Meson Loops in Relativistic Chiral SU(3) Effective Theory

MARSTON COPELAND, CHUENG JI, WALLY MELNITCHOUK

Why Chiral Effective Theory?

- Quantum Chromodynamics is notoriously difficult to solve.

- Coupling constant is not sufficiently small so at medium to low energies, so its non perturbative.

- QCD Lagrangian is invariant under chiral symmetry in massless quark limit.
  - Spontaneous chiral symmetry breaking leads to pseudoscalar Goldstone bosons (mesons).

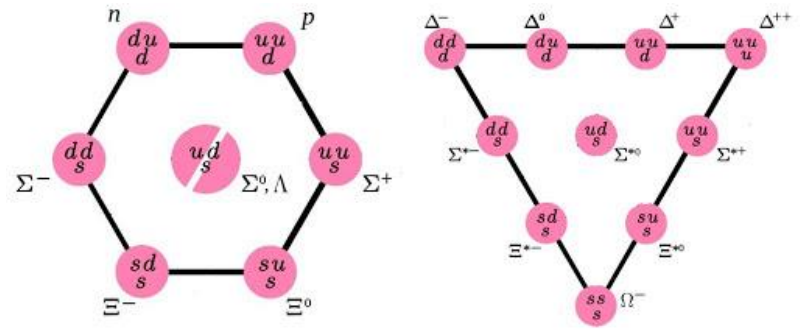
- This chiral symmetry can be exploited to construct an “effective field theory” of QCD.

Chiral SU(3) Effective Theory

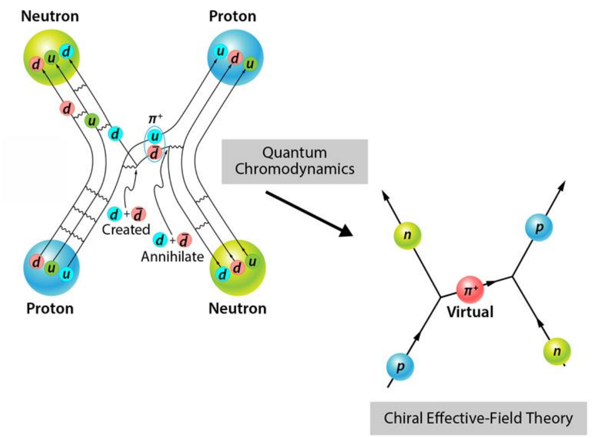
- Instead of interactions being mediated by gluons, they are mediated by pseudoscalar mesons.

- To investigate hadron structure, write Lagrangian density in terms of hadronic degrees of freedom.

- Mesons, Octet baryons, and decuplet baryons



http://sten.astronomycafe.net/quarks/



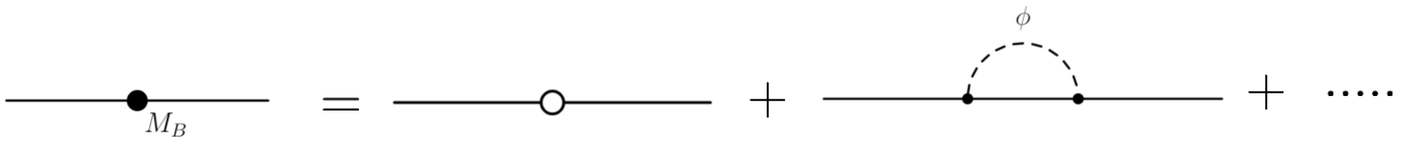
https://physics.aps.org/articles/v10/72

Introduction Meson Loops

- Field theoretic description of baryons (particles composed of three quarks) require higher order loop corrections.

- Meson loops are when baryons emit a virtual meson (particles composed of two quarks) at one space-time point and reabsorb them at another.

- Important contributions to the properties of baryons like masses, sigma terms, electromagnetic form factors, etc.



Abstract

- Investigate fundamental low-energy properties of baryons (e.g. self-energies, masses, and sigma terms) described within chiral effective theory.

- Calculate meson loop corrections to these properties using a relativistic formulation with finite range regularization for the full SU(3) octet and decuplet.

- Compare with lattice QCD data to determine fitting parameters. Calculate light and strange quark sigma terms using the Feynman-Hellman Theorem.

SU(3) Chiral Effective Lagrangian

- The lowest order Lagrangian describing the interaction of pseudoscalar mesons (ϕ) with octet (B) and decuplet (T_μ) baryons is given by

$$
\begin{aligned}
\mathcal{L}_{BT\phi} &= \mathrm{Tr}\big[\bar{B}(i\not{D} - M_B)B\big] - \frac{D}{2}\mathrm{Tr}\big[\bar{B}\gamma^\mu\gamma_5\{u_\mu, B\}\big] - \frac{F}{2}\mathrm{Tr}\big[\bar{B}\gamma^\mu\gamma_5[u_\mu, B]\big] \\
&+ \overline{T}^{ijk}_\mu(i\gamma^{\mu\nu\alpha}D_\alpha - M_T\gamma^{\mu\nu})T^{ijk}_\nu - \frac{\mathcal{C}}{2}\left[\epsilon^{ijk}\,\overline{T}^{ilm}_\mu\Theta^{\mu\nu}(u_\nu)^{lj}B^{mk} + \mathrm{h.c.}\right] \\
&- \frac{\mathcal{H}}{2}\overline{T}^{ijk}_\mu\gamma^\alpha\gamma_5(u_\alpha)^{kl}\,T^{ijl}_\mu + \frac{f_\phi^2}{4}\mathrm{Tr}\big[D_\mu U(D^\mu U)^\dagger\big]
\end{aligned}
$$

where

$$u = \exp\left(i\frac{\phi}{\sqrt{2}f_\phi}\right)$$

$$U = u^2$$

$$u_\mu = i(u^\dagger\partial_\mu u - u\partial_\mu u^\dagger) + (u^\dagger\lambda^a u - u\lambda^a u^\dagger)\nu^a_\mu$$

$$\Theta^{\mu\nu} = g^{\mu\nu} - \gamma^\mu\gamma^\nu$$

Baryon and Meson Fields

- The SU(3) octet and decuplet fields are given by

$$B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}$$

$$T_\mu = \left\{ \begin{pmatrix} \Delta^{++} & \frac{1}{\sqrt{3}}\Delta^+ & \frac{1}{\sqrt{3}}\Sigma^{*+} \\ \frac{1}{\sqrt{3}}\Delta^+ & \frac{1}{\sqrt{3}}\Delta^0 & \frac{1}{\sqrt{6}}\Sigma^{*0} \\ \frac{1}{\sqrt{3}}\Sigma^{*+} & \frac{1}{\sqrt{6}}\Sigma^{*0} & \frac{1}{\sqrt{3}}\Xi^{*0} \end{pmatrix}, \begin{pmatrix} \frac{1}{\sqrt{3}}\Delta^+ & \frac{1}{\sqrt{3}}\Delta^0 & \frac{1}{\sqrt{6}}\Sigma^{*0} \\ \frac{1}{\sqrt{3}}\Delta^0 & \Delta^- & \frac{1}{\sqrt{3}}\Sigma^{*-} \\ \frac{1}{\sqrt{6}}\Sigma^{*0} & \frac{1}{\sqrt{3}}\Sigma^{*-} & \frac{1}{\sqrt{3}}\Xi^{*-} \end{pmatrix}, \begin{pmatrix} \frac{1}{\sqrt{3}}\Sigma^{*+} & \frac{1}{\sqrt{6}}\Sigma^{*0} & \frac{1}{\sqrt{3}}\Xi^{*0} \\ \frac{1}{\sqrt{6}}\Sigma^{*0} & \frac{1}{\sqrt{3}}\Sigma^{*-} & \frac{1}{\sqrt{3}}\Xi^{*-} \\ \frac{1}{\sqrt{3}}\Xi^{*0} & \frac{1}{\sqrt{3}}\Xi^{*-} & \Omega^- \end{pmatrix} \right\}$$

- Pseudoscalar meson fields are

$$\phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \overline{K}^0 & -\frac{\sqrt{2}}{\sqrt{3}}\eta \end{pmatrix}$$

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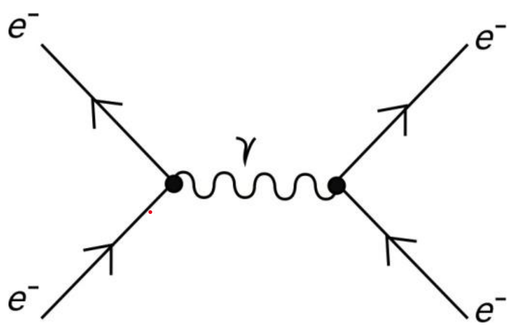
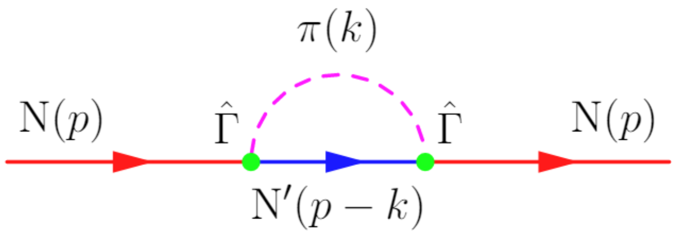
Feynman Rules

- External lines have a factor of $u(p)$ or $\overline{u}(p)$ (Dirac spinors).
- Internal lines have corresponding propagators, $S_H(p)$.
- Interactions have vertex factors, $\hat{\Gamma}_{\mathcal{B}\mathcal{B}'\phi}$.
- Need to sum over the external spins (results in a trace).
- For closed fermion loops, integrate over the intermediate momentum.

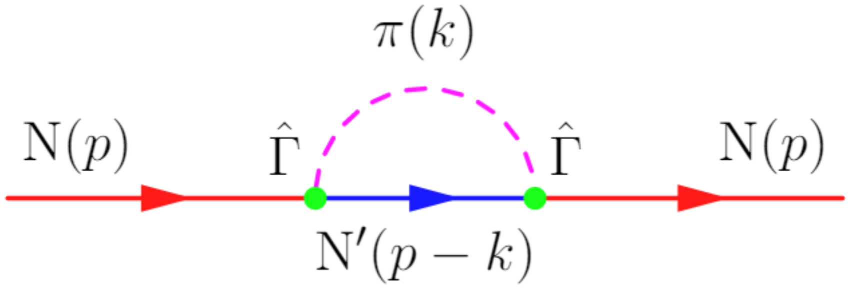


https://protonsforbreakfast.wordpress.com/2014/04/13/feynman-diagrams-are-maths-not-physics/

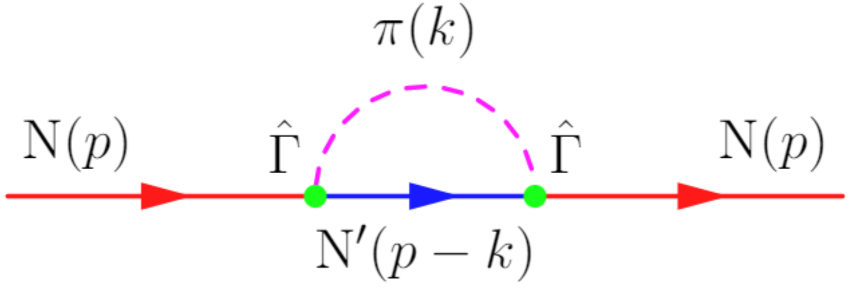


Simple Self-Energy Derivation

Nucleon to nucleon-pion loop



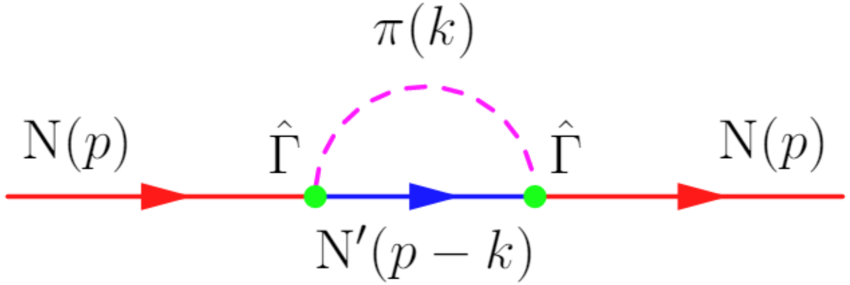
Simple Self-Energy Derivation



$$\hat{\Sigma}_{N\to N'\pi} = i\int \frac{d^4k}{(2\pi)^4}\hat{\Gamma}\cdot S_{N'}(p-k)\cdot\hat{\Gamma}\cdot S_\pi(k)$$

Nucleon to nucleon-pion self-energy operator

Simple Self-Energy Derivation



$$\hat{\Sigma}_{N\to N'\pi} = i\int \frac{d^4k}{(2\pi)^4}\hat{\Gamma}\cdot S_{N'}(p-k)\cdot\hat{\Gamma}\cdot S_\pi(k)$$

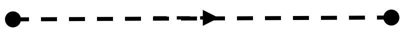
Nucleon to nucleon-pion self-energy

$$\Sigma_{N\to N'\pi} = \frac{1}{2}\sum_s \bar{u}(p,s)\hat{\Sigma}_{N\to N'\pi}u(p,s)$$

$$= \frac{1}{2}\mathrm{Tr}\left[\frac{\not{p}+M_N}{2M_N}\hat{\Sigma}_{N\to N'\pi}\right]$$

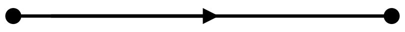
Propagators

Meson:



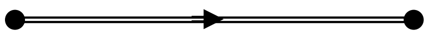
$$S_\phi(k) = \frac{i}{k^2 - m_\phi^2 + i\epsilon}$$

Octet baryon:



$$S_B(p-k) = i\frac{\not{p} - \not{k} + M_B}{(p-k)^2 - M_B^2 + i\epsilon}$$

Decuplet baryon:



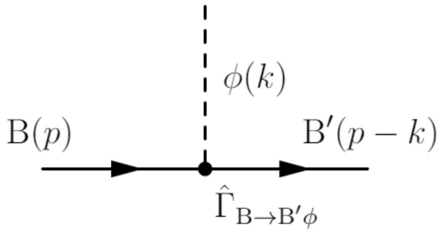
$$S_T^\mu(p-k) = -i\frac{\not{p} - \not{k} + M_T}{(p-k)^2 - M_T^2 + i\epsilon}\Lambda^\mu(p-k)$$

where

$$\Lambda_{\nu\mu}(p) = g_{\nu\mu} - \frac{1}{3}\gamma_\nu\gamma_\mu - \frac{1}{3M}(\gamma_\nu p_\mu - \gamma_\mu p_\nu) - \frac{2}{3M^2}p_\nu p_\mu$$

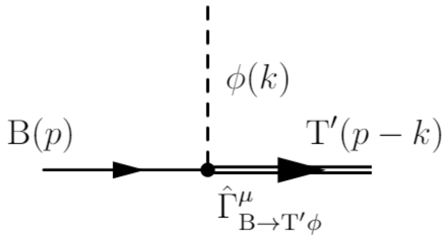
Interactions/Vertices

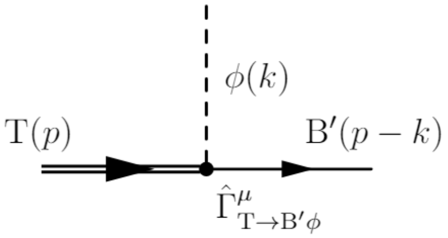
Octet to Octet:



$$\hat{\Gamma}_{BB'\phi} = \frac{C_{BB'\phi}}{f_\phi}\gamma_5 \not{k}$$

Octet to Decuplet:

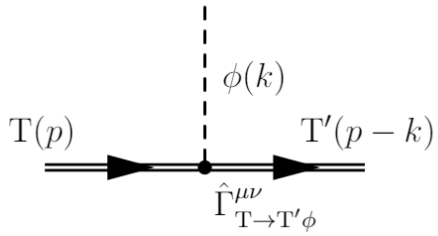




$$\hat{\Gamma}_{BT'\phi/TB'\phi} = \frac{C_{BT'\phi/TB'\phi}}{f_\phi}\Theta^{\mu\nu}k_\nu$$

$$\Theta^{\mu\nu} = g^{\mu\nu} - \gamma^\mu\gamma^\nu$$

Decuplet to Decuplet:



$$\hat{\Gamma}_{TT'\phi} = \frac{C_{TT'\phi}}{f_\phi}\epsilon^{\mu\sigma\alpha\beta}\gamma_\beta k_\alpha$$

SU(3) Chiral Effective Lagrangian

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&- \frac{\mathcal{H}}{2}\overline{T}^{ijk}_{\mu}\gamma^{\alpha}\gamma_5(u_{\alpha})^{kl}\,T^{ijl}_{\mu} + \frac{f_{\phi}^2}{4}\mathrm{Tr}\left[D_{\mu}U(D^{\mu}U)^{\dagger}\right]
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where

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$$\Theta^{\mu\nu} = g^{\mu\nu} - \gamma^{\mu}\gamma^{\nu}$$

Coupling Constants

external state B

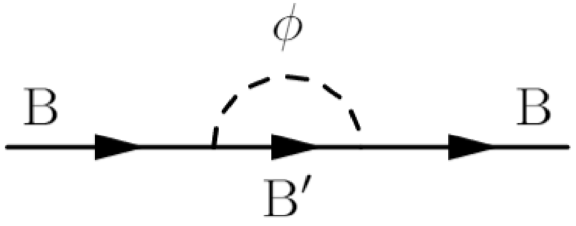
	$\boldsymbol{p}$		$\boldsymbol{\Lambda}$		$\boldsymbol{\Sigma^+}$		$\boldsymbol{\Xi^0}$	
$(\boldsymbol{B'}\phi)$	$(p\pi^0)$	$\frac{(D+F)}{2}$	(pK^-)	$-\frac{(D+3F)}{\sqrt{12}}$	$(p\overline{K}^0)$	$\frac{(D-F)}{\sqrt{2}}$	$(\Sigma^0\overline{K}^0)$	$\frac{(D+F)}{2}$
	$(n\pi^+)$	$\frac{(D+F)}{\sqrt{2}}$	(nK^0)	$-\frac{(D+3F)}{\sqrt{12}}$	(ΞK^+)	$\frac{(D+F)}{\sqrt{2}}$	(Σ^+K^-)	$\frac{(D+F)}{\sqrt{2}}$
	(Σ^+K^0)	$\frac{(D-F)}{\sqrt{2}}$	$(\Sigma^+\pi^-)$	$\frac{1}{\sqrt{3}}D$	$(\Sigma^+\pi^0)$	F	$(\Xi^0\pi^0)$	$\frac{(D-F)}{2}$
	(Σ^0K^+)	$\frac{(D-F)}{2}$	$(\Sigma^0\pi^0)$	$\frac{1}{\sqrt{3}}D$	$(\Sigma^0\pi^+)$	$-F$	$(\Xi^-\pi^-)$	$\frac{(D-F)}{\sqrt{2}}$
	(ΛK^+)	$-\frac{(D+3F)}{\sqrt{12}}$	$(\Sigma^-\pi^-)$	$\frac{1}{\sqrt{3}}D$	$(\Sigma^+\eta)$	$\frac{1}{\sqrt{3}}D$	$(\Xi^0\eta)$	$-\frac{(D+3F)}{\sqrt{12}}$
	$(p\eta)$	$-\frac{(D-3F)}{\sqrt{12}}$	(Ξ^0K^0)	$-\frac{(D-3F)}{\sqrt{12}}$	$(\Lambda\pi^+)$	$\frac{1}{\sqrt{3}}D$	$(\Lambda\overline{K}^0)$	$-\frac{(D-3F)}{\sqrt{12}}$
			$(\Xi^-K^+$	$-\frac{(D-3F)}{\sqrt{12}}$				
			$(\Lambda\eta)$	$\frac{1}{\sqrt{3}}D$				
$(\boldsymbol{T'}\phi)$	$(\Delta^{++}\pi^-)$	$\frac{1}{\sqrt{2}}\mathcal{C}$	$(\Sigma^{*+}\pi^-)$	$\frac{1}{2}\mathcal{C}$	$(\Delta^{++}K^-)$	$-\frac{1}{\sqrt{2}}\mathcal{C}$	$(\Sigma^{*+}K^-)$	$-\frac{1}{\sqrt{6}}\mathcal{C}$
	$(\Delta^+\pi^0)$	$-\frac{1}{\sqrt{3}}\mathcal{C}$	$(\Sigma^{*-}\pi^+)$	$\frac{1}{2}\mathcal{C}$	$(\Delta^+\overline{K}^0)$	$-\frac{1}{\sqrt{6}}\mathcal{C}$	$(\Sigma^{*0}\overline{K}^0)$	$-\frac{1}{\sqrt{12}}\mathcal{C}$
	$(\Delta^0\pi^+)$	$-\frac{1}{\sqrt{6}}\mathcal{C}$	$(\Sigma^{*0}\pi^0)$	$\frac{1}{2}\mathcal{C}$	$(\Sigma^{*+}\pi^0)$	$\frac{1}{\sqrt{12}}\mathcal{C}$	$(\Xi^{*0}\pi^0)$	$-\frac{1}{\sqrt{12}}\mathcal{C}$
	$(\Sigma^{*+}K^0)$	$\frac{1}{\sqrt{6}}\mathcal{C}$	$(\Xi^{*0}K^0)$	$-\frac{1}{2}\mathcal{C}$	$(\Sigma^{*0}\pi^+)$	$\frac{1}{\sqrt{12}}\mathcal{C}$	$(\Xi^{*-}\pi^+)$	$\frac{1}{\sqrt{6}}\mathcal{C}$
	$(\Sigma^{*0}K^+)$	$-\frac{1}{\sqrt{12}}\mathcal{C}$	$(\Xi^{*-}K^+)$	$\frac{1}{2}\mathcal{C}$	$(\Xi^{*0}K^+)$	$\frac{1}{\sqrt{6}}\mathcal{C}$	$(\Xi^{*0}\eta)$	$-\frac{1}{2}\mathcal{C}$
					$(\Sigma^{*+}\eta)$	$\frac{1}{2}\mathcal{C}$	(Ω^-K^+)	$\frac{1}{\sqrt{2}}\mathcal{C}$

external state T

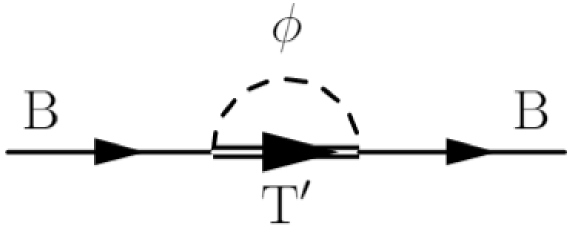
	$\boldsymbol{\Delta^+}$		$\boldsymbol{\Sigma^{*+}}$		$\boldsymbol{\Xi^{*0}}$		$\boldsymbol{\Omega^-}$	
$(\boldsymbol{B'}\phi)$	$(p\pi^0)$	$\frac{1}{\sqrt{6}}\mathcal{C}$	$(p\overline{K}^0)$	$\frac{1}{\sqrt{6}}\mathcal{C}$	(Σ^+K^-)	$\frac{1}{\sqrt{6}}\mathcal{C}$	(Ξ^0K^-)	$-\frac{1}{\sqrt{2}}\mathcal{C}$
	$(n\pi^+)$	$-\frac{1}{\sqrt{3}}\mathcal{C}$	(Ξ^0K^+)	$\frac{1}{\sqrt{6}}\mathcal{C}$	$(\Sigma^0\overline{K}^0)$	$-\frac{1}{\sqrt{12}}\mathcal{C}$	$(\Xi^-\overline{K}^0)$	$\frac{1}{\sqrt{2}}\mathcal{C}$
	(Σ^+K^0)	$-\frac{1}{\sqrt{6}}\mathcal{C}$	$(\Sigma^+\pi^0)$	$-\frac{1}{\sqrt{12}}\mathcal{C}$	$(\Xi^0\pi^0)$	$\frac{1}{\sqrt{12}}\mathcal{C}$		
	(Σ^0K^+)	$\frac{1}{\sqrt{3}}\mathcal{C}$	$(\Sigma^0\pi^+)$	$\frac{1}{\sqrt{12}}\mathcal{C}$	$(\Xi^-\pi^+)$	$-\frac{1}{\sqrt{6}}\mathcal{C}$		
			$(\Sigma^+\eta)$	$\frac{1}{2}\mathcal{C}$	$(\Xi^0\eta)$	$\frac{1}{2}\mathcal{C}$		
			$(\Lambda\pi^+)$	$-\frac{1}{2}\mathcal{C}$	$(\Lambda\overline{K}^0)$	$-\frac{1}{2}\mathcal{C}$		
$(\boldsymbol{T'}\phi)$	$(\Delta^{++}\pi^-)$	$\frac{1}{\sqrt{6}}\mathcal{H}$	$(\Delta^{++}K^-)$	$\frac{1}{\sqrt{6}}\mathcal{H}$	$(\Xi^{*-}\pi^+)$	$\frac{1}{\sqrt{18}}\mathcal{H}$	$(\Xi^{*-}\overline{K}^0)$	$\frac{1}{\sqrt{6}}\mathcal{H}$
	$(\Delta^+\pi^0)$	$\frac{1}{6}\mathcal{H}$	$(\Delta^+\overline{K}^0)$	$\frac{1}{\sqrt{18}}\mathcal{H}$	$(\Xi^{*0}\pi^0)$	$\frac{1}{6}\mathcal{H}$	$(\Xi^{*0}K^+)$	$\frac{1}{\sqrt{6}}\mathcal{H}$
	$(\Delta^0\pi^+)$	$\frac{\sqrt{2}}{3}\mathcal{H}$	$(\Xi^{*0}K^+)$	$\frac{\sqrt{2}}{3}\mathcal{H}$	$(\Xi^{*0}\eta)$	$-\frac{1}{\sqrt{12}}\mathcal{H}$	$(\Omega^-\eta)$	$-\frac{1}{\sqrt{3}}\mathcal{H}$
	$(\Delta^+\eta)$	$\frac{1}{\sqrt{12}}\mathcal{H}$	$(\Sigma^{*+}\pi^0)$	$\frac{1}{3}\mathcal{H}$	$(\Sigma^{*+}K^-)$	$\frac{\sqrt{2}}{3}\mathcal{H}$		
	$(\Sigma^{*+}K^0)$	$\frac{1}{\sqrt{18}}\mathcal{H}$	$(\Sigma^{*0}\pi+)$	$\frac{1}{3}\mathcal{H}$	$(\Sigma^{*0}\overline{K}^0)$	$\frac{1}{3}\mathcal{H}$		
	$(\Sigma^{*0}K^+)$	$\frac{1}{3}\mathcal{H}$			(Ω^-K^+)	$\frac{1}{\sqrt{6}}\mathcal{H}$		

Self-Energy Diagrams

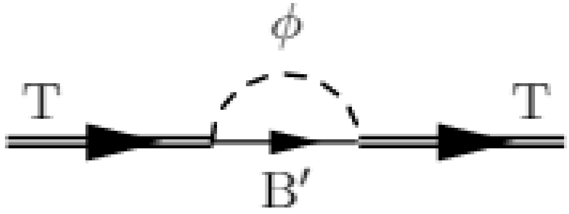
Octet to octet



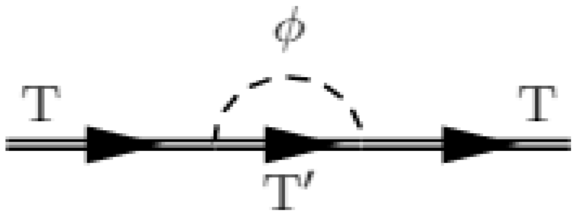
Octet to decuplet



Decuplet to octet



Decuplet to decuplet



Self-Energy Operators

Octet to octet

$$\hat{\Sigma}_{\mathrm{B}\to\mathrm{B}'\phi} = -i\left(\frac{C_{BB'\phi}}{f_\phi}\right)^2 \int \frac{d^4k}{(2\pi)^4}(\not{k}\gamma_5)\frac{i(\not{p}-\not{k}+M_{B'})}{(p-k)^2-M_{B'}^2+i\epsilon}$$
$$\times (\gamma_5\not{k})\frac{i}{k^2-m_\phi^2+i\epsilon}$$

Octet to decuplet

$$\hat{\Sigma}_{B\to T'\phi} = -i\left(\frac{C_{BT'\phi}}{f_\phi}\right)^2 \int \frac{d^4k}{(2\pi)^4}(g_{\mu\nu}-\gamma_\mu\gamma_\nu)k^\nu$$
$$\times [\frac{i(\not{p}-\not{k}+M_{T'})}{(p-k)^2-M_{T'}^2+i\epsilon}]\Lambda^{\mu\lambda}(p-k)(g_{\lambda\sigma}-\gamma_\lambda\gamma_\sigma)k^\sigma$$
$$\times \frac{i}{k^2-m_\phi+i\epsilon}$$

Decuplet to octet

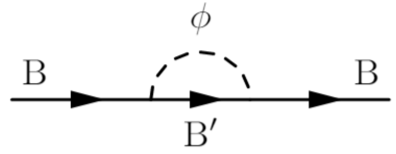
$$\hat{\Sigma}^{\mu\nu}_{\mathrm{T}\to\mathrm{B}'\phi} = -i\left(\frac{C_{TB'\phi}}{f_\phi}\right)^2 \int \frac{d^4k}{(2\pi)^4}((g^{\mu\alpha}-\gamma^\mu\gamma^\alpha)k_\alpha)$$
$$\times \frac{i(\not{p}-\not{k}+M_{B'})}{(p-k)^2-M_{B'}^2+i\epsilon}((g^{\nu\beta}-\gamma^\nu\gamma^\beta)k_\beta)$$
$$\times \frac{i}{k^2-m_\phi^2+i\epsilon}$$

Decuplet to decuplet

$$\hat{\Sigma}^{\mu\nu}_{T\to T'\phi} = -i\left(\frac{C_{TT'\phi}}{f_\phi}\right)^2 \int \frac{d^4k}{(2\pi)^4}(\epsilon^{\mu\sigma\alpha\beta}\gamma_\beta k_\alpha)$$
$$\times \left[\frac{i(\not{p}-\not{k}+M_{T'})}{(p-k)^2-M_{T'}^2+i\epsilon}\right]\Lambda_{\sigma\lambda}(p-k)(\epsilon^{\lambda\nu\rho\delta}\gamma_\delta k_\rho)$$
$$\times \frac{i}{k^2-m_\phi+i\epsilon}$$

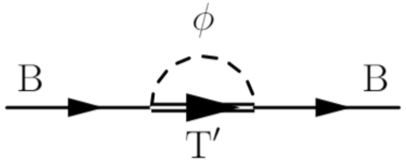
Definitions of Self-Energies

- Octet to octet



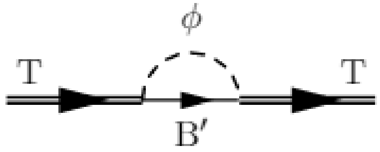
$$\Sigma_{\mathrm{B}\to\mathrm{B}'\phi} = \frac{1}{2}\mathrm{Tr}\Big[\frac{\not{p}+M_B}{2M_B}\hat{\Sigma}_{\mathrm{B}\to\mathrm{B}'\phi}\Big]$$

- Octet to decuplet



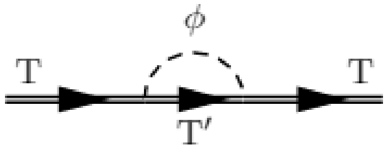
$$\Sigma_{\mathrm{B}\to\mathrm{T}'\phi} = \frac{1}{2}\mathrm{Tr}\Big[\frac{\not{p}+M_B}{2M_B}\hat{\Sigma}_{\mathrm{B}\to\mathrm{T}'\phi}\Big]$$

- Decuplet to octet



$$\Sigma_{\mathrm{T}\to\mathrm{B}'\phi} = \frac{1}{4}\mathrm{Tr}\Big[\frac{\not{p}+M_T}{2M_T}\Lambda_{\nu\mu}(p)\hat{\Sigma}^{\mu\nu}_{\mathrm{T}\to\mathrm{B}'\phi}\Big]$$

- Decuplet to decuplet



$$\Sigma_{\mathrm{T}\to\mathrm{T}'\phi} = \frac{1}{4}\mathrm{Tr}\Big[\frac{\not{p}+M_T}{2M_T}\Lambda_{\nu\mu}(p)\hat{\Sigma}^{\mu\nu}_{\mathrm{T}\to\mathrm{T}'\phi}\Big]$$

where

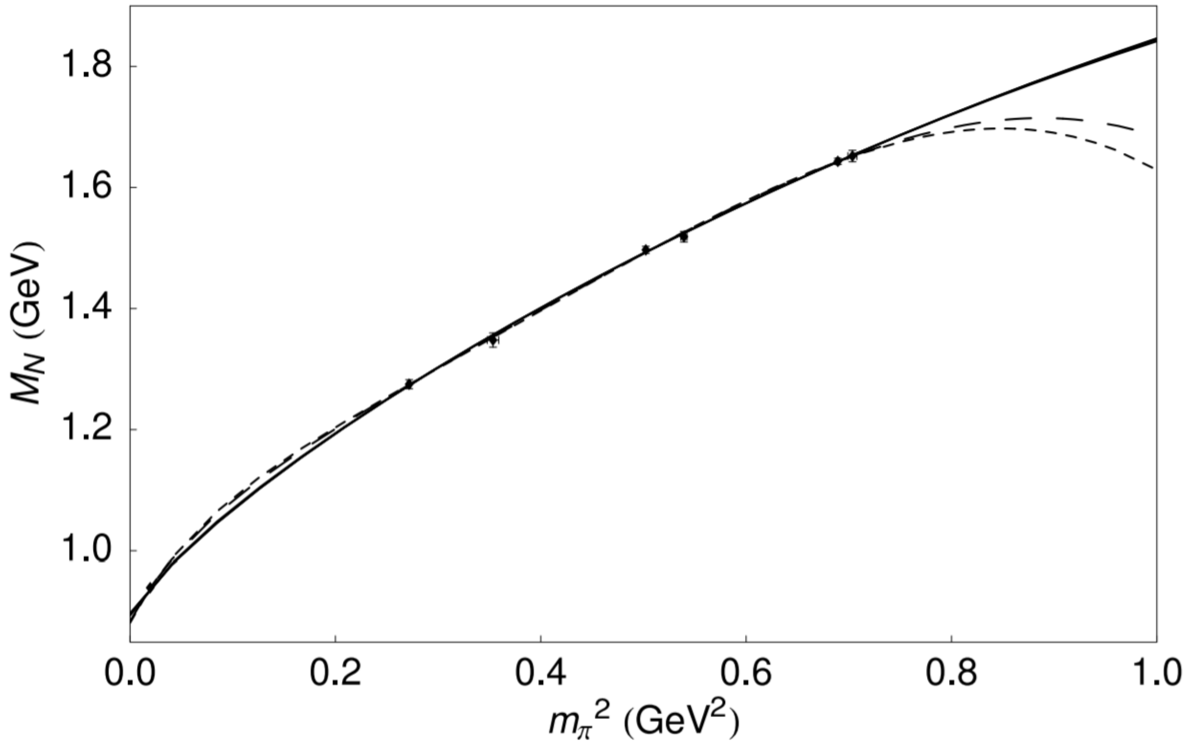
$$\Lambda_{\nu\mu}(p) = g_{\nu\mu} - \frac{1}{3}\gamma_\nu\gamma_\mu - \frac{1}{3M}(\gamma_\nu p_\mu - \gamma_\mu p_\nu) - \frac{2}{3M^2}p_\nu p_\mu$$

Regularization Prescription

- Integral over the meson momentum diverges.
- Need a regularization prescription to suppress the divergences.
- Options are dimensional regularization, Pauli-Villars, or finite range regularization (FRR).
- FRR offers some advantages over the others.

Convergence of Finite Range Regularization

- Ross Young et. al. studied the convergence properties of FRR in detail [1].
  - For HB χ-PT but results still apply here.
- Short dash is dim.-reg. , long dash is dim.-reg. with $\Delta\pi$ intermediate state included, and solid is FRR.
- Dim. reg. diverges badly at large m_π^2, but FRR doesn't!
- Especially useful for sigma term calculations.

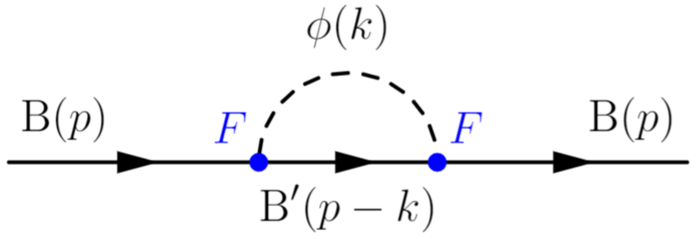


[1] R. D. Young, D. B. Leinweber, A. W. Thomas, Prog. Part. Nucl. Phys. 50 (2003) 399-417

Finite Range Regularization

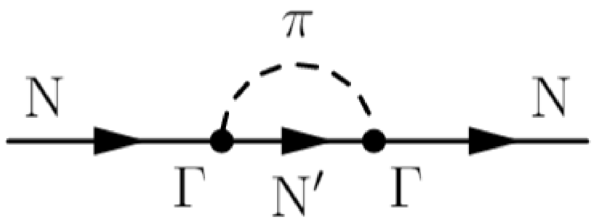
- Apply finite range regularization prescription in the form of a dipole form factor.
- Considers the finite structure of the baryons and offers better convergence than other prescriptions.
- Opposed to dimensional regularization, which is better for point like-particles

$$F(k,\Lambda) = \left(\frac{m_\phi^2 - \Lambda^2}{k^2 - \Lambda^2}\right)^2$$



$N \to N\pi$ Result

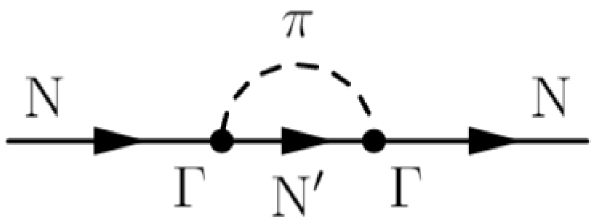
- Simplest result in all its glory is still long.
  - Some important terms show up though.



$$\Sigma_{N\to N\pi} = -\frac{g_A^2}{(8\pi f_\pi)^2}\frac{1}{M\Lambda^3(4M^2-\Lambda^2)^{5/2}}$$
$$\times\Bigg\{\Lambda\sqrt{4M^2-\Lambda^2}\Bigg[M^2\widetilde{\Lambda}^4\Big(m_\pi^2(\Lambda^2-10M^2)+2\Lambda^2(\Lambda^2-M^2)\Big)$$
$$+3\Lambda^2 m_\pi^2(4M^2-\Lambda^2)^2\Bigg(m_\pi^2\log\frac{m_\pi^2}{\Lambda^2}$$
$$+2m_\pi\sqrt{4M^2-m_\pi^2}\Bigg[\tan^{-1}\frac{m_\pi}{\sqrt{4M^2-m_\pi^2}}+\tan^{-1}\frac{2M^2-m_\pi^2}{m_\pi\sqrt{4M^2-m_\pi^2}}\Bigg]\Bigg)\Bigg]$$
$$+6\Big(2\Lambda^6M^4(\Lambda^2-2M^2)-\Lambda^2m_\pi^4(\Lambda^2-6M^2)(\Lambda^4-4\Lambda^2M^2+6M^4)$$
$$+2\Lambda^4M^2m_\pi^2(\Lambda^4-10\Lambda^2M^2+18M^4)-4M^6m_\pi^6\Big)\Bigg[\tan^{-1}\frac{\Lambda^2-2M^2}{\Lambda\sqrt{4M^2-\Lambda^2}}-\tan^{-1}\frac{\Lambda}{\sqrt{4M^2-\Lambda^2}}\Bigg]\Bigg\}.$$

$N \to N\pi$ Result

- Simplest result in all its glory is still long.
  - Some important terms show up though.



$$
\begin{aligned}
\Sigma_{N\to N\pi} = &-\frac{g_A^2}{(8\pi f_\pi)^2}\frac{1}{M\Lambda^3(4M^2-\Lambda^2)^{5/2}} \\
&\times\Bigg\{\Lambda\sqrt{4M^2-\Lambda^2}\Bigg[M^2\widetilde{\Lambda}^4\Big(m_\pi^2(\Lambda^2-10M^2)+2\Lambda^2(\Lambda^2-M^2)\Big) \\
&\quad+3\Lambda^2 m_\pi^2(4M^2-\Lambda^2)^2\Bigg(m_\pi^2\log\frac{m_\pi^2}{\Lambda^2} \\
&\qquad+2m_\pi\sqrt{4M^2-m_\pi^2}\left[\tan^{-1}\frac{m_\pi}{\sqrt{4M^2-m_\pi^2}}+\tan^{-1}\frac{2M^2-m_\pi^2}{m_\pi\sqrt{4M^2-m_\pi^2}}\right]\Bigg)\Bigg] \\
&\quad+6\Big(2\Lambda^6M^4(\Lambda^2-2M^2)-\Lambda^2m_\pi^4(\Lambda^2-6M^2)(\Lambda^4-4\Lambda^2M^2+6M^4) \\
&\qquad+2\Lambda^4M^2m_\pi^2(\Lambda^4-10\Lambda^2M^2+18M^4)-4M^6m_\pi^6\Big)\left[\tan^{-1}\frac{\Lambda^2-2M^2}{\Lambda\sqrt{4M^2-\Lambda^2}}-\tan^{-1}\frac{\Lambda}{\sqrt{4M^2-\Lambda^2}}\right]\Bigg\}.
\end{aligned}
$$

Leading Nonanalytic Behavior

- An important consistency check with the theory is to ensure that the self-energies have the same leading nonanalytic (LNA) terms predicted by QCD.

- "Nonanalytic terms" are odd powers of the meson masses or logarithms of the meson mass.

- Other analytic terms depend on the model and/or regularization prescription.

- Meson masses squared are proportional to the quark masses.

$$m_l \propto \frac{m_\pi^2}{2} \qquad m_s \propto m_K^2 - \frac{m_\pi^2}{2}.$$

LNA Results

- The LNA for all octet and decuplet baryons of the form

$$\Sigma^{\mathrm{LNA}}_{\mathcal{B}\mathcal{B}'\phi} = -\frac{2\mathcal{C}^{\mathrm{LNA}}_{\mathcal{B}\mathcal{B}'}}{\pi f_\phi^2}\, m_\phi^3, \qquad \Delta_{\mathcal{B}\mathcal{B}'} < m_\phi.$$

$$\Sigma^{\mathrm{LNA}}_{\mathcal{B}\mathcal{B}'\phi} = \frac{3\,\mathcal{C}^{\mathrm{LNA}}_{\mathcal{B}\mathcal{B}'}}{4\pi^2 f_\phi^2}\frac{m_\phi^4}{\Delta_{\mathcal{B}'\mathcal{B}}}\log m_\phi^2\,, \qquad \Delta_{\mathcal{B}'\mathcal{B}} > m_\phi.$$

$$\Delta_{\mathcal{B}'\mathcal{B}} = M_{\mathcal{B}'} - M_{\mathcal{B}}$$

- Reproduces the LNA of $N \to \pi N$, $N \to \pi\Delta$, $\Delta \to \pi N$, $\Delta \to \pi\Delta$ found in the literature [1].

- Can also now compute the LNA of any other self-energy.

$\mathcal{C}^{\mathrm{LNA}}_{\mathcal{B}\mathcal{B}'}$	B'	T'
B	$\frac{1}{16}C^2_{BB'\phi}$	$\frac{1}{24}C^2_{BT'\phi}$
T	$\frac{1}{48}C^2_{TB'\phi}$	$\frac{5}{144}C^2_{TT'\phi}$

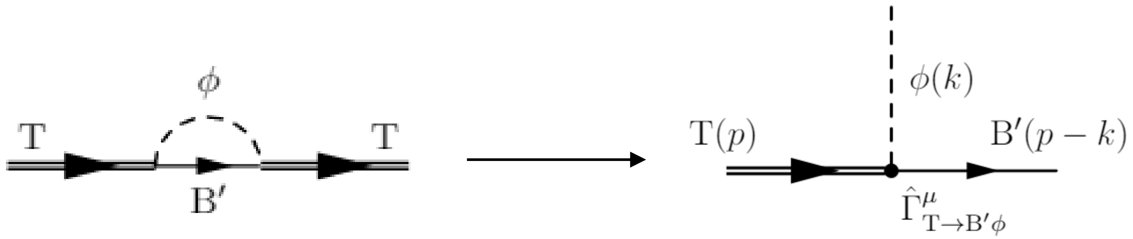
[2] V. Pascalutsa and M. Vanderhaeghen, Phys. Lett. B 636, 31 (2006).

Decay Rates

- If $M_{\mathcal{B}} > m_\phi + M_{\mathcal{B}'}$ then the self-energy gains an imaginary term, and the external baryon can decay into a baryon-meson pair via the strong interaction.

- The self-energies are related to the decay rates of baryons through $\Gamma_{\mathcal{B}\mathcal{B}'\phi} = -2\,\Im\mathrm{m}\,\Sigma_{\mathcal{B}\to\mathcal{B}'\phi}$ (like optical theorem).

- Kinematically $\Delta \to N\pi,\ \Sigma^* \to \Lambda\pi,\ \Sigma^* \to \Sigma\pi$, and $\Xi^* \to \Xi\pi$ are all allowed to decay.

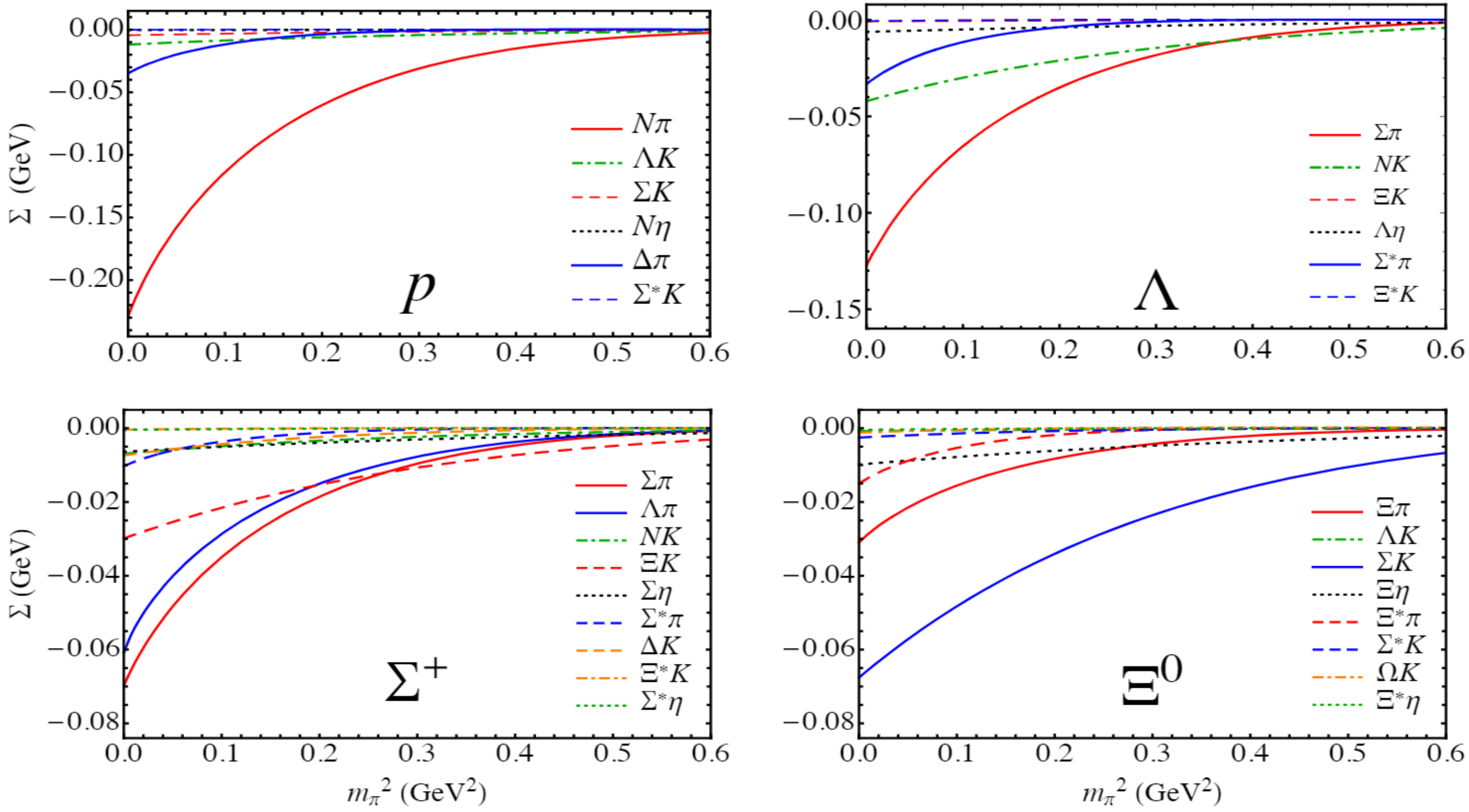


Decay Rate Equations

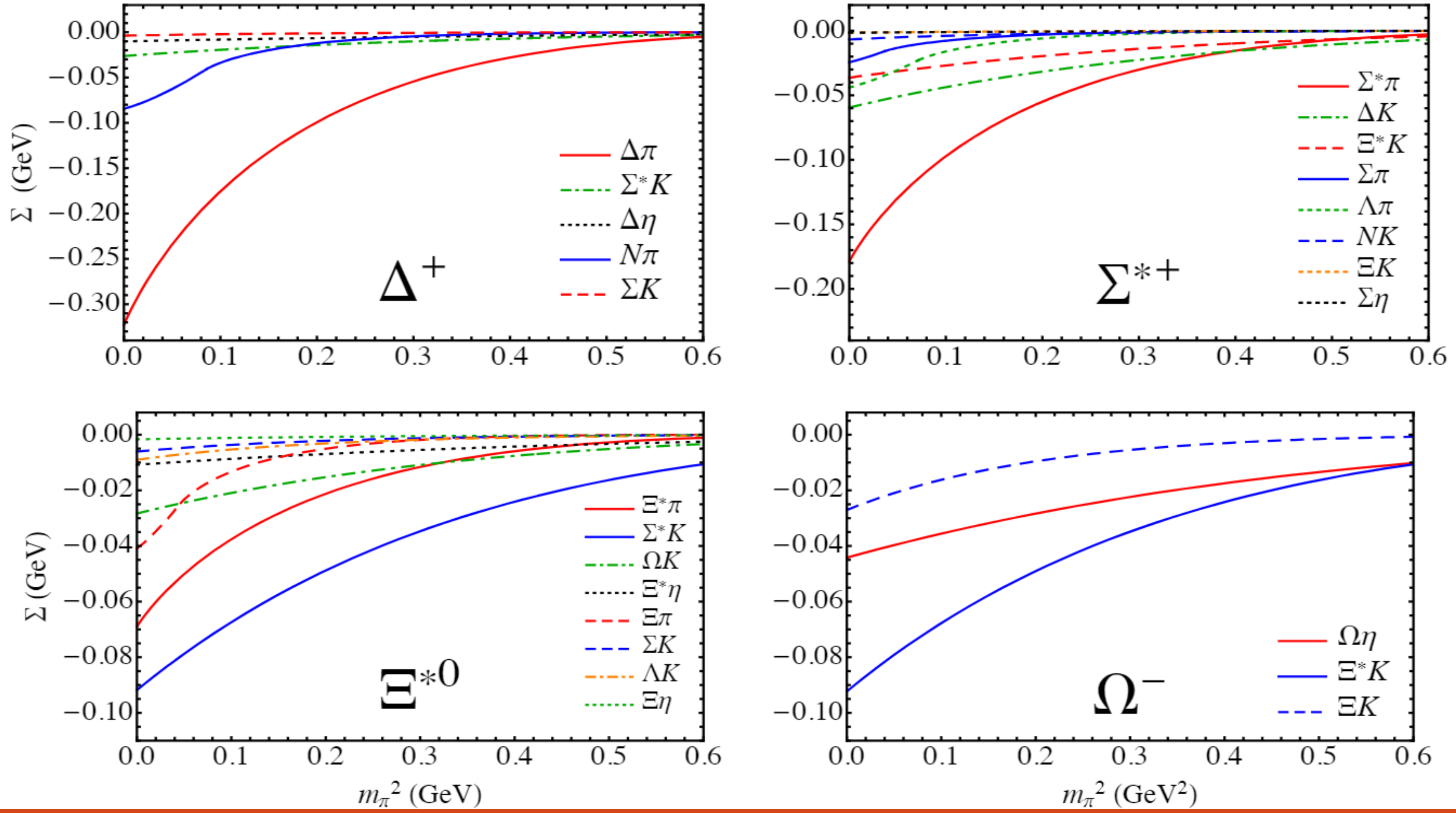
$$\Delta_{\mathcal{B}'\mathcal{B}} = M_{\mathcal{B}'} - M_{\mathcal{B}}$$
$$M_{\mathcal{B}\mathcal{B}'} = M_{\mathcal{B}'} + M_{\mathcal{B}}$$

$$\begin{aligned}
\Gamma_{BB'\phi} &= \frac{C^2_{BB'\phi}}{16\pi f_\phi^2}\frac{\overline{M}^2_{BB'}}{M_B^3}\left(\Delta^2_{B'B} - m_\phi^2\right)^{3/2}\left(\overline{M}^2_{BB'} - m_\phi^2\right)^{1/2} \\
\Gamma_{BT'\phi} &= \frac{C^2_{BT'\phi}}{72\pi f_\phi^2}\frac{1}{M_B^3 M_{T'}^2}\left(\Delta^2_{T'B} - m_\phi^2\right)^{3/2}\left(\overline{M}^2_{BT'} - m_\phi^2\right)^{5/2} \\
\Gamma_{TB'\phi} &= \frac{C^2_{TB'\phi}}{192\pi f_\phi^2}\frac{1}{M_T^5}\left(\Delta^2_{B'T} - m_\phi^2\right)^{3/2}\left(\overline{M}^2_{TB'} - m_\phi^2\right)^{5/2} \\
\Gamma_{TT'\phi} &= \frac{C^2_{TT'\phi}}{288\pi f_\phi^2}\frac{\overline{M}^2_{T'T}}{M_T^5 M_{T'}^2}\left(\Delta^2_{T'T} - m_\phi^2\right)^{3/2}\left(\overline{M}^2_{TT'} - m_\phi^2\right)^{1/2} \\
&\qquad \times\left[\left(\Delta^2_{T'T} - m_\phi^2 + M_T M_{T'}\right)^2 + 9M_T^2 M_{T'}^2\right].
\end{aligned}$$

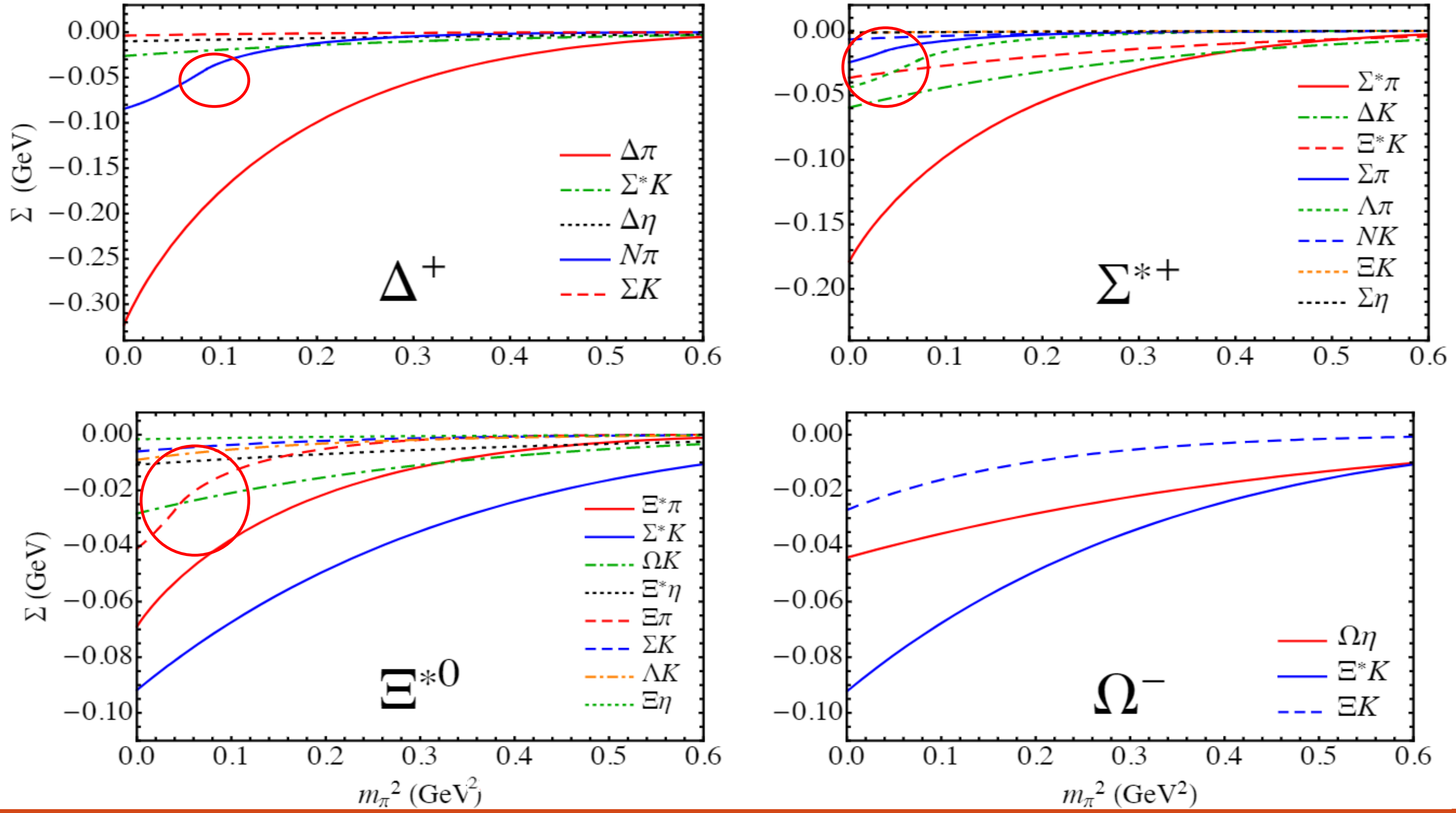
Octet as a Function of the Pion Mass



Decuplet as a Function of Pion Mass

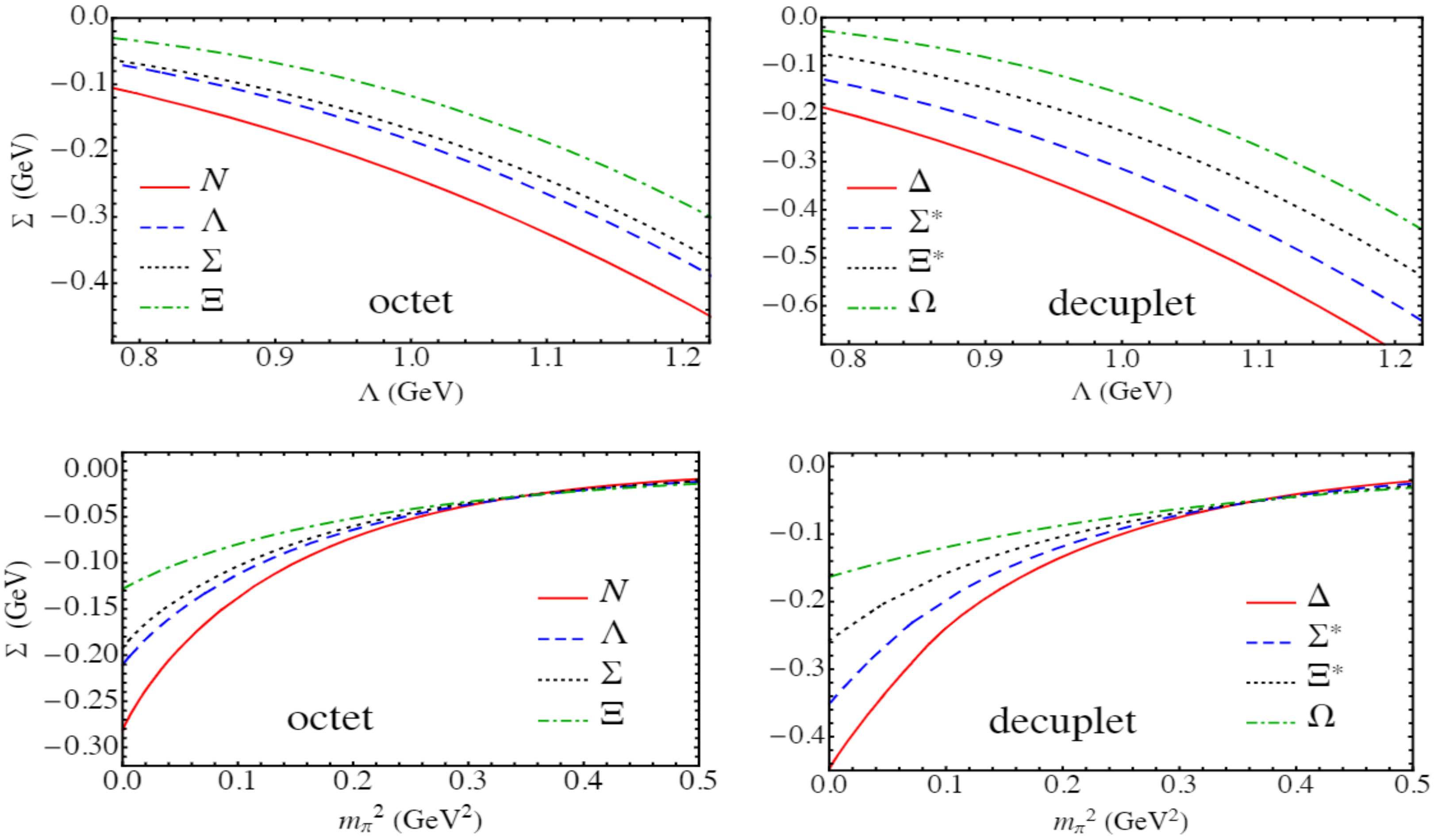


Decuplet as a Function of Pion Mass



$$M_{\mathcal{B}} > m_\phi + M_{\mathcal{B}'}$$

Total Self-Energies of Baryons



Applications of Self-Energies

- These results are presented in our paper [3].
- These meson loops give higher order corrections to many baryon properties.
- An immediate application of the self-energies is to determine higher order corrections to the masses.

[3] P. M. Copeland, C.-R. Ji, W. Melnichouk, Phys. Rev. D **103**, 094019 (2021).

Expansion of Baryon Mass

- Baryon masses can be expanded in powers of the quark mass (or meson mass squared):

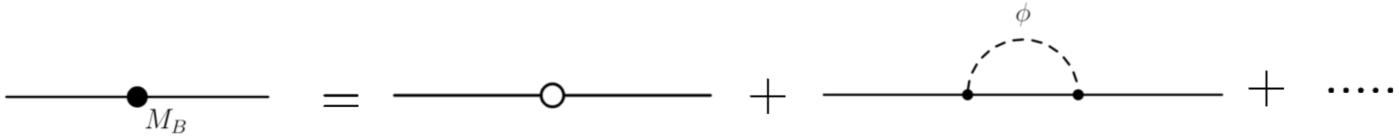
$$M_{\mathcal{B}} = M_{\mathcal{B}}^{(0)} + \delta M_{\mathcal{B}}^{(1)} + \delta M_{\mathcal{B}}^{(3/2)} + \ldots$$

- $M_{\mathcal{B}}^{(0)}$ is the baryon mass in the chiral limit.

- The linear correction term is : $\delta M_{\mathcal{B}}^{(1)} = -C_{\mathcal{B}l}^{(1)} m_l - C_{\mathcal{B}s}^{(1)} m_s$.

- Quark masses are proportional to meson masses squared:

$$m_l = (m_u + m_d)/2 \propto \frac{m_\pi^2}{2} \qquad m_s \propto m_K^2 - \frac{m_\pi^2}{2}.$$



SU(3) Symmetric Scheme

- Formally, the linear terms have SU(3) symmetric coefficients.

- The parameters and masses in the equations are all **shared.**
  - Otherwise, SU(3) symmetry is broken.
  - Requires a global fit to the data.

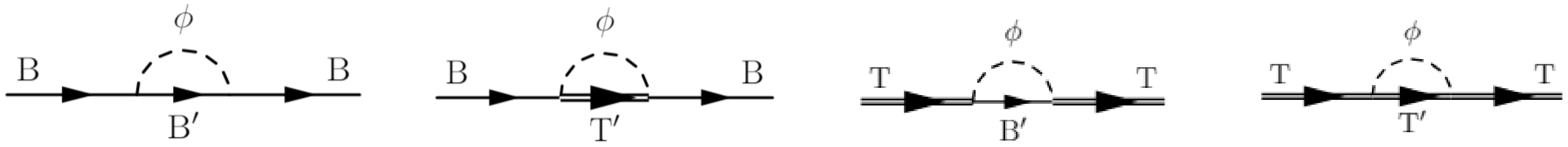
- Use the same average octet mass and average decuplet mass in all self-energy equations

B	$C^{(1)}_{\mathcal{B}_l}$	$C^{(1)}_{\mathcal{B}_s}$
N	$2\alpha + 2\beta + 4\sigma$	2σ
Λ	$\alpha + 2\beta + 4\sigma$	$\alpha + 2\sigma$
Σ	$\frac{5}{3}\alpha + \frac{2}{3}\beta + 4\sigma$	$\frac{1}{3}\alpha + \frac{4}{3}\beta + 2\sigma$
Ξ	$\frac{1}{3}\alpha + \frac{4}{3}\beta + 4\sigma$	$\frac{5}{3}\alpha + \frac{2}{3}\beta + 2\sigma$
Δ	$2\gamma - 4\overline{\sigma}$	$2\overline{\sigma}$
Σ^*	$\frac{4}{3}(\gamma - 3\overline{\sigma})$	$\frac{2}{3}(\gamma - 3\overline{\sigma})$
Ξ^*	$\frac{2}{3}(\gamma - 6\overline{\sigma})$	$\frac{2}{3}(2\gamma - 3\overline{\sigma})$
Ω	$2\gamma - 2\overline{\sigma}$	$4\overline{\sigma}$

[4] A. Walker-Loud, Nucl. Phys. **A747**, 476-507 (2005).

Renormalizing Self-Energies

- In the SU(3) scheme, the meson loop corrections are the renormalized baryon self-energies.
- Subtract off the analytic m_ϕ^0 and m_ϕ^2 terms: $\overline{\Sigma}_{\mathcal{B}\mathcal{B}'\phi} = \Sigma_{\mathcal{B}\mathcal{B}'\phi} - \left(\Sigma_{\mathcal{B}\mathcal{B}'\phi} \Big|_{m_\phi=0} \right) - m_\phi^2 \left(\frac{\partial \Sigma_{\mathcal{B}\mathcal{B}'\phi}}{\partial m_\phi^2} \Big|_{m_\phi=0} \right)$
- Sum all intermediate states: $\delta M_{\mathcal{B}}^{(3/2)} = \sum_{\mathcal{B}',\phi} \overline{\Sigma}_{\mathcal{B}\mathcal{B}'\phi}.$



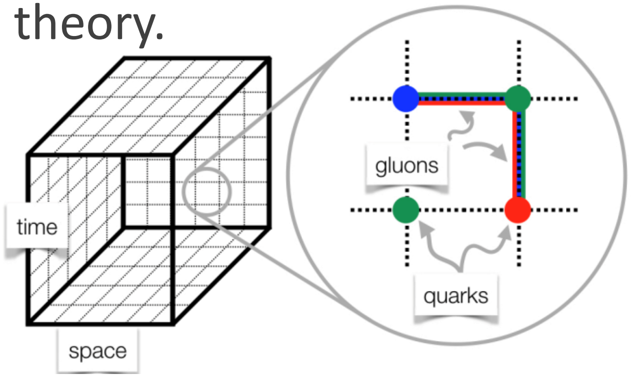
Generalized Scheme

- The SU(3) symmetric scheme may be seen as an over extension of the theory.
- To study model dependence, we also make a generalized scheme in which the coefficients of linear terms are arbitrary.
- The meson loop contributions are then the original self-energies (non-renormalized).
- No parameters are shared by any of the baryons, so the fits are performed individually.

$$\delta M_{\mathcal{B}}^{(1)} = -C_{\mathcal{B}l}^{(1)} m_l - C_{\mathcal{B}s}^{(1)} m_s \qquad \delta M_{\mathcal{B}}^{(3/2)} = \sum_{\mathcal{B}',\phi} \Sigma_{\mathcal{B}\mathcal{B}'\phi}.$$

Crash course on Lattice QCD

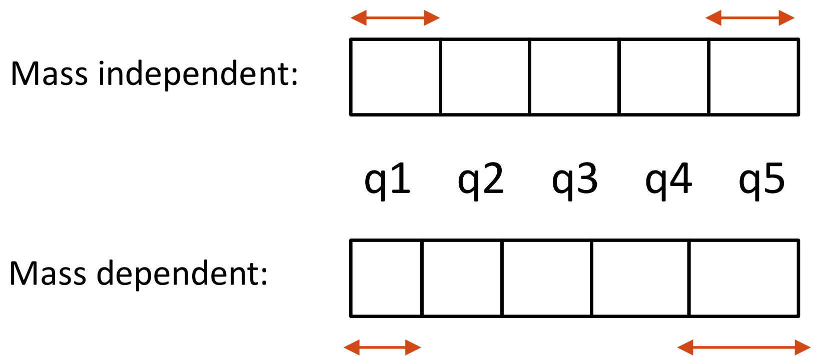
- Lattice QCD is a non perturbative way to make predictions from QCD.
- Input the action of QCD into a computer to simulate predictions from the theory.
- “Discretize” space-time and compute observables on the lattice.



- Very computationally expensive, typically done at larger than physical values of the pion mass.
- Need effective field theories to extrapolate to the physical point and to the continuum limit.

Scale Setting in Lattice QCD

- Lattice data are computed in dimensionless lattice units.
- Quantities are multiplied by the lattice spacing, “a” (ex: aM_N).
- Two different ways to think about the lattice spacing.



- Mass independent (MILS) - the lattice spacing stays the same at all values of the quark mass.
- Mass dependent (MDLS) - the lattice spacing changes with the quark mass in order to keep an external physical quantity the same at all simulation values (ex: Sommer scale).

Fitting to Lattice QCD Data

- Need to fit to lattice QCD to determine the fitting parameters.

- Use the PACS-CS [5] and QCDSF-UKQCD baryon mass data [6].

- Choose these data sets because they study all the baryon masses as function of the light quarks (up and down) AND the strange quark.
  - Important for extracting strange sigma terms.

[5] S. Aoki et al. [PACS-CS Collaboration], Phys Rev. **D79**, 034503 (2009).
[6] W. Bietenholz et al., [QCDSF-UKQCD Collaboration] Phys. Rev. **D84**, 054509 (2011).

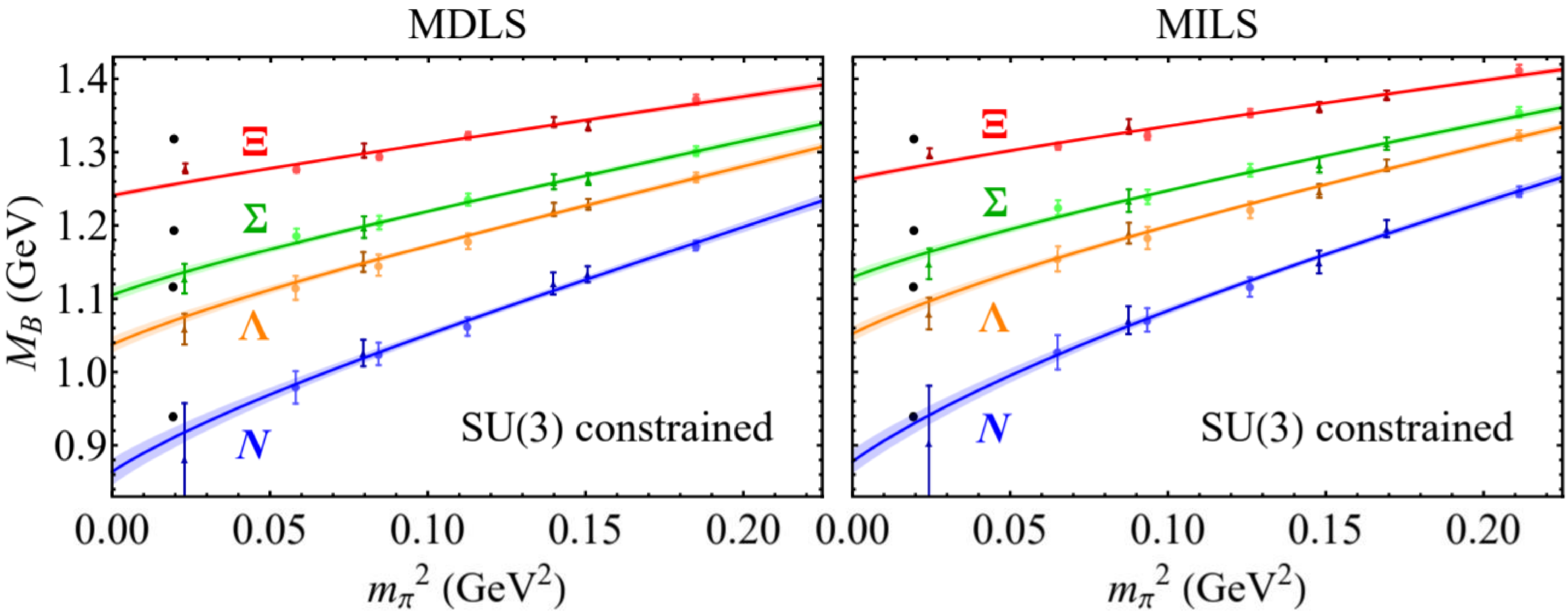
Fitting to Lattice QCD Data (cont'd)

- Fit in the light-strange quark mass planes by fitting the baryon masses to the array of pion, kaon data.

- Only consider $m_\phi^2 < 0.25$ GeV^2 (less susceptible to scale setting scheme).

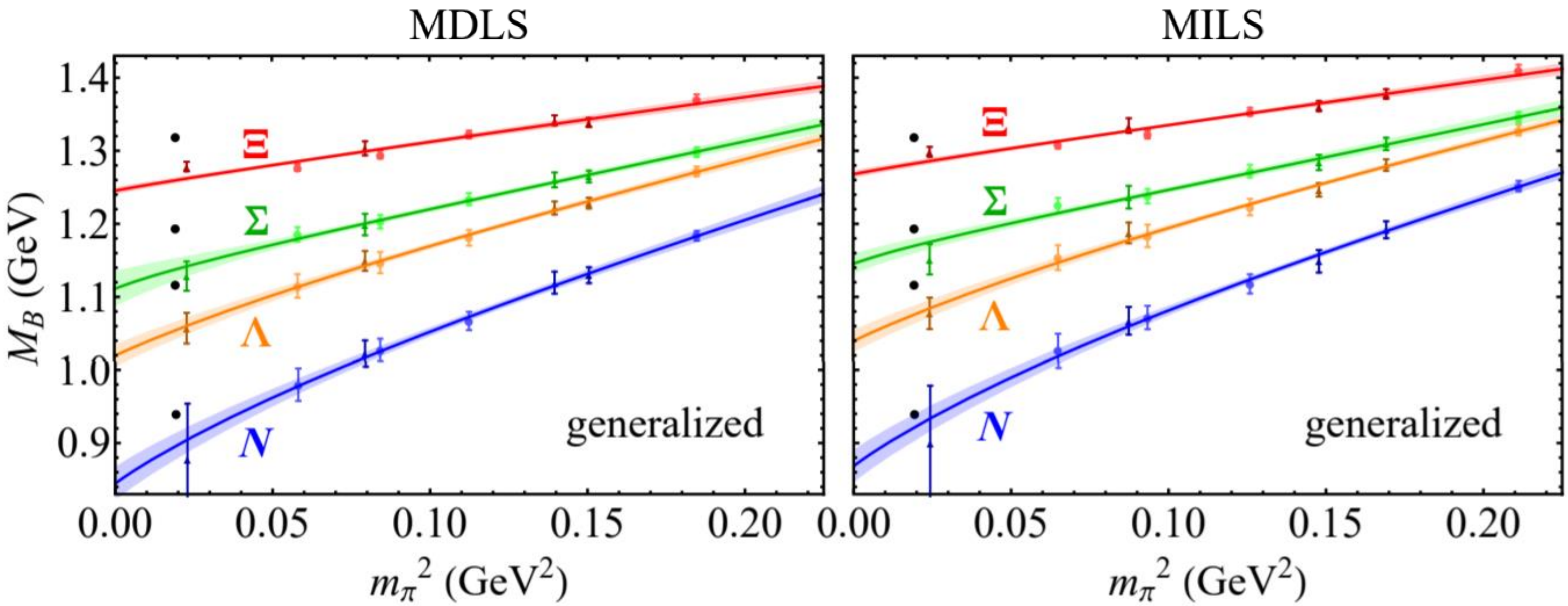
Octet fits – SU(3)

- Perform a simultaneous fit to determine the shared parameters $M_B^{(0)},\ \alpha,\ \beta,\ \sigma$ and also the Λ_B's.



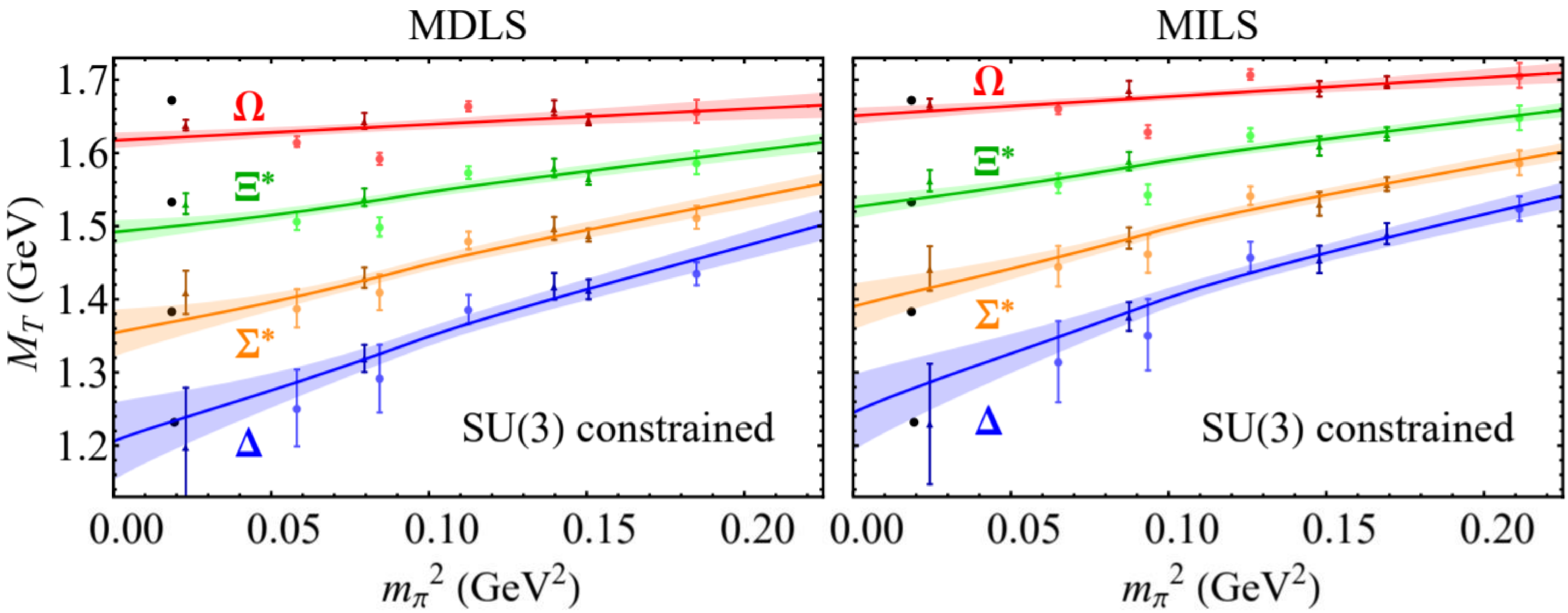
Octet fits - Generalized

- Perform a fit to determine the many individual parameters.



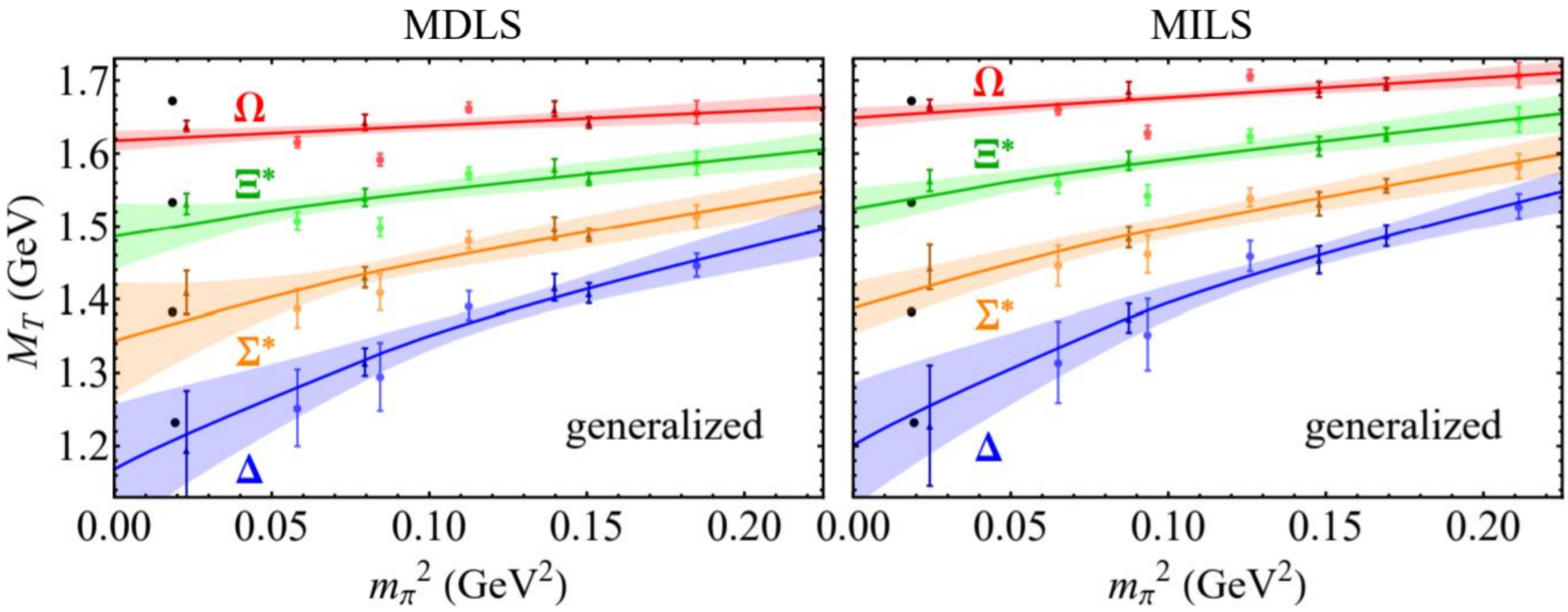
Decuplet Fits - SU(3)

- Perform a global fit to determine the shared parameters $M_T^{(0)}$, $\overline{\sigma}$, γ, and Λ_T‘s.



Decuplet Fits - Generalized

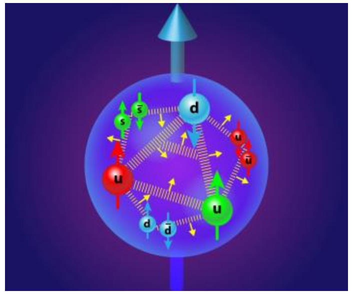
- Likewise, fit generalized expansion for decuplet to determine those parameters.



Sigma Terms

- Scalar matrix elements that quantify baryon mass dependence on the quark masses.

- Crucial for understanding chiral symmetry breaking, interpreting dark matter experiments, and understanding the origin of baryon masses.



- Use Feynman-Hellman theorem to determine sigma terms from masses.

$$\sigma_{\mathcal{B}q} = m_q \langle \mathcal{B} | \overline{q} q | \mathcal{B} \rangle \rightarrow \quad \sigma_{\mathcal{B}q} = m_q \frac{\partial M_{\mathcal{B}}}{\partial m_q}$$

$$\sigma_{\mathcal{B}l} = m_l \frac{\partial M_{\mathcal{B}}}{\partial m_l} = m_\pi^2 \frac{\partial M_{\mathcal{B}}}{\partial m_\pi^2}$$

$$\sigma_{\mathcal{B}l} = m_s \frac{\partial M_{\mathcal{B}}}{\partial m_s} = (m_K^2 - \frac{m_\pi^2}{2}) \frac{\partial M_{\mathcal{B}}}{\partial m_K^2}$$

[7] P. E. Shanahan, A. W. Thomas, R. D. Young, Phys. Rev. **D87,** 074503 (2013)

[8] R. D. Young, A. W. Thomas, Phys. Rev. **D81**, 034503 (2010).

Octet Sigma Term Results

- Plugging the fit parameters back into the mass expansions and taking the derivatives gives the light and strange sigma terms for the baryons.

	B	MDLS $\sigma_{B\ell}$ (MeV)	MDLS σ_{Bs} (MeV)	MDLS M_B (MeV)	MDLS χ^2_{dof}	MILS $\sigma_{B\ell}$ (MeV)	MILS σ_{Bs} (MeV)	MILS M_B (MeV)	MILS χ^2_{dof}
SU(3) constrained									
	N	41(4)	50(8)	910(14)	0.89	46(3)	49(8)	930(14)	0.78
	Λ	29(2)	203(7)	1 070(8)		32(2)	189(7)	1 089(8)	
	Σ	24(2)	263(7)	1 132(7)		26(2)	248(7)	1 158(7)	
	Ξ	14(1)	376(7)	1 257(3)		15(1)	353(6)	1 280(4)	
generalized									
	N	46(5)	68(15)	896(18)	0.82	47(4)	59(14)	921(18)	0.75
	Λ	32(7)	212(12)	1 055(12)		34(3)	200(13)	1 076(12)	
	Σ	23(8)	257(11)	1 138(15)		21(3)	228(22)	1 170(10)	
	Ξ	14(1)	379(16)	1 260(4)		14(1)	348(17)	1 283(4)	

Octet Sigma Term Results

- Plugging the fit parameters back into the mass expansions and taking the derivatives gives the light and strange sigma terms for the baryons.

	B	MDLS $\sigma_{B\ell}$ (MeV)	MDLS σ_{Bs} (MeV)	MDLS M_B (MeV)	MDLS χ^2_{dof}	MILS $\sigma_{B\ell}$ (MeV)	MILS σ_{Bs} (MeV)	MILS M_B (MeV)	MILS χ^2_{dof}
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generalized									
	N	46(5)	68(15)	896(18)	0.82	47(4)	59(14)	921(18)	0.75
	Λ	32(7)	212(12)	1 055(12)		34(3)	200(13)	1 076(12)	
	Σ	23(8)	257(11)	1 138(15)		21(3)	228(22)	1 170(10)	
	Ξ	14(1)	379(16)	1 260(4)		14(1)	348(17)	1 283(4)	

Octet Sigma Term Results

- Plugging the fit parameters back into the mass expansions and taking the derivatives gives the light and strange sigma terms for the baryons.

	B	MDLS				MILS			
		$\sigma_{B\ell}$ (MeV)	σ_{Bs} (MeV)	M_B (MeV)	χ^2_{dof}	$\sigma_{B\ell}$ (MeV)	σ_{Bs} (MeV)	M_B (MeV)	χ^2_{dof}
SU(3) constrained									
	N	41(4)	50(8)	910(14)		46(3)	49(8)	930(14)	
	Λ	29(2)	203(7)	1 070(8)	0.89	32(2)	189(7)	1 089(8)	0.78
	Σ	24(2)	263(7)	1 132(7)		26(2)	248(7)	1 158(7)	
	Ξ	14(1)	376(7)	1 257(3)		15(1)	353(6)	1 280(4)	
generalized									
	N	46(5)	68(15)	896(18)		47(4)	59(14)	921(18)	
	Λ	32(7)	212(12)	1 055(12)	0.82	34(3)	200(13)	1 076(12)	0.75
	Σ	23(8)	257(11)	1 138(15)		21(3)	228(22)	1 170(10)	
	Ξ	14(1)	379(16)	1 260(4)		14(1)	348(17)	1 283(4)	

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	Ξ	14(1)	379(16)	1 260(4)		14(1)	348(17)	1 283(4)	

Decuplet Sigma Term Results

- Plugging the fit parameters back into the mass expansions and taking the derivatives gives the light and strange sigma terms for the decuplet baryons.

	T	MDLS $\sigma_{T\ell}$ (MeV)	MDLS σ_{Ts} (MeV)	MDLS M_T (MeV)	MDLS χ^2_{dof}	MILS $\sigma_{T\ell}$ (MeV)	MILS σ_{Ts} (MeV)	MILS M_T (MeV)	MILS χ^2_{dof}
SU(3) constrained									
	Δ	26(13)	69(17)	1 240(38)	3.29	31(11)	64(15)	1 285(40)	2.70
	Σ^*	16(9)	198(18)	1 363(17)		19(7)	180(14)	1 407(20)	
	Ξ^*	8(5)	322(19)	1 499(9)		11(4)	292(16)	1 540(9)	
	Ω	4(2)	437(19)	1 645(10)		5(2)	398(20)	1 681(12)	
generalized									
	Δ	38(17)	88(50)	1 209(71)	4.39	40(15)	71(48)	1 245(70)	3.61
	Σ^*	23(23)	199(46)	1 368(55)		24(8)	171(46)	1 412(29)	
	Ξ^*	13(14)	321(39)	1 500(31)		14(6)	288(45)	1 539(23)	
	Ω	4(3)	435(25)	1 622(11)		5(3)	400(38)	1 655(12)	

Comparison with other σ_{qN}'s

- $\sigma_{\pi N}$ tension between pionic atom scattering experiments and lattice results/chiral extrapolations.
  - $\sigma_{\pi N}$ ~ 60 MeV (pionic atoms) vs. 45 MeV (lattice)

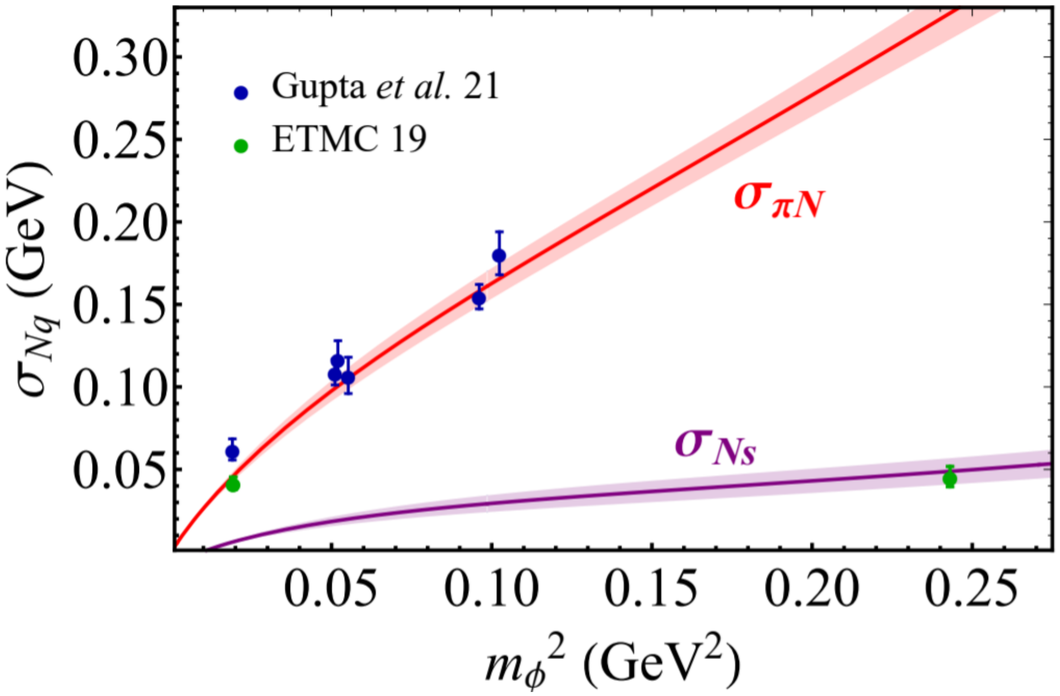
- Gupta et. al. [9] recently proposed considering excited states (πN and $\pi\pi N$) on lattice with the help of ordinary χ-PT and found some agreement between experimental results.

- [9] R. Gupta et. al., Phy. Rev. Lett. 127, 242002 (2021)

Comparison with Gupta et. al.

- When we compare with Gupta et. al. we find agreement at larger m_π^2 but not at the physical point.

- Would like to see a new analysis that uses full chiral SU(3) effective theory as guiding theory and also want to consider strange sigma terms.



- [10] R. Gupta et. al., Phy. Rev. Lett. 127, 242002 (2021)

Previous FRR analysis

- Shanahan et. al. [8] performed similar analysis on the octet using heavy baryon chiral effective theory.

- Similar light sigma term results ($\sigma_{\pi N}$ ~ 51(3)(6) MeV).

- However, σ_{Ns} ranged between 21(6)(0) and 59(6)(1) MeV for the MDLS and MILS schemes. respectively.

- Compare with our σ_{Ns} = 50(8) vs 49(8) MeV for the MDLS and MILS schemes.

- [8] P. E. Shanahan, A. W. Thomas, R. D. Young, Phys. Rev. **D87,** 074503 (2013).

Mass decompositions

- The origin of the proton’s mass is one of the most important and hotly debated topics right now.
- Several potential decompositions - from Hamiltonian, trace, or gravitational form factors.
- Sigma term contribution to the mass is the same regardless of scheme!

$$M_{\mathcal{B}} = \frac{\left\langle \mathcal{B} \right| \int d^3\mathbf{x} T^{00} \left| \mathcal{B} \right\rangle}{\langle \mathcal{B} | \mathcal{B} \rangle} = \frac{\langle \mathcal{B} | H_{QCD} | \mathcal{B} \rangle}{\langle \mathcal{B} | \mathcal{B} \rangle} = \sum_q \left(\langle x \rangle_q^E M_{\mathcal{B}} + \sigma_{\mathcal{B}q} \right) + \frac{1}{4} f_{\mathcal{B}a} M_{\mathcal{B}} + \frac{3}{4} \langle x \rangle_g M_{\mathcal{B}}$$

$$M_{\mathcal{B}} = \frac{\left\langle \mathcal{B} \right| T^{\mu}_{\mu} \left| \mathcal{B} \right\rangle}{\langle \mathcal{B} | \mathcal{B} \rangle} = \sum_q \sigma_{\mathcal{B}q} + f_{\mathcal{B}a} M_{\mathcal{B}}$$

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Sigma Term Contributions to Masses

- Define quark mass fraction, trace anomaly, and quark energy.

$$f_{\mathcal{B}q} = \frac{\sigma_{\mathcal{B}q}}{M_{\mathcal{B}}} \qquad f_{\mathcal{B}a} = 1 - \sum_q f_{\mathcal{B}q} \qquad \langle x \rangle^E_{\mathcal{B}q} = \frac{3}{4}\left(\langle x \rangle_{\mathcal{B}q} - f_{\mathcal{B}q}\right)$$

$\mathcal{B}$ (MeV)	$f_{B\ell}$	f_{Bs}	$f_{\mathcal{B}a}$	$\langle x \rangle^E_{\mathcal{B}q} + \frac{3}{4}\langle x \rangle_{\mathcal{B}g}$
$N(939)$	0.047(3)(3)	0.053(6)(1)	0.900(7)(3)	0.676(6)(3)
$\Lambda(1116)$	0.028(1)(2)	0.176(4)(6)	0.796(4)(6)	0.599(4)(6)
$\Sigma(1193)$	0.021(1)(1)	0.215(4)(6)	0.764(4)(6)	0.577(4)(6)
$\Xi(1318)$	0.011(1)(1)	0.277(4)(10)	0.712(4)(10)	0.533(4)(10)
$\Delta(1232)$	0.024(9)(2)	0.054(9)(2)	0.921(13)(3)	0.692(13)(3)
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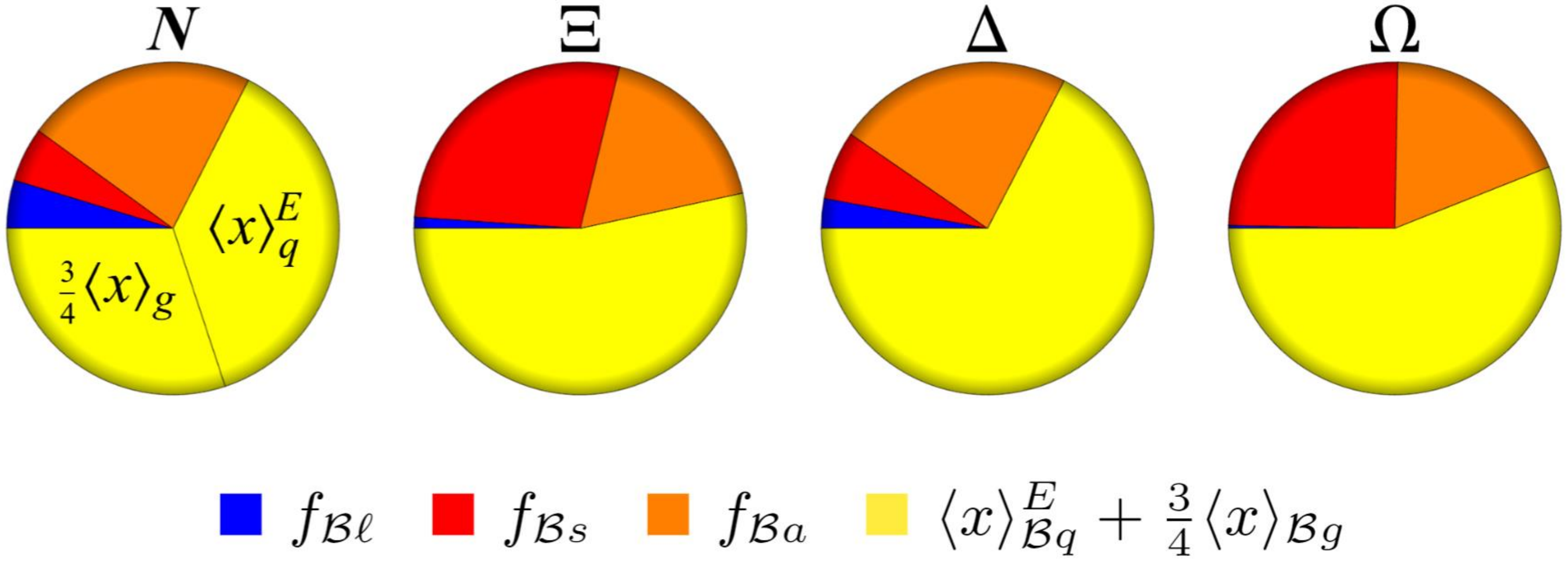
Decomposition of baryon masses

- Due to larger strange sigma terms, the trace anomaly and quark/gluon energy contributions decrease for strange baryons.



Summary

- Computed all octet and decuplet self-energies within relativistic SU(3) chiral effective theory using finite range regularization.
- Produced the LNA behavior for all self-energies.
- Numerically studied the self-energies as a function of the pion mass squared and the cuff off parameter.
- Expanded the baryon mass in powers of the quark masses (meson masses squared). Utilized baryon self-energies to get the loop corrections to the mass.
- Fit masses to PACS-CS lattice QCD data to determine the parameters in the mass expansion.

Summary

- Used the Feynman-Hellman theorem to derive the light and strange quark sigma terms for all SU(3) octet and decuplet baryons.

- Reduced uncertainty on σ_{Ns} from ≈20 MeV – 60MeV to σ_{Ns} = 50(6)(1).

Outlook

- Want a more comprehensive lattice study of
- Encourage lattice collaborations to compute quark and gluon energy/momentum fractions of other baryons.
- Compute heavy quark (c, b, t) sigma terms of other baryons to understand mass decomposition and trace anomaly at higher energy scales/$N_f = 6$ QCD.
- Apply new results in dark matter models.

References

- S. Scherer and M.R. Schindler, Lect. Notes Phys. 830, 1 (2012).
- V. Pascalutsa and M. Vanderhaeghen, Phys. Lett. B 636, 31 (2006).
- A. Walker-Loud, Nucl. Phys. **A747**, 476-507 (2005).
- P. M. Copeland, C.-R. Ji, W. Melnitchouk, arXiv:2012.07767v1 [hep-ph] (2020).

References (cont'd)

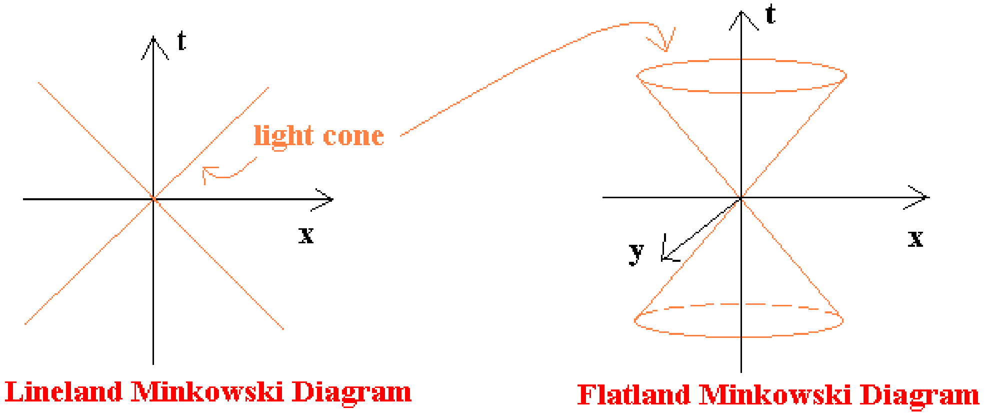
- S. Aoki et al. [PACS-CS Collaboration], Phys Rev. **D79**, 034503 (2009).
- P. E. Shanahan, A. W. Thomas, R. D. Young, Phys. Rev. **D87,** 074503 (2013).
- R. D. Young, A. W. Thomas, Phys. Rev. **D81**, 034503 (2010).

Light front-coordinates

- To evaluate the integrals, it is convenient to make a change of variables to light-front coordinates.

$$v^\mu = (v^+, v^-, \boldsymbol{v}_\perp)$$

- Longitudinal and transverse components: $v^\pm = v_0 \pm v_z$ $\qquad \boldsymbol{v}_\perp^2 = v_x^2 + v_y^2$



Strange quark correction to the data

- Strange quark mass used in lattice simulations are larger than physical.
- Since fits are in light AND strange quark planes, we can extrapolate down to the physical strange mass.