Complementarity of experimental and lattice QCD data on pion parton distributions

Patrick Barry

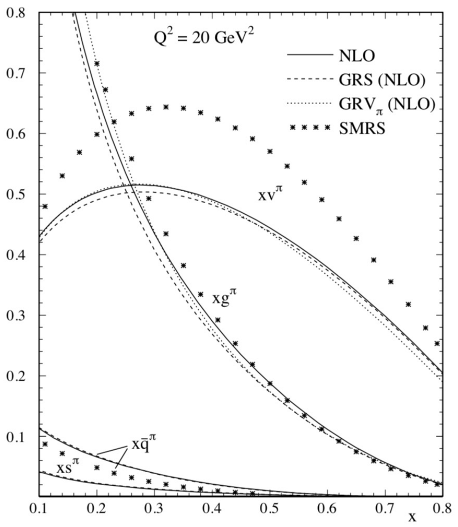
NCSU Group Meeting 2/18/2022

Background and Motivation

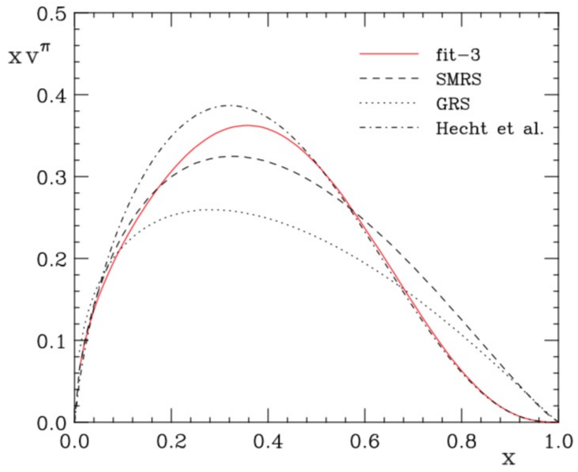
Pion structure

- Historically, pion distributions have been extracted from fixed target πA data
  - Drell-Yan (DY) $\pi A \to \mu^+\mu^- X$
  - Prompt photon $\pi A \to \gamma X$

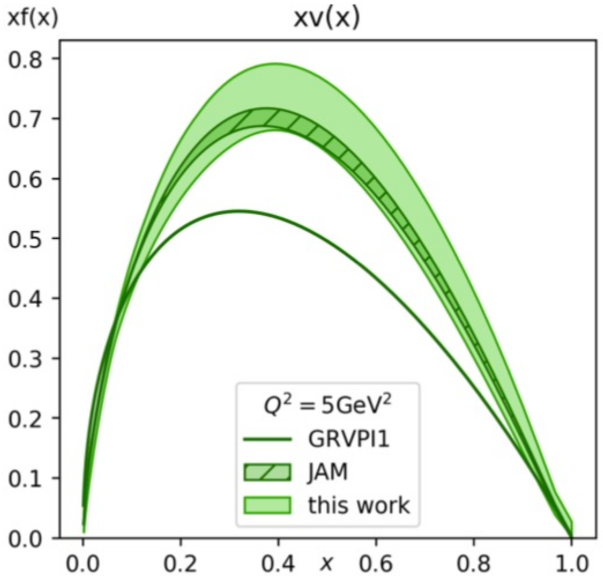
Owens attempted to use J/ψ production from πA scattering using CEM



GRS, GRV, and SMRS



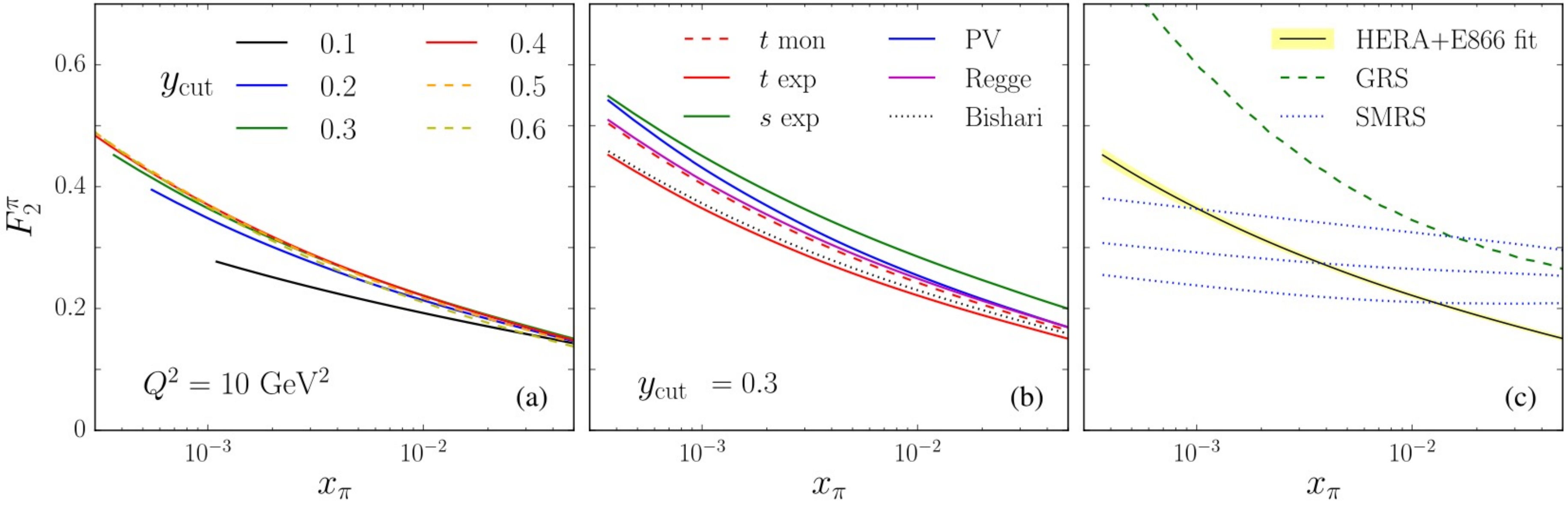
ASV valence PDF



xFitter

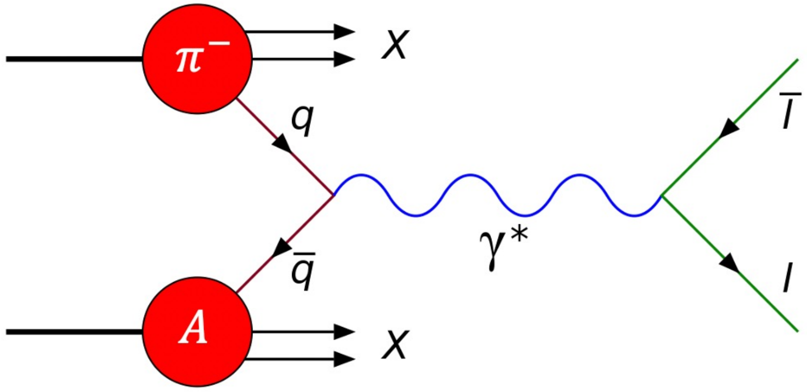
Introduction of leading neutron

- Description of leading neutron (LN) data through Sullivan process
- First phenomenological study on pion structure functions using by McKenney, et al.

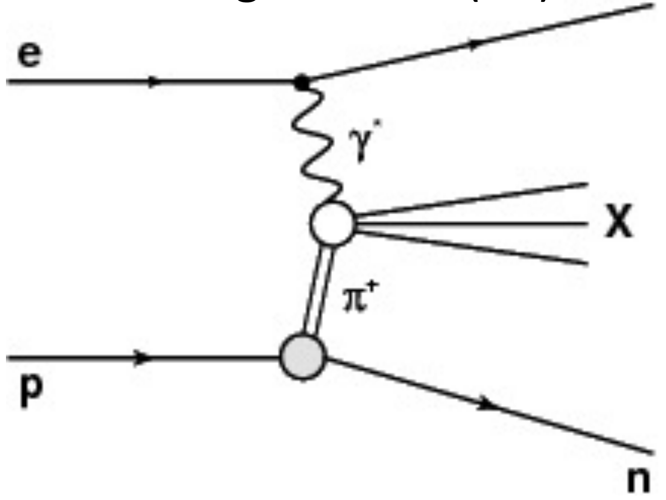


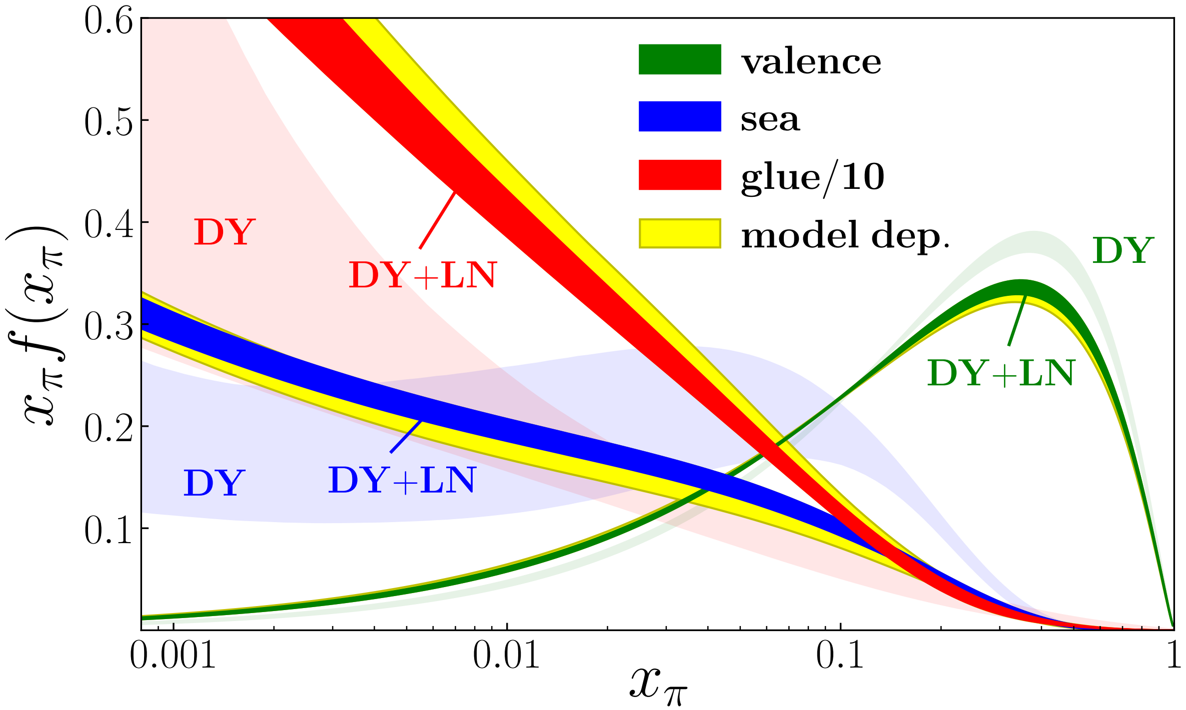
Experiments to probe pion structure

Drell-Yan (DY)



Leading Neutron (LN)





PHYSICAL REVIEW LETTERS **121,** 152001 (2018)

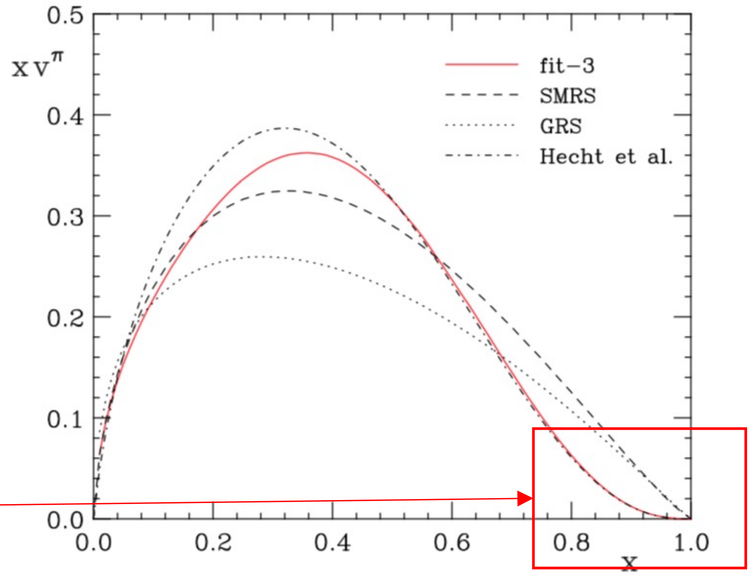
Featured in Physics

First Monte Carlo Global QCD Analysis of Pion Parton Distributions

P. C. Barry,[1] N. Sato,[2] W. Melnitchouk,[3] and Chueng-Ryong Ji[1]

Large-x_π behavior

- Generally, the parametrization lends a behavior as $x_\pi \rightarrow 1$ of the valence quark PDF of $q_v(x) \propto (1-x)^\beta$
- For a fixed order analysis, we find $\beta \approx 1$
- Debate whether $\beta = 1$ or $\beta = 2$
- Aicher, Schaefer Vogelsang (ASV) found $\beta = 2$ with threshold resummation



ASV valence PDF

Phys. Rev. Lett. **105**, 114023 (2011).

Counting Rules predict large-x_π behavior

Testing Quark Counting Rules

1. Structure functions

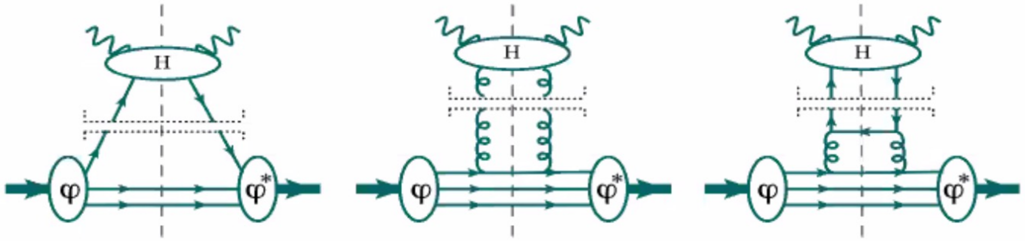
Threshold limit in DIS ⇨ x →1
Proved for exclusive and inclusive *processes*

Brodsky and Farrar, PRL31 and PRD11
Ezawa, Nuovo Cim. A23
Berger and Brodsky, PRL42
Soper, PRD15

$$F_2(x_B) \xrightarrow[x_B \to 1]{} (1 - x_B)^{2p-1+2|\lambda_q - \lambda_A|}$$

p=#spectators
λ_q & λ_A =helicities of active quark and target

2. Unpolarized PDFs



⇦ extended from SF — without λ

$$f(x) \xrightarrow[x_B \to 1]{} (1 - x_B)^{n=3} \qquad (1 - x_B)^{n>4} \qquad (1 - x_B)^{n>5}$$

Complementary testing to first principles
Scale dependent

Other claims

Controversy over pion valence DF

- Parton model prediction for the valence-quark DF of a spin-zero meson:

$$x \simeq 1 \Rightarrow q^{\pi}(x;\zeta_{\mathrm{H}}) \propto (1-x)^2$$

- The hadronic scale is not empirically accessible in Drell-Yan or DIS processes.
  (Matter of conditions necessary for data to be interpreted in terms of distribution functions.)

- For such processes, QCD-improvement of parton model leads to the following statement:
  At any scale for which experiment can be interpreted in terms of parton distributions, then

$$x \simeq 1 \Rightarrow q^{\pi}(x;\zeta) \propto (1-x)^{\beta=2+\gamma}, \gamma > 0$$

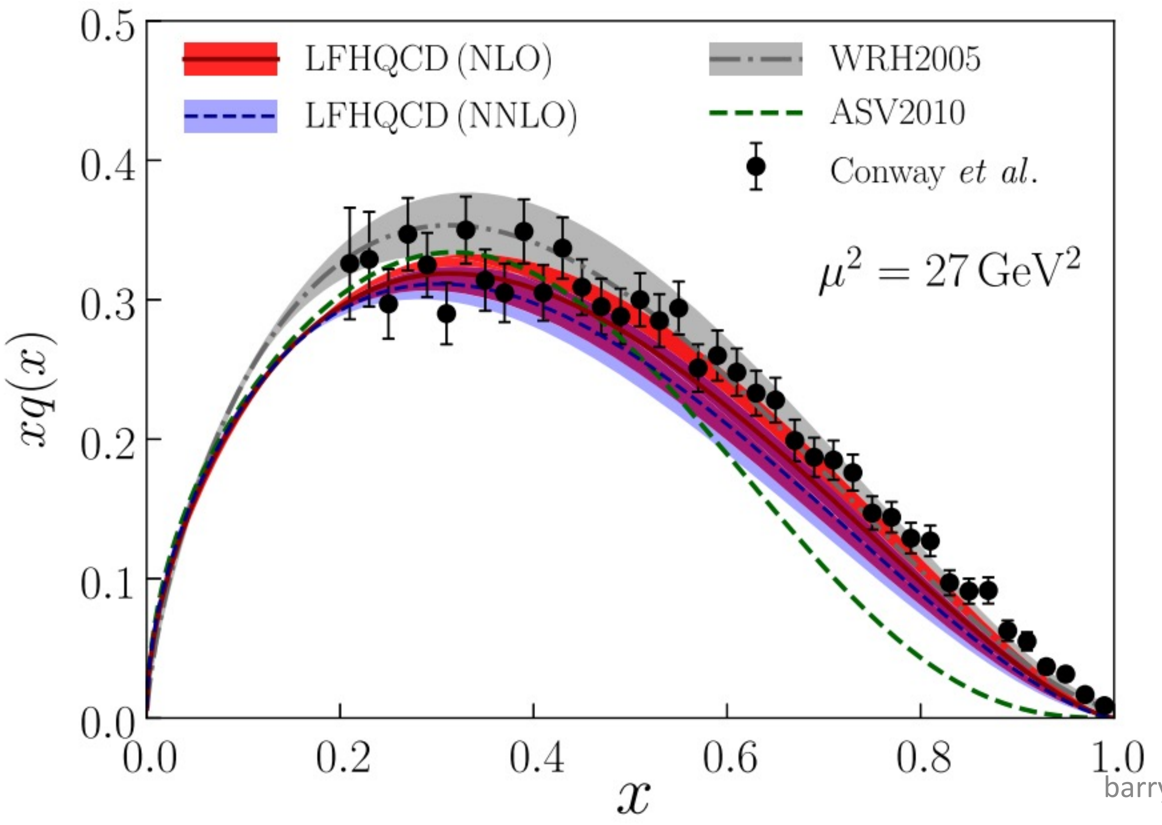
- Consequence
  - Any analysis of DY or DIS (or similar) experiment which returns a value of β <2 conflicts with QCD.

Light-Front Holographic QCD

Universality of Generalized Parton Distributions in Light-Front Holographic QCD

Guy F. de Téramond,[1] Tianbo Liu,[2,3] Raza Sabbir Sufian,[2] Hans Günter Dosch,[4] Stanley J. Brodsky,[5] and Alexandre Deur[2]

(HLFHS Collaboration)



Our results are in good agreement with the data analysis in Ref. [82] and consistent with the nucleon global fit results through the GPD universality described here. There is, however, a tension with the data analysis in [83] for $x \geq 0.6$ and with the Dyson-Schwinger results in [85], which incorporate the $(1-x)^2$ pQCD falloff at large x from hard gluon transfer to the spectator quarks. In contrast, our nonperturbative results falloff as $1-x$ from the leading twist-2 term in (20). A softer falloff $\sim(1-x)^{1.5}$ in Fig. 4 follows from DGLAP evolution. Our analysis incorporates the nonperturbative behavior of effective LFWFs in the limit of zero quark masses. However, if we include a nonzero quark mass in the LFWFs [28,86,87], the PDFs will be further suppressed at $x \to 1$.

Include Threshold Resummation in DY

- ASV analysis got $(1-x)^2$ behavior using threshold resummation, while all NLO analyses follow $(1-x)$

PHYSICAL REVIEW LETTERS **127,** 232001 (2021)

Global QCD Analysis of Pion Parton Distributions with Threshold Resummation

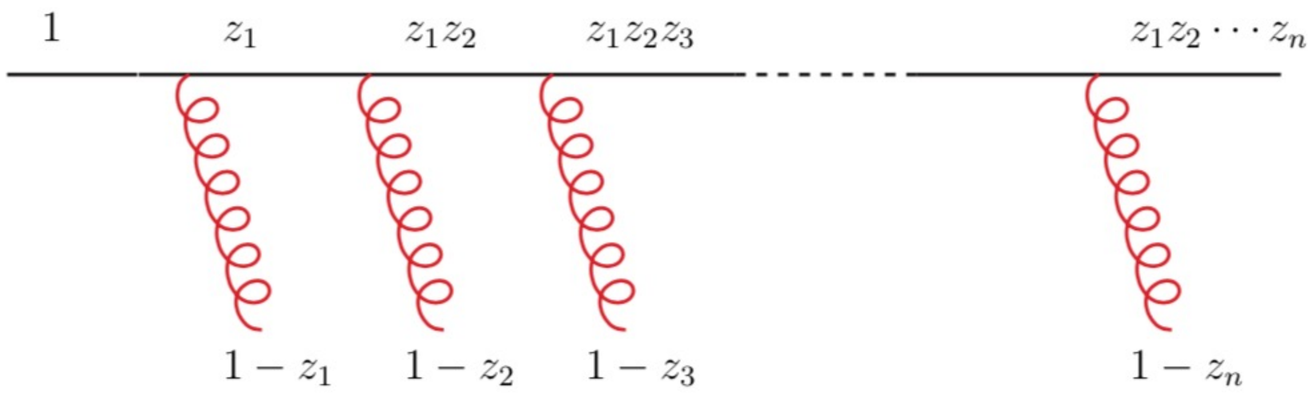
P. C. Barry,[1] Chueng-Ryong Ji,[2] N. Sato,[1] and W. Melnitchouk[1]

(JAM Collaboration)

[1]*Jefferson Lab, Newport News, Virginia 23606, USA*
[2]*Department of Physics, North Carolina State University, Raleigh, North Carolina 27695, USA*

(Received 18 August 2021; revised 22 October 2021; accepted 26 October 2021; published 29 November 2021)

Soft gluon resummation in DY



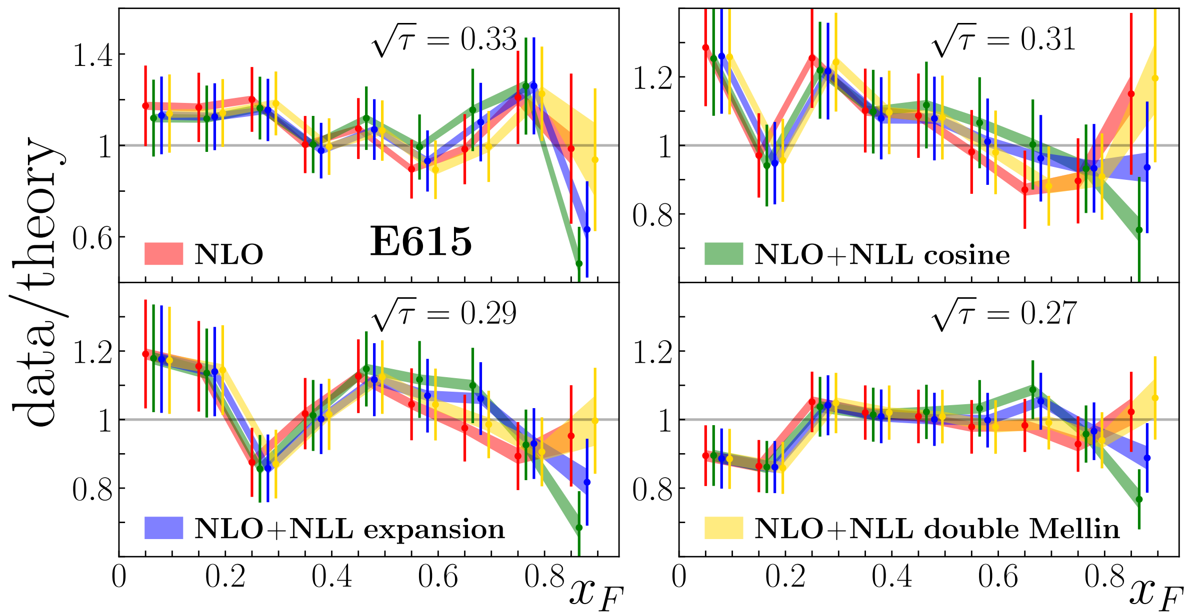
- Fixed-target Drell-Yan notoriously has large-x_F contamination of higher orders
- Large logarithms may spoil perturbation
- Focus on corrections to the most important $q\bar{q}$ channel
- Resum contributions to all orders of α_s

Methods of resummation

- Resummation is performed in conjugate space
- Drell-Yan data needs two transformations
- We can perform a Mellin-Fourier transform to account for the rapidity
  - A cosine appears while doing Fourier transform; options:
    1) Take first order expansion, cosine ≈ 1
    2) Keep cosine intact
- Can additionally perform a Double Mellin transform
- Explore the different methods and analyze effects
- Double Mellin transform is theoretically cleaner and sums up terms appropriately

Data and theory comparison

- Cosine method tends to overpredict the data at very large x_F
- Double Mellin method is qualitatively very similar to NLO
- Resummation is largely a high-x_F effect

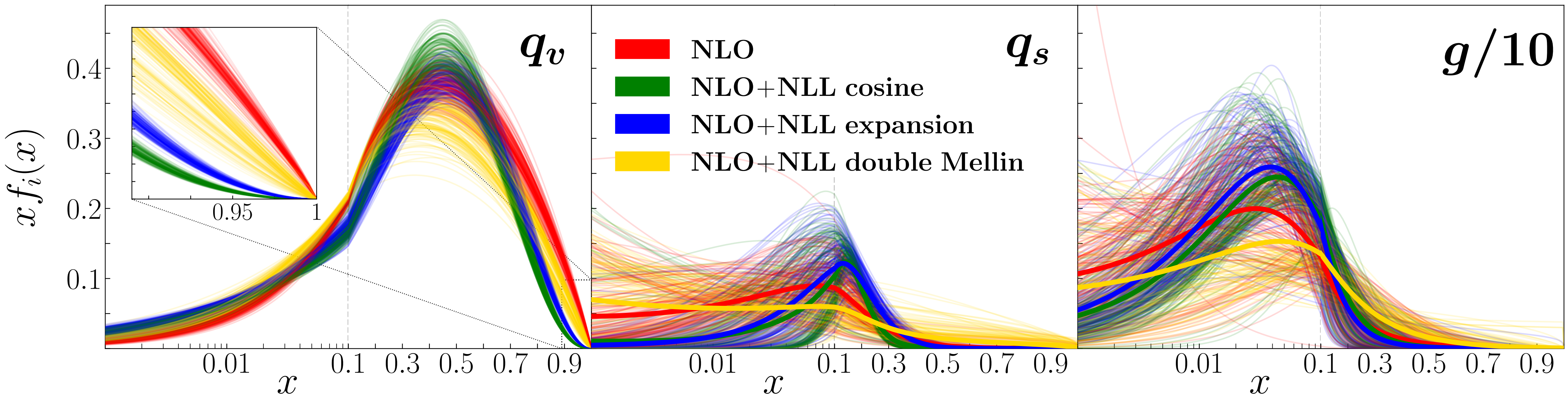


Current data do not distinguish between NLO and NLO+NLL

Method	χ^2/npts
NLO	0.85
NLO+NLL cosine	1.29
NLO+NLL expansion	0.95
NLO+NLL double Mellin	0.80

Slightly disfavored

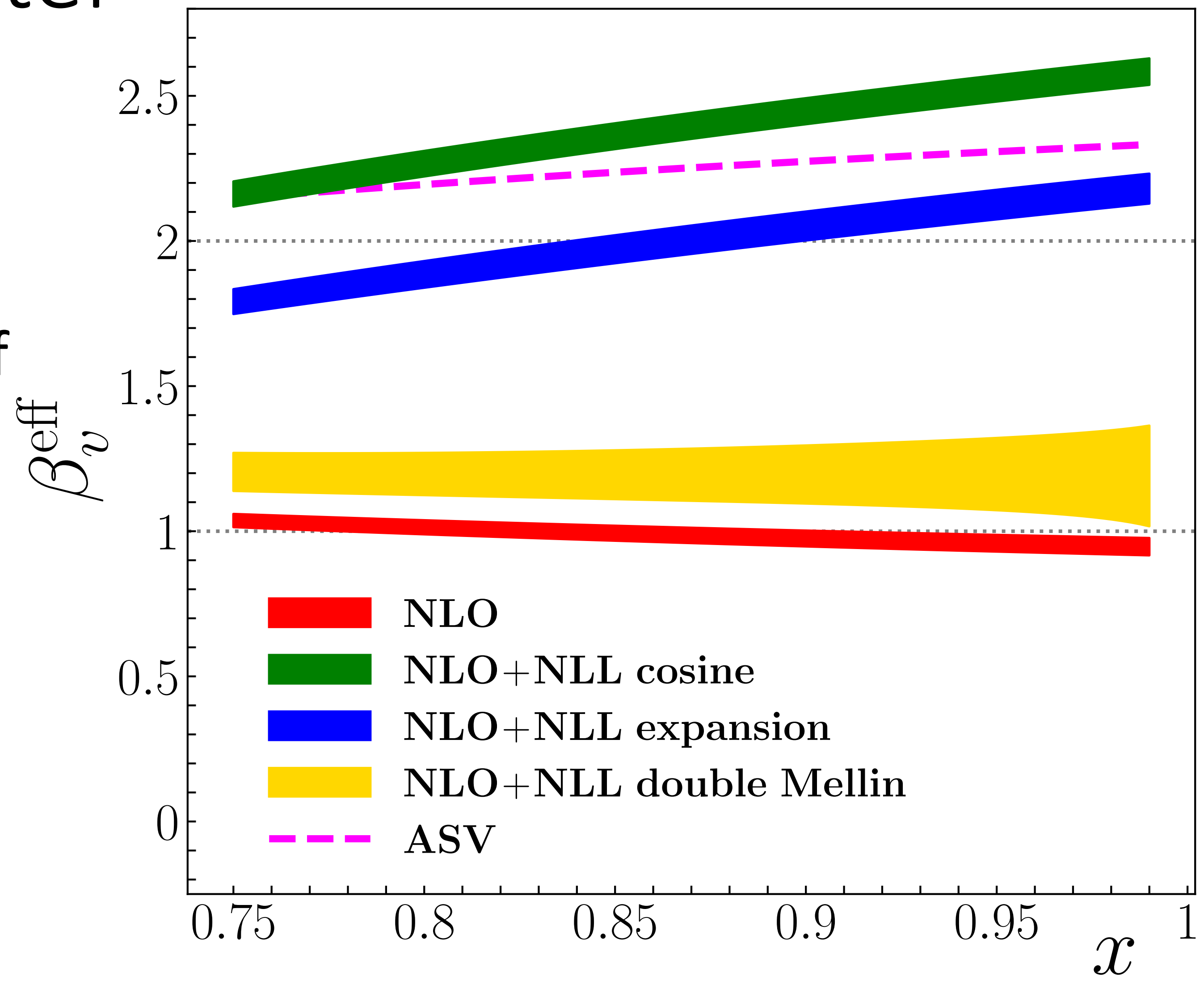
Resulting PDFs



- Large x behavior of q_v **highly sensitive** to method of resummation

Effective β_v parameter

- $q_v(x) \sim (1-x)^{\beta_v^{\text{eff}}}$ as $x \to 1$
- Threshold resummation does not give universal behavior of β_v^{eff}
- NLO and double Mellin give $\beta_v^{\text{eff}} \approx 1$
- Cosine and Expansion give $\beta_v^{\text{eff}} > 2$



Deriving resummation expressions – MF

Claim: yellow terms give rise to the resummation expressions

$$\frac{C_{q\bar{q}}}{e_q^2} = \delta(1-z)\,\frac{\delta(y)+\delta(1-y)}{2}\left[1+\frac{C_F\alpha_s}{\pi}\left(\frac{3}{2}\ln\frac{M^2}{\mu_f^2}+\frac{2\pi^2}{3}-4\right)\right]$$
$$+\frac{C_F\alpha_s}{\pi}\left\{\frac{\delta(y)+\delta(1-y)}{2}\left[(1+z^2)\left[\frac{1}{1-z}\ln\frac{M^2(1-z)^2}{\mu_f^2 z}\right]_+ +1-z\right]\right.$$
$$\left.+\frac{1}{2}\left[1+\frac{(1-z)^2}{z}y(1-y)\right]\left[\frac{1+z^2}{1-z}\left(\left[\frac{1}{y}\right]_+ +\left[\frac{1}{1-y}\right]_+\right)-2(1-z)\right]\right\}$$

Claim: Red terms are power suppressed in $(1-z)$ and wouldn't contribute to the same order as the yellow terms

Generalized Threshold resummation

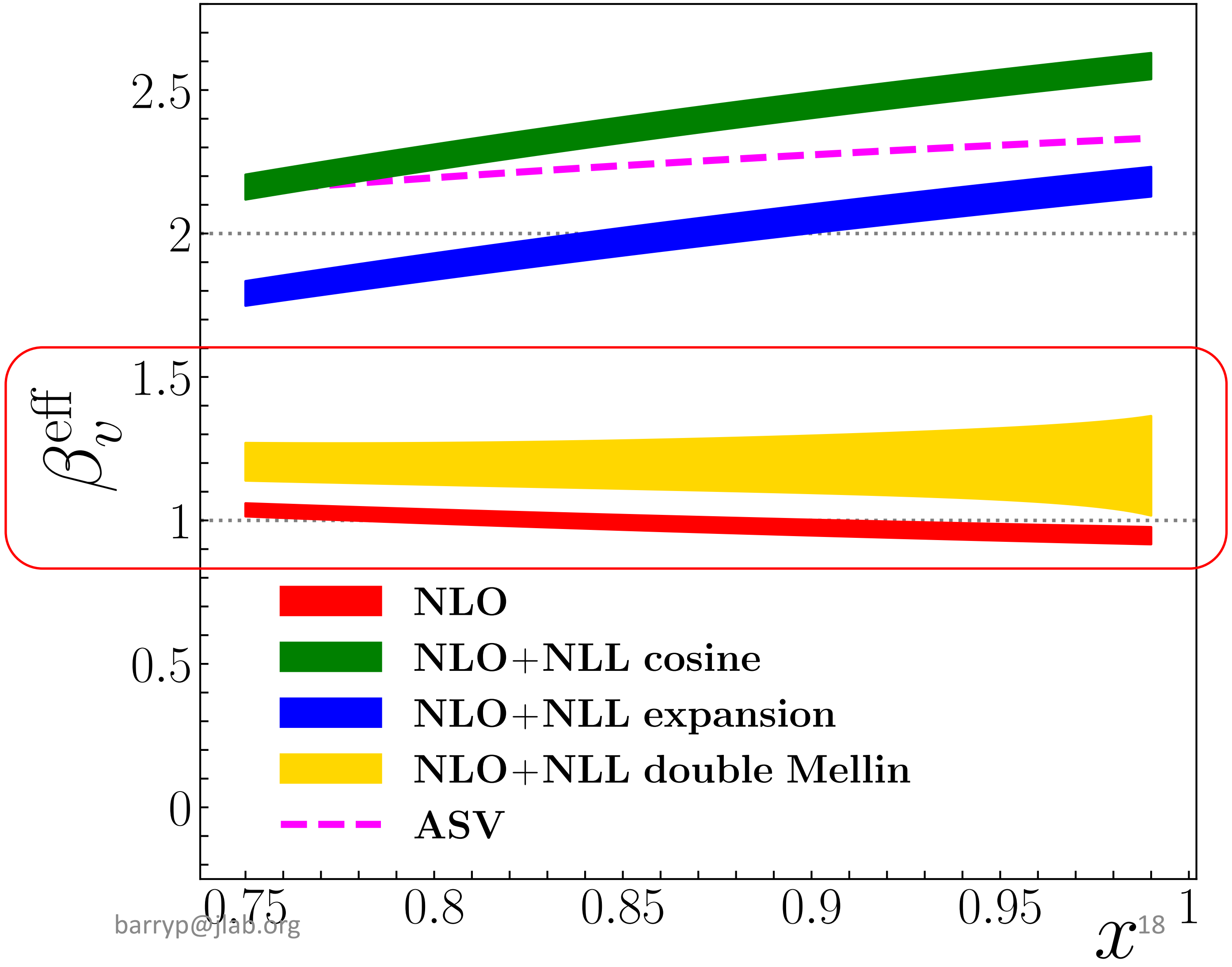
- Write the (z, y) coefficients in terms of (z_a, z_b), and for the red terms, you get:

$$\mathrm{d}z\,\mathrm{d}y\,\frac{1}{1-z}\left(\frac{1}{y}+\frac{1}{1-y}\right) = \mathrm{d}z_a \mathrm{d}z_b\,\frac{1}{(1-z_a)(1-z_b)}\left[1+\mathcal{O}(1-z_a, 1-z_b)\right].$$

- This is *not* power suppressed in $(1 - z_a)$ or $(1 - z_b)$ but instead the same order as the leading power in the soft limit
- Generalized threshold resummation in the soft limit does not agree with the MF methods

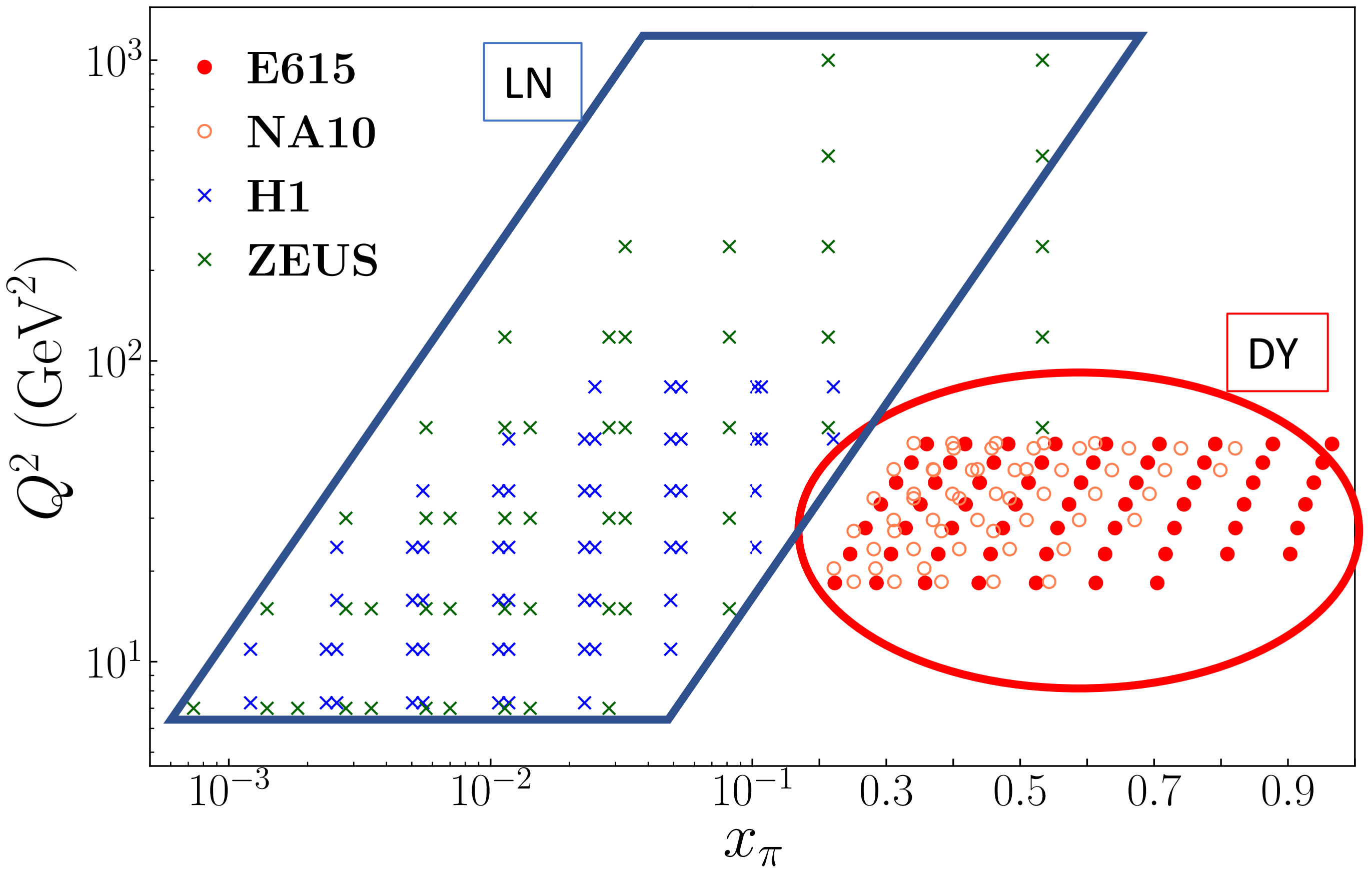
What we believe to be theoretically better

- Take more seriously the red and yellow
- $\beta_v^{\mathrm{eff}} \sim 1 - 1.2$, much closer to 1 than 2



Datasets -- Kinematics

- Not much data overlap
- Could be problems with factorization at high x_π - should we trust the data?
- Need more observables!



Lattice QCD observables

How to do it?

- Make use of good lattice cross sections and appropriate matching coefficients

$$\sigma_{n/h}(\omega, \xi^2) \equiv \langle h(p)|T\{\mathcal{O}_n(\xi)\}|h(p)\rangle$$
$$= \sum_i f_{i/h}(x, \mu^2) \otimes K_{n/i}(x\omega, \xi^2, \mu^2)$$
$$+ O(\xi^2 \Lambda^2_{\mathrm{QCD}}) \, ,$$

- Structure just like experimental cross sections – good for global analysis

Roadblocks

- Don’t have a definite answer to DY hard coefficients – NLO or NLO+NLL?
- Lattice QCD data intrinsically have systematic corrections associated with it that are *a priori* unknown
- Can we further distinguish?

Reduced pseudo-Ioffe time distributions

Observable

- Nonlocal matrix element of quark operators sandwiched between hadron states:

$$M^\alpha(p,z) \equiv \langle p|\, \bar{\psi}(0)\gamma^\alpha \mathcal{W}(z;0)\psi(z)\,|p\rangle$$

- When Fourier transformed and taking the $\alpha = +$ index, we recover the standard PDF

$$f_{q/A}(\xi) = \frac{1}{4\pi}\int \mathrm{d}x^- e^{-i\xi P^+ x^-} \langle P|\bar{\psi}(0,x^-,0_\perp)\,\gamma^+\,\mathcal{G}\,\psi(0,0,0_\perp)|P\rangle. \qquad (43)$$

What is done

"Ioffe time"

$\nu = p \cdot z$

- For generic z, α, and p, the Lorentz decomposition is

$$M^\alpha(z,p) = 2p^\alpha \mathcal{M}(\nu, z^2) + 2z^\alpha \mathcal{N}(\nu, z^2)$$

Ioffe time pseudo-distribution

- Lattice people will choose a *convenient* z, α to make calculation easier
- $z = (0,0,0,z_3)$ and $\alpha = 0$
- $M^0(z_3, p) = 2p^0 \mathcal{M}(\nu, z_3^2)$
- Then can extract the Ioffe time pseudo-distribution from calculated matrix elements

Observable

- Actual calculation is the **reduced** pseudo Ioffe time distribution (reduced pseudo-ITD)

$$\mathfrak{M}(\nu, z^2) = \frac{\mathcal{M}(\nu, z^2)}{\mathcal{M}(0, z^2)}$$

- The UV divergences arising from choosing the spacelike z cancel from taking the ratio at the rest frame $p_z = 0$ (light-like z does not have these divergences)
- Taking real part gives access to the valence quark distribution

Fitting the Data and Systematic Effects

$$\mathrm{Re}[\mathfrak{M}(\nu, z^2)] = \int_0^1 dx\, q_v(x, \mu_{\mathrm{lat}})\, \mathcal{C}^{\mathrm{Rp-ITD}}\left(x\nu, z^2, \mu_{\mathrm{lat}}\right) + z^2 B_1(\nu) + \frac{a}{|z|} P_1(\nu) + e^{-m_\pi (L-z)} F_1(\nu) + \ldots,$$

Valence quark distribution in pion

Wilson coefficients for matching

Systematic Effects to parametrize

- $z^2 B_1(\nu)$: power corrections
- $\frac{a}{|z|} P_1(\nu)$: lattice spacing errors
- $e^{-m_\pi (L-z)} F_1(\nu)$: finite volume corrections

Other potential systematic corrections the data is not sensitive to

Integration limits

- Notice the integral over x goes $0 \to 1$ – this is the case for general lattice matching
- However, the integral for experimental values goes from $x_{\mathrm{min}} \to 1$
- Because the sensitivity to threshold corrections to the short distance coefficient comes at large x where the PDF is sharply falling, the integration over the entire range of x is not sensitive to threshold regions
- Do not perform threshold resummation for lattice observables

Parametrizing the systematic effects

- Use a basis of Jacobi polynomials and Taylor expand

$$\mathrm{Re}B_1(\nu) = \sum_n \sigma_{0,n}(\nu)\, b_n,$$

$$\mathrm{Re}P_1(\nu) = \sum_n \sigma_{0,n}(\nu)\, p_n,$$

$$\mathrm{Re}F_1(\nu) = \sum_n \sigma_{0,n}(\nu)\, f_n,$$

$$\sigma_{0,n}(\nu) = \int_0^1 dx\; \cos(\nu x)\, x^a (1-x)^b\, J_n(x),$$

- Expanded b_n, p_n, f_n, which are free parameters in the fit

Begin at $n = 1$ to ensure at $\nu = 0$ the observable $== 1$

Current-current correlators

Current-current correlators

- Another type of observable from lattice currents (axial-vector)

$$\Sigma^{\mu\nu}_{VA}(z,p) = z^4 Z_V Z_A \langle p| \, [\bar{\psi}\gamma^\mu\psi](z)[\bar{\psi}\gamma^\nu\gamma^5\psi](0)|p\rangle + \big(V \leftrightarrow A\big),$$

- Where $Z_{V,A}$ are renormalization constants
- This can be expressed in two dimensionless quantities T_1 and T_2, which are functions of invariants ν and z^2
- Antisymmetric in $\mu \leftrightarrow \nu$
- Choosing the $\mu = 1$ and $\nu = 2$, we isolate T_1

Current-current correlator matching

$$T_1(\nu, z^2) = \int_0^1 \mathrm{d}x\, q_v(x, \mu_{\rm lat})\, \mathcal{C}^{\rm CC}(x\nu, z^2, \mu_{\rm lat}) + z^2 B_1(\nu) + aR_1(\nu) + \ldots,$$

- Very noisy data, fit a subset of the systematics to ensure PDF stability

$$R_1(\nu) = \sum_n \sigma_{0,n}(\nu)\, r_n,$$

- Sum starts at $n = 0$

Datasets available

Used in both Rp-ITD and CC correlators

ID	a (fm)	m_π (MeV)	β	$L^3 \times T$
a127m413	0.127(2)	413(4)	6.1	$24^3 \times 64$
a127m413L	0.127(2)	413(5)	6.1	$32^3 \times 96$
a94m358	0.094(1)	358(3)	6.3	$32^3 \times 64$
a94m278	0.094(1)	278(4)	6.3	$32^3 \times 64$

Scale setting/Methodology

Multiple scale problem

$$\sigma_{n/h}(\omega, \xi^2) \equiv \langle h(p)|T\{\mathcal{O}_n(\xi)\}|h(p)\rangle$$
$$= \sum_i f_{i/h}(x, \mu^2) \otimes K_{n/i}(x\omega, \xi^2, \mu^2)$$

- LHS (1st equation): Lattice QCD data are calculated using QCD and must be renormalized to the continuum limit and have renormalization constants – unlike experimental cross sections!!
  - Related with lattice spacing.
- RHS: two scales – renormalization scale to specify PDF, factorization scale to get hard coefficients

Convenient choice of scales

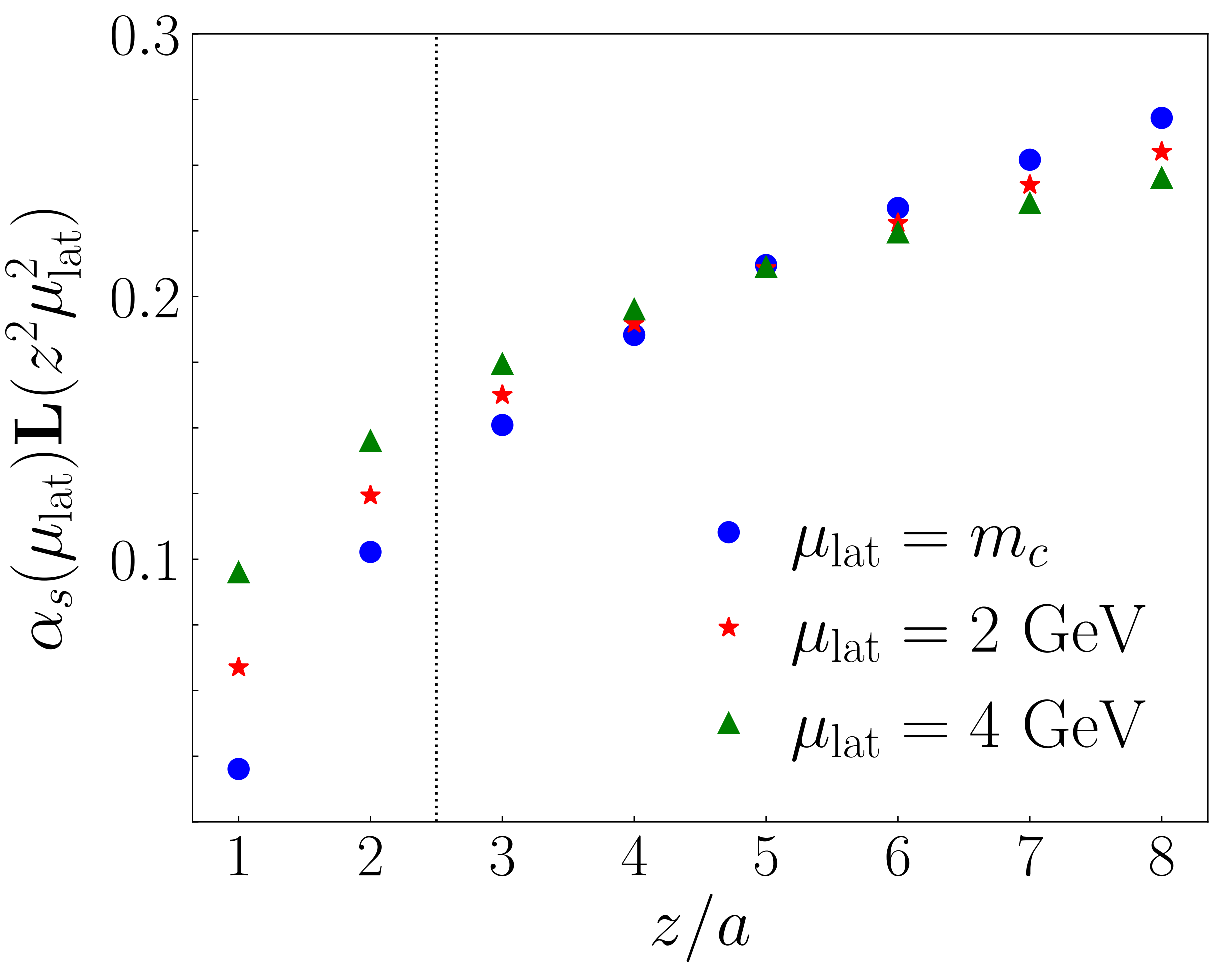
- Usually scales are chosen to ensure perturbative expansion is OK
- Hard coefficients for experimental cross sections usually have $\log(\frac{\mu^2}{Q^2})$, and a choice of $\mu = Q$ cancels the logs
- For Rp-ITD, terms like: $\log\left(z^2\mu^2 e^{2\gamma_E+1}/4\right)$
- CC: $\log(z^2\mu^2 e^{2\gamma_E}/4)$

Not so convenient for lattice data

- The values of z are so large, that the corresponding μ is below 1 GeV
- Equating $\mu_F = \mu_R$ would imply that $\alpha_S(\mu^2)$ is non-perturbative
- Alternative: set μ to be in a perturbative region and constant among all data

Perturbation expansion is OK

- At the expense of a small α_S, the product with the logarithm is under control
- Choose $\mu_{\text{lat}} = 2$ GeV unless otherwise specified



Methodology

Parametrization of PDFs

$$f(x,\mu_0^2) = \frac{N_f\, x^{\alpha_f}(1-x)^{\beta_f}(1+\gamma_f x^2)}{B(\alpha_f+2,\beta_f+1)+\gamma_f B(\alpha_f+4,\beta_f+1)},$$

Experimental data

$$\chi_{\mathrm{e}}^2(\boldsymbol{a},\mathrm{data}) = \sum_i \left[\frac{d_i^e - \sum_k r_k^e\, \beta_{k,i}^e - t_i^e(\boldsymbol{a})/n_e}{\alpha_i^e}\right]^2 + \left(\frac{1-n_e}{\delta n_e}\right)^2 + \sum_k \left(r_k^e\right)^2,$$

Lattice data

$$\chi_\lambda^2(\boldsymbol{a},\mathrm{data}) = \left(\boldsymbol{D}^\lambda - \boldsymbol{T}^\lambda(\boldsymbol{a})\right)^T V_\lambda^{-1}\left(\boldsymbol{D}^\lambda - \boldsymbol{T}^\lambda(\boldsymbol{a})\right).$$

Covariance matrix

Analysis Results

Reduced pseudo-ITD

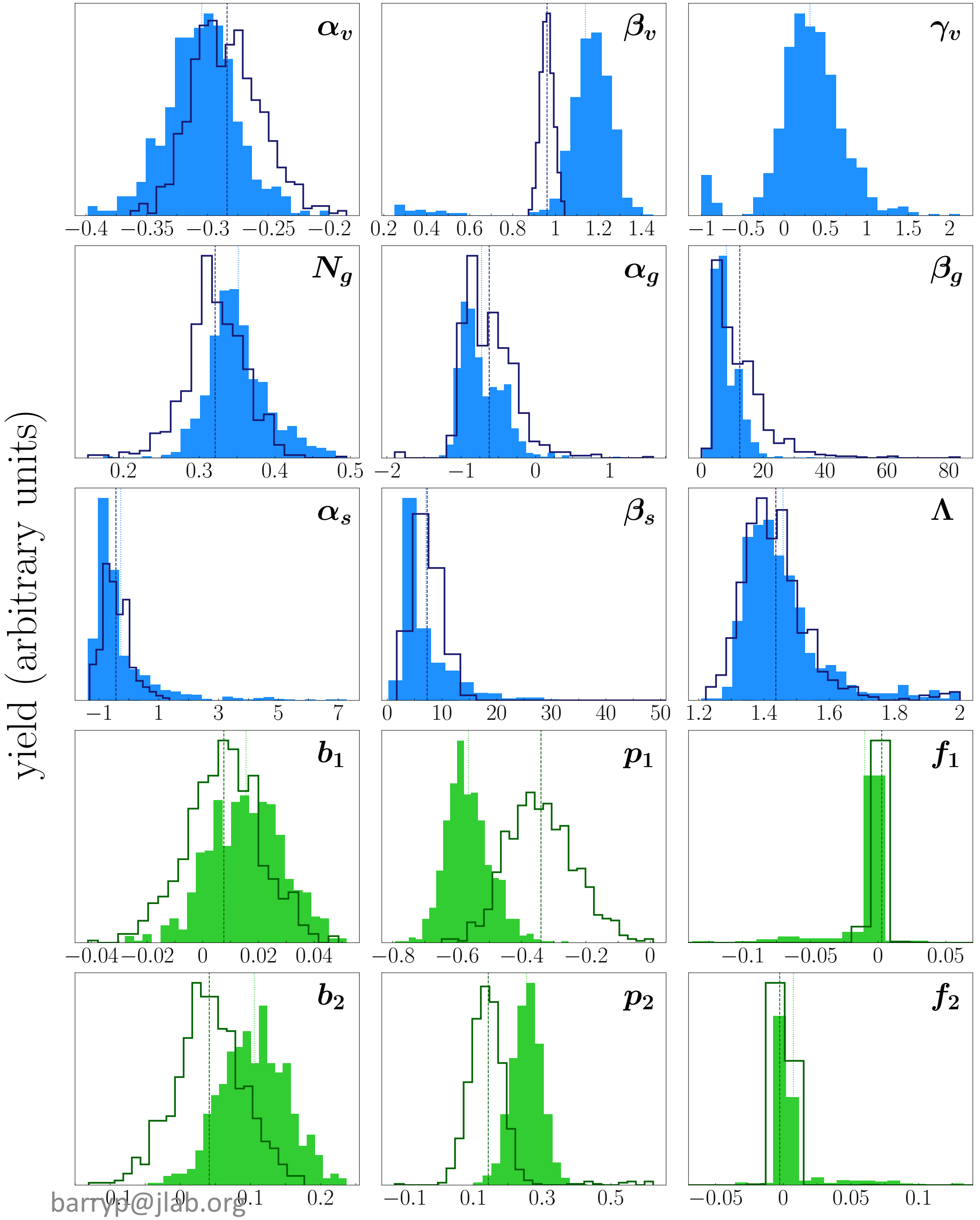
Goodness of fit

- Scenario A: experimental data alone
- Scenario B: experimental + lattice, no systematics
- Scenario C: experimental + lattice, with systematics

			Scenario A		Scenario B		Scenario C	
			NLO	+NLL$_{DY}$	NLO	+NLL$_{DY}$	NLO	+NLL$_{DY}$
Process	Experiment	N_{dat}	$\overline{\chi}^2$		$\overline{\chi}^2$		$\overline{\chi}^2$	
DY	E615	61	0.84	0.82	0.84	0.82	0.83	0.82
	NA10 (194 GeV)	36	0.53	0.53	0.52	0.54	0.53	0.55
	NA10 (286 GeV)	20	0.80	0.81	0.78	0.79	0.79	0.87
LN	H1	58	0.37	0.35	0.38	0.39	0.37	0.37
	ZEUS	50	1.49	1.48	1.60	1.69	1.59	1.60
Rp-ITD	a127m413L	18	–	–	1.05	1.06	1.05	1.06
	a127m413	8	–	–	1.97	2.63	1.15	1.42
Total		**251**	**0.81**	**0.80**	**0.89**	**0.92**	**0.86**	**0.87**

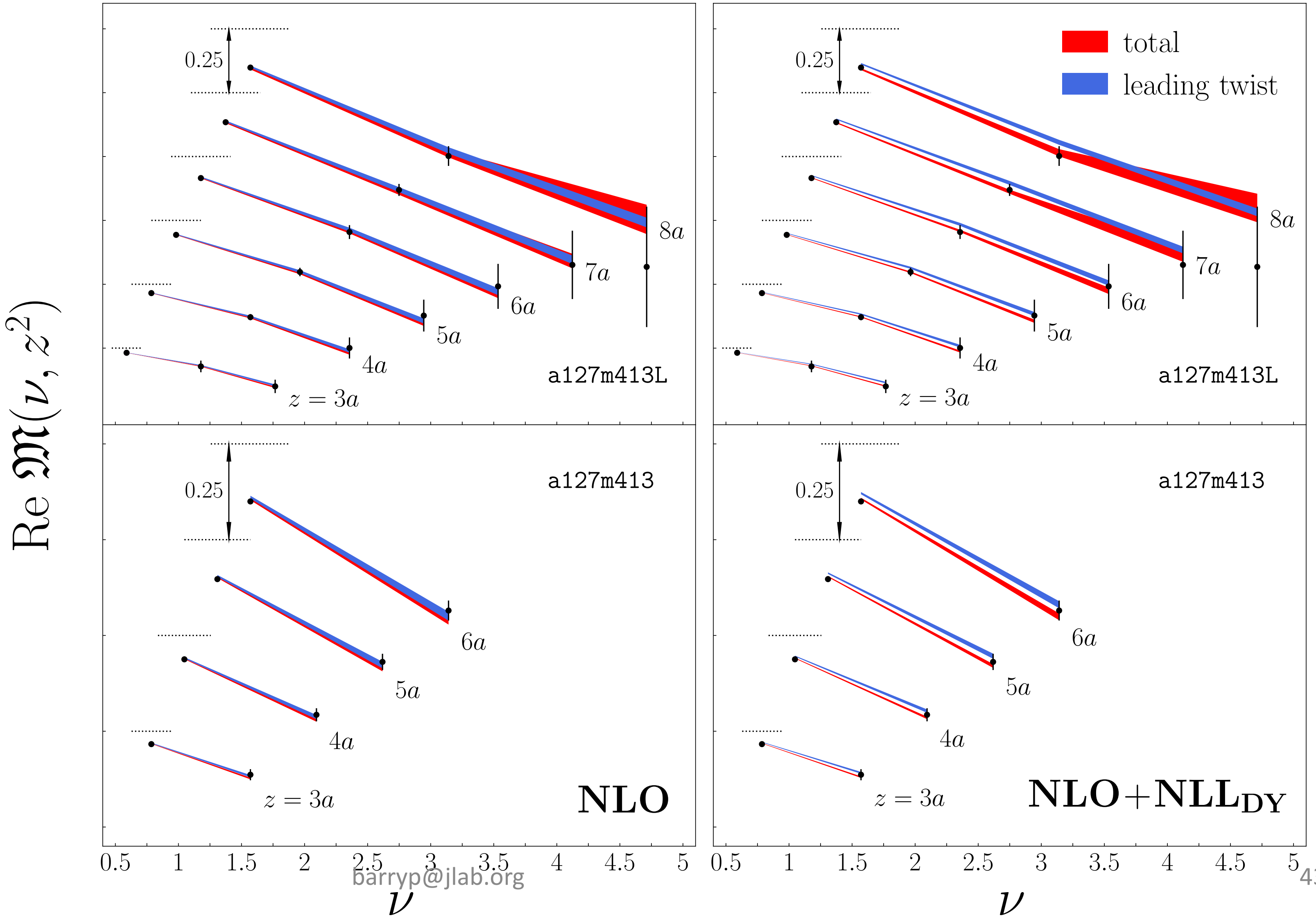
Histograms of parameters

- Outlined – NLO
- Filled – NLO+NLL$_{DY}$
- All distributions well peaked



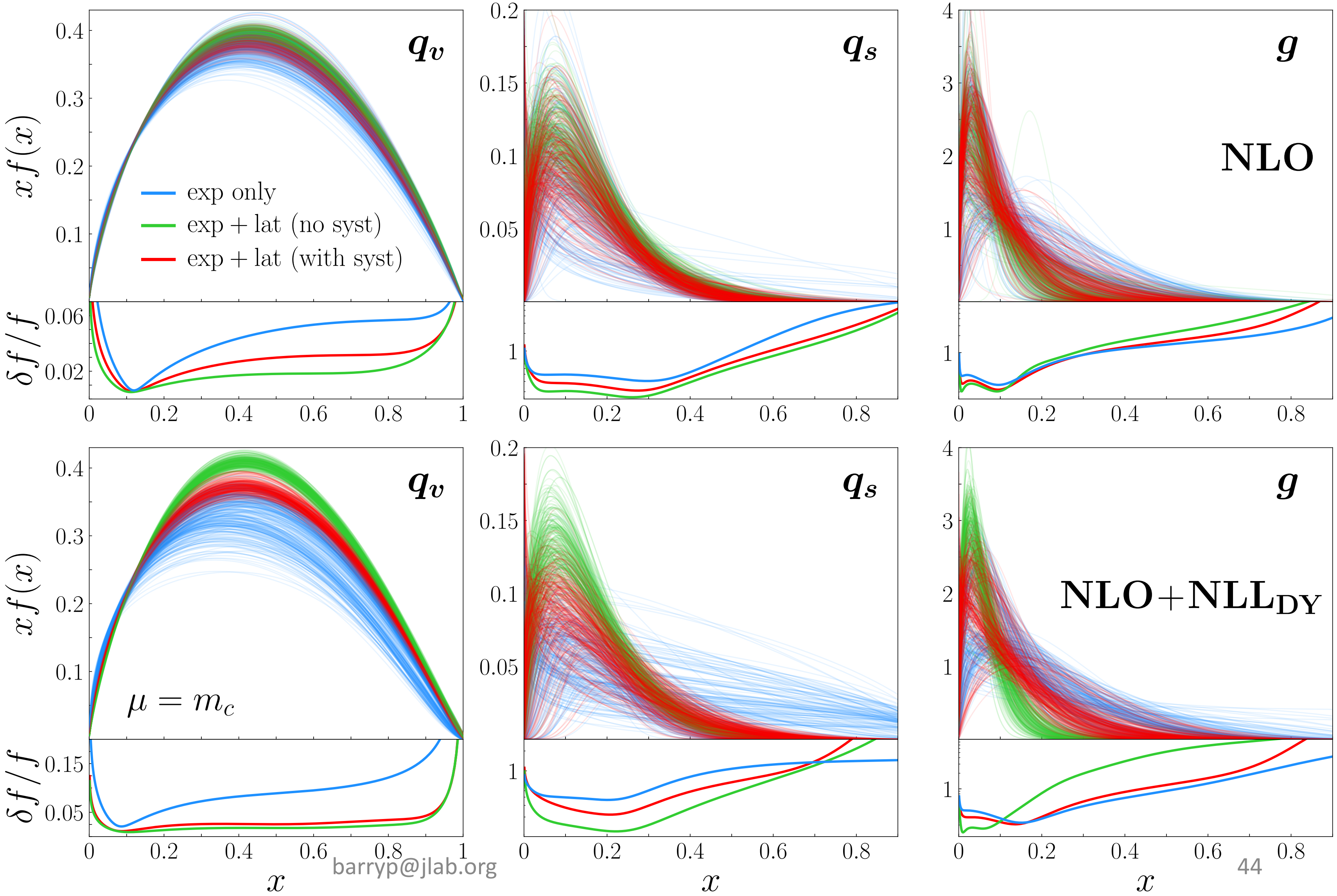
Agreement with the data

- Results from the full fit and isolating the leading twist term
- Difference between bands is the systematic correction



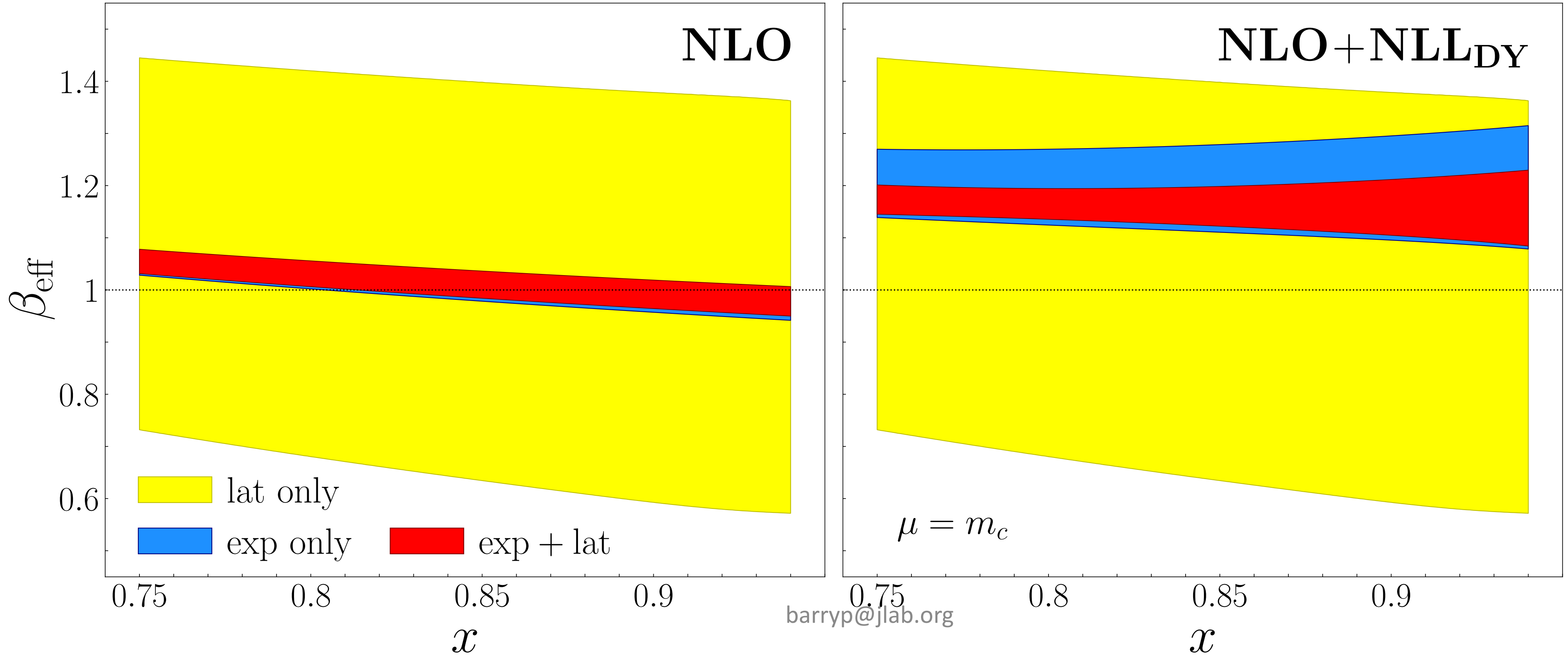
Resulting PDFs

- PDFs and relative uncertainties
- Including lattice reduces uncertainties
- NLO+NLL$_{DY}$ changes a lot – unstable under new data



Effective β from $(1-x)^{\beta_{\mathrm{eff}}}$

$$\beta_{\mathrm{eff}}(x,\mu) = \frac{\partial \log |q_v(x,\mu)|}{\partial \log(1-x)}$$

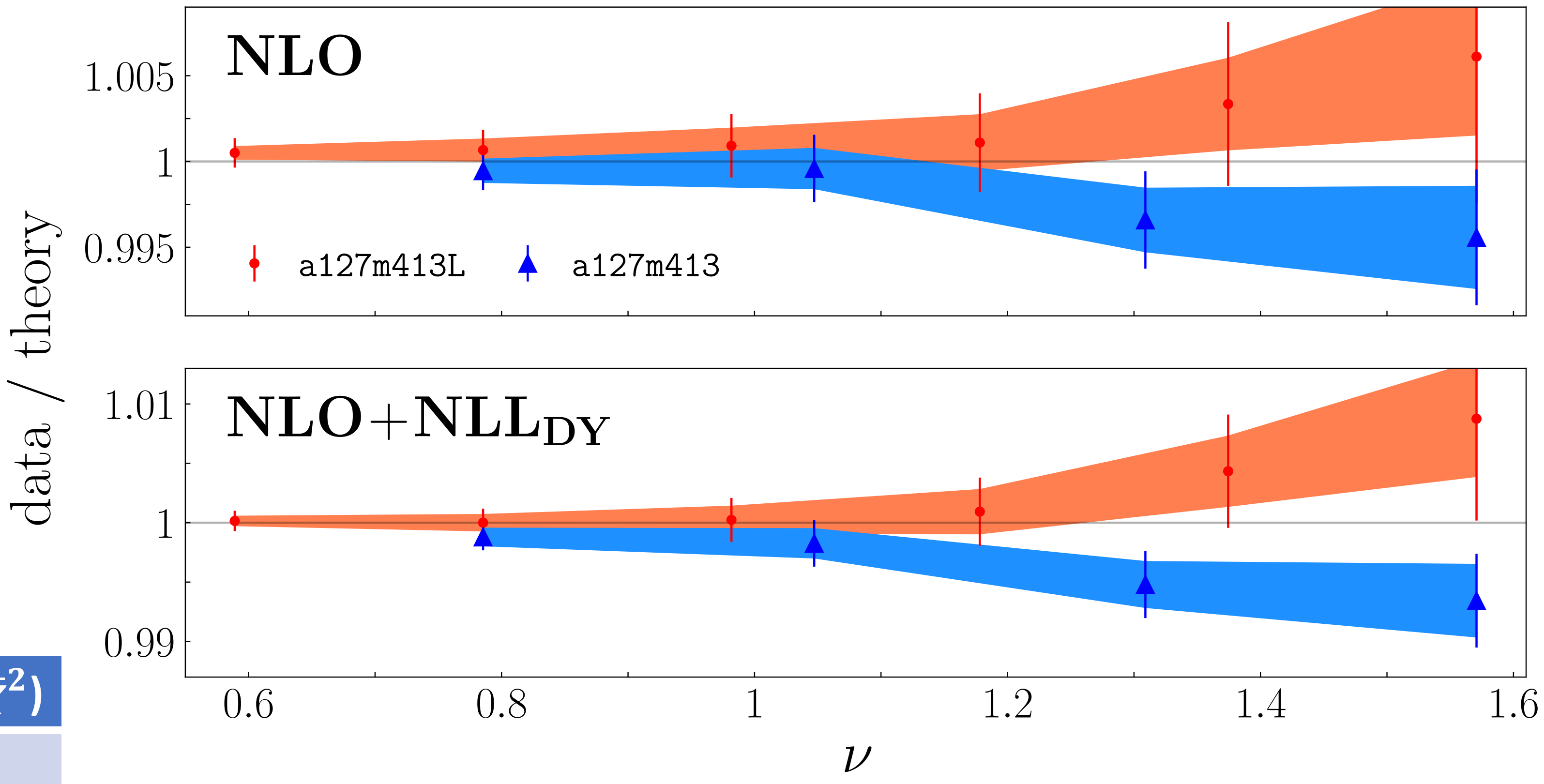


Fitting only the $p = 1$ points

- Most precise points, but not large range in Ioffe time
- Through analysis containing *only* lattice data, would not be sufficient to get a large x description of PDF
- Contrary to quasi-PDFs, which have correction terms $\propto \frac{1}{x^2(1-x)p_z}$

Data and theory comparison

- Each bin of z contains 3 momentum points, but only fitting to 1 momentum point
- Overall χ^2 are similar, but the fits to these are

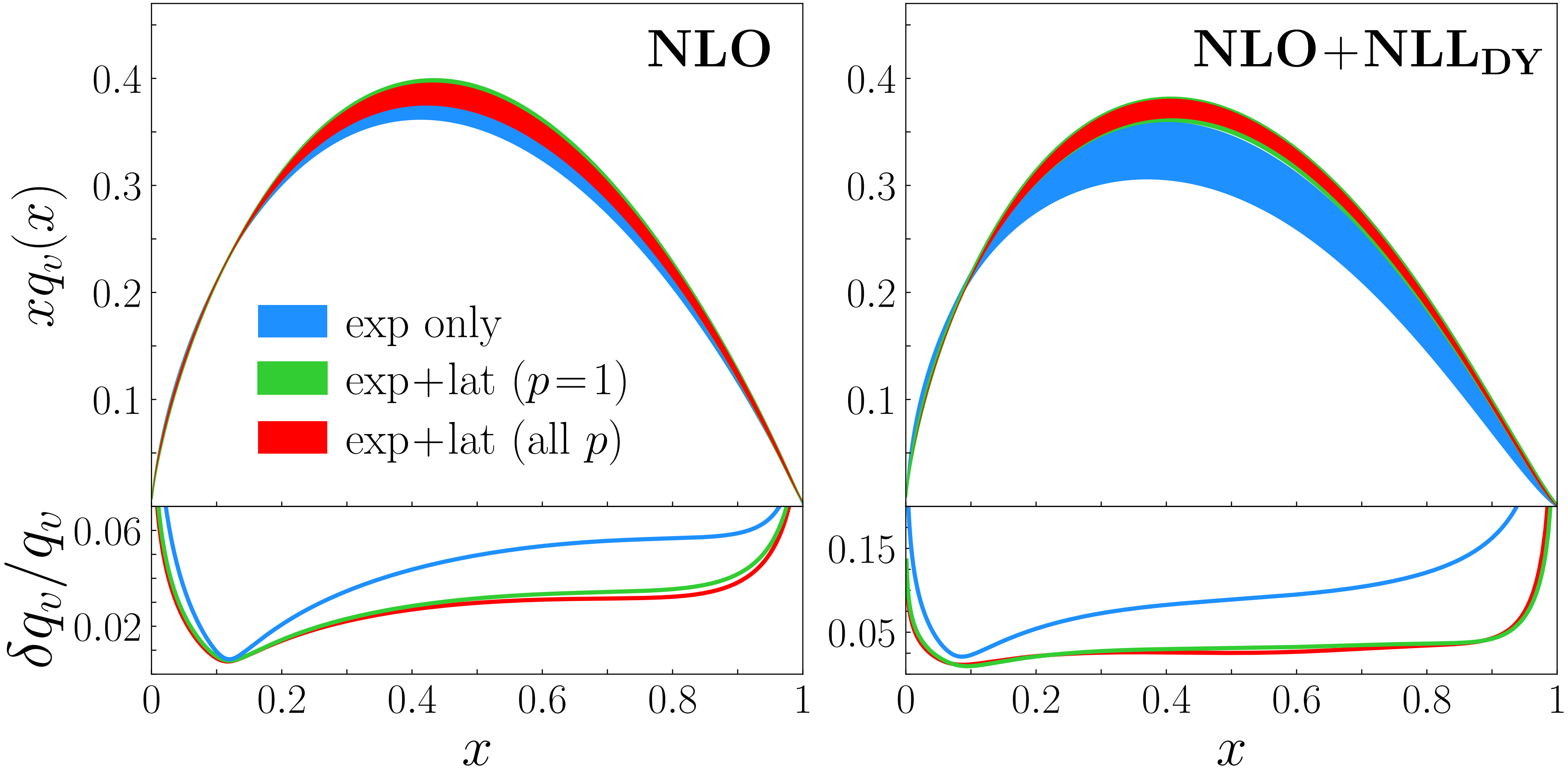


Dataset	NLO ($\bar{\chi}^2$)	NLO+NLL$_{DY}$ ($\bar{\chi}^2$)
a127m413L	0.76	0.81
a127m413	1.28	1.45

barryp@jlab.org

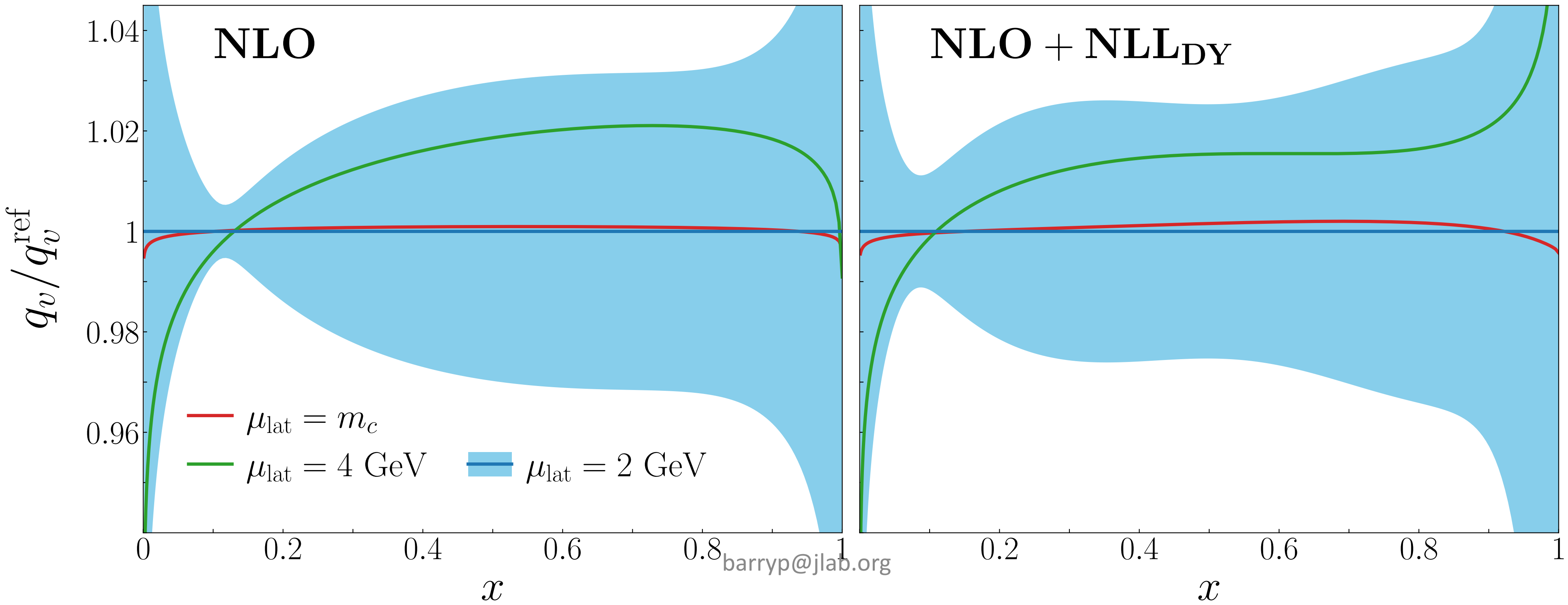
Resulting low-momentum PDFs

- These momentum points do entire job!



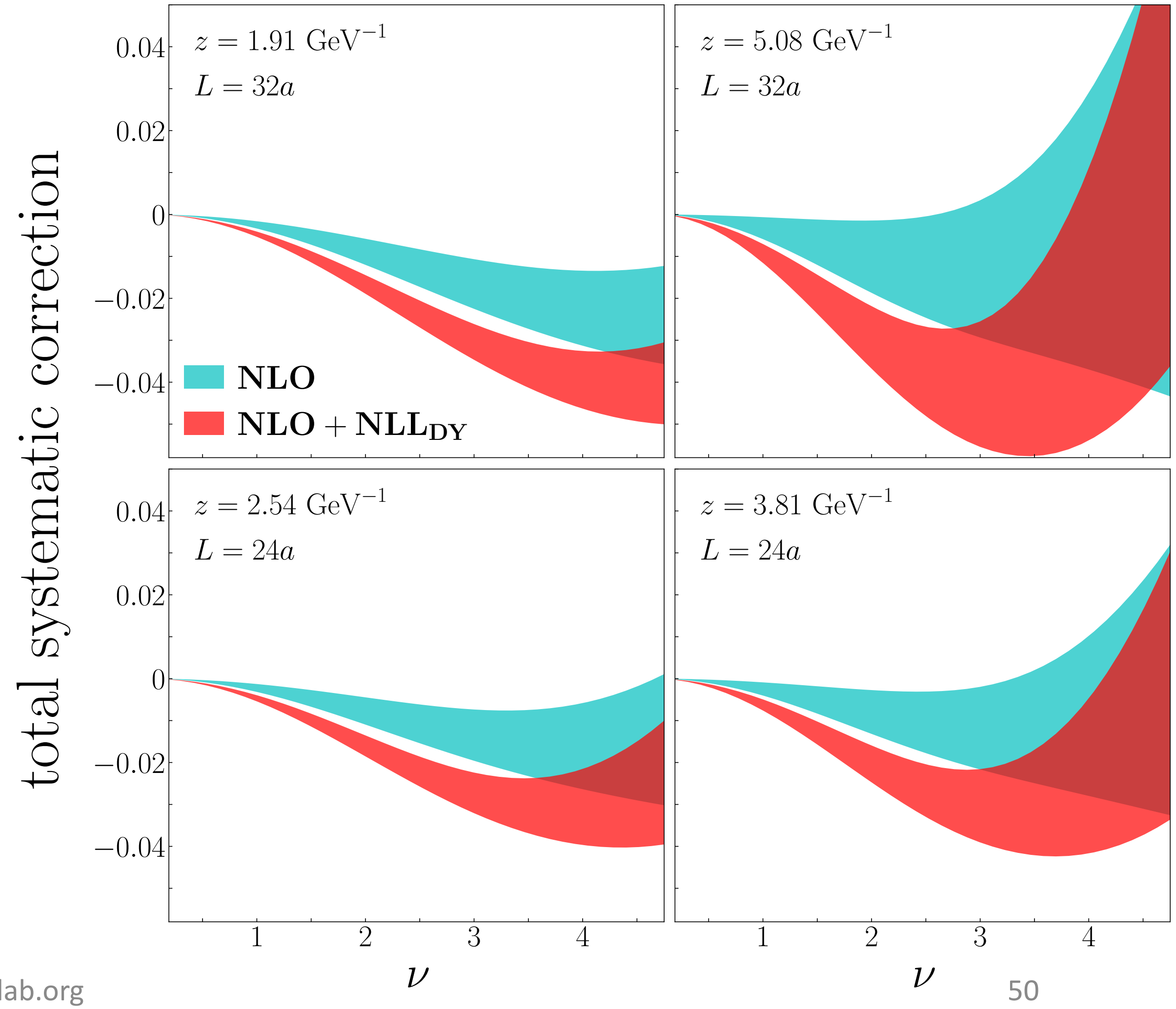
Scale Variation

- Do we capture systematic uncertainty from choosing $\mu_{\text{lat}} = 2$ GeV?
- Central values within uncertainty band – not a big issue



Quantifying Systematic Corrections

- Do systematic corrections agree within the DY theories?
  - No!
- Because the leading twist terms are well constrained by the experimental data, systematic corrections are "fudge factors"
- Have a min/max estimation for the systematic corrections

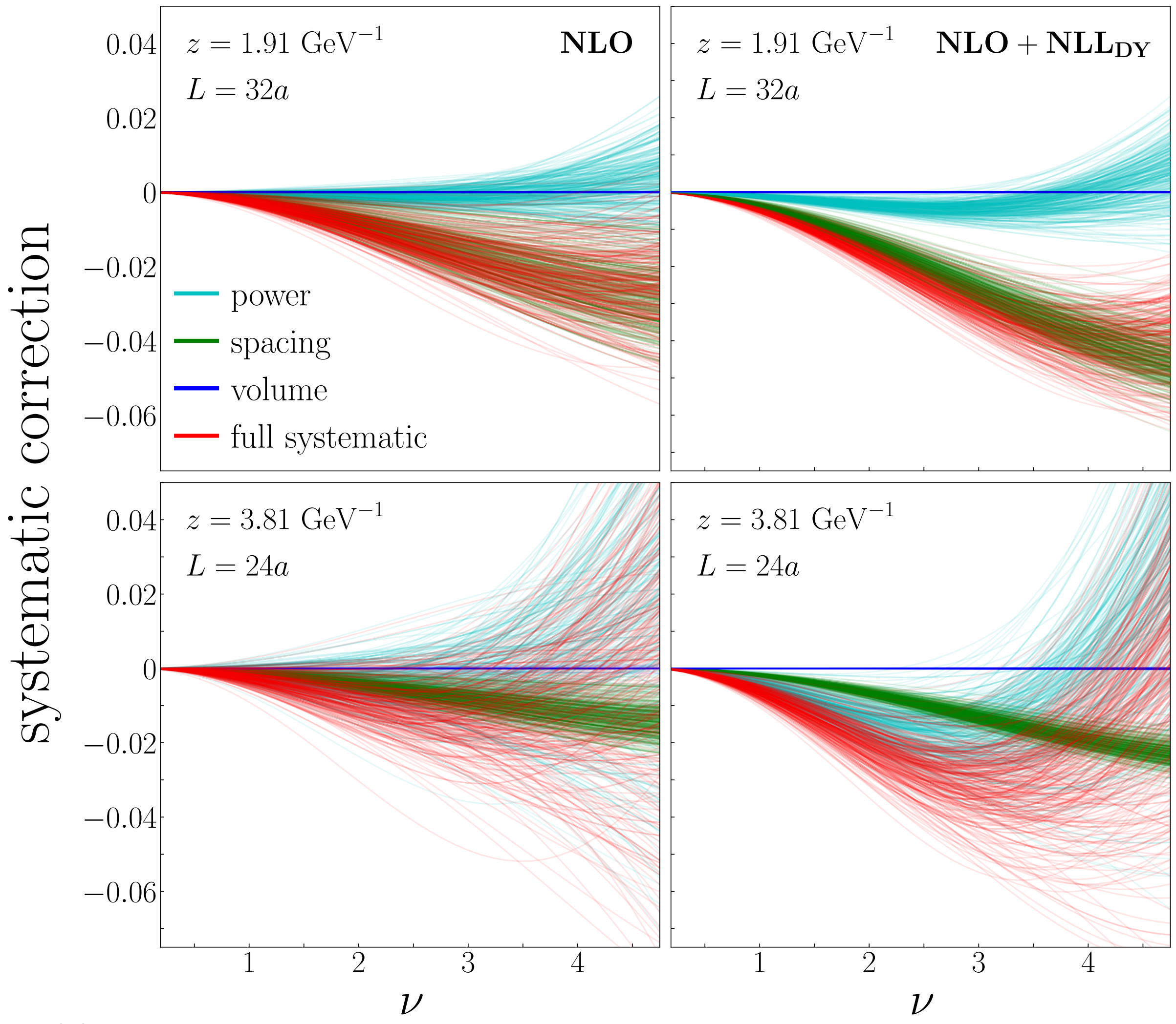


Quantifying individual systematics

- Breaking down by the 3 systematics

$$z^2 B_1(\nu) + \frac{a}{|z|} P_1(\nu) + e^{-m_\pi (L-z)} F_1(\nu)$$

- Depends on z values which of power or spacing corrections dominate
- Finite volume corrections don't matter



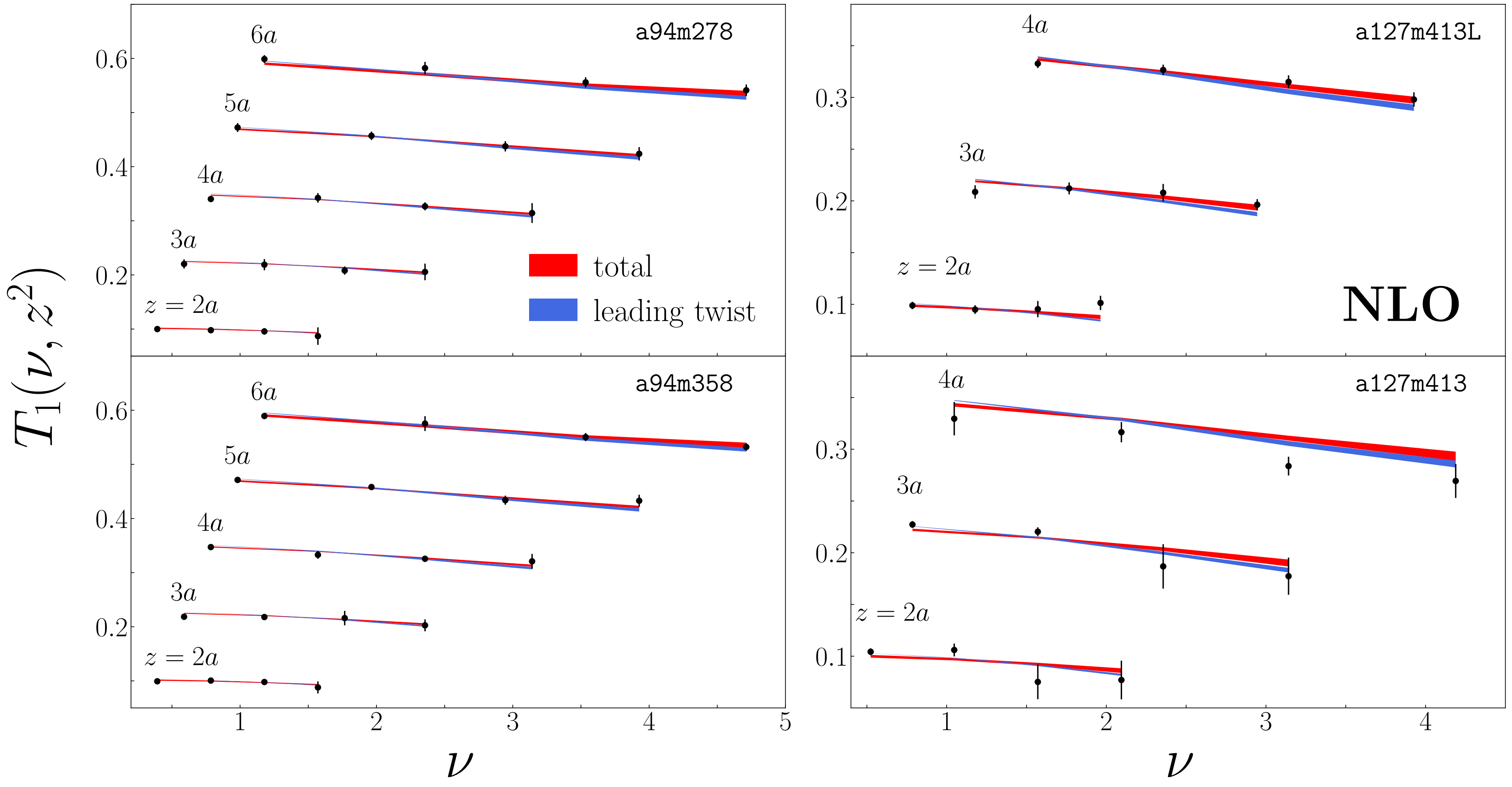
Current-current correlator analysis

Resulting χ^2

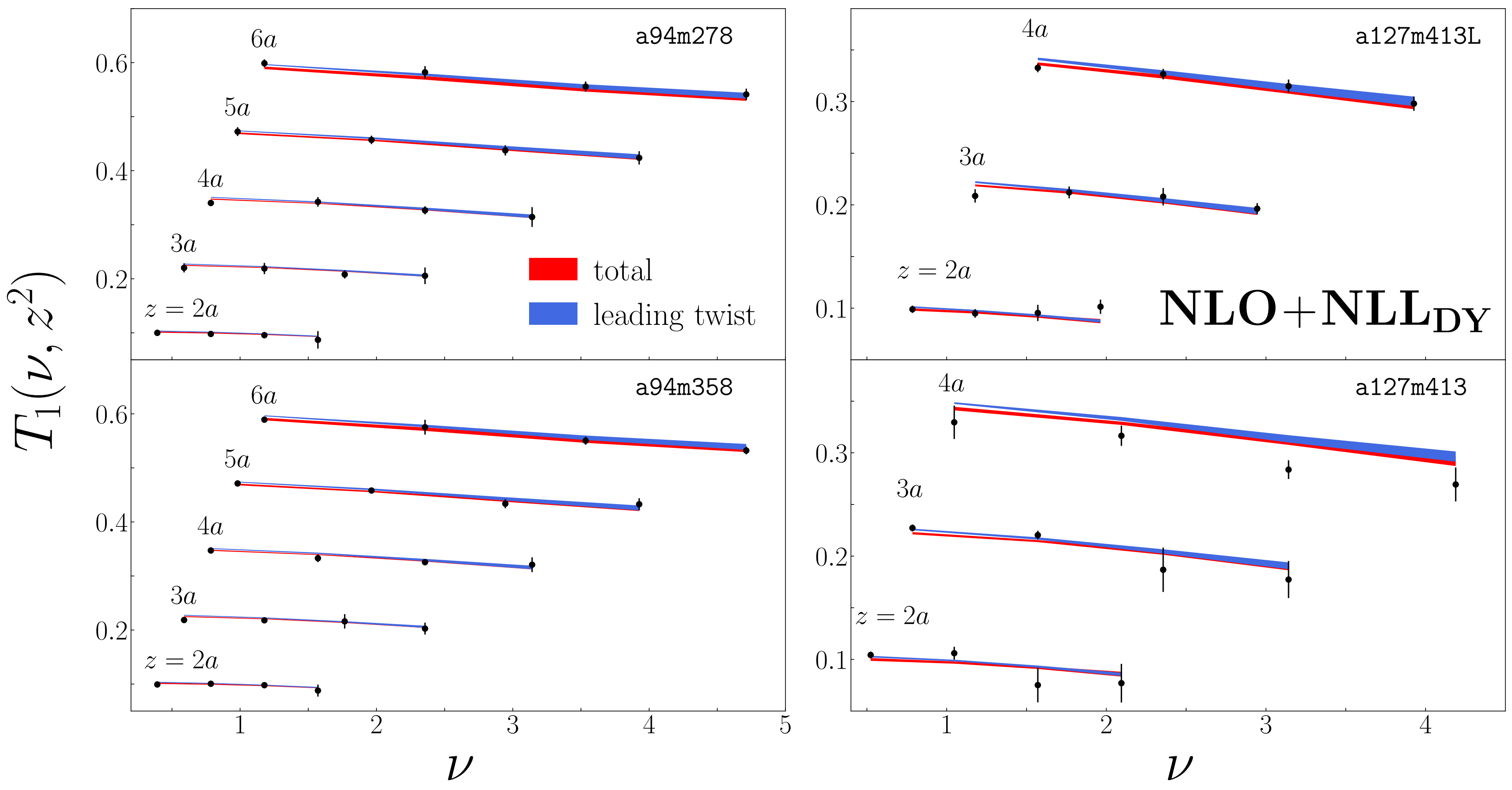
- With full systematics

Process	Experiment	N_{dat}	NLO $\overline{\chi}^2$	NLO+NLL$_{\text{DY}}$ $\overline{\chi}^2$
DY	E615 (x_F, Q)	61	0.83	0.81
	NA10 (194 GeV) (x_F, Q)	36	0.55	0.54
	NA10 (286 GeV) (x_F, Q)	20	0.85	0.86
LN	H1	58	0.37	0.35
	ZEUS	50	1.56	1.55
CC	a94m278	20	0.33	0.33
	a94m358	20	0.45	0.45
	a127m413L	12	0.72	0.77
	a127m413	12	1.98	1.90
Total		**289**	**0.81**	**0.80**

Agreement with data

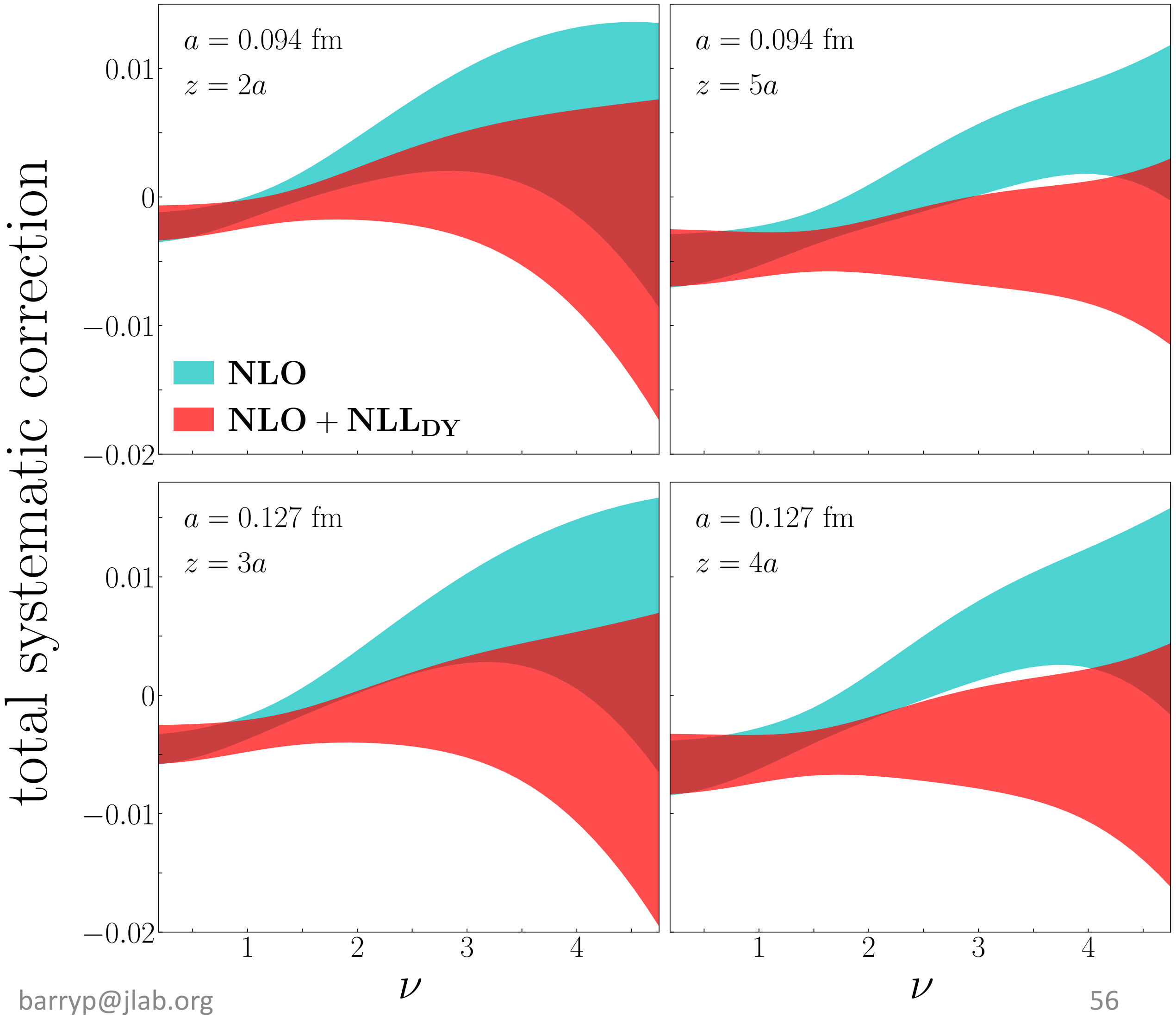


Agreement with data



Quantifying systematics – total

- Not guaranteed to be 0 and $\nu = 0$
- Different DY methods give different signs
- Large uncertainties at small z

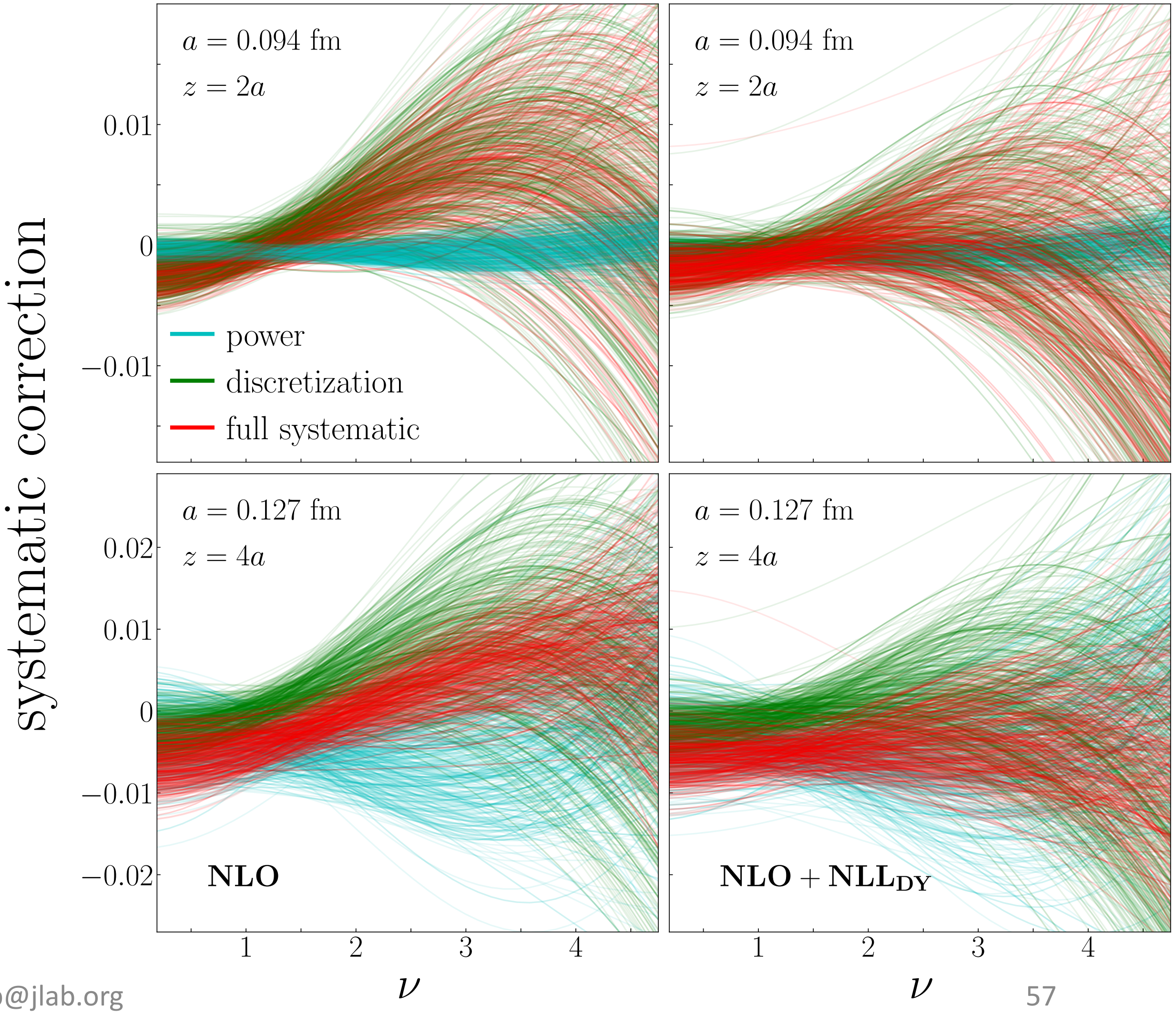


Quantifying systematics

- Each of two systematics

$$z^2 B_1(\nu) + aR_1(\nu)$$

- Some tension between the two types, effectively canceling



Conclusions

Conclusions and Outlook

- Need more observables to further distinguish between DY theories
- Large x behavior consistent with $\beta_{\text{eff}} \sim 1$ – from QCD calculations!
- Extend methodology to observables that are not well constrained by experimental data – helicity PDFs, transversity PDFs, GPDs, etc.