Manifestation of quantum correlations in the interpolating helicity amplitudes between the instant form dynamics and the light-front dynamics for the annihilation/production process of two spin-1 particles

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Quantum correlation of spins

$$\frac{1}{2} \otimes \frac{1}{2} = 1 \oplus 0$$

$$d^{(j)}_{m',m}(\beta) = < j, m' | exp(\frac{-i j_y \beta}{\hbar}) | j, m >$$

(Wigner-d function)

For spin-1 particles $(j = 1, m = 1, 0, -1)$

$$|1,1> = |\uparrow\uparrow>$$

$$|1,0> = \frac{1}{\sqrt{2}}(|\uparrow\downarrow \underline{+} \downarrow\uparrow>)$$

$$|1,-1> = |\downarrow\downarrow>$$

For spin-0 particles $(j = 0, m = 0)$

$$|0,0> = \frac{1}{\sqrt{2}}(|\uparrow\downarrow \underline{-} \downarrow\uparrow>)$$

$$d^{1/2}(\pi)|\uparrow> = |\downarrow>$$

$$d^{1/2}(\pi)|\downarrow> = \underline{-}|\uparrow>$$

⇒

spin-1 particles

$$|1,1> \longrightarrow |1,-1>$$

$$|1,0> \longrightarrow -|1,0>$$

$$|1,-1> \longrightarrow |1,1>$$

spin-0 particles

$$|0,0> \longrightarrow |0,0>$$

Interpolating Dynamic

$$\begin{pmatrix} x^{\hat{+}} \\ x^{\hat{1}} \\ x^{\hat{2}} \\ x^{\hat{-}} \end{pmatrix} = \begin{pmatrix} \cos(\delta) & 0 & 0 & \sin(\delta) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \sin(\delta) & 0 & 0 & -\cos(\delta) \end{pmatrix} \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}$$

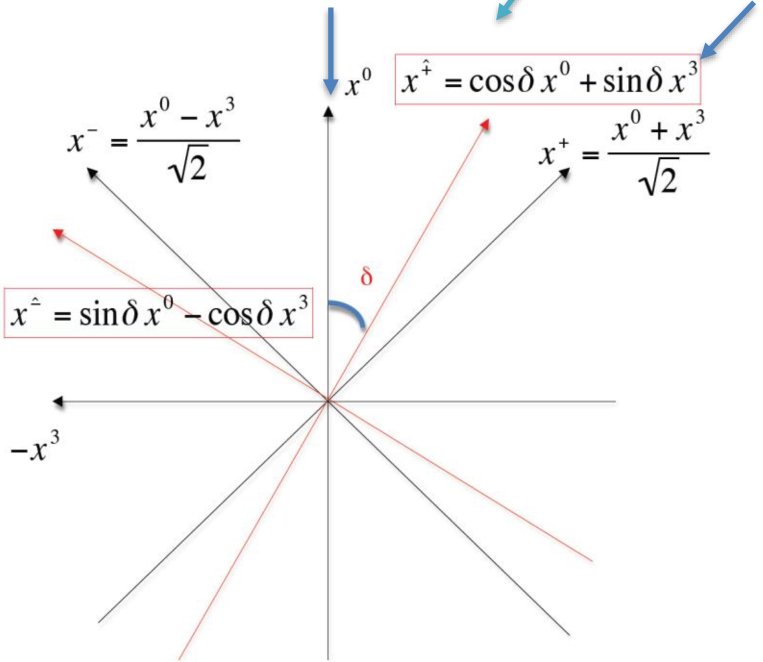
Not invariant under the boost

IFD ($\delta = 0$)

δ_c

LFD ($\delta = \frac{\pi}{4}$)

Invariant under the boost



Interpolation space time matrix

$$g_{\hat{\mu}\hat{\nu}} = \begin{bmatrix} \mathbb{C} & 0 & 0 & \mathbb{S} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \mathbb{S} & 0 & 0 & -\mathbb{C} \end{bmatrix}$$

Co-variant and contra-variant indices

$$x_{\hat{+}} = \mathbb{C}x^{\hat{+}} + \mathbb{S}x^{\hat{-}} \qquad x^{\hat{+}} = \mathbb{C}x_{\hat{+}} + \mathbb{S}x_{\hat{-}}$$
$$x_{\hat{-}} = \mathbb{S}x^{\hat{+}} + \mathbb{C}x^{\hat{-}} \qquad x^{\hat{-}} = \mathbb{S}x_{\hat{+}} + \mathbb{C}\, x_{\hat{-}}$$
$$x_j = -x^j, (j = 1,2)$$

Where $\mathbb{S} = Sin(2\delta)$
$\mathbb{C} = Cos(2\delta)$

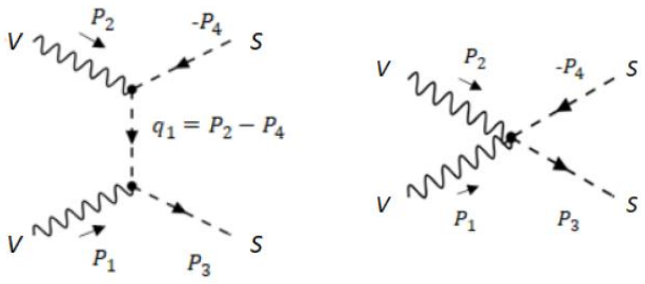
- We connect two relativistic dynamics, proposed by Dirac

P. A. M. Dirac, Rev. Mod. Phys. **21**, 392 (1949)

M. Jacob and G. Wick, Ann. Phys.**7**, 404 (1959)

C.-R. Ji, Z. Li, and B. Ma Phys. Rev. D **98**, 036017 (2018)

Scalar particle and its anti-particle production by two neutral massive spin-1 particles



(a) (b)

Fig. (a) t-channel Feynman diagram, the cross channel (u-channel) can be drawn by crossing the two final states' particles. Fig. (b) is drawn for the seagull channel.

v -> Vector particle (Spin-1)
s -> Scalar particle (Spin-0)

Interpolating helicity amplitudes

$$M_t^{\lambda_1\lambda_2} = (p_3 + q_1)^{\hat{\mu}} \varepsilon_{\hat{\mu}}(p_1, \lambda_1) \frac{1}{q_1^2 - m_s{}^2} (-p_4 + q_1)^{\hat{\nu}} \varepsilon_{\hat{\nu}}(p_2, \lambda_2)$$

$$M_u^{\lambda_1\lambda_2} = (-p_3 + q_2)^{\hat{\nu}} \varepsilon_{\hat{\nu}}(p_2, \lambda_2) \frac{1}{q_2^2 - m_s{}^2} (p_4 + q_2)^{\hat{\mu}} \varepsilon_{\hat{\mu}}(p_1, \lambda_1)$$

$$M_{se}^{\lambda_1\lambda_2} = -2 g_{\hat{\mu}\hat{\nu}} \varepsilon^{\hat{\mu}} (p_1, \lambda_1)\, \varepsilon^{\hat{\nu}} (p_2, \lambda_2)$$

Where $q_2 = p_2 - p_3$

The interpolating photon polarization vectors

$$\epsilon_{\hat{\mu}}(P,+) = -\frac{1}{\sqrt{2}\,\mathbb{P}}\left(\mathbb{S}|\boldsymbol{p}_{\perp}|, \frac{P_1 P_{\hat{-}} - iP_2\mathbb{P}}{|\mathbf{p}_{\perp}|}, \frac{P_2 P_{\hat{-}} + iP_1\mathbb{P}}{|\mathbf{p}_{\perp}|}, -\mathbb{C}\,|\boldsymbol{p}_{\perp}|\right)$$

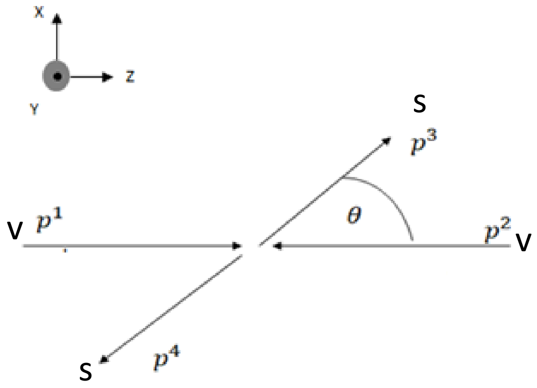
$$\epsilon_{\hat{\mu}}(P,-) = \frac{1}{\sqrt{2}\,\mathbb{P}}\left(\mathbb{S}|\boldsymbol{p}_{\perp}|, \frac{P_1 P_{\hat{-}} + iP_2\mathbb{P}}{|\mathbf{p}_{\perp}|}, \frac{P_2 P_{\hat{-}} - iP_1\mathbb{P}}{|\mathbf{p}_{\perp}|}, -\mathbb{C}\,|\boldsymbol{p}_{\perp}|\right)$$

$$\epsilon_{\hat{\mu}}(P,0) = \frac{P^{\hat{+}}}{m_v\mathbb{P}}\left(P_{\hat{+}} - \frac{m_\gamma^2}{P^{\hat{+}}}, P_1, P_2, P_{\hat{-}}\right)$$

Where, $\mathbb{P} = \sqrt{{P_{\hat{-}}}^2 + \mathbb{C}{\mathbf{p}_{\perp}}^2} = \sqrt{(P^{\hat{+}})^2 - \mathbb{C}{m_v}^2}$

$$|\mathbf{p}_{\perp}| = \sqrt{P_1^2 + P_2^2}$$

C.-R. Ji, Z. Li, and A. T. Suzuki, Phys. Rev. D **91**, 065020 (2015)



- Center of mass frame and the four momenta of the particles

$$p^1 = \{E_0, 0,0, P_v\}$$

$$p^2 = \{E_0, 0,0, -P_v\}$$

$$p^3 = \{E_0, P_s Sin(\theta), 0, P_s Cos(\theta)\}$$

$$p^4 = \{E_0, -P_s Sin(\theta), 0, -P_s Cos(\theta)\}$$

Lorentz Transformation

$$E = \sqrt{4E_0^2 + P_z^2} \qquad \alpha = \frac{E}{4E_0} \qquad \alpha\beta = \frac{P_z}{4E_0}$$

$$p_i'^0 = \alpha p_i^0 + \alpha\beta p_i^z$$

$$p_i'^z = \alpha p_i^z + \alpha\beta p_i^0$$

$$p_i'^{\perp} = p_i^{\perp}$$

Symmetries between covariant Helicity amplitudes

Mt + +	=	Mt - -	Mt 0 +	=	-(Mt 0 -)
Mu + +	=	Mu - -	Mu 0 +	=	-(Mu 0 -)
Mse + +	=	Mse - -	Mse 0 +	=	-(Mse 0 -)
Mt + -	=	Mt - +	Mt + 0	=	-(Mt – 0)
Mu + -	=	Mu - +	Mu + 0	=	-(Mu – 0)
Mse + -	=	Mse - +	Mse + 0	=	-(Mse – 0)

- t – u Symmetry

Mt + + (θ)	=	Mu ++ $(\pi-\theta)$
Mt + - (θ)	=	Mu - +$(\pi-\theta)$
Mt 0 + (θ)	=	Mu + 0 $(\pi-\theta)$
Mu 0 + (θ)	=	Mt + 0 $(\pi-\theta)$
Mt 00 (θ)	=	Mu 00 $(\pi-\theta)$

- Helicity amplitudes satisfy symmetry based on parity conservation

$$\mathrm{M}(-s',-h',-s,-h) = (-1)^{s'+h'-s-h} M(s',h',s,h)$$

$$\mathrm{M}(-h',-h) = (-1)^{h'-h} M(h',h)$$

Where h, h' → helicity values of initial state particles
s, s' → helicity values of final state particles

C-R . Ji, B.L.G.Bakker , International Journal of Modern Physics,(2013)

Generalized helicity operator

$$\mathfrak{J}_3 = \frac{1}{\mathbf{P}}(P_{\hat{-}} J_3 + P^1 \kappa^{\hat{2}} - P^2 \kappa^{\hat{1}})$$

$\delta \to 0$

$$\mathfrak{J}_3 \to \frac{\boldsymbol{P}.\boldsymbol{J}}{|\boldsymbol{P}|}$$

$\delta \to \frac{\pi}{4}$

$$\mathfrak{J}_3 \to J_3 + \frac{(P^1 E_2 - P^2 E_1)}{P^+}$$

If the particle is moving in the *z*- directions ($P^1 = P^2 = 0$)

$$\mathfrak{J}_3 = \frac{P_{\hat{-}} J_3}{|P_{\hat{-}}|}$$

- Helicity and phase changes are taken place, when $P_{\hat{-}} = 0$

Boost in z- directions

↓

Wigner-d rotation in 180^0 angles

Z. Li, M. An, and C.-R. Ji, , Phys. Rev. D **92**, 105014 (2015)

Critical interpolation angles

- The angle separate the branch that LFD belongs to from the branch that IFD belongs to.

$P_{\hat{=}}$ =0 (zero longitudinal momentum of the particle)

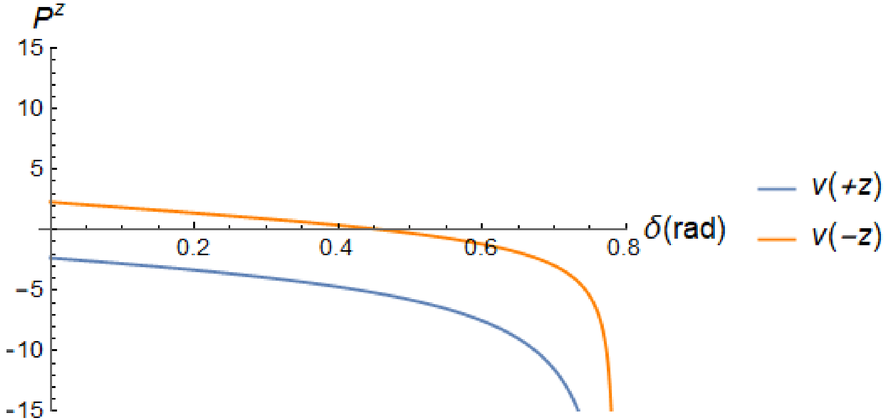
Spin 1 particle moving in *+z* direction

$$\delta_c = -ArcTan\left[\frac{\left(E_0 P_z + P_v\sqrt{4{E_0}^2 + {P_z}^2}\right)}{\left(P_v P_z + E_0\sqrt{4{E_0}^2 + {P_z}^2}\right)}\right]$$

Spin 1 particle moving in *-z* direction

$$\delta_c = -ArcTan\left[\frac{\left(E_0 P_z - P_v * \sqrt{4{E_0}^2 + {P_z}^2}\right)}{\left(-P_v P_z + E_0\sqrt{4{E_0}^2 + {P_z}^2}\right)}\right]$$

$E_0 = 2 \quad P_v = 1$



- Two boundaries correspond to the massive spin-1 particle move in +z and –z direction

Seagull Channel

- Contact interaction - Angular momentum is conserved without involving orbital angular momentum

V V → SS

\+ +

Mse^{-+} -Z

+Z Mse^{+-}

\+ -

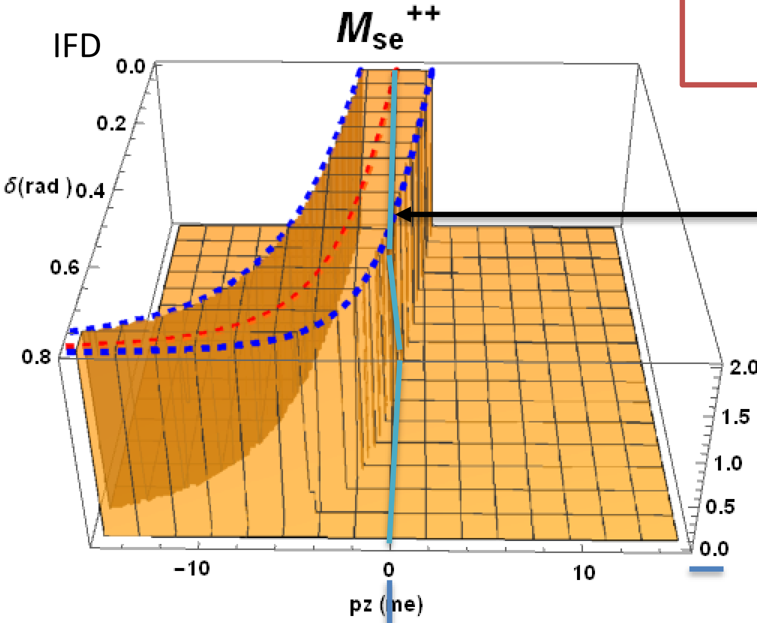
Mse^{--} -Z

+Z Mse^{++}

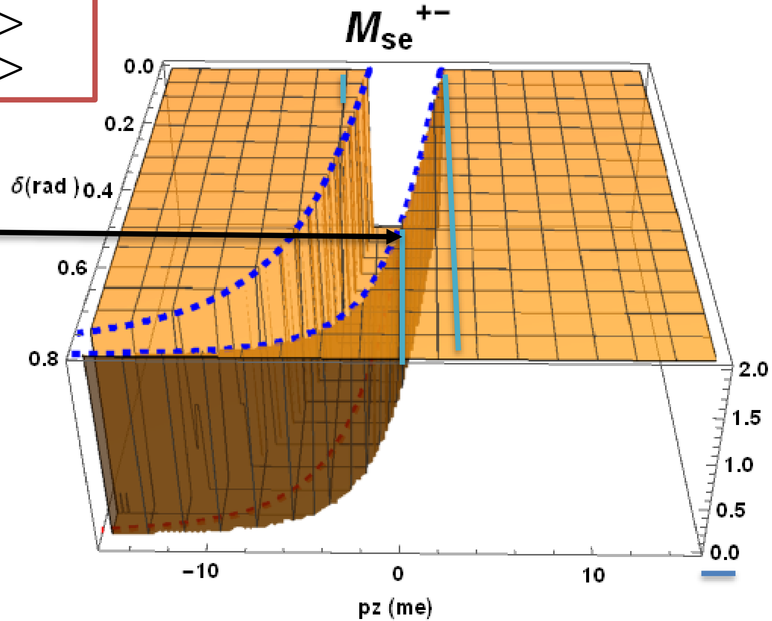
Helicity Change

$|1,1> \rightarrow |1,-1>$

$|1,-1> \rightarrow |1,1>$



δ_c



CMF

$Mse^{-+} = Mse^{-+}$

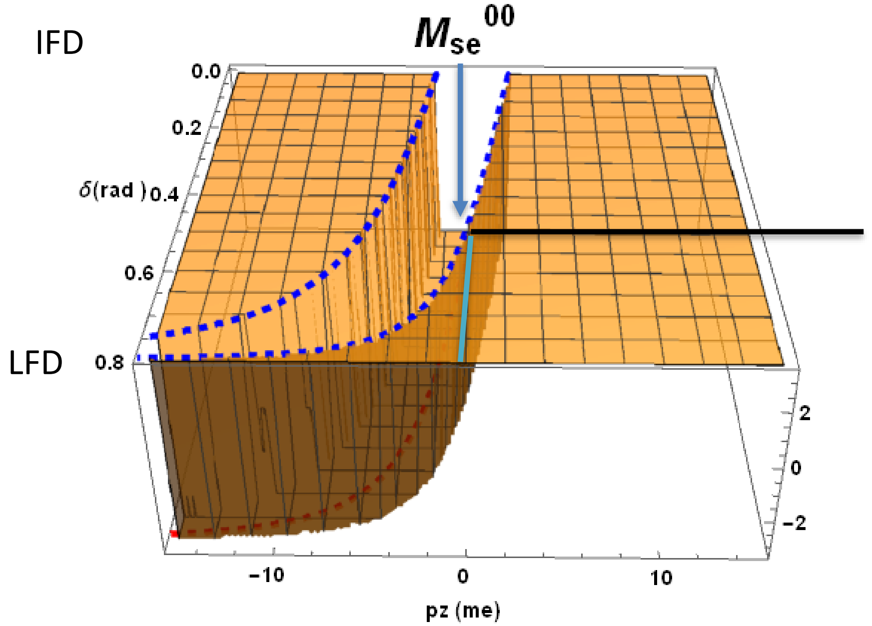
$E_0 = 2 \quad P_s = \sqrt{3}$

$Mse^{--} = Mse^{++}$

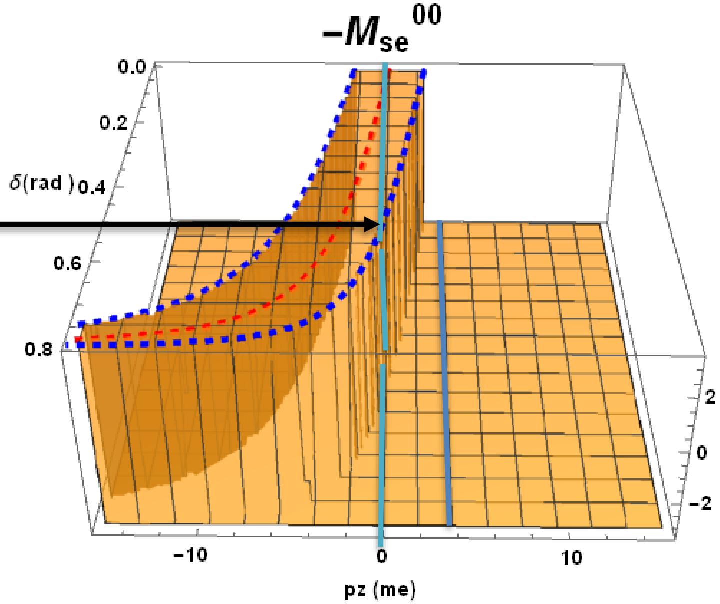
Seagull Channel

V V → SS

$|1,0> \rightarrow -|1,0> --$Phase Change due to Quantum Correlatio



δ_c

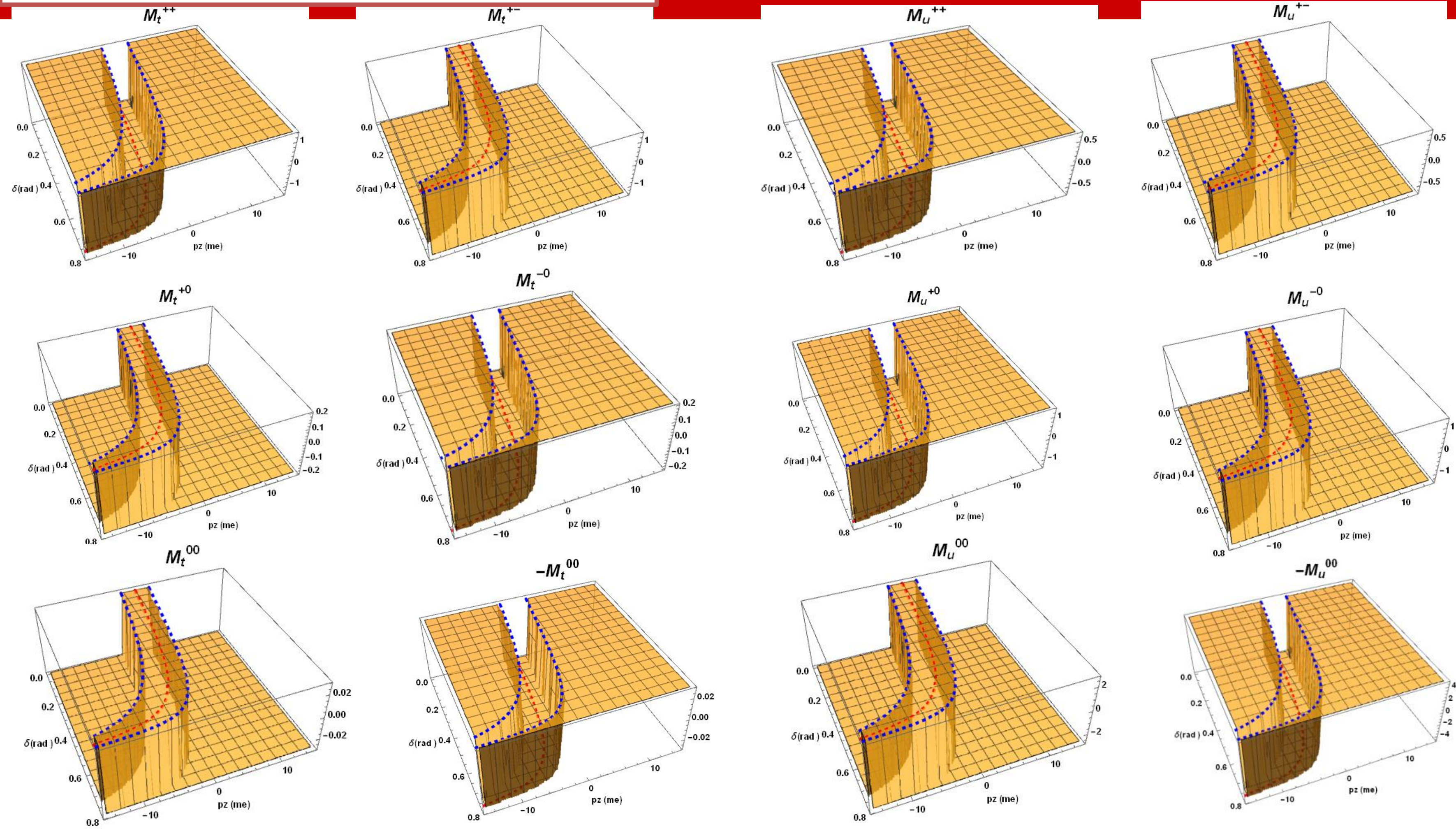


CMF $-Mse^{00}$ ← -z Mse^{00} → +z $-Mse^{00}$

- $Mse^{+0} = Mse^{-0} = Mse^{0+} = Mse^{0-} = 0$ → Do not satisfy conservation of total angular momentum in any frame

T and U channels helicity amplitudes

- Depend on orbital angular momentum involving impact parameter



$$E_0 = 2$$
$$P_s = \sqrt{3}$$
$$P_v = 1$$
$$\theta = \frac{\pi}{3}$$

Conclusions

- The quantum correlation effect manifests itself as the helicity boundary between the IFD and the LFD which appears in the helicity amplitudes of scattering processes by boosting the reference frame

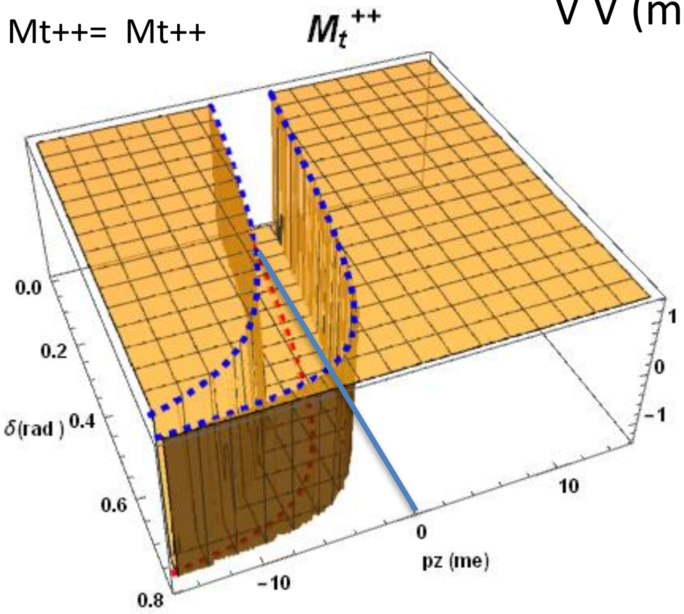
- Helicity amplitude do not change at the LF end and helicity boundaries always appear outside from it which confirm that the helicity in LFD in invariant under boost.

THANK YOU

BACKUP SLIDES

V V (massive) → SS

Mt++= Mt++



$|1,1> \rightarrow |1,-1>$

$|1,-1> \rightarrow |1,1>$

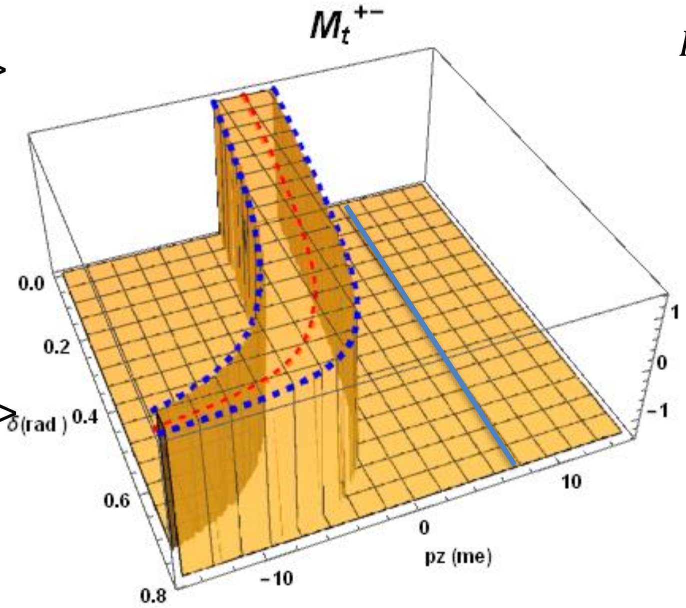
Mt+-= Mt-+

$E_0 = 2$

$P_s = \sqrt{3}$

$P_v = 1$

$\theta = \frac{\pi}{3}$

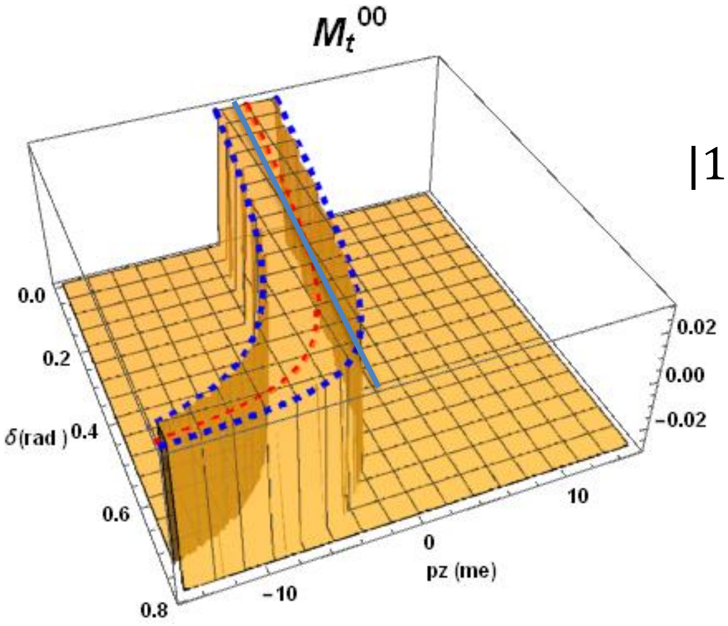


Mt-+ ← -z — Mt++ — +z → Mt+-

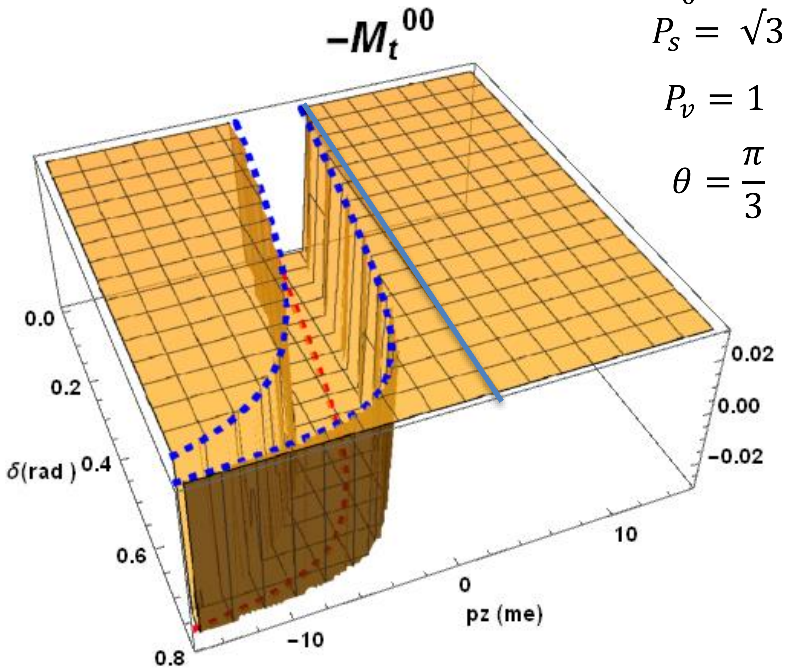
Mt-- ← -z — Mt+- — +z → Mt++

V V (massive) → SS

$E_0 = 2$
$P_s = \sqrt{3}$
$P_v = 1$
$\theta = \frac{\pi}{3}$



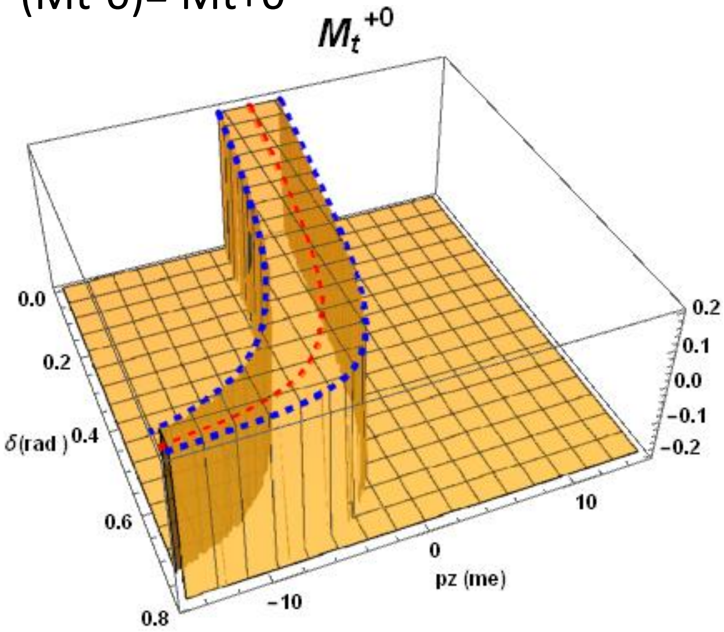
$|1,0> \rightarrow -|1,0>$



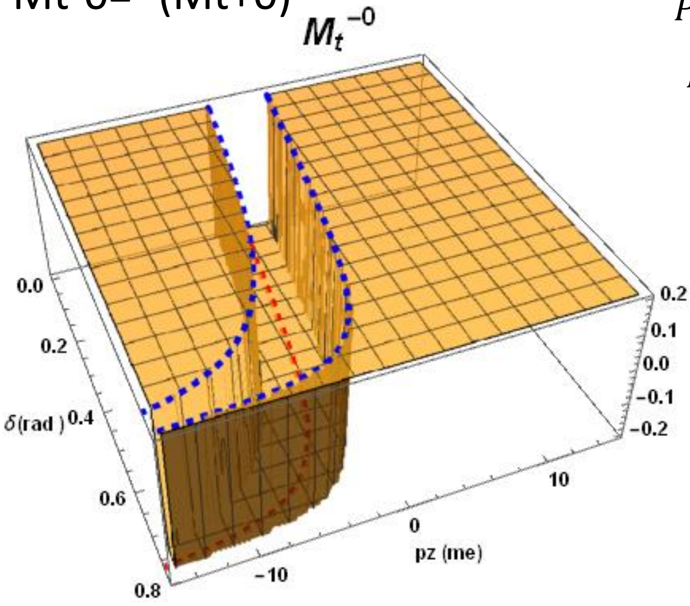
-Mt00 ← (-z) Mt00 (+z) → -Mt00

V V (massive) → SS

-(Mt-0)= Mt+0



Mt-0= -(Mt+0)

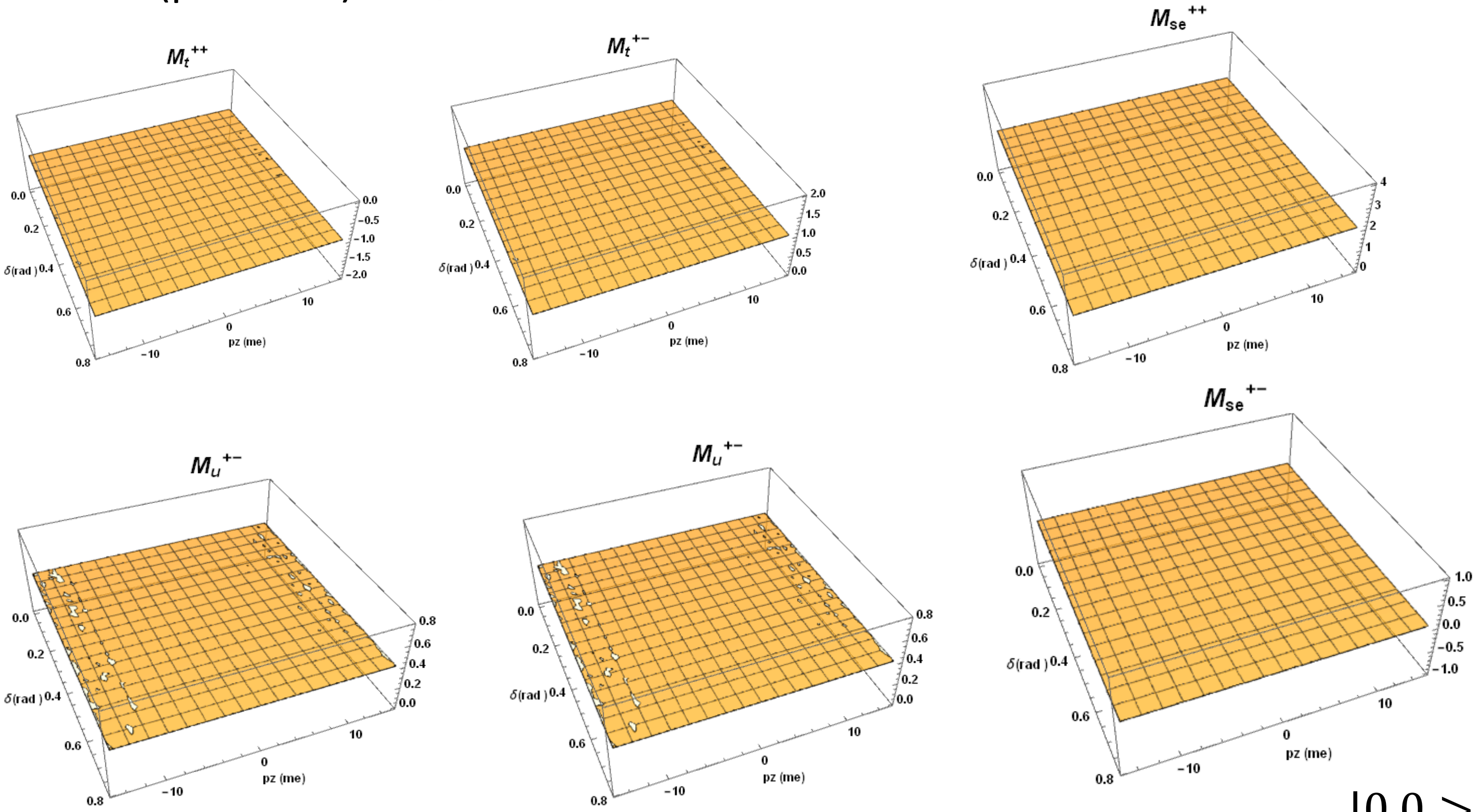


$E_0 = 2$
$P_s = \sqrt{3}$
$P_v = 1$
$\theta = \frac{\pi}{3}$

Mt-0 ← (-z) Mt+0 → (+z) -Mt+0

Mt+0 ← (-z) Mt-0 → (+z) -Mt-0

V V (photons) → SS



$$P_s = \sqrt{3}$$

$$\theta = \frac{\pi}{3}$$

$$E_0 = 2$$

$$|0,0> \rightarrow |0,0>$$