QC in zero-mode and helicity amplitudes in conjunction with LaMET

Deepasika Dayananda

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Out Line

1. Summary of the last discussion
   - -QC in the Spin
   - -QC in the Spinors and polarization vector

2. QC in the zero-mode
   - -Longitudinal momentum normalizing by the $\sqrt{\cos(2\delta)}$

3. Interpolating helicity amplitudes and large momentum

Quantum Correlation in spins

The angle must be rotated to get the same configuration as when it started change with the spin.

Spin- 0 $\rightarrow$ Any angle
Spin-1/2 $\rightarrow$ 720^0
Spin-1 $\rightarrow$ 360^0

- Fundamental characteristic that explains the differences of spins

For 180^0 $\longrightarrow$

Spin-0 particles

$$|0,0> \longrightarrow |0,0>$$

Spin-1/2 particles

$$|1/2,1/2> \longrightarrow |1/2,-1/2>$$
$$|1/2,-1/2> \longrightarrow -|1/2,1/2>$$

Spin-1 particles

$$|1,1> \longrightarrow |1,-1>$$
$$|1,0> \longrightarrow -|1,0>$$
$$|1,-1> \longrightarrow |1,1>$$

$\rightarrow$ Quantum – entangled states of spin-1/2

Main Goal $\longrightarrow$ Quantum correlation in the interpolating helicity amplitudes

$\downarrow$

QC in the interpolating helicity spinors and polarization vectors

Spinors and polarization vectors →

- Momentum of the particle
- Spin orientation

(0,J)$\oplus$ *(J,0)* chiral representation of the Lorentz group

- Spin- ½ spinors
- Spin- 1 spinors

4- vector representation

- Spin-1 polarization vectors

Spin- 1 spinors and Spin-1 polarization vectors:

An analogue between six-component Electromagnetic field and four- component electromagnetic potential

- We start from the rest frame and apply the relevant helicity transformation matrix to get the interpolating helicity spinors and polarization vectors

Helicity transformation matrix

$$T = T_{12}T_3 = e^{i\beta_1 \mathcal{K}^{\hat{1}} + i\beta_2 \mathcal{K}^{\hat{2}}} e^{-i\beta_3 K^3}$$

$$\mathcal{K}^{\hat{1}} = -K^1 \sin\delta - J^2 \cos\delta,$$

$$\mathcal{K}^{\hat{2}} = J^1 \cos\delta - K^2 \sin\delta,$$

$$(\delta \to 0),\ \mathcal{K}^{\hat{1}} \to -J^2,\ \mathcal{K}^{\hat{2}} \to J^1$$

$$(\delta \to \pi/4),\ \mathcal{K}^{\hat{1}} \to -E_1,\ \mathcal{K}^{\hat{2}} \to -E_2$$

We consider this transformation for spin-up

$$T = B(\boldsymbol{\eta})\mathcal{D}(\hat{\mathbf{m}}, \theta_s) = e^{-i\boldsymbol{\eta}\cdot\mathbf{K}} e^{-i\hat{\mathbf{m}}\cdot\mathbf{J}\theta_s},$$

$\mathcal{D}(\hat{\mathbf{m}}, \theta_s) = e^{-i\hat{\mathbf{m}}\cdot\mathbf{J}\theta_s}$ → Rotates the spin around the axis by a unit vector $\hat{m} = (-sin\varphi_s, cos\varphi_s, 0)$ by angle θ_s .

$B(\boldsymbol{\eta}) = e^{-i\boldsymbol{\eta}\cdot\mathbf{K}}$ → Boost to momentum **P**

$$\hat{n} = (sin\theta cos\varphi, sin\theta sin\varphi, cos\theta)$$

Dirac spinors and polarization vectors use only this boost operator .

- Depending on the spin and the representation Lorentz Group generators of rotation (J) and boost (K) change.

- Comparing two explicit T transformation matrices of spin-1/2 ,spin-1 spinors and spin-1 polarization vectors we find a same result

$$\mathrm{Cos}[\theta s] = \frac{\mathrm{Cos}[\alpha] + \mathrm{Cosh}[\beta 3] + \mathrm{Cos}[\alpha]\mathrm{Cosh}[\beta 3] - \mathrm{Cosh}[\eta]}{1 + \mathrm{Cosh}[\eta]}$$

Where;

$$\mathrm{Cosh}[\eta] = \frac{\big((\mathrm{Cos}[\delta]\mathrm{Cosh}[\beta 3] + \mathrm{Sin}[\delta]\mathrm{Sinh}[\beta 3])\mathrm{Cos}[\delta]\big) - \big(\mathrm{Sin}[\delta]\mathrm{Cos}[\alpha](\mathrm{Sin}[\delta]\mathrm{Cosh}[\beta 3] + \mathrm{Cos}[\delta]\mathrm{Sinh}[\beta 3])\big)}{\mathrm{Cos}[2\delta]}$$

$$\mathrm{Cos}[\phi s] = \frac{\beta 1}{\beta 1^2 + \beta 2^2} = \mathrm{Cos}[\phi] \qquad \mathrm{Sin}[\phi s] = \frac{\beta 2}{\beta 1^2 + \beta 2^2} = \mathrm{Sin}[\phi]$$

$$e^{-\beta_3} = \frac{P^{\hat{+}} - \mathbb{P}}{M(\cos\delta - \sin\delta)}$$

$$e^{\beta_3} = \frac{P^{\hat{+}} + \mathbb{P}}{M(\sin\delta + \cos\delta)}$$

Without loss and generality, we can make $\phi = \phi s = 0$

$$\cos\alpha = \frac{P_{\hat{-}}}{\mathbb{P}}$$

First, we fix the particle's initial momentum direction as +z ($\theta = 0$) , to find out the spin orientation and quantum correlation

We can simplify

$$\mathbb{P} = \sqrt{P_{\hat{-}}^2 + \mathbf{P}_{\perp}^2 \mathbb{C}}$$

$$\theta_s = 0 \ , \ \cos\alpha = P_{\hat{-}}/\mathbb{P} \to 1 \qquad \theta_s \to \pi \ , \ \cos\alpha = P_{\hat{-}}/\mathbb{P} \to -1$$

Longitudinal momentum of the particle

$$P_{\hat{-}} = \Big[(P_v P^z + E_0\sqrt{\bar{E}^2 + (P^z)^2})\sin\delta + (E_0 P^z + P_v\sqrt{\bar{E}^2 + (P^z)^2})\cos\delta\Big]/\bar{E}$$

P_v = Momentum of the particle in the rest frame
E_0 =Energy of the particle in the rest frame
$\bar{E} = E_0$, (Single particle system)

Example

Spin-1 spinors in the rest frame , Chiral representation

$$U^1(0) = \begin{bmatrix} \sqrt{M} \\ 0 \\ 0 \\ \sqrt{M} \\ 0 \\ 0 \end{bmatrix} \qquad U^{-1}(0) = \begin{bmatrix} 0 \\ 0 \\ \sqrt{M} \\ 0 \\ 0 \\ \sqrt{M} \end{bmatrix} \qquad U^0(0) = \begin{bmatrix} 0 \\ \sqrt{M} \\ 0 \\ 0 \\ \sqrt{M} \\ 0 \end{bmatrix}$$

$$P^R = P^1 + iP^2, \qquad P^L = P^1 - iP^2, \qquad A = \cos\delta,\ B = -\sin\delta \qquad \mathbb{X} \equiv \frac{P^{\hat{+}} - \mathbb{P}}{\mathbb{C}} = \frac{P^{\hat{+}} - \sqrt{(P^{\hat{+}})^2 - M^2\mathbb{C}}}{\mathbb{C}}$$

$$u_H^{(+1)} = \frac{1}{2\sqrt{M\mathbb{P}^2}} \begin{pmatrix} \frac{(P_{\hat{-}}+\mathbb{P})(P^{\hat{+}}+\mathbb{P})}{(A-B)} \\ \sqrt{2}P^R(P^{\hat{+}}+\mathbb{P}) \\ \frac{(A-B)(P^R)^2(P^{\hat{+}}+\mathbb{P})}{(P_{\hat{-}}+\mathbb{P})} \\ (A-B)(P_{\hat{-}}+\mathbb{P})\mathbb{X} \\ \sqrt{2}P^R(P^{\hat{+}}-\mathbb{P}) \\ \frac{(A+B)(P^R)^2(P^{\hat{+}}-\mathbb{P})}{(P_{\hat{-}}+\mathbb{P})} \end{pmatrix}, \quad u_H^{(-1)} = \frac{1}{2\sqrt{M\mathbb{P}^2}} \begin{pmatrix} \frac{(A+B)(P^L)^2(P^{\hat{+}}-\mathbb{P})}{(P_{\hat{-}}+\mathbb{P})} \\ -\sqrt{2}P^L(P^{\hat{+}}-\mathbb{P}) \\ (A-B)(P_{\hat{-}}+\mathbb{P})\mathbb{X} \\ \frac{(A-B)(P^L)^2(P^{\hat{+}}+\mathbb{P})}{(P_{\hat{-}}+\mathbb{P})} \\ -\sqrt{2}P^L(P^{\hat{+}}+\mathbb{P}) \\ \frac{(P_{\hat{-}}+\mathbb{P})(P^{\hat{+}}+\mathbb{P})}{(A-B)} \end{pmatrix} \quad u_H^{(0)} = \sqrt{\frac{M}{2\mathbb{P}^2}} \begin{pmatrix} -(A+B)P^L \\ \sqrt{2}P_{\hat{-}} \\ (A-B)P^R \\ (-A+B)P^L \\ \sqrt{2}P_{\hat{-}} \\ (A+B)P^R \end{pmatrix}$$

When $P_{\hat{-}} > 0$, we do not find the singularities in $u_H^{(+1)}, u_H^{(-1)}$ equations

When $P_{\hat{-}} < 0$, to avoid singularities

$$\mathbb{X} = \frac{P^{\hat{+}} - \mathbb{P}}{\mathbb{C}} = \frac{M^2}{P^{\hat{+}} + \mathbb{P}} \qquad \frac{\mathbb{P} - P_{\hat{-}}}{\mathbf{P}_\perp^2\mathbb{C}} = \frac{1}{\mathbb{P} + P_{\hat{-}}}$$

$$P_R/|P_\perp| \longrightarrow 1 \qquad P_L/|P_\perp| \longrightarrow 1$$

<u>QC in spin-1 spinor</u>

$$U^{+1}(P_{\hat{-}} < 0) \Rightarrow U^{-1}(P_{\hat{-}} > 0)$$
$$U^{0}(P_{\hat{-}} < 0) \Rightarrow -U^{0}(P_{\hat{-}} > 0)$$
$$U^{-1}(P_{\hat{-}} < 0) \Rightarrow U^{+1}(P_{\hat{-}} > 0)$$

QC in Spin-1/2 spinors

$$U^{+1/2}(P_{\hat{-}} < 0) \Rightarrow U^{-1/2}(P_{\hat{-}} > 0)$$
$$U^{-1/2}(P_{\hat{-}} < 0) \Rightarrow -U^{+1/2}(P_{\hat{-}} > 0)$$

$$u_H^{(1/2)}(P) = \begin{pmatrix} \sqrt{\frac{P_{\hat{-}}+\mathbb{P}}{2\mathbb{P}}}\sqrt{\frac{P^{\hat{+}}+\mathbb{P}}{(\sin\delta+\cos\delta)}} \\ P^R\sqrt{\frac{\sin\delta+\cos\delta}{2\mathbb{P}(\mathbb{P}+P_{\hat{-}})}}\sqrt{P^{\hat{+}}+\mathbb{P}} \\ \sqrt{\frac{P_{\hat{-}}+\mathbb{P}}{2\mathbb{P}}}\sqrt{\frac{P^{\hat{+}}-\mathbb{P}}{(\cos\delta-\sin\delta)}} \\ P^R\sqrt{\frac{\cos\delta-\sin\delta}{2\mathbb{P}(\mathbb{P}+P_{\hat{-}})}}\sqrt{P^{\hat{+}}-\mathbb{P}} \end{pmatrix} \qquad u_H^{(-1/2)}(P) = \begin{pmatrix} -P^L\sqrt{\frac{\cos\delta-\sin\delta}{2\mathbb{P}(\mathbb{P}+P_{\hat{-}})}}\sqrt{P^{\hat{+}}-\mathbb{P}} \\ \sqrt{\frac{P_{\hat{-}}+\mathbb{P}}{2\mathbb{P}}}\sqrt{\frac{P^{\hat{+}}-\mathbb{P}}{(\cos\delta-\sin\delta)}} \\ -P^L\sqrt{\frac{\sin\delta+\cos\delta}{2\mathbb{P}(\mathbb{P}+P_{\hat{-}})}}\sqrt{P^{\hat{+}}+\mathbb{P}} \\ \sqrt{\frac{P_{\hat{-}}+\mathbb{P}}{2\mathbb{P}}}\sqrt{\frac{P^{\hat{+}}+\mathbb{P}}{(\sin\delta+\cos\delta)}} \end{pmatrix}$$

QC in polarization vectors

$$\epsilon^{+1}(P_{\hat{-}} < 0) \Rightarrow \epsilon^{-1}(P_{\hat{-}} > 0)$$
$$\epsilon^{0}(P_{\hat{-}} < 0) \Rightarrow -\epsilon^{0}(P_{\hat{-}} > 0)$$
$$\epsilon^{-1}(P_{\hat{-}} < 0) \Rightarrow \epsilon^{+1}(P_{\hat{-}} > 0)$$

Interpolating Longitudinal Momentum

- Initial direction of particles' momentum
- Boost of the frame
- Interpolation angle

We consider two particles' system ,

$$P_1 = (E_0, 0, 0, P_v)$$
$$P_2 = (E_0, 0, 0, -P_v)$$

Lorentz transformation for a composite system

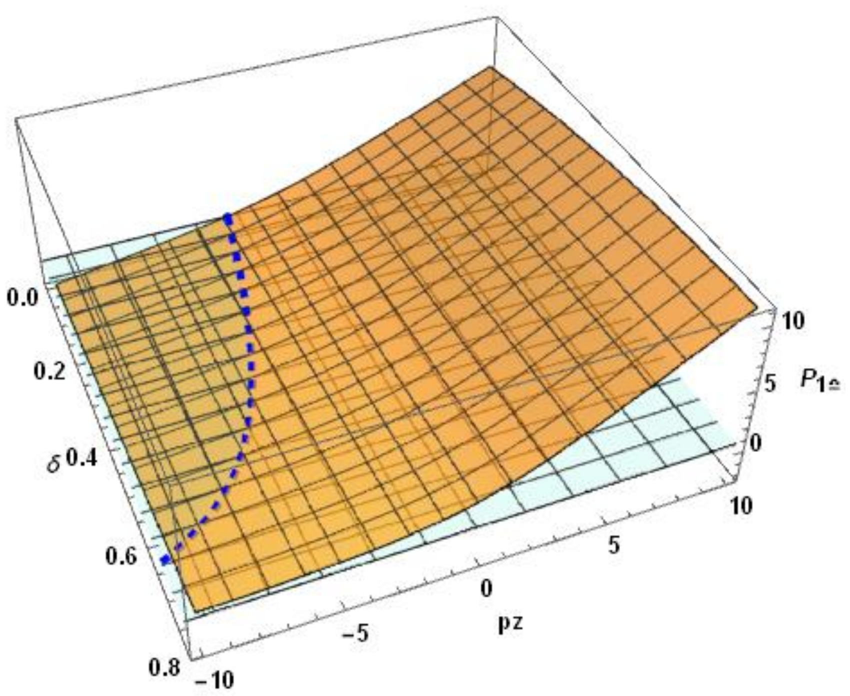
$$P_i^{\prime 0} = \gamma P_i^0 + \gamma\beta P_i^3 \qquad \gamma = E/\bar{E}$$
$$P_i^{\prime 3} = \gamma P_i^3 + \gamma\beta P_i^0 \qquad \gamma\beta = P_z/\bar{E}$$
$$P_i^{\prime\perp} = P_i^{\perp}$$

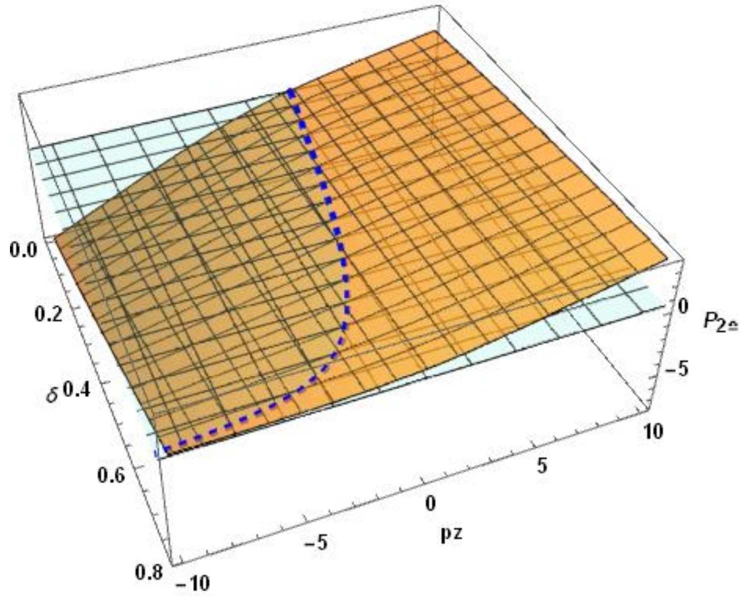
$$P_{1\hat{z}} = \left[\left(P_v P^z + E_0\sqrt{\bar{E}^2 + (P^z)^2}\right)\sin\delta + \left(E_0 P^z + P_v\sqrt{\bar{E}^2 + (P^z)^2}\right)\cos\delta\right]/\bar{E}$$

$$P_{2\hat{z}} = \left[\left(-P_v P^z + E_0\sqrt{\bar{E}^2 + (P^z)^2}\right)\sin\delta + \left(E_0 P^z - P_v\sqrt{\bar{E}^2 + (P^z)^2}\right)\cos\delta\right]/\bar{E}$$

We see $0 \leq \delta < \frac{\pi}{4}$ range $P_{1\widehat{-}}$ and $P_{2\widehat{-}}$ can get any real value, but exactly at the LF we do not see $P_{\widehat{-}} < 0$ values , since $P_{\widehat{-}} \to P^{+}$ at $\delta = \frac{\pi}{4}$.

→ $P_{1\widehat{-}} = 0$, and $P_{2\widehat{-}} = 0$, $P_1^{+} \to 0$ and $P_2^{+} \to 0$





$$\tan(\delta_{c1}) = -\frac{E_0 P^z + P_v\sqrt{\bar{E}^2 + (P^z)^2}}{P_v P^z + E_0\sqrt{\bar{E}^2 + (P^z)^2}}$$

$$\tan(\delta_{c2}) = -\frac{E_0 P^z - P_v\sqrt{\bar{E}^2 + (P^z)^2}}{-P_v P^z + E_0\sqrt{\bar{E}^2 + (P^z)^2}}$$

It seems that the QC is accumulated in the zero-mode

$$P^z \to -\infty$$

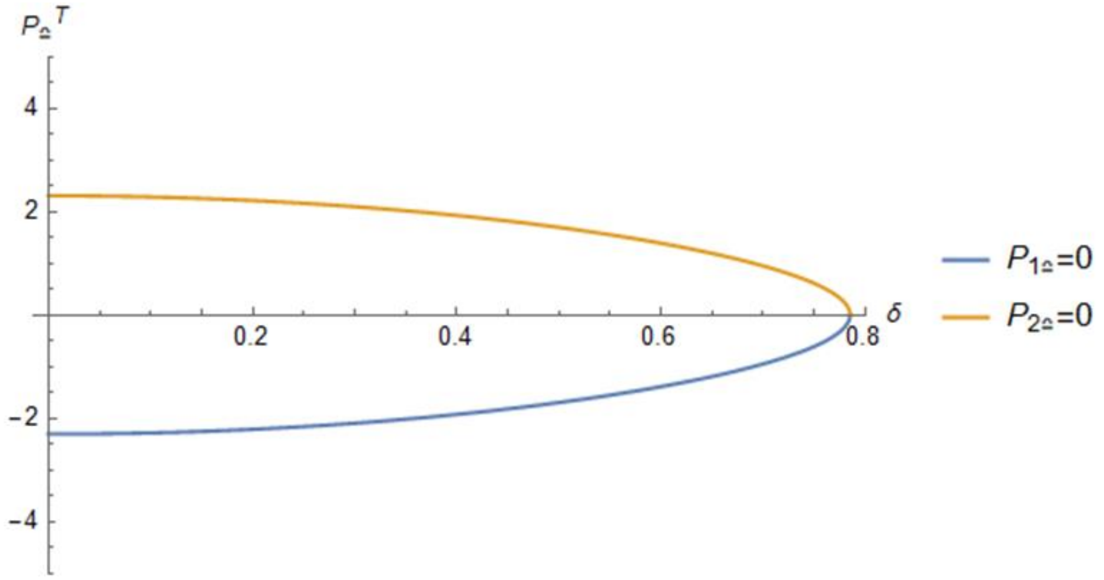
To see the QC in the zero-mode we consider total longitudinal momentum of the system

$$(P_{\underline{\hat{}}})^T = \sqrt{\bar{E}^2 + (P^z)^2}\,\sin\delta + P^z\cos\delta$$

$$\Rightarrow P_i^z = \frac{P_{\underline{\hat{}}}^T\cos\delta - \sin\delta\sqrt{(P_{\underline{\hat{}}}^T)^2 + \bar{E}^2\cos 2\delta}}{\cos 2\delta}$$

and

$$P_{ii}^z = \frac{P_{\underline{\hat{}}}^T\cos\delta + \sin\delta\sqrt{(P_{\underline{\hat{}}}^T)^2 + \bar{E}^2\cos 2\delta}}{\cos 2\delta}$$



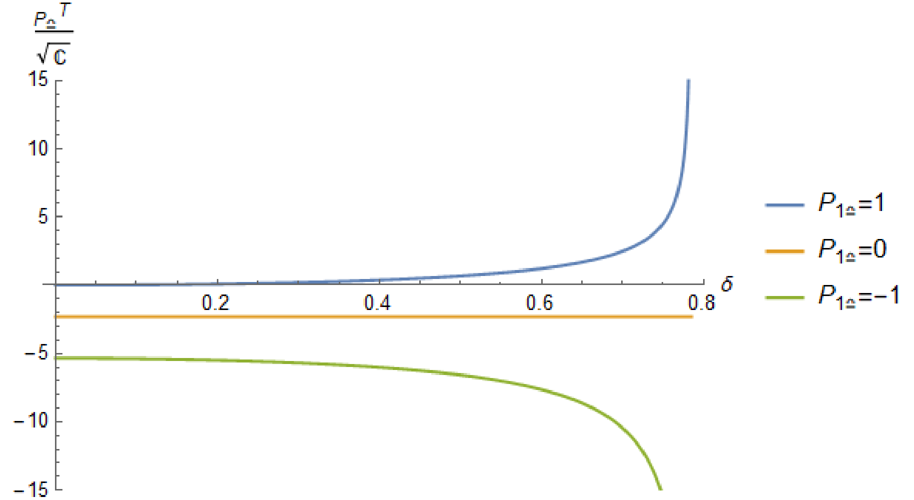
Zero-mode of the light front

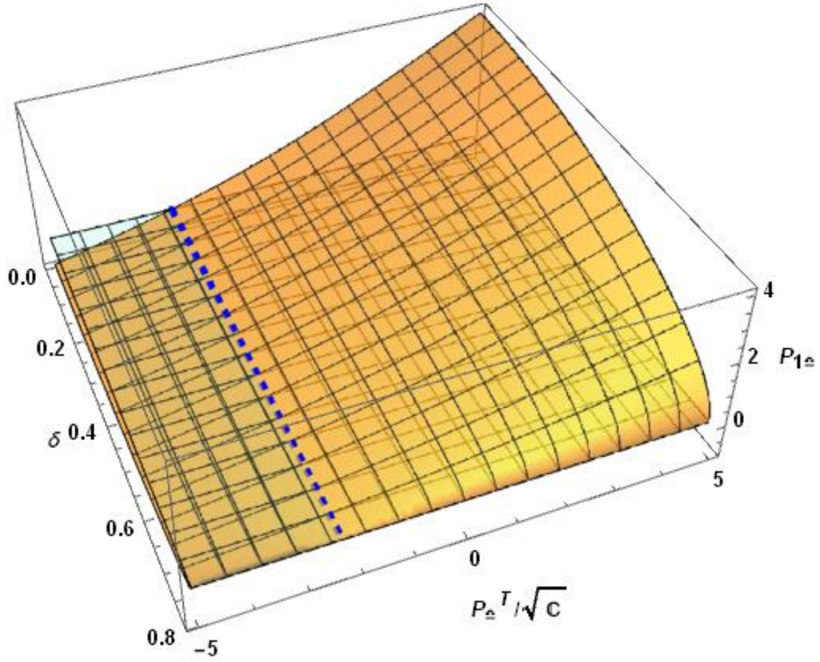
$$P_i^z \to -\infty\,,\ (P_{\underline{\hat{}}})^T \to 0$$

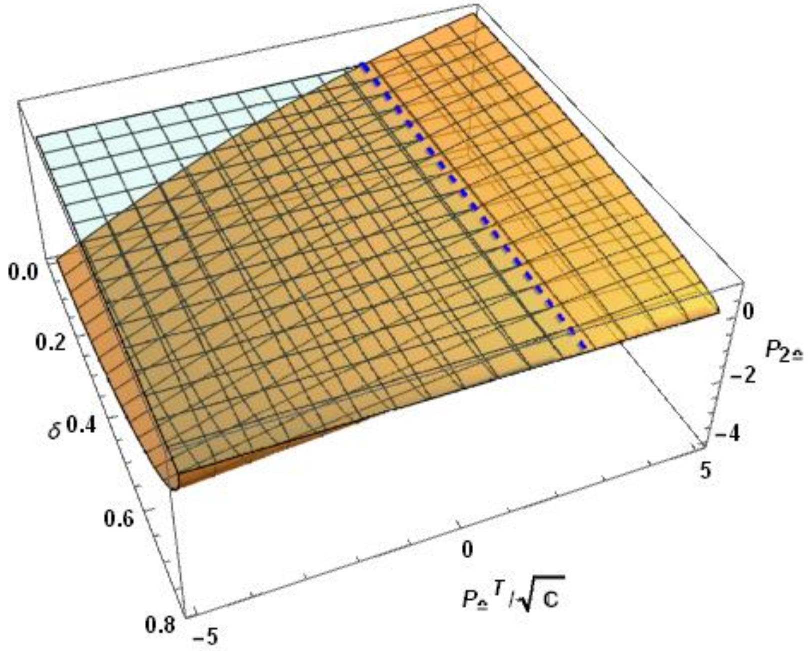
This further confirm that the QC accumulated in the zero-mode

Scanning the zero-mode

At $\delta = \frac{\pi}{4}$ $\quad \frac{(P_{\hat{-}})^T}{\sqrt{\mathbb{C}}}$ → Finite Value





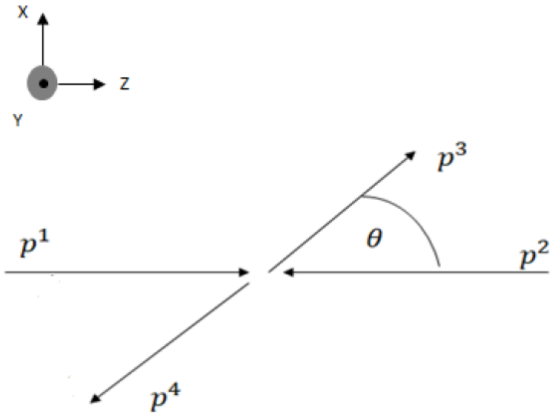


$$\left(\frac{(P_{\hat{-}})^T}{\sqrt{\mathbb{C}}}\right)_{c1} = -\frac{\bar{E}P_v}{M} = -2.3094$$

$$\left(\frac{(P_{\hat{-}})^T}{\sqrt{\mathbb{C}}}\right)_{c2} = \frac{\bar{E}P_v}{M} = 2.3094$$

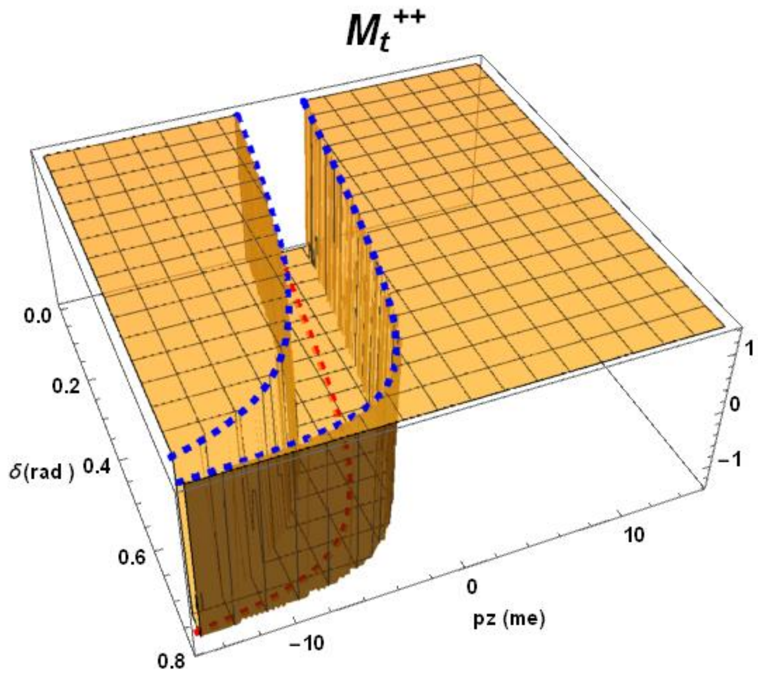
$$\bar{E} = 2E_0, E_0 = 2, P_v = 1, M = \sqrt{3}$$

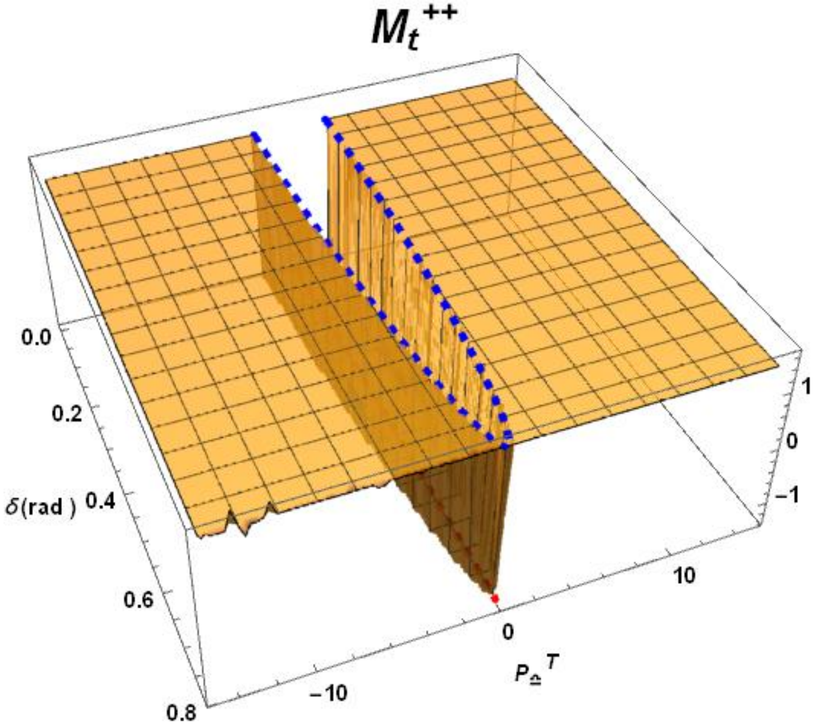
Quantum correlation boundaries in the interpolating helicity amplitudes

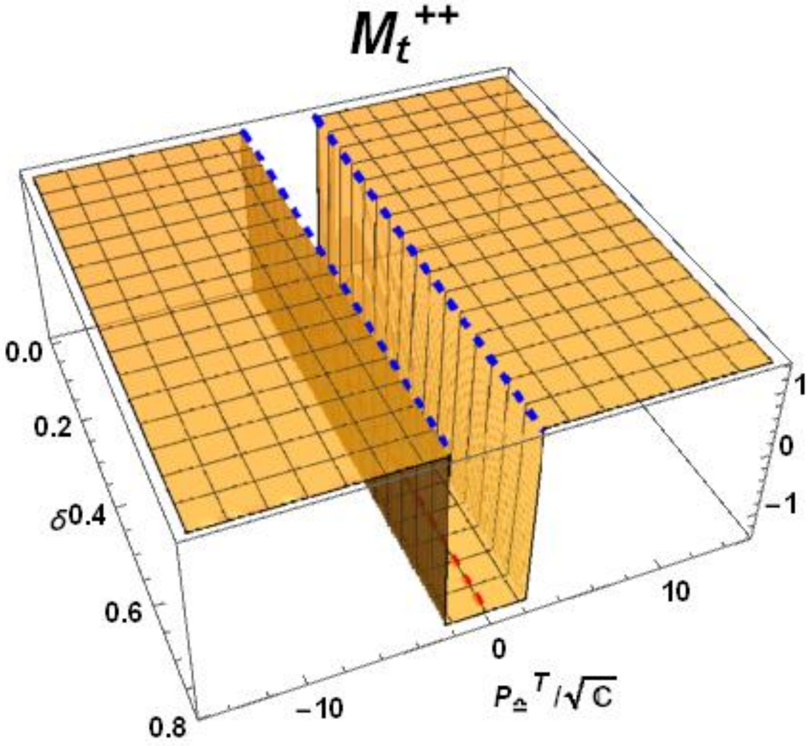


Examples

VV → SS





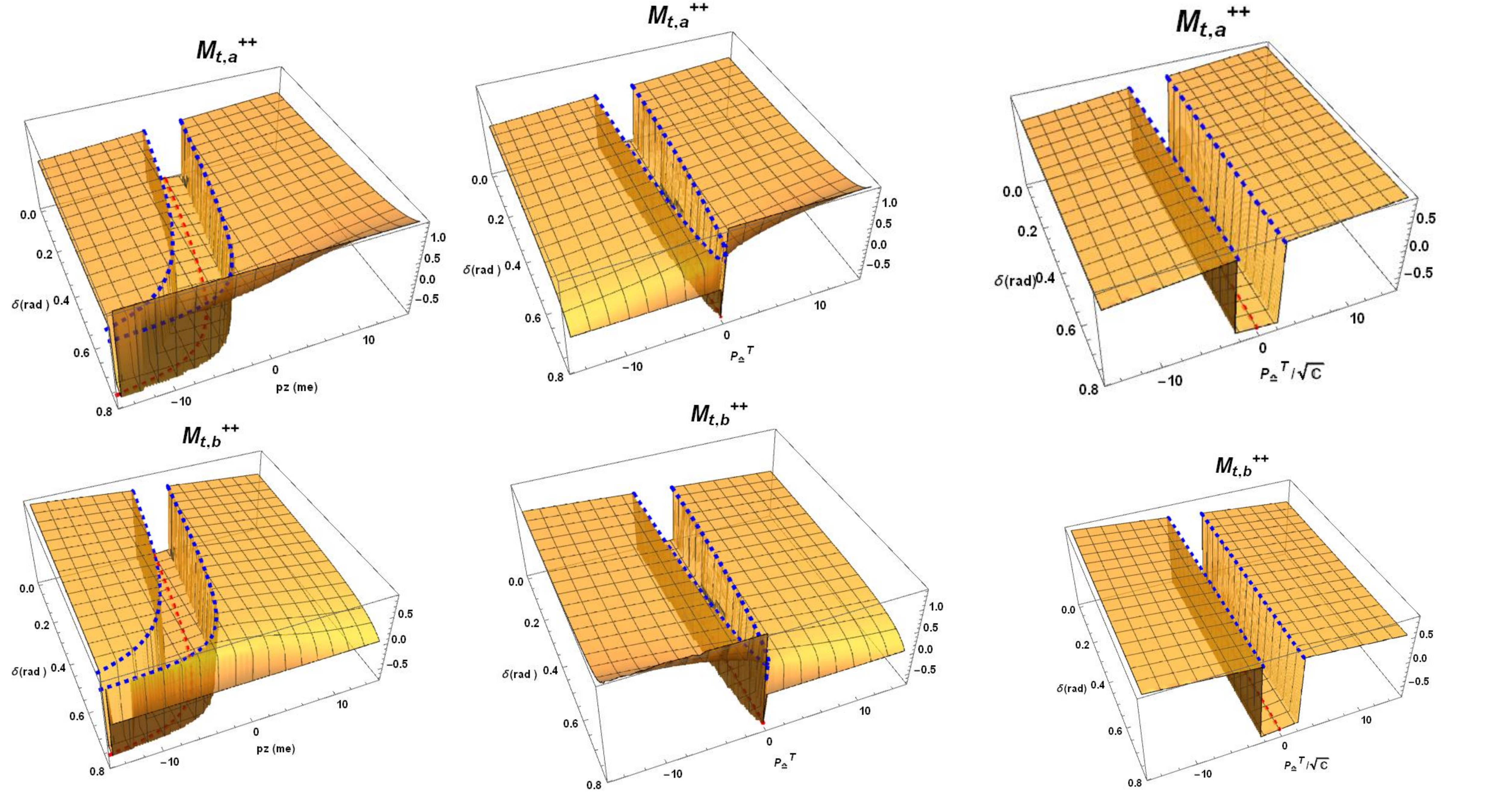


$\theta = \pi/3$ $\quad E_0 = 2\,, P_v = 1\,, P_s = \sqrt{3}$

Covariant scalar propagator

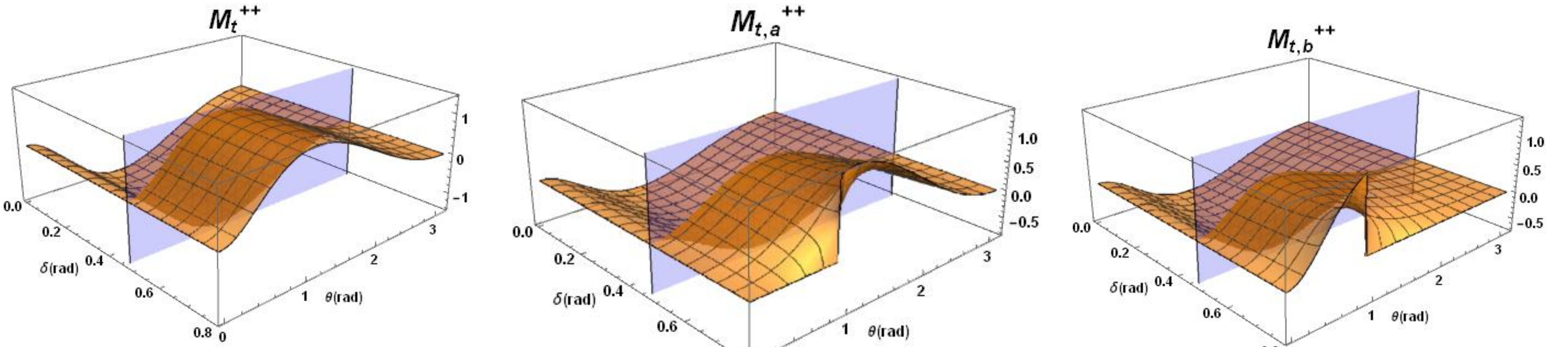
Time-ordered interpolating helicity amplitudes

$\theta = \pi/3$ $E_0 = 2\,, P_v = 1\,, P_s = \sqrt{3}$

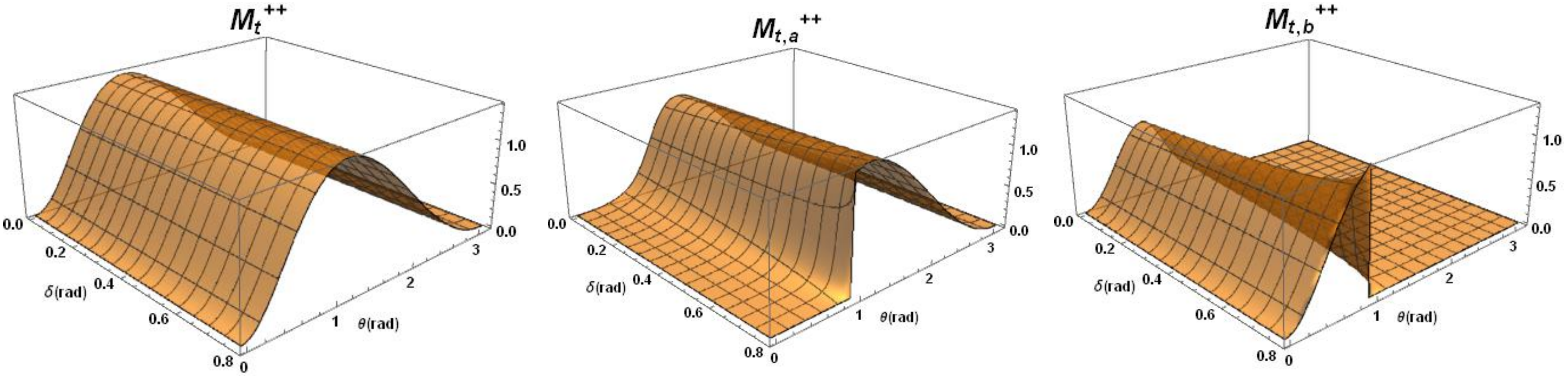


Angular Distributions

$P^z = 0$

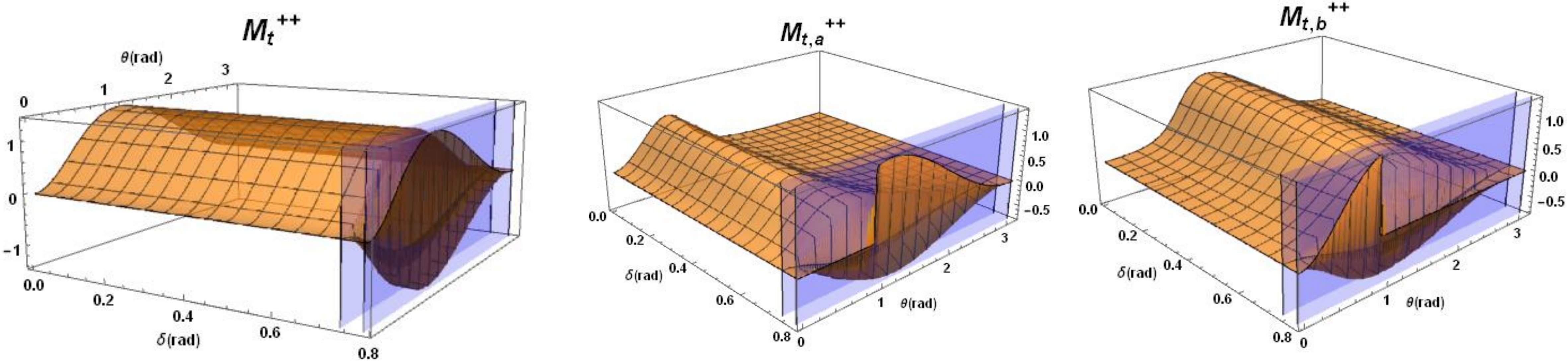


$P^z = 15$



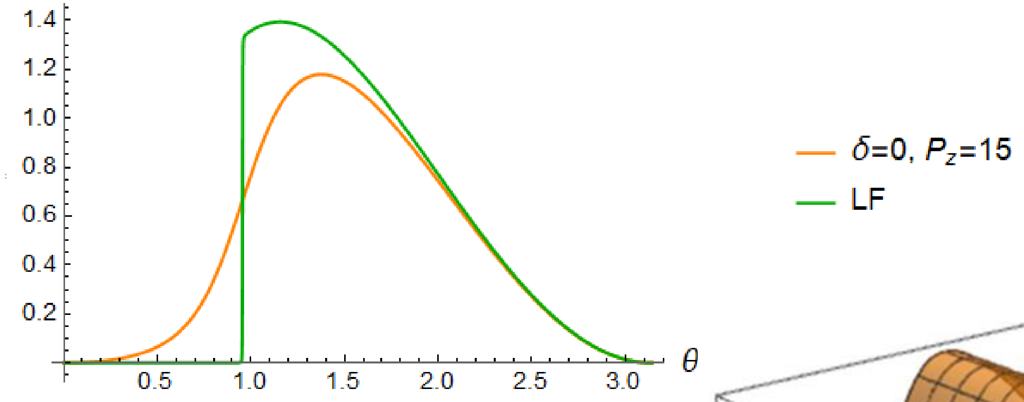
- When we boost to the positive high momentum helicity amplitude can mimic the light-front results and time-ordered results very with interpolation angle prominently .

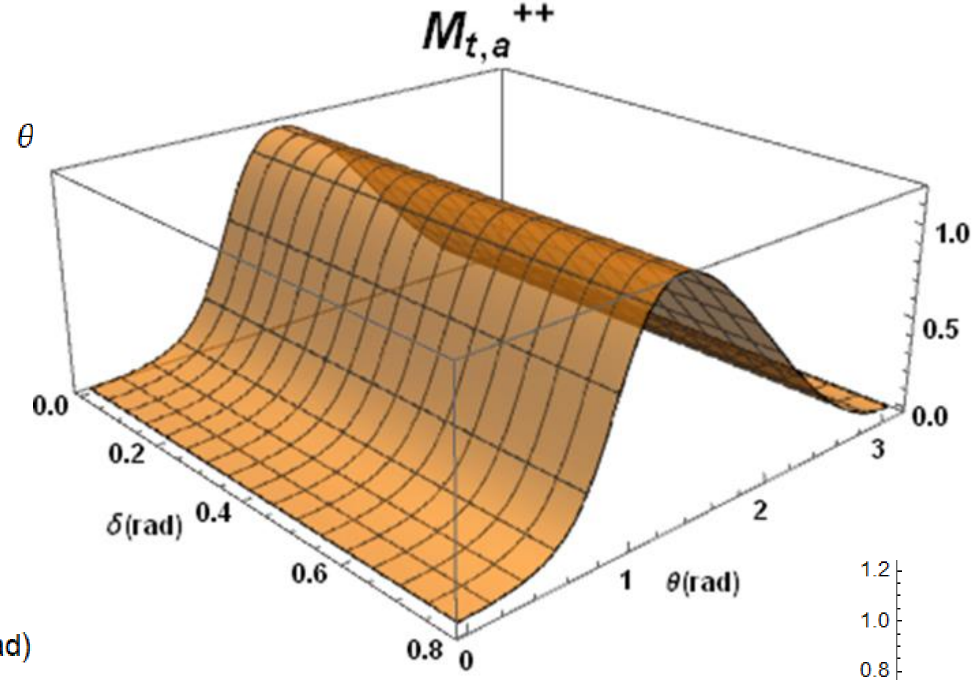
$P^z = -15$

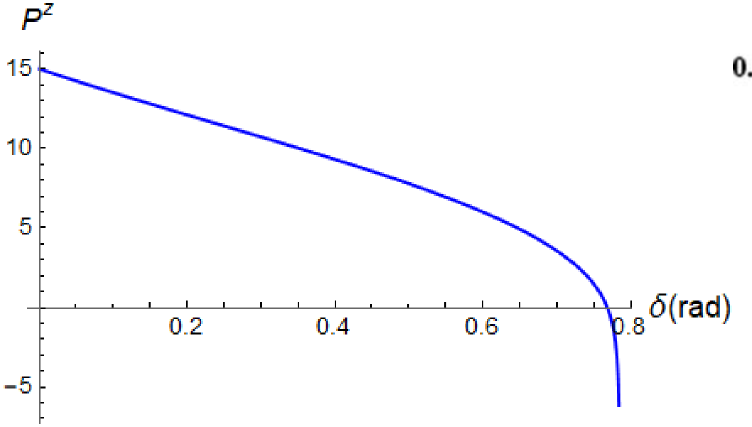


- When we boost to the negative high momentum time-ordered helicity amplitudes can not mimic the light-front results
- Therefore, we only consider the positive higher momentum to compare with the LaMET program

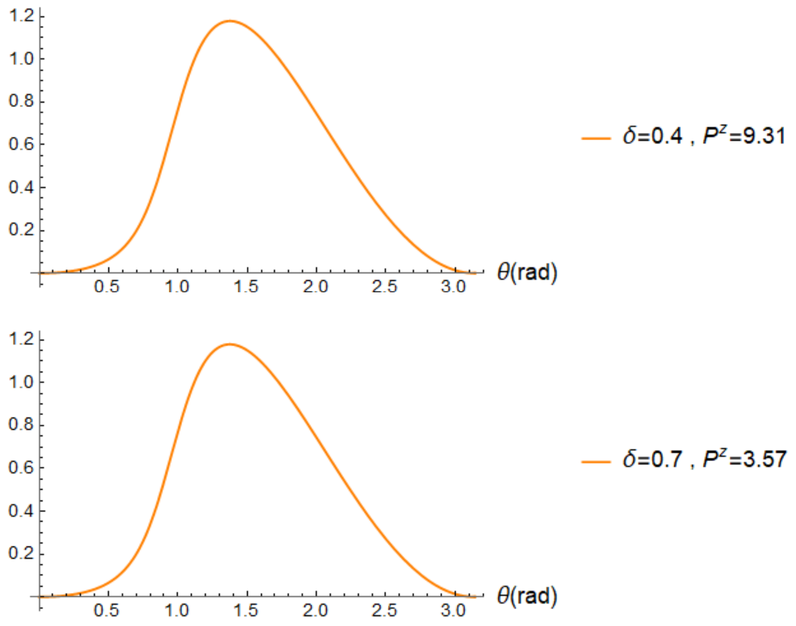
$M_{t,a}^{++}$ time-ordered helicity amplitude consider as an example

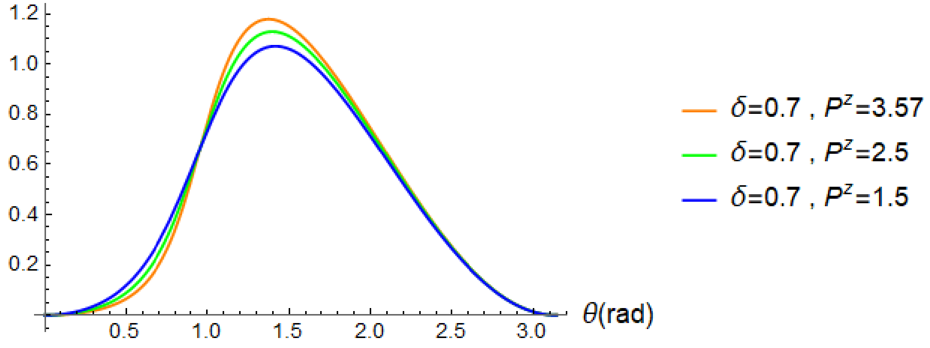






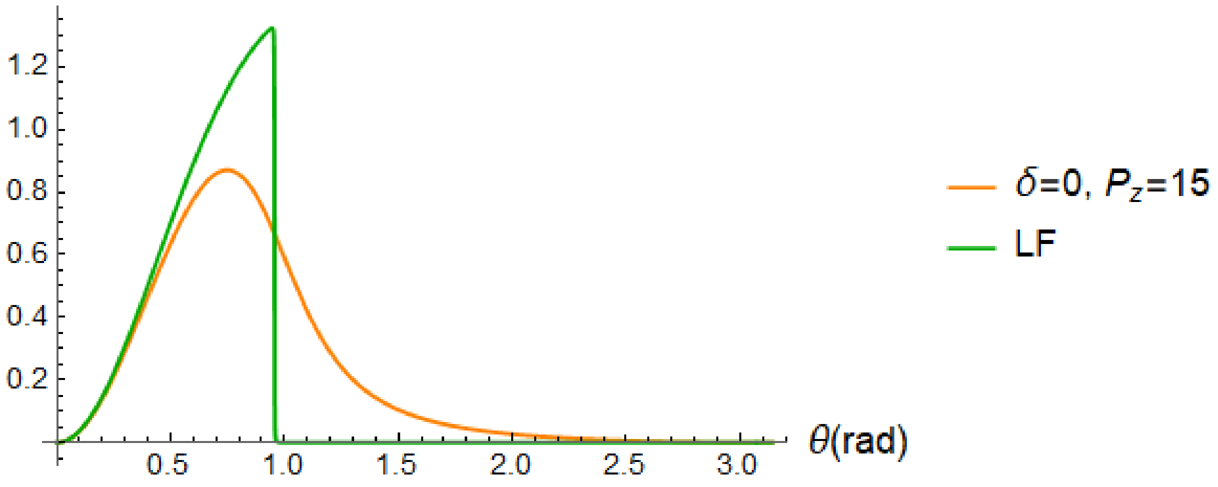
$$\frac{(P_{\hat{-}})^T}{\sqrt{\mathbb{C}}} = \frac{\sqrt{\bar{E}^2 + (P^z)^2}\sin\delta + P^z\cos\delta}{\sqrt{\cos 2\delta}} = 15$$

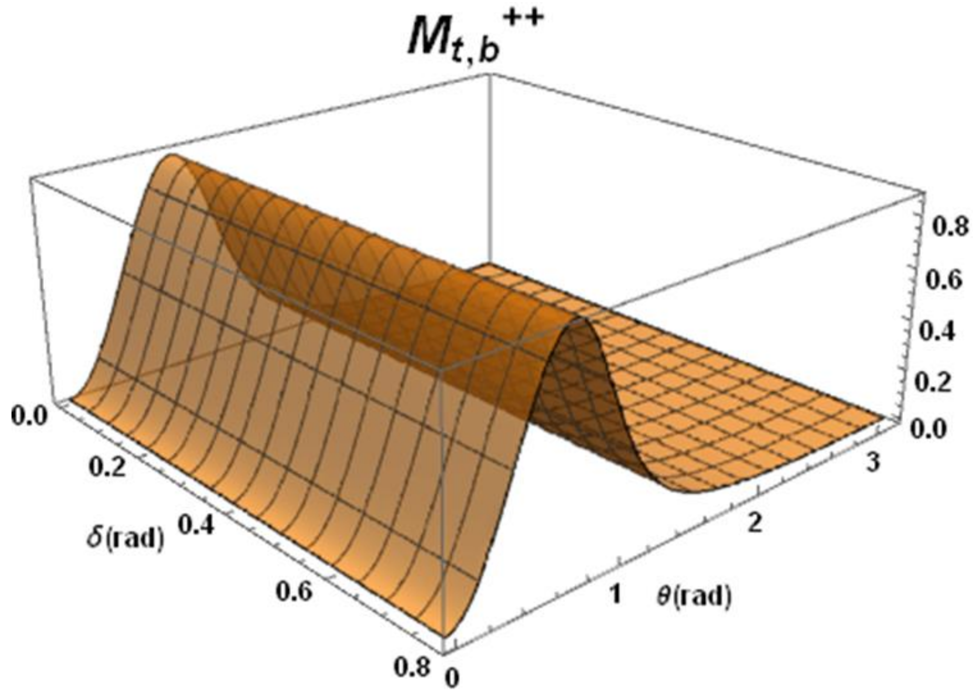




Checking for some other P^z values

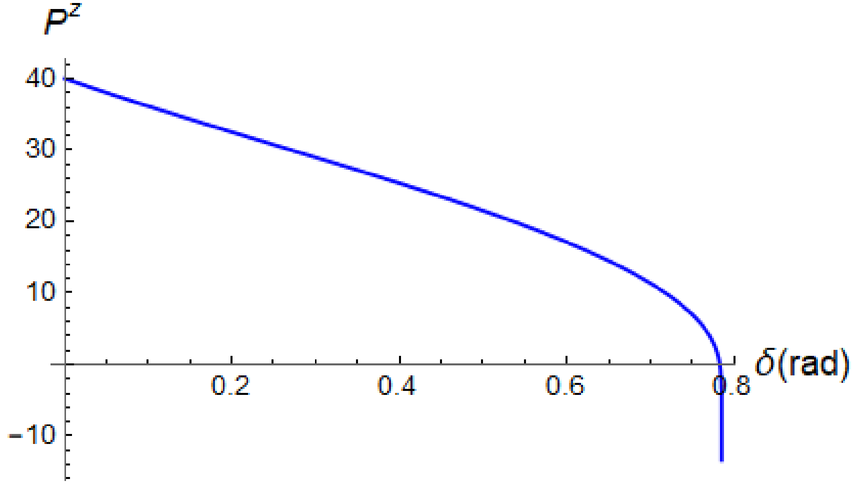
$M_{t,b}^{++}$ time-ordered helicity amplitude consider as an example

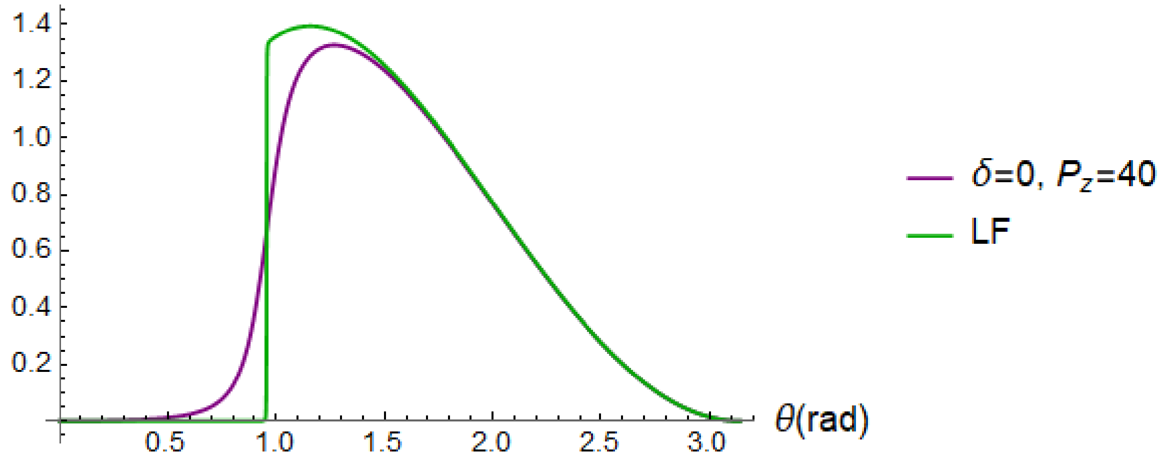


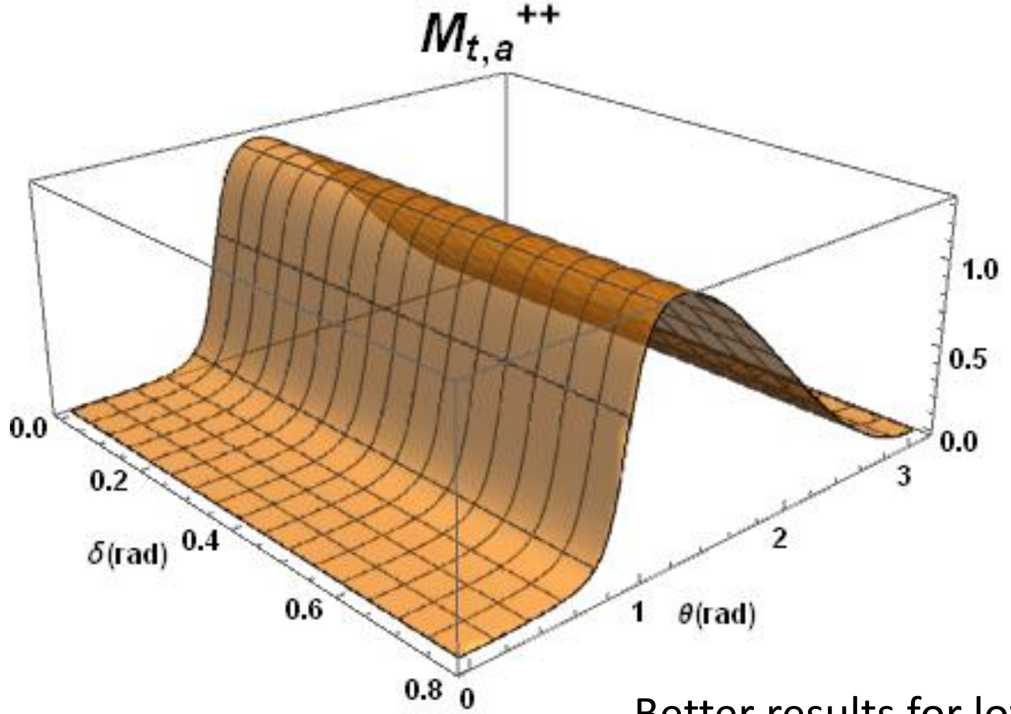


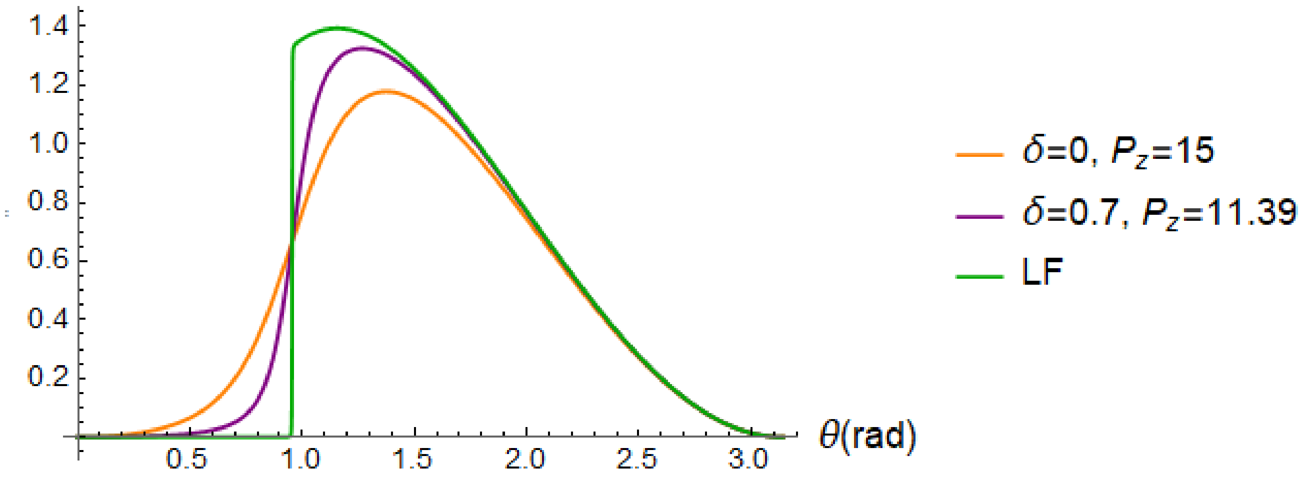
$$\frac{(P_{\hat{-}})^T}{\sqrt{\mathbb{C}}} = \frac{\sqrt{\bar{E}^2 + (P^z)^2}\sin\delta + P^z\cos\delta}{\sqrt{\cos 2\delta}} = 15$$

$$\frac{P_{\doteq}^{T}}{\sqrt{\mathbb{C}}} = 40$$







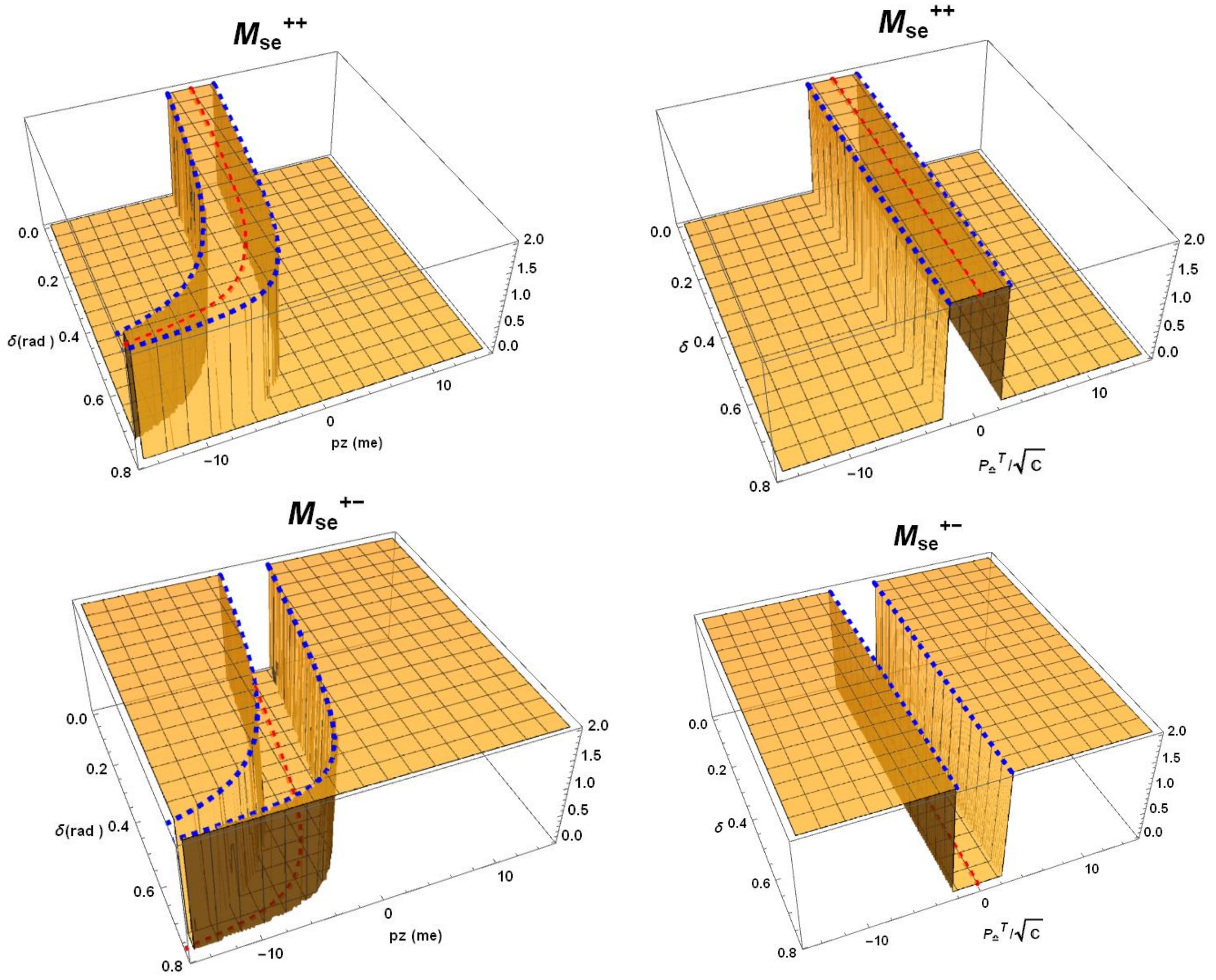


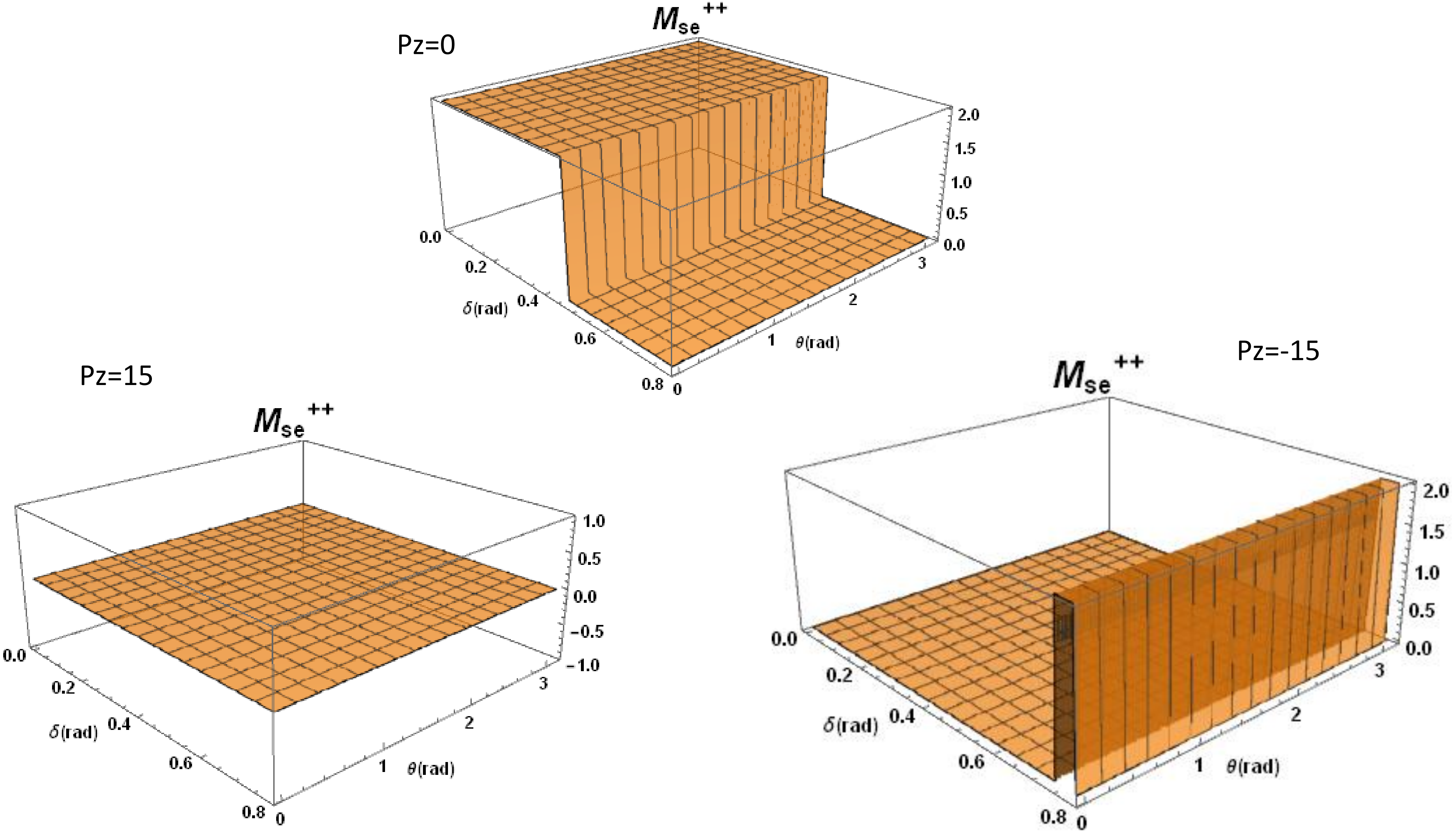
Better results for low momentum values if we use the interpolation angle dependence

CONCLUSION

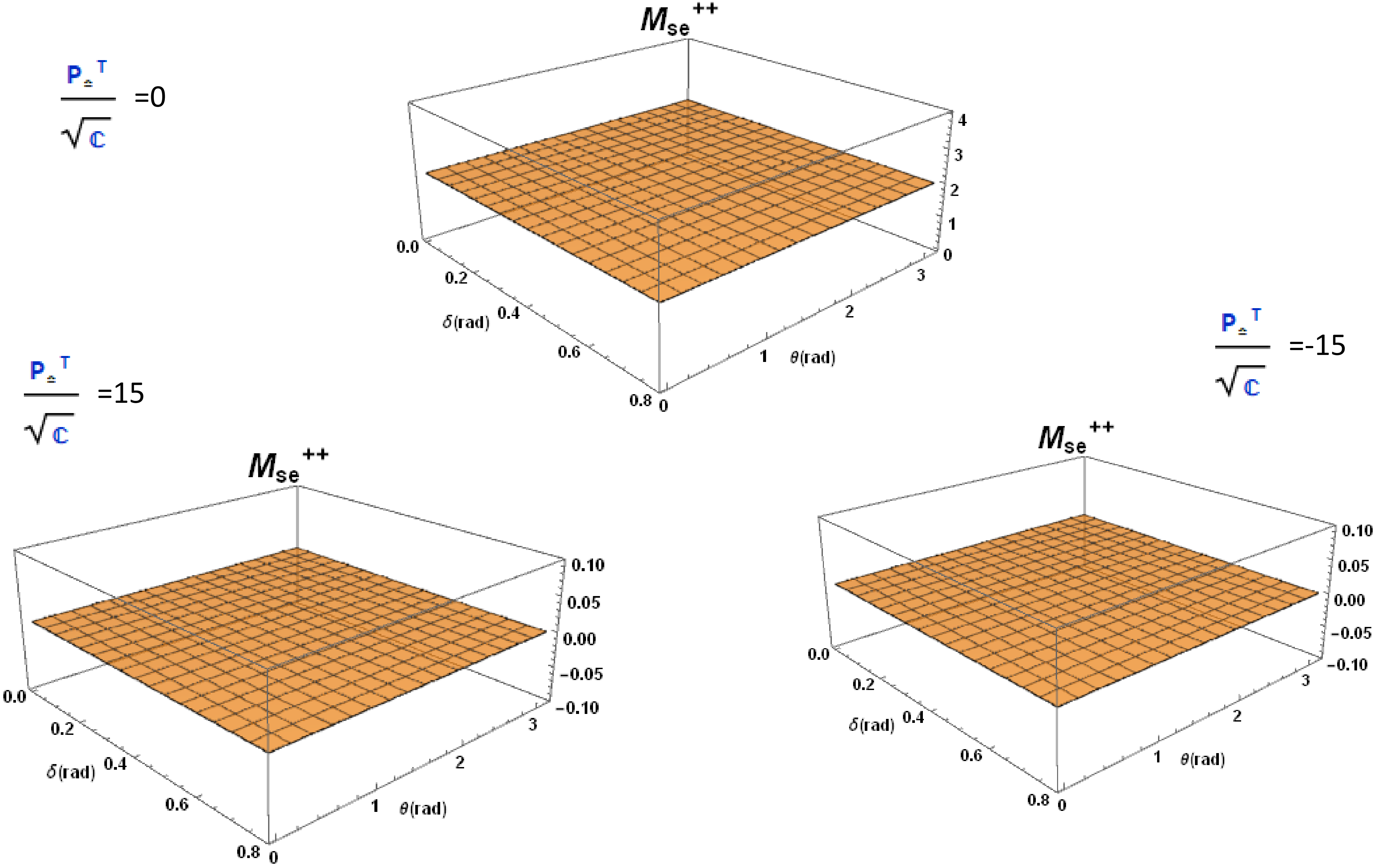
- We confirm QC in spins , interpolating spin-1/2 spinors , interpolating spin-1 spinors and polarization vectors.
- We discuss the conditions which enable us to see the QC for all reference frame and for all interpolation angle
- Specially we show that LF QC appears in the zero-mode . (Quantum entanglement in the LF)
- Large momentum can avoid using the interpolation angle dependence

Thank you

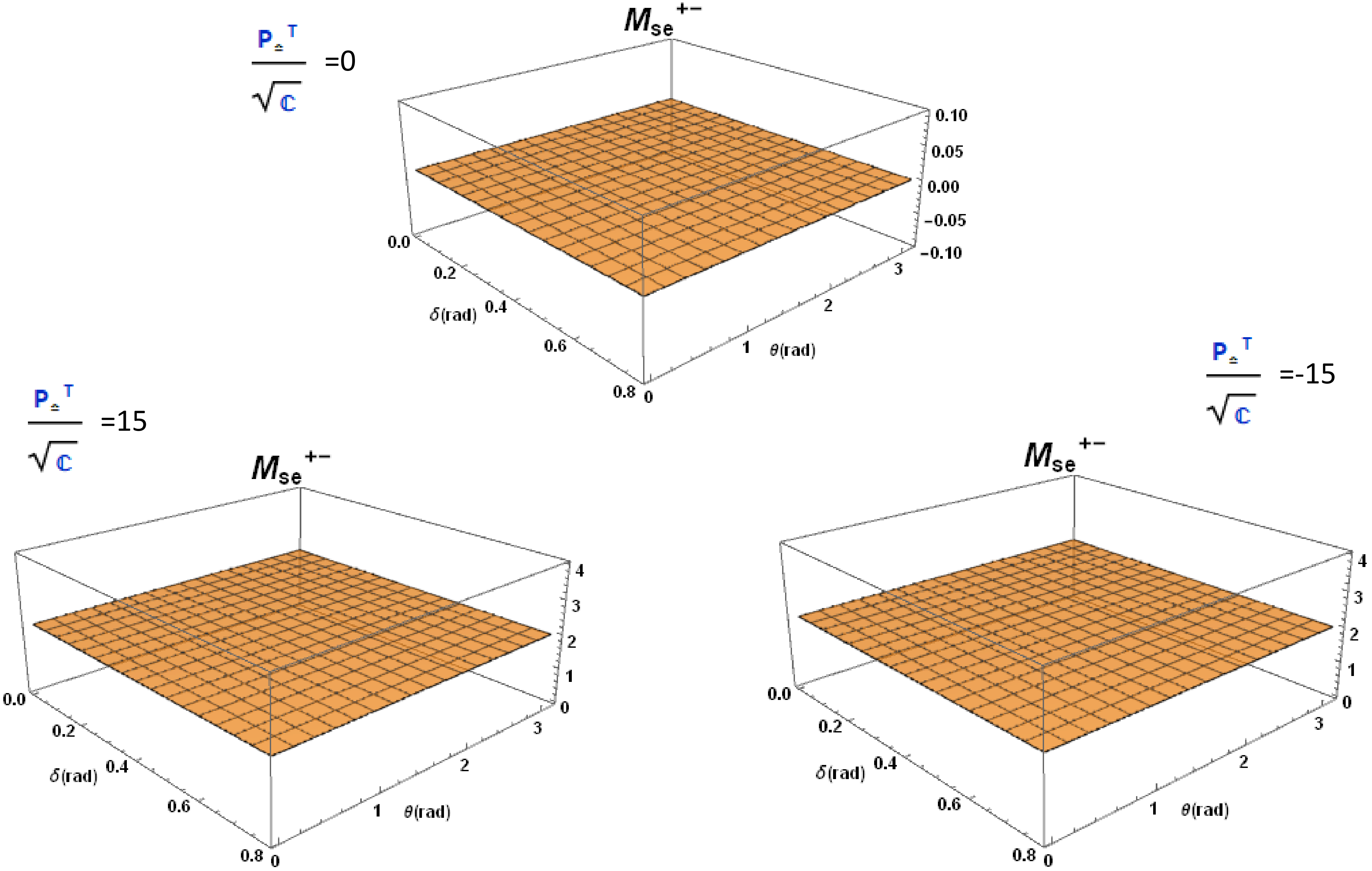

M_{se}^{++}
0.0
0.2
δ(rad) 0.4
0.6
0.8
−10
0
10
pz (me)
2.0
1.5
1.0
0.5
0.0
M_{se}^{++}
0.0
0.2
δ 0.4
0.6
0.8
−10
0
10
$P_{\perp}^{T}/\sqrt{C}$
2.0
1.5
1.0
0.5
0.0
M_{se}^{+-}
0.0
0.2
δ(rad) 0.4
0.6
0.8
−10
0
10
pz (me)
2.0
1.5
1.0
0.5
0.0
M_{se}^{+-}
0.0
0.2
δ 0.4
0.6
0.8
−10
0
10
$P_{\perp}^{T}/\sqrt{C}$
2.0
1.5
1.0
0.5
0.0



Pz=0
M_{se}^{++}
2.0
1.5
1.0
0.5
0.0
0.0
0.2
0.4
0.6
0.8
δ(rad)
0
1
2
3
θ(rad)
Pz=15
M_{se}^{++}
1.0
0.5
0.0
−0.5
−1.0
0.0
0.2
0.4
0.6
0.8
δ(rad)
0
1
2
3
θ(rad)
Pz=-15
M_{se}^{++}
2.0
1.5
1.0
0.5
0.0
0.0
0.2
0.4
0.6
0.8
δ(rad)
0
1
2
3
θ(rad)

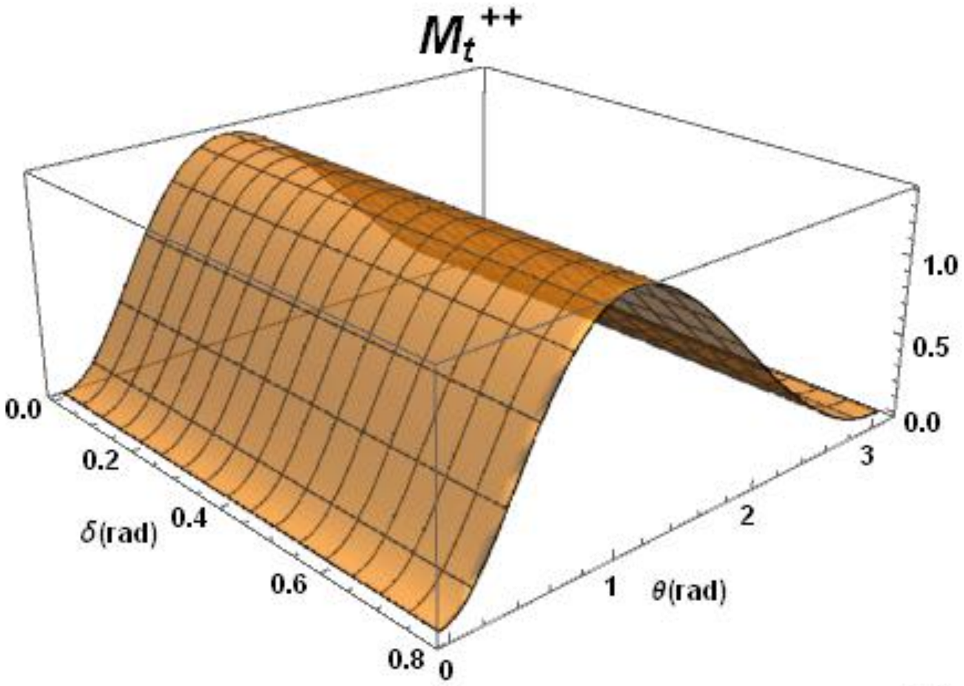


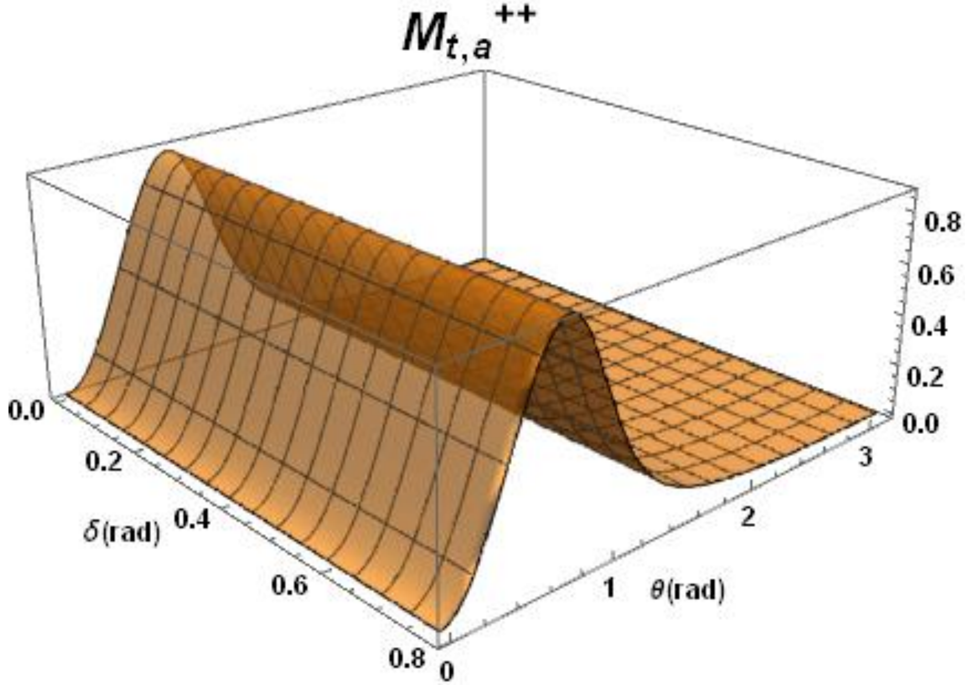
$\frac{P_{\pm}^{T}}{\sqrt{\mathbb{C}}}$ =0
M_{se}^{++}
0.0
0.2
0.4
0.6
0.8
δ(rad)
0
1
2
3
θ(rad)
0
1
2
3
4
$\frac{P_{\pm}^{T}}{\sqrt{\mathbb{C}}}$ =15
M_{se}^{++}
0.10
0.05
0.00
-0.05
-0.10
$\frac{P_{\pm}^{T}}{\sqrt{\mathbb{C}}}$ =-15
M_{se}^{++}
0.10
0.05
0.00
-0.05
-0.10

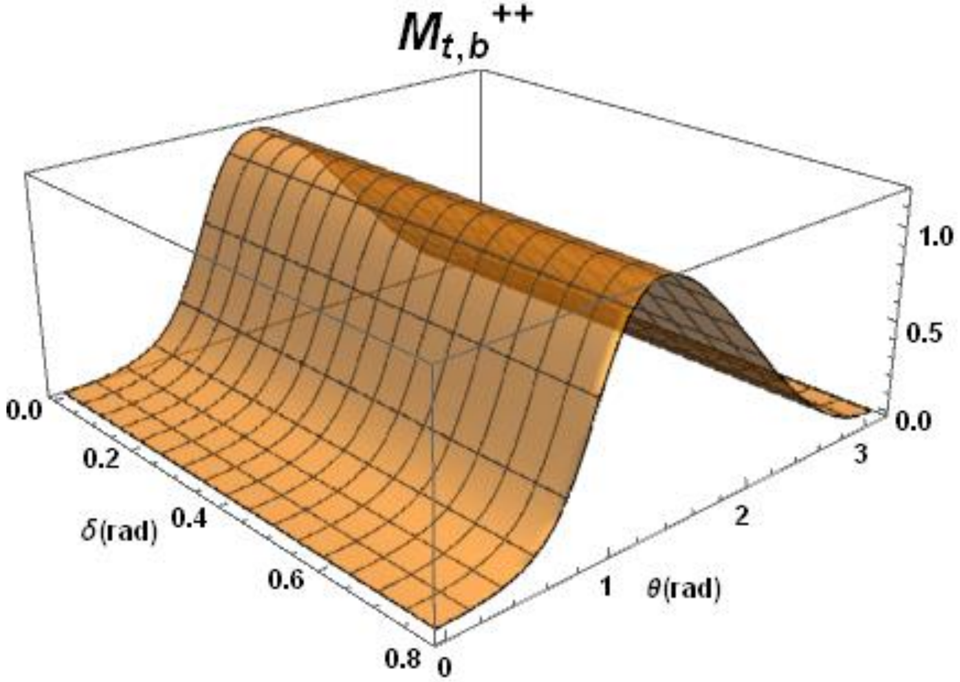


$\frac{P_{\pm}^{T}}{\sqrt{\mathbb{C}}}$ =0
M_{se}^{+-}
0.10
0.05
0.00
-0.05
-0.10
0.0
0.2
0.4
0.6
0.8
δ(rad)
0
1
2
3
θ(rad)
$\frac{P_{\pm}^{T}}{\sqrt{\mathbb{C}}}$ =15
M_{se}^{+-}
4
3
2
1
0
δ(rad)
θ(rad)
$\frac{P_{\pm}^{T}}{\sqrt{\mathbb{C}}}$ =-15
M_{se}^{+-}
4
3
2
1
0
δ(rad)
θ(rad)

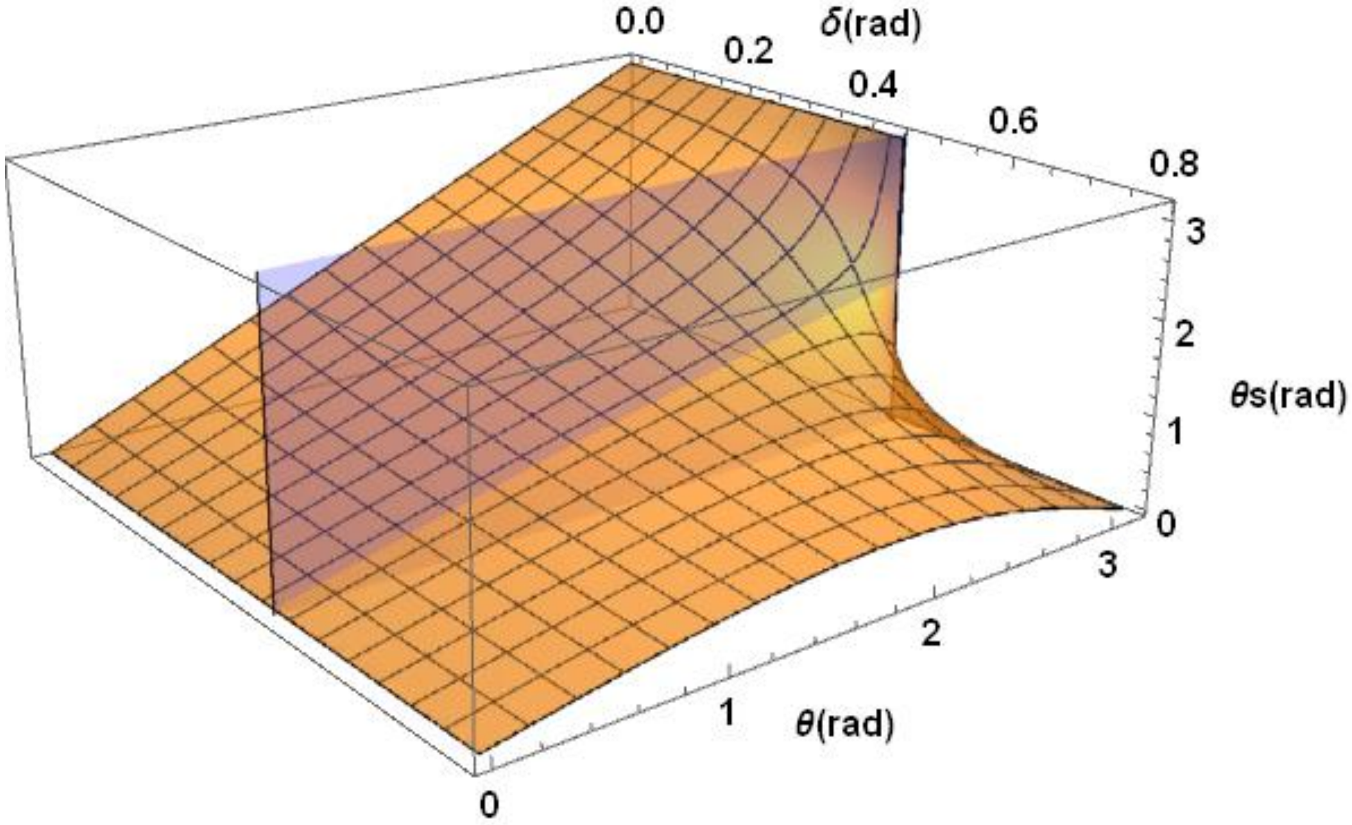
$\frac{P_{\pm}^{T}}{\sqrt{C}}$ =-15

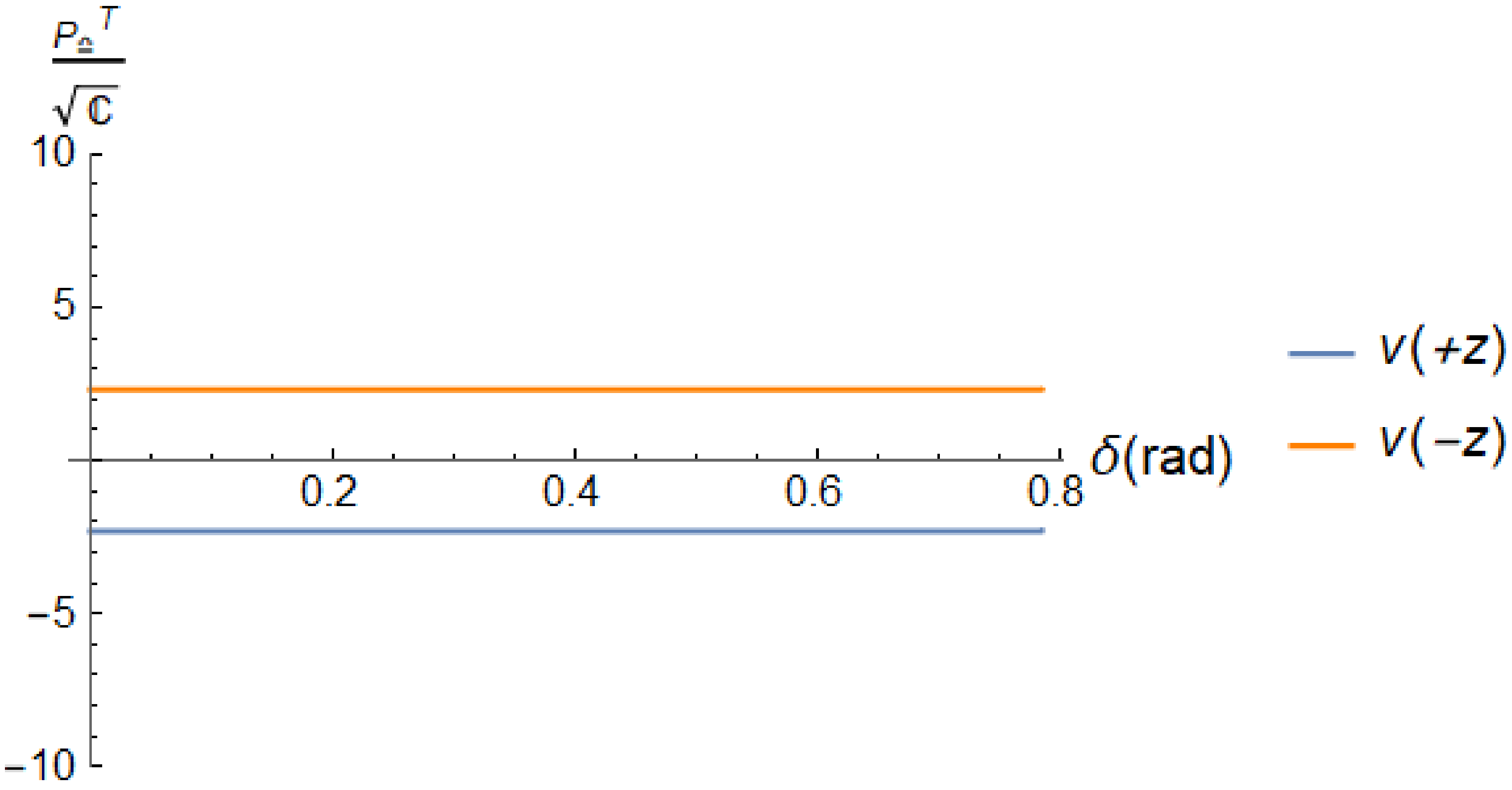




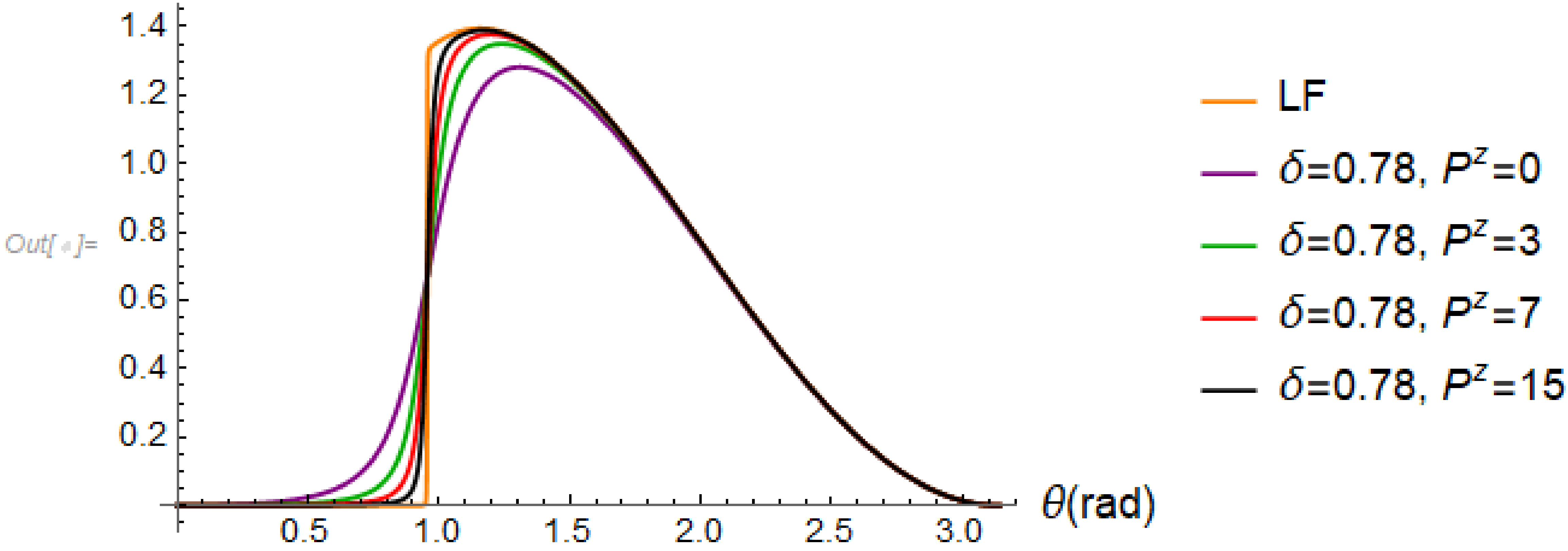


According to the Jacob and Wick helicity define in the IFD and the helicity define in the LFD , this whole image shows the helicity + scenario.





$\frac{P_{\triangleq}{}^{T}}{\sqrt{c}}$
10
5
−5
−10
0.2
0.4
0.6
0.8
δ(rad)
v(+z)
v(−z)



Out[ ]=
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.5
1.0
1.5
2.0
2.5
3.0
θ(rad)
LF
δ=0.78, P^z=0
δ=0.78, P^z=3
δ=0.78, P^z=7
δ=0.78, P^z=15