Critical interpolation angles and boundaries

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HELICITY SPINOR FOR ANY INTERPOLATION ANGLE

Initial state at rest $|0; j, m>$

A spin projection along the z direction

$$J_3|0; j, m >= \ m|0; j, m>$$

Generalized helicity spinor state in each interpolation angle for a particle of spin *j* moving with momentum *p* and helicity *m*.

$$|p; j, m>_{\delta} = T|0; j, m>$$

Transformation matrix $T = e^{i\beta_1 \kappa^{\hat{1}} + i\beta_2 \kappa^{\hat{2}} - i\beta_3 K^{\hat{3}}}$

$$T = T_{12}T_3 = e^{i\beta_1 \mathcal{K}^{\hat{1}} + i\beta_2 \mathcal{K}^{\hat{2}}} e^{-i\beta_3 K^3}$$

$$P^{\hat{+}} = (\cos\delta \cosh\beta_3 + \sin\delta \sinh\beta_3)\, M,$$

$$P^{\hat{1}} = \beta_1 \frac{\sin\alpha}{\alpha} (\sin\delta \cosh\beta_3 + \cos\delta \sinh\beta_3)\, M,$$

$$P^{\hat{2}} = \beta_2 \frac{\sin\alpha}{\alpha} (\sin\delta \cosh\beta_3 + \cos\delta \sinh\beta_3)\, M,$$

$$P^{\hat{-}} = \frac{\mathbb{S} P^{\hat{+}} - P_{\hat{-}}}{\mathbb{C}},$$

$$\mathcal{K}^{\hat{1}} = -K^1 \sin\delta - J^2 \cos\delta,$$

$$\mathcal{K}^{\hat{2}} = J^1 \cos\delta - K^2 \sin\delta,$$

$$\cos\alpha = \frac{P_{\hat{-}}}{\mathbb{P}}$$

Interpolating Helicity Spinors Between the Instant Form and the Light-front Form

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New spin Operator for a moving particle- Generalized helicity operator

$$\mathfrak{J}_i = T J_i T^{-1}$$

$$\mathfrak{J}_3|p;j,m\rangle_\delta = T J_i T^{-1} T|0;j,m\rangle = m|p;j,m\rangle_\delta$$

$\mathfrak{J}_3$ In terms of the particle's momentum

$$\mathfrak{J}_3 = \frac{1}{\mathbb{P}}(P_{\hat{-}} J_3 + P^1 \kappa^{\hat{2}} - P^2 \kappa^{\hat{1}})$$

$$\mathbb{P} = \sqrt{P_{\hat{-}}{}^2 + \mathbb{C}\mathbf{p}_{\perp}{}^2} = \sqrt{(P^{\hat{+}})^2 - \mathbb{C}M^2} \quad ; \mathbb{C} = Cos(2\delta)$$

Instant Form Limit

$$\kappa^{\hat{1}} \to -J^2, \kappa^{\hat{2}} \to J^1, P_{\hat{-}} \to P^3, \mathbb{P} \to \sqrt{(P^0)^2 - M^2} = |\mathbf{P}|$$

$$\mathfrak{J}_3 \to \frac{\mathbf{P}.\mathbf{J}}{|\mathbf{P}|}$$

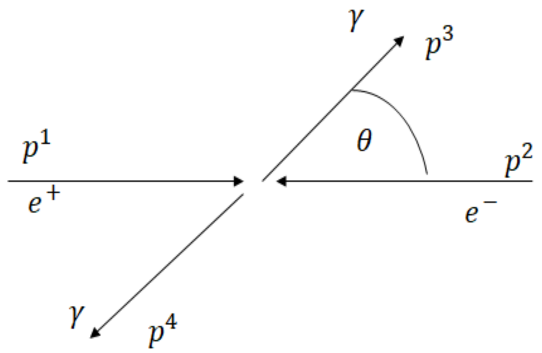
Light Front Limit

$$\kappa^{\hat{1}} \to -E_1, \kappa^{\hat{2}} \to -E_2, P_{\hat{-}} \to P^+, \mathbb{P} \to P^+$$

$$\mathfrak{J}_3 \to \frac{(P^2 E_1 - P^1 E_2)}{P^+}$$

If the particle is moving in the +*z* or −*z* direction ($P^1 = P^2 = 0$)

$$\mathfrak{J}_3 = \frac{1}{\mathbb{P}}(P_{\hat{-}} J_3)$$



Critical interpolation angle condition ; $P_{\hat{-}} = 0$

$$\delta_p^{\pm} = -ArcTan\left[\frac{\left(E0 * pz + pe * \sqrt{4E0^2 + pz^2}\right)}{\left(pe * pz + E0 * \sqrt{4E0^2 + pz^2}\right)}\right]$$

$$\delta_e^{\pm} = -ArcTan\left[\frac{\left(E0 * pz - pe * \sqrt{4E0^2 + pz^2}\right)}{\left(-pe * pz + E0 * \sqrt{4E0^2 + pz^2}\right)}\right]$$

$$p^1 = \{E_0, 0,0, P_e\}$$

$$p^2 = \{E_0, 0,0, -P_e\}$$

$$p^3 = \{E_0, P_\gamma Sin(\theta), 0, P_\gamma Cos(\theta)\}$$

$$p^4 = \{E_0, -P_\gamma Sin(\theta), 0, -P_\gamma Cos(\theta)\}$$

Interpolating quantum electrodynamics between instant and front forms

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Interpolating Helicity Spinors Between the Instant Form and the Light-front Form

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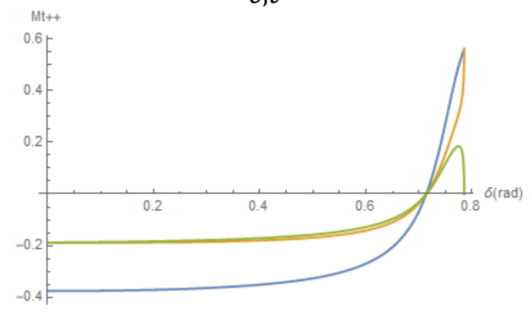
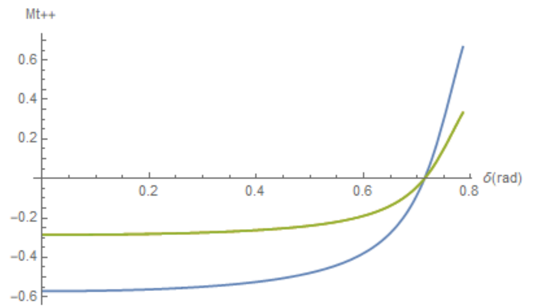
"$\pi^+\pi^-$" $\rightarrow$ $\rho^+\rho^-$

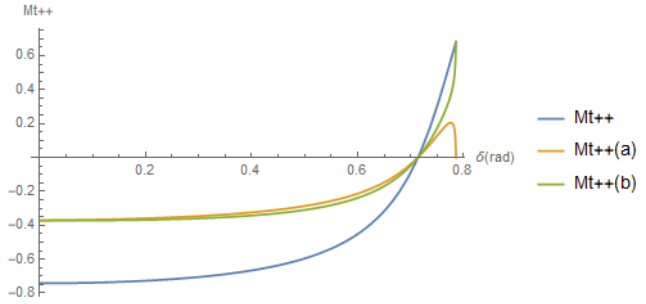
$\theta = \theta_{c,t} - 0.1$



$\theta = \theta_{c,t}$



$\theta = \theta_{c,t} + 0.1$



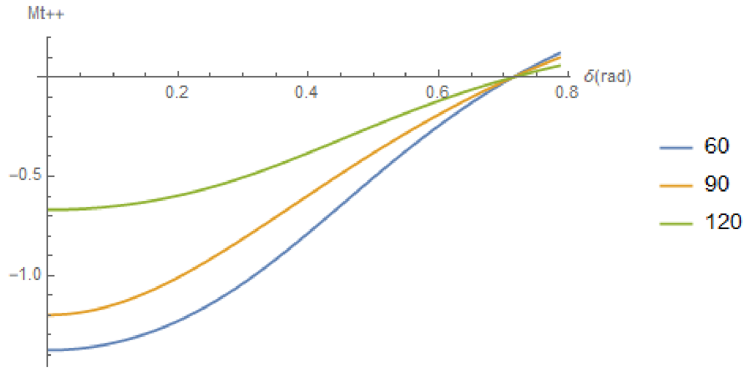
$E_0 = 2m_e$

$P_\gamma = \sqrt{3.5}m_e$

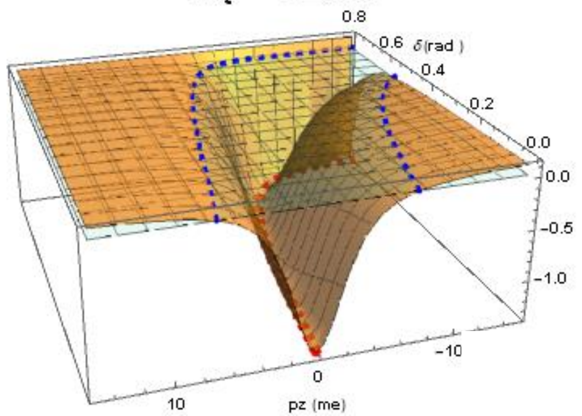
$P_e = \sqrt{3}m_e$

$\theta_{c,t}$ = critical annihilation angle

$P_\gamma = m_e$

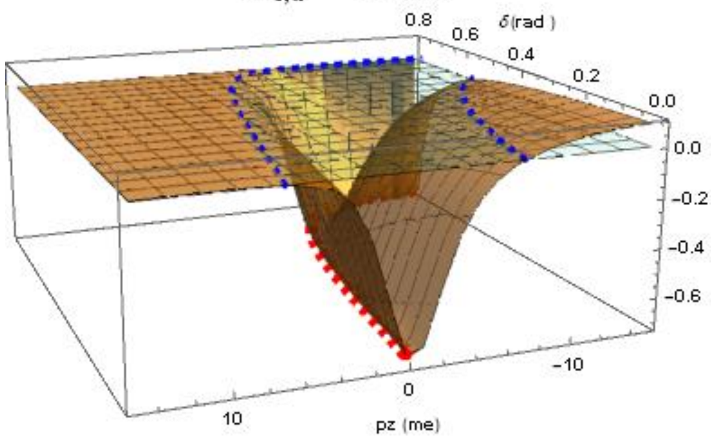


$^{\pi}M_t^{++}$ $\theta=\pi/3$

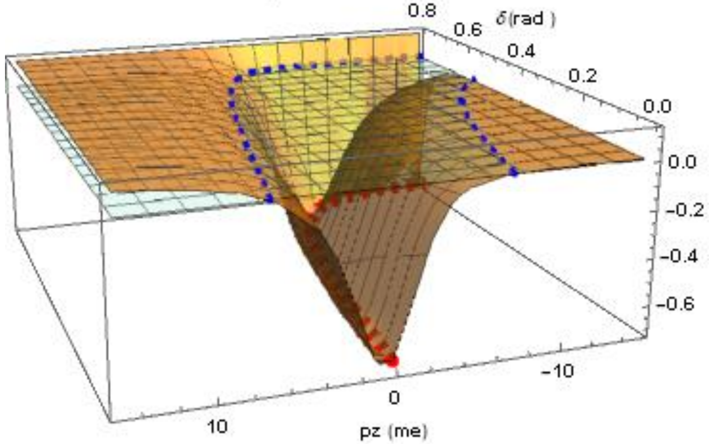


It seems there is a connection to the helicity.

$^{\pi}M_{t,a}^{++}$ $\theta=\pi/3$



$^{\pi}M_{t,b}^{++}$ $\theta=\pi/3$



$$\delta = ArcTan\left(\frac{P_e}{E_0}\right)$$

$$= 0.713724$$

$$M_t = (-p_1 + q_1)^{\hat{\mu}} \varepsilon^*_{\hat{\mu}}(p_3, \lambda_3) \frac{1}{q_1^2 - m^2} (p_2 + q_1)^{\hat{\nu}} \varepsilon^*_{\hat{\nu}}(p_4, \lambda_4)$$

$$(-p_1 + q_1)^{\hat{\mu}} \varepsilon^*_{\hat{\mu}}(p_3, \lambda_3) = -2\, (p_1)^{\hat{\mu}} \varepsilon^*_{\hat{\mu}}(p_3, \lambda_3) = 0 \quad \rightarrow (\delta_p^{\pm}, \delta_{pt}^{00})$$

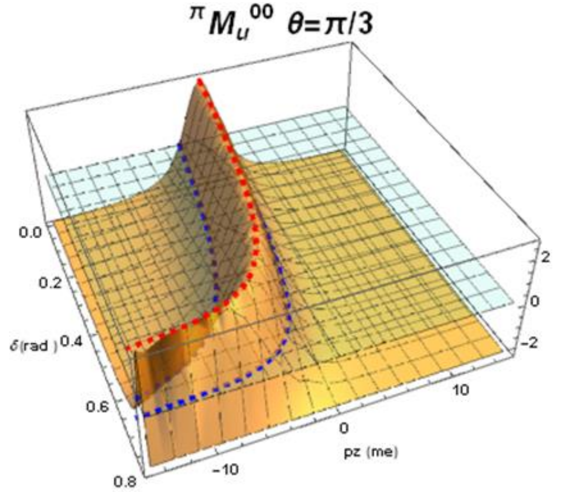
$$(p_2 + q_1)^{\hat{\nu}} \varepsilon^*_{\hat{\nu}}(p_4, \lambda_4) = 2(p_2)^{\hat{\nu}} \varepsilon^*_{\hat{\nu}}(p_4, \lambda_4) = 0 \quad \rightarrow (\delta_e^{\pm}, \delta_{et}^{00})$$

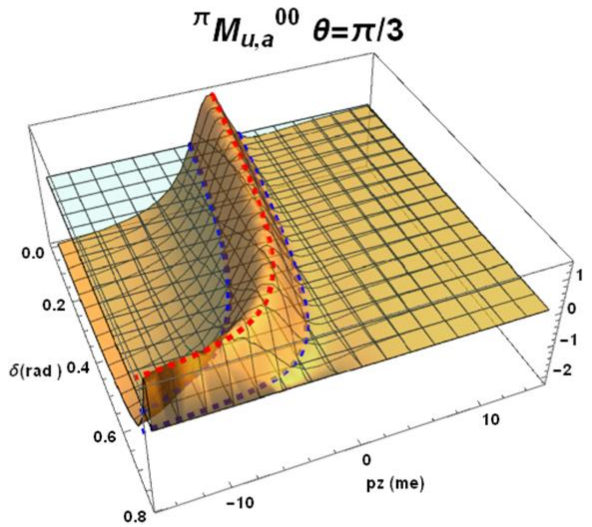
$$(p_1)^{\hat{\mu}} \varepsilon^*_{\hat{\mu}}(p_3, \pm) = (p_1)_{\hat{+}} \varepsilon^{*\hat{+}}(p_3, \pm) + (p_1)_{\hat{-}} \varepsilon^{*\hat{-}}(p_3, \pm) \quad \rightarrow P_{\hat{-}} = 0$$

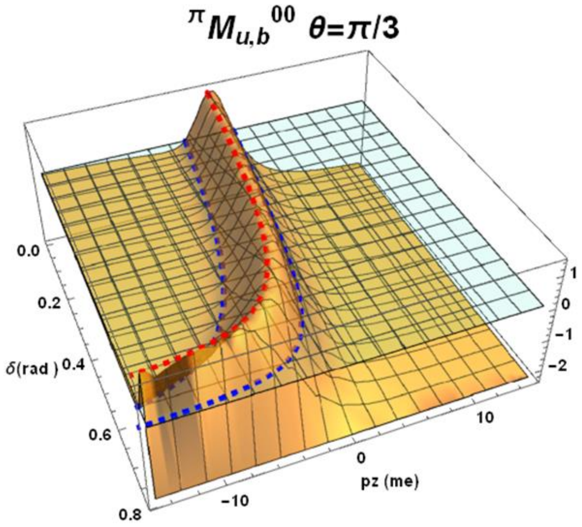
Transverse Interpolating polarization vector gauge conditions : $A^{\hat{+}} = 0$ and $\partial_{\hat{-}} A_{\hat{-}} + \partial_{\perp} . A_{\perp}$ =0

Light-front gauge : $A^{+} = 0$

Coulomb gauge in IFD : $\nabla . A = 0$







Does helicity 0 change ?

$$\delta_{et}^{00}(\theta) = \delta_{eu}^{00}(\pi - \theta)$$

$$\delta_{pt}^{00}(\theta) = \delta_{pu}^{00}(\pi - \theta)$$

$$Et = E0^2 p\gamma Cos[\theta] - pe(E0^2 - p\gamma^2 Sin[\theta]^2)$$

$$Pt = E0 p\gamma(p\gamma - pe Cos[\theta])$$

$$Eu = E0^2 p\gamma Cos[\theta] + pe(E0^2 - p\gamma^2 Sin[\theta]^2)$$

$$Pu = -E0 p\gamma(p\gamma + pe Cos[\theta])$$

$$\delta_{pt}^{00} = -ArcTan\left[\frac{\left(Et * pz + pt * \sqrt{4E0^2 + pz^2}\right)}{\left(pt * pz + Et * \sqrt{4E0^2 + pz^2}\right)}\right]$$

$$\delta_{et}^{00} = -ArcTan\left[\frac{\left(Et * pz - pt * \sqrt{4E0^2 + pz^2}\right)}{\left(-pt * pz + Et * \sqrt{4E0^2 + pz^2}\right)}\right]$$

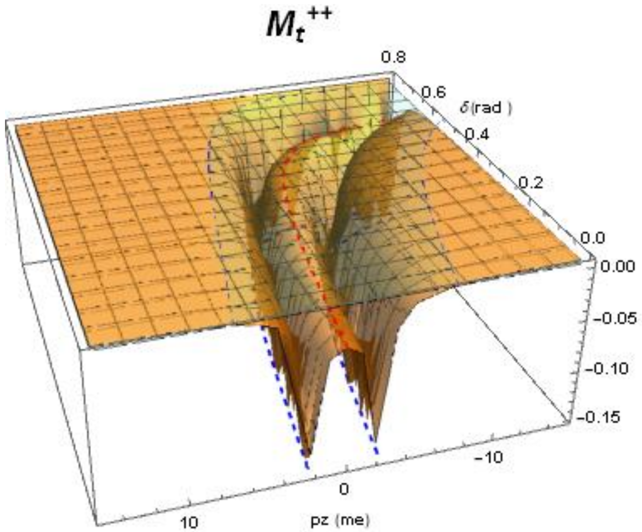
$$\delta_{pu}^{00} = -ArcTan\left[\frac{\left(Eu * pz + pu * \sqrt{4E0^2 + pz^2}\right)}{\left(pu * pz + Eu * \sqrt{4E0^2 + pz^2}\right)}\right]$$

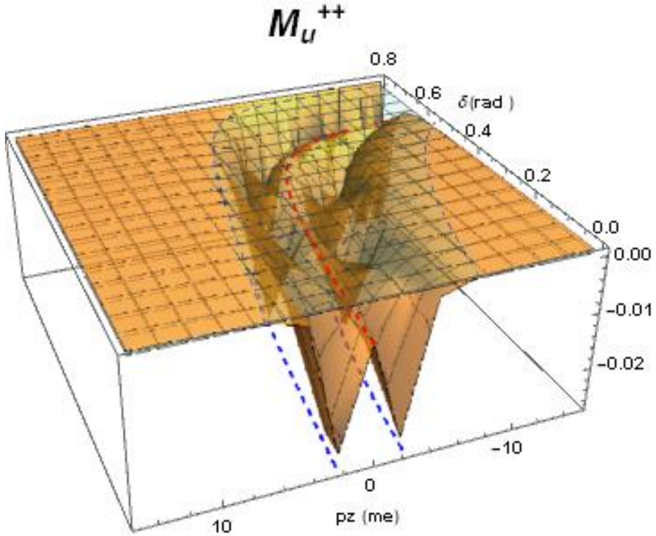
$$\delta_{eu}^{00} = -ArcTan\left[\frac{\left(Eu * pz - pu * \sqrt{4E0^2 + pz^2}\right)}{\left(-pu * pz + Eu * \sqrt{4E0^2 + pz^2}\right)}\right]$$

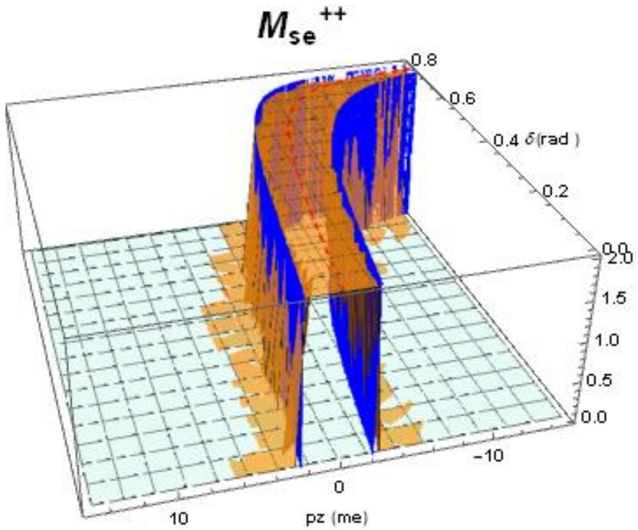
Annihilation angle $\theta \rightarrow 0$

$$\delta_{pt}^{00} = \delta_{pu}^{00} = -ArcTan\left[\frac{\left(E0*pz+p\gamma*\sqrt{4E0^2+pz^2}\right)}{\left(p\gamma*pz+E0*\sqrt{4E0^2+pz^2}\right)}\right] \qquad \delta_{et}^{00} = \delta_{eu}^{00} = -ArcTan\left[\frac{\left(E0*pz-p\gamma*\sqrt{4E0^2+pz^2}\right)}{\left(-p\gamma*pz+E0*\sqrt{4E0^2+pz^2}\right)}\right]$$

Critical interpolation angles from the helicity conditions for rhp-mesons

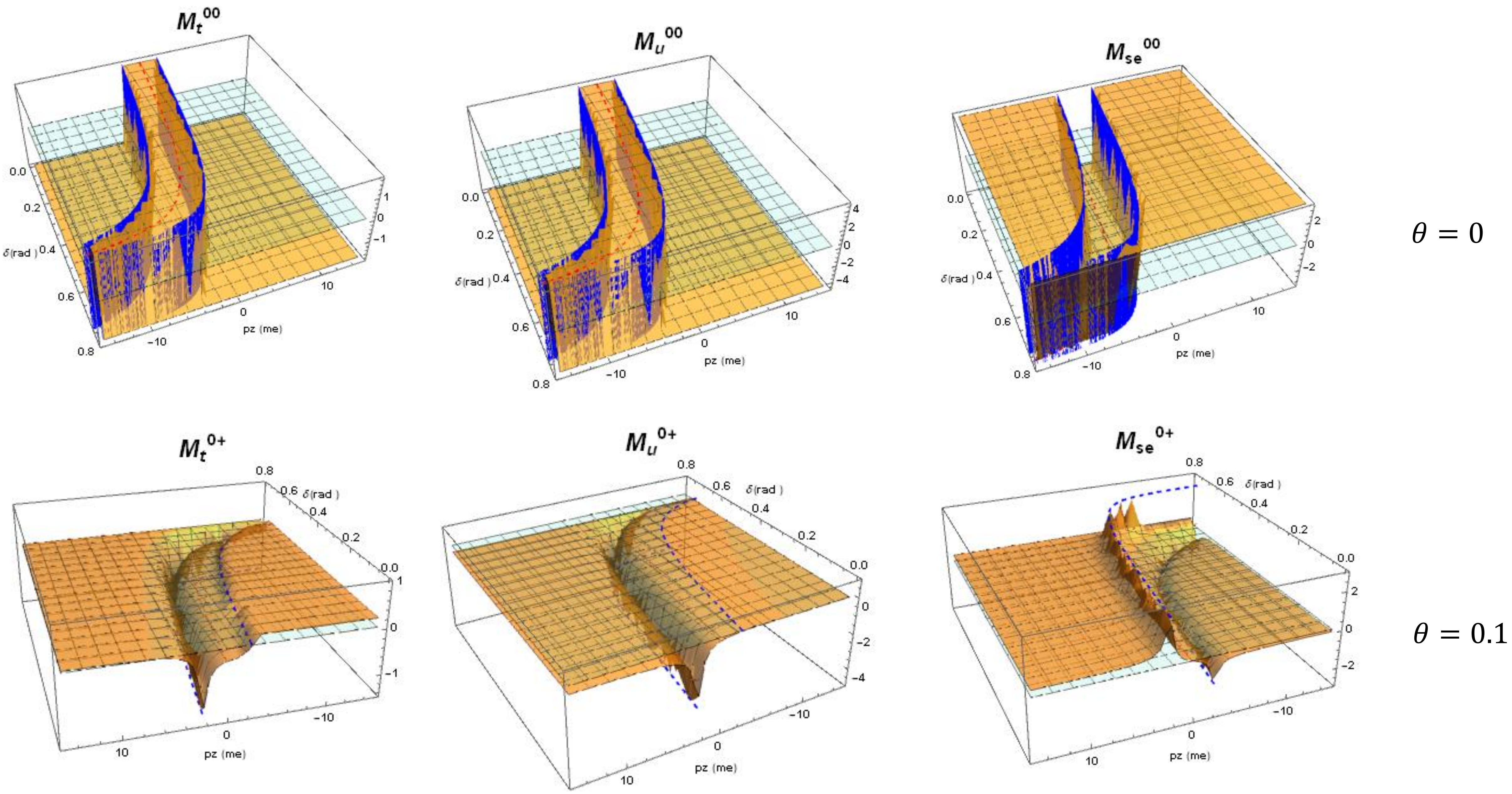






$\theta = 0.1$

$\theta = 0$

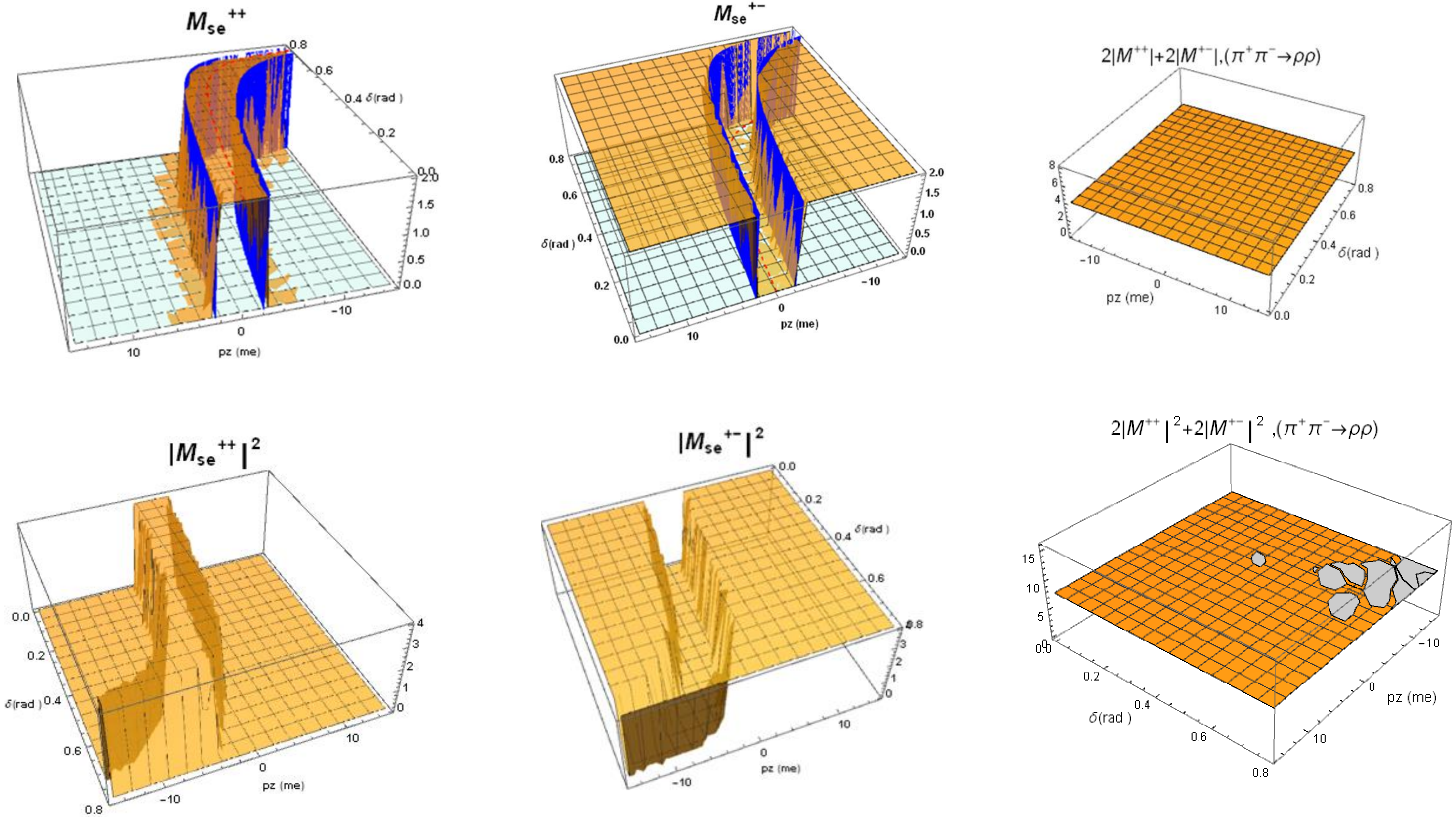


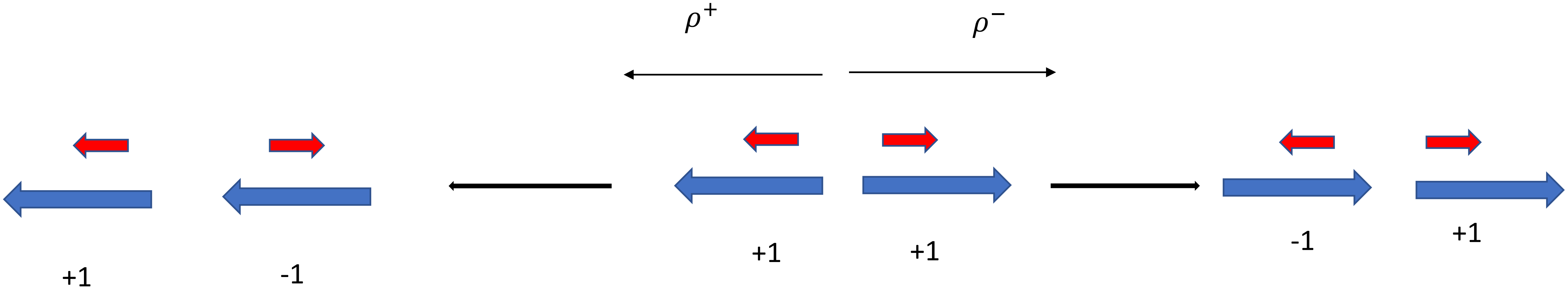
M_t^{00}
M_u^{00}
M_{se}^{00}
$\theta = 0$
M_t^{0+}
M_u^{0+}
M_{se}^{0+}
$\theta = 0.1$
δ(rad)
pz (me)

<u>"$\pi^+\pi^-$" $\rightarrow$ $\rho^+\rho^-$ process</u>

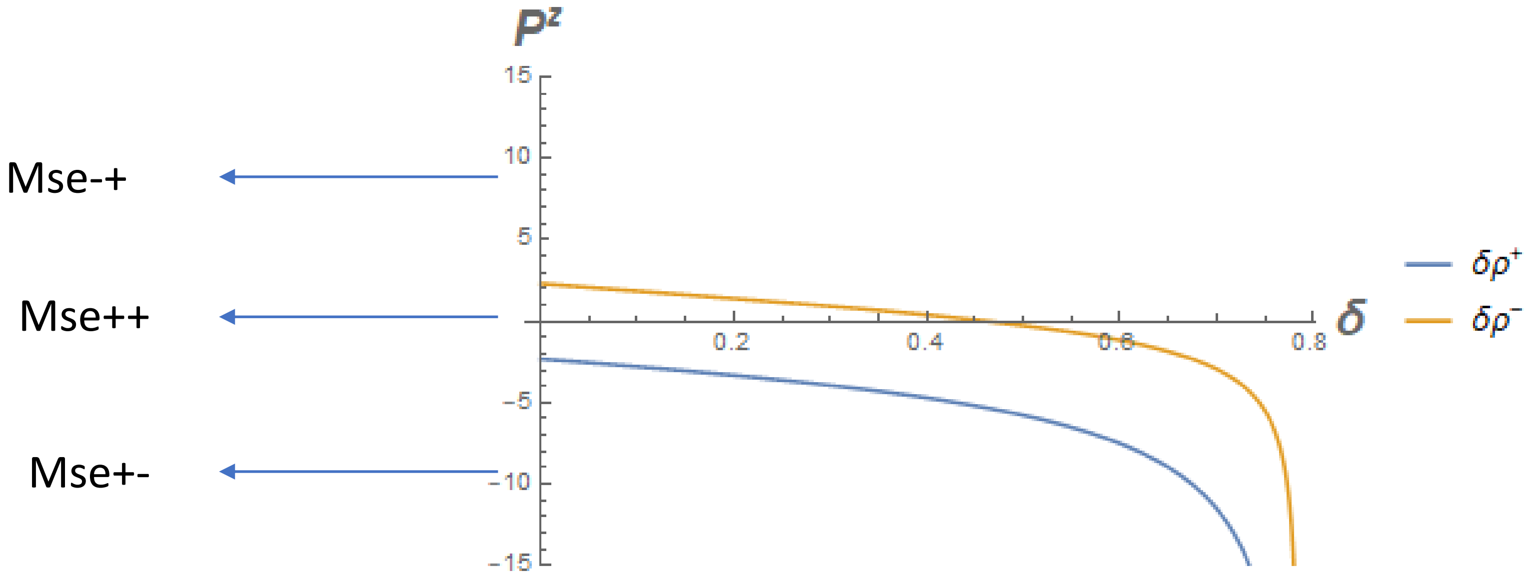
Mse++=Mse--, Mse+-=Mse-+,

$\theta = 0$

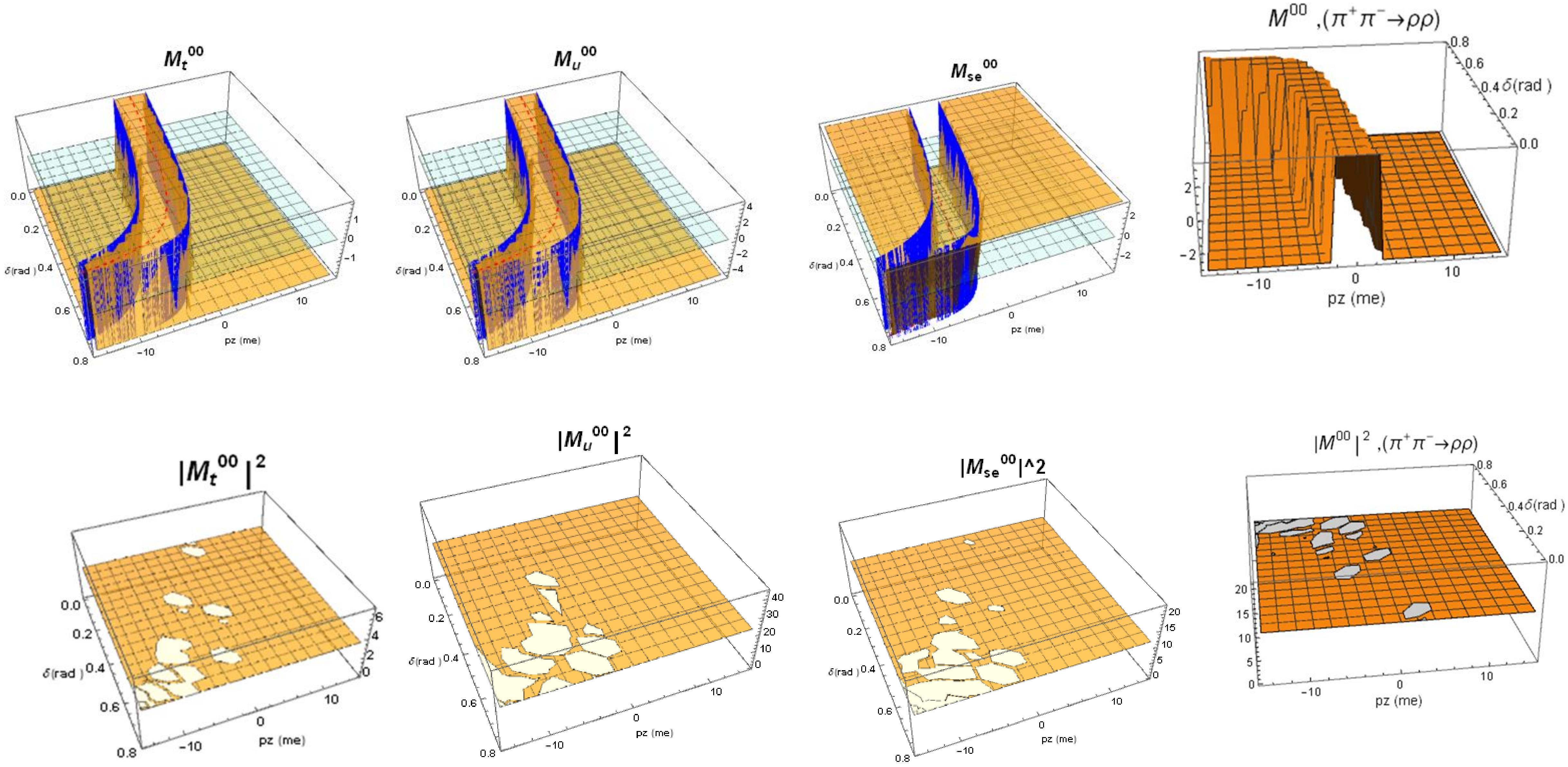




ρ^+
ρ^-
+1
-1
+1
+1
-1
+1



Mse-+
Mse++
Mse+-
P^z
15
10
5
-5
-10
-15
0.2
0.4
0.6
0.8
δ
$\delta\rho^+$
$\delta\rho^-$



M_t^{00}
M_u^{00}
M_{se}^{00}
$M^{00}, (\pi^+\pi^- \to \rho\rho)$
$|M_t^{00}|^2$
$|M_u^{00}|^2$
$|M_{se}^{00}|$^2
$|M^{00}|^2, (\pi^+\pi^- \to \rho\rho)$
pz (me)
δ(rad)

The longitudinal photon polarization vector

$$\epsilon_{\hat{\mu}}(P,0) = \frac{P^{\widehat{+}}}{m_\gamma \mathbb{P}} \left(P_{\widehat{+}} - \frac{m_\gamma^2}{P^{\widehat{+}}}, P_1, P_2, P_{\widehat{-}}\right)$$

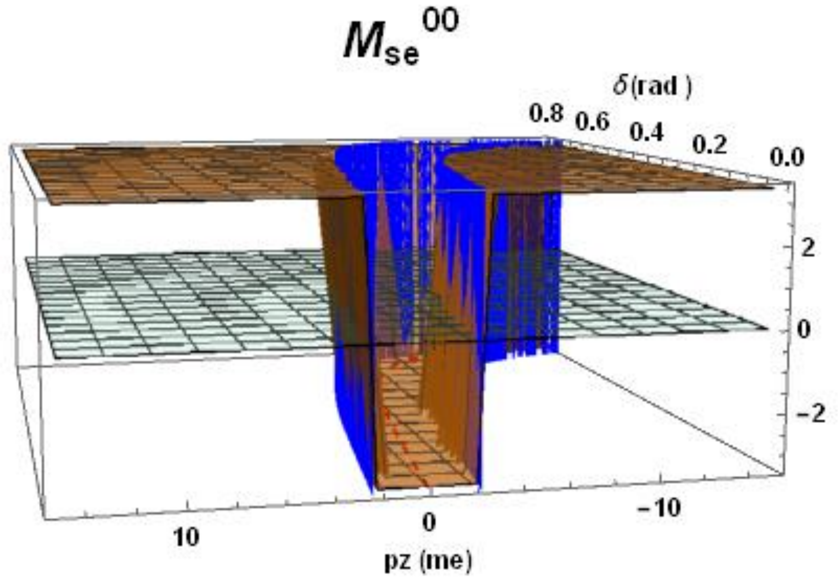
$$\mathbb{P} = \sqrt{P_{\widehat{-}}{}^2 + \mathbb{C}\mathbf{p}_{\perp}{}^2}$$

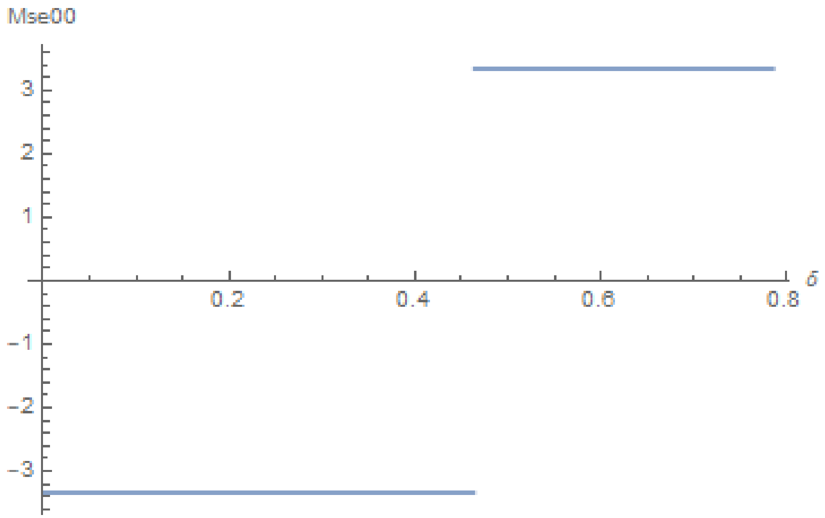
$$\text{Mse00} = \frac{2(\text{E0}^2 + \text{p}\gamma^2)(-\text{p}\gamma\text{Cos}[\delta] + \text{E0Sin}[\delta])}{(\text{E0}^2 - \text{p}\gamma^2)\text{Abs}[\text{p}\gamma\text{Cos}[\delta] - \text{E0Sin}[\delta]]}$$

Phase shift condition $P_{\widehat{-}} = 0$

$\text{Pz} \rightarrow 0$

$$\delta \rightarrow Tan^{-1}\left|\frac{P_\rho}{E0}\right| = 0.463648$$





Wigner Rotation

$S = 0$ Particle

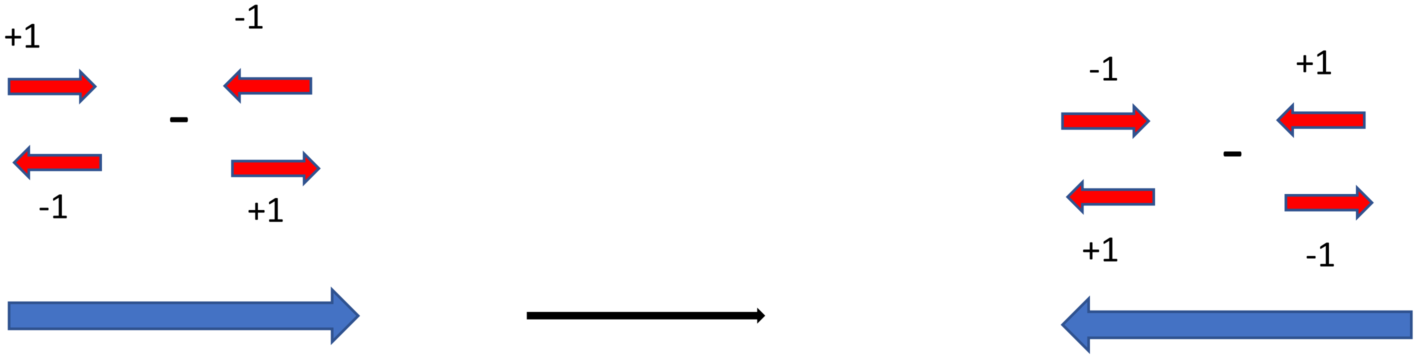
$$|0,0> \rightarrow |\frac{\uparrow\downarrow - \downarrow\uparrow}{\sqrt{2}}>$$

$$d^{\frac{1}{2}}(\beta) = e^{\frac{-i\sigma_y \beta}{2}}$$

$$d^{\frac{1}{2}}(\pi)|\uparrow> = e^{\frac{-i\sigma_y \pi}{2}}|\uparrow>$$

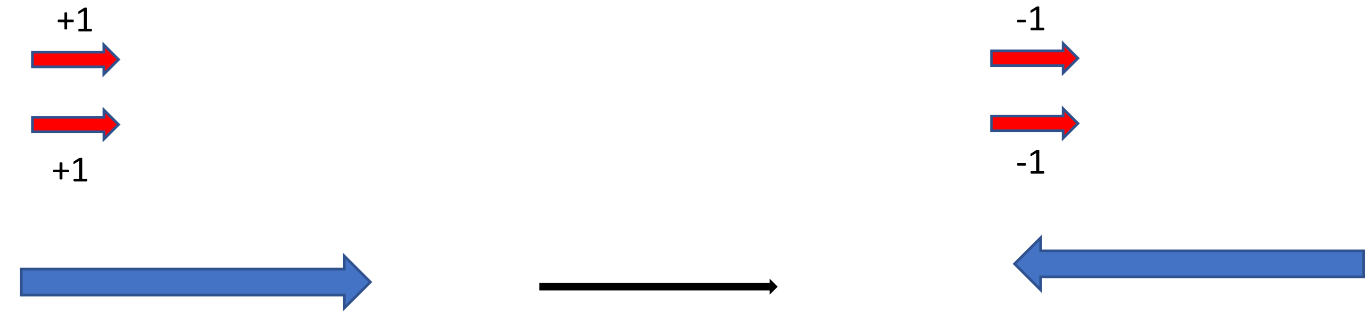
$$d^{\frac{1}{2}}(\pi)|\uparrow> = |\downarrow>$$

$$d^{\frac{1}{2}}(\pi)|\downarrow> = -|\uparrow>$$

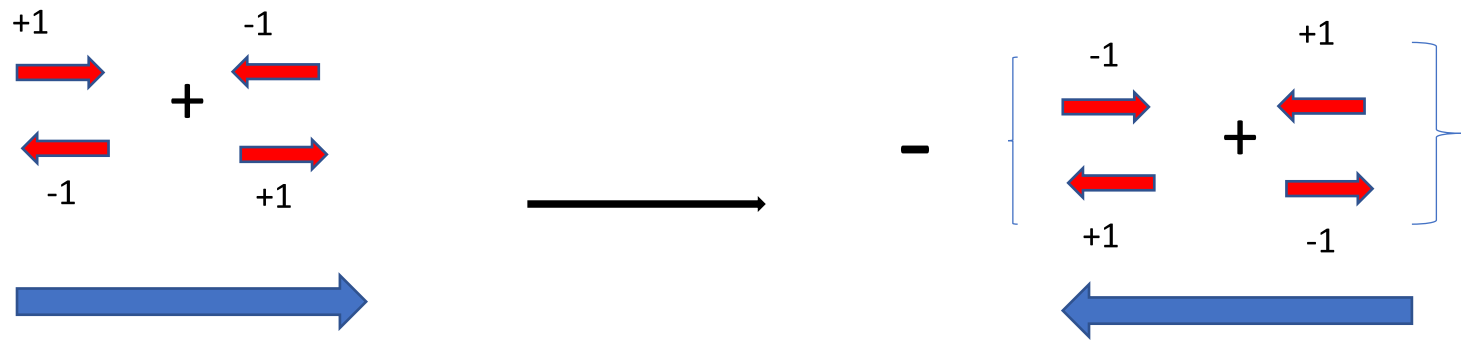


$S = 1$

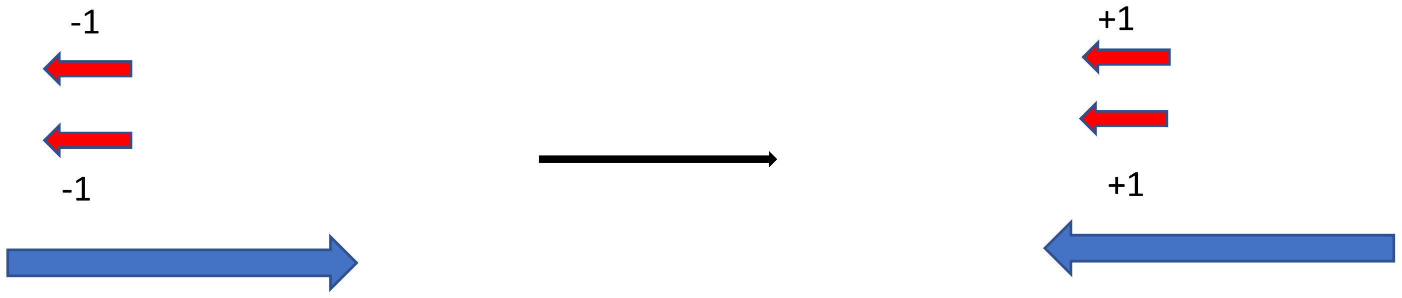
$|1,1> \rightarrow |\uparrow\uparrow>$



$|1,0> \rightarrow |\frac{\uparrow\downarrow+\downarrow\uparrow}{\sqrt{2}}>$



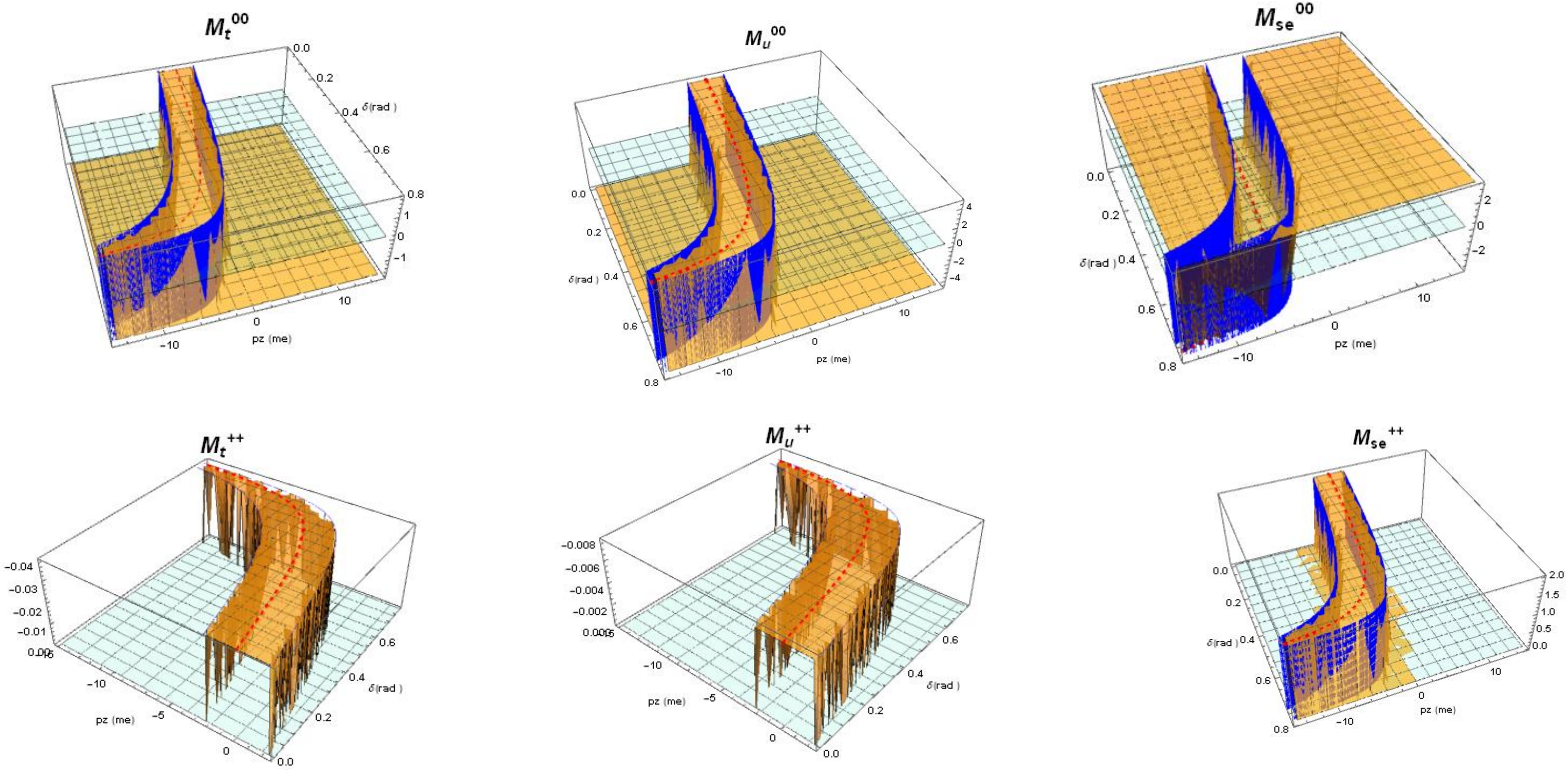
$|1,-1> \rightarrow |\downarrow\downarrow>$

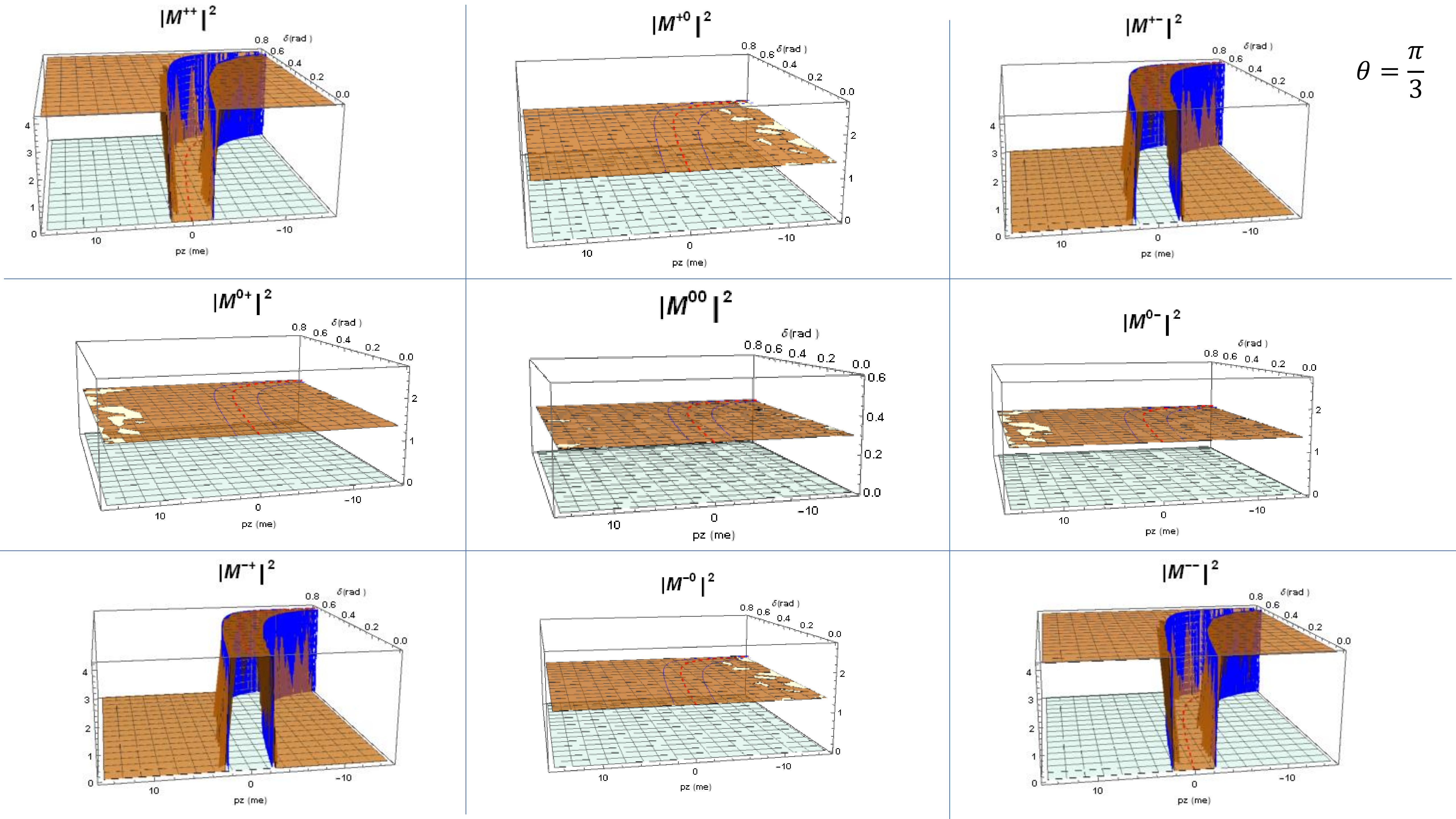


$\rho^+\rho^- \rightarrow$ "$\pi^+\pi^-$"

$$\theta = \frac{\pi}{3}$$

- Initial particles have spins.
- Clear boundaries can be seen in any annihilation angle

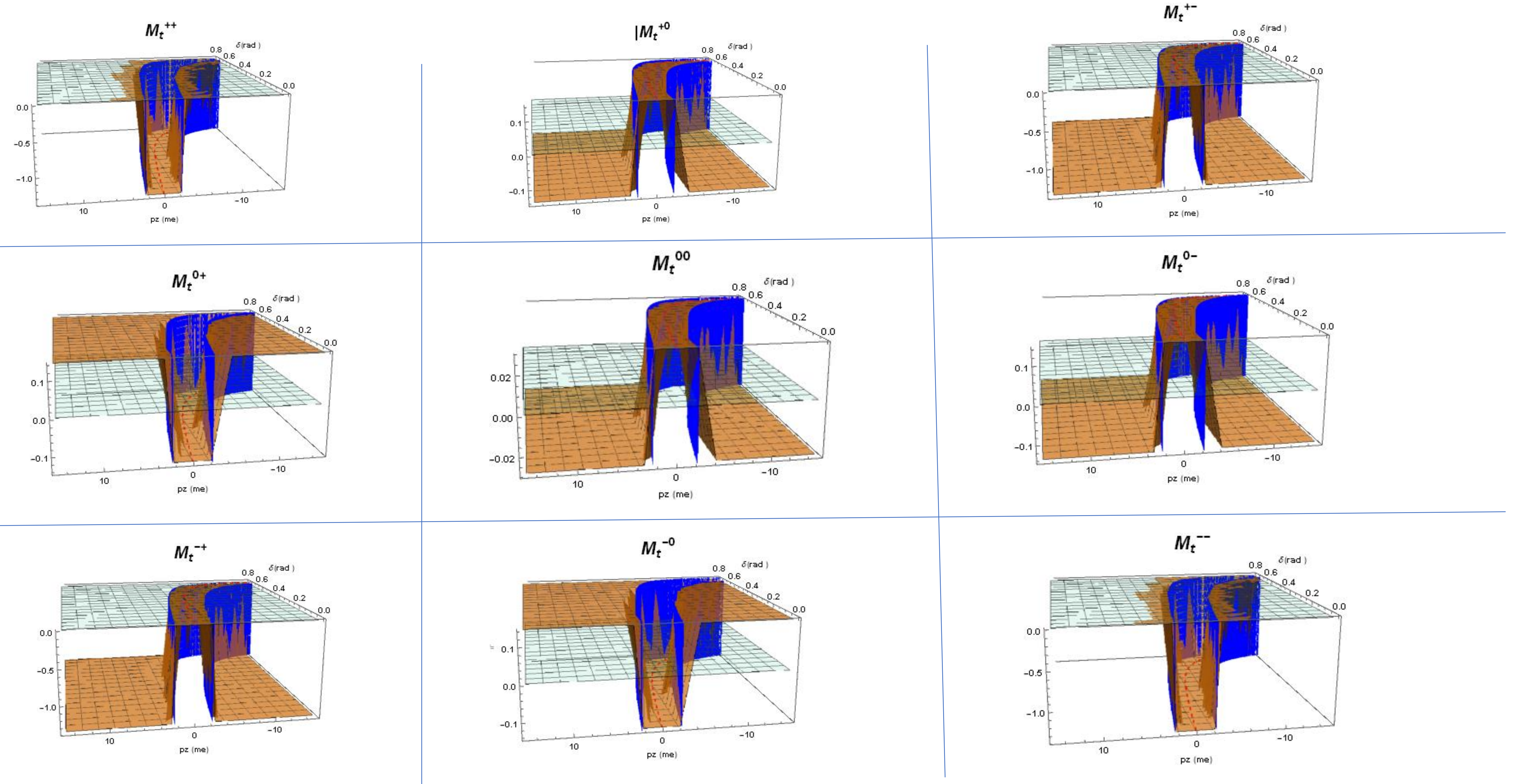




$|M^{++}|^2$
$|M^{+0}|^2$
$|M^{+-}|^2$
$\theta = \frac{\pi}{3}$
$|M^{0+}|^2$
$|M^{00}|^2$
$|M^{0-}|^2$
$|M^{-+}|^2$
$|M^{-0}|^2$
$|M^{--}|^2$
δ(rad)
pz (me)

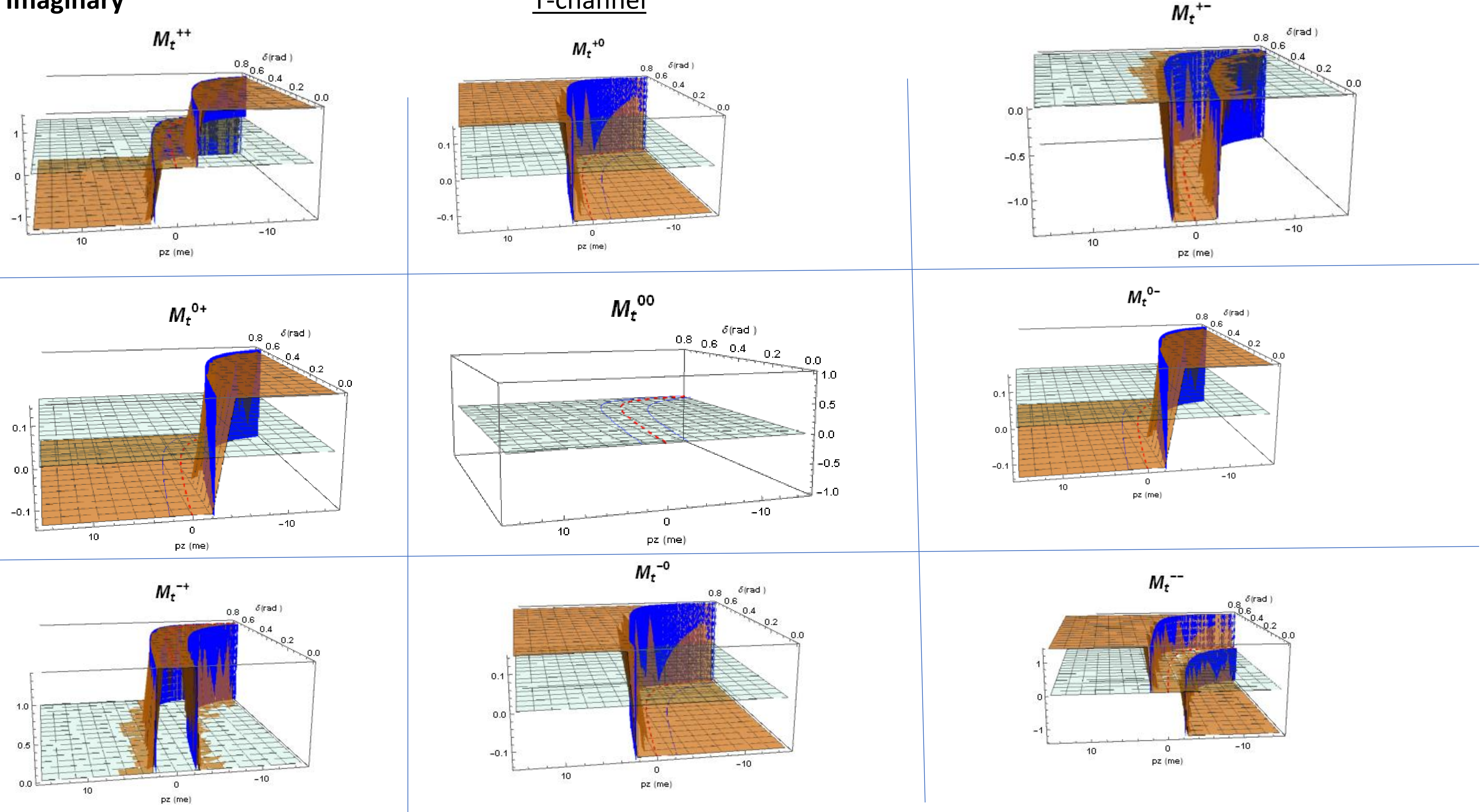
REAL

<u>T-channel</u>



Imaginary

<u>T-channel</u>



Summary

$“\pi^+\pi^-" \rightarrow \rho^+\rho^-$ *process*

- Out going photon polarization vectors are transverse ,$_{\pi^+}P_{\bumpeq} = 0$ and $_{\pi^-}P_{\bumpeq} = 0$ conditions make helicity amplitudes zero (Gauge condition)- Vanishing theorems in helicity amplitude in unbroken gauge theory.
- When $\theta = 0$, $_{\rho^+}P_{\bumpeq} = 0$ and $_{\rho^-}P_{\bumpeq} = 0$ conditions create clear abrupt changes in the helicity amplitudes (Helicity condition)
- Out going photon polarization vectors are longitudinal, $_{\rho^+}P_{\bumpeq} = 0$ and $_{\rho^-}P_{\bumpeq} = 0$ conditions create phase changes in the helicity amplitudes

$\rho^+\rho^- \rightarrow “\pi^+\pi^-"$ *process*

- Helicity amplitudes have real parts as well as imaginary parts
- $_{\rho^+}P_{\bumpeq} = 0$ and $_{\rho^-}P_{\bumpeq} = 0$ conditions create clear abrupt changes in the helicity amplitudes for any annihilation angle (General helicity condition)
- Out going photon polarization vectors are longitudinal, $_{\rho^+}P_{\bumpeq} = 0$ and $_{\rho^-}P_{\bumpeq} = 0$ conditions create phase changes in the helicity amplitudes

Thank you

$$M = \varepsilon_\mu^*(p_3, \lambda_3)\varepsilon_\nu^*(p_4, \lambda_4)[J^{t_{\mu\nu}} + J^{u_{\mu\nu}} + J^{se_{\mu\nu}}]$$

$$\left.\begin{aligned}
J^{t_{\mu\nu}} &= (-p_1 + q_1)^\mu \frac{1}{q_1^2 - m^2} (p_2 + q_1)^\nu \\
J^{u_{\mu\nu}} &= (-p_1 + q_2)^\nu \frac{1}{q_2^2 - m^2} (p_2 + q_2)^\mu \\
J^{se_{\mu\nu}} &= -2 g_{\mu\nu}
\end{aligned}\right\} \quad T^{\mu\nu}$$

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Selection rules for helicity amplitudes in massive gauge theories

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After a rediscussion of the vanishing theorems for helicity amplitudes in unbroken gauge theories, we study the case of spontaneously broken gauge theories at high energy. The vanishing theorems generalize to a definite pattern of m/E suppression of the amplitudes that vanish in the massless case, where E is the energy scale of the process and m is the mass of the gauge vectors. We use only elementary arguments, and as an application we show how these methods can be employed to understand some aspects of the effective W approximation in the polarized case.

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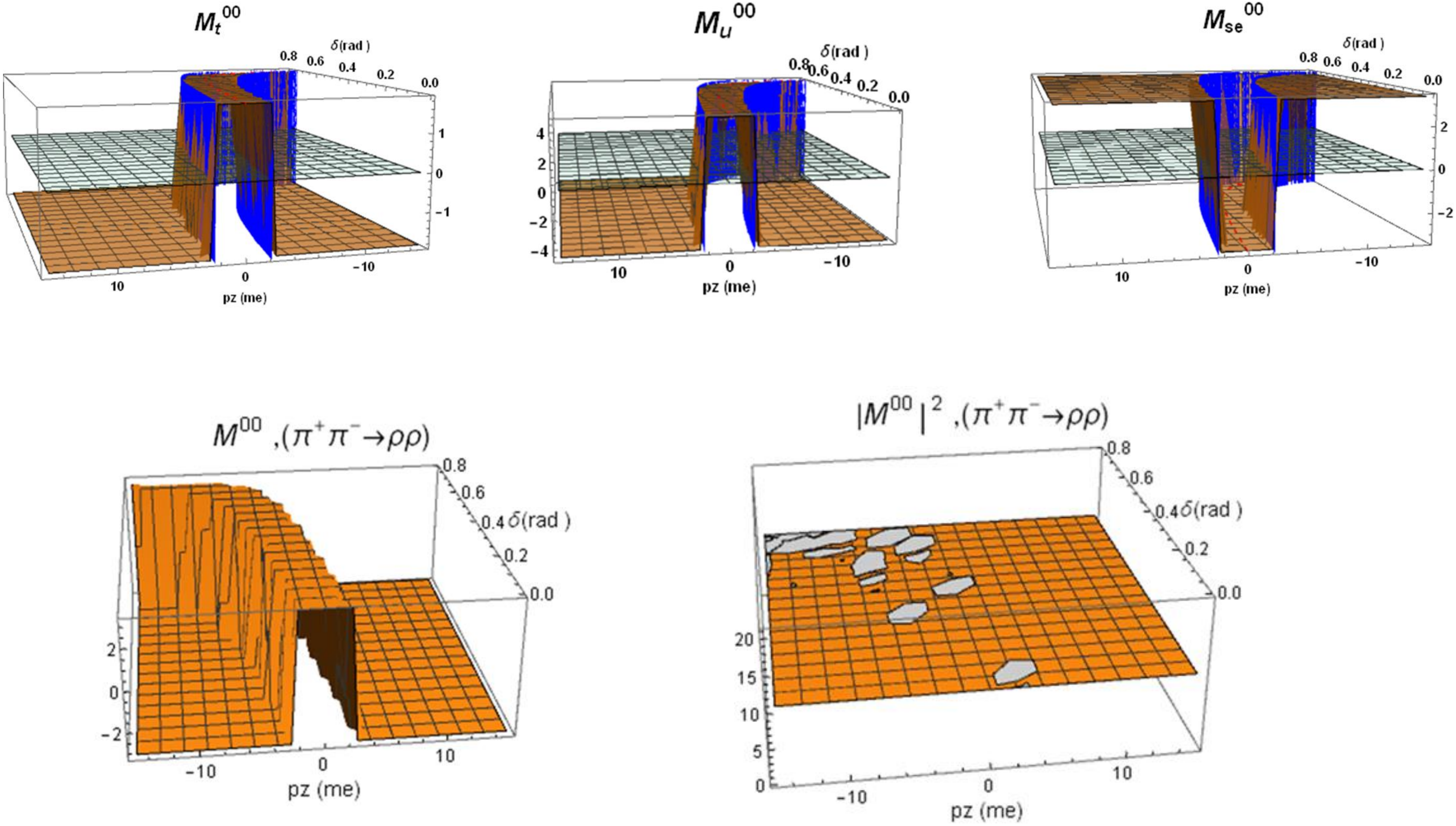
M_t^{00}
δ(rad)
0.8 0.6 0.4 0.2 0.0
1 0 −1
10 0 −10
pz (me)
M_u^{00}
δ(rad)
0.8 0.6 0.4 0.2 0.0
4 2 0 −2 −4
10 0 −10
pz (me)
M_{se}^{00}
δ(rad)
0.8 0.6 0.4 0.2 0.0
2 0 −2
10 0 −10
pz (me)
M^{00} ,($\pi^+\pi^- \to \rho\rho$)
0.8 0.6 0.4 0.2 0.0
δ(rad)
2 0 −2
−10 0 10
pz (me)
$|M^{00}|^2$,($\pi^+\pi^- \to \rho\rho$)
0.8 0.6 0.4 0.2 0.0
δ(rad)
20 15 10 5 0
−10 0 10
pz (me)