Quantum Correlation and Spin Orientation

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Group meeting

03- 12-2021

Outline

- Quantum correlation of spins
- Quantum correlation of helicity spinors
    - -Spin- ½ spinors
    - -Spin -1 spinors
- Quantum correlation manifestation in helicity amplitudes
- Spin orientation of spin-1 helicity spinors
- Relation between spin-1 spinors and photon polarization vectors
    - -Helicity relation
    - -Dirac relation

Quantum - Correlation of spins

The tensor product of two fundamental spin ½ particles can be written as the direct sum of spin-1 and spin-0

$$\frac{1}{2} \otimes \frac{1}{2} = 1 \oplus 0$$

$$d^{(j)}_{m',m}(\beta) = < j, m' | exp(\frac{-i j_y \beta}{\hbar}) | j, m >$$

(Wigner-d function)

For spin-1 particles $(j = 1, m = 1, 0, -1)$

$$|1,1> = |\uparrow\uparrow>$$

$$|1,0> = \frac{1}{\sqrt{2}}(|\uparrow\downarrow \underline{+} \downarrow\uparrow>)$$

$$|1,-1> = |\downarrow\downarrow>$$

For spin-0 particles $(j = 0, m = 0)$

$$|0,0> = \frac{1}{\sqrt{2}}(|\uparrow\downarrow \underline{-} \downarrow\uparrow>)$$

$$d^{1/2}(\pi)|\uparrow> = |\downarrow>$$

$$d^{1/2}(\pi)|\downarrow> = \underline{-}|\uparrow>$$

➡

spin-1 particles

$$|1,1> \longrightarrow |1,-1>$$

$$|1,0> \longrightarrow -|1,0>$$

$$|1,-1> \longrightarrow |1,1>$$

spin-0 particles

$$|0,0> \longrightarrow |0,0>$$

Spin - ½ helicity spinors

Spinors in the rest frame , Chiral representation

$$u^{(1/2)}(0) = \begin{pmatrix} \sqrt{M} \\ 0 \\ \sqrt{M} \\ 0 \end{pmatrix}, \quad u^{(-1/2)}(0) = \begin{pmatrix} 0 \\ \sqrt{M} \\ 0 \\ \sqrt{M} \end{pmatrix},$$

Normalization

$$\bar{u}^{(\lambda)}u^{(\lambda)} = 2M$$

Helicity transformation matrix

$$T = T_{12}T_3 = e^{i\beta_1\mathcal{K}^{\hat{1}}+i\beta_2\mathcal{K}^{\hat{2}}}e^{-i\beta_3 K^3}$$

$$\mathcal{K}^{\hat{1}} = -K^1\sin\delta - J^2\cos\delta,$$

$$\mathcal{K}^{\hat{2}} = J^1\cos\delta - K^2\sin\delta,$$

$$(\delta \to 0),\ \mathcal{K}^{\hat{1}} \to -J^2,\ \mathcal{K}^{\hat{2}} \to J^1$$

$$(\delta \to \pi/4),\ \mathcal{K}^{\hat{1}} \to -E_1,\ \mathcal{K}^{\hat{2}} \to -E_2$$

Spin- ½ helicity spinors

The six Lorentz group generators of rotation (J) and boost (K) in chiral representation for spin-1/2

$$\mathbf{J}_C = \frac{1}{2}\begin{pmatrix} \boldsymbol{\sigma} & 0 \\ 0 & \boldsymbol{\sigma} \end{pmatrix} \qquad \mathbf{K}_C = \frac{i}{2}\begin{pmatrix} \boldsymbol{\sigma} & 0 \\ 0 & -\boldsymbol{\sigma} \end{pmatrix}$$

$$\mathbf{J}_C + i\mathbf{K}_C = \begin{pmatrix} 0 & 0 \\ 0 & \boldsymbol{\sigma} \end{pmatrix}$$

$$\mathbf{J}_C - i\mathbf{K}_C = \begin{pmatrix} \boldsymbol{\sigma} & 0 \\ 0 & 0 \end{pmatrix}$$

Pauli matrices for spin (1/2)

$$\sigma_1 = \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$
$$\sigma_2 = \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$
$$\sigma_3 = \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

$$\mathbf{A} = \frac{1}{2}(\mathbf{J} + i\mathbf{K}),$$
$$\mathbf{B} = \frac{1}{2}(\mathbf{J} - i\mathbf{K}),$$

$$(0, j) \rightarrow \mathbf{J} = -i\mathbf{K} \quad (\mathbf{A} = 0),$$
$$(j, 0) \rightarrow \mathbf{J} = i\mathbf{K} \quad (\mathbf{B} = 0),$$

A and **B** each generates a corresponding *SU(2)* group algebra. The above specific combination between rotation and boost may suggest the two decoupled helical motions of the particle that may be denotes as the right-handed vs left-handed chirality

Spin- ½ helicity spinors

We write spinors in the *(0,J)*⊕ *(J,0)* chiral representation of the Lorentz group due to a clear decoupling between the right-handed and left-handed components .

In this representation transformation matrix T ⟶

$$T = \begin{pmatrix} T_R & 0 \\ 0 & T_L \end{pmatrix}$$

(0,1/2)⊕ *(1/2,0)* chiral representation

- (0,1/2) representation A=0 $\mathbf{K} = i\mathbf{J} = i\boldsymbol{\sigma}/2$ $\mathbf{b}_\perp = (\beta_1 \sin\delta + i\beta_2 \cos\delta, -i\beta_1 \cos\delta + \beta_2 \sin\delta)$

$$T_R = e^{\mathbf{b}_\perp \cdot \boldsymbol{\sigma}_\perp/2} e^{\beta_3 \sigma_3/2}$$

$$T_{12R} = e^{\mathbf{b}_\perp \cdot \boldsymbol{\sigma}_\perp/2} = \cos\left(\frac{\alpha}{2}\right) \cdot I + \frac{\mathbf{b}_\perp \cdot \boldsymbol{\sigma}_\perp}{\alpha} \sin\left(\frac{\alpha}{2}\right)$$

$$(\mathbf{b}_\perp \cdot \boldsymbol{\sigma}_\perp)^2 = -\alpha^2 \cdot I$$

$$\sigma_3^2 = I \qquad \alpha = \sqrt{\mathbb{C}(\beta_1^2 + \beta_2^2)}$$

$$T_{3R} = e^{\beta_3 \sigma_3/2} = \begin{pmatrix} e^{\beta_3/2} & 0 \\ 0 & e^{-\beta_3/2} \end{pmatrix}$$

Spin- ½ helicity spinors

$$T_R = \begin{pmatrix} \cos\frac{\alpha}{2} e^{\frac{\beta_3}{2}} & -\frac{\beta_L(\cos\delta - \sin\delta)}{\alpha} \sin\frac{\alpha}{2} e^{-\frac{\beta_3}{2}} \\ \frac{\beta_R(\sin\delta + \cos\delta)}{\alpha} \sin\frac{\alpha}{2} e^{\frac{\beta_3}{2}} & \cos\frac{\alpha}{2} e^{-\frac{\beta_3}{2}} \end{pmatrix}$$

$$\beta_R = \beta_1 + i\beta_2 \qquad \beta_L = \beta_1 - i\beta_2$$

- (1/2,0) representation B=0 $\quad \mathbf{K} = -i\mathbf{J} = -i\boldsymbol{\sigma}/2$

$$T_L = e^{-\mathbf{b}_\perp^* \cdot \boldsymbol{\sigma}_\perp/2} e^{-\beta_3 \sigma_3/2}$$

$$T_{12L} = e^{-\mathbf{b}_\perp^* \cdot \boldsymbol{\sigma}_\perp/2} = \cos\left(\frac{\alpha}{2}\right) \cdot I - \frac{\mathbf{b}_\perp^* \cdot \boldsymbol{\sigma}_\perp}{\alpha} \sin\left(\frac{\alpha}{2}\right) \qquad T_{3L} = e^{-\beta_3\sigma_3/2} = \begin{pmatrix} e^{-\beta_3/2} & 0 \\ 0 & e^{\beta_3/2} \end{pmatrix}.$$

Spin-½ helicity spinors

$$T_L = \begin{pmatrix} \cos\frac{\alpha}{2}\, e^{-\frac{\beta_3}{2}} & -\frac{\beta_L(\sin\delta+\cos\delta)}{\alpha}\sin\frac{\alpha}{2}\, e^{\frac{\beta_3}{2}} \\ \frac{\beta_R(\cos\delta-\sin\delta)}{\alpha}\sin\frac{\alpha}{2}\, e^{-\frac{\beta_3}{2}} & \cos\frac{\alpha}{2}\, e^{\frac{\beta_3}{2}} \end{pmatrix}$$

After applying T- transformation to the rest spinors

$$u_H^{(1/2)}(\beta) = \sqrt{M}\begin{pmatrix} \cos\frac{\alpha}{2}\, e^{\beta_3/2} \\ \frac{\beta_R(\sin\delta+\cos\delta)}{\alpha}\sin\frac{\alpha}{2}\, e^{\beta_3/2} \\ \cos\frac{\alpha}{2}\, e^{-\beta_3/2} \\ \frac{\beta_R(\cos\delta-\sin\delta)}{\alpha}\sin\frac{\alpha}{2}\, e^{-\beta_3/2} \end{pmatrix} \qquad u_H^{(-1/2)}(\beta) = \sqrt{M}\begin{pmatrix} -\frac{\beta_L(\cos\delta-\sin\delta)}{\alpha}\sin\frac{\alpha}{2}\, e^{-\beta_3/2} \\ \cos\frac{\alpha}{2}\, e^{-\beta_3/2} \\ -\frac{\beta_L(\sin\delta+\cos\delta)}{\alpha}\sin\frac{\alpha}{2}\, e^{\beta_3/2} \\ \cos\frac{\alpha}{2}\, e^{\beta_3/2} \end{pmatrix}$$

Spin-½ helicity spinors

We can write $\beta_1, \beta_2, \beta_3$ and α in terms of the momentum as follows

$$\cos\alpha = \frac{P_{\hat{-}}}{\mathbb{P}} \qquad \sin\alpha = \frac{\sqrt{\mathbf{P}_\perp^2 \mathbb{C}}}{\mathbb{P}} \qquad e^{\beta_3} = \frac{P^{\hat{+}} + \mathbb{P}}{M(\sin\delta + \cos\delta)} \qquad e^{-\beta_3} = \frac{P^{\hat{+}} - \mathbb{P}}{M(\cos\delta - \sin\delta)} \qquad \frac{\beta_j}{\alpha} = \frac{P^j}{\sqrt{\mathbf{P}_\perp^2 \mathbb{C}}}, \qquad (j = 1, 2)$$

$$u_H^{(1/2)}(P) = \begin{pmatrix} \sqrt{\frac{P_{\hat{-}}+\mathbb{P}}{2\mathbb{P}}}\sqrt{\frac{P^{\hat{+}}+\mathbb{P}}{(\sin\delta+\cos\delta)}} \\ P^R\sqrt{\frac{\sin\delta+\cos\delta}{2\mathbb{P}(\mathbb{P}+P_{\hat{-}})}}\sqrt{P^{\hat{+}}+\mathbb{P}} \\ \sqrt{\frac{P_{\hat{-}}+\mathbb{P}}{2\mathbb{P}}}\sqrt{\frac{P^{\hat{+}}-\mathbb{P}}{(\cos\delta-\sin\delta)}} \\ P^R\sqrt{\frac{\cos\delta-\sin\delta}{2\mathbb{P}(\mathbb{P}+P_{\hat{-}})}}\sqrt{P^{\hat{+}}-\mathbb{P}} \end{pmatrix} \qquad u_H^{(-1/2)}(P) = \begin{pmatrix} -P^L\sqrt{\frac{\cos\delta-\sin\delta}{2\mathbb{P}(\mathbb{P}+P_{\hat{-}})}}\sqrt{P^{\hat{+}}-\mathbb{P}} \\ \sqrt{\frac{P_{\hat{-}}+\mathbb{P}}{2\mathbb{P}}}\sqrt{\frac{P^{\hat{+}}-\mathbb{P}}{(\cos\delta-\sin\delta)}} \\ -P^L\sqrt{\frac{\sin\delta+\cos\delta}{2\mathbb{P}(\mathbb{P}+P_{\hat{-}})}}\sqrt{P^{\hat{+}}+\mathbb{P}} \\ \sqrt{\frac{P_{\hat{-}}+\mathbb{P}}{2\mathbb{P}}}\sqrt{\frac{P^{\hat{+}}+\mathbb{P}}{(\sin\delta+\cos\delta)}} \end{pmatrix}$$

$$P^{\hat{R}} = P^{\hat{1}} + iP^{\hat{2}} \qquad P^{\hat{L}} = P^{\hat{1}} - iP^{\hat{2}}.$$

$$\mathbb{P}^2 = (P^{\hat{+}})^2 - M^2\mathbb{C} = P_{\hat{-}}^2 + \mathbf{P}_\perp^2\mathbb{C},$$

$$\begin{pmatrix} x^{\hat{+}} \\ x^{\hat{1}} \\ x^{\hat{2}} \\ x^{\hat{-}} \end{pmatrix} = \begin{pmatrix} \cos\delta & 0 & 0 & \sin\delta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \sin\delta & 0 & 0 & -\cos\delta \end{pmatrix} \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}$$

Spin-1 helicity spinors

Spinors in the rest frame , Chiral representation

$$U^1(0) = \begin{bmatrix} \sqrt{M} \\ 0 \\ 0 \\ \sqrt{M} \\ 0 \\ 0 \end{bmatrix} \quad U^0(0) = \begin{bmatrix} 0 \\ \sqrt{M} \\ 0 \\ 0 \\ \sqrt{M} \\ 0 \end{bmatrix} \quad U^{-1}(0) = \begin{bmatrix} 0 \\ 0 \\ \sqrt{M} \\ 0 \\ 0 \\ \sqrt{M} \end{bmatrix}$$

The six Lorentz group generators of rotation (J) and boost (K) in chiral representation for spin-1

$$\mathrm{J_c} = \begin{bmatrix} \sigma & 0 \\ 0 & \sigma \end{bmatrix} \qquad \mathrm{K_c} = i\begin{bmatrix} \sigma & 0 \\ 0 & -\sigma \end{bmatrix}$$

Pauli matrices for spin-1

$$\sigma_1 = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\sigma_2 = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{bmatrix}$$

$$\sigma_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$T_R = e^{\mathbf{b}_\perp \cdot \boldsymbol{\sigma}_\perp} \; e^{\beta_3 \sigma_3} \qquad T_L = e^{-\mathbf{b}_\perp^* \cdot \boldsymbol{\sigma}_\perp} \; e^{-\beta_3 \sigma_3}$$

$$(\sigma_3)^1 = (\sigma_3)^3 \qquad T_{3R} = \mathrm{I} + (\sigma_3.\sigma_3) * (\mathrm{Cosh}[\beta 3] - 1) + \sigma_3 * \mathrm{Sinh}[\beta 3]$$

$$(\sigma_3)^2 = (\sigma_3)^4 \qquad T_{3L} = \mathrm{I} + (\sigma_3.\sigma_3) * (\mathrm{Cosh}[\beta 3] - 1) - \sigma_3 * \mathrm{Sinh}[\beta 3]$$

Define

$$b_R = \frac{i(b_{\perp}.\sigma_{\perp})}{\alpha} \qquad b_L = \frac{i(-b_{\perp}^{*}.\sigma_{\perp})}{\alpha}$$

$$T_{12R} = \mathrm{I} + (b_R.b_R) * (\mathrm{Cos}[\alpha] - 1) - i b_R \mathrm{Sin}[\alpha]$$

$$T_{12L} = \mathrm{I} + (b_L.b_L) * (\mathrm{Cos}[\alpha] - 1) - i b_L \mathrm{Sin}[\alpha]$$

$$(b_R)^1 = (b_R)^3 \qquad (b_L)^1 = (b_L)^3$$

$$(b_R)^2 = (b_R)^4 \qquad (b_L)^2 = (b_L)^4$$

$$\mathrm{TR}=\begin{pmatrix} e^{\beta 3}\mathrm{Cos}\left[\frac{\alpha}{2}\right]^2 & -\frac{\mathrm{Sin}[\alpha](\mathrm{Cos}[\delta]-\mathrm{Sin}[\delta])\beta_L}{\sqrt{2}\alpha} & \frac{e^{-\beta 3}\mathrm{Sin}\left[\frac{\alpha}{2}\right]^2(\mathrm{Cos}[\delta]-\mathrm{Sin}[\delta])^2\beta_L^2}{\alpha^2} \\ \frac{e^{\beta 3}\mathrm{Sin}[\alpha](\mathrm{Cos}[\delta]+\mathrm{Sin}[\delta])\beta_R}{\sqrt{2}\alpha} & \mathrm{Cos}[\alpha] & -\frac{e^{-\beta 3}\mathrm{Sin}[\alpha](\mathrm{Cos}[\delta]-\mathrm{Sin}[\delta])\beta_L}{\sqrt{2}\alpha} \\ \frac{e^{\beta 3}\mathrm{Sin}\left[\frac{\alpha}{2}\right]^2(\mathrm{Cos}[\delta]+\mathrm{Sin}[\delta])^2\beta_R^2}{\alpha^2} & \frac{\mathrm{Sin}[\alpha](\mathrm{Cos}[\delta]+\mathrm{Sin}[\delta])\beta_R}{\sqrt{2}\alpha} & e^{-\beta 3}\mathrm{Cos}\left[\frac{\alpha}{2}\right]^2 \end{pmatrix}$$

$$\mathrm{TL}=\begin{pmatrix} e^{-\beta 3}\mathrm{Cos}\left[\frac{\alpha}{2}\right]^2 & -\frac{\mathrm{Sin}[\alpha](\mathrm{Cos}[\delta]+\mathrm{Sin}[\delta])\beta_L}{\sqrt{2}\alpha} & \frac{e^{\beta 3}\mathrm{Sin}\left[\frac{\alpha}{2}\right]^2(\mathrm{Cos}[\delta]+\mathrm{Sin}[\delta])^2\beta_L^2}{\alpha^2} \\ \frac{e^{-\beta 3}\mathrm{Sin}[\alpha](\mathrm{Cos}[\delta]-\mathrm{Sin}[\delta])\beta_R}{\sqrt{2}\alpha} & \mathrm{Cos}[\alpha] & -\frac{e^{\beta 3}\mathrm{Sin}[\alpha](\mathrm{Cos}[\delta]+\mathrm{Sin}[\delta])\beta_L}{\sqrt{2}\alpha} \\ \frac{e^{-\beta 3}\mathrm{Sin}\left[\frac{\alpha}{2}\right]^2(\mathrm{Cos}[\delta]-\mathrm{Sin}[\delta])^2\beta_R^2}{\alpha^2} & \frac{\mathrm{Sin}[\alpha](\mathrm{Cos}[\delta]-\mathrm{Sin}[\delta])\beta_R}{\sqrt{2}\alpha} & e^{\beta 3}\mathrm{Cos}\left[\frac{\alpha}{2}\right]^2 \end{pmatrix}$$

$$U^{(+1)}{}_H(\beta) = \begin{pmatrix} e^{\beta 3}\sqrt{M}\,\mathrm{Cos}\left[\frac{\alpha}{2}\right]^2 \\ \frac{e^{\beta 3}\sqrt{M}\,\mathrm{Sin}[\alpha](\mathrm{Cos}[\delta]+\mathrm{Sin}[\delta])\beta_R}{\sqrt{2}\alpha} \\ \frac{e^{\beta 3}\sqrt{M}\,\mathrm{Sin}\left[\frac{\alpha}{2}\right]^2(\mathrm{Cos}[\delta]+\mathrm{Sin}[\delta])^2\beta_R^2}{\alpha^2} \\ e^{-\beta 3}\sqrt{M}\,\mathrm{Cos}\left[\frac{\alpha}{2}\right]^2 \\ \frac{e^{-\beta 3}\sqrt{M}\,\mathrm{Sin}[\alpha](\mathrm{Cos}[\delta]-\mathrm{Sin}[\delta])\beta_R}{\sqrt{2}\alpha} \\ \frac{e^{-\beta 3}\sqrt{M}\,\mathrm{Sin}\left[\frac{\alpha}{2}\right]^2(\mathrm{Cos}[\delta]-\mathrm{Sin}[\delta])^2\beta_R^2}{\alpha^2} \end{pmatrix} \quad U^{(-1)}{}_H(\beta) = \begin{pmatrix} \frac{e^{-\beta 3}\sqrt{M}\,\mathrm{Sin}\left[\frac{\alpha}{2}\right]^2(\mathrm{Cos}[\delta]-\mathrm{Sin}[\delta])^2\beta_L^2}{\alpha^2} \\ -\frac{e^{-\beta 3}\sqrt{M}\,\mathrm{Sin}[\alpha](\mathrm{Cos}[\delta]-\mathrm{Sin}[\delta])\beta_L}{\sqrt{2}\alpha} \\ e^{-\beta 3}\sqrt{M}\,\mathrm{Cos}\left[\frac{\alpha}{2}\right]^2 \\ \frac{e^{\beta 3}\sqrt{M}\,\mathrm{Sin}\left[\frac{\alpha}{2}\right]^2(\mathrm{Cos}[\delta]+\mathrm{Sin}[\delta])^2\beta_L^2}{\alpha^2} \\ -\frac{e^{\beta 3}\sqrt{M}\,\mathrm{Sin}[\alpha](\mathrm{Cos}[\delta]+\mathrm{Sin}[\delta])\beta_L}{\sqrt{2}\alpha} \\ e^{\beta 3}\sqrt{M}\,\mathrm{Cos}\left[\frac{\alpha}{2}\right]^2 \end{pmatrix} \quad U^{(0)}{}_H(\beta) = \begin{pmatrix} -\frac{\sqrt{M}\,\mathrm{Sin}[\alpha](\mathrm{Cos}[\delta]-\mathrm{Sin}[\delta])\beta_L}{\sqrt{2}\alpha} \\ \sqrt{M}\,\mathrm{Cos}[\alpha] \\ \frac{\sqrt{M}\,\mathrm{Sin}[\alpha](\mathrm{Cos}[\delta]+\mathrm{Sin}[\delta])\beta_R}{\sqrt{2}\alpha} \\ -\frac{\sqrt{M}\,\mathrm{Sin}[\alpha](\mathrm{Cos}[\delta]+\mathrm{Sin}[\delta])\beta_L}{\sqrt{2}\alpha} \\ \sqrt{M}\,\mathrm{Cos}[\alpha] \\ \frac{\sqrt{M}\,\mathrm{Sin}[\alpha](\mathrm{Cos}[\delta]-\mathrm{Sin}[\delta])\beta_R}{\sqrt{2}\alpha} \end{pmatrix}$$

$$u_H^{(+1)} = \frac{1}{2\sqrt{M\mathbb{P}^2}} \begin{pmatrix} \frac{(P_{\hat{-}}+\mathbb{P})(P^{\hat{+}}+\mathbb{P})}{(A-B)} \\ \sqrt{2}P^R(P^{\hat{+}}+\mathbb{P}) \\ \frac{(A-B)(P^R)^2(P^{\hat{+}}+\mathbb{P})}{(P_{\hat{-}}+\mathbb{P})} \\ (A-B)(P_{\hat{-}}+\mathbb{P})\mathbb{X} \\ \sqrt{2}P^R(P^{\hat{+}}-\mathbb{P}) \\ \frac{(A+B)(P^R)^2(P^{\hat{+}}-\mathbb{P})}{(P_{\hat{-}}+\mathbb{P})} \end{pmatrix}, \quad u_H^{(-1)} = \frac{1}{2\sqrt{M\mathbb{P}^2}} \begin{pmatrix} \frac{(A+B)(P^L)^2(P^{\hat{+}}-\mathbb{P})}{(P_{\hat{-}}+\mathbb{P})} \\ -\sqrt{2}P^L(P^{\hat{+}}-\mathbb{P}) \\ (A-B)(P_{\hat{-}}+\mathbb{P})\mathbb{X} \\ \frac{(A-B)(P^L)^2(P^{\hat{+}}+\mathbb{P})}{(P_{\hat{-}}+\mathbb{P})} \\ -\sqrt{2}P^L(P^{\hat{+}}+\mathbb{P}) \\ \frac{(P_{\hat{-}}+\mathbb{P})(P^{\hat{+}}+\mathbb{P})}{(A-B)} \end{pmatrix} \quad u_H^{(0)} = \sqrt{\frac{M}{2\mathbb{P}^2}} \begin{pmatrix} -(A+B)P^L \\ \sqrt{2}P_{\hat{-}} \\ (A-B)P^R \\ (-A+B)P^L \\ \sqrt{2}P_{\hat{-}} \\ (A+B)P^R \end{pmatrix}$$

$$\bar{u}^{(\lambda)}u^{(\lambda)} = 2M$$

General Spinor Operator

$$\mathcal{J}_3 = \mathrm{T}\, J_3\, T^{-1}$$

$$\mathcal{J}_3 = J_3 \cos\alpha + (\beta_1 \mathcal{K}^{\hat{2}} - \beta_2 \mathcal{K}^{\hat{1}}) \frac{\sin\alpha}{\alpha}.$$

$$\mathcal{J}_3 = \frac{1}{\mathbb{P}} (P_{\hat{-}} J_3 + P^1 \mathcal{K}^{\hat{2}} - P^2 \mathcal{K}^{\hat{1}}),$$

Spin - 1/2

$$\mathcal{J}_3{}^{(\frac{1}{2})} U_H^{1/2} = +\frac{1}{2} U_H^{1/2}$$

$$\mathcal{J}_3{}^{(\frac{1}{2})} U_H^{-1/2} = -\frac{1}{2} U_H^{-1/2}$$

Spin - 1

$$\mathcal{J}_3{}^{(1)} U_H^1 = +1 U_H^1$$

$$\mathcal{J}_3{}^{(1)} U_H^0 = 0 * U_H^0$$

$$\mathcal{J}_3{}^{(1)} U_H^{-1} = -1 U_H^{-1}$$

$$\mathcal{J}_3 |p; j, m\rangle_\delta = T J_3 T^{-1} T |0; j, m\rangle = m |p; j, m\rangle_\delta$$

Quantum Correlation of Spinors

Define

$$RB1 = e^{i\beta\,\widehat{\mathcal{K}^1}}$$

$$\mathcal{RB}\mathbf{1} = \mathrm{T}\,.\,RB1.T^{-1}$$

$$\mathcal{K}^{\hat{1}} = -K^1 \sin\delta - J^2 \cos\delta,$$

$RB1$ is also in chiral representation

IFD, $\delta \rightarrow 0$ when $\beta \rightarrow \pi$

Spin - 1/2

$$\mathcal{RB}\mathbf{1}^{(\frac{1}{2})} U_H^{1/2} = U_H^{-1/2}$$

$$\mathcal{RB}\mathbf{1}^{(\frac{1}{2})} U_H^{-1/2} = -U_H^{1/2}$$

Spin - 1

$$\mathcal{RB}\mathbf{1}^{(1)} U_H^{1} = U_H^{-1}$$

$$\mathcal{RB}\mathbf{1}^{(1)} U_H^{0} = -U_H^{0}$$

$$\mathcal{RB}\mathbf{1}^{(1)} U_H^{-1} = U_H^{1}$$

* Spinors act similar to spins in the IFD

$$RB1 = e^{i\beta\widehat{\mathcal{K}^1}} \qquad \alpha_1 \to \sqrt{\beta^2 Cos[2\delta]}$$

$$RB1^1 = \begin{pmatrix} \text{Cos}\left[\frac{\alpha_1}{2}\right]^2 & -\frac{\beta(\text{Cos}[\delta]-\text{Sin}[\delta])\text{Sin}[\alpha_1]}{\sqrt{2}\alpha_1} & \frac{\beta^2(\text{Cos}[\delta]-\text{Sin}[\delta])^2\text{Sin}\left[\frac{\alpha_1}{2}\right]^2}{\alpha_1^2} & 0 & 0 & 0 \\ \frac{\beta(\text{Cos}[\delta]+\text{Sin}[\delta])\text{Sin}[\alpha_1]}{\sqrt{2}\alpha_1} & \text{Cos}[\alpha_1] & -\frac{\beta(\text{Cos}[\delta]-\text{Sin}[\delta])\text{Sin}[\alpha_1]}{\sqrt{2}\alpha_1} & 0 & 0 & 0 \\ \frac{\beta^2(\text{Cos}[\delta]+\text{Sin}[\delta])^2\text{Sin}\left[\frac{\alpha_1}{2}\right]^2}{\alpha_1^2} & \frac{\beta(\text{Cos}[\delta]+\text{Sin}[\delta])\text{Sin}[\alpha_1]}{\sqrt{2}\alpha_1} & \text{Cos}\left[\frac{\alpha_1}{2}\right]^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \text{Cos}\left[\frac{\alpha_1}{2}\right]^2 & -\frac{\beta(\text{Cos}[\delta]+\text{Sin}[\delta])\text{Sin}[\alpha_1]}{\sqrt{2}\alpha_1} & \frac{\beta^2(\text{Cos}[\delta]+\text{Sin}[\delta])^2\text{Sin}\left[\frac{\alpha_1}{2}\right]^2}{\alpha_1^2} \\ 0 & 0 & 0 & \frac{\beta(\text{Cos}[\delta]-\text{Sin}[\delta])\text{Sin}[\alpha_1]}{\sqrt{2}\alpha_1} & \text{Cos}[\alpha_1] & -\frac{\beta(\text{Cos}[\delta]+\text{Sin}[\delta])\text{Sin}[\alpha_1]}{\sqrt{2}\alpha_1} \\ 0 & 0 & 0 & \frac{\beta^2(\text{Cos}[\delta]-\text{Sin}[\delta])^2\text{Sin}\left[\frac{\alpha_1}{2}\right]^2}{\alpha_1^2} & \frac{\beta(\text{Cos}[\delta]-\text{Sin}[\delta])\text{Sin}[\alpha_1]}{\sqrt{2}\alpha_1} & \text{Cos}\left[\frac{\alpha_1}{2}\right]^2 \end{pmatrix}$$

$$RB1^{1/2} = \begin{pmatrix} \text{Cos}\left[\frac{\alpha_1}{2}\right] & -\frac{\beta(\text{Cos}[\delta]-\text{Sin}[\delta])\text{Sin}\left[\frac{\alpha_1}{2}\right]}{\alpha_1} & 0 & 0 \\ \frac{\beta(\text{Cos}[\delta]+\text{Sin}[\delta])\text{Sin}\left[\frac{\alpha_1}{2}\right]}{\alpha_1} & \text{Cos}\left[\frac{\alpha_1}{2}\right] & 0 & 0 \\ 0 & 0 & \text{Cos}\left[\frac{\alpha_1}{2}\right] & -\frac{\beta(\text{Cos}[\delta]+\text{Sin}[\delta])\text{Sin}\left[\frac{\alpha_1}{2}\right]}{\alpha_1} \\ 0 & 0 & \frac{\beta(\text{Cos}[\delta]-\text{Sin}[\delta])\text{Sin}\left[\frac{\alpha_1}{2}\right]}{\alpha_1} & \text{Cos}\left[\frac{\alpha_1}{2}\right] \end{pmatrix}$$

Special feature

$$RB1 = e^{i\beta\,\widehat{\mathcal{K}^1}} \qquad\qquad RB2 = e^{i\beta\,\widehat{\mathcal{K}^2}}$$

When $\beta \to 2\pi\sqrt{\sec[2\delta]}$

Spin - 1/2

$$RB1^{1/2} = -\mathrm{I}$$

$$RB2^{1/2} = -\mathrm{I}$$

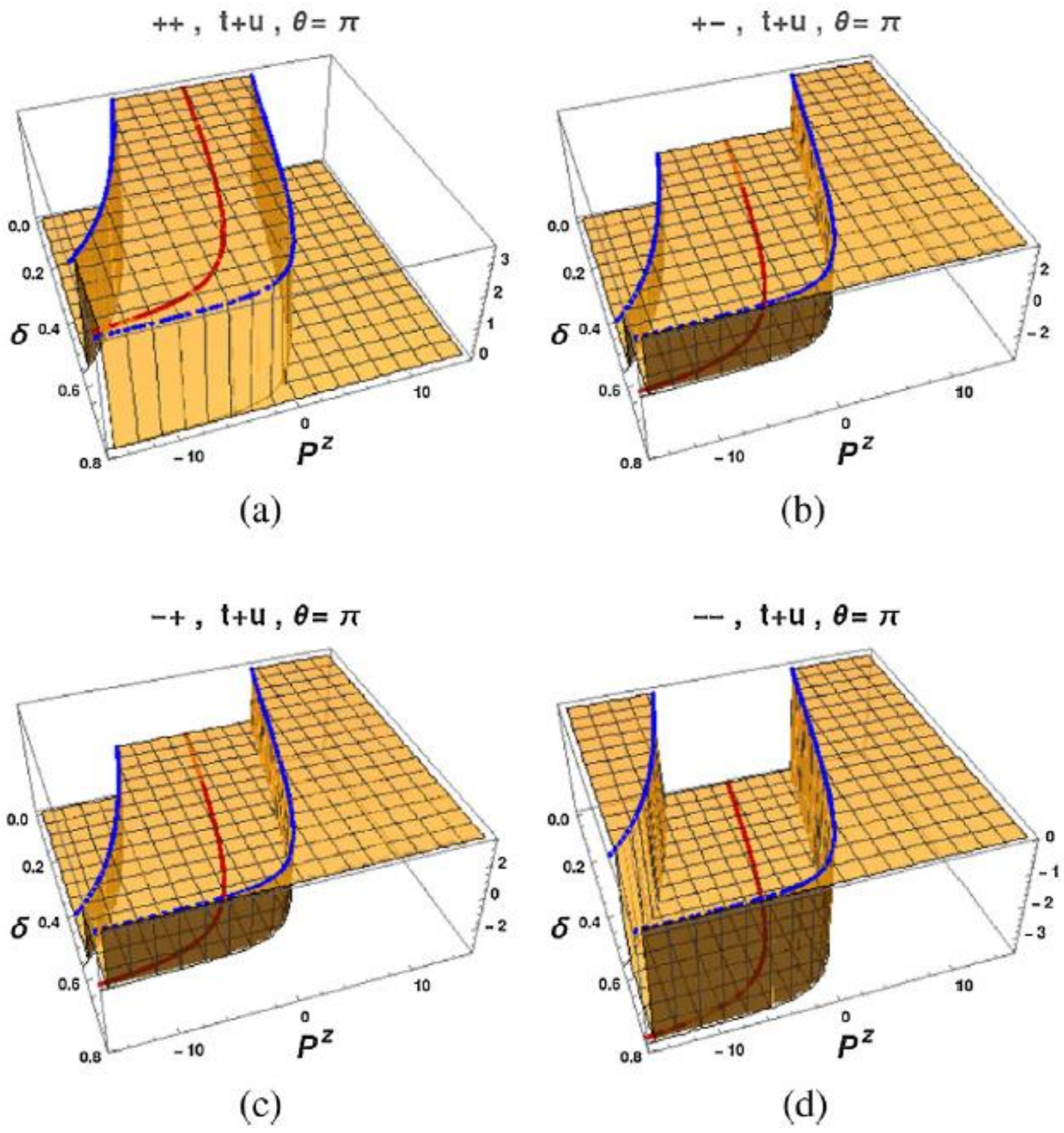
Spin - 1

$$RB1^{1} = \mathrm{I}$$

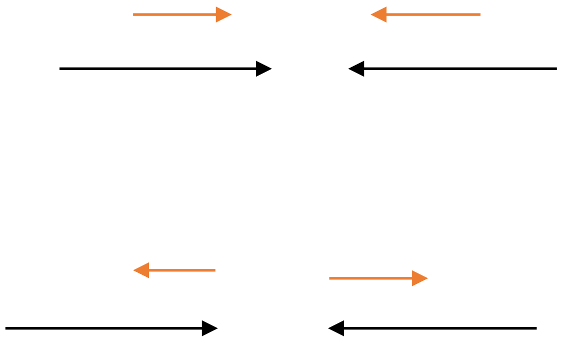
$$RB2^{1} = \mathrm{I}$$

To get the initial state again for the spin-1/2 particle , we have to consider $\beta \to 4\pi\sqrt{\sec[2\delta]}$

- Since this is valid for any δ value this confirm that quantum correlation doesn't depend on δ
- When $\delta \to \frac{\pi}{4}$, we can see $\beta \to \infty$, Since in the light front limit β is no longer an angle.
- It seems like to see the quantum-correlation in the light front limit we have to consider infinite momentum.



$e^+e^- \to SS$



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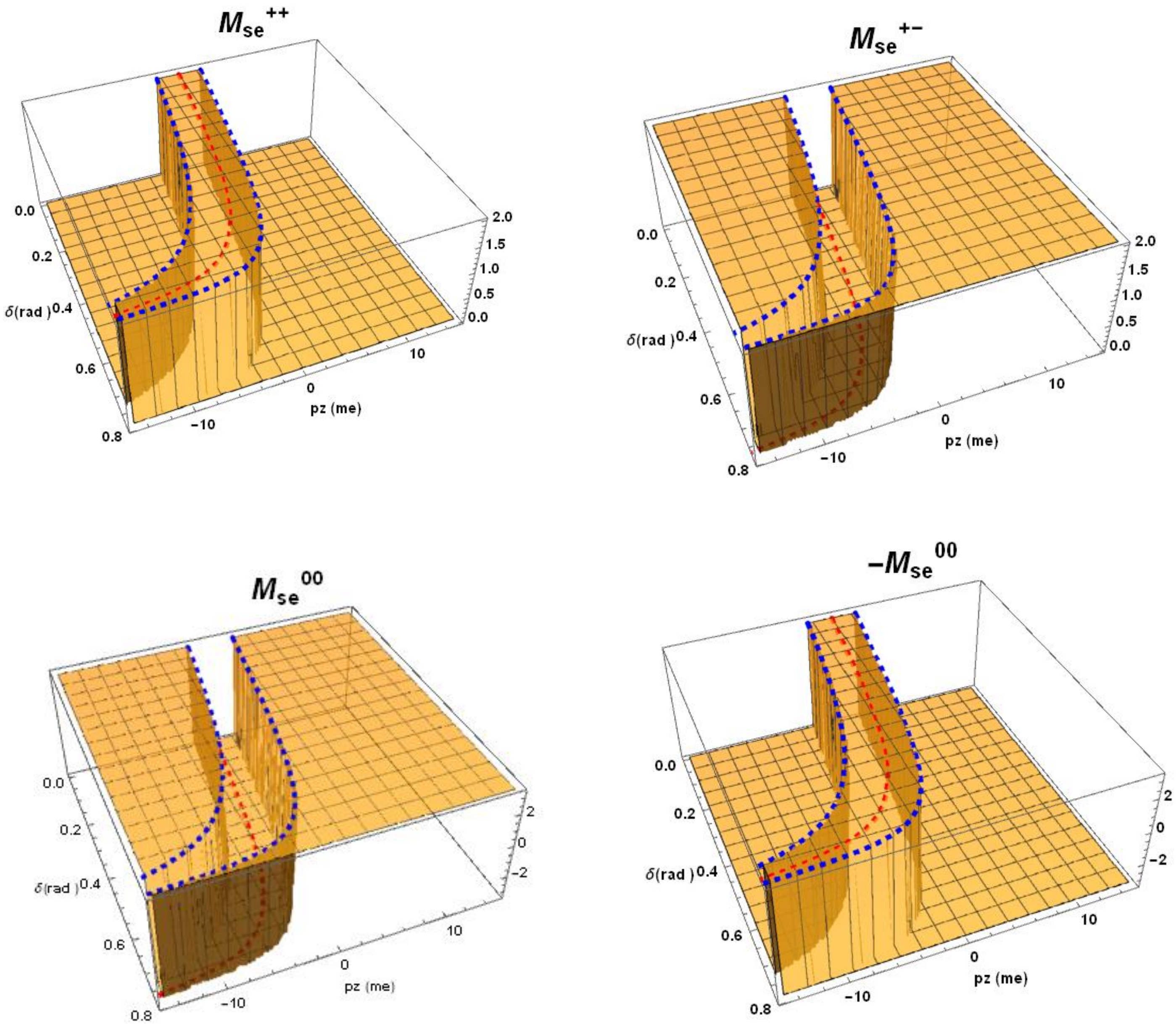


$$VV \rightarrow SS$$

$$Mse^{++} = Mse^{--}$$

$$Mse^{+-} = Mse^{-+}$$

Spin- Orientation for spin-1 spinors

Direction of momentum **P** as (θ, φ) and direction of **S** as (θ_s, φ_s)

We consider this transformation for spin-up spinor

$$T = B(\boldsymbol{\eta})\mathcal{D}(\hat{\mathbf{m}}, \theta_s) = e^{-i\boldsymbol{\eta}\cdot\mathbf{K}} e^{-i\hat{\mathbf{m}}\cdot\mathbf{J}\theta_s},$$

$\mathcal{D}(\hat{\mathbf{m}}, \theta_s) = e^{-i\hat{\mathbf{m}}\cdot\mathbf{J}\theta_s}$ ⟶ Rotates the spin around the axis by a unit vector $\hat{m} = (-Sin(\varphi_s), Cos(\varphi_s), 0)$ by angle θ_s .

$B(\boldsymbol{\eta}) = e^{-i\boldsymbol{\eta}\cdot\mathbf{K}}$ ⟶ Boost the spinor to momentum **P**

In the *(0,1)*⊕ *(1,0)* chiral representation we have

$$\mathcal{D}(\hat{\mathbf{m}}, \theta_s) = e^{-i\hat{\mathbf{m}}\cdot\mathbf{J}\theta_s} = \begin{pmatrix} e^{-i\hat{\mathbf{m}}\cdot\boldsymbol{\sigma}\theta_s} & 0 \\ 0 & e^{-i\hat{\mathbf{m}}\cdot\boldsymbol{\sigma}\theta_s} \end{pmatrix},$$

$$B(\boldsymbol{\eta}) = e^{-i\boldsymbol{\eta}\cdot\mathbf{K}} = \begin{pmatrix} e^{\boldsymbol{\eta}\cdot\boldsymbol{\sigma}} & 0 \\ 0 & e^{-\boldsymbol{\eta}\cdot\boldsymbol{\sigma}} \end{pmatrix}.$$

$\hat{m}.\sigma = a \qquad a^2 = a^4 \qquad a^1 = a^3$

$$e^{-i\hat{m}.\sigma\theta_s} = \mathrm{I} + (a^2)(\mathrm{Cos}[\theta_s] - 1) - ia\mathrm{Sin}[\theta_s]$$

$$\hat{n} = (Sin(\theta)Cos(\varphi), Sin(\theta)Sin(\varphi), Cos(\theta))$$

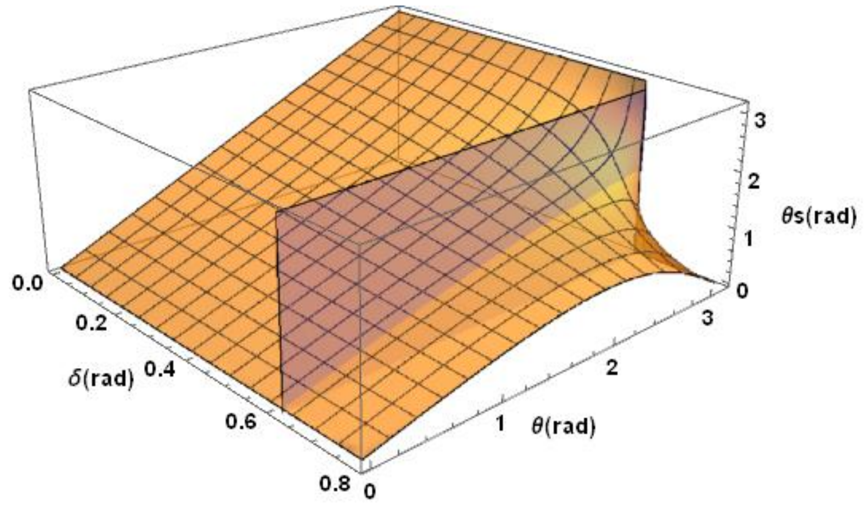
$\hat{n}.\sigma = b \qquad b^2 = b^4 \qquad b^1 = b^3$

$$e^{\pm\eta.\sigma} = \mathrm{I} + (b^2)(\mathrm{Cosh}[\eta] - 1) \pm ib\mathrm{Sinh}[\eta]$$

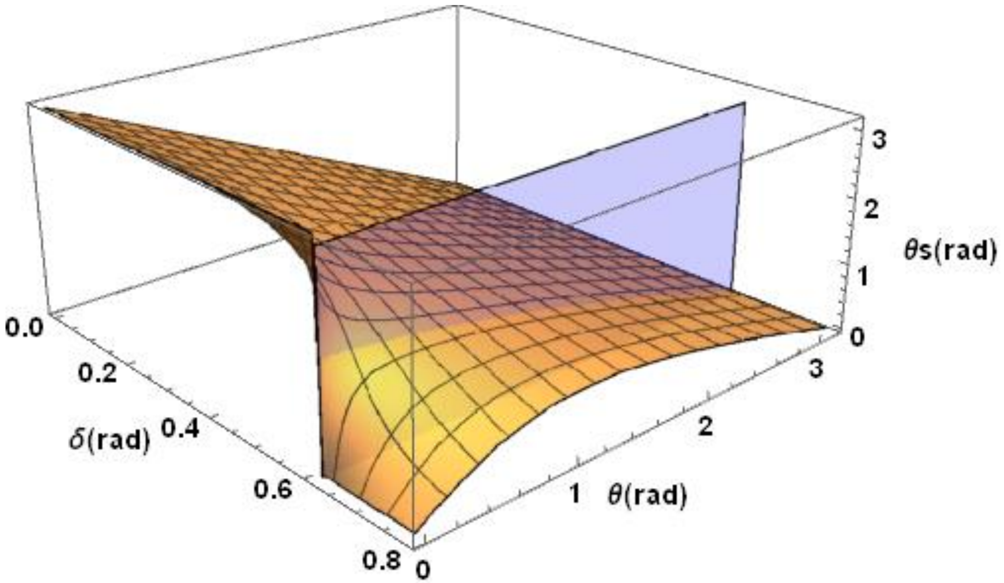
$$\mathrm{Cos}[\theta s] = \frac{\mathrm{Cos}[\alpha] + \mathrm{Cosh}[\beta 3] + \mathrm{Cos}[\alpha]\mathrm{Cosh}[\beta 3] - \mathrm{Cosh}[\eta]}{1 + \mathrm{Cosh}[\eta]}$$

$$\mathrm{Cos}[\phi s] = \frac{\beta 1}{\beta 1^2 + \beta 2^2} \qquad \mathrm{Sin}[\phi s] = \frac{\beta 2}{\beta 1^2 + \beta 2^2}$$

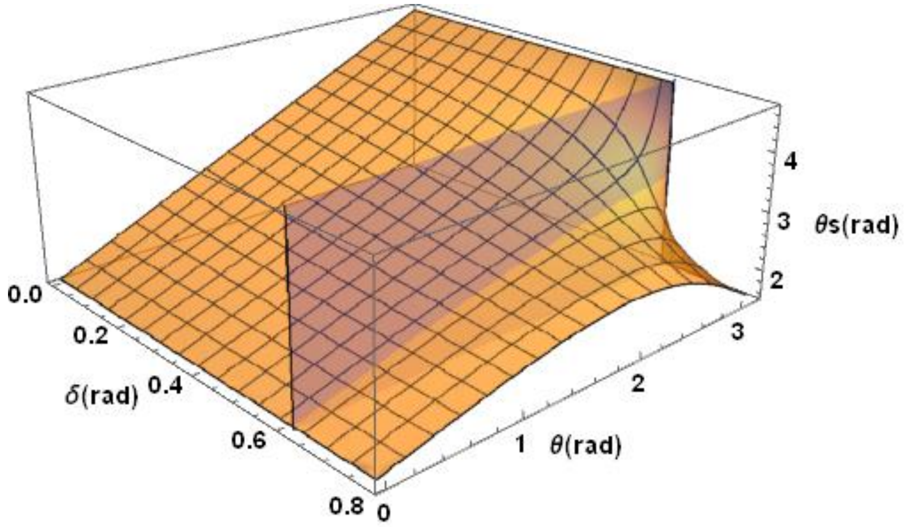
U_H^1



U_H^{-1}



U_H^0



Helicity Photon Polarization Vectors

4- vector representation

Rest Frame

$$\epsilon(\pm) = \mp\frac{1}{\sqrt{2}}(0,1,\pm i,0), \qquad \epsilon(0) = (0,0,0,1)$$

$$K_1 = \begin{pmatrix} 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad K_2 = \begin{pmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad K_3 = \begin{pmatrix} 0 & 0 & 0 & i \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix},$$

$$J_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix}, \quad J_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & i \\ 0 & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{pmatrix}, \quad J_3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

After applying T-transformation

$\epsilon^\mu = (\epsilon^0, \epsilon^1, \epsilon^2, \epsilon^3)$ basis

$$\epsilon^\mu(\beta,+) = \begin{pmatrix} -\frac{(\beta 1 + i\beta 2)\mathrm{Sin}[\alpha]\mathrm{Sin}[\delta]}{\sqrt{2}\alpha} \\ \frac{i\beta 2 - \beta 1\mathrm{Cos}[\alpha]}{\sqrt{2}(\beta 1 - i\beta 2)} \\ \frac{-i\beta 1 - \beta 2\mathrm{Cos}[\alpha]}{\sqrt{2}(\beta 1 - i\beta 2)} \\ \frac{(\beta 1 + i\beta 2)\mathrm{Cos}[\delta]\mathrm{Sin}[\alpha]}{\sqrt{2}\alpha} \end{pmatrix}$$

$$\epsilon^\mu(\beta,-) = \begin{pmatrix} \frac{(\beta 1 - i\beta 2)\mathrm{Sin}[\alpha]\mathrm{Sin}[\delta]}{\sqrt{2}\alpha} \\ \frac{i\beta 2 + \beta 1\mathrm{Cos}[\alpha]}{\sqrt{2}(\beta 1 + i\beta 2)} \\ \frac{-i\beta 1 + \beta 2\mathrm{Cos}[\alpha]}{\sqrt{2}(\beta 1 + i\beta 2)} \\ -\frac{(\beta 1 - i\beta 2)\mathrm{Cos}[\delta]\mathrm{Sin}[\alpha]}{\sqrt{2}\alpha} \end{pmatrix}$$

$$\epsilon^{\mu}(\beta,0) = \begin{pmatrix} \frac{\text{Cos}[\delta](\text{Sin}[\delta]\text{Cosh}[\beta3] + \text{Cos}[\delta]\text{Sinh}[\beta3]) - \text{Sin}[\delta]\text{Sin}[\alpha](\text{Cos}[\delta]\text{Cosh}[\beta3] + \text{Sin}[\delta]\text{Sinh}[\beta3])}{\text{Cos}[2\delta]} \\ \frac{\beta1\text{Sin}[\alpha](\text{Cos}[\delta]\text{Cosh}[\beta3] + \text{Sin}[\delta]\text{Sinh}[\beta3])}{\alpha} \\ \frac{\beta2\text{Sin}[\alpha](\text{Cos}[\delta]\text{Cosh}[\beta3] + \text{Sin}[\delta]\text{Sinh}[\beta3])}{\alpha} \\ \frac{\text{Cos}[\delta]\text{Sin}[\alpha](\text{Cos}[\delta]\text{Cosh}[\beta3] + \text{Sin}[\delta]\text{Sinh}[\beta3]) - \text{Sin}[\delta](\text{Sin}[\delta]\text{Cosh}[\beta3] + \text{Cos}[\delta]\text{Sinh}[\beta3])}{\text{Cos}[2\delta]} \end{pmatrix}$$

Relation between spin-1 spinors and the photon polarization vectors

$$C = \sum_{\lambda} -(U^{\mu}(P,\lambda) \otimes \epsilon^{*}_{\mu}(P,\lambda)$$

$$C = -(U^{\mu}(P,+)\otimes\epsilon^{*}_{\mu}(P,+) -(U^{\mu}(P,-)\otimes\epsilon^{*}_{\mu}(P,-) -(U^{\mu}(P,0)\otimes\epsilon^{*}_{\mu}(P,0)$$

If I multiply C by ϵ^{μ}(P,+)

$U^{\mu}(P,+)$=C $\in^{\mu}$ $(P,+)$

$$\epsilon^{*}(P,\lambda)\cdot\epsilon(P,\lambda') = -\delta_{\lambda\lambda'}$$

$$C = \begin{pmatrix} \frac{P1 - iP2}{\sqrt{2}\sqrt{M}} & -\frac{P0 + P3}{\sqrt{2}\sqrt{M}} & \frac{i(P0 + P3)}{\sqrt{2}\sqrt{M}} & \frac{P1 - iP2}{\sqrt{2}\sqrt{M}} \\ -\frac{P3}{\sqrt{M}} & -\frac{iP2}{\sqrt{M}} & \frac{iP1}{\sqrt{M}} & \frac{P0}{\sqrt{M}} \\ -\frac{P1 + iP2}{\sqrt{2}\sqrt{M}} & \frac{P0 - P3}{\sqrt{2}\sqrt{M}} & \frac{i(P0 - P3)}{\sqrt{2}\sqrt{M}} & \frac{P1 + iP2}{\sqrt{2}\sqrt{M}} \\ \frac{P1 - iP2}{\sqrt{2}\sqrt{M}} & \frac{-P0 + P3}{\sqrt{2}\sqrt{M}} & \frac{i(P0 - P3)}{\sqrt{2}\sqrt{M}} & -\frac{P1 - iP2}{\sqrt{2}\sqrt{M}} \\ -\frac{P3}{\sqrt{M}} & \frac{iP2}{\sqrt{M}} & -\frac{iP1}{\sqrt{M}} & \frac{P0}{\sqrt{M}} \\ -\frac{P1 + iP2}{\sqrt{2}\sqrt{M}} & \frac{P0 + P3}{\sqrt{2}\sqrt{M}} & \frac{i(P0 + P3)}{\sqrt{2}\sqrt{M}} & -\frac{P1 + iP2}{\sqrt{2}\sqrt{M}} \end{pmatrix}$$

Electric and magnetic field tensor and gauge field .

Electric and magnetic field tensor

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}$$

We can see correspondence of boost operators and electric fields $K \leftrightarrow E$ and rotation operators and magnetic fields $J \leftrightarrow B$

Poincare Matrix

$$M^{\mu\nu} = \begin{pmatrix} 0 & K^1 & K^2 & K^3 \\ -K^1 & 0 & J^3 & -J^2 \\ -K^2 & -J^3 & 0 & J^1 \\ -K^3 & J^2 & -J^1 & 0 \end{pmatrix}$$

We can write down a matrix which connects a 4 degrees of freedom polarization vectors in (1/2,1/2) Lorentz group and a six-component spin -1 spinor (1,0)+(0,1)

$$\begin{pmatrix} E_x \\ E_y \\ E_z \\ B_x \\ B_y \\ B_z \end{pmatrix} = -i \begin{pmatrix} p^1 & -p^0 & 0 & 0 \\ p^2 & 0 & -p^0 & 0 \\ p^3 & 0 & 0 & -p^0 \\ 0 & 0 & -p^3 & p^2 \\ 0 & p^3 & 0 & -p^1 \\ 0 & -p^2 & p^1 & 0 \end{pmatrix} \begin{pmatrix} \phi \\ A_x \\ A_y \\ A_z \end{pmatrix}$$

$F^{\mu\nu}$ can be separated in to right-handed and left-handed as $E + iB \, \epsilon \, (1,0)$ and $E - iB \, \epsilon \, (0,1)$

We can find the component of the u- spinor by expressing the electric field and magnetic field in terms of the spherical harmonics +, - ,0

$$u(p) = i\sqrt{2m} \begin{bmatrix} E^- - iB^- \\ -(E^3 - iB^3) \\ -(E^+ - iB^+) \\ E^- + iB^- \\ -(E^3 + iB^3) \\ -(E^+ + iB^+) \end{bmatrix}$$

Where $E^{\pm} = (E^1 \pm iE^2)/\sqrt{2}$

$$B^{\pm} = (B^1 \pm iB^2)/\sqrt{2}$$

$$\ell = 1 \quad \begin{cases} Y_{\ell=1}^{m=-1}(\theta, \phi) = \sqrt{\frac{3}{8\pi}} \sin\theta e^{-i\phi} & \sqrt{\frac{3}{8\pi}} \frac{x - iy}{r} \\ Y_1^0(\theta, \phi) = \sqrt{\frac{3}{4\pi}} \cos\theta & \sqrt{\frac{3}{4\pi}} \frac{z}{r} \\ Y_1^{+1}(\theta, \phi) = -\sqrt{\frac{3}{8\pi}} \sin\theta e^{i\phi} & -\sqrt{\frac{3}{8\pi}} \frac{x + iy}{r} \end{cases}$$

Dirac Spinor : Boosting the initial state at rest that has a spin projection along the z-direction to the state with the desired momentum $\vec{P}$

→ The spin direction of the Dirac spinor is in general not align to the moving direction.

Dirac Photon polarization vectors

$$\epsilon^{\mu}(\mathrm{P},+)=\frac{-1}{m(\mathrm{p0}+m)}\begin{bmatrix}(\mathrm{p0}+m)\,(\mathrm{p1}+i\mathrm{p2})/\sqrt{2}\\(\mathrm{p0}+m)\,m/\sqrt{2}+\mathrm{p1}\,(\mathrm{p1}+i\mathrm{p2})/\sqrt{2}\\i(\mathrm{p0}+m)\,m/\sqrt{2}+\mathrm{p2}\,(\mathrm{p1}+i\mathrm{p2})/\sqrt{2}\\\mathrm{p3}\,(\mathrm{p1}+i\mathrm{p2})/\sqrt{2}\end{bmatrix}$$

$$\epsilon^{\mu}(\mathrm{P},0)=\frac{1}{m(\mathrm{p0}+m)}\begin{bmatrix}(\mathrm{p0}+m)\mathrm{p3}\\\mathrm{p1p3}\\\mathrm{p2p3}\\(\mathrm{p0}+m)m+\mathrm{p3}^2\end{bmatrix}$$

$$\epsilon^{\mu}(\mathrm{P},-)=\frac{1}{m(\mathrm{p0}+m)}\begin{bmatrix}(\mathrm{p0}+m)\,(\mathrm{p1}-i\mathrm{p2})/\sqrt{2}\\(\mathrm{p0}+m)\,m/\sqrt{2}+\mathrm{p1}\,(\mathrm{p1}-i\mathrm{p2})/\sqrt{2}\\-i(\mathrm{p0}+m)\,m/\sqrt{2}+\mathrm{p2}\,(\mathrm{p1}-i\mathrm{p2})/\sqrt{2}\\\mathrm{p3}\,(\mathrm{p1}-i\mathrm{p2})/\sqrt{2}\end{bmatrix}$$

Lorenz Gauge condition satisfied -> $\partial_{\mu}A^{\mu}=0$

Dirac spin-1 spinors in chiral basis

$$U^{\mu}(P,+) = \sqrt{2m}\begin{pmatrix} \frac{(m+\mathrm{p0}+\mathrm{p3})^2}{2\sqrt{2}m(m+\mathrm{p0})} \\ \frac{(\mathrm{p1}+\mathrm{ip2})(m+\mathrm{p0}+\mathrm{p3})}{2m(m+\mathrm{p0})} \\ \frac{(\mathrm{p1}+\mathrm{ip2})^2}{2\sqrt{2}m(m+\mathrm{p0})} \\ \frac{(m+\mathrm{p0}-\mathrm{p3})^2}{2\sqrt{2}m(m+\mathrm{p0})} \\ -\frac{(\mathrm{p1}+\mathrm{ip2})(m+\mathrm{p0}-\mathrm{p3})}{2m(m+\mathrm{p0})} \\ \frac{(\mathrm{p1}+\mathrm{ip2})^2}{2\sqrt{2}m(m+\mathrm{p0})} \end{pmatrix}$$

$$U^{\mu}(P,0) = \sqrt{2m}\begin{pmatrix} \frac{(\mathrm{p1}-i\mathrm{p2})(m+\mathrm{p0}+\mathrm{p3})}{2m(m+\mathrm{p0})} \\ \frac{m\mathrm{p0}+\mathrm{p0}^2-\mathrm{p3}^2}{\sqrt{2}m(m+\mathrm{p0})} \\ \frac{(\mathrm{p1}+i\mathrm{p2})(m+\mathrm{p0}-\mathrm{p3})}{2m(m+\mathrm{p0})} \\ -\frac{(\mathrm{p1}-i\mathrm{p2})(m+\mathrm{p0}-\mathrm{p3})}{2m(m+\mathrm{p0})} \\ \frac{m\mathrm{p0}+\mathrm{p0}^2-\mathrm{p3}^2}{\sqrt{2}m(m+\mathrm{p0})} \\ -\frac{(\mathrm{p1}+i\mathrm{p2})(m+\mathrm{p0}+\mathrm{p3})}{2m(m+\mathrm{p0})} \end{pmatrix}$$

$$U^{\mu}(P,-) = \sqrt{2m}\begin{pmatrix} \frac{(\mathrm{p1}-\mathrm{ip2})^2}{2\sqrt{2}m(m+\mathrm{p0})} \\ \frac{(\mathrm{p1}-\mathrm{ip2})(m+\mathrm{p0}-\mathrm{p3})}{2m(m+\mathrm{p0})} \\ \frac{(m+\mathrm{p0}-\mathrm{p3})^2}{2\sqrt{2}m(m+\mathrm{p0})} \\ \frac{(\mathrm{p1}-\mathrm{ip2})^2}{2\sqrt{2}m(m+\mathrm{p0})} \\ -\frac{(\mathrm{p1}-\mathrm{ip2})(m+\mathrm{p0}+\mathrm{p3})}{2m(m+\mathrm{p0})} \\ \frac{(m+\mathrm{p0}+\mathrm{p3})^2}{2\sqrt{2}m(m+\mathrm{p0})} \end{pmatrix}$$

We change the basis of photon polarization vectors

$$\begin{pmatrix} x^{\hat{+}} \\ x^{\hat{1}} \\ x^{\hat{2}} \\ x^{\hat{-}} \end{pmatrix} = \begin{pmatrix} \cos\delta & 0 & 0 & \sin\delta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \sin\delta & 0 & 0 & -\cos\delta \end{pmatrix} \begin{pmatrix} x^{0} \\ x^{1} \\ x^{2} \\ x^{3} \end{pmatrix}$$

$$\begin{bmatrix} U^1 \\ U^2 \\ U^3 \\ U^4 \\ U^5 \\ U^6 \end{bmatrix} = \begin{pmatrix} \frac{(\mathrm{P1}-i\mathrm{P2})(\mathrm{Cos}[\delta]+\mathrm{Sin}[\delta])}{\sqrt{2}\sqrt{M}} & -\frac{\mathrm{P0}+\mathrm{P3}}{\sqrt{2}\sqrt{M}} & \frac{i(\mathrm{P0}+\mathrm{P3})}{\sqrt{2}\sqrt{M}} & -\frac{(\mathrm{P1}-i\mathrm{P2})(\mathrm{Cos}[\delta]-\mathrm{Sin}[\delta])}{\sqrt{2}\sqrt{M}} \\ \frac{-\mathrm{P3}\mathrm{Cos}[\delta]+\mathrm{P0}\mathrm{Sin}[\delta]}{\sqrt{M}} & -\frac{i\mathrm{P2}}{\sqrt{M}} & \frac{i\mathrm{P1}}{\sqrt{M}} & -\frac{\mathrm{P0}\mathrm{Cos}[\delta]+\mathrm{P3}\mathrm{Sin}[\delta]}{\sqrt{M}} \\ -\frac{(\mathrm{P1}+i\mathrm{P2})(\mathrm{Cos}[\delta]-\mathrm{Sin}[\delta])}{\sqrt{2}\sqrt{M}} & \frac{\mathrm{P0}-\mathrm{P3}}{\sqrt{2}\sqrt{M}} & \frac{i(\mathrm{P0}-\mathrm{P3})}{\sqrt{2}\sqrt{M}} & -\frac{(\mathrm{P1}+i\mathrm{P2})(\mathrm{Cos}[\delta]+\mathrm{Sin}[\delta])}{\sqrt{2}\sqrt{M}} \\ \frac{(\mathrm{P1}-i\mathrm{P2})(\mathrm{Cos}[\delta]-\mathrm{Sin}[\delta])}{\sqrt{2}\sqrt{M}} & \frac{-\mathrm{P0}+\mathrm{P3}}{\sqrt{2}\sqrt{M}} & \frac{i(\mathrm{P0}-\mathrm{P3})}{\sqrt{2}\sqrt{M}} & \frac{(\mathrm{P1}-i\mathrm{P2})(\mathrm{Cos}[\delta]+\mathrm{Sin}[\delta])}{\sqrt{2}\sqrt{M}} \\ \frac{-\mathrm{P3}\mathrm{Cos}[\delta]+\mathrm{P0}\mathrm{Sin}[\delta]}{\sqrt{M}} & \frac{i\mathrm{P2}}{\sqrt{M}} & -\frac{i\mathrm{P1}}{\sqrt{M}} & -\frac{\mathrm{P0}\mathrm{Cos}[\delta]+\mathrm{P3}\mathrm{Sin}[\delta]}{\sqrt{M}} \\ -\frac{(\mathrm{P1}+i\mathrm{P2})(\mathrm{Cos}[\delta]+\mathrm{Sin}[\delta])}{\sqrt{2}\sqrt{M}} & \frac{\mathrm{P0}+\mathrm{P3}}{\sqrt{2}\sqrt{M}} & \frac{i(\mathrm{P0}+\mathrm{P3})}{\sqrt{2}\sqrt{M}} & \frac{(\mathrm{P1}+i\mathrm{P2})(\mathrm{Cos}[\delta]-\mathrm{Sin}[\delta])}{\sqrt{2}\sqrt{M}} \end{pmatrix} \begin{bmatrix} \varepsilon^{\hat{+}} \\ \varepsilon^{\hat{1}} \\ \varepsilon^{\hat{2}} \\ \varepsilon^{\hat{-}} \end{bmatrix}$$

$$\begin{aligned} E^{\hat{1}} &= K^1 \cos\delta + J^2 \sin\delta, \\ E^{\hat{2}} &= K^2 \cos\delta - J^1 \sin\delta, \\ F^{\hat{1}} &= K^1 \sin\delta - J^2 \cos\delta, \\ F^{\hat{2}} &= K^2 \sin\delta + J^1 \cos\delta, \end{aligned}$$

$$M^{\hat{\mu}\hat{\nu}} = \mathcal{R}^{\hat{\mu}}_{\alpha} M^{\alpha\beta} \mathcal{R}^{\hat{\nu}}_{\beta} = \begin{pmatrix} 0 & E^{\hat{1}} & E^{\hat{2}} & -K^3 \\ -E^{\hat{1}} & 0 & J^3 & -F^{\hat{1}} \\ -E^{\hat{2}} & -J^3 & 0 & -F^{\hat{2}} \\ K^3 & F^{\hat{1}} & F^{\hat{2}} & 0 \end{pmatrix}$$

$$F^{\hat{\mu}\hat{\nu}} = \mathcal{R}^{\hat{\mu}}{}_{\alpha} F^{\alpha\beta} \mathcal{R}^{\hat{\nu}}{}_{\beta}$$

$$C = \begin{pmatrix} \frac{P1 - iP2}{\sqrt{2}\sqrt{M}} & -\frac{P0 + P3}{\sqrt{2}\sqrt{M}} & \frac{i(P0 + P3)}{\sqrt{2}\sqrt{M}} & \frac{P1 - iP2}{\sqrt{2}\sqrt{M}} \\ -\frac{P3}{\sqrt{M}} & -\frac{iP2}{\sqrt{M}} & \frac{iP1}{\sqrt{M}} & \frac{P0}{\sqrt{M}} \\ -\frac{P1 + iP2}{\sqrt{2}\sqrt{M}} & \frac{P0 - P3}{\sqrt{2}\sqrt{M}} & \frac{i(P0 - P3)}{\sqrt{2}\sqrt{M}} & \frac{P1 + iP2}{\sqrt{2}\sqrt{M}} \\ \frac{P1 - iP2}{\sqrt{2}\sqrt{M}} & \frac{-P0 + P3}{\sqrt{2}\sqrt{M}} & \frac{i(P0 - P3)}{\sqrt{2}\sqrt{M}} & -\frac{P1 - iP2}{\sqrt{2}\sqrt{M}} \\ -\frac{P3}{\sqrt{M}} & \frac{iP2}{\sqrt{M}} & -\frac{iP1}{\sqrt{M}} & \frac{P0}{\sqrt{M}} \\ -\frac{P1 + iP2}{\sqrt{2}\sqrt{M}} & \frac{P0 + P3}{\sqrt{2}\sqrt{M}} & \frac{i(P0 + P3)}{\sqrt{2}\sqrt{M}} & -\frac{P1 + iP2}{\sqrt{2}\sqrt{M}} \end{pmatrix}$$

$$\begin{pmatrix} x^{\hat{+}} \\ x^{\hat{1}} \\ x^{\hat{2}} \\ x^{\hat{-}} \end{pmatrix} = \begin{pmatrix} \cos\delta & 0 & 0 & \sin\delta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \sin\delta & 0 & 0 & -\cos\delta \end{pmatrix} \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}$$

$$\begin{bmatrix} U^1 \\ U^2 \\ U^3 \\ U^4 \\ U^5 \\ U^6 \end{bmatrix} = \begin{pmatrix} \frac{(P1 - iP2)(\mathrm{Cos}[\delta] + \mathrm{Sin}[\delta])}{\sqrt{2}\sqrt{M}} & -\frac{P0 + P3}{\sqrt{2}\sqrt{M}} & \frac{i(P0 + P3)}{\sqrt{2}\sqrt{M}} & -\frac{(P1 - iP2)(\mathrm{Cos}[\delta] - \mathrm{Sin}[\delta])}{\sqrt{2}\sqrt{M}} \\ \frac{-P3\mathrm{Cos}[\delta] + P0\mathrm{Sin}[\delta]}{\sqrt{M}} & -\frac{iP2}{\sqrt{M}} & \frac{iP1}{\sqrt{M}} & -\frac{P0\mathrm{Cos}[\delta] + P3\mathrm{Sin}[\delta]}{\sqrt{M}} \\ -\frac{(P1 + iP2)(\mathrm{Cos}[\delta] - \mathrm{Sin}[\delta])}{\sqrt{2}\sqrt{M}} & \frac{P0 - P3}{\sqrt{2}\sqrt{M}} & \frac{i(P0 - P3)}{\sqrt{2}\sqrt{M}} & -\frac{(P1 + iP2)(\mathrm{Cos}[\delta] + \mathrm{Sin}[\delta])}{\sqrt{2}\sqrt{M}} \\ \frac{(P1 - iP2)(\mathrm{Cos}[\delta] - \mathrm{Sin}[\delta])}{\sqrt{2}\sqrt{M}} & \frac{-P0 + P3}{\sqrt{2}\sqrt{M}} & \frac{i(P0 - P3)}{\sqrt{2}\sqrt{M}} & \frac{(P1 - iP2)(\mathrm{Cos}[\delta] + \mathrm{Sin}[\delta])}{\sqrt{2}\sqrt{M}} \\ \frac{-P3\mathrm{Cos}[\delta] + P0\mathrm{Sin}[\delta]}{\sqrt{M}} & \frac{iP2}{\sqrt{M}} & -\frac{iP1}{\sqrt{M}} & -\frac{P0\mathrm{Cos}[\delta] + P3\mathrm{Sin}[\delta]}{\sqrt{M}} \\ -\frac{(P1 + iP2)(\mathrm{Cos}[\delta] + \mathrm{Sin}[\delta])}{\sqrt{2}\sqrt{M}} & \frac{P0 + P3}{\sqrt{2}\sqrt{M}} & \frac{i(P0 + P3)}{\sqrt{2}\sqrt{M}} & \frac{(P1 + iP2)(\mathrm{Cos}[\delta] - \mathrm{Sin}[\delta])}{\sqrt{2}\sqrt{M}} \end{pmatrix} \begin{bmatrix} \varepsilon^{\hat{+}} \\ \varepsilon^{\hat{1}} \\ \varepsilon^{\hat{2}} \\ \varepsilon^{\hat{-}} \end{bmatrix}$$

Conclusion

- We confirm that the quantum correlation (QC) of spin can be seen in the spinors too.
- To see the quantum correlation in the light from , we should consider infinite momentum of the particle.
- We show the QC of spin-1/2 spinors in the helicity amplitudes of scattering process
- We found the critical interpolation of spin-1 spinor by investigating the spin orientation.
- We were able to connect the spn-1 helicity spinors to the helicity photon polarization vector.

Thank you