

# SPINORS AND PHOTON POLARIZATION VECTORS

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# OUTLINE

- Representations
  - Standard and Chiral
- Helicity and Dirac Spinors
- Electric and magnetic field tensor and Gauge field
- Results and Difficulties
- Future

## Standard representation (S) and Chiral representation (C)

The transformation between two representation S and C can be made by the transformation matrix S

$$S = S^\dagger = \frac{1}{\sqrt{2}} \begin{pmatrix} I & I \\ I & -I \end{pmatrix}$$

Where  $I$  is the  $2 \times 2$  identity matrix for spin (1/2) representation and  $3 \times 3$  identity matrix for spin 1 representation

Gamma matrices are related by

$$\begin{aligned} \gamma_S^\mu &= S \gamma_C^\mu S^\dagger, \\ \gamma_C^\mu &= S^\dagger \gamma_S^\mu S, \end{aligned}$$

Spinors are related by

$$\begin{aligned} u_S(p) &= S u_C(p), \\ u_C(p) &= S^\dagger u_S(p), \end{aligned}$$

Pauli matrices for spin 1

$$\begin{aligned} \sigma_x &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \\ \sigma_y &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix} \\ \sigma_z &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \end{aligned}$$

Pauli matrices for spin (1/2)

$$\begin{aligned} \sigma_1 = \sigma_x &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ \sigma_2 = \sigma_y &= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \\ \sigma_3 = \sigma_z &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \end{aligned}$$

Gamma matrices in chiral basis

$$\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & -\sigma^i \\ \sigma^i & 0 \end{pmatrix}$$

The six Lorentz group generators of rotation (J) and boost (K)

$$\mathbf{J}_S = \mathbf{J}_C = \frac{1}{2} \begin{pmatrix} \boldsymbol{\sigma} & 0 \\ 0 & \boldsymbol{\sigma} \end{pmatrix}, \quad \mathbf{K}_S = \frac{i}{2} \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ \boldsymbol{\sigma} & 0 \end{pmatrix}, \mathbf{K}_C = \frac{i}{2} \begin{pmatrix} \boldsymbol{\sigma} & 0 \\ 0 & -\boldsymbol{\sigma} \end{pmatrix},$$

Spin (1/2)

The combination between rotation and boost suggest the two decoupled helical motion of the particle that can be denotes as the right -handed and left- handed chirality

$$\mathbf{A} = \frac{1}{2}(\mathbf{J} + i\mathbf{K}),$$
$$\mathbf{B} = \frac{1}{2}(\mathbf{J} - i\mathbf{K}),$$

## Helicity Spinor and Dirac Spinor

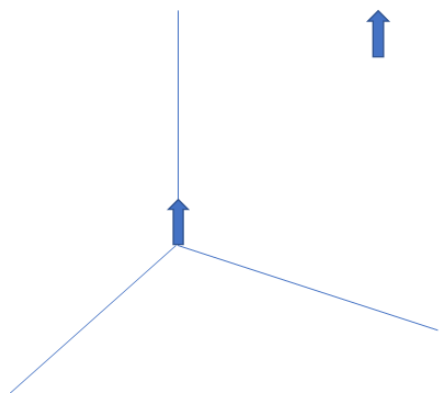
Helicity Spinor : Starting with the state at rest having a spin projection along the z-direction equal to the desired helicity , then boosting it in the z-direction to get the desired magnitude of momentum  $|\vec{P}|$ , and then rotating it subsequently to get the momentum and spin projection in the desired direction.

→ Direction of the helicity spinor always align to the momentum direction

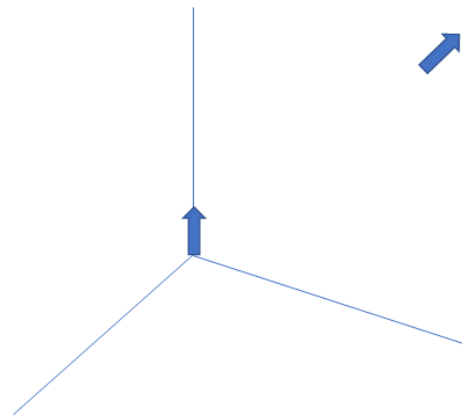
$$T = T_{12}T_3 = e^{i\beta_1\mathcal{K}^1+i\beta_2\mathcal{K}^2} e^{-i\beta_3K^3}$$

Dirac Spinor : Boosting the initial state at rest that has a spin projection along the z-direction to the state with the desired momentum  $\vec{P}$

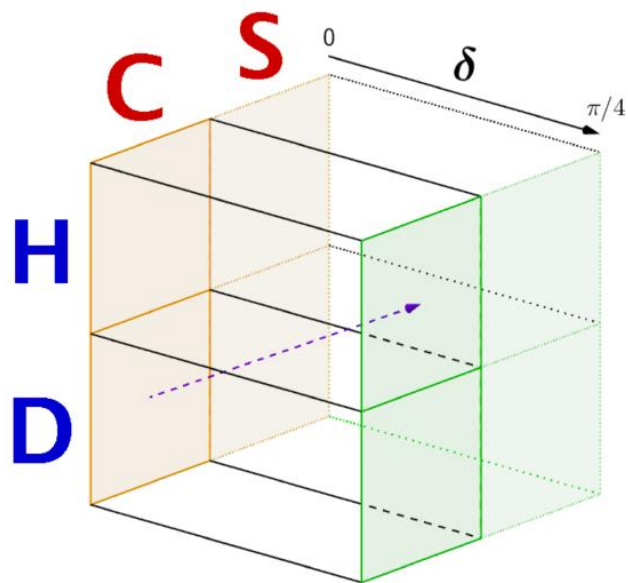
→ The spin direction of the Dirac spinor is in general not align to the moving direction.



Dirac



Helicity



Dashed Purple line -Melosh transformation between instant form Dirac spinor connect to the Light front form helicity spinor

## Electric and magnetic field tensor and gauge field .

Electric and magnetic field tensor

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}$$

We can see correspondence of  
boost operators and electric  
fields  $K \leftrightarrow E$  and rotation  
operators and magnetic fields  
 $J \leftrightarrow B$

Poincare Matrix

$$M^{\mu\nu} = \begin{pmatrix} 0 & K^1 & K^2 & K^3 \\ -K^1 & 0 & J^3 & -J^2 \\ -K^2 & -J^3 & 0 & J^1 \\ -K^3 & J^2 & -J^1 & 0 \end{pmatrix}$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

Gauge fields

$$A^\mu(x) = \int \frac{d^3k}{\sqrt{2(2\pi)^3\omega_k}} \varepsilon^\mu(k) e^{-ik \cdot x}$$

We can write down a matrix which connects a 4 degrees of freedom polarization vectors in  $(1/2,1/2)$  Lorentz group and a six-component spin -1 spinor  $(1,0)+(0,1)$

$$\begin{pmatrix} E_x \\ E_y \\ E_z \\ B_x \\ B_y \\ B_z \end{pmatrix} = -i \begin{pmatrix} p^1 & -p^0 & 0 & 0 \\ p^2 & 0 & -p^0 & 0 \\ p^3 & 0 & 0 & -p^0 \\ 0 & 0 & -p^3 & p^2 \\ 0 & p^3 & 0 & -p^1 \\ 0 & -p^2 & p^1 & 0 \end{pmatrix} \begin{pmatrix} \phi \\ A_x \\ A_y \\ A_z \end{pmatrix}$$

Similar to the spin  $(1/2)$  **A** and **B** notation

$F^{\mu\nu}$  can be separated in to right-handed and left-handed as  $E + iB \in (1,0)$  and  $E - iB \in (0,1)$

We can find the component of the u- spinor by expressing the electric field and magnetic field in terms of the spherical harmonics +, -, 0



$$u(p) = \frac{i}{\sqrt{2}m} \begin{bmatrix} E^- - iB^- \\ -(E^3 - iB^3) \\ -(E^+ - iB^+) \\ E^- + iB^- \\ -(E^3 + iB^3) \\ -(E^+ + iB^+) \end{bmatrix}$$

$$u_1 = \frac{-(P_0 + P_3)\epsilon_1 + i(P_0 + P_3)\epsilon_2 + (P_1 - iP_2)(\epsilon_0 + \epsilon_3)}{2m}$$

$$u_2 = \frac{-P_3\epsilon_0 - iP_2\epsilon_1 + iP_1\epsilon_2 + P_0\epsilon_3}{\sqrt{2}m}$$

$$u_3 = \frac{(P_0 - P_3)\epsilon_1 + i(P_0 - P_3)\epsilon_2 + (P_1 + iP_2)(-\epsilon_0 + \epsilon_3)}{2m}$$

Where  $E^\pm = (E^1 \pm iE^2)/\sqrt{2}$

$$B^\pm = (B^1 \pm iB^2)/\sqrt{2}$$

$$u_4 = \frac{(-P_0 + P_3)\epsilon_1 + i(P_0 - P_3)\epsilon_2 + (P_1 - iP_2)(\epsilon_0 - \epsilon_3)}{2m}$$

$$u_5 = \frac{-P_3\epsilon_0 + iP_2\epsilon_1 - iP_1\epsilon_2 + P_0\epsilon_3}{\sqrt{2}m}$$

$$u_6 = \frac{(P_0 + P_3)\epsilon_1 + i(P_0 + P_3)\epsilon_2 + (P_1 + iP_2)(-\epsilon_0 - \epsilon_3)}{2m}$$

$$\ell=1 \left\{ \begin{array}{ll} Y_{\ell=1}^{m=-1}(\theta, \phi) = \sqrt{\frac{3}{8\pi}} \sin\theta e^{-i\phi} & \sqrt{\frac{3}{8\pi}} \frac{x-iy}{r} \\ Y_1^0(\theta, \phi) = \sqrt{\frac{3}{4\pi}} \cos\theta & \sqrt{\frac{3}{4\pi}} \frac{z}{r} \\ Y_1^{+1}(\theta, \phi) = -\sqrt{\frac{3}{8\pi}} \sin\theta e^{i\phi} & -\sqrt{\frac{3}{8\pi}} \frac{x+iy}{r} \end{array} \right.$$

$$\begin{bmatrix} U^1 \\ U^2 \\ U^3 \\ U^4 \\ U^5 \\ U^6 \end{bmatrix} = \frac{1}{2m} \begin{pmatrix} p_1 - Ip_2 & -p_0 - p_3 & I(p_0 + p_3) & p_1 - Ip_2 \\ -p_3\sqrt{2} & -Ip_2\sqrt{2} & Ip_1\sqrt{2} & p_0\sqrt{2} \\ -p_1 - Ip_2 & p_0 - p_3 & I(p_0 - p_3) & p_1 + Ip_2 \\ p_1 - Ip_2 & -p_0 + p_3 & I(p_0 - p_3) & -p_1 + Ip_2 \\ -p_3\sqrt{2} & Ip_2\sqrt{2} & -Ip_1\sqrt{2} & p_0\sqrt{2} \\ -p_1 - Ip_2 & p_0 + p_3 & I(p_0 + p_3) & -p_1 - Ip_2 \end{pmatrix} \begin{bmatrix} \epsilon^0 \\ \epsilon^1 \\ \epsilon^2 \\ \epsilon^3 \end{bmatrix} \quad U^\mu = C \epsilon^\mu$$

### Dirac Photon polarization vectors in chiral basis

$$\epsilon^\mu(P, +) = \frac{-1}{m(p_0+m)} \begin{bmatrix} (p_0 + m) (p_1 + ip_2)/\sqrt{2} \\ (p_0 + m) m/\sqrt{2} + p_1 (p_1 + ip_2)/\sqrt{2} \\ i(p_0 + m) m/\sqrt{2} + p_2 (p_1 + ip_2)/\sqrt{2} \\ p_3 (p_1 + ip_2)/\sqrt{2} \end{bmatrix}$$

$$\epsilon^\mu(P, 0) = \frac{1}{m(p_0+m)} \begin{bmatrix} (p_0 + m)p_3 \\ p_1p_3 \\ p_2p_3 \\ (p_0 + m)m + p_3^2 \end{bmatrix}$$

$$\epsilon^\mu(P, -) = \frac{1}{m(p_0 + m)} \begin{bmatrix} (p_0 + m) (p_1 - ip_2)/\sqrt{2} \\ (p_0 + m) m/\sqrt{2} + p_1 (p_1 - ip_2)/\sqrt{2} \\ -i(p_0 + m) m/\sqrt{2} + p_2 (p_1 - ip_2)/\sqrt{2} \\ p_3 (p_1 - ip_2)/\sqrt{2} \end{bmatrix}$$

Lorenz Gauge condition  
satisfied  $\rightarrow \partial_\mu A^\mu = 0$

## Dirac spin-1 spinors in chiral basis

$$\begin{aligned}
 U^\mu(P, +) &= \left\{ \begin{array}{l} \frac{(m + p_0 + p_3)^2}{2\sqrt{2}m(m + p_0)} \\ \frac{(p_1 + ip_2)(m + p_0 + p_3)}{2m(m + p_0)} \\ \frac{(p_1 + ip_2)^2}{2\sqrt{2}m(m + p_0)} \\ \frac{(m + p_0 - p_3)^2}{2\sqrt{2}m(m + p_0)} \\ \frac{(p_1 + ip_2)(m + p_0 - p_3)}{2m(m + p_0)} \\ \frac{(p_1 + ip_2)^2}{2\sqrt{2}m(m + p_0)} \end{array} \right\} \\
 U^\mu(P, 0) &= \left\{ \begin{array}{l} \frac{(p_1 - ip_2)(m + p_0 + p_3)}{2m(m + p_0)} \\ \frac{mp_0 + p_0^2 - p_3^2}{\sqrt{2}m(m + p_0)} \\ \frac{(p_1 + ip_2)(m + p_0 - p_3)}{2m(m + p_0)} \\ \frac{(p_1 - ip_2)(m + p_0 - p_3)}{2m(m + p_0)} \\ \frac{mp_0 + p_0^2 - p_3^2}{\sqrt{2}m(m + p_0)} \\ \frac{(p_1 + ip_2)(m + p_0 + p_3)}{2m(m + p_0)} \end{array} \right\} \\
 U^\mu(P, -) &= \left\{ \begin{array}{l} \frac{(p_1 - ip_2)^2}{2\sqrt{2}m(m + p_0)} \\ \frac{(p_1 - ip_2)(m + p_0 - p_3)}{2m(m + p_0)} \\ \frac{(m + p_0 - p_3)^2}{2\sqrt{2}m(m + p_0)} \\ \frac{(p_1 - ip_2)^2}{2\sqrt{2}m(m + p_0)} \\ \frac{(p_1 - ip_2)(m + p_0 + p_3)}{2m(m + p_0)} \\ \frac{(m + p_0 + p_3)^2}{2\sqrt{2}m(m + p_0)} \end{array} \right\}
 \end{aligned}$$

$$C = \sum_{\lambda} -(U^{\mu}(P, \lambda) \otimes \epsilon_{\mu}^{*}(P, \lambda))$$

$$C = -(U^{\mu}(P, +) \otimes \epsilon_{\mu}^{*}(P, +)) - (U^{\mu}(P, -) \otimes \epsilon_{\mu}^{*}(P, -)) - (U^{\mu}(P, 0) \otimes \epsilon_{\mu}^{*}(P, 0))$$

If I multiply C by  $\epsilon^{\mu}(P, +)$

$$U^{\mu}(P, +) = C \epsilon^{\mu}(P, +)$$

Properties of photon polarization vector

$$\epsilon^{*}(P, \lambda) \cdot \epsilon(P, \lambda') = -\delta_{\lambda\lambda'}$$

If the two vectors have dimensions  $n$  and  $m$ , then their outer product is an  $n \times m$  matrix.

$$\mathbf{u} \otimes \mathbf{v} = \mathbf{A} = \begin{bmatrix} u_1 v_1 & u_1 v_2 & \dots & u_1 v_n \\ u_2 v_1 & u_2 v_2 & \dots & u_2 v_n \\ \vdots & \vdots & \ddots & \vdots \\ u_m v_1 & u_m v_2 & \dots & u_m v_n \end{bmatrix}$$

$$\bar{C} = \sum_{\lambda} (\epsilon^{\mu}(P, \lambda) \otimes \bar{U}(P, \lambda))$$

$$\bar{U}(P, \lambda) = \gamma_0 U^*(P, \lambda)$$

$$\gamma_0 = \begin{pmatrix} 0_3 & I_3 \\ I_3 & 0_3 \end{pmatrix}$$

$$\bar{C} = \begin{pmatrix} -\frac{p_1 + ip_2}{2m} & \frac{p_3}{\sqrt{2}m} & \frac{p_1 - ip_2}{2m} & -\frac{p_1 + ip_2}{2m} & \frac{p_3}{\sqrt{2}m} & \frac{p_1 - ip_2}{2m} \\ \frac{-p_0 + p_3}{2m} & -\frac{ip_2}{\sqrt{2}m} & \frac{p_0 + p_3}{2m} & -\frac{p_0 + p_3}{2m} & \frac{ip_2}{\sqrt{2}m} & \frac{p_0 - p_3}{2m} \\ -\frac{i(p_0 - p_3)}{2m} & \frac{ip_1}{\sqrt{2}m} & -\frac{i(p_0 + p_3)}{2m} & -\frac{i(p_0 + p_3)}{2m} & \frac{ip_1}{\sqrt{2}m} & -\frac{i(p_0 - p_3)}{2m} \\ -\frac{p_1 + ip_2}{2m} & \frac{p_0}{\sqrt{2}m} & -\frac{p_1 - ip_2}{2m} & \frac{p_1 + ip_2}{2m} & \frac{p_0}{\sqrt{2}m} & \frac{p_1 - ip_2}{2m} \end{pmatrix}$$

$$\epsilon^{\mu}(P, \lambda) = \bar{C} U^{\mu}(P, \lambda)$$

$$C\bar{C} = \sum_{\lambda} -(U^{\mu}(P, \lambda) \otimes \epsilon_{\mu}^*(P, \lambda) \cdot (\epsilon^{\mu}(P, \lambda) \otimes \bar{U}(P, \lambda))) = \sum_{\lambda} (U^{\mu}(P, \lambda) \otimes \bar{U}(P, \lambda))$$

$$C\bar{C} = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{(p_0 + p_3)^2}{2m^2} & \frac{(p_1 - ip_2)(p_0 + p_3)}{\sqrt{2}m^2} & \frac{(p_1 - ip_2)^2}{2m^2} \\ 0 & \frac{1}{2} & 0 & \frac{(p_1 + ip_2)(p_0 + p_3)}{\sqrt{2}m^2} & -\frac{1}{2} + \frac{p_0^2 - p_3^2}{m^2} & \frac{(p_1 - ip_2)(p_0 - p_3)}{\sqrt{2}m^2} \\ 0 & 0 & \frac{1}{2} & \frac{(p_1 + ip_2)^2}{2m^2} & \frac{(p_1 + ip_2)(p_0 - p_3)}{\sqrt{2}m^2} & \frac{(p_0 - p_3)^2}{2m^2} \\ \frac{(p_0 - p_3)^2}{2m^2} & -\frac{(p_1 - ip_2)(p_0 - p_3)}{\sqrt{2}m^2} & \frac{(p_1 - ip_2)^2}{2m^2} & \frac{1}{2} & 0 & 0 \\ -\frac{(p_1 + ip_2)(p_0 - p_3)}{\sqrt{2}m^2} & -\frac{1}{2} + \frac{p_0^2 - p_3^2}{m^2} & -\frac{(p_1 - ip_2)(p_0 + p_3)}{\sqrt{2}m^2} & 0 & \frac{1}{2} & 0 \\ \frac{(p_1 + ip_2)^2}{2m^2} & -\frac{(p_1 + ip_2)(p_0 + p_3)}{\sqrt{2}m^2} & \frac{(p_0 + p_3)^2}{2m^2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

$$\sum_{\lambda} (U^{\mu}(0, \lambda) \otimes \bar{U}(0, \lambda)) = \frac{(I + \gamma_0)}{2}$$

Spin (1/2)

$$\sum_{\lambda} u(p, \lambda) \bar{u}(p, \lambda) = (\not{p} + m)/2m$$

Spin one

$$(\not{p} + m)/2m = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{p_0 + p_3}{2m} & \frac{p_1 - ip_2}{2\sqrt{2}m} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{p_1 + ip_2}{2\sqrt{2}m} & \frac{p_0}{2m} & \frac{p_1 - ip_2}{2\sqrt{2}m} \\ 0 & 0 & \frac{1}{2} & 0 & \frac{p_1 + ip_2}{2\sqrt{2}m} & \frac{p_0 - p_3}{2m} \\ \frac{p_0 - p_3}{2m} & -\frac{p_1 - ip_2}{2\sqrt{2}m} & 0 & \frac{1}{2} & 0 & 0 \\ -\frac{p_1 + ip_2}{2\sqrt{2}m} & \frac{p_0}{2m} & -\frac{p_1 - ip_2}{2\sqrt{2}m} & 0 & \frac{1}{2} & 0 \\ 0 & -\frac{p_1 + ip_2}{2\sqrt{2}m} & \frac{p_0 + p_3}{2m} & 0 & 0 & \frac{1}{2} \end{pmatrix} \neq \sum_{\lambda} (U^{\mu}(P, \lambda) \otimes \bar{U}(P, \lambda))$$

## Rest Frame

- Spin one spinors

$$U^+ = \left\{ \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}$$

$$U^- = \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}} \right\}$$

$$U^0 = \left\{ 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\}$$

- Photon polarization vectors

$$\epsilon^+ = \left\{ 0, -\frac{1}{\sqrt{2}}, -\frac{i}{\sqrt{2}}, 0 \right\}$$

$$\epsilon^- = \left\{ 0, \frac{1}{\sqrt{2}}, -\frac{i}{\sqrt{2}}, 0 \right\}$$

$$\epsilon^0 = \{0, 0, 0, 1\}$$

$$C = \begin{bmatrix} 0 & -\frac{1}{2} & \frac{i}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{2} & \frac{i}{2} & 0 \\ 0 & -\frac{1}{2} & \frac{i}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{2} & \frac{i}{2} & 0 \end{bmatrix}$$

## Interpolating Helicity Spinors and Photon polarization vectors in chiral basis

$$\begin{aligned}\epsilon_{\hat{\mu}}^+(P) &= -\frac{1}{\sqrt{2}\mathbb{P}} \\ &\times \left( S|\mathbf{P}_{\perp}|, \frac{P_1P_{\perp} - iP_2\mathbb{P}}{|\mathbf{P}_{\perp}|}, \frac{P_2P_{\perp} + iP_1\mathbb{P}}{|\mathbf{P}_{\perp}|}, -C|\mathbf{P}_{\perp}| \right), \\ \epsilon_{\hat{\mu}}^-(P) &= \frac{1}{\sqrt{2}\mathbb{P}} \\ &\times \left( S|\mathbf{P}_{\perp}|, \frac{P_1P_{\perp} + iP_2\mathbb{P}}{|\mathbf{P}_{\perp}|}, \frac{P_2P_{\perp} - iP_1\mathbb{P}}{|\mathbf{P}_{\perp}|}, -C|\mathbf{P}_{\perp}| \right), \\ \epsilon_{\hat{\mu}}^0(P) &= \frac{P^{\dagger}}{M\mathbb{P}} \left( P_{\dagger} - \frac{M^2}{P^{\dagger}}, P_1, P_2, P_{\perp} \right),\end{aligned}$$

Conditions

$$\begin{aligned}\epsilon^{\dagger} &= \mathbb{C}\epsilon_{\dagger} + \mathbb{S}\epsilon_{\perp} = 0, \\ \epsilon_{\perp}(\lambda)P_{\perp} + \epsilon_{\perp}(\lambda)\mathbf{P}_{\perp}\mathbb{C} &= 0, \\ A^{\dagger} &= 0 \text{ and } \partial_{\perp}A_{\perp} + \partial_{\perp} \cdot \mathbf{A}_{\perp}\mathbb{C} = 0.\end{aligned}$$

$$U_H = C_H \in_H$$

$$C = \sum_{\lambda} -(U^{\mu}(P, \lambda) \otimes \epsilon_{\hat{\mu}}^*(P, \lambda))$$

$$u_H^{(+1)} = \frac{1}{2\sqrt{M\mathbb{P}^2}} \begin{pmatrix} \frac{(P_{\perp} + \mathbb{P})(P^{\dagger} + \mathbb{P})}{(A - B)} \\ \frac{\sqrt{2}P^R(P^{\dagger} + \mathbb{P})}{(A - B)(P^R)^2(P^{\dagger} + \mathbb{P})} \\ (P_{\perp} + \mathbb{P}) \\ \frac{(A - B)(P_{\perp} + \mathbb{P})\mathbb{X}}{\sqrt{2}P^R(P^{\dagger} - \mathbb{P})} \\ \frac{(A + B)(P^R)^2(P^{\dagger} - \mathbb{P})}{(P_{\perp} + \mathbb{P})} \end{pmatrix},$$

$$u_H^{(-1)} = \frac{1}{2\sqrt{M\mathbb{P}^2}} \begin{pmatrix} \frac{(A + B)(P^L)^2(P^{\dagger} - \mathbb{P})}{(P_{\perp} + \mathbb{P})} \\ -\sqrt{2}P^L(P^{\dagger} - \mathbb{P}) \\ \frac{(A - B)(P_{\perp} + \mathbb{P})\mathbb{X}}{(A - B)(P^L)^2(P^{\dagger} + \mathbb{P})} \\ (P_{\perp} + \mathbb{P}) \\ -\sqrt{2}P^L(P^{\dagger} + \mathbb{P}) \\ \frac{(P_{\perp} + \mathbb{P})(P^{\dagger} + \mathbb{P})}{(A - B)} \end{pmatrix},$$

$$u_H^{(0)} = \sqrt{\frac{M}{2\mathbb{P}^2}} \begin{pmatrix} -(A + B)P^L \\ \sqrt{2}P_{\perp} \\ (A - B)P^R \\ (-A + B)P^L \\ \sqrt{2}P_{\perp} \\ (A + B)P^R \end{pmatrix}.$$

$$A = \cos \delta, \quad B = -\sin \delta$$

$$\mathbb{X} \equiv \frac{P^{\dagger} - \mathbb{P}}{\mathbb{C}} = \frac{P^{\dagger} - \sqrt{(P^{\dagger})^2 - M^2}\mathbb{C}}{\mathbb{C}}$$

$$P^R = P^1 + iP^2$$

$$P^L = P^1 - iP^2$$

$$P^+ = (P^0 + P^3)/\sqrt{2}$$

Spinors are all normalized,  $\bar{U}U = 2M$



## Light front spin-1 spinors in Standard representation

Spin-1 hadronic spinors with even spinor parity:

$$\mathcal{U}_{+1}(p^\mu) = \frac{1}{2} \begin{bmatrix} p^+ + (m^2/p^+) \\ \sqrt{2}p_r \\ p_r^2/p^+ \\ p^+ - (m^2/p^+) \\ \sqrt{2}p_r \\ p_r^2/p^+ \end{bmatrix}, \quad \mathcal{U}_0(p^\mu) = m\sqrt{1/2} \begin{bmatrix} -p_l/p^+ \\ \sqrt{2} \\ p_r/p^+ \\ p_l/p^+ \\ 0 \\ p_r/p^+ \end{bmatrix}, \quad \mathcal{U}_{-1}(p^\mu) = \frac{1}{2} \begin{bmatrix} p_l^2/p^+ \\ -\sqrt{2}p_l \\ p^+ + (m^2/p^+) \\ -p_l^2/p^+ \\ \sqrt{2}p_l \\ -p^+ + (m^2/p^+) \end{bmatrix}. \quad (\text{B2})$$

$$P^+ = (P^0 + P^3)$$

### Front-form spinors in the Weinberg-Soper formalism and generalized Melosh transformations for any spin

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## Generalized Melosh Transformation for spinors

Connects Dirac spinors with generalized helicity spinors

$$\Omega_{\lambda\rho}^{[u_D u_H]} = \frac{1}{2M} \bar{u}_H^{(\rho)}(P) u_D^{(\lambda)}(P) = \mathcal{D}(\hat{\mathbf{m}}, -\theta_s)_{\rho\lambda} \quad \rightarrow \quad \text{Transpose of the Wigner rotation matrix}$$

$$\Omega(i) = \begin{pmatrix} \omega(i) & 0 \\ 0 & \omega(i) \end{pmatrix}.$$

$$\cos \frac{\theta_s}{2} = \frac{1}{2} \sqrt{\frac{2M}{E+M}} \sqrt{\frac{P_z + \mathbb{P}}{2\mathbb{P}}} \left( \sqrt{\frac{P^{\hat{z}} + \mathbb{P}}{M(\sin \delta + \cos \delta)}} + \sqrt{\frac{M(\sin \delta + \cos \delta)}{P^{\hat{z}} + \mathbb{P}}} \right)$$

Spin 1

$$\omega(1) = \begin{pmatrix} \cos^2 \frac{\theta_s}{2} & -\frac{e^{i\phi_s} \sin \theta_s}{\sqrt{2}} & e^{2i\phi_s} \sin^2 \frac{\theta_s}{2} \\ \frac{e^{-i\phi_s} \sin \theta_s}{\sqrt{2}} & \cos \theta_s & -\frac{e^{i\phi_s} \sin \theta_s}{\sqrt{2}} \\ e^{-2i\phi_s} \sin^2 \frac{\theta_s}{2} & \frac{e^{-i\phi_s} \sin \theta_s}{\sqrt{2}} & \cos^2 \frac{\theta_s}{2} \end{pmatrix}.$$

$$\cos \phi_s = \frac{P^1}{\sqrt{\mathbf{P}_1^2}} \quad \sin \phi_s = \frac{P^2}{\sqrt{\mathbf{P}_1^2}}$$

$$U_D = \Omega U_H$$

$\Omega \rightarrow 6 \times 6$  matrix

$$\epsilon_D = \overline{C}_D U_D$$

$\overline{C}_D \rightarrow 4 \times 6$  matrix

$$\epsilon_D = \overline{C}_D \Omega U_H$$

$\overline{C}_{DH} \rightarrow 4 \times 6$  matrix

$$\epsilon_D = \overline{C}_{DH} U_H$$

Possibility

If I can calculate generalized transformation matrix that connect  $\epsilon_D$  and  $\epsilon_H$

$$\epsilon_D = \Omega_\epsilon \epsilon_H$$

$\Omega_\epsilon \rightarrow 4 \times 4$  matrix

$$\epsilon_H = \Omega_\epsilon^{-1} \overline{C}_{DH} U_H$$

THANK YOU