Scalar electron and scalar positron pair annihilation to two photons

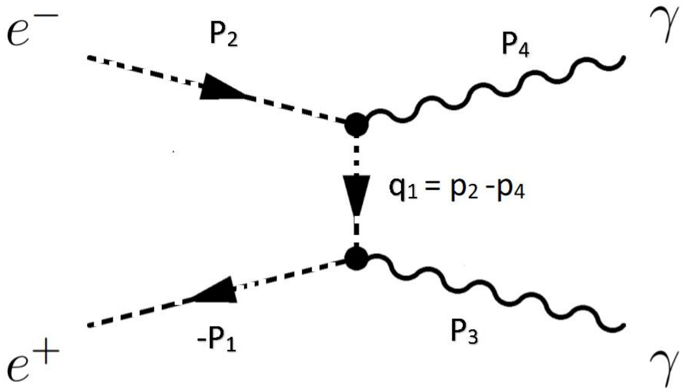
$$"\, e^{-}e^{+}\," \rightarrow \gamma\gamma$$

11-02-2018

Deepasika Dayananda

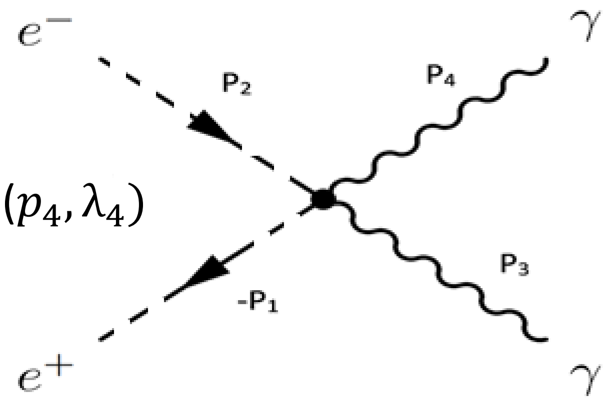
Lowest –order Covariant annihilation diagrams

t-Channel



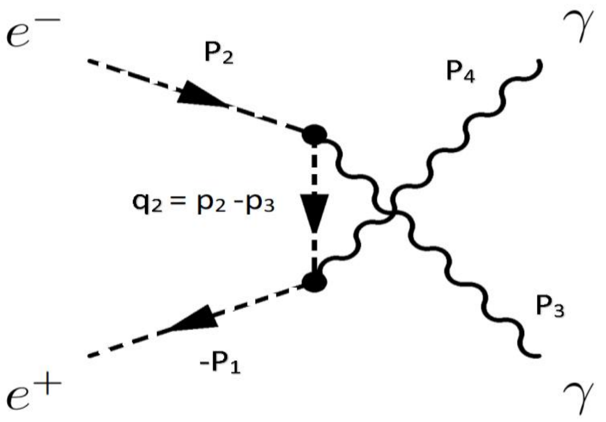
$$M_t = (-p_1 + q_1)^{\mu} \varepsilon_{\mu}^{*}(p_3, \lambda_3) \frac{1}{q_1^2 - m^2} (p_2 + q_1)^{\nu} \varepsilon_{\nu}^{*}(p_4, \lambda_4)$$

Seagull



$$M_{se} = -2 g_{\mu\nu} \epsilon^{*\mu}(p_3, \lambda_3) \varepsilon^{*\nu} (p_4, \lambda_4)$$

u-Channel



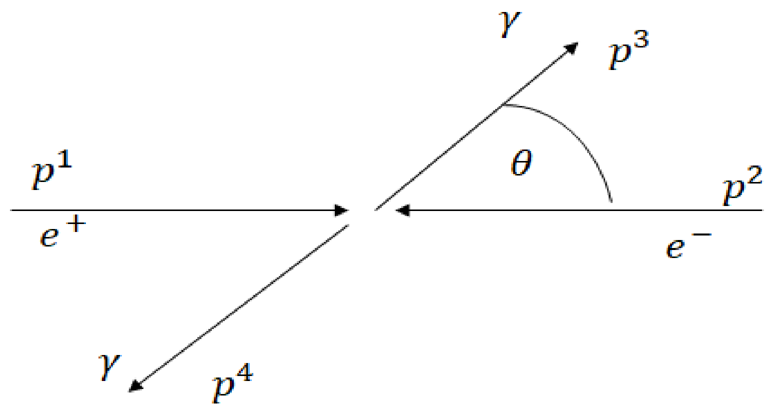
$$M_u = (-p_1 + q_2)^{\nu} \varepsilon_{\nu}^{*}(p_4, \lambda_4) \frac{1}{q_2^2 - m^2} (p_2 + q_2)^{\mu} \varepsilon_{\mu}^{*}(p_3, \lambda_3)$$

Covariant Feynman amplitude

$$M = M_t + M_u + 2M_{se}$$

$$M \quad = (-p_1 \; + q_1)^{\mu} \varepsilon_{\mu}^{*}(p_3, \lambda_3) \frac{1}{q_1^2 - m^2} (p_2 \; + q_1)^{\nu} \varepsilon_{\nu}^{*}(p_4, \lambda_4)$$
$$+ (-p_1 \; + q_2)^{\nu} \varepsilon_{\nu}^{*}(p_4, \lambda_4) \frac{1}{q_2^2 - m^2} (p_2 \; + q_2)^{\mu} \; \varepsilon_{\mu}^{*}(p_3, \lambda_3 \;)$$
$$-2 g_{\mu\nu} \epsilon^{*\mu}(p_3, \lambda_3) \varepsilon^{*\nu} \; (p_4, \lambda_4)$$

Center of mass kinematics



$$p^1 = \{E_0, 0{,}0, P_e\}$$

$$p^2 = \{E_0, 0{,}0, -P_e\}$$

$$p^3 = \{E_0, E_0 Sin(\theta), 0, E_0 Cos(\theta)\}$$

$$p^4 = \{E_0, -E_0 Sin(\theta), 0, -E_0 Cos(\theta)\}$$

Photon polarization vectors

Instant form

$$\epsilon_\mu(p,+) = -\frac{1}{\sqrt{2}\,|\vec{p}|}\,\left(0, \frac{-p_x p_z + i p_y|\vec{p}|}{|p_\perp|}, \frac{-p_y p_z - i p_x|\vec{p}|}{|p_\perp|}, |p_\perp|\right)$$

$$\epsilon_\mu(p,-) = \frac{1}{\sqrt{2}\,|\vec{p}|}\,\left(0, \frac{-p_x p_z - i p_y|\vec{p}|}{|p_\perp|}, \frac{-p_y p_z + i p_x|\vec{p}|}{|p_\perp|}, |p_\perp|\right)$$

$$g_{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Constraints

$$\epsilon_\mu(p,\lambda)p^\mu = 0$$

$$\epsilon^*(p,\lambda).\epsilon(p,\lambda') = -\delta_{\lambda\lambda'}$$

Chueng-Ryong Ji, Alfredo Takashi Suzuki PhysRevD.87.065015

Instant form covariant helicity amplitude

$$M_t^{++} = \frac{-2p_e^2 Sin^2(\theta)}{E_0^2 + m^2 + p_e^2 - 2E_0 P_e Cos(\theta)} = M_t^{--}$$

$$M_u^{++} = \frac{-2p_e^2 Sin^2(\theta)}{E_0^2 + m^2 + p_e^2 + 2E_0 P_e Cos(\theta)} = M_u^{--}$$

$$M_{se}^{++} = M_{se}^{--} = 2$$

$$M_t^{+-} = \frac{2p_e^2 Sin^2(\theta)}{E_0^2 + m^2 + p_e^2 - 2E_0 P_e Cos(\theta)} = M_t^{-+}$$

$$M_u^{+-} = \frac{2p_e^2 Sin^2(\theta)}{E_0^2 + m^2 + p_e^2 + 2E_0 P_e Cos(\theta)} = M_u^{-+}$$

$$M_{se}^{+-} = M_{se}^{-+} = 0$$

Cross section of a $2 \rightarrow 2$ process

$$\frac{d\sigma}{d\Omega} \propto |M|^2$$

$$|M|^2 = \sum_{\lambda_1,\lambda_2} |M_t^{\lambda_1,\lambda_2} + M_u^{\lambda_1,\lambda_2} + M_{se}^{\lambda_1,\lambda_2}|^2$$

$$= 4\left[1 + \left[1 - \frac{2p_e^2 Sin^2(\theta)}{E_0^2 - p_e^2 Cos^2(\theta)}\right]^2\right]$$

$$; E_0^2 = m^2 + p_e^2$$

Calculating cross section as a function of Mandelstam variable.

$$M \;= (-p_1 + q_1)^{\mu}\varepsilon^*_{\mu}(p_3,\lambda_3)\frac{1}{q_1^2-m^2}(p_2 + q_1)^{\nu}\varepsilon^*_{\nu}(p_4,\lambda_4)$$
$$+ (-p_1 + q_2)^{\nu}\varepsilon^*_{\nu}(p_4,\lambda_4)\frac{1}{q_2^2-m^2}(p_2 + q_2)^{\mu}\,\varepsilon^*_{\mu}(p_3,\lambda_3)$$
$$-2g_{\mu\nu}\epsilon^{*\mu}(p_3,\lambda_3)\varepsilon^{*\nu}(p_4,\lambda_4)$$

$$s = (p_1 + p_2)^2$$

$$t = (p_3 - p_1)^2 = q_1^2$$

$$u = (p_4 - p_1)^2 = q_2^2$$

$$M = \varepsilon^*_{\mu}(p_3,\lambda_3)\varepsilon^*_{\nu}(p_4,\lambda_4)[J^{t_{\mu\nu}} + J^{u_{\mu\nu}} + J^{se_{\mu\nu}}]$$

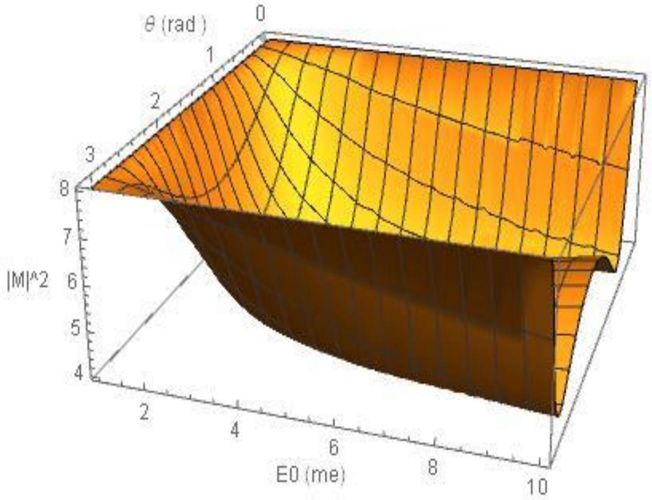
$$s + t + u = 2m^2$$

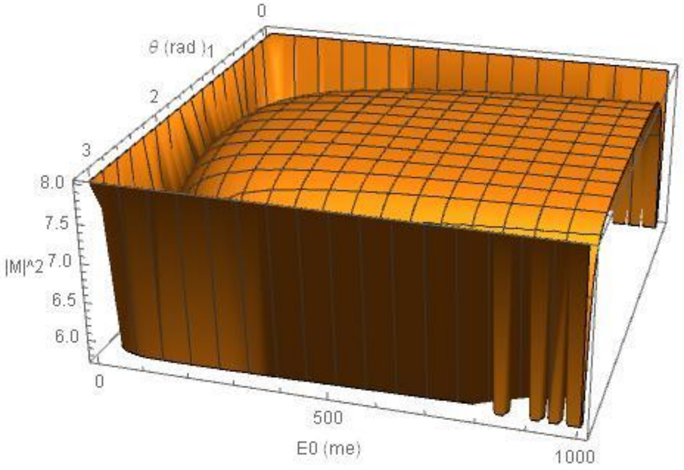
$$\sum_{\lambda}\varepsilon^*_{\mu}(p,\lambda)\epsilon_{\mu'}(p,\lambda) = -g_{\mu\mu'}$$

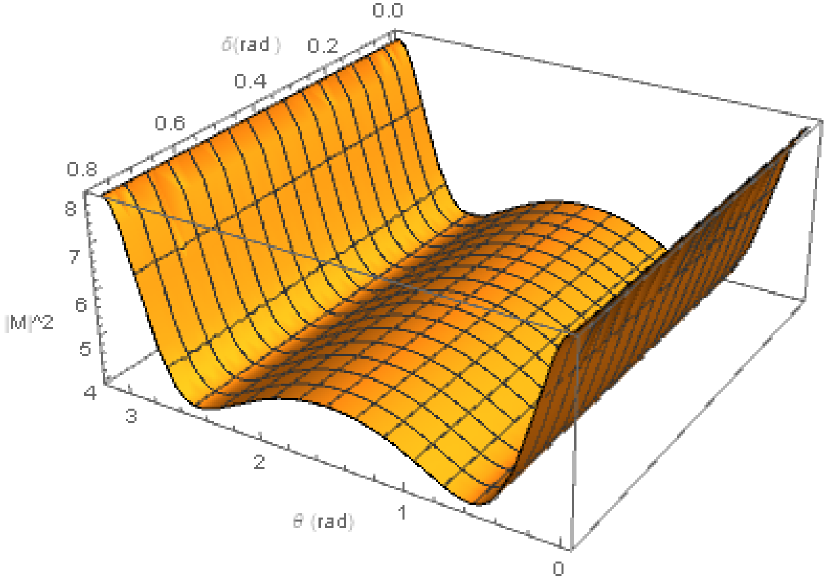
$$|M|^2 = [J^t_{\mu\nu} + J^u_{\mu\nu} + J^{se}_{\mu\nu}]\,[J^{t_{\mu\nu}} + J^{u_{\mu\nu}} + J^{se_{\mu\nu}}]$$

$$|M|^2 = 4\left[\left[\frac{t+m^2}{t-m^2}\right]^2 + \left[\frac{u+m^2}{u-m^2}\right]^2 + 4\left[\frac{(t+m^2)(u+m^2)-2tu}{(t-m^2)(u-m^2)}\right] + 4\right]$$

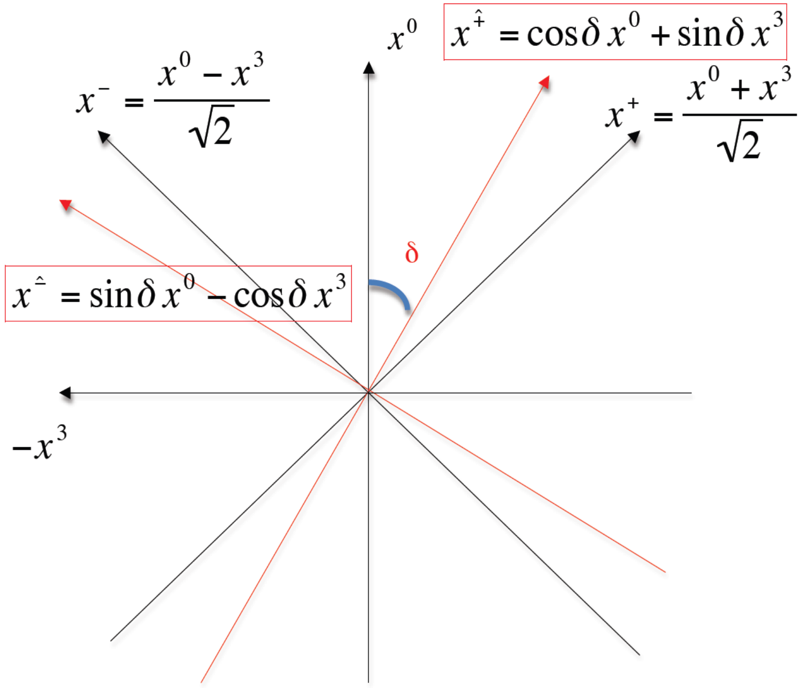
$$= 4\left[1 + \left[1 - \frac{2Sin^2(\theta)}{\frac{E_0^2}{E_0^2 - m_e^2} - Cos^2(\theta)}\right]^2\right]$$







Interpolating Dynamic



$$\begin{pmatrix} x^{\hat{+}} \\ x^{\hat{1}} \\ x^{\hat{2}} \\ x^{\hat{-}} \end{pmatrix} = \begin{pmatrix} cos(\delta) & 0 & 0 & \sin(\delta) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \sin(\delta) & 0 & 0 & -cos(\delta) \end{pmatrix} \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}$$

$$g_{\hat{\mu}\hat{\nu}} = \begin{bmatrix} cos(2\delta) & 0 & 0 & \sin(2\delta) \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \sin(2\delta) & 0 & 0 & -cos(2\delta) \end{bmatrix}$$

Interpolating helicity amplitude

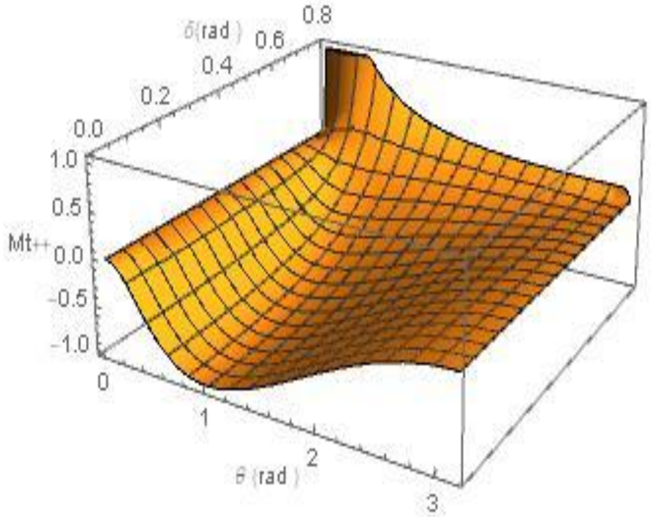
$$M_t^{\hat{+}\hat{+}} = \frac{2(-p_e^2 Cos^2(\delta) + E_0^2 Sin^2(\delta))Sin^2(\theta)}{Abs[Cos^2(\delta) - Cos^2(\theta)Sin^2(\delta)][E_0^2 + m^2 + p_e^2 - 2E_0 P_e Cos(\theta)]} = M_t^{\hat{-}\hat{-}}$$

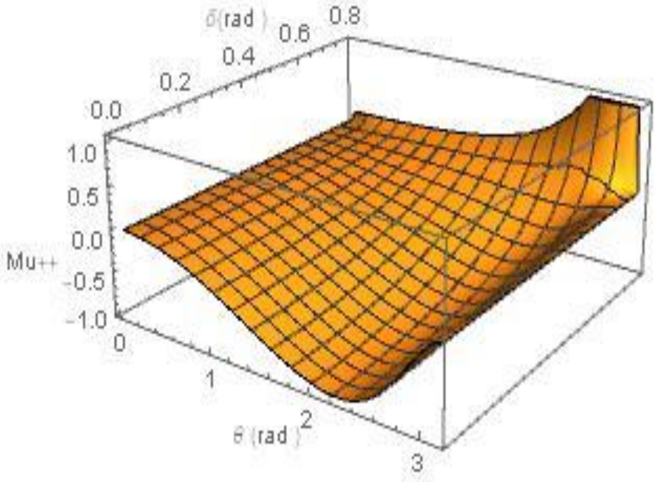
$$E_0 = 2m_e$$

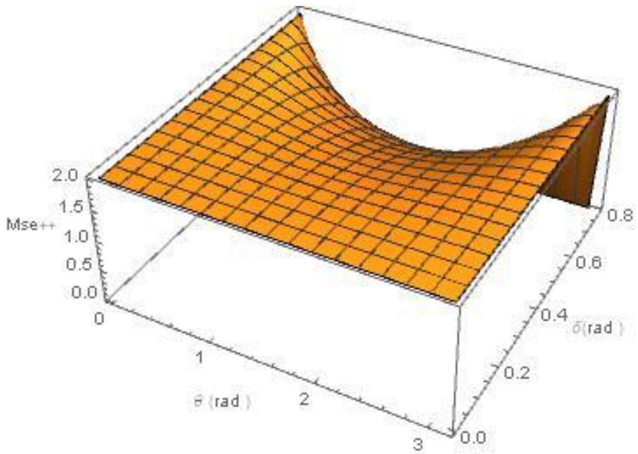
$$P_e = \sqrt{3}m_e$$

$$M_u^{\hat{+}\hat{+}} = \frac{2(-p_e^2 Cos^2(\delta) + E_0^2 Sin^2(\delta))Sin^2(\theta)}{Abs[Cos^2(\delta) - Cos^2(\theta)Sin^2(\delta)][E_0^2 + m^2 + p_e^2 + 2E_0 P_e Cos(\theta)]} = M_u^{\hat{-}\hat{-}}$$

$$M_{se}^{\hat{+}\hat{+}} = \left[1 - \left[\frac{Sin^2(\delta)[2 + Sin^2(\theta)] - 1}{Abs[Cos^2(\delta) - Cos^2(\theta)Sin^2(\delta)]}\right]\right] = M_{se}^{\hat{-}\hat{-}}$$



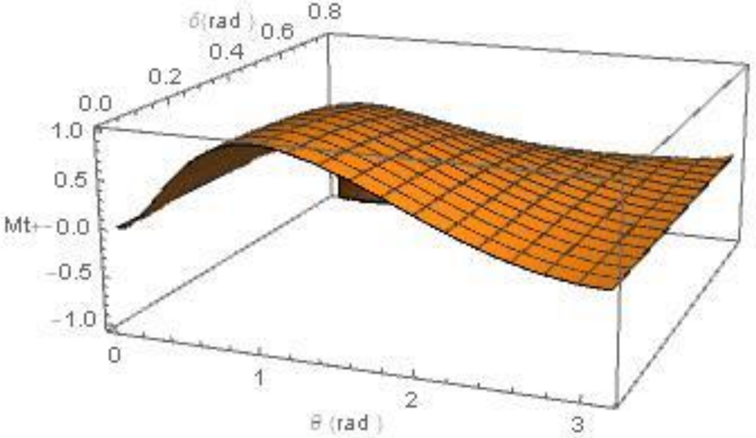


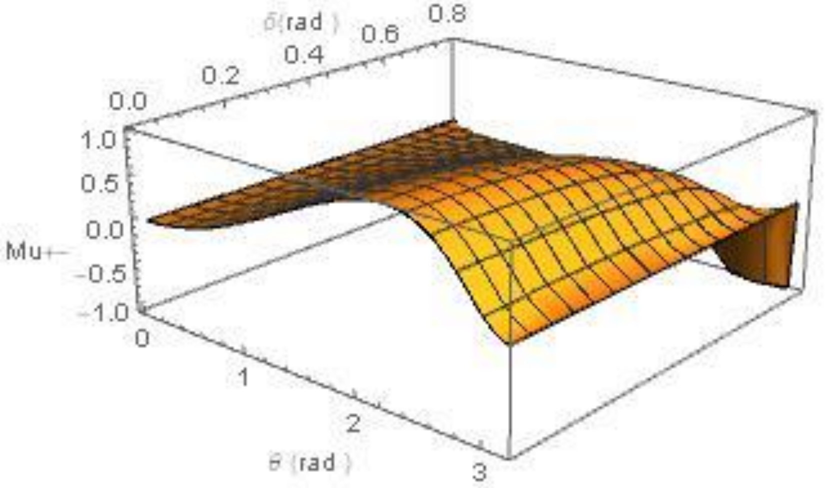


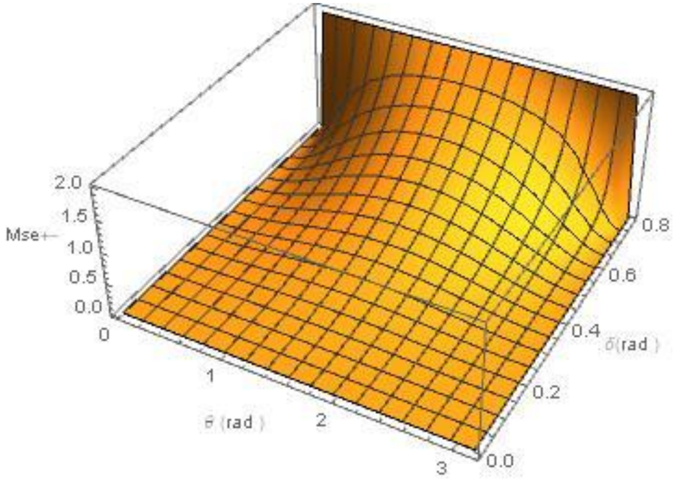
$$M_t^{\hat{+}\hat{-}} = \frac{2(p_e^2 Cos^2(\delta) - E_0^2 Sin^2(\delta))Sin^2(\theta)}{Abs[Cos^2(\delta) - Cos^2(\theta)Sin^2(\delta)][E_0^2 + m^2 + p_e^2 - 2E_0P_eCos(\theta)]} = M_t^{\hat{-}\hat{+}}$$

$$M_u^{\hat{+}\hat{-}} = \frac{2(p_e^2 Cos^2(\delta) - E_0^2 Sin^2(\delta))Sin^2(\theta)}{Abs[Cos^2(\delta) - Cos^2(\theta)Sin^2(\delta)][E_0^2 + m^2 + p_e^2 + 2E_0P_eCos(\theta)]} = M_u^{\hat{-}\hat{+}}$$

$$M_{se}^{\hat{+}\hat{-}} = \left[1 + \left[\frac{Sin^2(\delta)[2 + Sin^2(\theta)] - 1}{Abs[Cos^2(\delta) - Cos^2(\theta)Sin^2(\delta)]}\right]\right] = M_{se}^{\hat{-}\hat{+}}$$





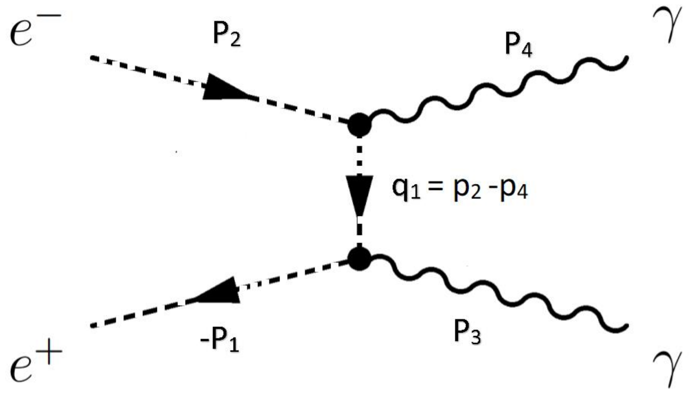


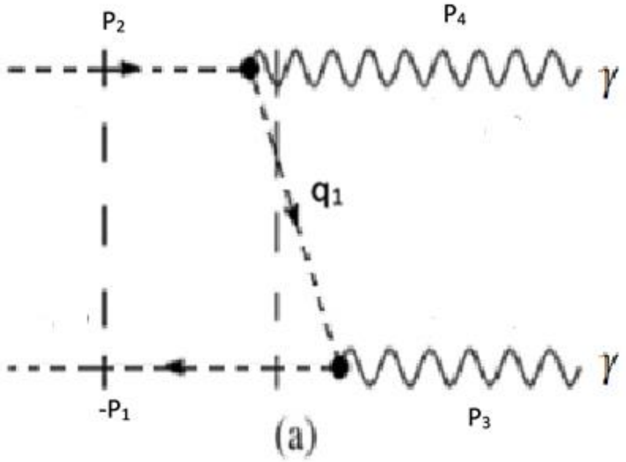
t-Channel

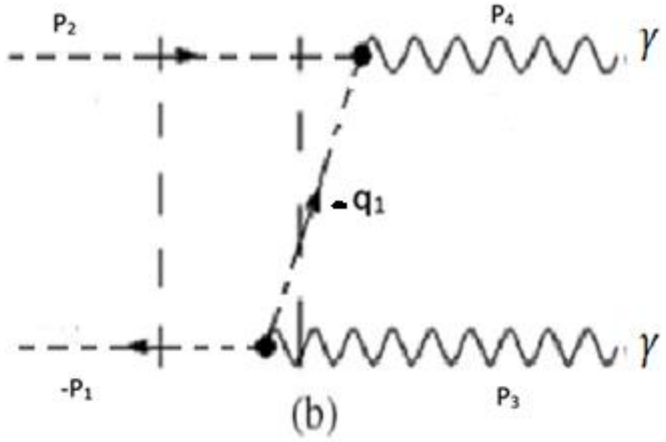
Covariant Propagator

$$\Sigma = \frac{1}{q_1^2 - m^2}$$

Time ordered amplitudes







$$\Sigma = \Sigma_a^{\mathrm{IFD}} + \Sigma_b^{\mathrm{IFD}}$$

$$\Sigma_a^{\mathrm{IFD}} = \frac{1}{2q_{0n}^0 (q_1^0 - q_{0n}^0)}$$

$$\Sigma_b^{\mathrm{IFD}} = -\frac{1}{2q_{0n}^0 (q_1^0 + q_{0n}^0)}$$

$$q_1^0 = p_2^0 - p_4^0$$

On-shell condition $q_{0n}^0 = \sqrt{(\overrightarrow{q_1}^2 + m^2)}$; $\vec{q}_1 = \vec{p}_2 - \vec{p}_4$

Time ordering in the interpolation dynamics

$$\Sigma_a^{\delta} = \frac{C}{2Q^{\hat{+}}\,(q^{\hat{+}} - Q^{\hat{+}})}$$

$$\Sigma_b^{\delta} = -\frac{C}{2Q^{\hat{+}}\,(q^{\hat{+}} + Q^{\hat{+}})}$$

$$\Sigma = \Sigma_a^{\delta} + \Sigma_b^{\delta} = \frac{1}{q_1^2 - m^2}$$

$$q^{\hat{+}} = p_2^{\hat{+}} - p_4^{\hat{+}}$$

$$C = Cos(2\delta) \qquad S = Sin(2\delta)$$

On-shell condition

4-momentum of the intermediate scaler $\quad Q_{\hat{\mu}} = (Q_{\hat{+}}\,, q_1\,, q_2\,, Q_{\hat{-}})$

Dispersion relation $\quad Q^2 = m^2$

$$C Q_{\hat{+}}^2 + 2S Q_{\hat{-}} Q_{\hat{+}} - C Q_{\hat{-}}^2 - \vec{q}^{\,2}{}_{\perp} - m^2 = 0$$

When $C \neq 0$, the quadratic equation has two solutions

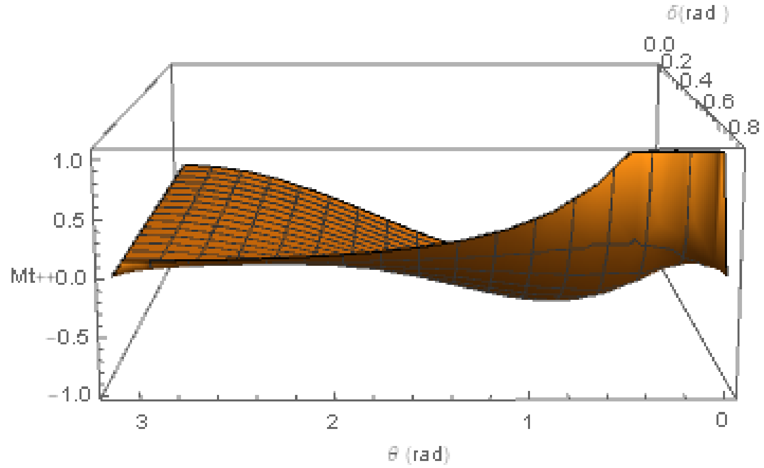
$$Q_{\hat{+}} = \frac{-SQ_{\hat{-}} \pm \sqrt{Q_{\hat{-}}^2 + C(\vec{q}^{\,2}_{\perp} + m^2)}}{C}$$

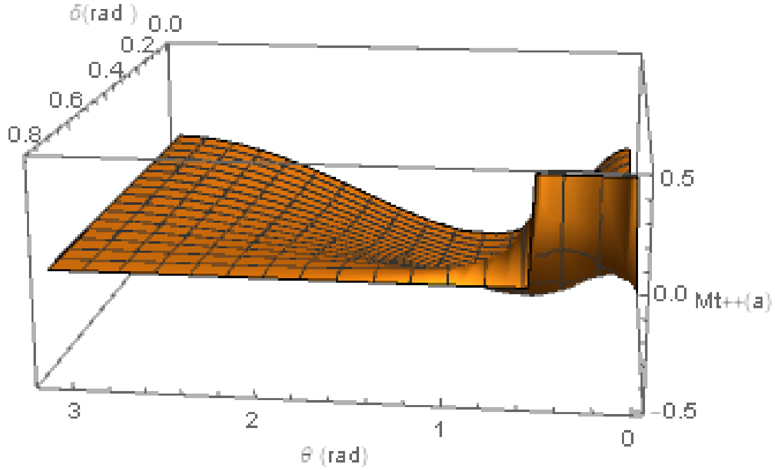
But, $Q^{\hat{+}} = CQ_{\hat{+}} + SQ_{\hat{-}}$ $\rightarrow$ $Q^{\hat{+}} = \pm\sqrt{Q_{\hat{-}}^2 + C(\vec{q}^{\,2}_{\perp} + m^2)}$

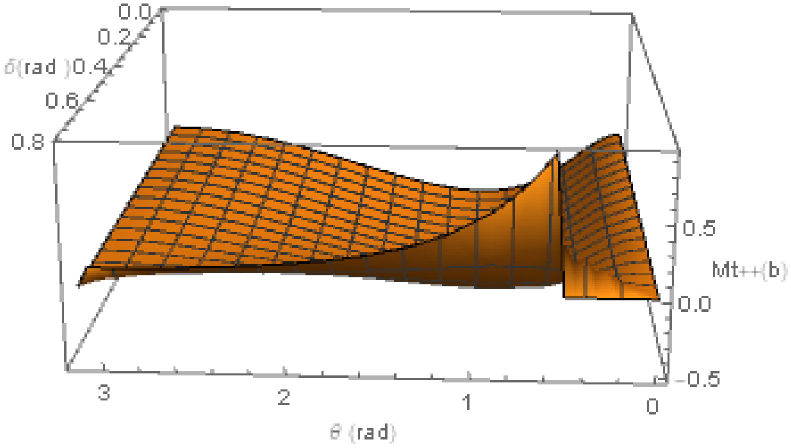
When $C = 0$ (Light Front), quadratic equation becomes a linear equation

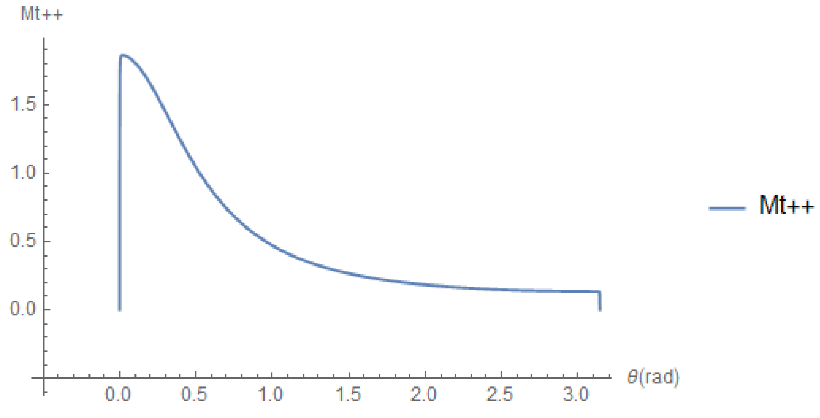
$$Q_{\hat{+}} = \frac{\vec{q}^{\,2}_{\perp} + m^2}{2Q_{\hat{-}}}$$

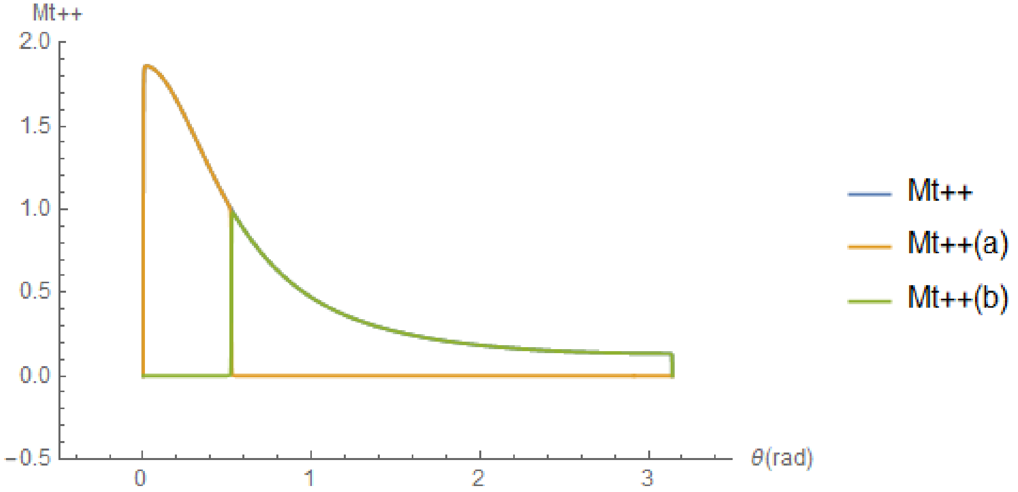
t-channel time ordering process











$$E_0 = 2m_e$$

$$P_e = \sqrt{3}m_e$$

$$\delta = 0.785398 \sim \frac{\pi}{4}$$

$$\theta_c = 0.523599 \sim \frac{\pi}{6}$$

Critical annihilation angle

$$q_1 = p_2 - p_4 = (0, E_0 Sin(\theta), 0, -P_e + E_0 Cos(\theta)\ )$$

$$q_1^+ = \frac{1}{\sqrt{2}}(-P_e + E_0 Cos(\theta))$$

$q_1^+ > 0 \rightarrow$Forward $\quad$ $q_1^+ < 0 \rightarrow$ Backward

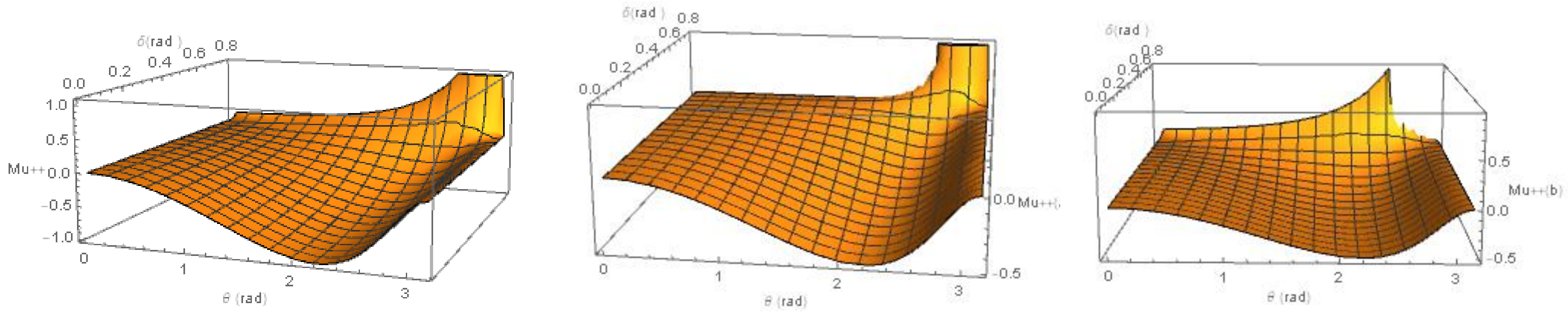
$$q_1^+ = 0 \quad \rightarrow \quad \theta_{c,t} = ArcCos\left(\frac{P_e}{E_0}\right)$$

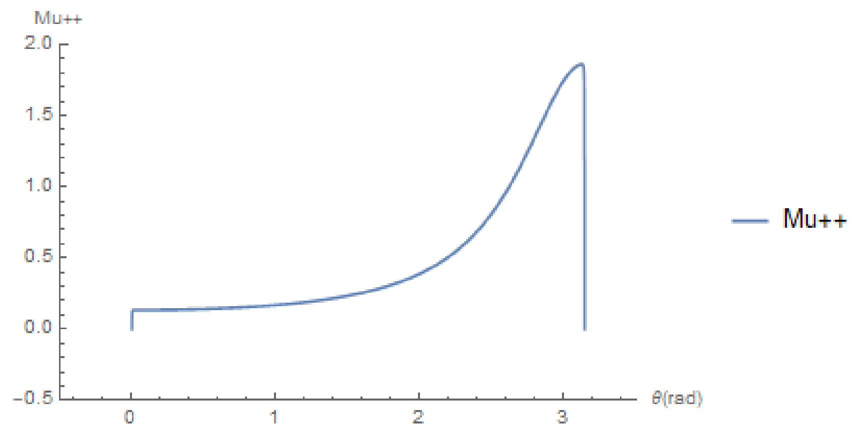
In this case $\theta_{c,t} = ArcCos\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

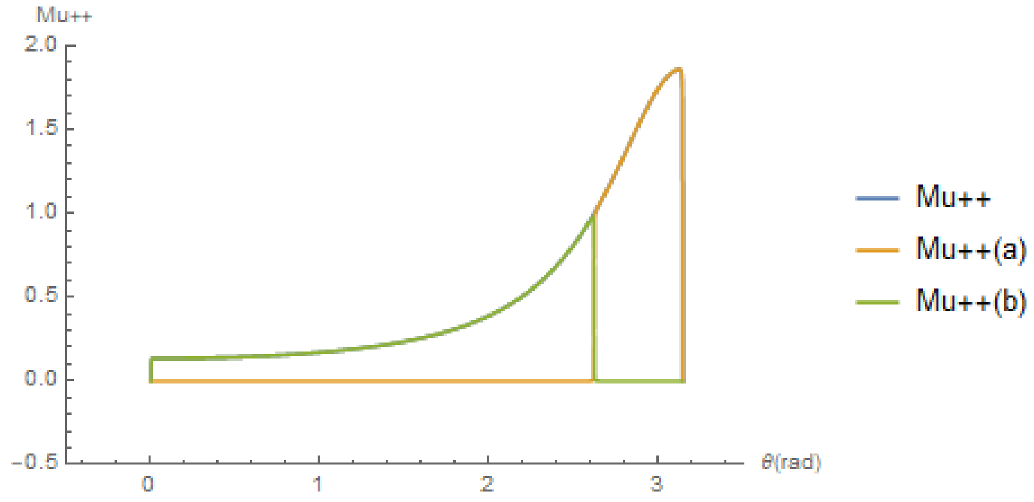
$$q_2^+ = 0 \quad \rightarrow \quad \theta_{c,u} = ArcCos\left(\frac{-P_e}{E_0}\right)$$

In this case $\theta_{c,t} = ArcCos\left(\frac{-\sqrt{3}}{2}\right) = \frac{5\pi}{6}$

u-channel time ordering process







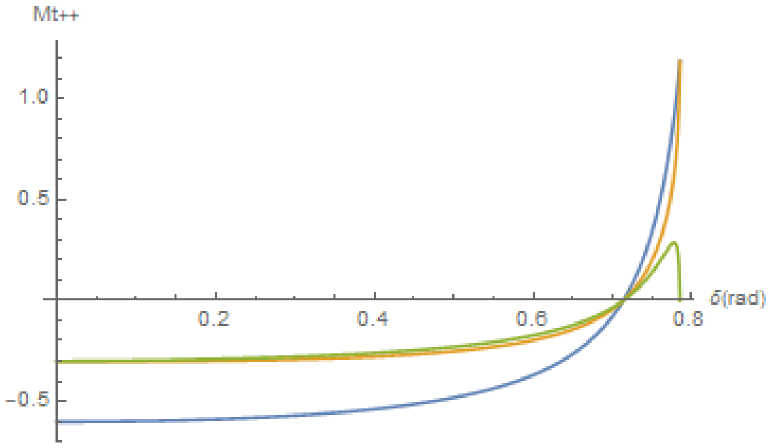
$$E_0 = 2m_e$$

$$P_e = \sqrt{3}m_e$$

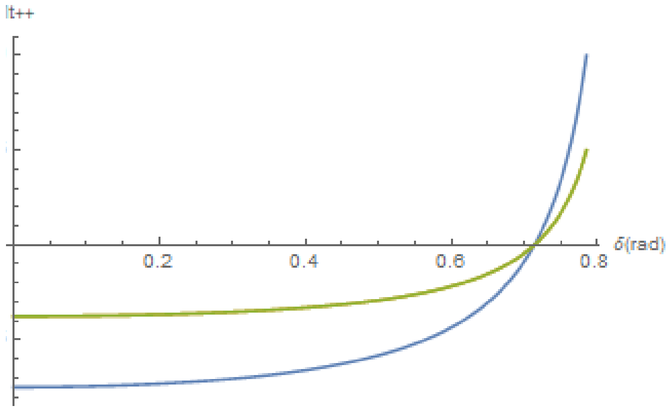
$$\delta = 0.785398 \sim \frac{\pi}{4}$$

$$\theta_{cu} = 2.61799 \sim \frac{5\pi}{6}$$

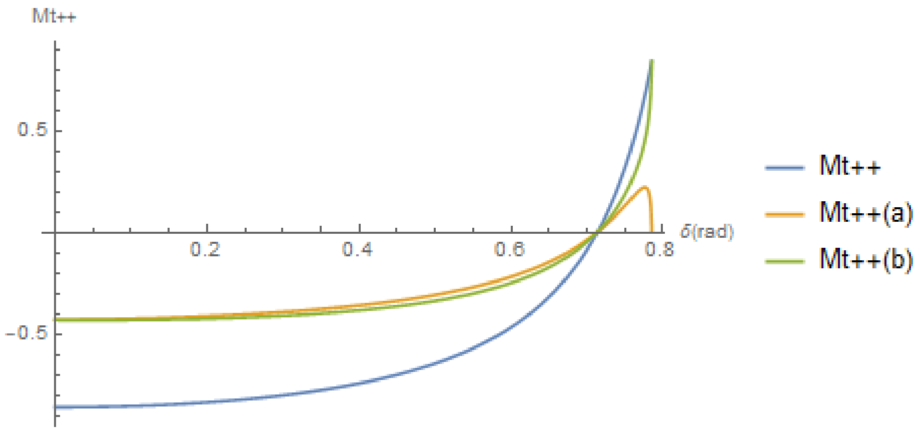
Critical interpolation angle



$$\theta = \frac{\pi}{6} - 0.1$$



$$\theta = \frac{\pi}{6} = \theta_{c,t}$$



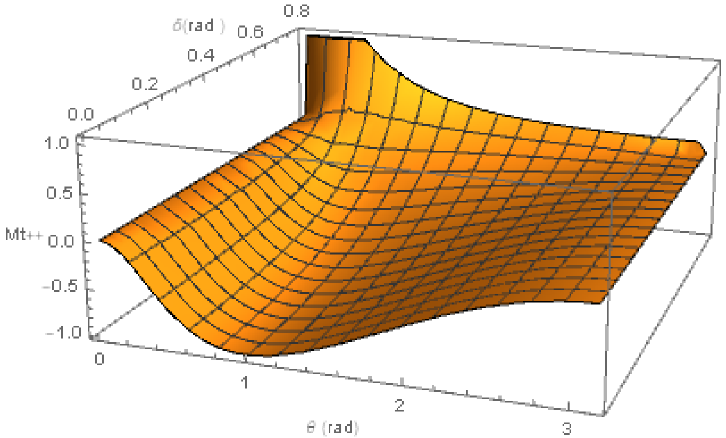
$$\theta = \frac{\pi}{6} + 0.1$$

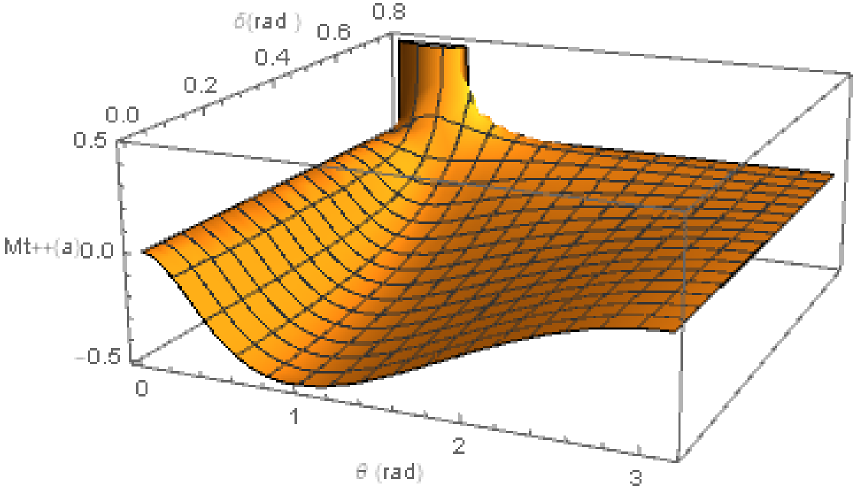
$$\delta_c = ArcTan\left(\frac{|\boldsymbol{P}|}{E_0}\right)$$

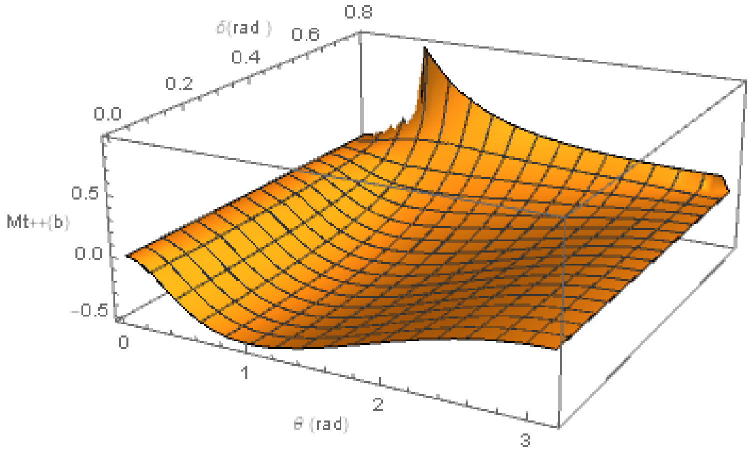
In this case= $\delta_c = ArcTan\left(\frac{|\boldsymbol{P}|}{E_0}\right) = 0.713724$

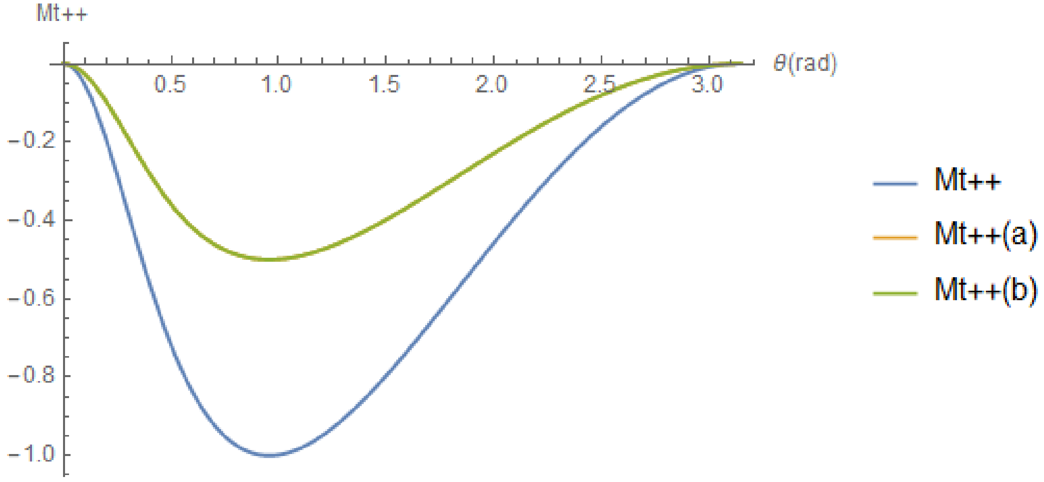
$|\boldsymbol{P}| =$ Magnitude of the three momentum

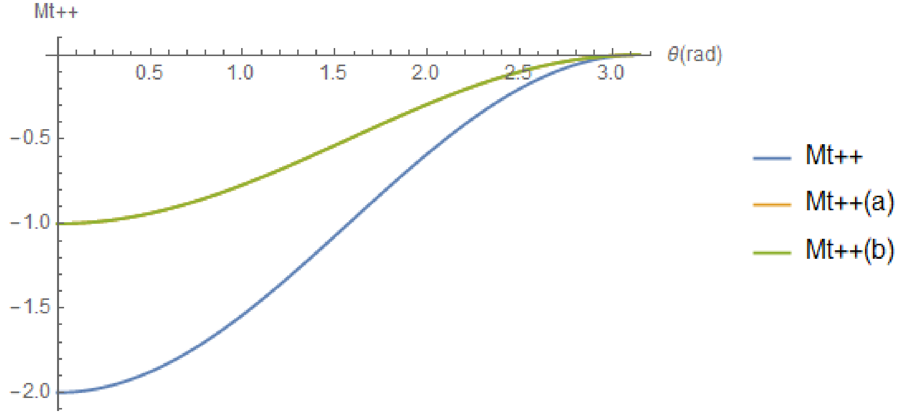
Chiral symmetry $\delta \to 0$





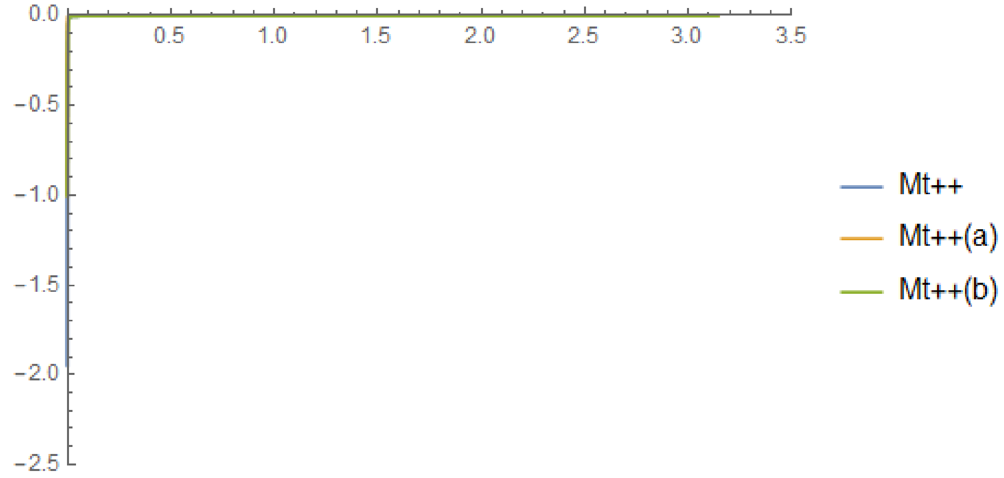


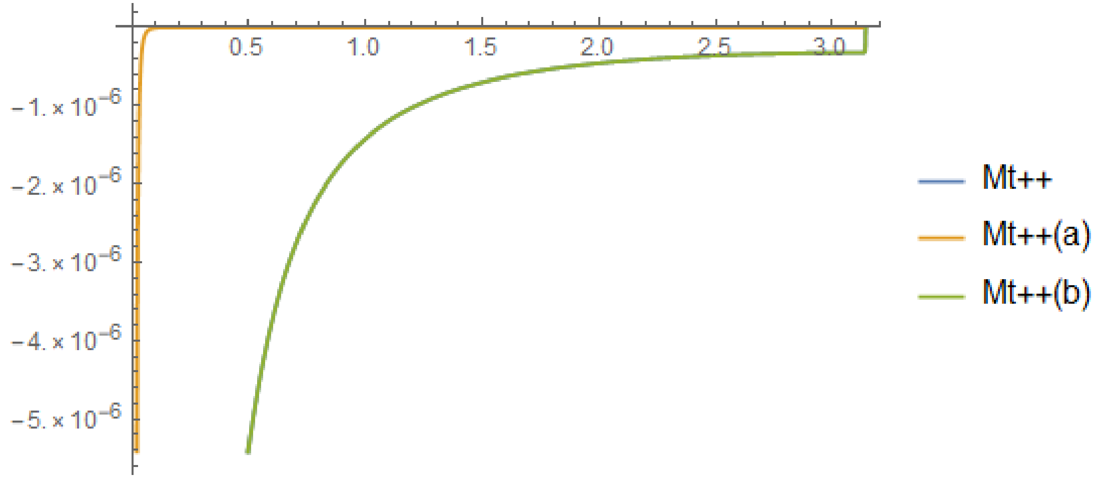


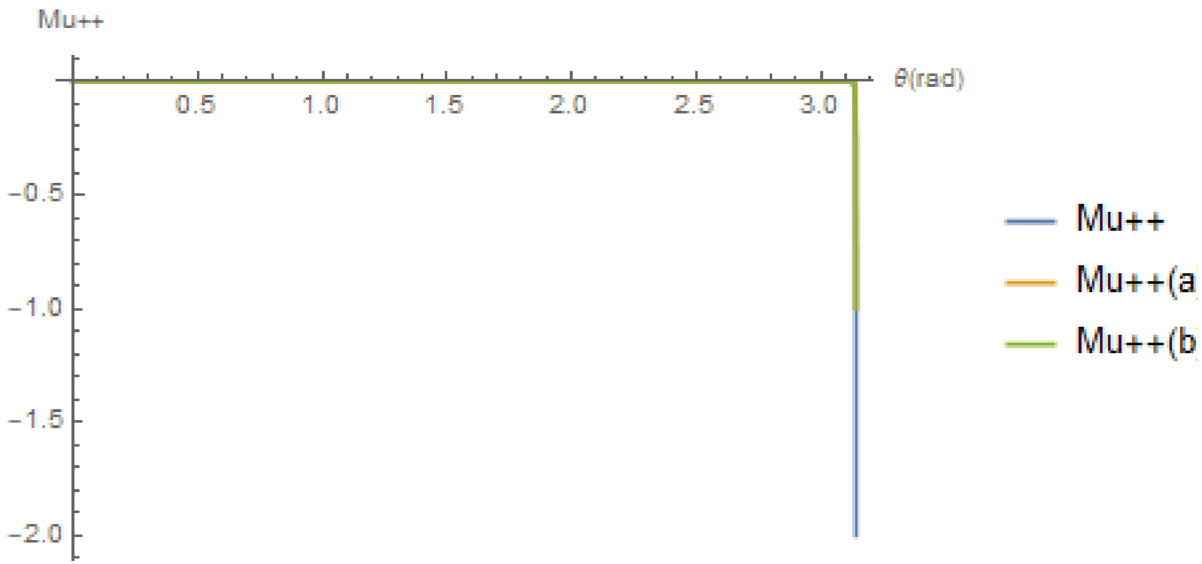


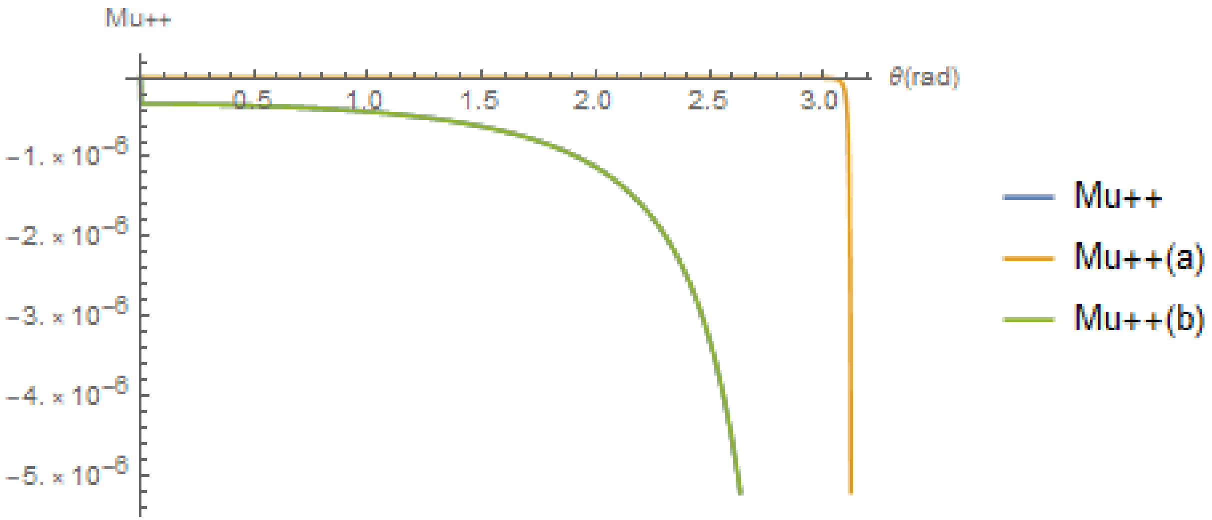
$m_e \to 0$

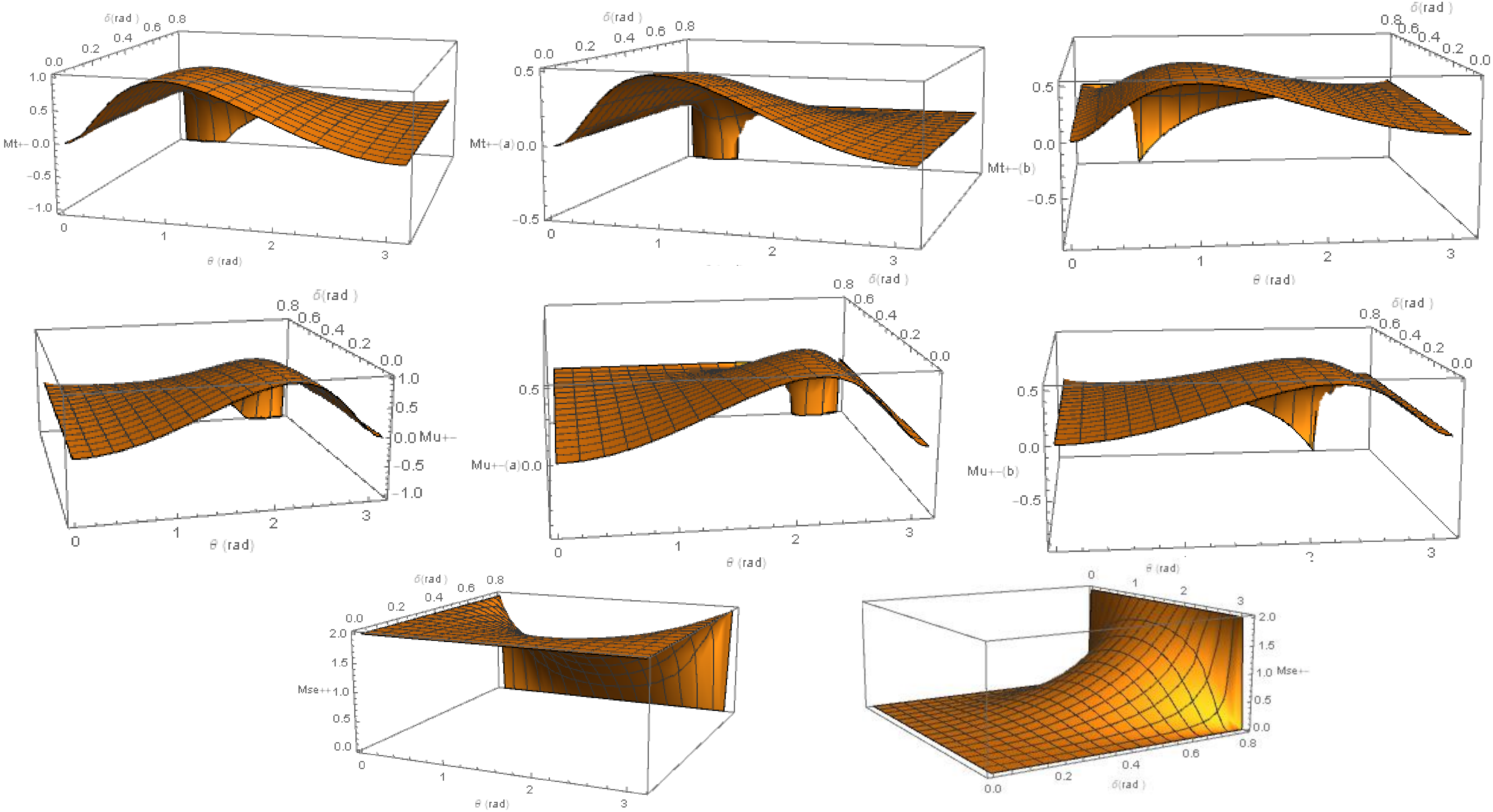
Chiral symmetry $\delta \to \pi/4$



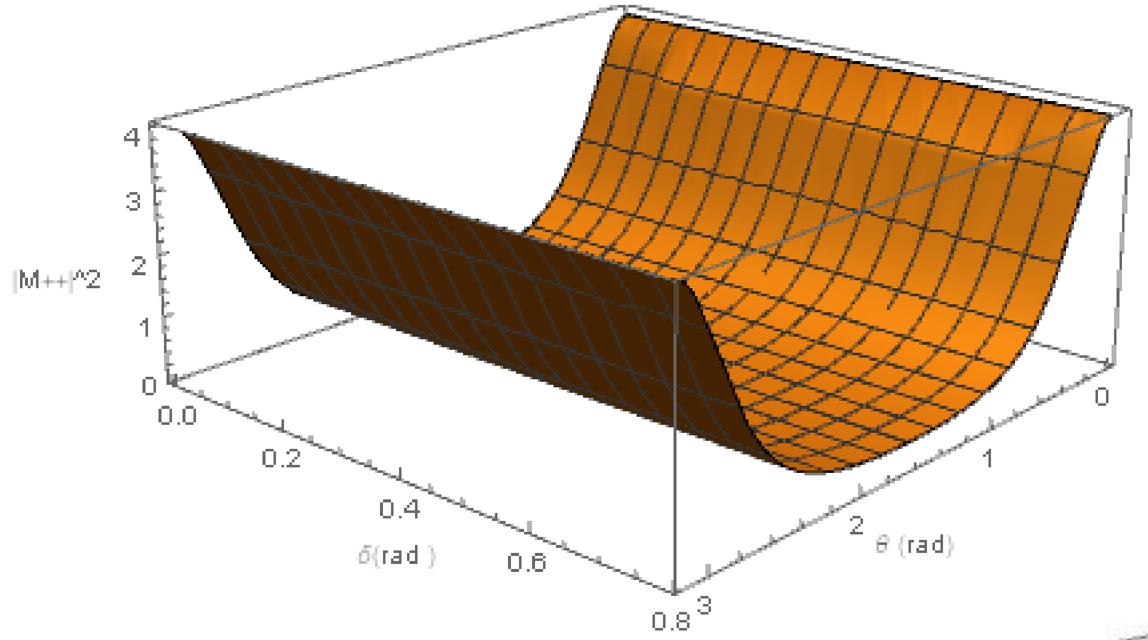


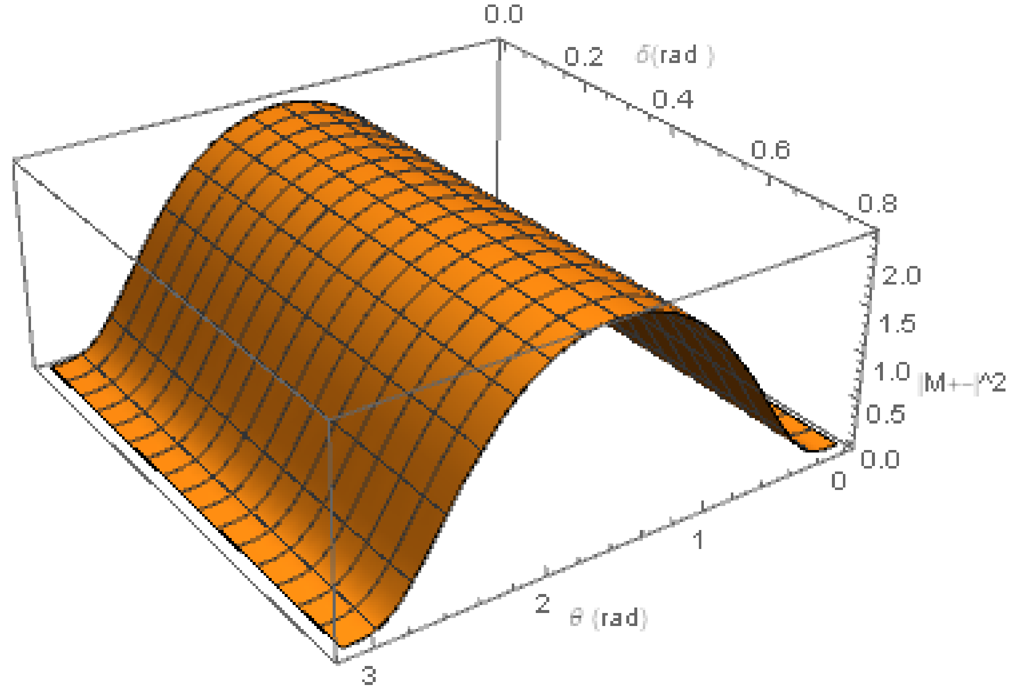


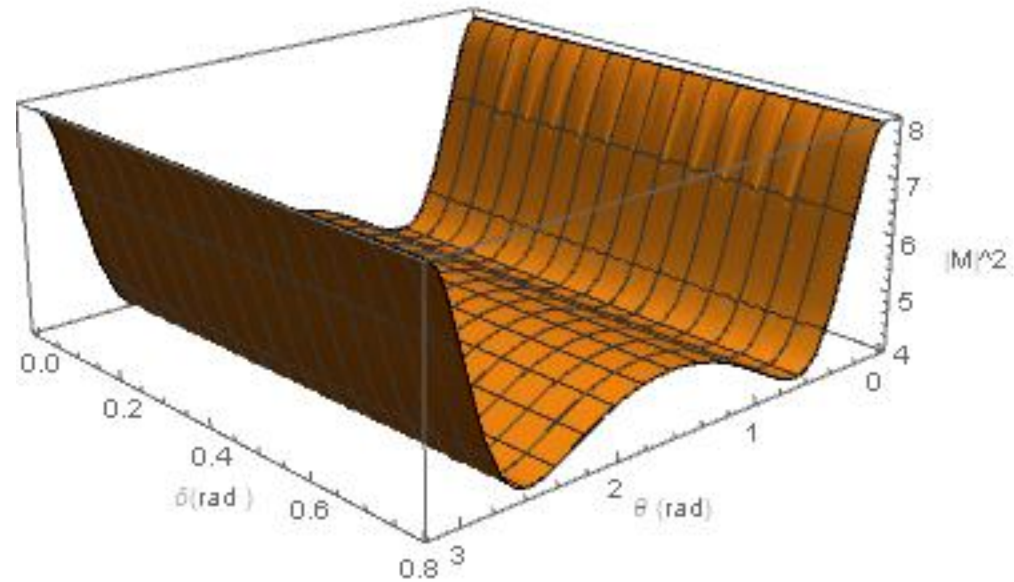




Mt+-
Mt+-(a)
Mt+-(b)
Mu+-
Mu+-(a)
Mu+-(b)
Mse++
Mse+-
θ (rad)
δ(rad)

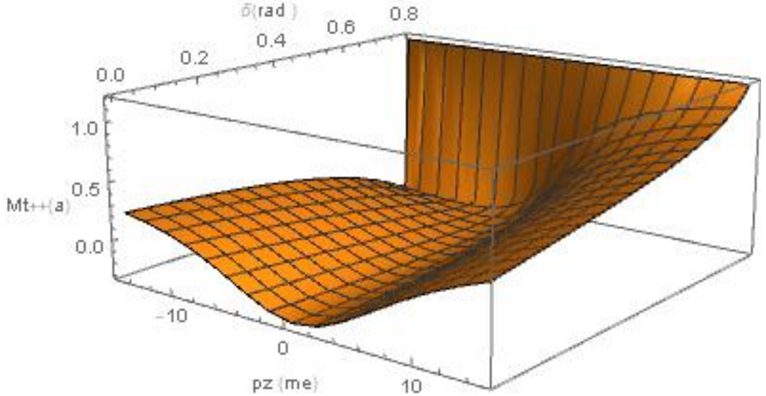

|M++|^2
0
1
2
3
4
0.0
0.2
0.4
0.6
0.8
δ(rad)
θ (rad)
0
1
2
3

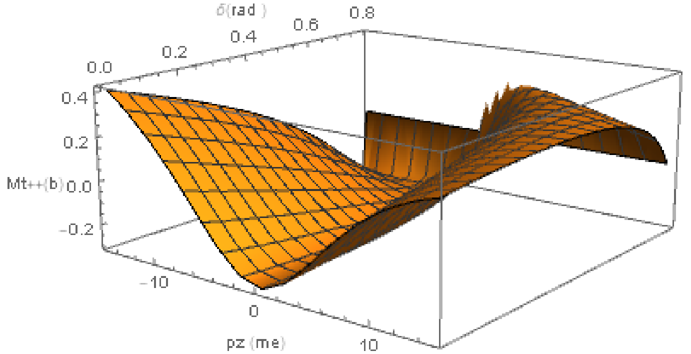

|M+-|^2
0.0
0.5
1.0
1.5
2.0
δ(rad)
0.0
0.2
0.4
0.6
0.8
θ (rad)
0
1
2
3

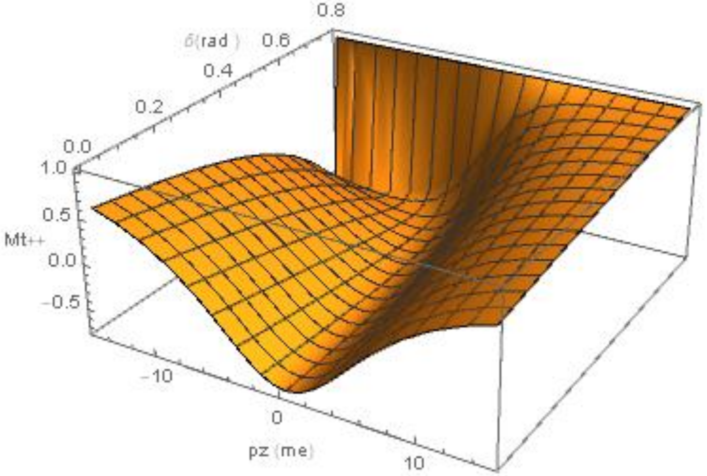

|M|^2
4
5
6
7
8
δ(rad)
0.0
0.2
0.4
0.6
0.8
θ (rad)
0
1
2
3

Boosted Frame

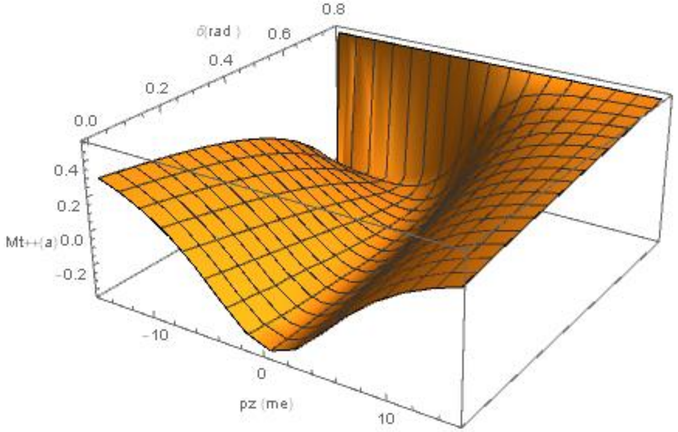
$$\theta = \frac{\pi}{6} - 0.1$$

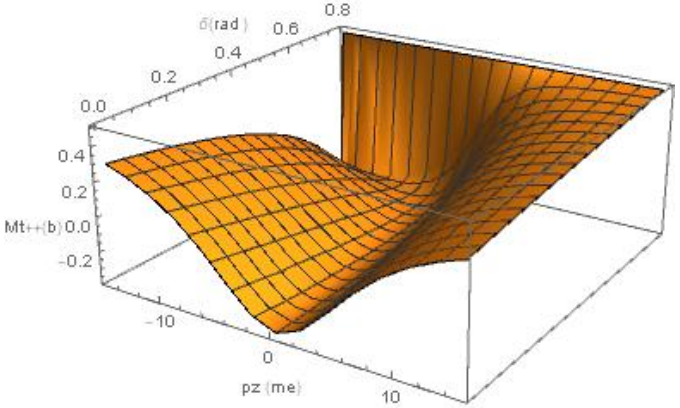




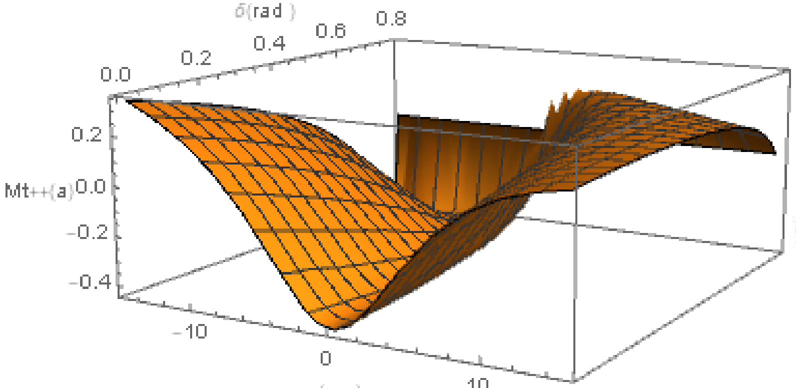


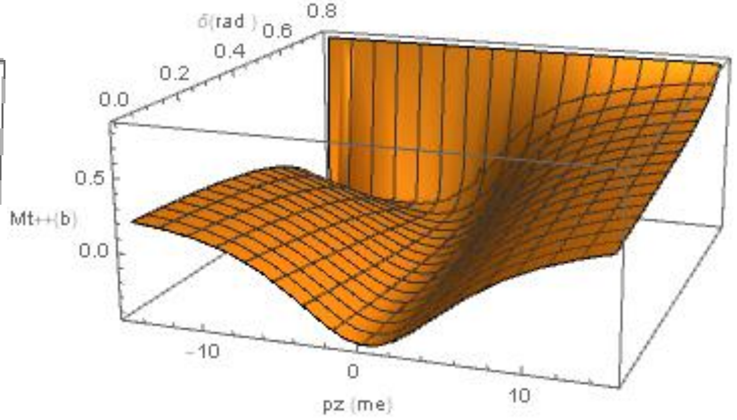
$$\theta = \frac{\pi}{6} = \theta_{c,t}$$





$$\theta = \frac{\pi}{6} + 0.1$$





Thank you