Scalar Particle and anti-particle annihilation and creating two bosons and its reverse process.

Analogue

$"e^+e^-" \rightarrow \gamma\gamma / "\pi^+\pi^-" \rightarrow \rho^0\rho^0$

$\gamma\gamma" \rightarrow e^+e^-" / \rho^0\rho^0 \rightarrow "\pi^+\pi^-"$

Deepasika Dayananda

06-19-2020

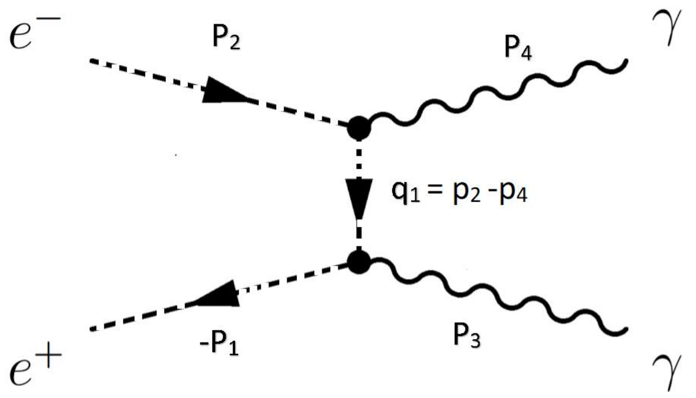
OUTLINE

- Feynman Diagrams and amplitudes
- Interpolation gauge polarization vectors and kinematics dependence
- Symmetries between helicity amplitudes
- Wigner d-Function for spin 1 particles
- Helicity and phase change
- Cross section
- Time orders
- Critical annihilation angle
- Different way of calculating

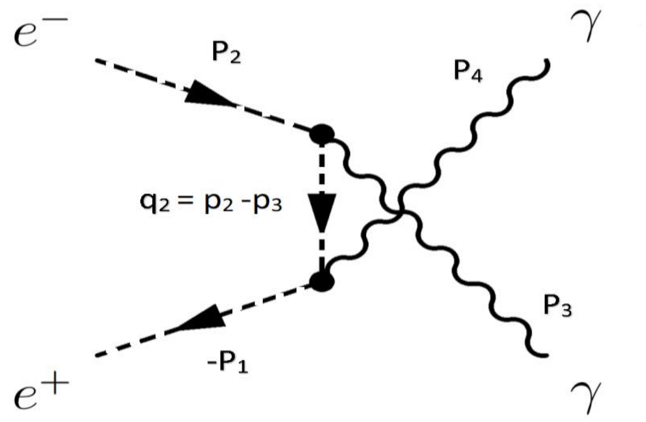
Lowest –order Covariant Feynman diagrams

"e^+e^-" $\rightarrow \gamma\gamma$/ “$\pi^+\pi^-$ ” $\rightarrow$ $\rho^0\rho^0$

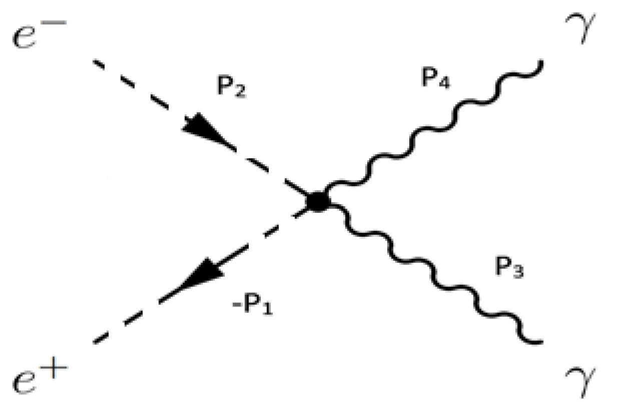
t-Channel



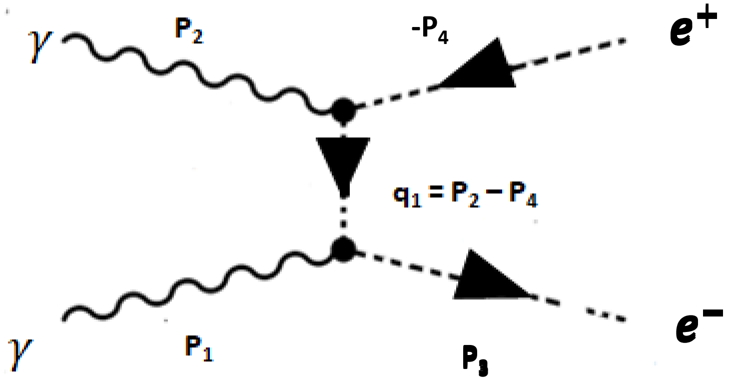
u-Channel

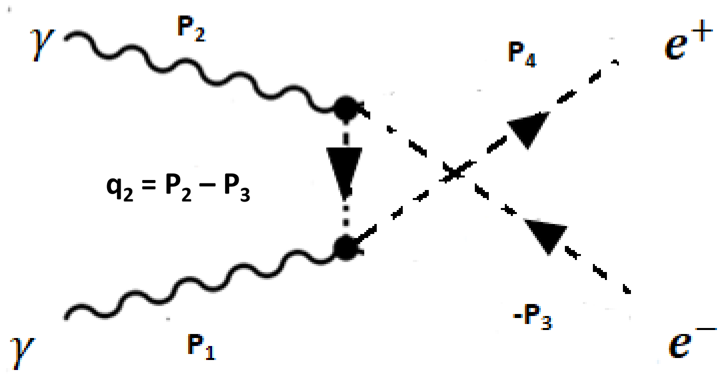


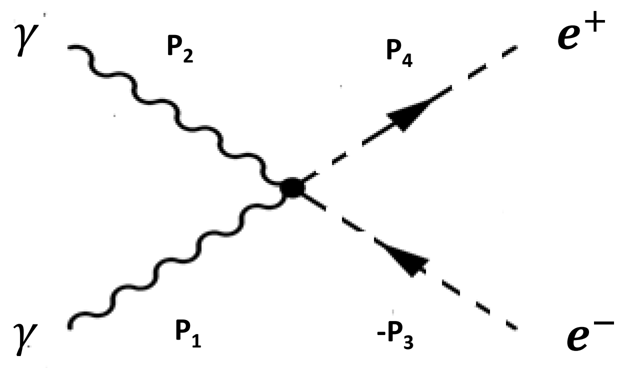
Seagull



$\gamma\gamma \rightarrow$ "$e^+ \ e^-$" $/\rho^0\rho^0 \rightarrow$ “$\pi^+\pi^-$ ”







$"e^+e^-" \rightarrow \gamma\gamma / “\pi^+\pi^- " \rightarrow \rho^0\rho^0$

$$M_t = (-p_1 + q_1)^\mu \varepsilon_\mu^*(p_3, \lambda_3) \frac{1}{q_1^2 - m^2} (p_2 + q_1)^\nu \varepsilon_\nu^*(p_4, \lambda_4)$$

$$M_u = (-p_1 + q_2)^\nu \varepsilon_\nu^*(p_4, \lambda_4) \frac{1}{q_2^2 - m^2} (p_2 + q_2)^\mu \varepsilon_\mu^*(p_3, \lambda_3)$$

$$M_{se} = -2 g_{\mu\nu} \epsilon^{*\mu}(p_3, \lambda_3) \varepsilon^{*\nu}(p_4, \lambda_4)$$

$\gamma\gamma \rightarrow "e^+ e^-" / \rho^0\rho^0 \rightarrow “\pi^+\pi^- "$

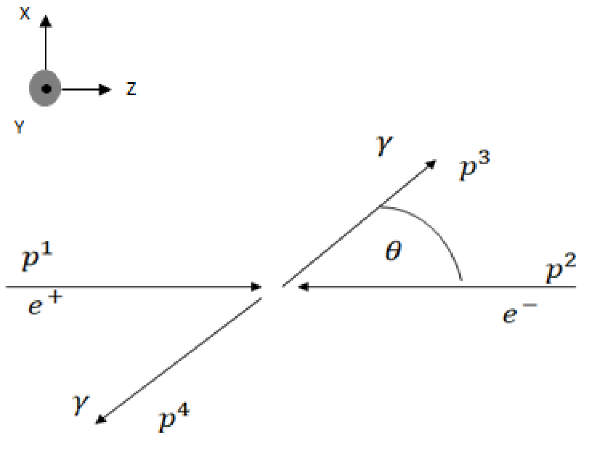
$$M_t = (p_3 + q_1)^\mu \varepsilon_\mu(p_1, \lambda_1) \frac{1}{q_1^2 - m^2} (-p_4 + q_1)^\nu \varepsilon_\nu(p_2, \lambda_2)$$

$$M_u = (-p_3 + q_2)^\nu \varepsilon_\nu(p_2, \lambda_2) \frac{1}{q_2^2 - m^2} (p_4 + q_2)^\mu \varepsilon_\mu(p_1, \lambda_1)$$

$$M_{se} = -2 g_{\mu\nu} \varepsilon^\mu (p_1, \lambda_1) \varepsilon^\nu (p_2, \lambda_2)$$

Center of mass kinematics

$"e^+e^-" \rightarrow \gamma\gamma / "\pi^+\pi^-" \rightarrow \rho^0\rho^0$



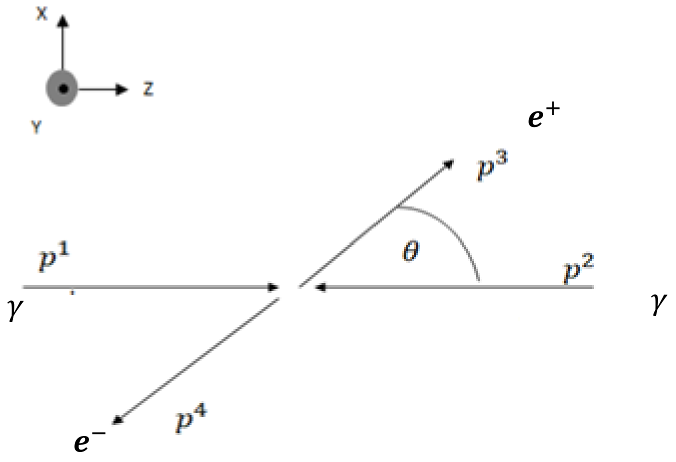
$p^1 = \{E_0, 0,0, P_e\}$

$p^2 = \{E_0, 0,0, -P_e\}$

$p^3 = \{E_0, P_\gamma Sin(\theta), 0, P_\gamma Cos(\theta)\}$

$p^4 = \{E_0, -P_\gamma Sin(\theta), 0, -P_\gamma Cos(\theta)\}$

$\gamma\gamma \rightarrow "e^+ e^-" / \rho^0\rho^0 \rightarrow "\pi^+\pi^-"$



$p^1 = \{E_0, 0,0, P_\gamma\}$

$p^2 = \{E_0, 0,0, -P_\gamma\}$

$p^3 = \{E_0, P_e Sin(\theta), 0, P_e Cos(\theta)\}$

$p^4 = \{E_0, -P_e Sin(\theta), 0, -P_e Cos(\theta)\}$

Lorentz Transformation

$E = \sqrt{4E_0^2 + P_z^2} \qquad \alpha = \frac{E}{4E_0} \qquad \alpha\beta = \frac{P_z}{4E_0}$

$$p_i'^0 = \alpha p_i^0 + \alpha\beta p_i^z$$

$$p_i'^z = \alpha p_i^z + \alpha\beta p_i^0$$

$$p_i'^\perp = p_i^\perp$$

The interpolating photon polarization vectors

$$\epsilon_{\widehat{\mu}}(P,+) = -\frac{1}{\sqrt{2}\,\mathbb{P}}\ (\mathbb{S}|\boldsymbol{p}_{\perp}|, \frac{P_1 P_{\widehat{-}} - iP_2\mathbb{P}}{|\mathbf{p}_{\perp}|}, \frac{P_2 P_{\widehat{-}} + iP_1\mathbb{P}}{|\mathbf{p}_{\perp}|}, \text{-}\ \mathbb{C}\ |\boldsymbol{p}_{\perp}|)$$

$$\epsilon_{\widehat{\mu}}(P,-) = \frac{1}{\sqrt{2}\,\mathbb{P}}\ (\mathbb{S}|\boldsymbol{p}_{\perp}|, \frac{P_1 P_{\widehat{-}} + iP_2\mathbb{P}}{|\mathbf{p}_{\perp}|}, \frac{P_2 P_{\widehat{-}} - iP_1\mathbb{P}}{|\mathbf{p}_{\perp}|}, \text{-}\ \mathbb{C}\ |\boldsymbol{p}_{\perp}|)$$

$$\epsilon_{\widehat{\mu}}(P,0) = \frac{P^{\widehat{+}}}{m_\gamma \mathbb{P}}\ (P_{\widehat{+}} - \frac{m_\gamma^2}{P^{\widehat{+}}}, P_1, P_2, P_{\widehat{-}})$$

Constraints

$$\epsilon_{\widehat{\mu}}(p,\lambda)p^{\widehat{\mu}} = 0$$

$$\epsilon^*(p,\lambda).\,\epsilon(p,\lambda'\,) = -\delta_{\lambda\lambda'}$$

Where $\mathbb{S} = Sin(2\delta)$

$\mathbb{C} = Cos(2\delta)$

$$\mathbb{P} = \sqrt{P_{\widehat{-}}{}^2 + \mathbb{C}\mathbf{p}_{\perp}{}^2} = \sqrt{(P^{\widehat{+}})^2 - \mathbb{C}m_\gamma{}^2}$$

$$|\mathbf{p}_{\perp}| = \sqrt{P_1^2 + P_2^2}$$

C-R ji,Ziyue Li and A.T.Suzuki ,Phy Rev D 91 06065020 (2014)

Polarization vectors in the rest frame $(\epsilon^0, \epsilon^1, \epsilon^2, \epsilon^3)$.

Helicity transformation matrix

$$\epsilon(\pm) = \mp \frac{1}{\sqrt{2}}(0, 1, \pm i, 0), \qquad \epsilon(0) = (0, 0, 0, 1),$$

$$T = T_{12} T_3 = e^{i\beta_1 \mathcal{K}^{\hat{1}} + i\beta_2 \mathcal{K}^{\hat{2}}} e^{-i\beta_3 K^3},$$

$$\epsilon_{\hat{\mu}}(P,+) = -\begin{bmatrix} \frac{\sin\delta}{\sqrt{2}}\left(\frac{\beta_1 \sin\alpha}{\alpha} - \frac{i\beta_2 \sin\alpha}{\alpha}\right) \\ \frac{\mathbb{C}}{\sqrt{2}}\left(\frac{\beta_2^2 + \beta_1^2 \cos\alpha}{\alpha^2} + \frac{i\beta_1\beta_2(-1+\cos\alpha)}{\alpha^2}\right) \\ \frac{\mathbb{C}}{\sqrt{2}}\left(\frac{\beta_1\beta_2(-1+\cos\alpha)}{\alpha^2} + \frac{i(\beta_1^2 + \beta_2^2 \cos\alpha)}{\alpha^2}\right) \\ -\frac{\cos\delta}{\sqrt{2}}\left(\frac{\beta_1 \sin\alpha}{\alpha} + \frac{i\beta_2 \sin\alpha}{\alpha}\right) \end{bmatrix}$$

$$\epsilon_{\hat{\mu}}(P,-) = \begin{bmatrix} \frac{\sin\delta}{\sqrt{2}}\left(\frac{\beta_1 \sin\alpha}{\alpha} + \frac{i\beta_2 \sin\alpha}{\alpha}\right) \\ \frac{\mathbb{C}}{\sqrt{2}}\left(\frac{\beta_2^2 + \beta_1^2 \cos\alpha}{\alpha^2} - \frac{i\beta_1\beta_2(-1+\cos\alpha)}{\alpha^2}\right) \\ \frac{\mathbb{C}}{\sqrt{2}}\left(\frac{\beta_1\beta_2(-1+\cos\alpha)}{\alpha^2} - \frac{i(\beta_1^2 + \beta_2^2 \cos\alpha)}{\alpha^2}\right) \\ -\frac{\cos\delta}{\sqrt{2}}\left(\frac{\beta_1 \sin\alpha}{\alpha} - \frac{i\beta_2 \sin\alpha}{\alpha}\right) \end{bmatrix}$$

$$\alpha = \sqrt{\mathbb{C}(\beta_1^2 + \beta_2^2)}$$

$$\frac{\beta_j}{\alpha} = \frac{P^j}{\sqrt{\mathbf{P}_\perp^2 \mathbb{C}}}, \quad (j = 1, 2),$$

$$\sin\alpha = \frac{\sqrt{\mathbf{P}_\perp^2 \mathbb{C}}}{\mathbb{P}},$$

$$\cos\alpha = \frac{P_{\hat{-}}}{\mathbb{P}},$$

$$
\epsilon_{\hat{\mu}}(P,0) = \begin{bmatrix} \frac{\cos\delta(\sin\delta\cosh\beta_3 + \cos\delta\sinh\beta_3) - \sin\delta\cos\alpha(\cos\delta\cosh\beta_3 + \sin\delta\sinh\beta_3)}{\mathbb{C}} \\ \frac{\beta_1 \sin\alpha(\sin\delta\cosh\beta_3 + \cos\delta\sinh\beta_3)}{\alpha} \\ \frac{\beta_2 \sin\alpha(\sin\delta\cosh\beta_3 + \cos\delta\sinh\beta_3)}{\alpha} \\ \frac{\cos\delta\cos\alpha(\cos\delta\cosh\beta_3 + \sin\delta\sinh\beta_3) - \sin\delta(\sin\delta\cosh\beta_3 + \cos\delta\sinh\beta_3)}{\mathbb{C}} \end{bmatrix}
$$

$$
\sin\delta\cosh\beta_3 + \cos\delta\sinh\beta_3 = \frac{\mathbb{P}}{M},
$$

$$
\cos\delta\cosh\beta_3 + \sin\delta\sinh\beta_3 = \frac{P^{\hat{+}}}{M},
$$

PHYSICAL REVIEW D **91,** 065020 (2015)

Electromagnetic gauge field interpolation between the instant form and the front form of the Hamiltonian dynamics

$$\epsilon_{\hat{\mu}}(P,+) = -\frac{1}{\sqrt{2}\mathbb{P}}\left(\sin\delta|\mathbf{P}_\perp|, \frac{P^1 P_{\hat{-}} - iP^2\mathbb{P}}{|\mathbf{P}_\perp|}, \frac{P^2 P_{\hat{-}} + iP^1\mathbb{P}}{|\mathbf{P}_\perp|}, -\cos\delta|\mathbf{P}_\perp|\right), \tag{A4}$$

$$\epsilon_{\hat{\mu}}(P,-) = \frac{1}{\sqrt{2}\mathbb{P}}\left(\sin\delta|\mathbf{P}_\perp|, \frac{P^1 P_{\hat{-}} + iP^2\mathbb{P}}{|\mathbf{P}_\perp|}, \frac{P^2 P_{\hat{-}} - iP^1\mathbb{P}}{|\mathbf{P}_\perp|}, -\cos\delta|\mathbf{P}_\perp|\right), \tag{A5}$$

$$\epsilon_{\hat{\mu}}(P,0) = \frac{1}{M\mathbb{P}}\left(\frac{\mathbb{P}^2\cos\delta - P_{\hat{-}}P^{\hat{+}}\sin\delta}{\mathbb{C}}, P^1 P^{\hat{+}}, P^2 P^{\hat{+}}, \frac{P^{\hat{+}}P_{\hat{-}}\cos\delta - \mathbb{P}^2\sin\delta}{\mathbb{C}}\right). \tag{A6}$$

The polarization vectors listed above are written in the form $\epsilon_{\hat{\mu}}(P,\lambda) = (\epsilon^0, \epsilon^1, \epsilon^2, \epsilon^3)$. We then change the basis to $(\epsilon_{\hat{+}}, \epsilon_1, \epsilon_2, \epsilon_{\hat{-}})$ using the relations listed in Eq. (7). Finally, we obtain the polarization vectors given by Eq. (22),

$$\epsilon_{\hat{\mu}}(P,+) = -\frac{1}{\sqrt{2}\mathbb{P}}\left(\mathbb{S}|\mathbf{P}_\perp|, \frac{P_1 P_{\hat{-}} - iP_2\mathbb{P}}{|\mathbf{P}_\perp|}, \frac{P_2 P_{\hat{-}} + iP_1\mathbb{P}}{|\mathbf{P}_\perp|}, -\mathbb{C}|\mathbf{P}_\perp|\right), \tag{A7a}$$

$$\epsilon_{\hat{\mu}}(P,-) = \frac{1}{\sqrt{2}\mathbb{P}}\left(\mathbb{S}|\mathbf{P}_\perp|, \frac{P_1 P_{\hat{-}} + iP_2\mathbb{P}}{|\mathbf{P}_\perp|}, \frac{P_2 P_{\hat{-}} - iP_1\mathbb{P}}{|\mathbf{P}_\perp|}, -\mathbb{C}|\mathbf{P}_\perp|\right), \tag{A7b}$$

$$\epsilon_{\hat{\mu}}(P,0) = \frac{P^{\hat{+}}}{M\mathbb{P}}\left(P_{\hat{+}} - \frac{M^2}{P^{\hat{+}}}, P_1, P_2, P_{\hat{-}}\right), \tag{A7c}$$

: $(\epsilon^0, \epsilon^1, \epsilon^2, \epsilon^3)$.

Polarization vectors move in $\pm$z-directions only

+z-direction= rest frame

$$\epsilon_{\hat{\mu}}(P,\pm) = \mp \frac{1}{\sqrt{2}}\left(0, \frac{P_{\hat{-}}}{|P_{\hat{-}}|}, \pm i \frac{P_{\hat{-}}}{|P_{\hat{-}}|}, 0\right) \longrightarrow \epsilon_{\hat{\mu}}(P,\pm) = \mp \frac{1}{\sqrt{2}}(0,1,\pm i10)$$

$$\epsilon_{\hat{\mu}}(P,0) = \frac{1}{\mathbb{C} m_\gamma |P_{\hat{-}}|}\left(|P_{\hat{-}}|^2 Cos\delta - P_{\hat{-}}P^{\hat{+}} Sin\delta, 0, 0, P_{\hat{-}}P^{\hat{+}} Cos\delta - |P_{\hat{-}}|^2 Sin\delta\right)$$

$$\epsilon_{\hat{\mu}}(P,\lambda) = (\epsilon_{\hat{+}}, \epsilon_1, \epsilon_2, \epsilon_{\hat{-}})$$

$$\begin{pmatrix} x_{\hat{+}} \\ x_{\hat{1}} \\ x_{\hat{2}} \\ x_{\hat{-}} \end{pmatrix} = \begin{pmatrix} \cos\delta & 0 & 0 & -\sin\delta \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \sin\delta & 0 & 0 & \cos\delta \end{pmatrix} \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}.$$

$$\epsilon_{\hat{\mu}}(P,\pm) = \pm \frac{1}{\sqrt{2}}\left(0, \frac{P_{\hat{-}}}{|P_{\hat{-}}|}, \pm i \frac{P_{\hat{-}}}{|P_{\hat{-}}|}, 0\right)$$

$$\epsilon_{\hat{\mu}}(P,0) = \frac{P^{\hat{+}}}{m_\gamma |P_{\hat{-}}|}\left(P_{\hat{+}} - \frac{m_\gamma^2}{P^{\hat{+}}}, 0, 0, P_{\hat{-}}\right)$$

Observe symmetries between covariant Helicity amplitudes

- Helicity amplitudes satisfy symmetry based on parity conservation

$$H(-s', -h', -s, -h) = (-1)^{s'+h'-s-h} H(s', h', s, h)$$

$$H(-h', -h) = (-1)^{h'-h} H(h', h)$$

Mt + + = Mt - -

Mu + + = Mu - -

Mse + + = Mse - -

Mt + - = Mt - +

Mu + - = Mu - +

Mse + - = Mse - +

Mt 0 + = -(Mt 0 -)

Mu 0 + = -(Mu 0 -)

Mse 0 + = -(Mse 0 -)

Mt + 0 = -(Mt – 0)

Mu + 0 = -(Mu – 0)

Mse + 0 = -(Mse – 0)

- All amplitudes satisfy t-u symmetry

Mt + + (θ) = Mu ++ ($\pi - \theta$)
Mt + - (θ) = Mu + - ($\pi - \theta$)

Mt 00 (θ) = Mu 00 ($\pi - \theta$)
Mt 0 + (θ) = Mu + 0 ($\pi - \theta$)
Mt + 0 (θ) = Mu0 + ($\pi - \theta$)
Mt 0 - (θ) = Mu - 0 ($\pi - \theta$)
Mt - 0 (θ) = Mu0 - ($\pi - \theta$)

❖ Corresponding time order amplitudes also satisfy above motioned all symmetries

QUANTUM CORRELATION

Wigner-d Function for spin 1

$$d^{(j)}_{m',m}(\beta) = < j, m'|exp(\frac{-i j_y \beta}{\hbar})|j, m >$$

$$d^1(\beta) = \begin{bmatrix} \frac{1}{2}(1+Cos\beta) & -\frac{1}{\sqrt{2}}Sin\beta & \frac{1}{2}(1-Cos\beta) \\ \frac{1}{\sqrt{2}}Sin\beta & Cos\beta & -\frac{1}{\sqrt{2}}Sin\beta \\ \frac{1}{2}(1-Cos\beta) & \frac{1}{\sqrt{2}}Sin\beta & \frac{1}{2}(1+Cos\beta) \end{bmatrix}$$

Eigen states

$$|1,+1> = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$|1,0> = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$|1,-1> = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

Rotation in 180^0 angles ($\beta = \pi$)

$|1,+1> \rightarrow |1,-1>$ ⇒ Helicity state change

$|1,0> \rightarrow -|1,0>$ ⇒ Phase change

$|1,-1> \rightarrow |1,+1>$ ⇒ Helicity state change

Generalized helicity operator

$$\mathfrak{I}_3 = \frac{1}{\mathbb{P}}(P_{\hat{-}} J_3 + P^1 \kappa^{\hat{2}} - P^2 \kappa^{\hat{1}})$$

If the particle is moving in the *+z* or *–z* direction ($P^1 = P^2 = 0$)

$$\mathfrak{I}_3 = \frac{1}{\mathbb{P}}(P_{\hat{-}} J_3)$$

→ Wigner-d rotation in 180^0 angles

- Helicity and phase changes point can be found when $P_{\hat{-}} = 0$

HELICITY AND PHASE CHANGE

$\gamma\gamma \rightarrow "e^+ e^-" / \rho^0\rho^0 \rightarrow "\pi^+\pi^-"$

$"e^+e^-" \rightarrow \gamma\gamma / "\pi^+\pi^-" \rightarrow \rho^0\rho^0$

- Two outgoing spin 1 particles (Final particles)
- When annihilation angle zero ($P^1 = P^2$=0, $\theta = 0$) one particle is moving in +z direction and other one is moving in –z direction

- Two incoming Spin 1 particles (Initial particles)
- For any annihilation angle one particle is moving in +z direction and other one is moving in –z direction

- Helicity amplitudes depend on the reference frames and interpolation angles
- The Landscape of helicity amplitude should exhibit two helicity boundaries when helicity or phase changes are taken place as two particles moving $\pm$z direction
- These boundaries separate the branch that LFD belongs to from the branch that IFD belongs and corresponding interpolation angles are called critical interpolation angle

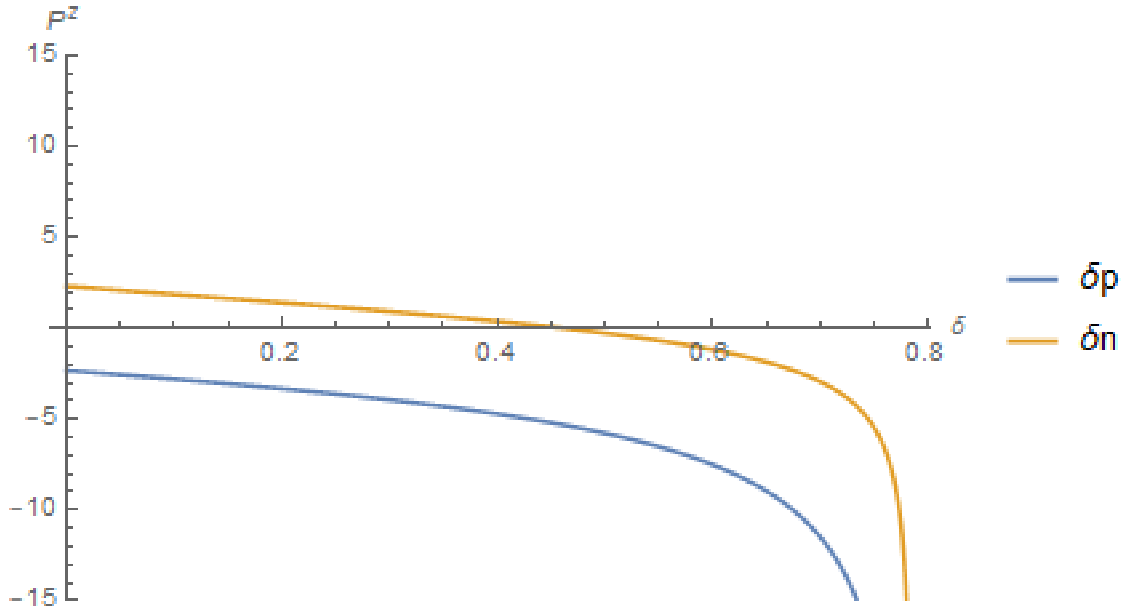
Critical interpolation angle

Spin 1 particle moving in +z- direction

$$\delta_p = -ArcTan\left[\frac{\left(E0*pz+p\gamma*\sqrt{4E0^2+pz^2}\right)}{\left(p\gamma*pz+E0*\sqrt{4E0^2+pz^2}\right)}\right]$$

Spin 1 particle moving in -z- direction

$$\delta_n = -ArcTan\left[\frac{\left(E0*pz-p\gamma*\sqrt{4E0^2+pz^2}\right)}{\left(-p\gamma*pz+E0*\sqrt{4E0^2+pz^2}\right)}\right]$$

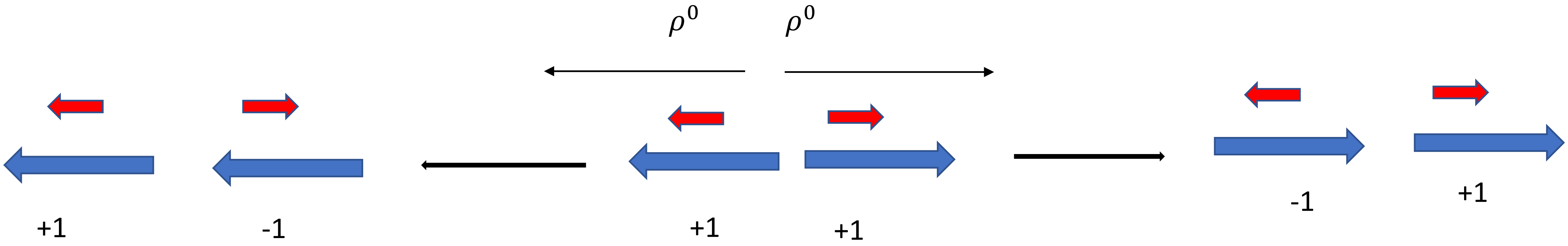


- When annihilation angle is equal to zero ($\theta = 0$).

$"e^+e^-" \rightarrow \gamma\gamma$ and $\gamma\gamma \rightarrow "e^+ e^-"$ processes → All t and u channel amplitudes goes to zero (only Seagle amplitude remain)

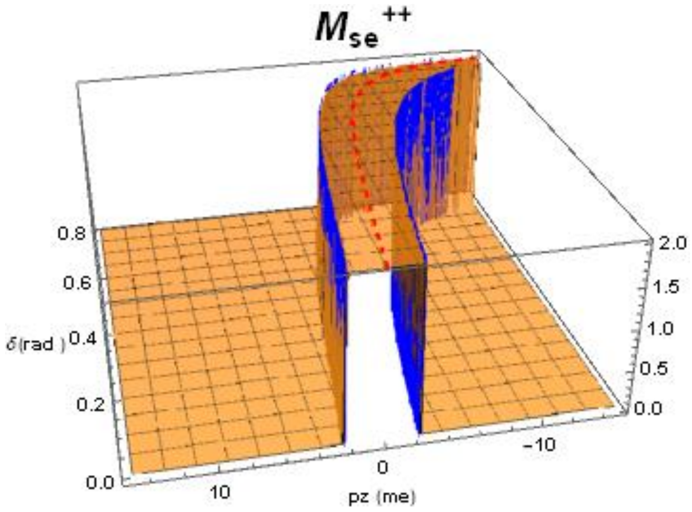
$“\pi^+\pi^-" \rightarrow \rho^0\rho^0$ and $\rho^0\rho^0 \rightarrow “\pi^+\pi^-“$ proceses → All t and u channel amplitudes goes to zero except the amplitudes with both outgoing (/incoming) polarization vectors are longitudinal (Mt00 and Mu00) and all Seagle

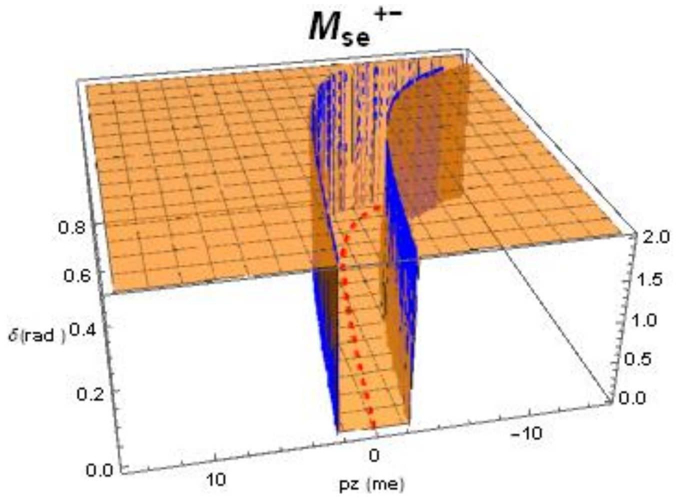
$“\pi^+\pi^-" \rightarrow \rho^0\rho^0$ ($\theta = 0$).



Helicity Changes

Mse++ =Mse - - , Mse+ - = Mse - + (symmetry based on parity conservation)





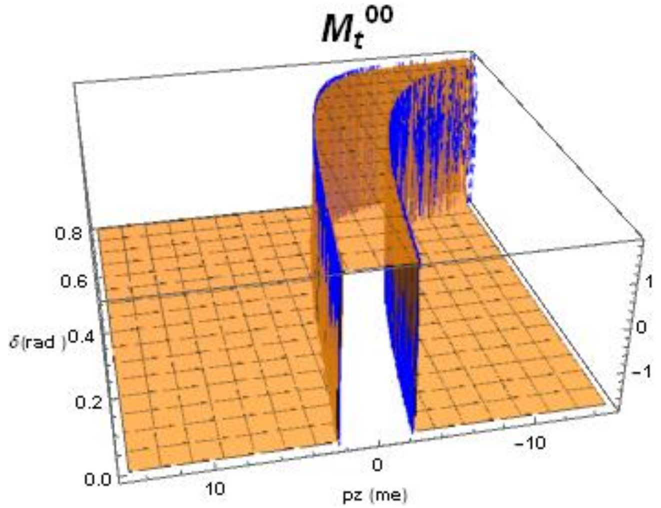
$E_0 = 2m_e$

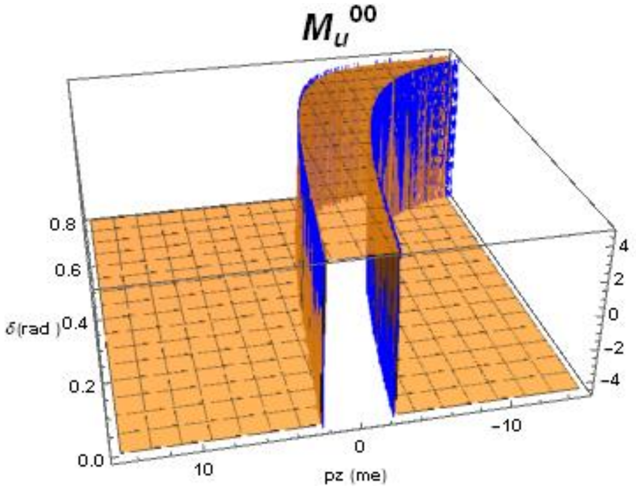
$\theta = 0$

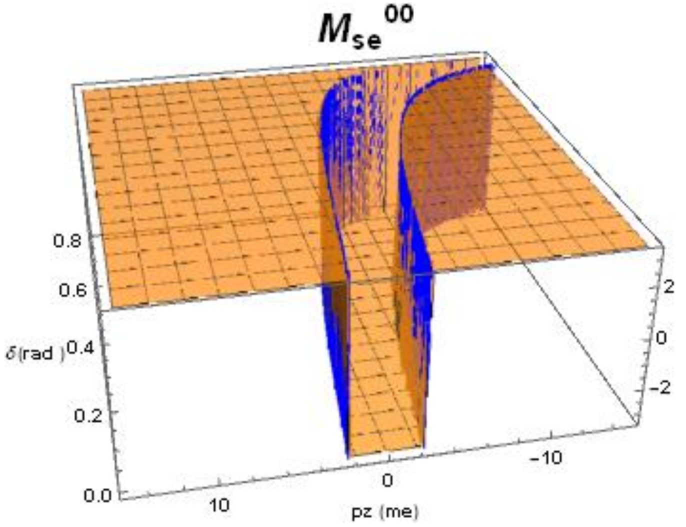
$P_\gamma = m_e$

$P_e = \sqrt{3}m_e$

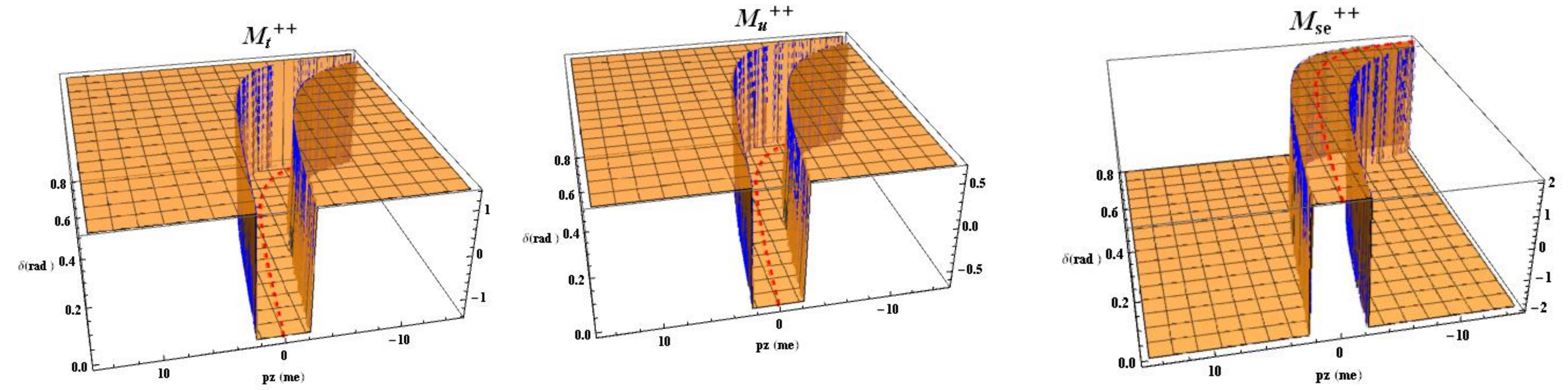
Phase changes







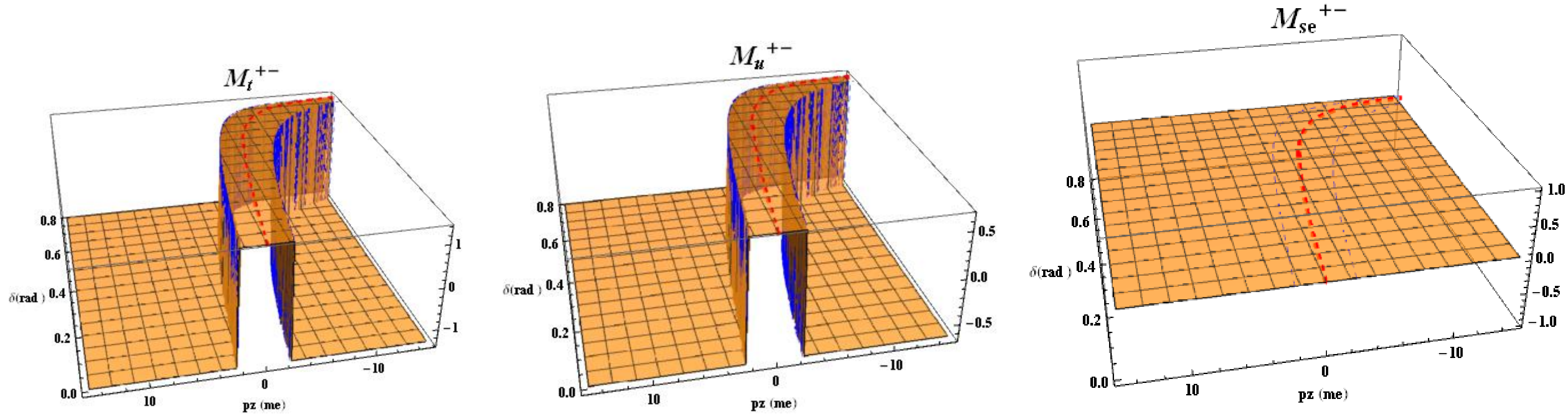
$\rho^0\rho^0 \rightarrow$ “$\pi^+\pi^-$ “ (No any annihilation angle condition should be satisfied to see clear boundaries)

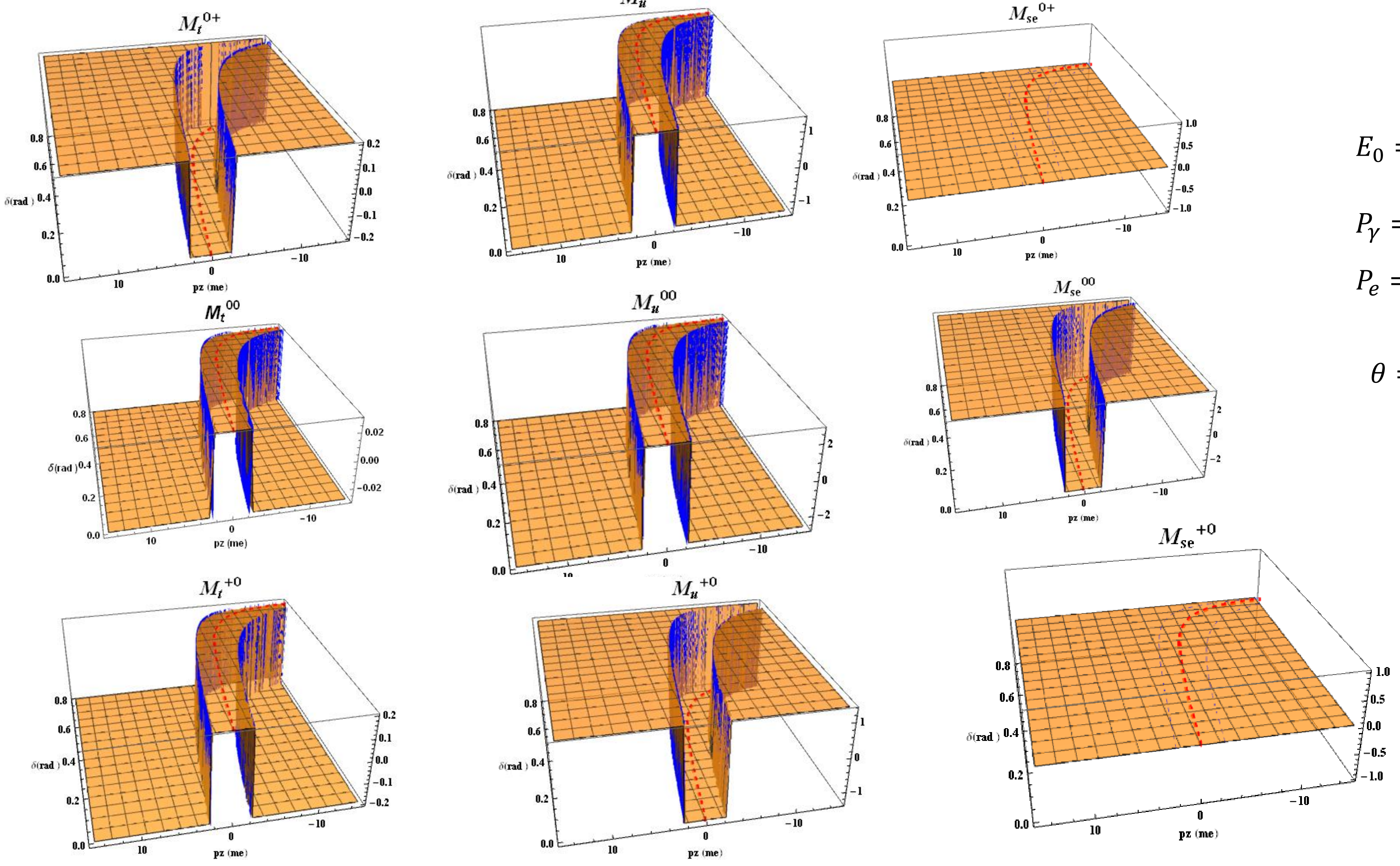


$E_0 = 2m_e$

$P_\gamma = m_e$

$P_e = \sqrt{3}m_e$





$$E_0 = 2m_e$$

$$P_\gamma = m_e$$

$$P_e = \sqrt{3}m_e$$

$$\theta = \frac{\pi}{3}$$

$\gamma\gamma \rightarrow$ "$e^+ e^-$"

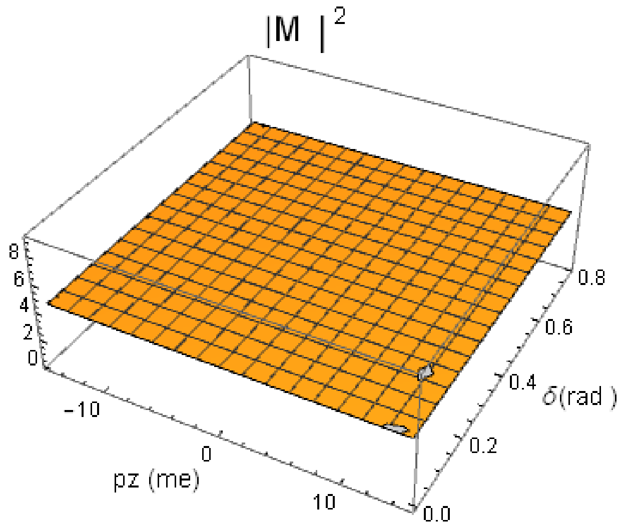
$\rho^0\rho^0 \rightarrow$ "$\pi^+\pi^-$"

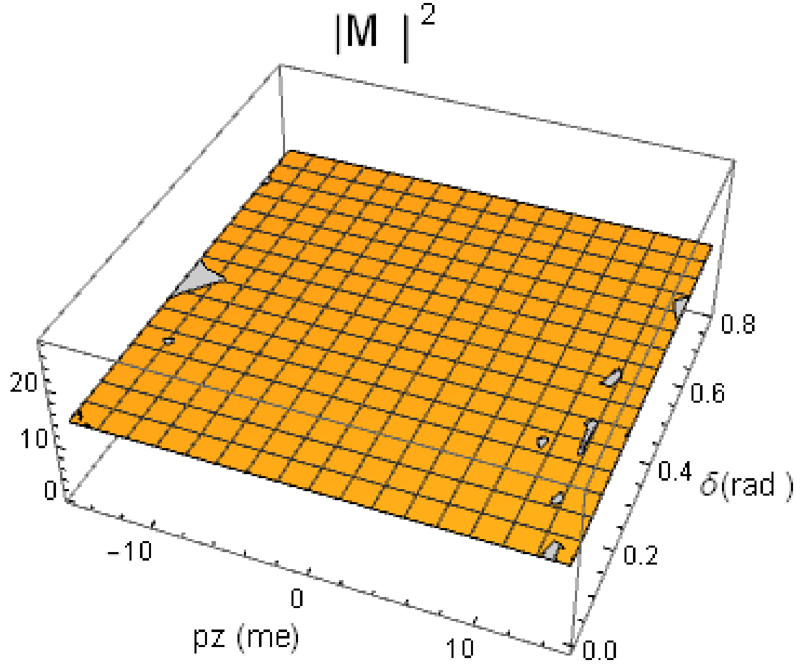
$$E_0 = 2m_e$$

$$P_\gamma = m_e$$

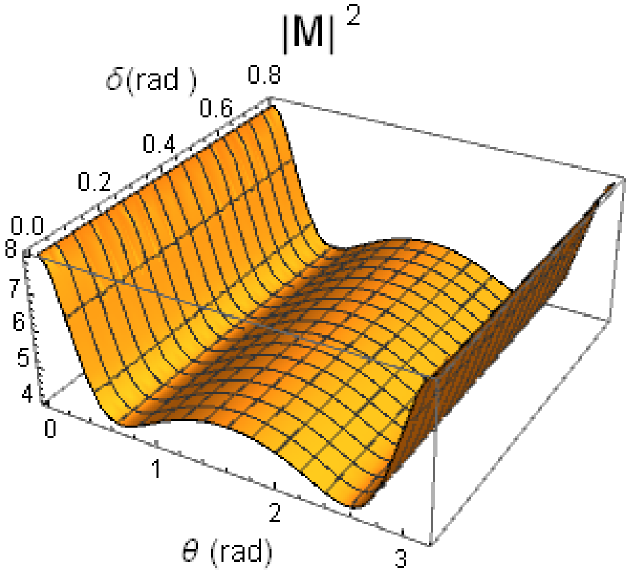
$$P_e = \sqrt{3}m_e$$

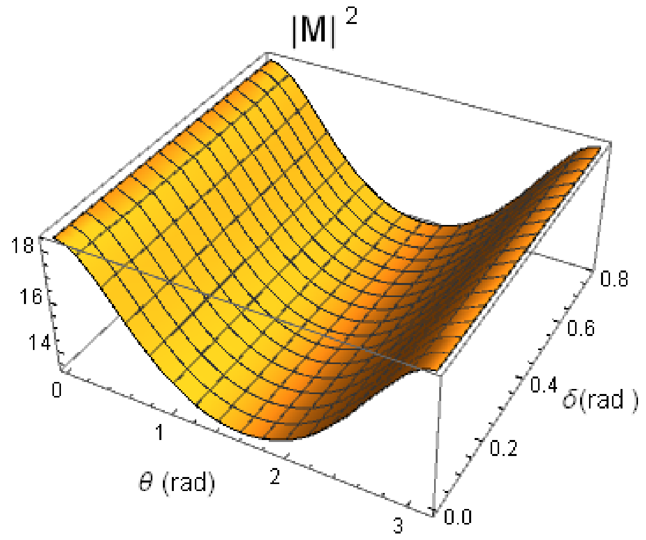
$$\theta = \frac{\pi}{3}$$

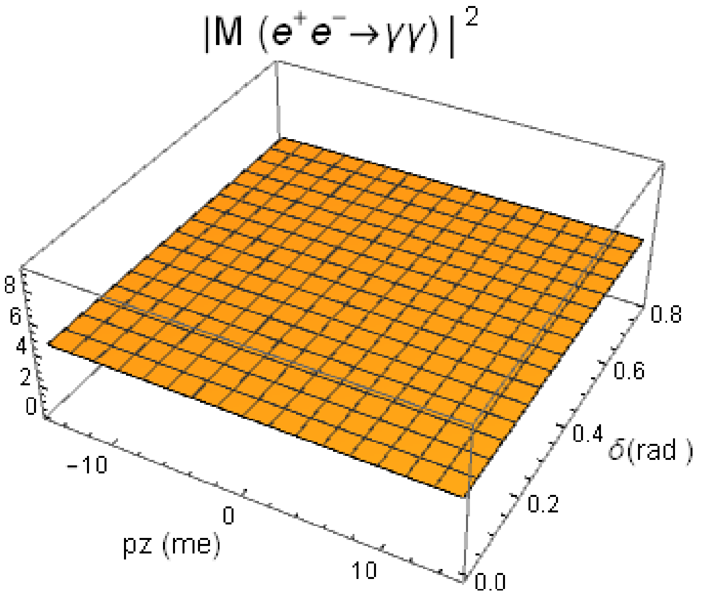




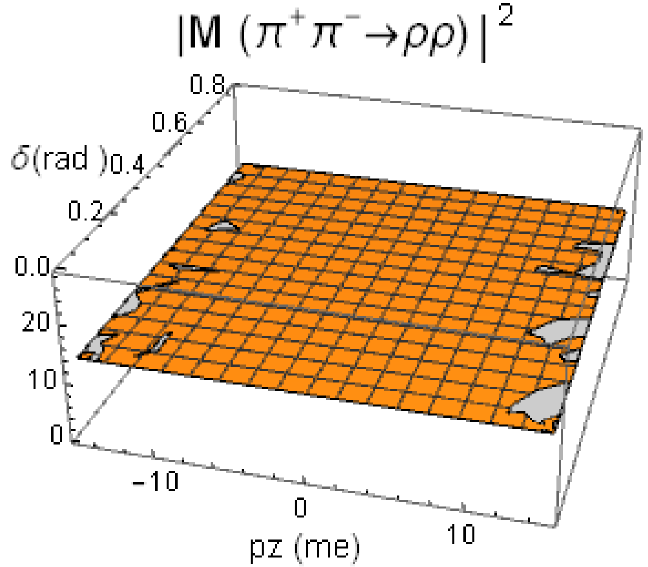
$$P_z = 0$$







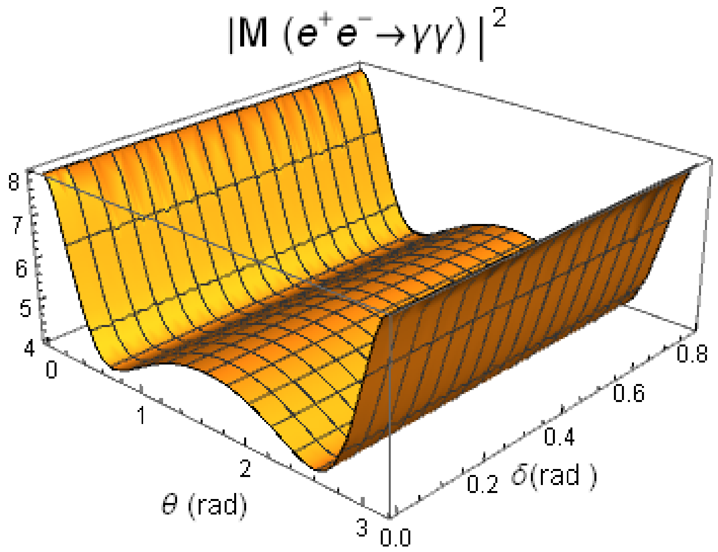
$\theta = \frac{\pi}{3}$



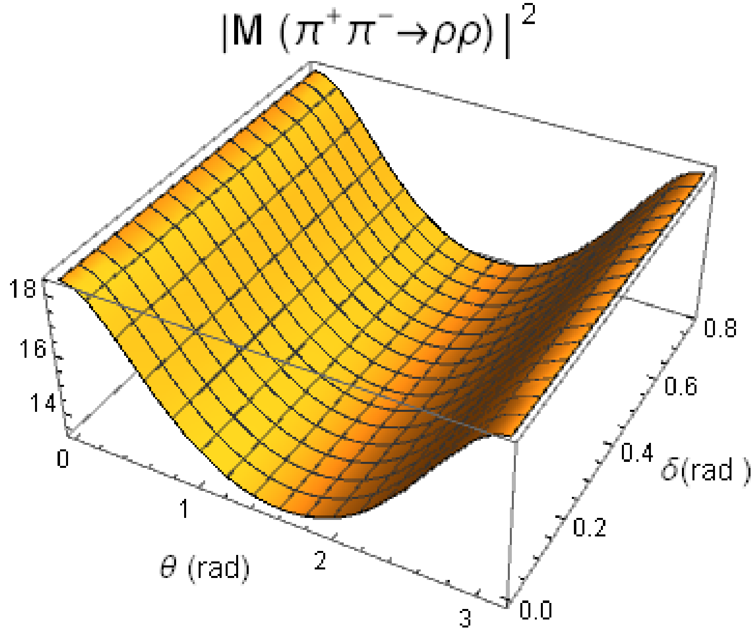
$E_0 = 2m_e$

$P_\gamma = m_e$

$P_e = \sqrt{3}m_e$



$P_z = 0$



Cross-Section of the processes using Mandelstam variable.

$$|M|^2 = \sum_{\lambda_1,\lambda_2} |M_t^{\lambda_1,\lambda_2} + M_u^{\lambda_1,\lambda_2} + M_{se}^{\lambda_1,\lambda_2}|^2$$

- Two Scalar mesons annihilation in to two rho mesons

$$|M_\rho^2| = \left[\left[\frac{2t + 2m_e^2 - m_\gamma^2}{(t - m_e^2)}\right]^2 + \left[\frac{2u + 2m_e^2 - m_\gamma^2}{(u - m_e^2)}\right]^2 + 2\left[\frac{\left[t + u + 2m_e^2 - 3m_\gamma^2\right]^2}{(t - m_e^2)(u - m_e^2)}\right]\right]$$

$$-4\left[\left[\frac{5t + u + 2m_e^2 - 4m_\gamma^2}{(t - m_e^2)}\right] + \left[\frac{5u + t + 2m_e^2 - 4m_\gamma^2}{(u - m_e^2)}\right]\right] + 16$$

- Scaler electron and proton annihilation in to two photons

$$|M|^2 = 4\left[\left[\frac{t + m_e^2}{t - m_e^2}\right]^2 + \left[\frac{u + m_e^2}{u - m_e^2}\right]^2 + 4\left[\frac{(t + m_e^2)(u + m_e^2) - 2tu}{(t - m_e^2)(u - m_e^2)}\right] + 4\right]$$

- Two Scalar mesons annihilation in to two rho mesons

$$|M_\rho^2| = 4\left[\left(1-\frac{2pe^2(E0^2+p\gamma^2)Sin[\theta]^2}{(E0^2+p\gamma^2)^2-4pe^2p\gamma^2Cos[\theta]^2}\right)^2+\frac{8pe^4(E0^2+p\gamma^2)^2Sin[\theta]^4}{\left((E0^2+p\gamma^2)^2-4pe^2p\gamma^2Cos[\theta]^2\right)^2}+\right]$$

$$4\left[\frac{(E0^2-p\gamma^2)^2(E0^2+p\gamma^2-4pe^2Cos[\theta]^2)^2}{\left((E0^2+p\gamma^2)^2-4pe^2p\gamma^2Cos[\theta]^2\right)^2}+\frac{8E0^2pe^4(E0^2-p\gamma^2)Sin[2\theta]^2}{\left((E0^2+p\gamma^2)^2-4pe^2p\gamma^2Cos[\theta]^2\right)^2}\right]$$

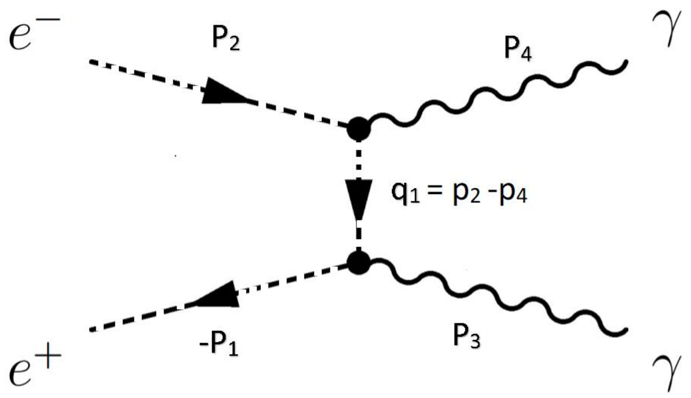
- Scaler electron and proton annihilation in to two photons

$$|M|^2 = 4\left[1+\left[1-\frac{2p_e^2Sin^2(\theta)}{E_0^2-p_e^2Cos^2(\theta)}\right]^2\right]$$

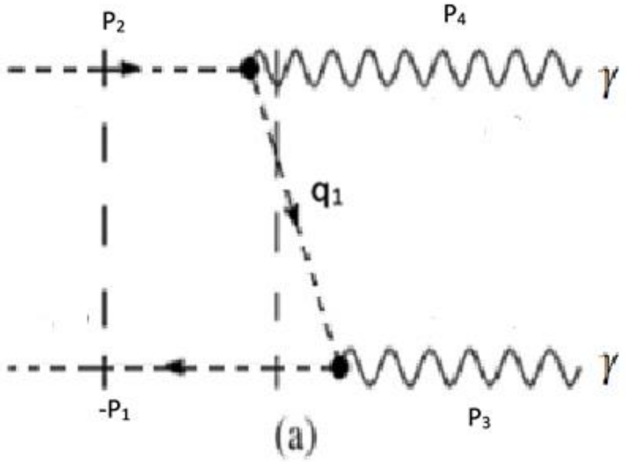
Time ordering in the interpolation dynamics

t-Channel

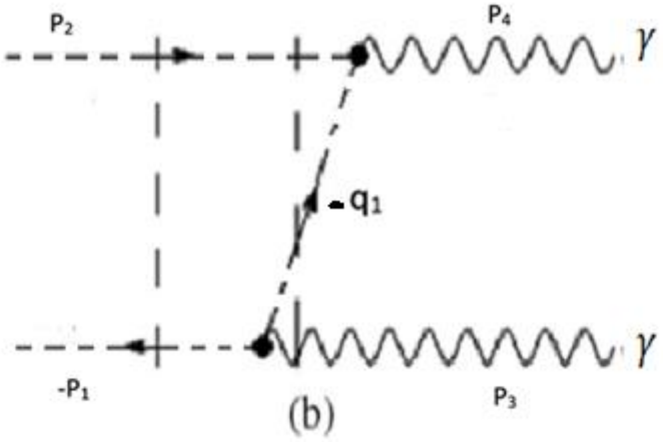
Covariant Propagator



Forward moving



Backward moving



$$\Sigma = \Sigma_a^{\delta} + \Sigma_b^{\delta} = \frac{1}{q_1^2 - m^2}$$

$$\Sigma_a^{\delta} = \frac{C}{2Q^{\hat{+}}\,(q^{\hat{+}} - Q^{\hat{+}})}$$

$$\Sigma_b^{\delta} = -\frac{C}{2Q^{\hat{+}}\,(q^{\hat{+}} + Q^{\hat{+}})}$$

Where: $C = Cos(2\delta)\,,\;\; S = Sin(2\delta),\;\; q^{\hat{+}} = p_2^{\hat{+}} - p_4^{\hat{+}},\quad Q^{\hat{+}} = \pm\sqrt{Q_{\hat{-}}^2 + C(\vec{q}^{\,2}{}_{\perp} + m^2)}$

Critical annihilation angle

"e^+e^-" → $\gamma\gamma$/ "$\pi^+\pi^-$" → $\rho^0\rho^0$

$$q_1^+ = \frac{-\frac{pepz}{2E0} + \frac{pzp\gamma Cos[\theta]}{2E0}}{\sqrt{2}} + \frac{-\frac{pe\sqrt{4E0^2+pz^2}}{2E0} + \frac{\sqrt{4E0^2+pz^2}p\gamma Cos[\theta]}{2E0}}{\sqrt{2}}$$

$$q_2^+ = \frac{-\frac{pepz}{2E0} - \frac{pzp\gamma Cos[\theta]}{2E0}}{\sqrt{2}} + \frac{-\frac{pe\sqrt{4E0^2+pz^2}}{2E0} - \frac{\sqrt{4E0^2+pz^2}p\gamma Cos[\theta]}{2E0}}{\sqrt{2}}$$

$\gamma\gamma$ → "$e^+ e^-$" /$\rho^0\rho^0$ → "$\pi^+\pi^-$"

$$q_1^+ = \frac{\left(\frac{pz}{\sqrt{2}} + \frac{\sqrt{4E0^2+pz^2}}{\sqrt{2}}\right)(p\gamma - peCos[\theta])}{2E0}$$

$$q_2^+ = -\frac{\left(\frac{pz}{\sqrt{2}} + \frac{\sqrt{4E0^2+pz^2}}{\sqrt{2}}\right)(p\gamma + peCos[\theta])}{2E0}$$

$q_1^+ = 0 \rightarrow \theta_{c,t} = ArcCos\left(\frac{P_e}{P_\gamma}\right)$

$q_2^+ = 0 \rightarrow \theta_{c,u} = -ArcCos\left(\frac{P_e}{P_\gamma}\right)$

$q_1^+ > 0$ →Forward

$q_2^+ > 0$ →Forward

$\rightarrow \exists\ Cos(\theta_c) \leftrightarrow P_e < P_\gamma$

$q_1^+ < 0$ → Backward

$q_2^+ < 0$ → Backward

$\theta_{c,t} = ArcCos\left(\frac{P_\gamma}{P_e}\right)$

$\theta_{c,u} = -ArcCos\left(\frac{P_\gamma}{P_e}\right)$

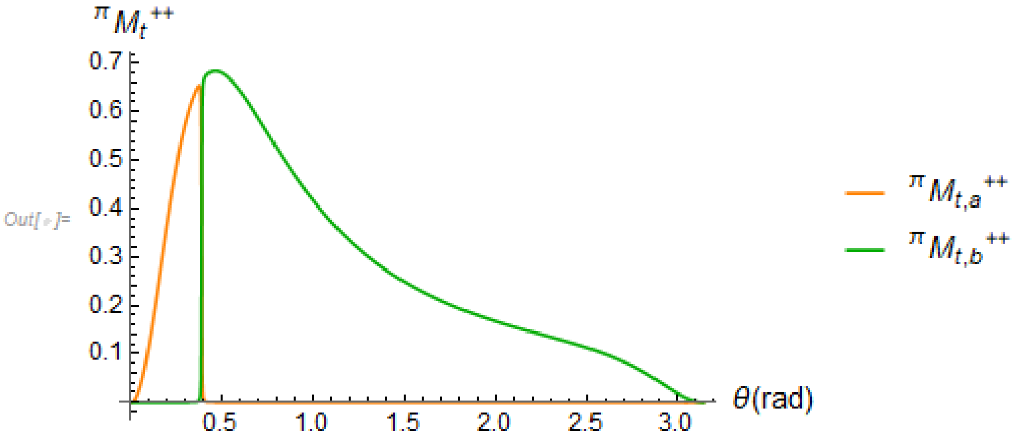
$\rightarrow \exists\ Cos(\theta_c) \leftrightarrow P_\gamma < P_e$

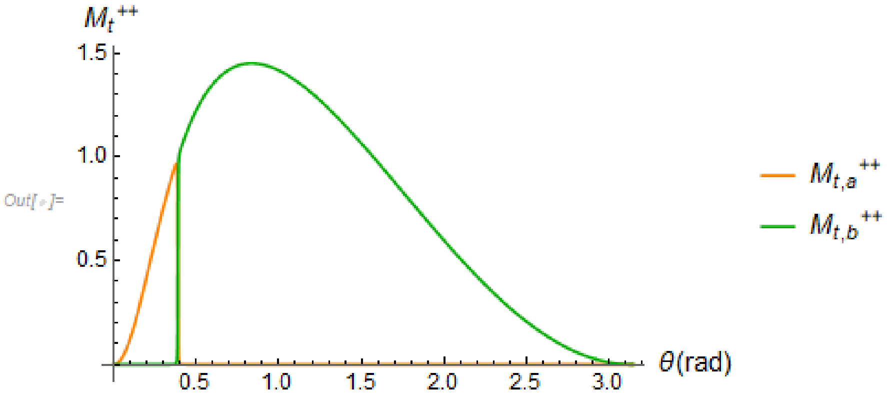
$$E_0 = 2m_e$$

$“\pi^+\pi^-" \rightarrow \rho^0\rho^0$

$$\delta = 0.785398 \sim \frac{\pi}{4}$$

$\rho^0\rho^0 \rightarrow “\pi^+\pi^-"$

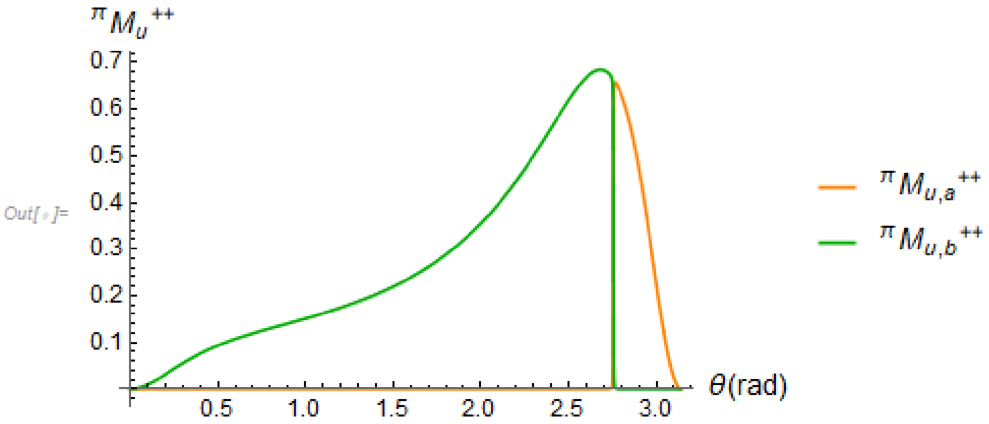


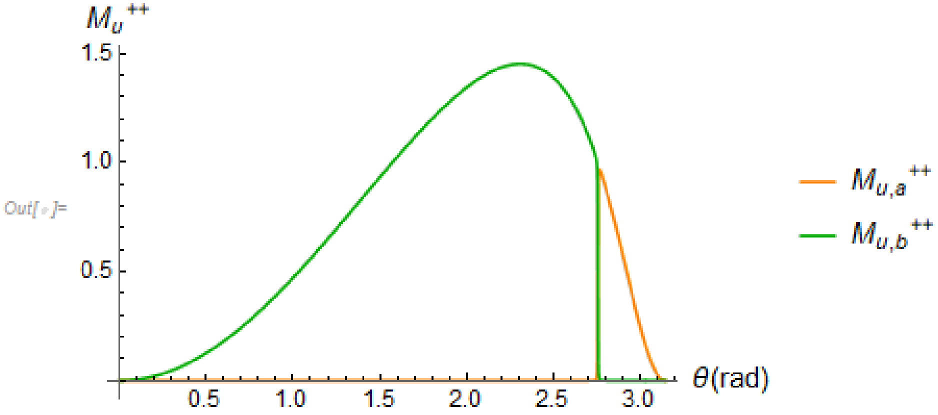


$P_e = \sqrt{3}m_e \quad P_\gamma = \sqrt{3.5}m_e$

$$\theta_{c,t} = ArcCos\left(\sqrt{\frac{3}{3.5}}\right) = 0.387597$$

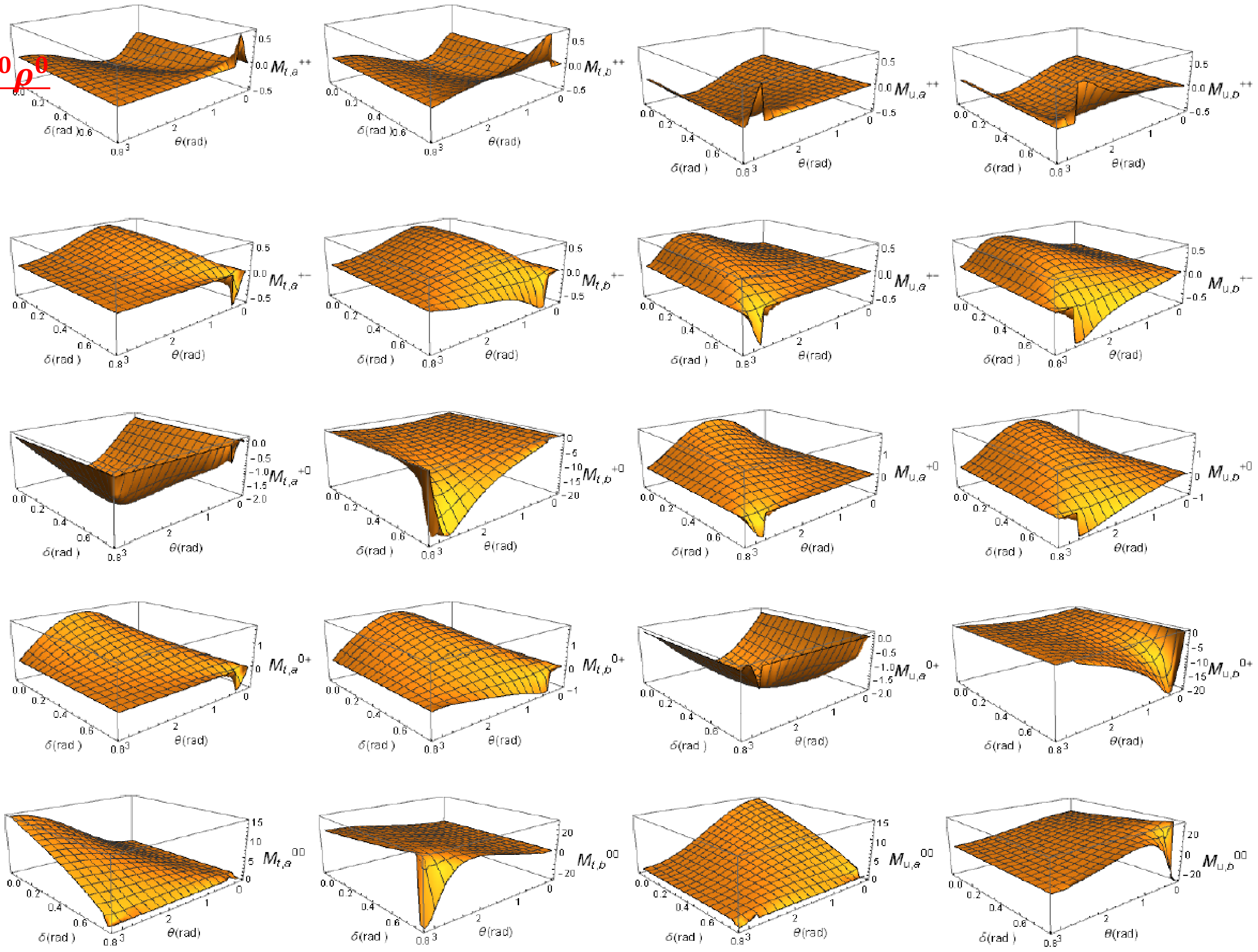
$P_\gamma = \sqrt{3}m_e \quad P_e = \sqrt{3.5}m_e$





$$\theta_{c,u} = -ArcCos\left(\sqrt{\frac{3}{3.5}}\right) = 2.754$$

$“\pi^+\pi^-” \rightarrow \rho^0\rho^0$

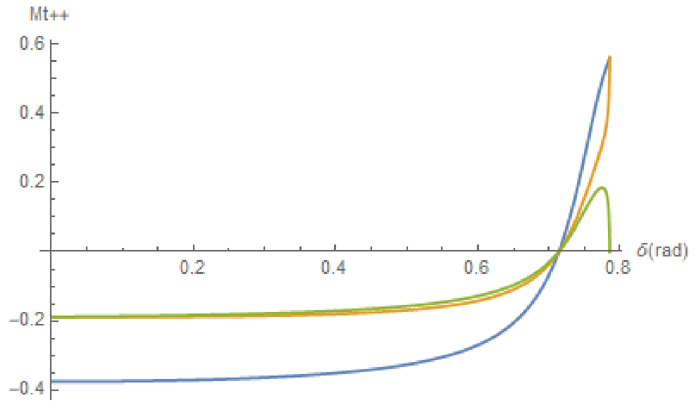


“$\pi^+\pi^-$” $\rightarrow$ $\rho^0\rho^0$

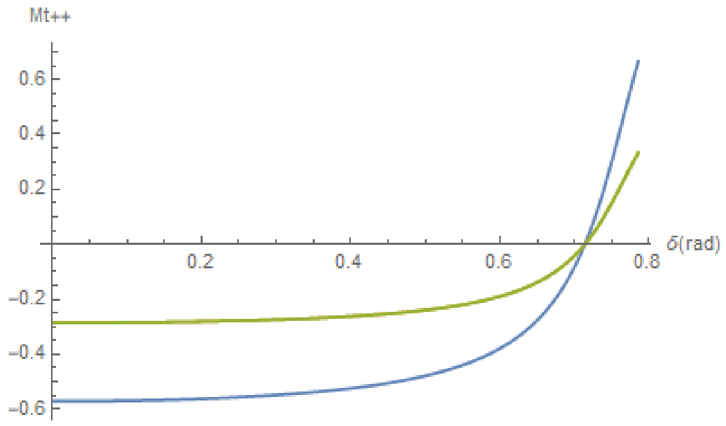
$E_0 = 2m_e$ $P_z = 0$ (Center of mass frame) $P_\gamma = \sqrt{3.5}m_e$ $P_e = \sqrt{3}m_e$

$\theta_{c,t}$ = critical annihilation angle

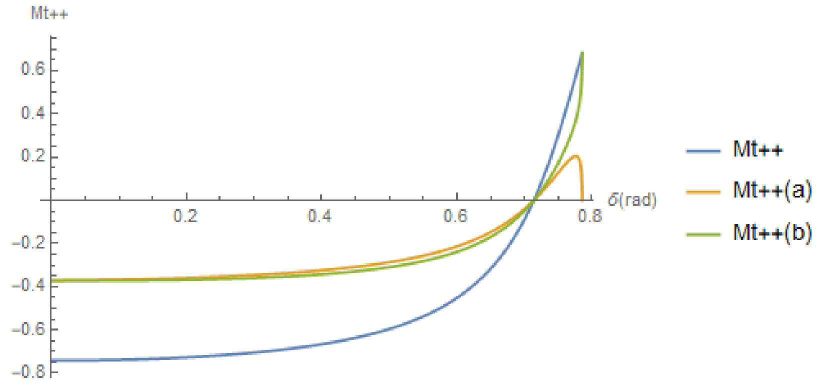
$\theta = \theta_{c,t} - 0.1$



$\theta = \theta_{c,t}$

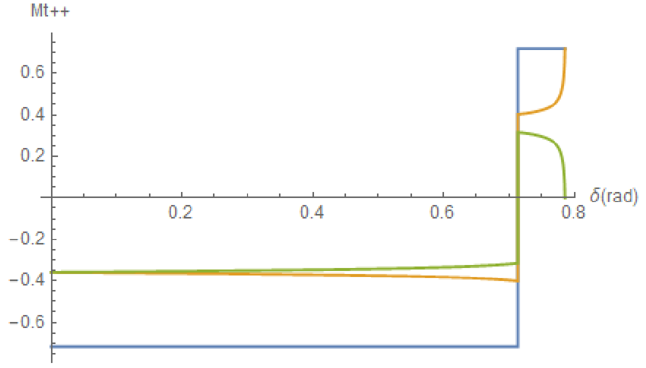


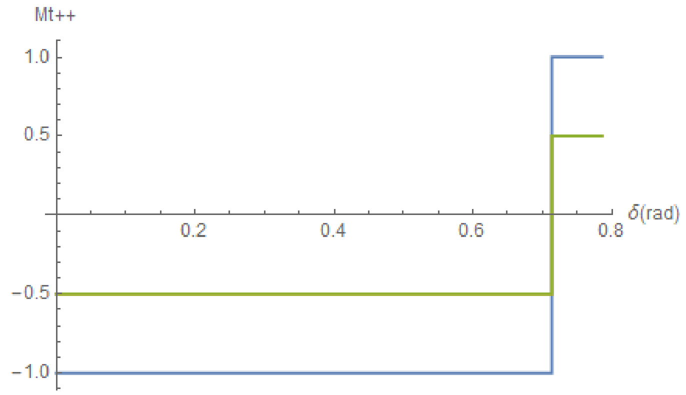
$\theta = \theta_{c,t} + 0.1$

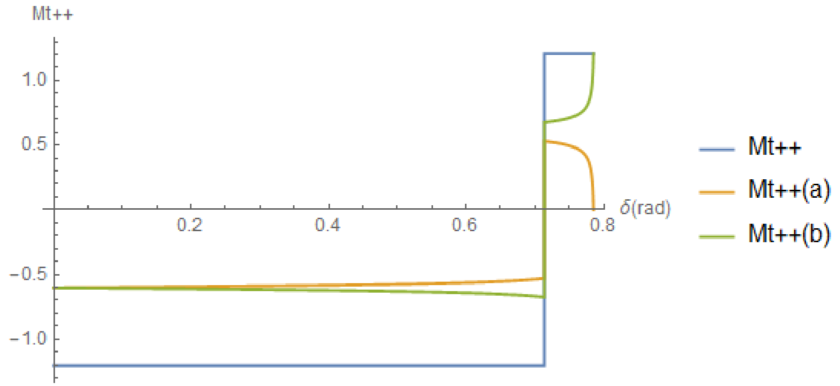


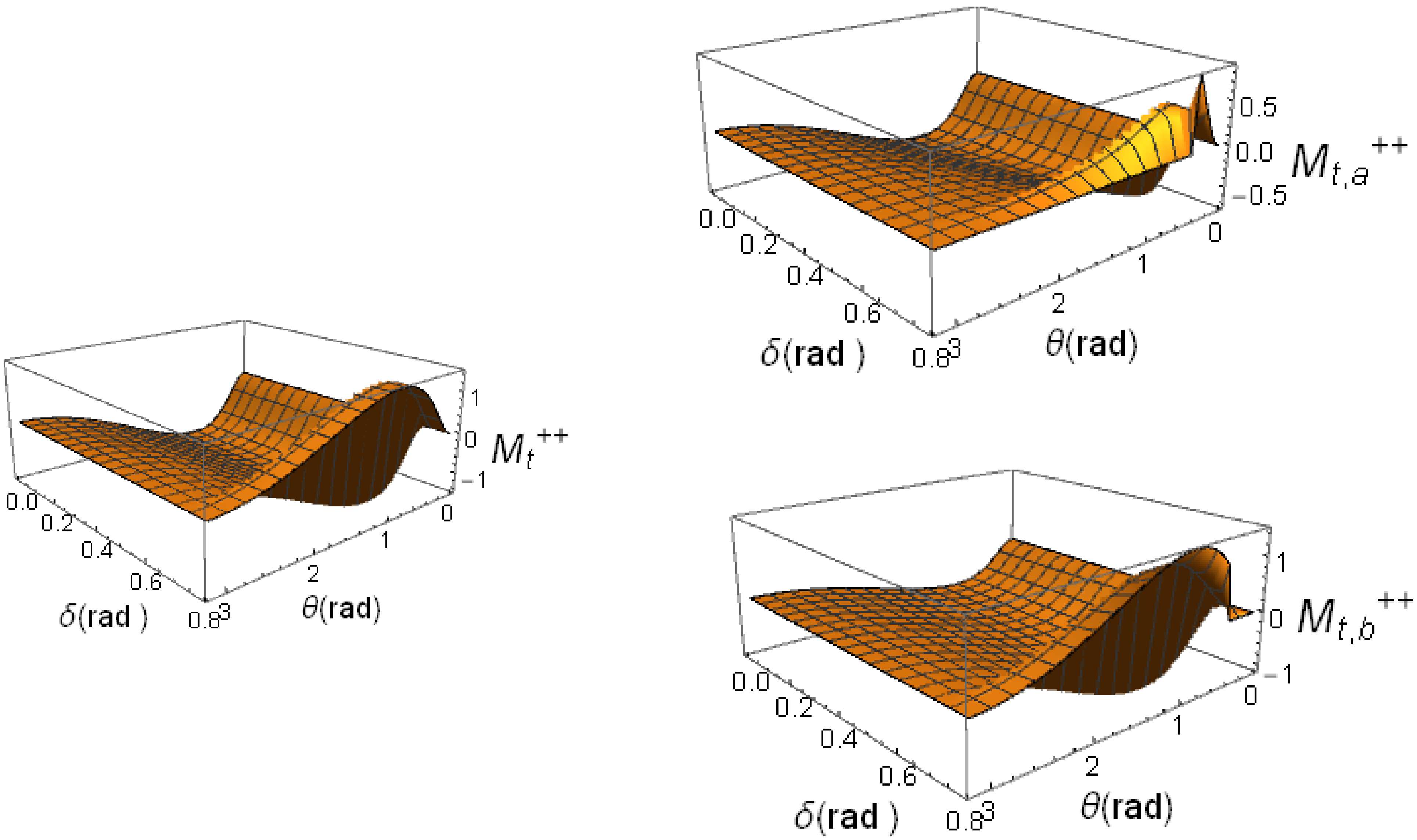
$\rho^0\rho^0$ $\rightarrow$ “$\pi^+\pi^-$”

$P_\gamma = \sqrt{3}m_e$ $P_e = \sqrt{3.5}m_e$









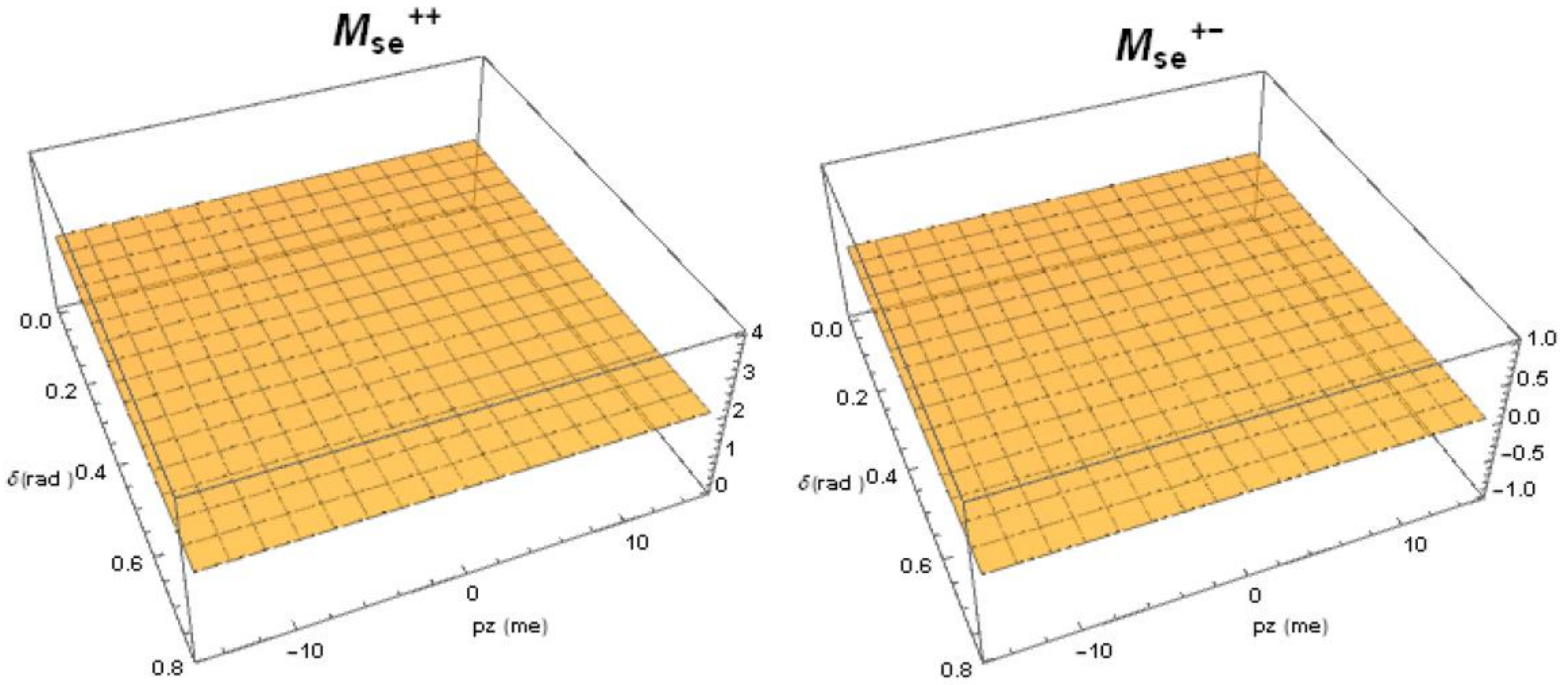
$M_{t,a}^{++}$
0.5
0.0
-0.5
0.0
0.2
0.4
0.6
0.8
δ(rad)
3
2
1
0
θ(rad)
M_t^{++}
1
0
-1
0.0
0.2
0.4
0.6
0.8
δ(rad)
3
2
1
0
θ(rad)
$M_{t,b}^{++}$
1
0
-1
0.0
0.2
0.4
0.6
0.8
δ(rad)
3
2
1
0
θ(rad)

Conclusion

- Choice of kinematics has high dependence on helicity amplitude
- Even though we have better understanding about the scaler particle annihilation and creation boson process we are not clear about the reverse processes
- Time order amplitudes share same features in both processes
- Total Probability independent of the kinematics and the process (Forward or Reverse)

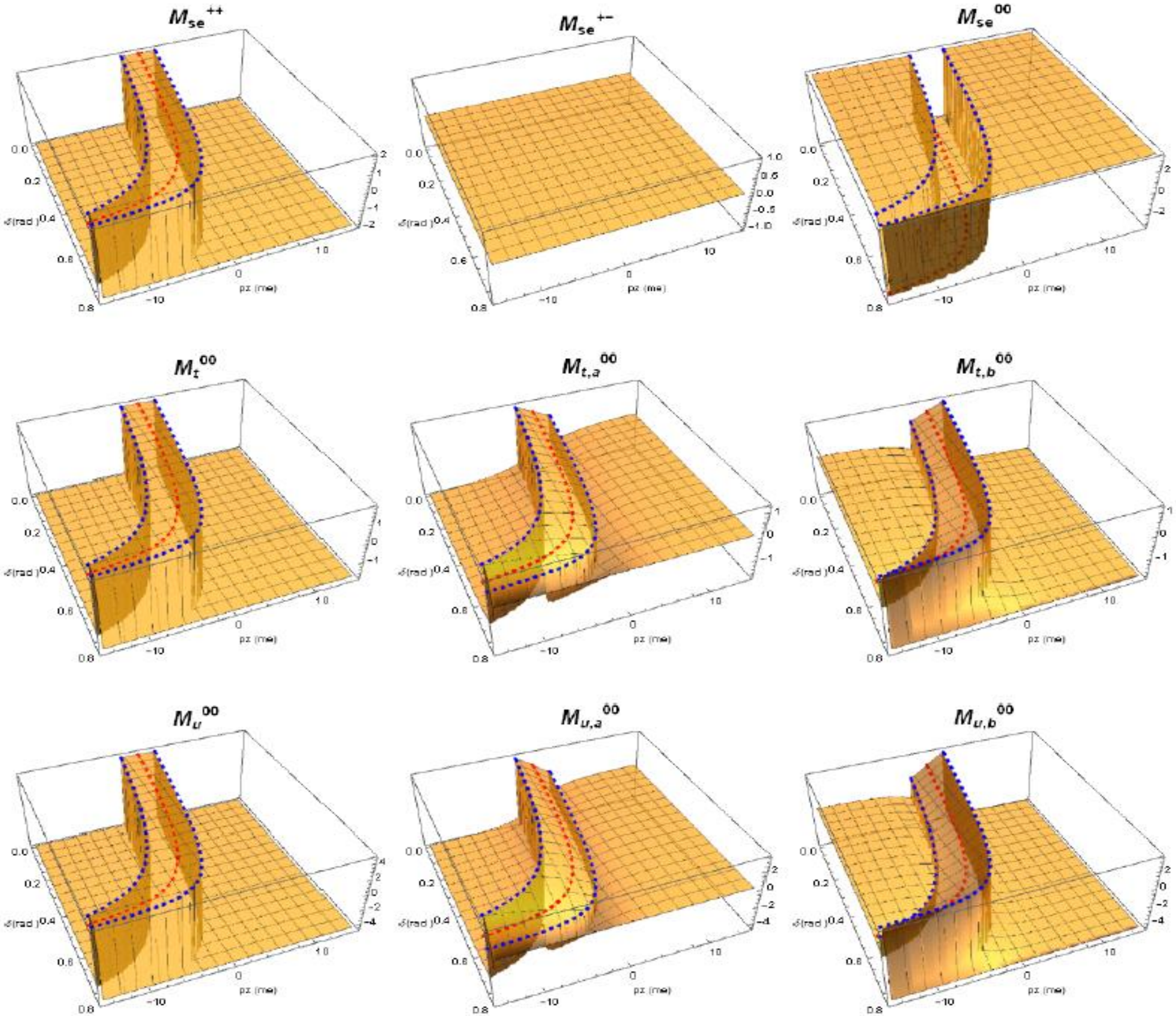
"e^+e^-" $\rightarrow \gamma\gamma$

$\theta = 0$



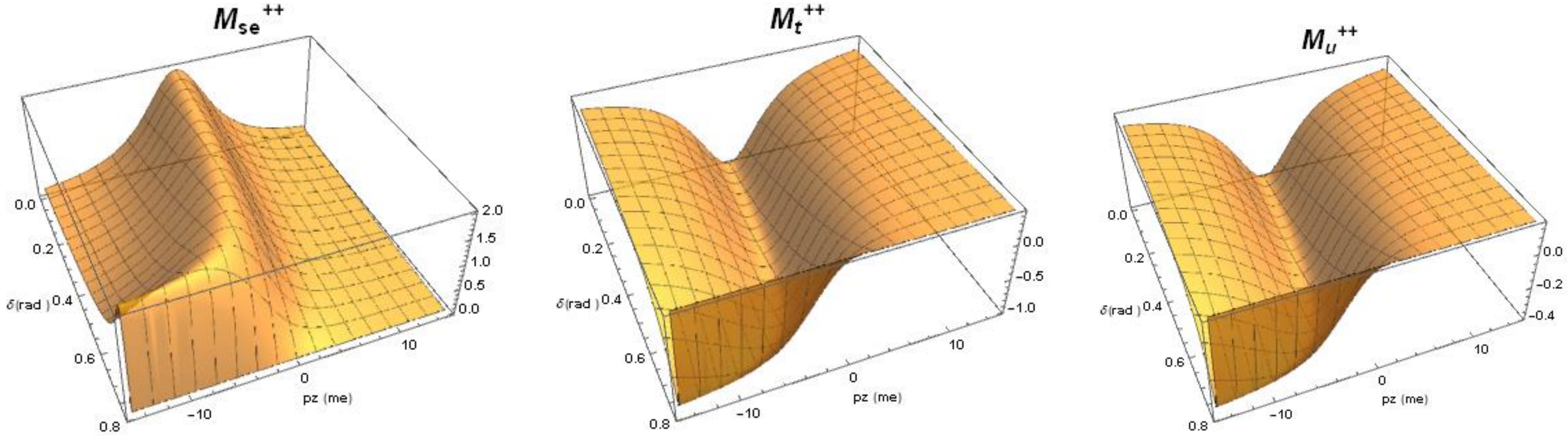
All t and u channel helicity amplitudes goes to zero

“$\pi^+\pi^-$ ” $\rightarrow$ $\rho^0\rho$

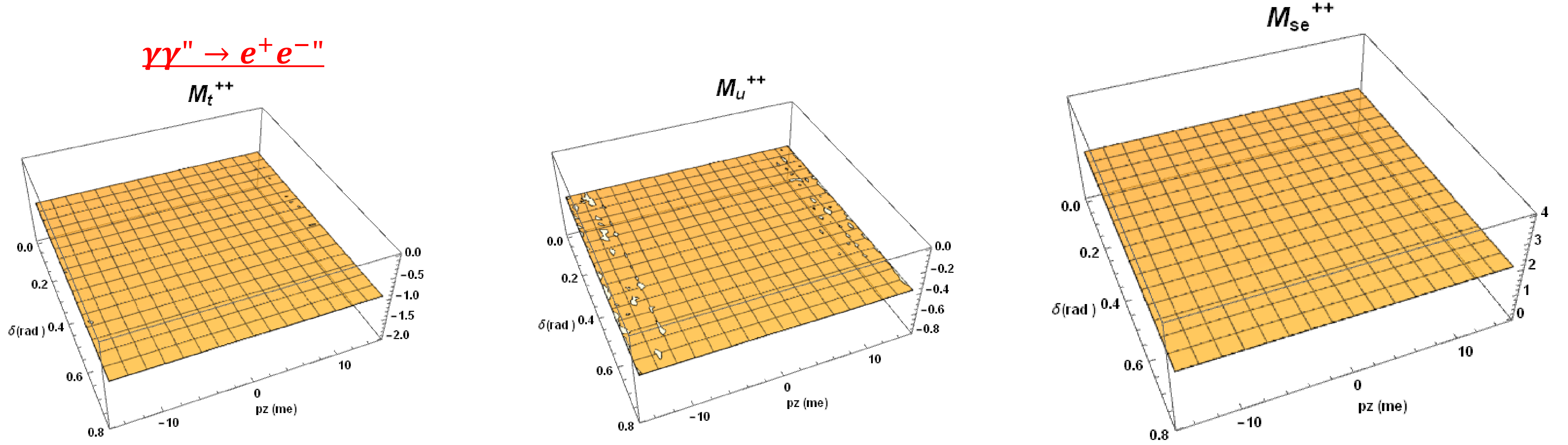


$\theta = \frac{\pi}{3}$

"e^+e^-" $\to \gamma\gamma$

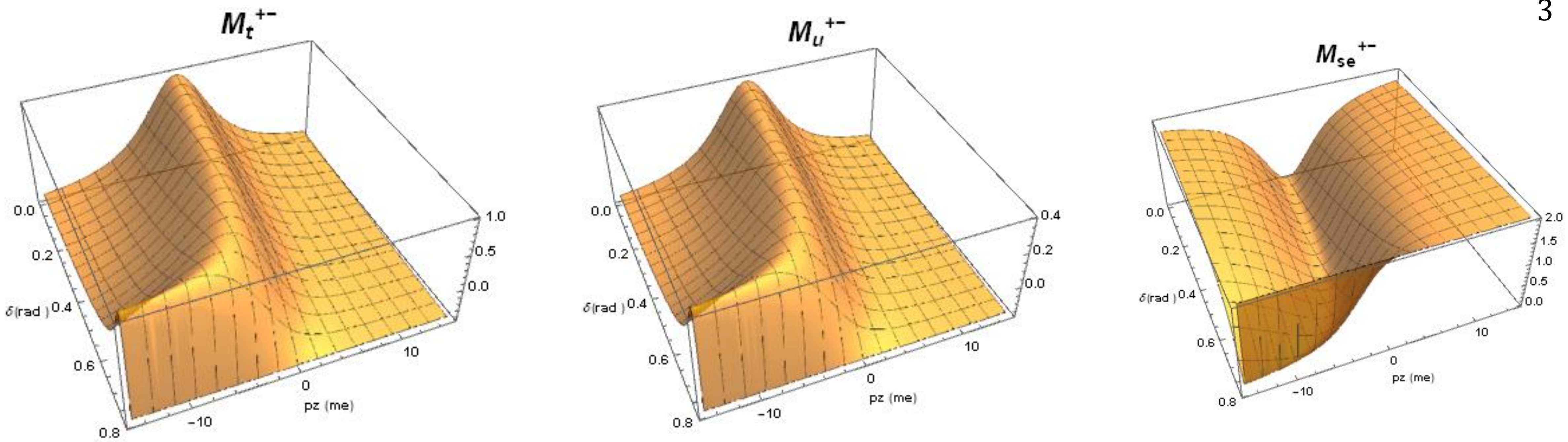


$\gamma\gamma$" $\to e^+e^-$"



"e^+e^-" $\rightarrow \gamma\gamma$

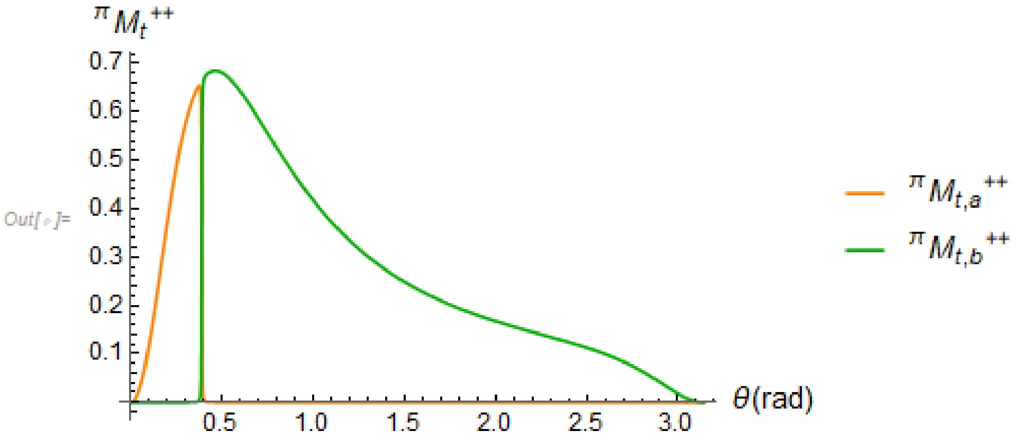
$\theta = \frac{\pi}{3}$



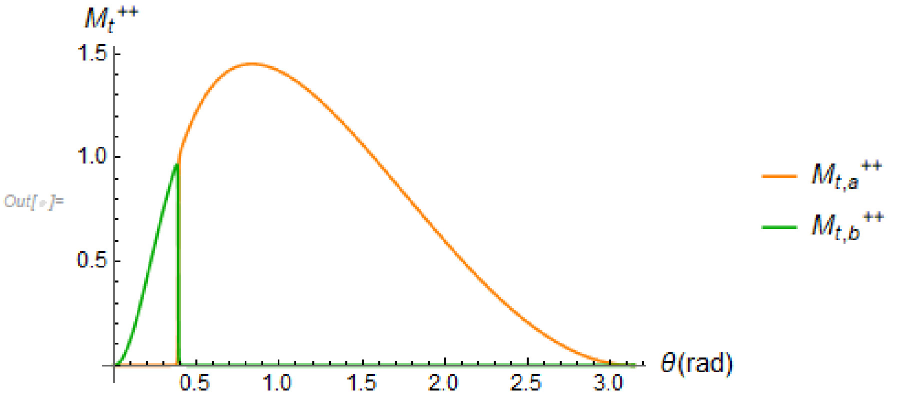
$$E_0 = 2m_e$$

$$\delta = 0.785398 \sim \frac{\pi}{4}$$

"$\pi^+\pi^-$" → $\rho^0\rho^0$



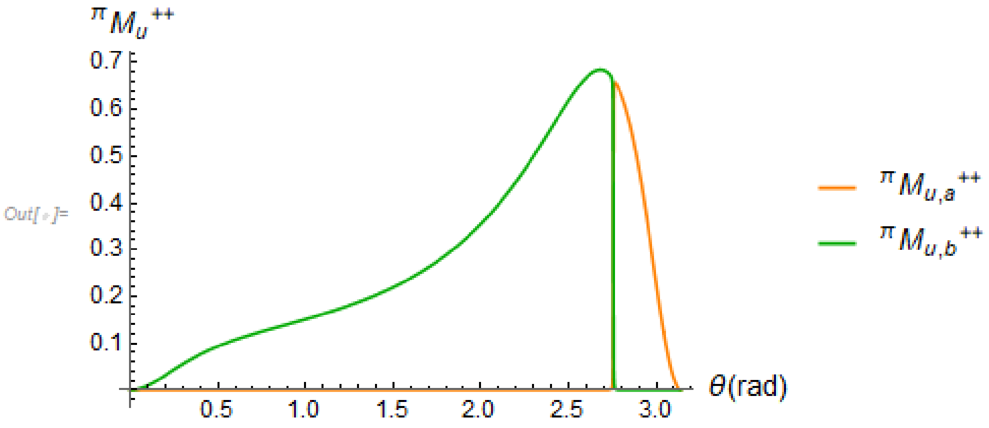
$\rho^0\rho^0$ → "$\pi^+\pi^-$"

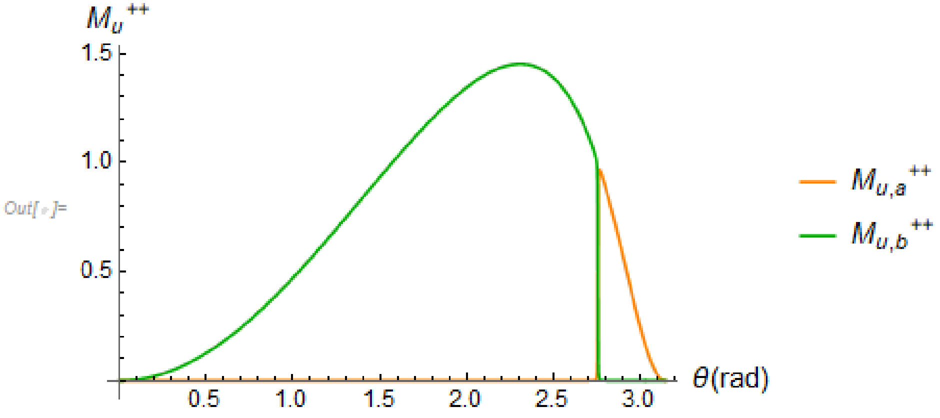


$P_e = \sqrt{3}m_e \quad P_\gamma = \sqrt{3.5}m_e$

$$\theta_{c,t} = ArcCos\left(\sqrt{\frac{3}{3.5}}\right) = 0.387597$$

$P_\gamma = \sqrt{3}m_e \quad P_e = \sqrt{3.5}m_e$





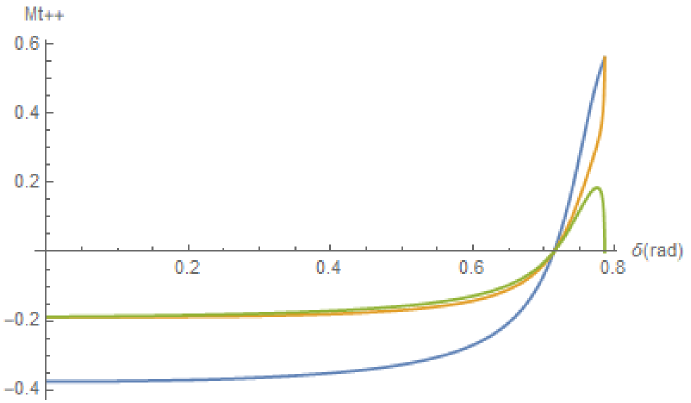
$$\theta_{c,u} = -ArcCos\left(\sqrt{\frac{3}{3.5}}\right) = 2.754$$

“$\pi^+\pi^-$” → $\rho^0\rho^0$

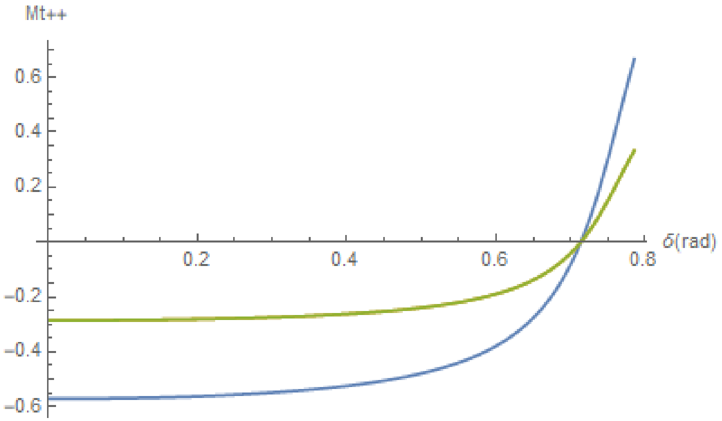
$E_0 = 2m_e$ $\quad P_z = 0$ (*Center of mass frame*) $\quad P_\gamma = \sqrt{3.5}m_e \quad P_e = \sqrt{3}m_e$

$\theta_{c,t}$ = critical annihilation angle

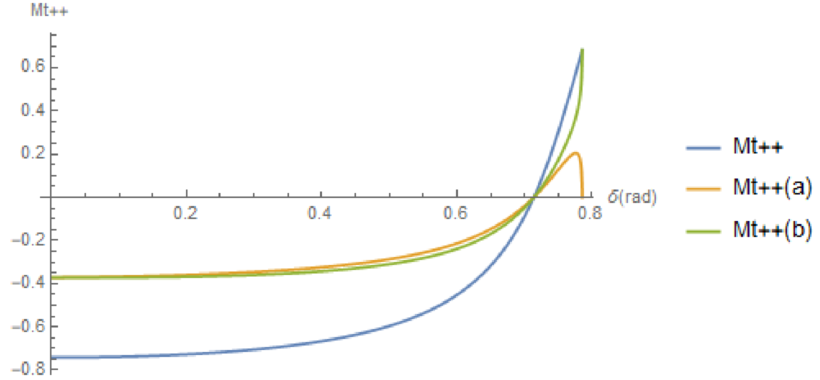
$\theta = \theta_{c,t} - 0.1$



$\theta = \theta_{c,t}$

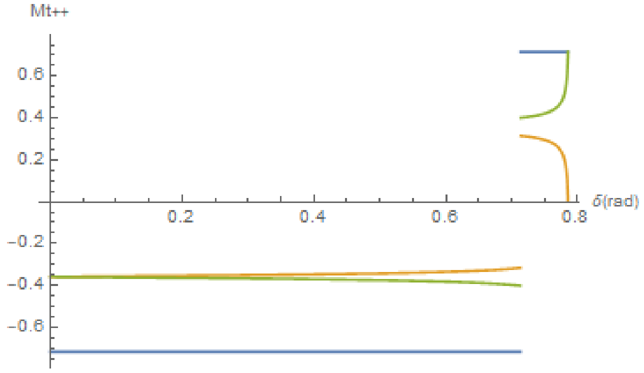


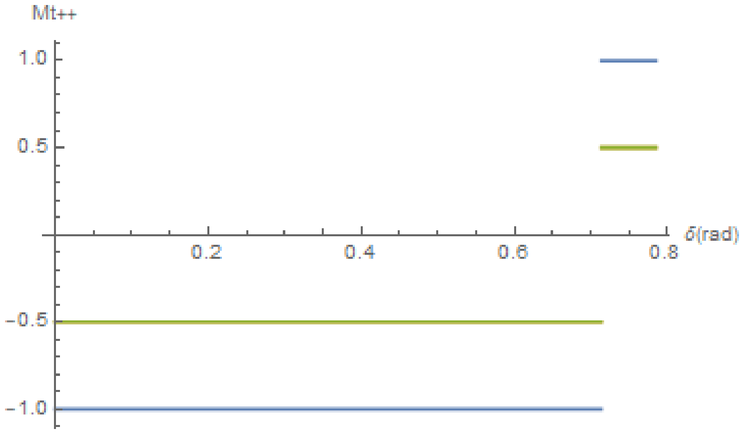
$\theta = \theta_{c,t} + 0.1$

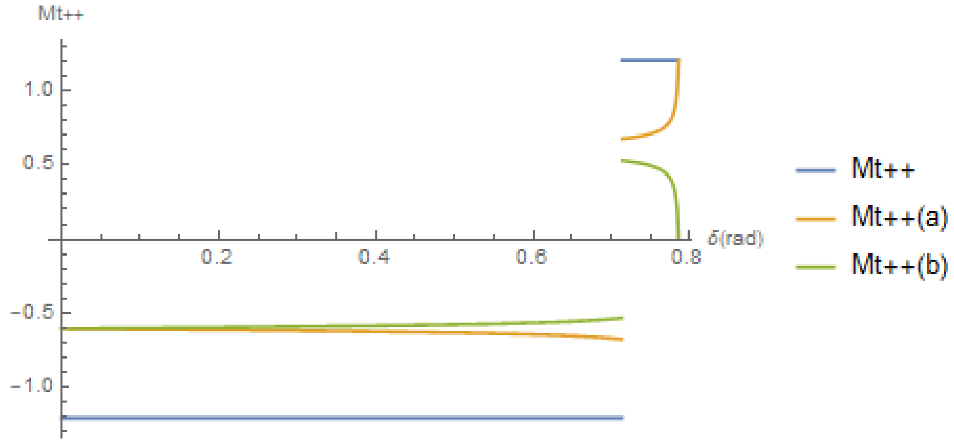


$\rho^0\rho^0$ → “$\pi^+\pi^-$”

$P_\gamma = \sqrt{3}m_e \quad P_e = \sqrt{3.5}m_e$







$$M_t = (-p_1 + q_1)^\mu \varepsilon_\mu^*(p_3,\lambda_3)\frac{1}{q_1^2-m^2}(p_2 + q_1)^\nu \varepsilon_\nu^*(p_4,\lambda_4)$$

$$(-p_1 + q_1)^\mu \varepsilon_\mu^*(p_3,\lambda_3) = -2\,(p_1)^\mu \varepsilon_\mu^*(p_3,\lambda_3) \quad \rightarrow (\delta_p^\pm, \delta_{pt}^{00})$$

$$(p_2 + q_1)^\nu \varepsilon_\nu^*(p_4,\lambda_4) = 2(p_2)^\nu \varepsilon_\nu^*(p_4,\lambda_4) \quad \rightarrow (\delta_e^\pm, \delta_{et}^{00})$$

$$M_u = (-p_1 + q_2)^\nu \varepsilon_\nu^*(p_4,\lambda_4)\frac{1}{q_2^2-m^2}(p_2 + q_2)^\mu \varepsilon_\mu^*(p_3,\lambda_3)$$

$$(-p_1 + q_2)^\nu \varepsilon_\nu^*(p_4,\lambda_4) = -2(p_1)^\nu \varepsilon_\nu^*(p_4,\lambda_4) \quad \rightarrow (\delta_p^\pm, \delta_{pu}^{00})$$

$$(p_2 + q_2)^\mu \varepsilon_\mu^*(p_3,\lambda_3) = 2\,(p_2)^\mu \varepsilon_\mu^*(p_3,\lambda_3) \quad \rightarrow (\delta_e^\pm, \delta_{eu}^{00})$$

Ex

- A positive scalar meson projection to the polarization vector of the rho meson gives $\delta_p^\pm$ or δ_{pt}^{00} critical angles depending on whether polarization vectors are transverse or longitudinal respectively.

Critical scattering angle

"e^+e^-" → $\gamma\gamma$/ "$\pi^+\pi^-$" → $\rho^0\rho^0$

$$q_1^+ = \frac{-\frac{pepz}{2E0} + \frac{pzp\gamma Cos[\theta]}{2E0}}{\sqrt{2}} + \frac{-\frac{pe\sqrt{4E0^2+pz^2}}{2E0} + \frac{\sqrt{4E0^2+pz^2}p\gamma Cos[\theta]}{2E0}}{\sqrt{2}}$$

$$q_2^+ = \frac{-\frac{pepz}{2E0} - \frac{pzp\gamma Cos[\theta]}{2E0}}{\sqrt{2}} + \frac{-\frac{pe\sqrt{4E0^2+pz^2}}{2E0} - \frac{\sqrt{4E0^2+pz^2}p\gamma Cos[\theta]}{2E0}}{\sqrt{2}}$$

$\gamma\gamma$ → "$e^+ e^-$" / $\rho^0\rho^0$ → "$\pi^+\pi^-$"

$$q_1^+ = \frac{\left(\frac{pz}{\sqrt{2}} + \frac{\sqrt{4E0^2+pz^2}}{\sqrt{2}}\right)(p\gamma - peCos[\theta])}{2E0}$$

$$q_2^+ = -\frac{\left(\frac{pz}{\sqrt{2}} + \frac{\sqrt{4E0^2+pz^2}}{\sqrt{2}}\right)(p\gamma + peCos[\theta])}{2E0}$$

$q_1^+ > 0$ →Forward $\quad$ $q_1^+ < 0$ → Backward

$q_2^+ > 0$ →Forward $\quad$ $q_2^+ < 0$ → Backward

"e^+e^-" $\rightarrow \gamma\gamma$

$$q_1^+ = 0 \quad \rightarrow \quad \theta_{c,t} = ArcCos\left(\frac{P_e}{E_0}\right)$$

$$q_2^+ = 0 \quad \rightarrow \quad \theta_{c,u} = -ArcCos\left(\frac{P_e}{E_0}\right)$$

$$\rightarrow \exists\ Cos(\theta_c) \leftrightarrow P_e < E_0$$

$\gamma\gamma \rightarrow$ "$e^+\ e^-$"

$$\theta_{c,t} = ArcCos\left(\frac{E_0}{P_e}\right)$$

$$\theta_{c,u} = -ArcCos\left(\frac{E_0}{P_e}\right)$$

$$\rightarrow \exists\ Cos(\theta_c) \leftrightarrow E_0 < P_e$$

"$\pi^+\pi^-$" $\rightarrow \rho^0\rho^0$

$$q_1^+ = 0 \quad \rightarrow \quad \theta_{c,t} = ArcCos\left(\frac{P_\pi}{P_\rho}\right)$$

$$q_2^+ = 0 \quad \rightarrow \quad \theta_{c,u} = -ArcCos\left(\frac{P_\pi}{P_\rho}\right)$$

$$\rightarrow \exists\ Cos(\theta_c) \leftrightarrow P_\pi < P_\rho$$

$\rho^0\rho^0 \rightarrow$ "$\pi^+\pi^-$"

$$\theta_{c,t} = ArcCos\left(\frac{P_\rho}{P_\pi}\right)$$

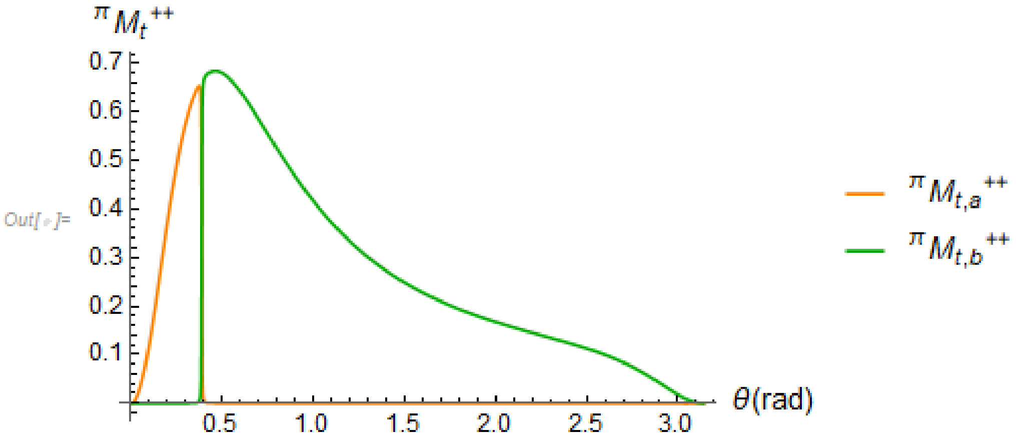
$$\theta_{c,u} = -ArcCos\left(\frac{P_\rho}{P_\pi}\right)$$

$$\rightarrow \exists\ Cos(\theta_c) \leftrightarrow P_\rho < P_\pi$$

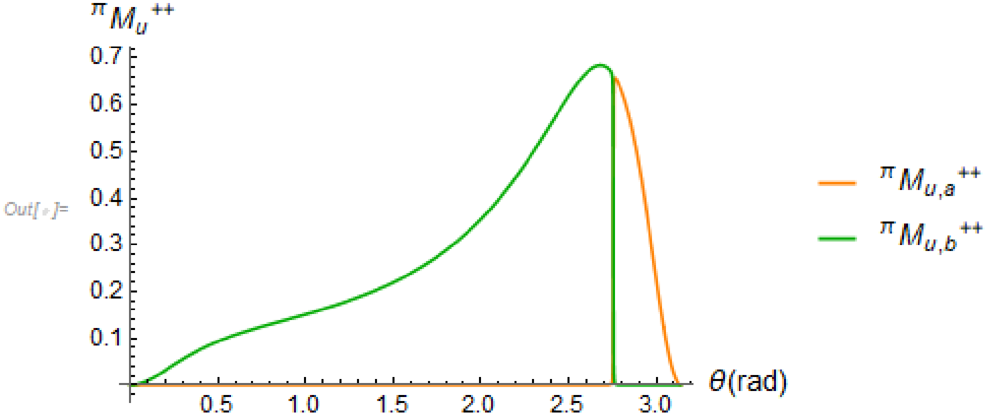
$$E_0 = 2m_e$$

$$\delta = 0.785398 \sim \frac{\pi}{4}$$

“$\pi^+\pi^-$” → $\rho^0\rho^0$



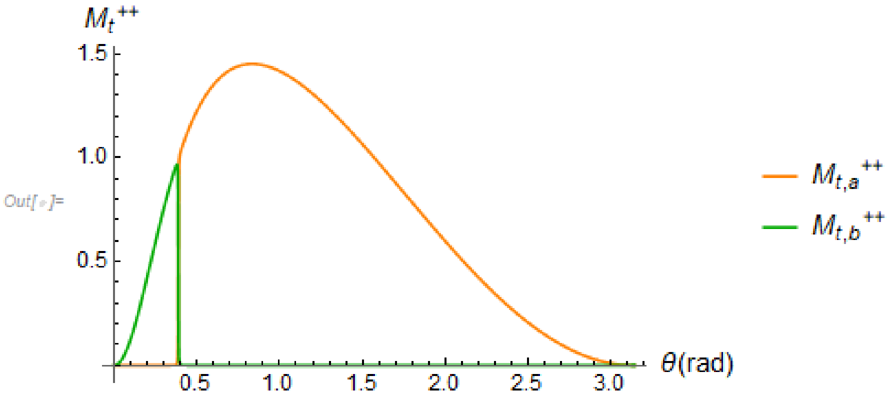
$P_e = \sqrt{3}m_e \quad P_\gamma = \sqrt{3.5}m_e$



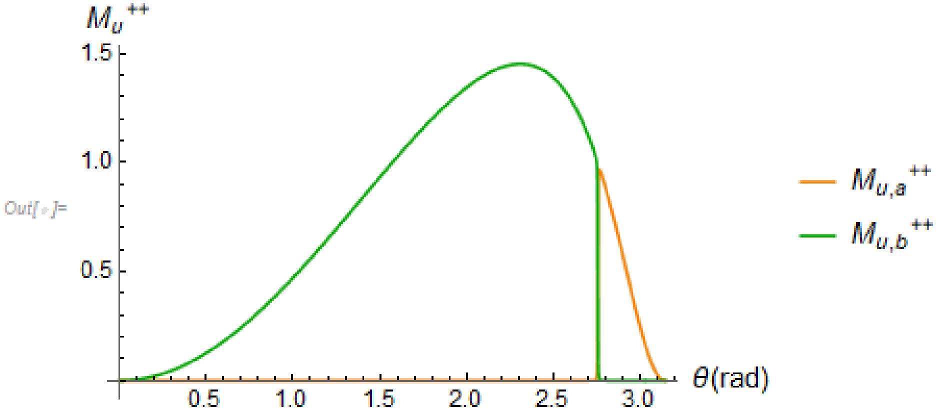
$$\theta_{c,t} = ArcCos\left(\sqrt{\frac{3}{3.5}}\right) = 0.387597$$

$$\theta_{c,u} = -ArcCos\left(\sqrt{\frac{3}{3.5}}\right) = 2.754$$

$\rho^0\rho^0$ → “$\pi^+\pi^-$”



$P_\gamma = \sqrt{3}m_e \quad P_e = \sqrt{3.5}m_e$



Thank you