Review and Plan for 2022

Chueng-Ryong Ji
North Carolina State University

Group Meeting on January 7, 2022

Outline

- Review of Tasks

  - Patrick: Global QCD Analysis of Pion PDF

  - Bailing: Interpolating ‘tHooft Model

  - Andrew: Virtual Meson Production in 3+1 Dimensions

  - Deepasika: Quantum Correlation in Light-Front Zero-Modes

  - Hari: Conformal Poincare Symmetry

- Plan for 2022

  - Big Picture

  - Where are we?

  - What can we contribute best?

  - Works to do

PHYSICAL REVIEW LETTERS **127,** 232001 (2021)

Global QCD Analysis of Pion Parton Distributions with Threshold Resummation

P. C. Barry,[1] Chueng-Ryong Ji,[2] N. Sato,[1] and W. Melnitchouk[1]

(JAM Collaboration)

[1]*Jefferson Lab, Newport News, Virginia 23606, USA*
[2]*Department of Physics, North Carolina State University, Raleigh, North Carolina 27695, USA*

(Received 18 August 2021; revised 22 October 2021; accepted 26 October 2021; published 29 November 2021)

We perform the first global QCD analysis of pion valence, sea quark, and gluon distributions within a Bayesian Monte Carlo framework with threshold resummation on Drell-Yan cross sections at next-to-leading log accuracy. Exploring various treatments of resummation, we find that the large-x asymptotics of the valence quark distribution $\sim(1-x)^{\beta_v}$ can differ significantly, with β_v ranging from ≈ 1 to > 2.5 at the input scale. Regardless of the specific implementation, however, the resummation induced redistribution of the momentum between valence quarks and gluons boosts the total momentum carried by gluons to $\approx 40\%$, increasing the gluon contribution to the pion mass to ≈ 40 MeV.

DOI: 10.1103/PhysRevLett.127.232001

Featured in Physics

First Monte Carlo Global QCD Analysis of Pion Parton Distributions

P. C. Barry, N. Sato, W. Melnitchouk, and Chueng-Ryong Ji (Jefferson Lab Angular Momentum (JAM) Collaboration)

Phys. Rev. Lett. **121**, 152001 (2018) – Published 10 October 2018

Physics Synopsis: More Gluons in the Pion

Teaser: A combined analysis of collider data finds that the gluon contribution to the pion is three times larger than earlier estimates.

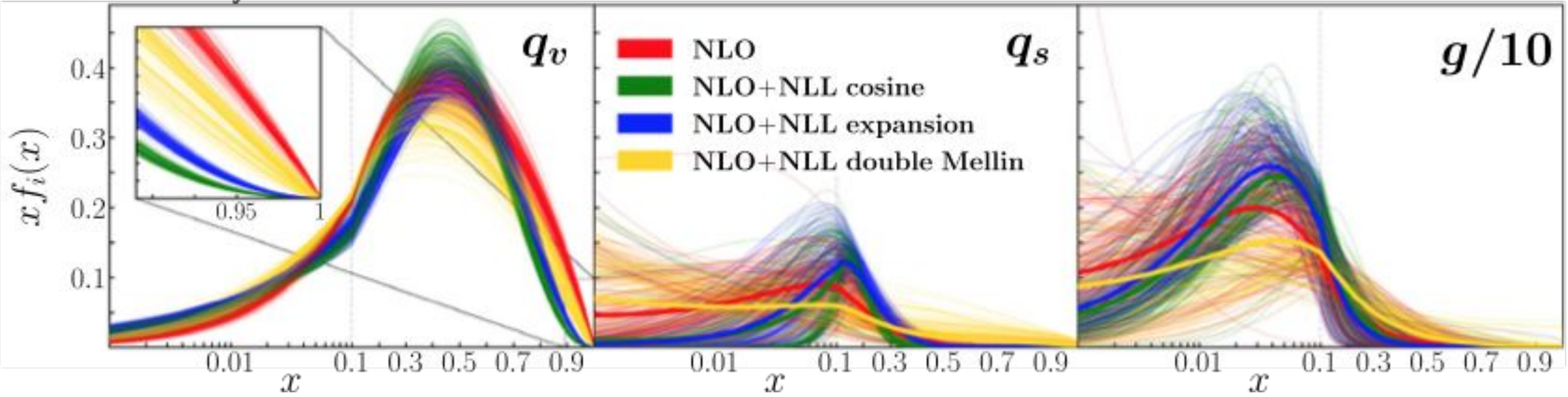
physicsworld

PARTICLE AND NUCLEAR RESEARCH UPDATE

Gluons account for much more pion momentum than previously thought

19 Oct 2018

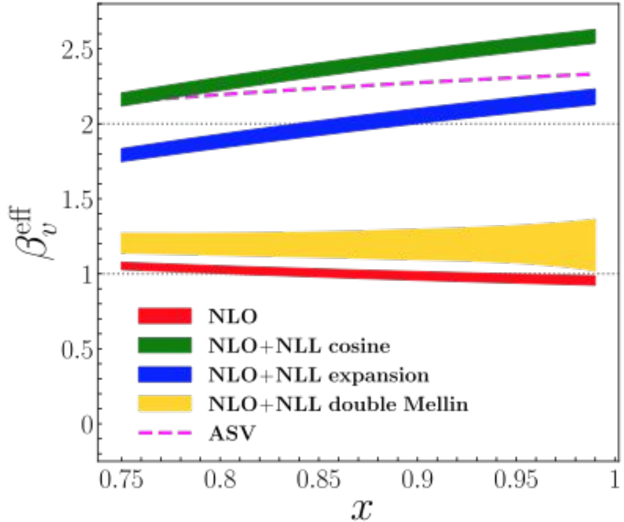
$$\frac{d^2\sigma}{d\tau dY} = \frac{4\pi\alpha^2}{9\tau S^2}\sum_{ij}\int dz \int dy C_{ij}(z,y,\mu/Q)\, f_i^{\pi}(x_\pi,\mu) f_j^{A}(x_A,\mu), \qquad f_i(x,\mu_0;\boldsymbol{a}_i) = N_i x^{\alpha_i}(1-x)^{\beta_i}(1+\gamma_i x^2)$$



$$\frac{d^2\sigma}{d\tau dY} = \int_{-\infty}^{\infty}\frac{dM}{2\pi}e^{-iMY}\int_{C_N^{\mathrm{MP}}}\frac{dN}{2\pi i}\tau^{-N}\sigma_{\mathrm{MF}}(N,M)$$

$$= \int_{C_M^{\mathrm{MP}}}\frac{dM}{2\pi i}(x_A^0)^{-M}\int_{C_N^{\mathrm{MP}}}\frac{dN}{2\pi i}(x_\pi^0)^{-N}\sigma_{\mathrm{DM}}(N,M).$$

$$\beta_v^{\mathrm{eff}}(x,\mu) = \frac{\partial \log|q_v(x,\mu)|}{\partial \log(1-x)}$$



Interpolating 't Hooft model between instant and front forms

Bailing Ma and Chueng-Ryong Ji

Department of Physics, North Carolina State University, Raleigh, North Carolina 27695-8202, USA

(Received 21 May 2021; accepted 15 June 2021; published 16 August 2021)

The 't Hooft model, i.e., the two-dimensional quantum chromodynamics in the limit of infinite number of colors, is interpolated by an angle parameter δ between $\delta = 0$ for the instant form dynamics (IFD) and $\delta = \pi/4$ for the light-front dynamics (LFD). With this parameter δ, we formulate the interpolating mass gap equation which takes into account the nontrivial vacuum effect on the bare fermion mass to find the dressed fermion mass. Our interpolating mass gap solutions not only reproduce the previous IFD result at $\delta = 0$ as well as the previous LFD result at $\delta = \pi/4$ but also link them together between the IFD and LFD results with the δ parameter. We find the interpolation angle independent characteristic energy function which satisfies the energy-momentum dispersion relation of the dressed fermion, identifying the renormalized fermion mass function and the wave function renormalization factor. The renormalized fermion condensate is also found independent of δ, indicating the persistence of the nontrivial vacuum structure even in the LFD. Using the dressed fermion propagator interpolating between IFD and LFD, we derive the corresponding quark-antiquark bound-state equation in the interpolating formulation verifying its agreement with the previous bound-state equations in the IFD and LFD at $\delta = 0$ and $\delta = \pi/4$, respectively. The mass spectra of mesons bearing the feature of the Regge trajectories are found independent of the δ-parameter reproducing the previous results in LFD and IFD for the equal mass quark and antiquark bound states. The Gell-Mann-Oakes-Renner relation for the pionic ground-state in the zero fermion mass limit is confirmed indicating that the spontaneous breaking of the chiral symmetry occurs in the 't Hooft model regardless of the quantization for $0 \leq \delta \leq \pi/4$. We obtain the corresponding bound-state wave functions and discuss their reference frame dependence with respect to the frame independent LFD result. Applying them for the computation of the so-called quasi-parton distribution functions now in the interpolating formulation between IFD and LFD, we note a possibility of utilizing not only the reference frame dependence but also the interpolation angle dependence to get an alternative effective approach to the LFD-like results.

DOI: 10.1103/PhysRevD.104.036004

PHYSICAL REVIEW D **98,** 036017 (2018)

Interpolating quantum electrodynamics between instant and front forms

Chueng-Ryong Ji, Ziyue Li, and Bailing Ma
Department of Physics, North Carolina State University, Raleigh, North Carolina 27695-8202, USA

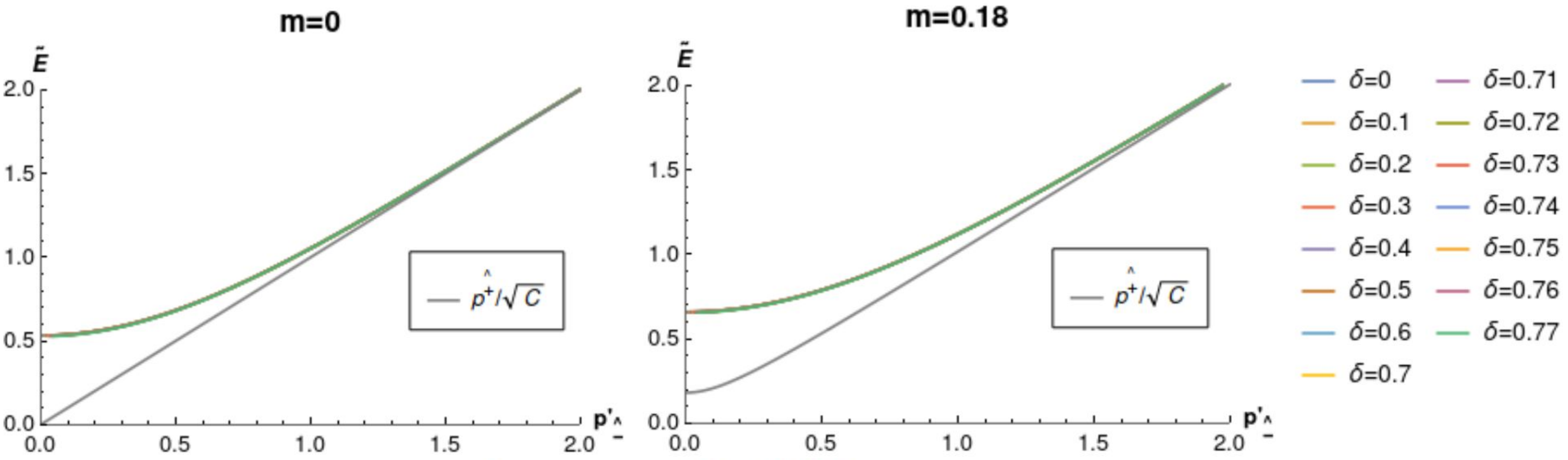
Alfredo Takashi Suzuki
Physics Department, La Sierra University, Riverside, California 92505, USA
and Instituto de Física Teórica-UNESP Universidade Estadual Paulista,
Rua Dr. Bento Teobaldo Ferraz, 271—Bloco II—01140-070, São Paulo, SP, Brazil

(Received 15 May 2018; published 22 August 2018)

The instant form and the front form of relativistic dynamics proposed by Dirac in 1949 can be linked by an interpolation angle parameter δ spanning between the instant form dynamics (IFD) at $\delta = 0$ and the front form dynamics, which is now known as the light-front dynamics (LFD) at $\delta = \pi/4$. We present the formal derivation of the interpolating quantum electrodynamics (QED) in the canonical field theory approach and discuss the constraint fermion degree of freedom, which appears uniquely in the LFD. The constraint component of the fermion degrees of freedom in LFD results in the instantaneous contribution to the fermion propagator, which is genuinely distinguished from the ordinary equal-time forward and backward propagation of relativistic fermion degrees of freedom. As discussed recently, the helicity of the on-mass-shell fermion spinors in LFD is also distinguished from the ordinary Jacob-Wick helicity in the IFD with respect to whether the helicity depends on the reference frame or not. To exemplify the characteristic difference of the fermion propagator between IFD and LFD, we compute the helicity amplitudes of typical QED processes such as $e^+e^- \to \gamma\gamma$ and $e\gamma \to e\gamma$ and present the whole landscape of the scattering amplitudes in terms of the frame dependence or the scattering angle dependence with respect to the interpolating angle dependence. Our analysis clarifies any conceivable confusion in the prevailing notion of the equivalence between the infinite momentum frame approach and the LFD.

DOI: 10.1103/PhysRevD.98.036017

Mass Gap Solutions



$$\tilde{E}(0) = \frac{F(0)E(0)}{\sqrt{\mathbb{C}}} = M(0)$$

m	0	0.045	0.18	0.749	1.00	2.11	4.23
$M(0)$	0.532778	0.563644	0.659112	1.10105	1.31167	2.30969	4.34358
$F(0)$	−0.495173	−0.584175	−0.987673	4.11079	2.17976	1.22134	1.05526

$$m \lesssim 0.56$$

Parton Distribution Functions (PDFs)

$$q_n(x) = \int_{-\infty}^{+\infty} \frac{d\xi^-}{4\pi} e^{-ixP^+\xi^-}$$

$$\times \langle P_n^-, P^+ | \bar{\psi}(\xi^-)\gamma^+ \mathcal{W}[\xi^-, 0]\psi(0) | P_n^-, P^+ \rangle_C,$$

$$\mathcal{W}[\xi^-, 0] = \mathcal{P}\left[\exp\left(-ig_s \int_0^{\xi^-} d\eta^- A^+(\eta^-)\right)\right]$$

A^+=0 Gauge in LFD

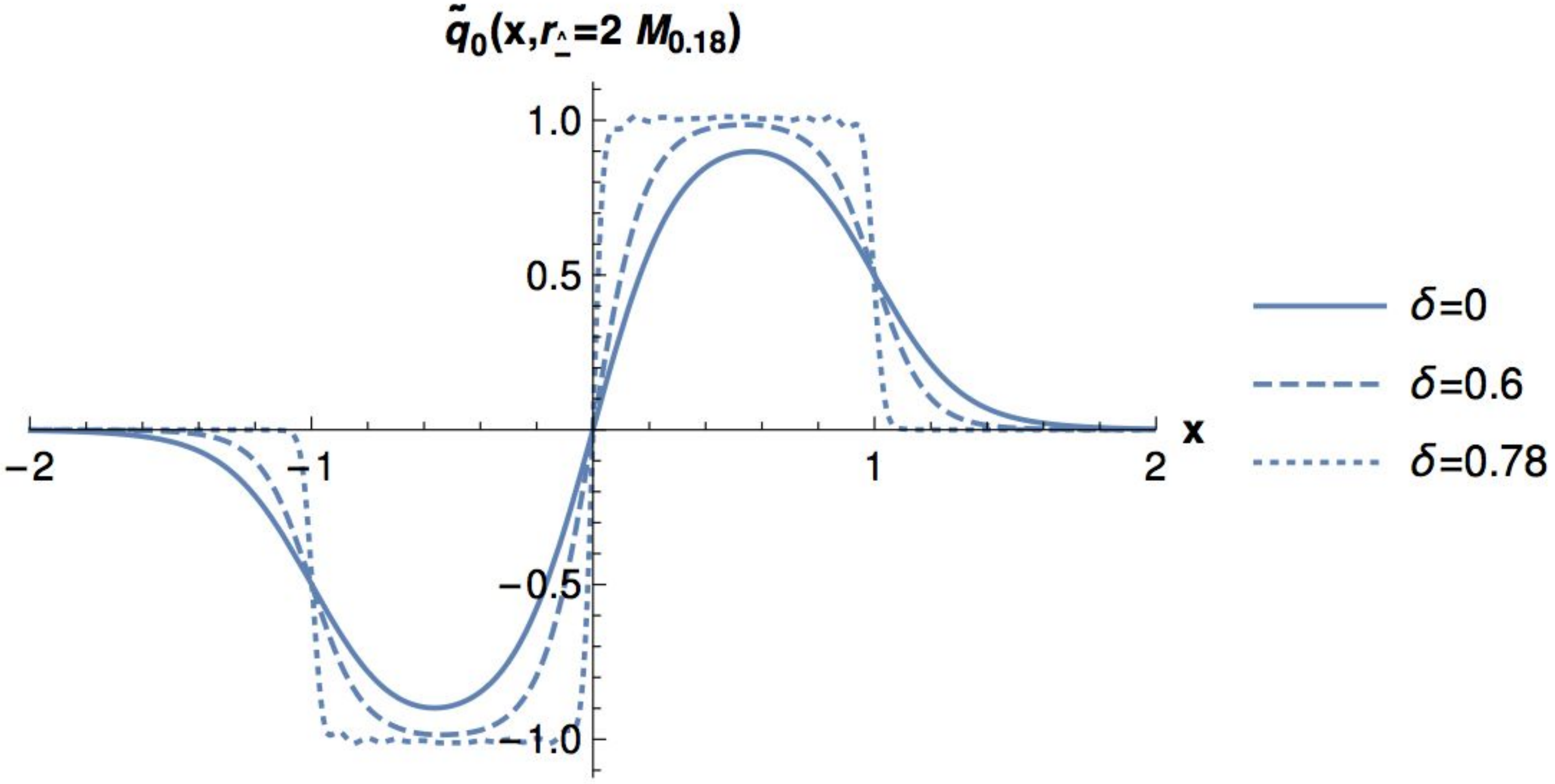
Quasi-PDFs

$$\tilde{q}_{(n)}(r_{\hat{-}}, x) = \int_{-\infty}^{+\infty} \frac{dx^{\hat{-}}}{4\pi} \mathrm{e}^{ix^{\hat{-}} r_{\hat{-}}}$$

$$\times < r^{\hat{+}}_{(n)}, r_{\hat{-}} \mid \bar{\psi}(x^{\hat{-}}) \gamma_{\hat{-}} \mathcal{W}[x^{\hat{-}}, 0] \psi(0) \mid r^{\hat{+}}_{(n)}, r_{\hat{-}} >_C,$$

$$\mathcal{W}[x^{\hat{-}}, 0] = \mathcal{P}\left[\exp\left(-ig \int_0^{x^{\hat{-}}} dx'^{\hat{-}} A_{\hat{-}}(x'^{\hat{-}})\right)\right]$$

Interpolating dynamics



$\tilde{q}_0(x, r_\perp = 2\ M_{0.18})$
1.0
0.5
−0.5
−1.0
−2
−1
1
2
x
δ=0
δ=0.6
δ=0.78

PHYSICAL REVIEW D **99**, 116008 (2019)

Beam spin asymmetry in the electroproduction of a pseudoscalar meson or a scalar meson off the scalar target

Chueng-Ryong Ji,[1] Ho-Meoyng Choi,[2] Andrew Lundeen,[1] and Bernard L. G. Bakker[3]

[1]*Department of Physics, North Carolina State University, Raleigh, North Carolina 27695-8202, USA*
[2]*Department of Physics, Teachers College, Kyungpook National University, Daegu 41566, Korea*
[3]*Faculteit der Bètawetenschappen, Vrije Universiteit, Amsterdam, Netherlands*

(Received 25 February 2019; published 12 June 2019)

We discuss the electroproduction of a pseudoscalar (0^{-+}) meson or a scalar (0^{++}) meson off the scalar target. The most general formulation of the differential cross section for the 0^{-+} or 0^{++} meson process involves only one or two hadronic form factors, respectively, on a scalar target. The Rosenbluth-type separation of the differential cross section provides the explicit relation between the hadronic form factors and the different parts of the differential cross section in a completely model-independent manner. The absence of the beam spin asymmetry for the pseudoscalar meson production provides a benchmark for the experimental data analysis. The measurement of the beam spin asymmetry for the scalar meson production may also provide a unique opportunity not only to explore the imaginary part of the hadronic amplitude in the general formulation but also to examine the significance of the chiral-odd generalized parton distribution (GPD) contribution in the leading-twist GPD formulation.

DOI: 10.1103/PhysRevD.99.116008

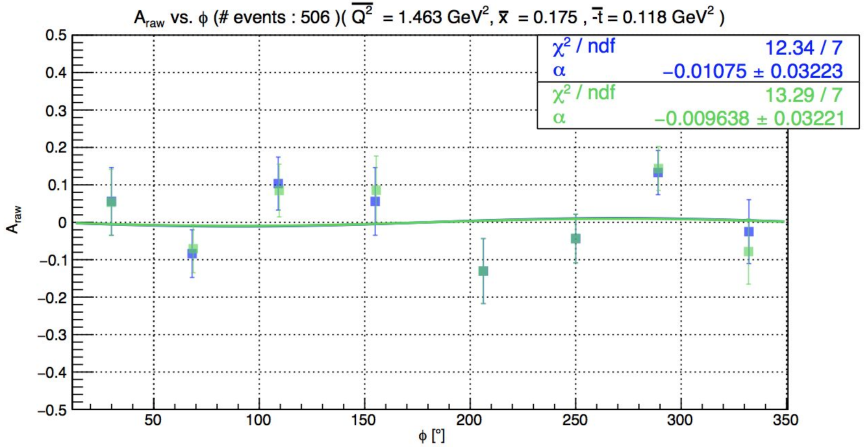
$$J^{\mu}_{PS} = F_{PS}\epsilon^{\mu\nu\alpha\beta}q_{\nu}\bar{P}_{\alpha}\Delta_{\beta}$$

$$\begin{aligned}\mathcal{H}_{\mu\nu} &= J^{\dagger}_{\mu}J_{\nu} \\ &= |F_{PS}|^2\epsilon_{\mu\alpha\beta\gamma}\epsilon_{\nu\alpha'\beta'\gamma'}q^{\alpha}\bar{P}^{\beta}\Delta^{\gamma}q^{\alpha'}\bar{P}^{\beta'}\Delta^{\gamma'} \\ &= \mathcal{H}_{\nu\mu}\end{aligned}$$

$$\epsilon^{\mu\nu\alpha\beta}k_{\alpha}k'_{\beta}\mathcal{H}_{\mu\nu} = 0$$

$$\frac{d\sigma^{PS}_{h=+1} - d\sigma^{PS}_{h=-1}}{d\sigma^{PS}_{h=+1} + d\sigma^{PS}_{h=-1}} = 0$$

$$\begin{aligned}J^{\mu}_{S} = &F_1(q^2\Delta^{\mu} - q\cdot\Delta q^{\mu}) \\ &+ F_2[(\bar{P}\cdot q + q^2)\Delta^{\mu} - q\cdot\Delta(\bar{P}^{\mu} + q^{\mu})]\end{aligned}$$



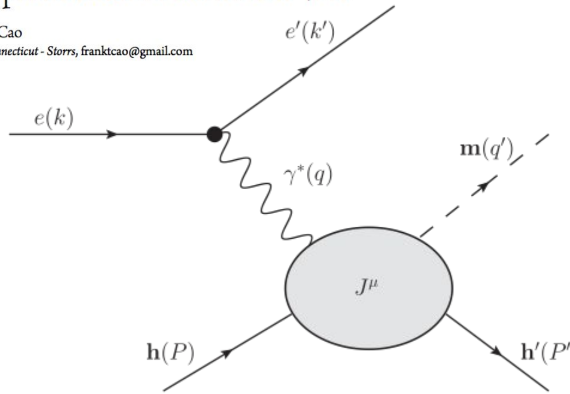
$$A_{LU}(\phi) = A^{90^\circ}_{LU}\sin\phi$$

$$A^{90^\circ}_{LU} = -1.08 \pm 3.22\,(\text{stat.}) \pm 2.83\,(\text{sys.})\,\%$$

4-26-2019

Beam-Spin Asymmetry of Exclusive Coherent Electroproduction of the π0 Off 4He

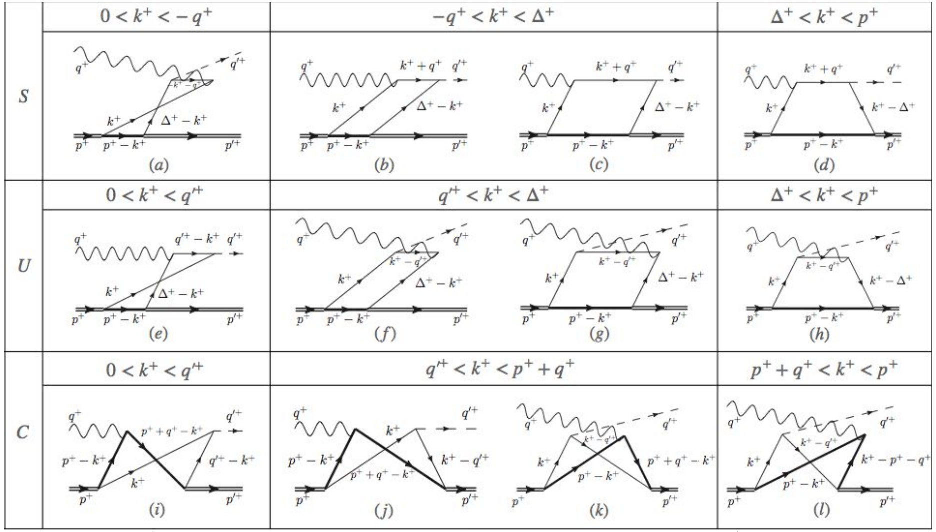
Frank Thanh Cao
University of Connecticut - Storrs, franktcao@gmail.com

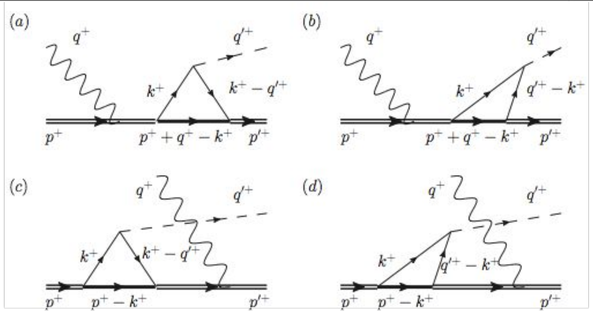


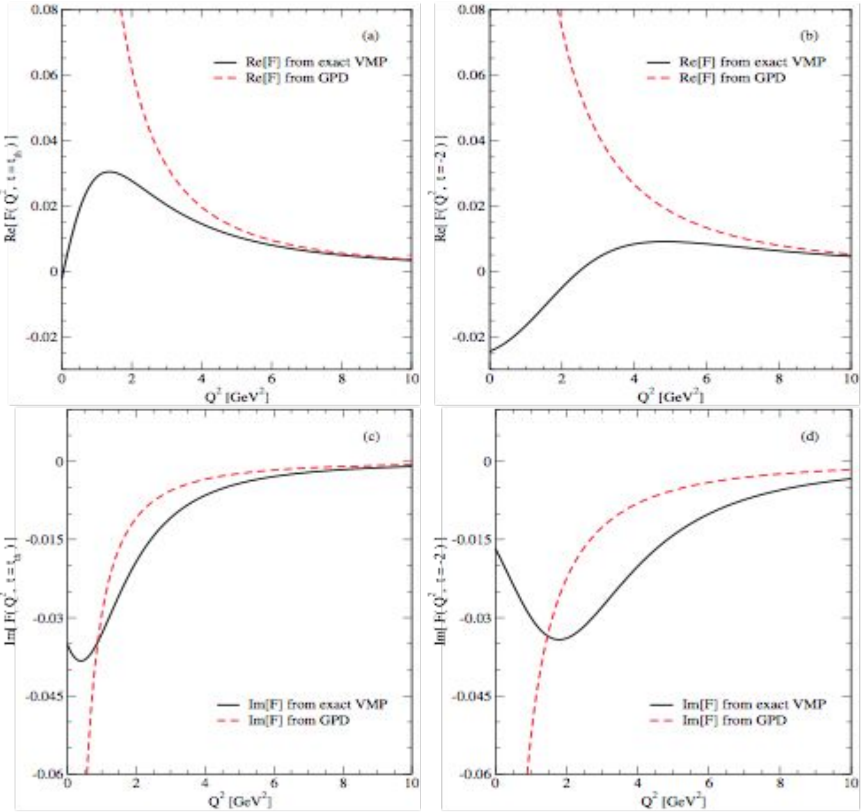
Analysis of virtual meson production in solvable (1+1) dimensional scalar field theory

Yongwoo Choi,[1,*] Ho-Meoyng Choi,[2,†] Chueng-Ryong Ji,[3,‡] and Yongseok Oh[1,4,§]

arXiv:2112.04837v1 [hep-ph] 9 Dec 2021







Manifestation of quantum correlation in the interpolating helicity amplitudes

e-HUGS 17th June 2021

Deepasika Dayananda[1], Chueng-Ryong Ji[2]

Interpolating Lorentz force equation and solution between the Instant Form Dynamics and Light-Front Dynamics

Light Cone

December 2nd 2021

For spin-1 particles ($j = 1, m = 1, 0, -1$)

$$|1,1> = |\uparrow\uparrow>$$
$$|1,0> = \frac{1}{\sqrt{2}}(|\uparrow\downarrow + \downarrow\uparrow>)$$
$$|1,-1> = |\downarrow\downarrow>$$

For spin-0 particles ($j = 0, m = 0$)

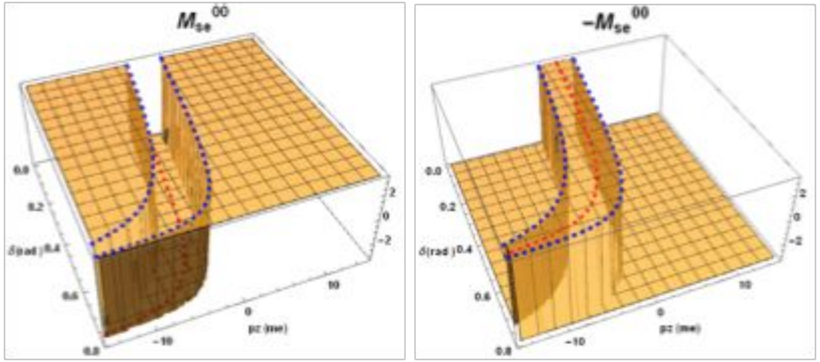
$$|0,0> = \frac{1}{\sqrt{2}}(|\uparrow\downarrow - \downarrow\uparrow>)$$

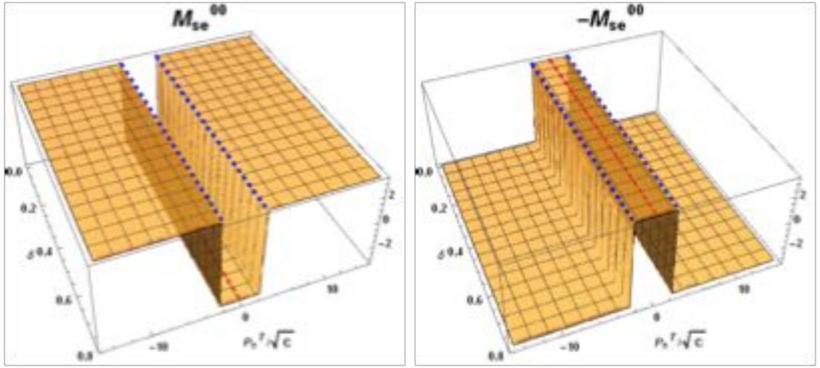
spin-1 particles

$$|1,1> \longrightarrow |1,-1>$$
$$|1,0> \longrightarrow -|1,0>$$
$$|1,-1> \longrightarrow |1,1>$$

spin-0 particles

$$|0,0> \longrightarrow |0,0>$$





Lorentz Force Equation

$$m\frac{d\,U^{\mu}(\tau)}{d\,\tau} = qF^{\mu\nu}U_{\nu}(\tau)$$

$$F^{\hat{\mu}\,\hat{\nu}} = \begin{pmatrix} 0 & -ExCos[\delta] - BySin[\delta] & -EyCos[\delta] + BxSin[\delta] & Ez \\ ExCos[\delta] + BySin[\delta] & 0 & -Bz & -ByCos[\delta] + ExSin[\delta] \\ EyCos[\delta] - BxSin[\delta] & Bz & 0 & BxCos[\delta] + EySin[\delta] \\ -Ez & ByCos[\delta] - ExSin[\delta] & -BxCos[\delta] - EySin[\delta] & 0 \end{pmatrix}$$

$$M_{\hat{\mu}\hat{\nu}} = \begin{pmatrix} 0 & \mathcal{D}^{\hat{1}} & \mathcal{D}^{\hat{2}} & K^{3} \\ -\mathcal{D}^{\hat{1}} & 0 & J^{3} & -\mathcal{K}^{\hat{1}} \\ -\mathcal{D}^{\hat{2}} & -J^{3} & 0 & -\mathcal{K}^{\hat{2}} \\ -K^{3} & \mathcal{K}^{\hat{1}} & \mathcal{K}^{\hat{2}} & 0 \end{pmatrix}$$

- Since kinematic operators leave the time-invariant, their usage is beneficial in describing the characteristics of the motion with a simpler time-variant expression.

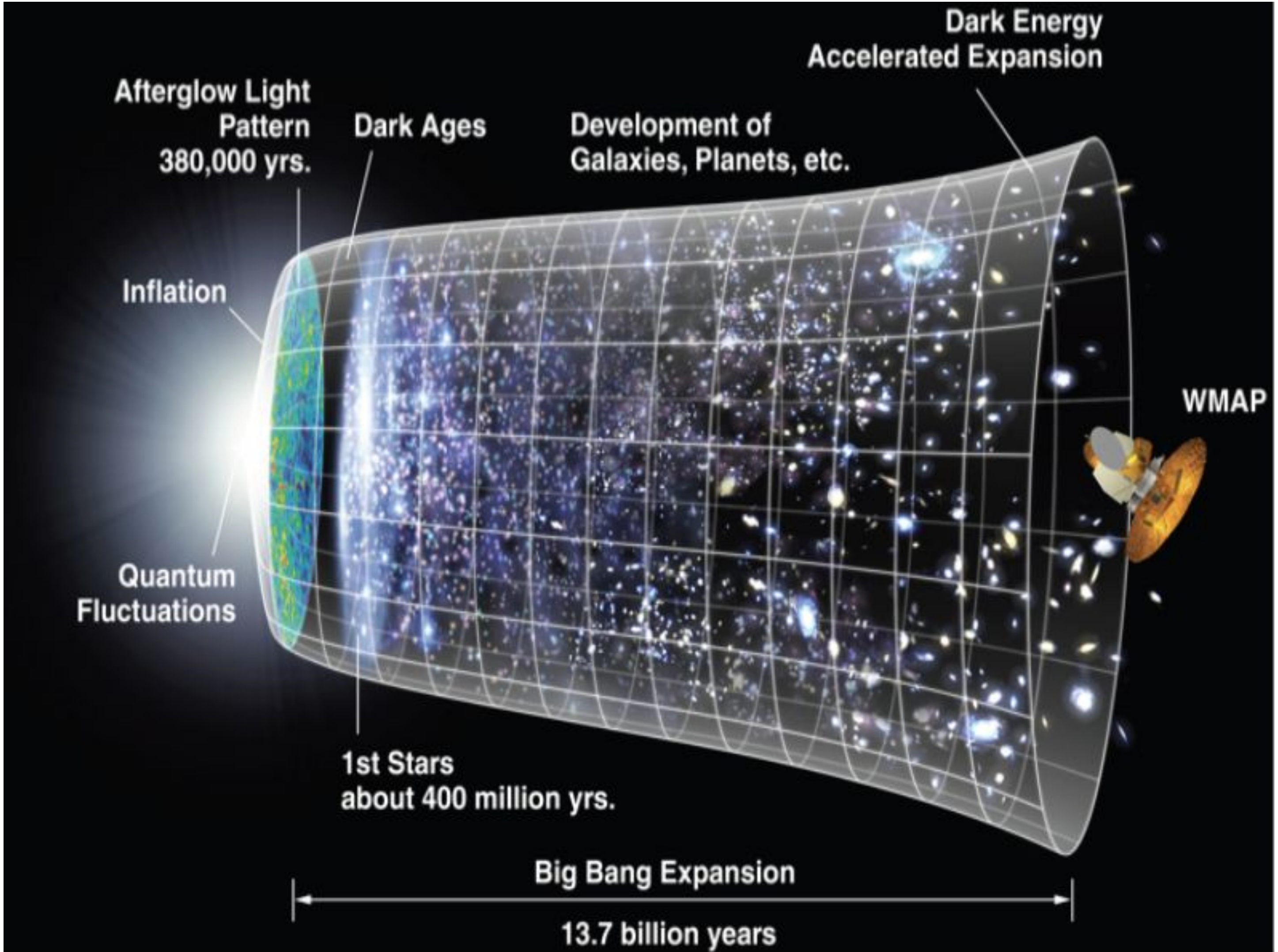
Interpolation angle	Kinematic	Dynamic
$\delta = 0$	$\mathcal{K}^{\hat{1}} = -J^2,\ \mathcal{K}^{\hat{2}} = J^1,\ J^3, P^1, P^2, P^3,\ D$	$\mathcal{D}^{\hat{1}} = -K^1,\ \mathcal{D}^{\hat{2}} = -K^2,\ K^3, P^0,\ \mathfrak{K}_0,\ \mathfrak{K}_1,\ \mathfrak{K}_2,\ \mathfrak{K}_3$
$0 \leq \delta < \pi/4$	$\mathcal{K}^{\hat{1}}, \mathcal{K}^{\hat{2}}, J^3, P^1, P^2, P_{\hat{-}},\ D$	$\mathcal{D}^{\hat{1}}, \mathcal{D}^{\hat{2}}, K^3, P_{\hat{+}},\ \mathfrak{K}_{\hat{+}},\ \mathfrak{K}_{\hat{1}},\ \mathfrak{K}_{\hat{2}},\ \mathfrak{K}_{\hat{-}}$
$\delta = \pi/4$	$\mathcal{K}^{\hat{1}} = -E^1,\ \mathcal{K}^{\hat{2}} = -E^2,\ J^3, K^3, P^1, P^2, P_-,\ D,\ \mathfrak{K}_-$	$\mathcal{D}^{\hat{1}} = -F^1,\ \mathcal{D}^{\hat{2}} = -F^2,\ P_+,\ \mathfrak{K}_+,\ \mathfrak{K}_1,\ \mathfrak{K}_2$

	$P_{\hat{+}}$	$P_{\hat{1}}$	$P_{\hat{2}}$	$K^{\hat{3}}$	$\mathcal{D}^{\hat{1}}$	$\mathcal{D}^{\hat{2}}$	$J^{\hat{3}}$	$\mathcal{K}^{\hat{1}}$	$\mathcal{K}^{\hat{2}}$	$P_{\hat{-}}$	$\mathfrak{K}_{\hat{+}}$	$\mathfrak{K}_{\hat{1}}$	$\mathfrak{K}_{\hat{2}}$	$\mathfrak{K}_{\hat{-}}$	D
$P_{\hat{+}}$	0	0	0	$i\left(\mathbb{C}P_{\hat{-}} - \mathbb{S}P_{\hat{+}}\right)$	$i\mathbb{C}P_{\hat{1}}$	$i\mathbb{C}P_{\hat{2}}$	0	$i\mathbb{S}P_{\hat{1}}$	$i\mathbb{S}P_{\hat{2}}$	0	$-2i\mathbb{C}D$	$-2i\mathcal{D}^{\hat{1}}$	$-2i\mathcal{K}^{\hat{2}}$	$-2i(\mathbb{S}D - K^{\hat{3}})$	$-iP_{\hat{+}}$
$P_{\hat{1}}$	0	0	0	0	$iP_{\hat{+}}$	0	$-iP_{\hat{2}}$	$iP_{\hat{-}}$	0	0	$2i\mathcal{D}^{\hat{1}}$	$2iD$	$-2iJ^{\hat{3}}$	$2i\mathcal{K}^{\hat{1}}$	$-iP_{\hat{1}}$
$P_{\hat{2}}$	0	0	0	0	0	$iP_{\hat{+}}$	$iP_{\hat{1}}$	0	$iP_{\hat{-}}$	0	$2i\mathcal{D}^{\hat{2}}$	$2iJ^{\hat{3}}$	$2iD$	$2i\mathcal{K}^{\hat{2}}$	$-iP_{\hat{2}}$
$K^{\hat{3}}$	$-i\left(\mathbb{C}P_{\hat{-}} - \mathbb{S}P_{\hat{+}}\right)$	0	0	0	$i\mathbb{S}\mathcal{D}^{\hat{1}} - i\mathbb{C}\mathcal{K}^{\hat{1}}$	$i\mathbb{S}\mathcal{D}^{\hat{2}} - i\mathbb{C}\mathcal{K}^{\hat{2}}$	0	$-i\mathbb{S}\mathcal{K}^{\hat{1}} - i\mathbb{C}\mathcal{D}^{\hat{1}}$	$-i\mathbb{S}\mathcal{K}^{\hat{2}} - i\mathbb{C}\mathcal{D}^{\hat{2}}$	$-i\left(\mathbb{S}P_{\hat{-}} + \mathbb{C}P_{\hat{+}}\right)$	$i\left(\mathbb{S}\mathfrak{K}_{\hat{+}} - \mathbb{C}\mathfrak{K}_{\hat{-}}\right)$	0	0	$-i\left(\mathbb{C}\mathfrak{K}_{\hat{+}} + \mathbb{S}\mathfrak{K}_{\hat{-}}\right)$	0
$\mathcal{D}^{\hat{1}}$	$-i\mathbb{C}P_{\hat{1}}$	$-iP_{\hat{+}}$	0	$-i\mathbb{S}\mathcal{D}^{\hat{1}} + i\mathbb{C}\mathcal{K}^{\hat{1}}$	0	$-i\mathbb{C}J^{\hat{3}}$	$-i\mathcal{D}^{\hat{2}}$	$-iK^{\hat{3}}$	$-i\mathbb{S}J^{\hat{3}}$	$-i\mathbb{S}P_{\hat{1}}$	$-i\mathbb{C}\mathfrak{K}_{\hat{1}}$	$-i\mathfrak{K}_{\hat{+}}$	0	$-i\mathbb{S}\mathfrak{K}_{\hat{1}}$	0
$\mathcal{D}^{\hat{2}}$	$-i\mathbb{C}P_{\hat{2}}$	0	$-iP_{\hat{+}}$	$-i\mathbb{S}\mathcal{D}^{\hat{2}} + i\mathbb{C}\mathcal{K}^{\hat{2}}$	$i\mathbb{C}J^{\hat{3}}$	0	$i\mathcal{D}^{\hat{1}}$	$i\mathbb{S}J^{\hat{3}}$	$-iK^{\hat{3}}$	$-i\mathbb{S}P_{\hat{2}}$	$-i\mathbb{C}\mathfrak{K}_{\hat{2}}$	0	$-i\mathfrak{K}_{\hat{+}}$	$-i\mathbb{S}\mathfrak{K}_{\hat{2}}$	0
$J^{\hat{3}}$	0	$iP_{\hat{2}}$	$-iP_{\hat{1}}$	0	$i\mathcal{D}^{\hat{2}}$	$-i\mathcal{D}^{\hat{1}}$	0	$i\mathcal{K}^{\hat{2}}$	$-i\mathcal{K}^{\hat{1}}$	0	0	$i\mathfrak{K}_{\hat{2}}$	$-i\mathfrak{K}_{\hat{1}}$	0	0
$\mathcal{K}^{\hat{1}}$	$-i\mathbb{S}P_{\hat{1}}$	$-iP_{\hat{-}}$	0	$i\mathbb{S}\mathcal{K}^{\hat{1}} + i\mathbb{C}\mathcal{D}^{\hat{1}}$	$iK^{\hat{3}}$	$-i\mathbb{S}J^{\hat{3}}$	$-i\mathcal{K}^{\hat{2}}$	0	$i\mathbb{C}J^{\hat{3}}$	$i\mathbb{C}P_{\hat{1}}$	$-i\mathbb{S}\mathfrak{K}_{\hat{1}}$	$-i\mathfrak{K}_{\hat{-}}$	0	$i\mathbb{C}\mathfrak{K}_{\hat{1}}$	0
$\mathcal{K}^{\hat{2}}$	$-i\mathbb{S}P_{\hat{2}}$	0	$-iP_{\hat{-}}$	$i\mathbb{S}\mathcal{K}^{\hat{2}} + i\mathbb{C}\mathcal{D}^{\hat{2}}$	$i\mathbb{S}J^{\hat{3}}$	$iK^{\hat{3}}$	$i\mathcal{K}^{\hat{1}}$	$-i\mathbb{C}J^{\hat{3}}$	0	$i\mathbb{C}P_{\hat{2}}$	$-i\mathbb{S}\mathfrak{K}_{\hat{2}}$	0	$-i\mathfrak{K}_{\hat{-}}$	$i\mathbb{C}\mathfrak{K}_{\hat{2}}$	0
$P_{\hat{-}}$	0	0	0	$i\left(\mathbb{S}P_{\hat{-}} + \mathbb{C}P_{\hat{+}}\right)$	$i\mathbb{S}P_{\hat{1}}$	$i\mathbb{S}P_{\hat{2}}$	0	$-i\mathbb{C}P_{\hat{1}}$	$-i\mathbb{C}P_{\hat{2}}$	0	$-2i(\mathbb{S}D + K^{\hat{3}})$	$-2i\mathcal{K}^{\hat{1}}$	$-2i\mathcal{K}^{\hat{2}}$	$2i\mathbb{C}D$	$-iP_{\hat{-}}$
$\mathfrak{K}_{\hat{+}}$	$2i\mathbb{C}D$	$-2i\mathcal{D}^{\hat{1}}$	$-2i\mathcal{D}^{\hat{2}}$	$-i\left(\mathbb{S}\mathfrak{K}_{\hat{+}} - \mathbb{C}\mathfrak{K}_{\hat{-}}\right)$	$i\mathbb{C}\mathfrak{K}_{\hat{1}}$	$i\mathbb{C}\mathfrak{K}_{\hat{2}}$	0	$i\mathbb{S}\mathfrak{K}_{\hat{1}}$	$i\mathbb{S}\mathfrak{K}_{\hat{2}}$	$2i(\mathbb{S}D + K^{\hat{3}})$	0	0	0	0	$i\mathfrak{K}_{\hat{+}}$
$\mathfrak{K}_{\hat{1}}$	$2i\mathcal{D}^{\hat{1}}$	$-2iD$	$-2iJ^{\hat{3}}$	0	$i\mathfrak{K}_{\hat{+}}$	0	$-i\mathfrak{K}_{\hat{2}}$	$i\mathfrak{K}_{\hat{-}}$	0	$2i\mathcal{K}^{\hat{1}}$	0	0	0	0	$i\mathfrak{K}_{\hat{1}}$
$\mathfrak{K}_{\hat{2}}$	$2i\mathcal{K}^{\hat{2}}$	$2iJ^{\hat{3}}$	$-2iD$	0	0	$i\mathfrak{K}_{\hat{+}}$	$i\mathfrak{K}_{\hat{1}}$	0	$i\mathfrak{K}_{\hat{-}}$	$2i\mathcal{K}^{\hat{2}}$	0	0	0	0	$i\mathfrak{K}_{\hat{2}}$
$\mathfrak{K}_{\hat{-}}$	$2i(\mathbb{S}D - K^{\hat{3}})$	$-2i\mathcal{K}^{\hat{1}}$	$-2i\mathcal{K}^{\hat{2}}$	$i\left(\mathbb{C}\mathfrak{K}_{\hat{+}} + \mathbb{S}\mathfrak{K}_{\hat{-}}\right)$	$i\mathbb{S}\mathfrak{K}_{\hat{1}}$	$i\mathbb{S}\mathfrak{K}_{\hat{2}}$	0	$-i\mathbb{C}\mathfrak{K}_{\hat{1}}$	$-i\mathbb{C}\mathfrak{K}_{\hat{2}}$	$-2i\mathbb{C}D$	0	0	0	0	$i\mathfrak{K}_{\hat{-}}$
D	$iP_{\hat{+}}$	$iP_{\hat{1}}$	$iP_{\hat{2}}$	0	0	0	0	0	0	$iP_{\hat{-}}$	$-i\mathfrak{K}_{\hat{+}}$	$-i\mathfrak{K}_{\hat{1}}$	$-i\mathfrak{K}_{\hat{2}}$	$-i\mathfrak{K}_{\hat{-}}$	0

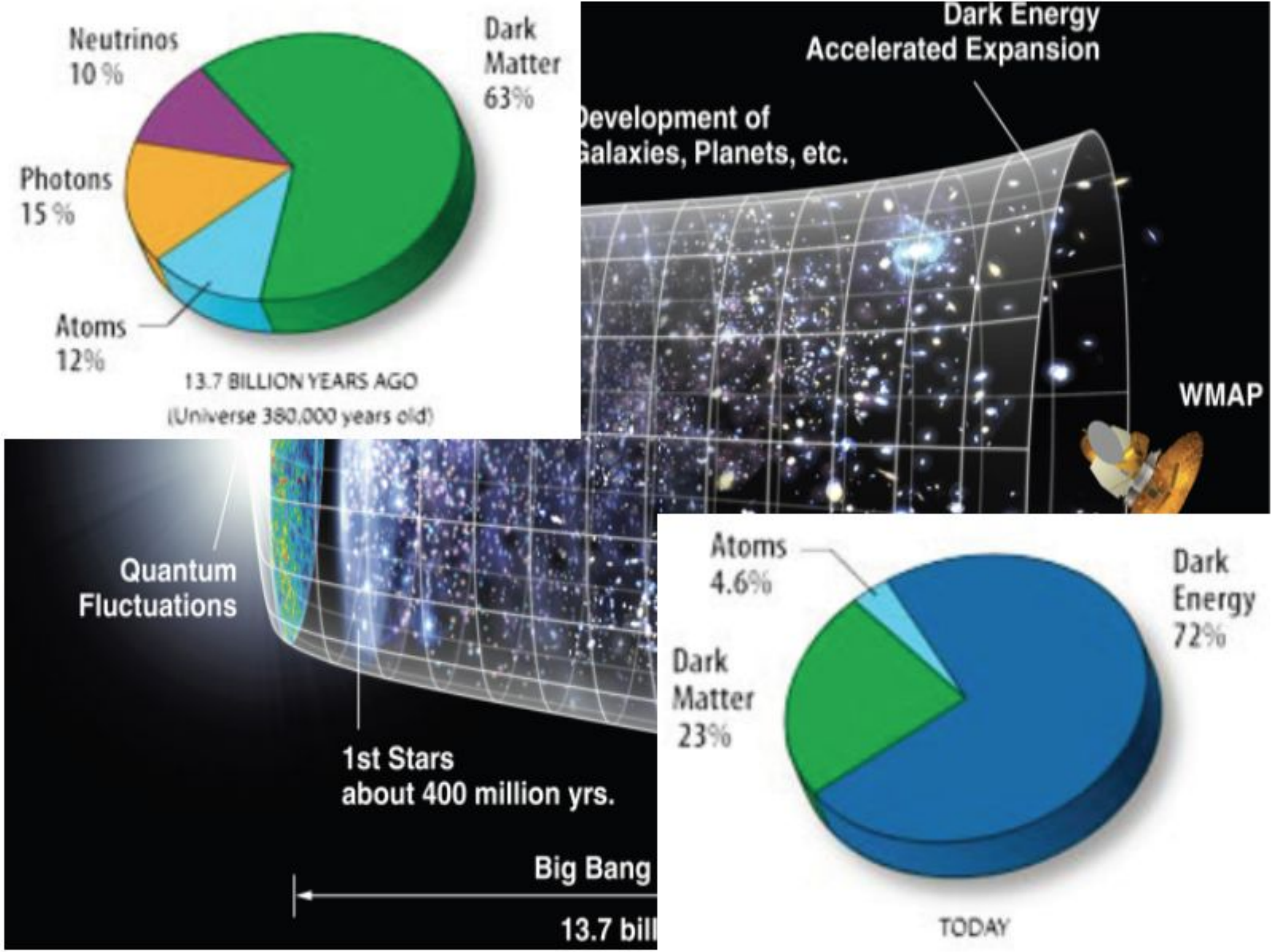
Light Cone 2021: Physics of Hadrons on the Light Front

Interpolating conformal algebra between the instant form and the front form of relativistic dynamics

Chueng-Ryong Ji[1], Harleen Dahiya[2], and Hariprashad Ravikumar[3]



Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
380,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
WMAP
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.7 billion years


Neutrinos 10 %
Dark Matter 63%
Photons 15 %
Atoms 12%
13.7 BILLION YEARS AGO
(Universe 380,000 years old)
Dark Energy
Accelerated Expansion
Development of Galaxies, Planets, etc.
WMAP
Quantum Fluctuations
1st Stars about 400 million yrs.
Big Bang
13.7 bill
Atoms 4.6%
Dark Energy 72%
Dark Matter 23%
TODAY

Eleven Science Questions for the 21st Century

- What is Dark Matter?
- What is the nature of Dark Energy?
- How did the Universe begin?
- Did Einstein have the last word on Gravity?
- What are the masses of the Neutrinos, and how have they shaped the evolution of the Universe?
- How do Cosmic Accelerators work and what are they accelerating?
- Are Protons unstable?
- What are the new states of matter at exceedingly High Density and Temperature?
- Are there Additional Space-Time Dimensions?
- How were the elements from Iron to Uranium made?
- Is a new theory of Matter and Light needed at the Highest Energies?

Nuclear Science

[Today and for the Next Decade]

General goal (from U.S. Long Range Plan):
Explain the origin, evolution, and structure of the visible matter of the universe—the matter that makes up stars, planets, and human life itself.

Frontiers:

- **Quantum Chromodynamics** (QCD) and **Hadrons**

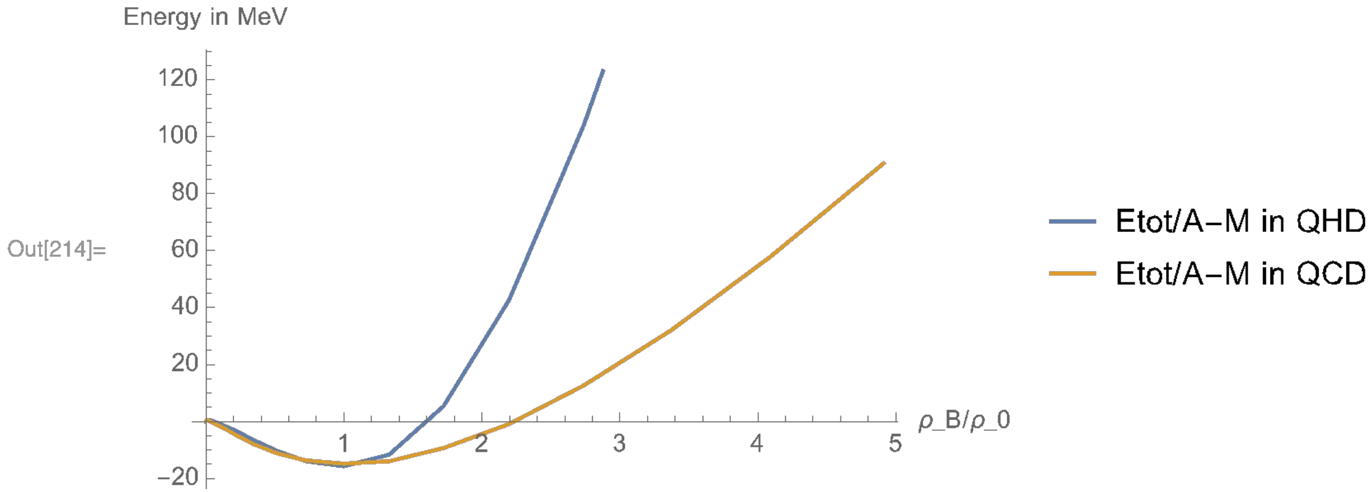
  [**LHC, RHIC, JLab, JPARC, $e^+ e^-$ (Beijing, DAPHNE, KEKB, Novosibirsk), . . ., FAIR**]

- **Fundamental Symmetries** and **Neutrinos**

  [neutrinos, double-beta decay, low-energy Standard Model studies, edm's, . . .]

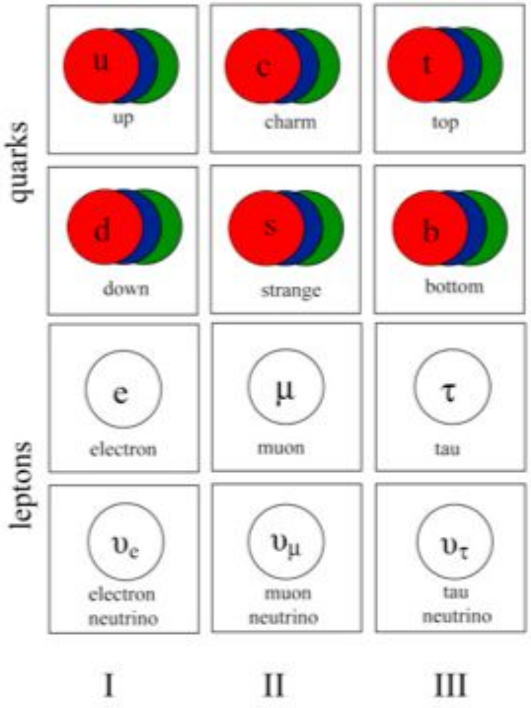
- **Physics of Nuclei** and **Nuclear Astrophysics**

Stability of Nuclear Matter



- **Relativity is crucial for the stabilization of nuclear matter.**
- **Point nucleon in QHD yields so large incompressibility.**
- **Proton has quarks and gluons inside.**
- **Quantum Chromodynamics (QCD) governs them.**

Standard Model



$$Q_f$$

$$3 \times 2/3$$

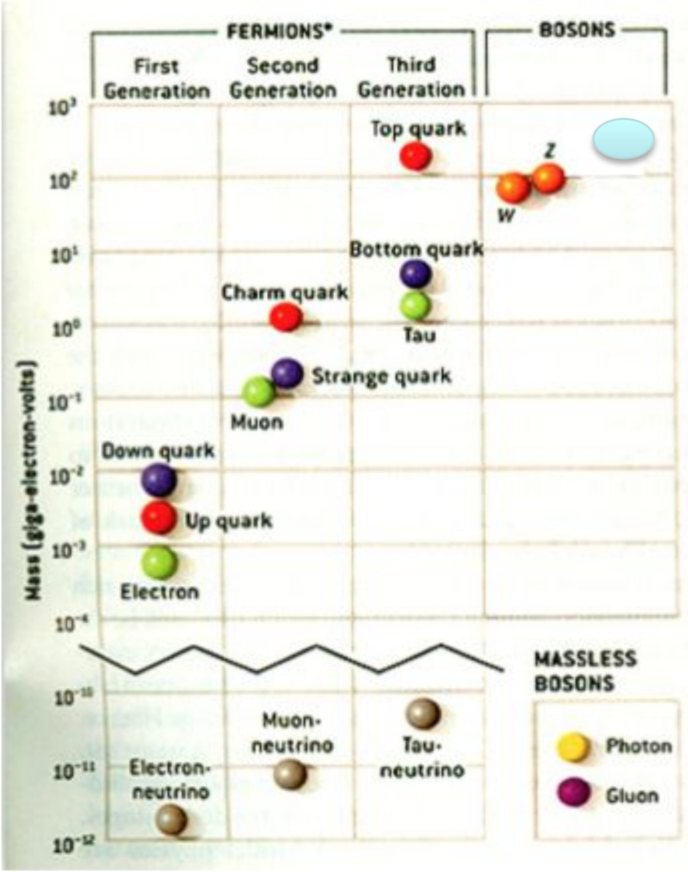
$$3 \times -1/3$$

$$-1$$

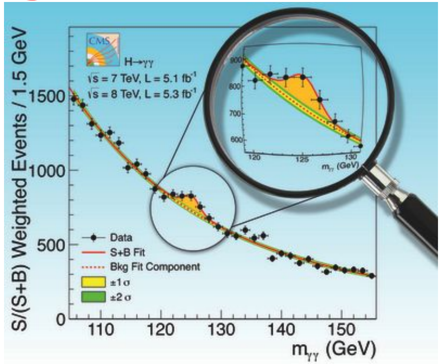
$$0$$

Anomaly Free Condition $\sum_f Q_f = 0$

The Elementary Particles



CERN,
July 4, 2012
Higgs!



The Large Hadron Collider carves a huge circle 17 miles in circumference (marked in white) beneath the French-Swiss border (dotted line; France is at top).

Nucleon is not a point-like particle.
It is made out of “partons”.

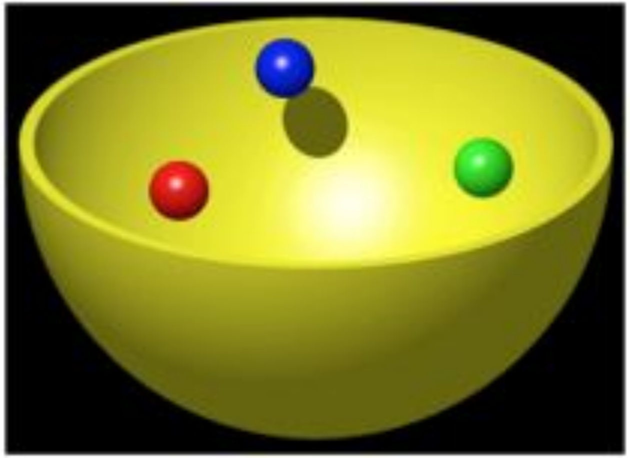
- valence quarks
- sea quarks
- gluons

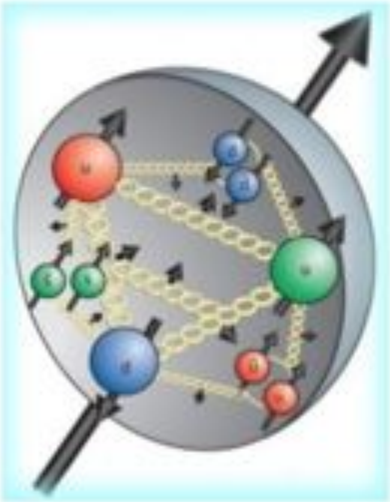
How can we recover basic properties of nucleon from those of its constituents?

- mass
- charge density/radius
- spin contents

$$M_p = 938.272046 \pm 0.000021\ MeV$$

$$M_n = 939.565379 \pm 0.000021\ MeV$$

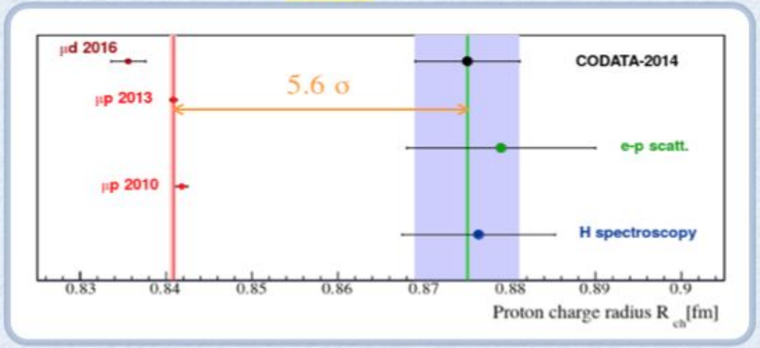


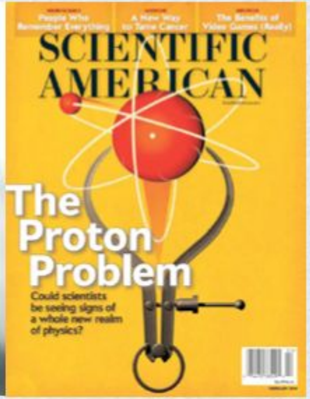


$$m_u = 2.3^{+0.7}_{-0.5}\ MeV \quad ; \quad m_d = 4.8^{+0.7}_{-0.3}\ MeV$$

Proton radius puzzle

2016





μH data: $R_E = 0.8409 \pm 0.0004$ fm

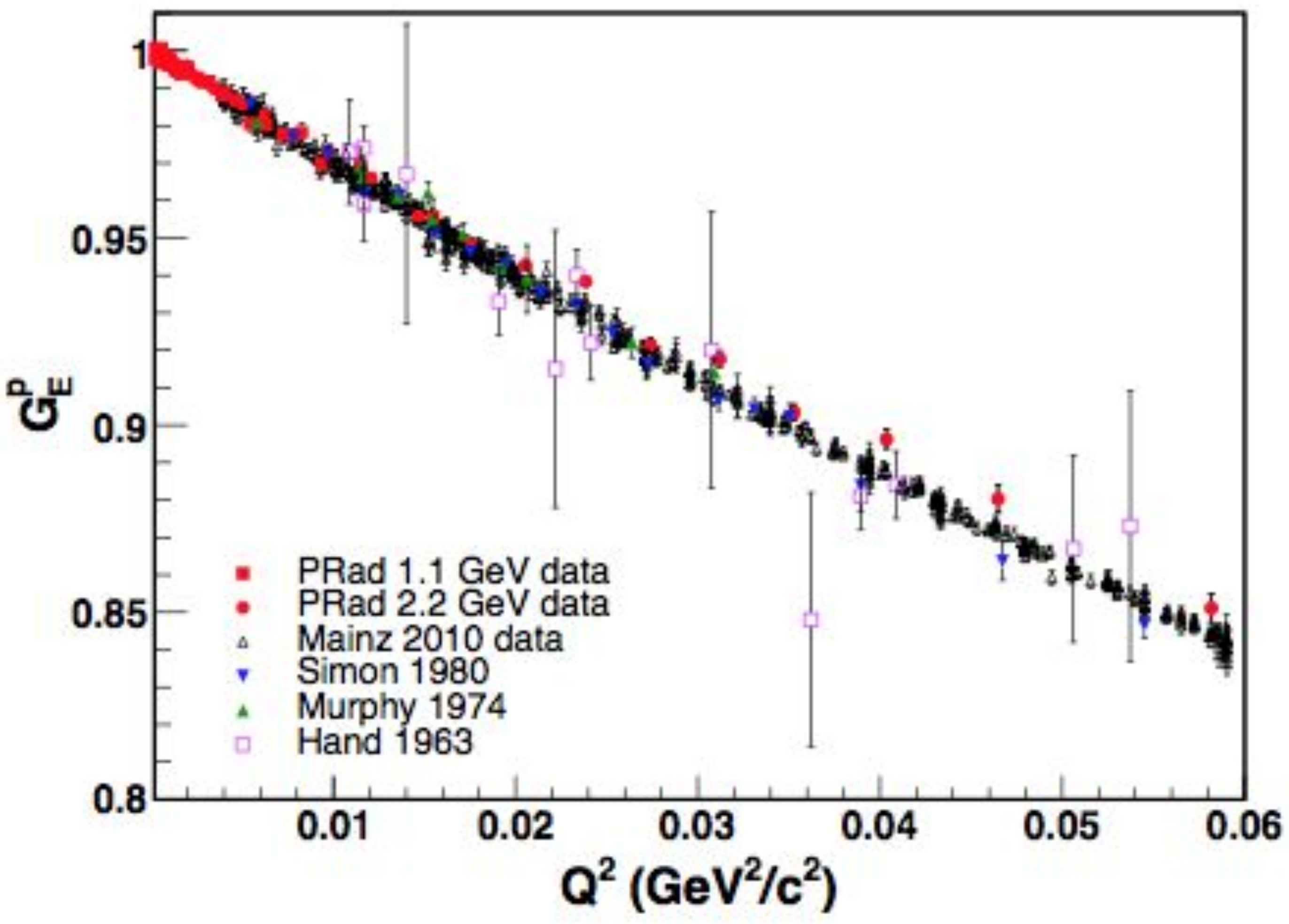
Pohl et al.(2010)

Antognini et al.(2013)

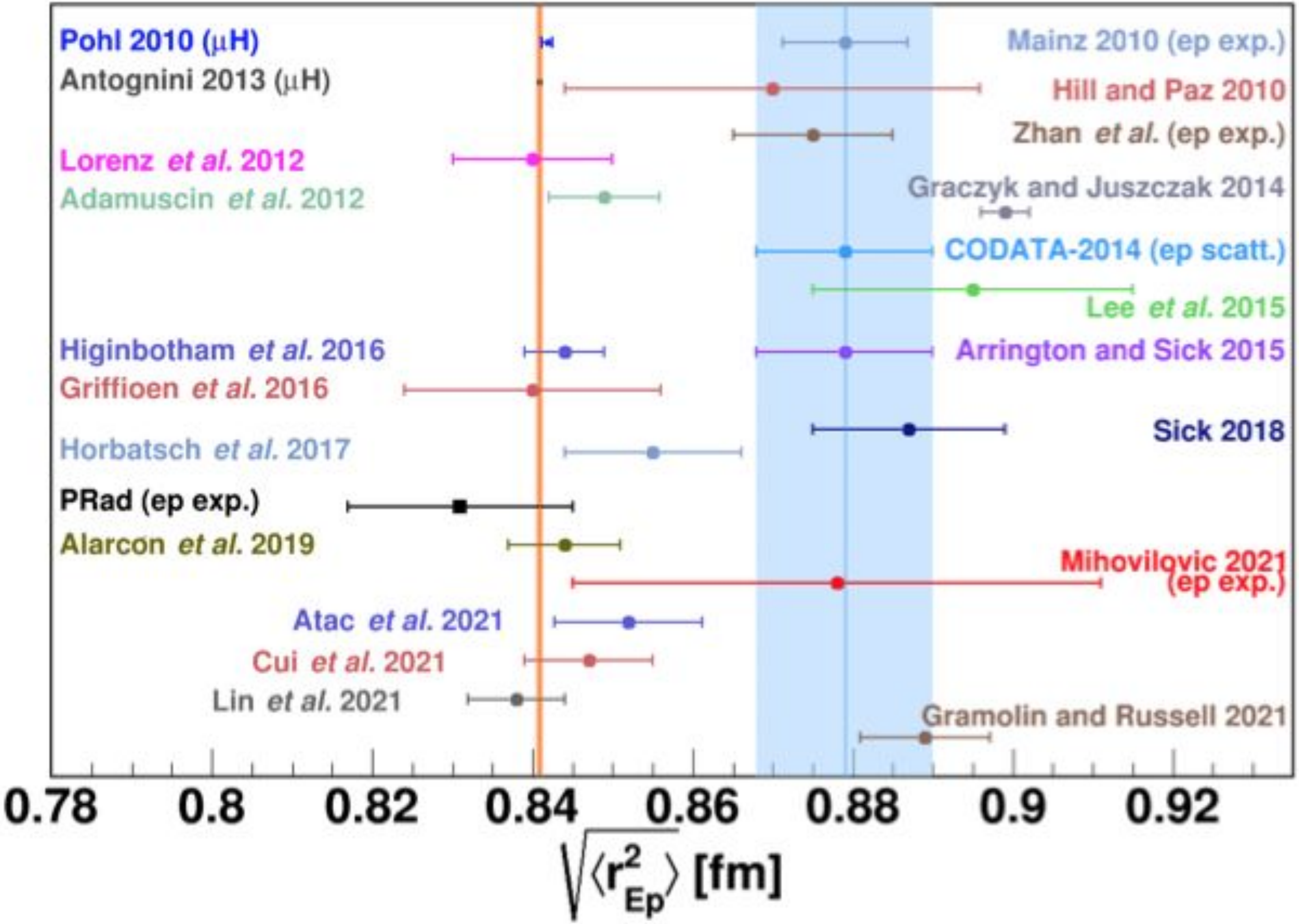
5.6 σ difference

ep data: $R_E = 0.8775 \pm 0.0051$ fm

CODATA(2014)



PRad 1.1 GeV data
PRad 2.2 GeV data
Mainz 2010 data
Simon 1980
Murphy 1974
Hand 1963
G_E^p
1
0.95
0.9
0.85
0.8
0.01
0.02
0.03
0.04
0.05
0.06
Q^2 (GeV^2/c^2)



Pohl 2010 (μH)
Antognini 2013 (μH)
Lorenz et al. 2012
Adamuscin et al. 2012
Higinbotham et al. 2016
Griffioen et al. 2016
Horbatsch et al. 2017
PRad (ep exp.)
Alarcon et al. 2019
Atac et al. 2021
Cui et al. 2021
Lin et al. 2021
Mainz 2010 (ep exp.)
Hill and Paz 2010
Zhan et al. (ep exp.)
Graczyk and Juszczak 2014
CODATA-2014 (ep scatt.)
Lee et al. 2015
Arrington and Sick 2015
Sick 2018
Mihovilovic 2021 (ep exp.)
Gramolin and Russell 2021
0.78
0.8
0.82
0.84
0.86
0.88
0.9
0.92
$\sqrt{\langle r_{Ep}^2 \rangle}$ [fm]

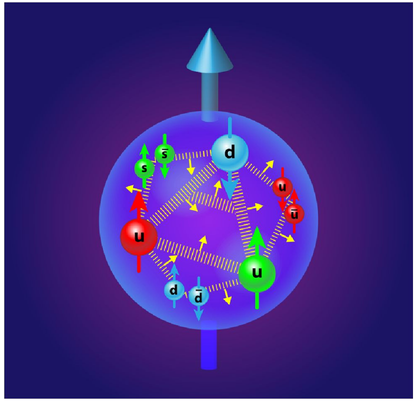
The Proton Spin

$$\frac{1}{2} = \Delta\Sigma + \Delta G + L_q + L_G$$

DSSV

PRL 113 (2014) 012001

$$\int_{0.05}^{1} \Delta g(x)dx = 0.2^{+0.06}_{-0.07}$$



(APS/Alan Stonebraker)

NNPDF

NPB 887 (2015) 276

$$\int_{0.05}^{0.5} \Delta g(x)dx = 0.23 \pm 0.07$$

Physics

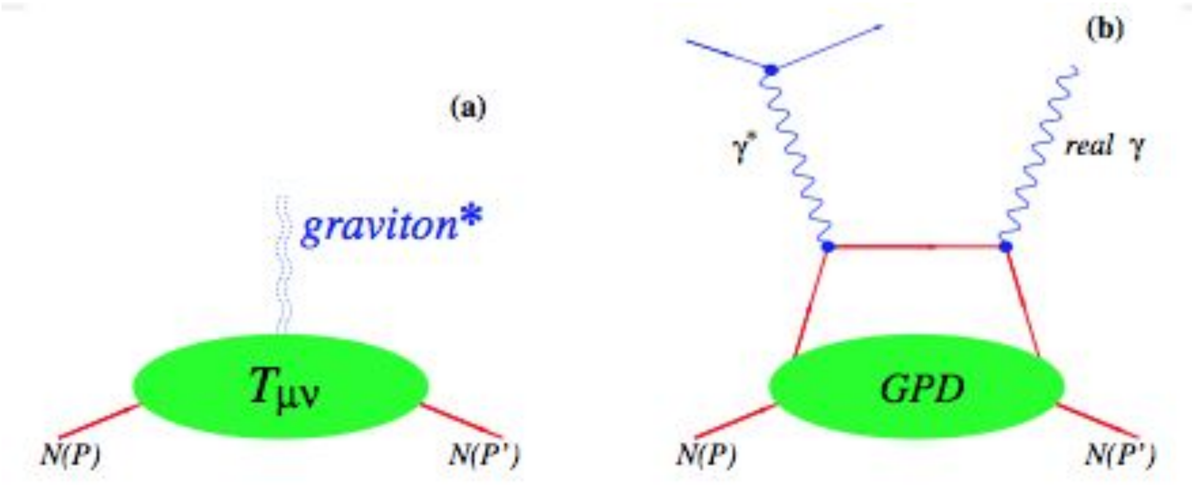
VIEWPOINT

Spinning Gluons in the Proton

Computer simulations indicate that about 50% of the proton's spin comes from the spin of the gluons that bind its quark constituents.

by Steven D. Bass

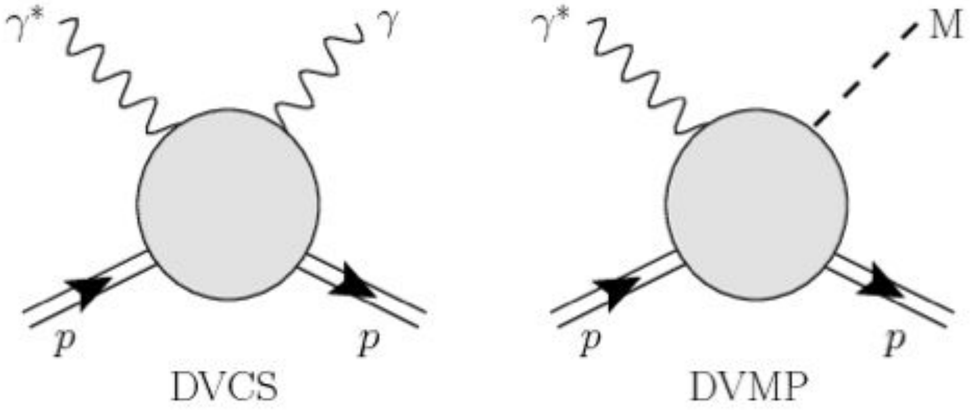
Mechanical Properties of Hadrons



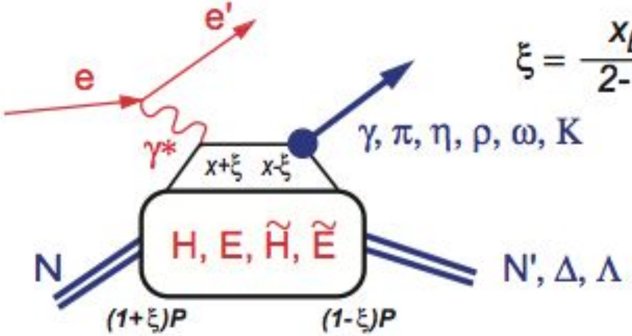
$$\langle p'|T^a_{\mu\nu}(0)|p\rangle = \bar{u}'\left[A^a(t)\,\frac{P_\mu P_\nu}{M_N} + J^a(t)\,\frac{i\,P_{\{\mu}\sigma_{\nu\}\rho}\Delta^\rho}{2M_N} + D^a(t)\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4M_N} + M_N\,\bar{c}^a(t)g_{\mu\nu}\right]u$$

$a = g, Q$ (gluon or quark parts)

$A^a(t)$	$J^a(t)$	$D^a(t)$	$\bar{c}^a(t)$
$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$
δg^{00}	δg^{0i}	δg^{ij}	non − conservation of EMT pieces
$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$
Mass	Spin	deformation of space = elastic properties of N	
$\sum_a A^a(0) = 1$	$\sum_a J^a(0) = \frac{1}{2}$		$\sum_a \bar{c}^a(t) = 0$



Q^2 >> M^2, |t|,..



$$\xi = \frac{x_B}{2 - x_B}$$

H, E - unpolarized, $\widetilde{H}$, $\widetilde{E}$ - polarized GPD

The GPDs Define Nucleon Structure

Developing predictions for tests at the new and upgraded hadron experimental facilities

JLAB,

LHC,

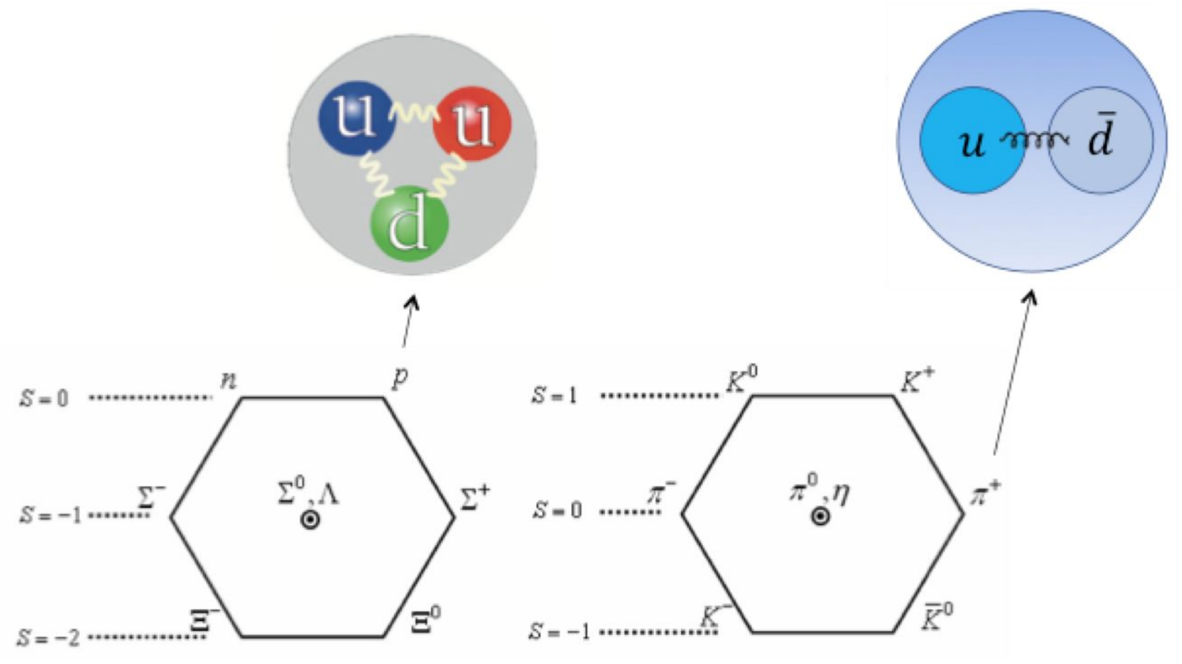
J-PARC,

GSI-FAIR.

EIC CD-1 is Approved

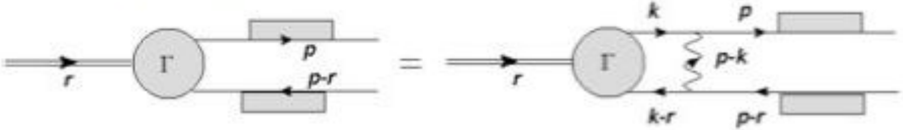
June 25, 2021

How do we understand the Quark Model in Quantum Chromodynamics as we do the Atomic Model in Quantum Electrodynamics?



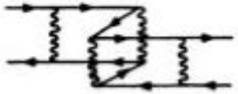
BOUND-STATE EQUATION

$$\Gamma(r,p) = \frac{i\lambda}{2\pi}\int \frac{dk_{\hat{-}}\,dk_{\hat{+}}}{(p_{\hat{-}} - k_{\hat{-}})^2} S(p)\gamma^{\hat{+}}\Gamma(r,k)\gamma^{\hat{+}}S(p-r)$$



$$\left[-r_{\hat{+}} + \frac{-\mathbb{S}p_{\hat{-}} + E(p_{\hat{-}})}{\mathbb{C}} + \frac{\mathbb{S}(p_{\hat{-}} - r_{\hat{-}}) + E(p_{\hat{-}} - r_{\hat{-}})}{\mathbb{C}}\right]\hat{\phi}_+(r_{\hat{-}}, p_{\hat{-}})$$
$$= \lambda\int \frac{dk_{\hat{-}}}{(p_{\hat{-}} - k_{\hat{-}})^2}\left[C(p_{\hat{-}}, k_{\hat{-}}, r_{\hat{-}})\hat{\phi}_+(r_{\hat{-}}, k_{\hat{-}}) - S(p_{\hat{-}}, k_{\hat{-}}, r_{\hat{-}})\hat{\phi}_-(r_{\hat{-}}, k_{\hat{-}})\right],$$

$$\left[r_{\hat{+}} + \frac{-\mathbb{S}(p_{\hat{-}} - r_{\hat{-}}) + E(p_{\hat{-}} - r_{\hat{-}})}{\mathbb{C}} + \frac{\mathbb{S}p_{\hat{-}} + E(p_{\hat{-}})}{\mathbb{C}}\right]\hat{\phi}_-(r_{\hat{-}}, p_{\hat{-}})$$
$$= \lambda\int \frac{dk_{\hat{-}}}{(p_{\hat{-}} - k_{\hat{-}})^2}\left[C(p_{\hat{-}}, k_{\hat{-}}, r_{\hat{-}})\hat{\phi}_-(r_{\hat{-}}, k_{\hat{-}}) - S(p_{\hat{-}}, k_{\hat{-}}, r_{\hat{-}})\hat{\phi}_+(r_{\hat{-}}, k_{\hat{-}})\right].$$

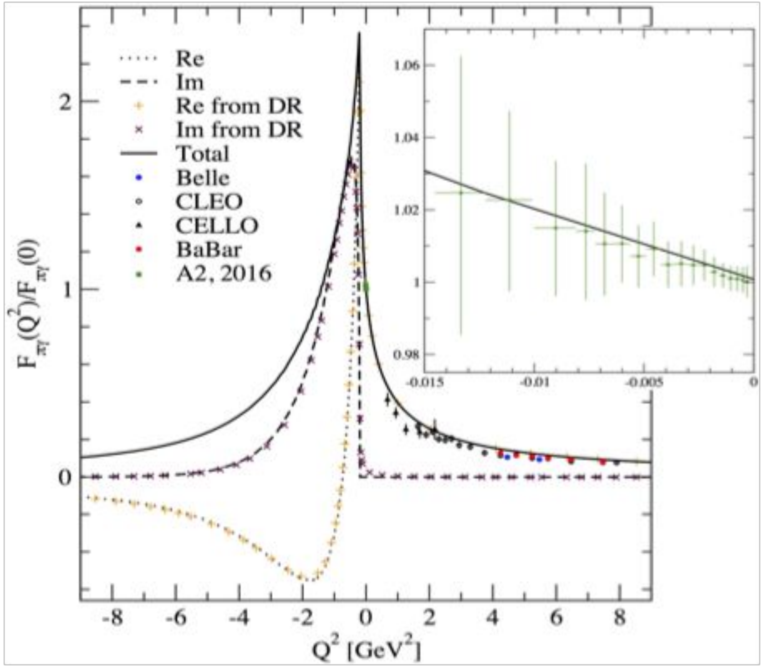


LFD $$\left[\mathcal{M}^2 - \frac{m^2 - 2\lambda}{x} - \frac{m^2 - 2\lambda}{1-x}\right]\phi(x) = -2\lambda\int_0^1 \frac{dy}{(x-y)^2}\phi(y)$$

Results appear to support the relativistic constituent quark model picture, e.g. PHYSICAL REVIEW C **92**, 055203 (2015)

Variational analysis of mass spectra and decay constants ···

Ho-Meoyng Choi,[1] Chueng-Ryong Ji,[2] Ziyue Li,[2] and Hui-Young Ryu[1]

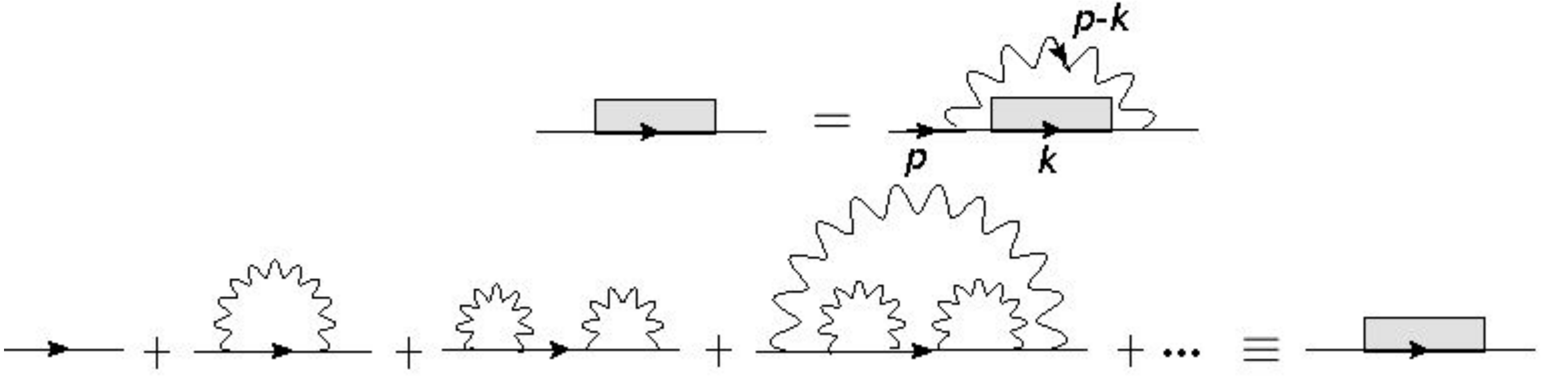


Both spacelike and timelike form factors can be computed in LFQM.

H.-M. Choi, H.-Y. Ryu, C.-R.Ji, PRD96,056008(2017); PRD99,076012(2019)

How about the physics in the timelike region?

- Mass of electron vs. quark in the timelike region?
- Euclidean vs. Minkowski spaces?
- Spacelike vs. Timelike regions?
- Free vs. Confined particles?
- Mass gap equation in timelike region?



$$\equiv i\,\Sigma(p)$$

$$\Sigma(p_-) = i\frac{\lambda}{2\pi}\int \frac{dk_- dk_+}{(p_- - k_-)^2}\gamma^+ \frac{1}{\not{k} - m - \Sigma(k_-) + i\epsilon}\gamma^+$$

Fermion Propagator

Free Propagator

$$S_f(p) = \frac{1}{\not{p} - m + i\varepsilon}$$

$$\Sigma(p) = \Sigma_s(p) + \Sigma_v(p)\not{p}$$

Interacting Propagator

$$S(p) = \frac{1}{\not{p} - m - \sum(p) + i\varepsilon}$$

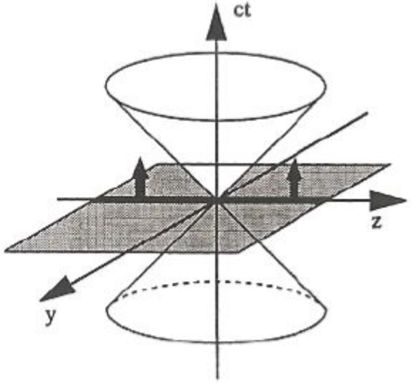
$$= \frac{F(p)}{\not{p} - M(p) + i\varepsilon}$$

$$F(p) = \left(1 - \Sigma_v(p)\right)^{-1}$$ “Wave function renormalization factor”

$$M(p) = \frac{m + \Sigma_s(p)}{1 - \Sigma_v(p)}$$ “Renormalized fermion mass function”

IFD

Instant Form Dynamics

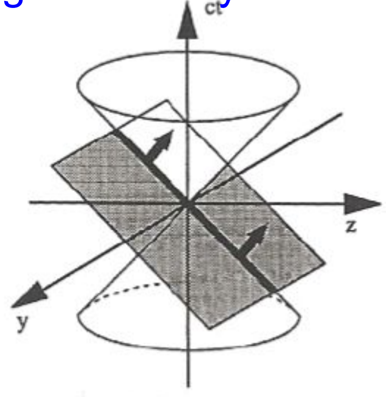


The instant form

1949

LFD

Light-Front Dynamics



The front form

Traditional approach evolved from NR dynamics

Close contact with Euclidean space

T-dept QFT, LQCD, IMF, etc.

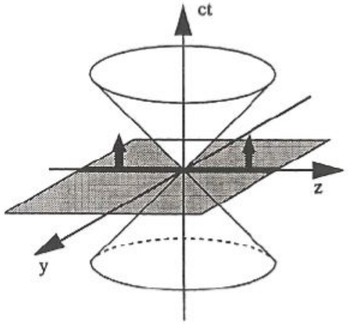
Innovative approach for relativistic dynamics

Strictly in Minkowski space

DIS, PDFs, DVCS, GPDs, etc.

IFD

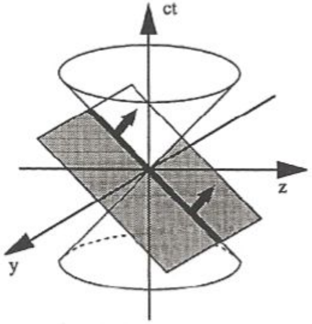
Instant Form Dynamics



1949

LFD

Light-Front Dynamics



Can IFD and LFD be linked?

K. Hornbostel, PRD45,3781(1992)
“Nontrivial vacua from equal time to the light cone”

.....

C.Ji, Z.Li,B.Ma & A.Suzuki, PRD98,036017(2018) “Interpolating quantum electrodynamics between instant and front forms”
B.Ma & C.Ji, PRD103,036004(2021) “Interpolating ‘tHooft model between instant and front forms”

Extended Wick Rotation

$$p^0 \to \tilde{P}^0 = ip^0 \quad (\delta = 0)$$

$$For \quad 0 < \delta < \pi / 4,$$

$$p^{\hat{+}} / \sqrt{C} \to \tilde{P}^{\hat{+}} / \sqrt{C} = ip^{\hat{+}} / \sqrt{C}.$$

Correspondence to Euclidean Space

$$p'^2_{\underline{\wedge}} = p^2_{\underline{\wedge}} / C \leftrightarrow -\tilde{P}^2$$

Works to do

- Impact study of global QCD analyses in EIC and JLab-TDIS
- VMP in 3+1 D for the feasibility study of GPDs
- Timelike transition form factor in ‘tHooft model
- Quantum correlation of LF zero-modes
- Study of interpolation utility in solving dynamics
- Interpolating conformal symmetry