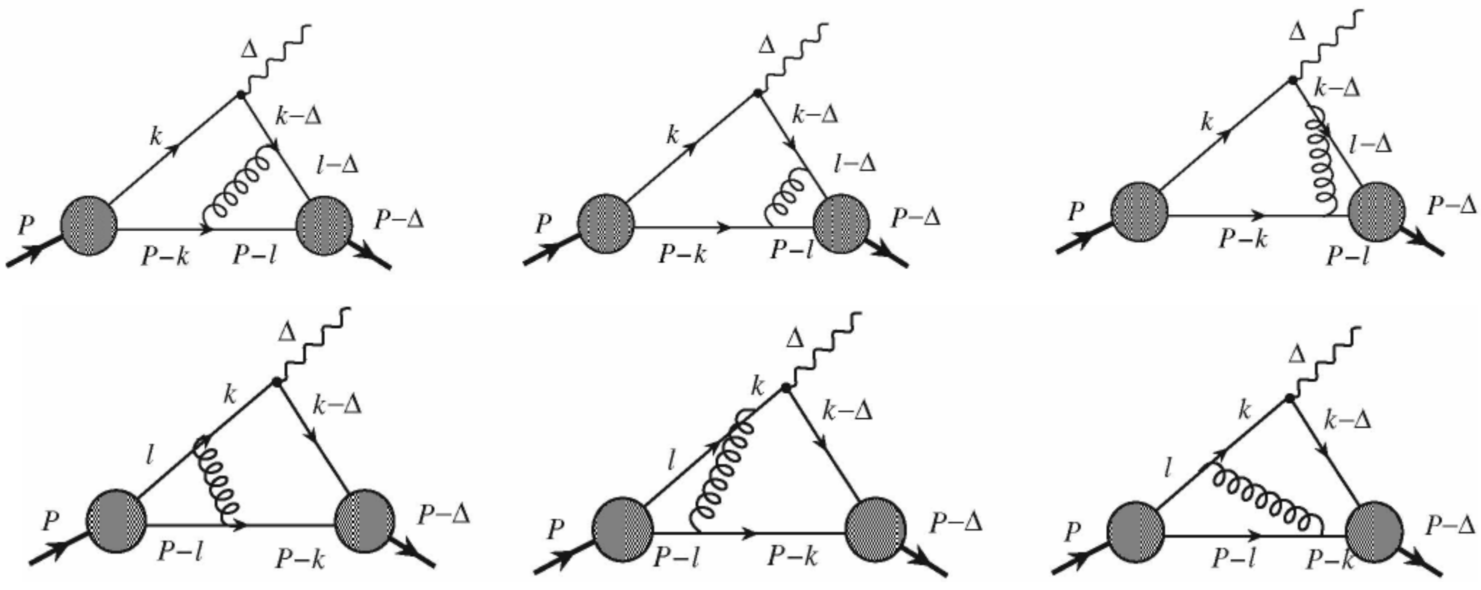
$$F_\pi(t) = \int_\zeta^1 \frac{dx}{1-\zeta/2}\mathcal{F}_\pi^{val}(\zeta,x,t) + \int_0^\zeta \frac{dx}{1-\zeta/2}\mathcal{F}_\pi^{nv}(\zeta,x,t)$$







1
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1



Δ
k
k−Δ
l−Δ
P
P−k
P−l
P−Δ
Δ
k
k−Δ
l−Δ
P
P−k
P−l
P−Δ
Δ
k
k−Δ
l−Δ
P
P−k
P−l
P−Δ
Δ
k
k−Δ
l
P
P−l
P−k
P−Δ
Δ
k
k−Δ
l
P
P−l
P−k
P−Δ
Δ
k
k−Δ
l
P
P−l
P−k
P−Δ

Conclusion and Outlook

- EMT of hadron provides essential properties of hadron such as mass, spin, pressure and shear modulus, etc.
- D-term carries the elastic property of hadron and gives an access to the details of strong forces inside the hadron.
- GPDs may be utilized to provide the EMT information of hadrons although extraction of GPDs from data requires severe constraints in the kinematic region of data collection.

- Much more careful study of GPDs is required in the interpretation of experimental data.
- BSA indicates the imaginary part of GPDs or general CFFs.
- GPDs at the crossover point between DGLAP and ERBL regions must be continuous and analytic. Otherwise, the real amplitude would blow up logarithmically divergence.
- To yield BSA, GPDs at the crossover point must be nonzero.

- D term is effectively provided by the ERBL region.
- Reduction of GPDs from the most general hadronic tensor amplitudes can be used to benchmark the kinematic constraints of GPDs formulation.
- Evolution of GPDs should be also carefully investigated: See arXiv: 2104.03836v1 about the deconvolution problem of deeply virtual Compton scattering by V. Bertone, H. Dutrieux,C. Mezrag,H. Moutarde, and P. Sznajder.