Two Scalar mesons annihilation in to two rho mesons

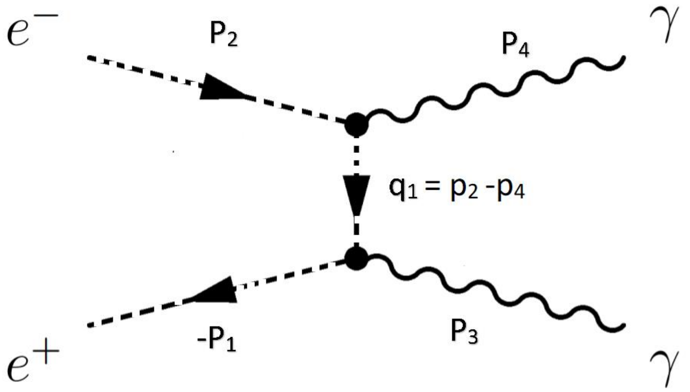
Group meeting

26th April 2019

Deepasika Dayananda

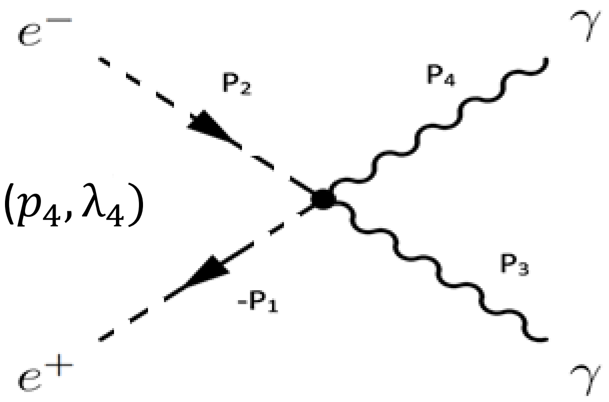
Lowest –order Covariant annihilation diagrams

t-Channel



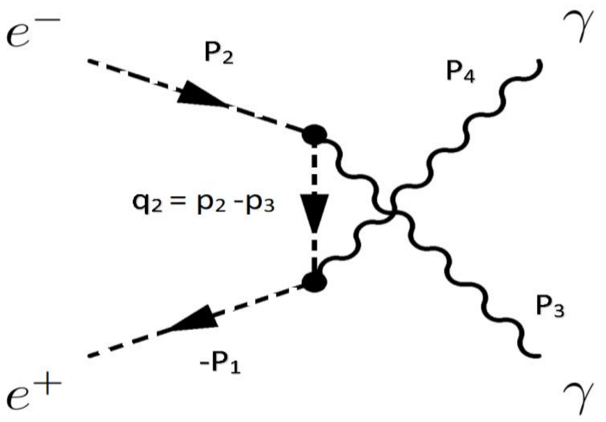
$$M_t = (-p_1 + q_1)^{\mu} \varepsilon^*_{\mu}(p_3, \lambda_3) \frac{1}{q_1^2 - m^2} (p_2 + q_1)^{\nu} \varepsilon^*_{\nu}(p_4, \lambda_4)$$

Seagull



$$M_{se} = -2 g_{\mu\nu} \epsilon^{*\mu}(p_3, \lambda_3) \varepsilon^{*\nu}(p_4, \lambda_4)$$

u-Channel



$$M_u = (-p_1 + q_2)^{\nu} \varepsilon^*_{\nu}(p_4, \lambda_4) \frac{1}{q_2^2 - m^2} (p_2 + q_2)^{\mu} \varepsilon^*_{\mu}(p_3, \lambda_3)$$

The interpolating photon polarization vectors

$$\epsilon_{\hat{\mu}}(P,+) = -\frac{1}{\sqrt{2}\,\mathbb{P}}\left(\mathbb{S}|\boldsymbol{p}_{\perp}|, \frac{P_1 P_{\hat{-}} - iP_2\mathbb{P}}{|\mathbf{p}_{\perp}|}, \frac{P_2 P_{\hat{-}} + iP_1\mathbb{P}}{|\mathbf{p}_{\perp}|}, -\mathbb{C}\,|\boldsymbol{p}_{\perp}|\right)$$

$$\epsilon_{\hat{\mu}}(P,-) = \frac{1}{\sqrt{2}\,\mathbb{P}}\left(\mathbb{S}|\boldsymbol{p}_{\perp}|, \frac{P_1 P_{\hat{-}} + iP_2\mathbb{P}}{|\mathbf{p}_{\perp}|}, \frac{P_2 P_{\hat{-}} - iP_1\mathbb{P}}{|\mathbf{p}_{\perp}|}, -\mathbb{C}\,|\boldsymbol{p}_{\perp}|\right)$$

$$\epsilon_{\hat{\mu}}(P,0) = \frac{P^{\hat{+}}}{m_{\gamma}\mathbb{P}}\left(P_{\hat{+}} - \frac{m_{\gamma}^2}{P^{\hat{+}}}, P_1, P_2, P_{\hat{-}}\right)$$

Where
$$\mathbb{S} = Sin(2\delta)$$
$$\mathbb{C} = Cos(2\delta)$$
$$\mathbb{P} = \sqrt{{P_{\hat{-}}}^2 + \mathbb{C}{\mathbf{p}_{\perp}}^2} = \sqrt{(P^{\hat{+}})^2 - \mathbb{C}{m_{\gamma}}^2}$$

Constraints

$$\epsilon_{\hat{\mu}}(p,\lambda)p^{\hat{\mu}} = 0$$

$$\epsilon^*(p,\lambda).\,\epsilon(p,\lambda') = -\delta_{\lambda\lambda'}$$

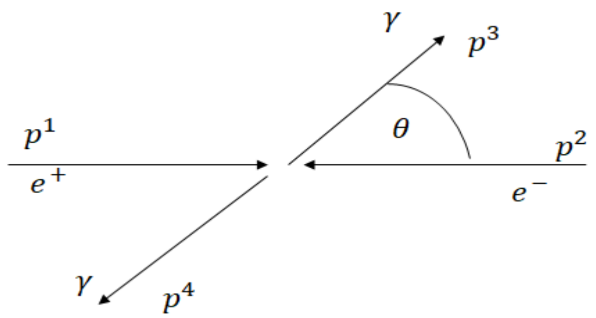
C-R ji,Ziyue Li and A.T.Suzuki ,Phy Rev D 91 06065020 (2014)

Covariant Feynman amplitude

$$M = M_t + M_u + M_{se}$$

$$
\begin{aligned}
M \quad = & (-p_1 + q_1)^{\mu} \varepsilon_{\mu}^{*}(p_3, \lambda_3) \frac{1}{q_1^2 - m^2} (p_2 + q_1)^{\nu} \varepsilon_{\nu}^{*}(p_4, \lambda_4) \\
& + (-p_1 + q_2)^{\nu} \varepsilon_{\nu}^{*}(p_4, \lambda_4) \frac{1}{q_2^2 - m^2} (p_2 + q_2)^{\mu} \varepsilon_{\mu}^{*}(p_3, \lambda_3) \\
& -2 g_{\mu\nu} \epsilon^{*\mu}(p_3, \lambda_3) \varepsilon^{*\nu}(p_4, \lambda_4)
\end{aligned}
$$

Center of mass kinematics



$$p^1 = \{E_0, 0{,}0, P_e\}$$

$$p^2 = \{E_0, 0{,}0, -P_e\}$$

$$p^3 = \{E_0, P_\gamma Sin(\theta), 0, P_\gamma Cos(\theta)\}$$

$$p^4 = \{E_0, -P_\gamma Sin(\theta), 0, -P_\gamma Cos(\theta)\}$$

Lorentz Transformation

$$E = \sqrt{4E_0^2 + P_z^2} \qquad \alpha = \frac{E}{4E_0^2} \qquad \alpha\beta = \frac{P_z}{4E_0^2}$$

$$p_i'^{0} = \alpha p_i^{0} + \alpha\beta p_i^{z}$$

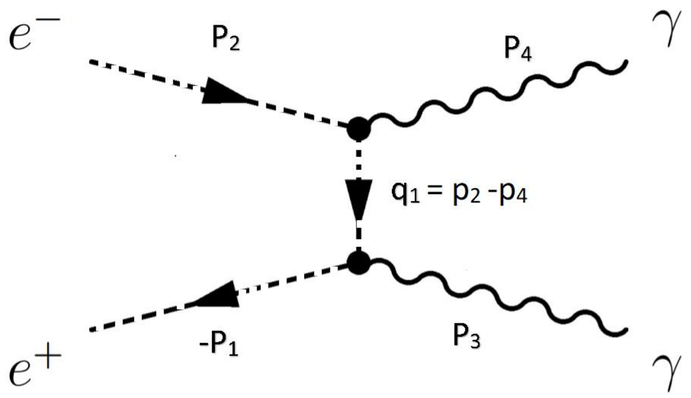
$$p_i'^{z} = \alpha p_i^{z} + \alpha\beta p_i^{0}$$

$$p_i'^{\perp} = p_i^{\perp}$$

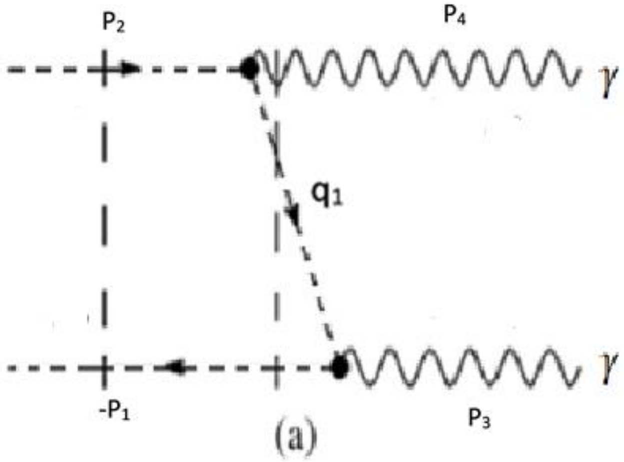
Time ordering in the interpolation dynamics

t-Channel

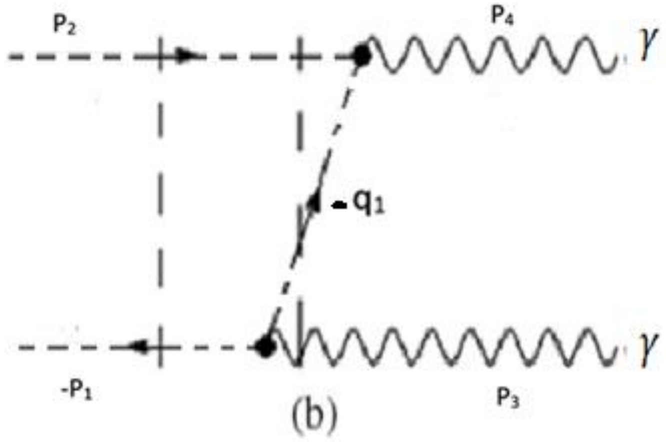
Covariant Propagator



Forward moving



Backward moving



$$\Sigma = \Sigma_a^\delta + \Sigma_b^\delta = \frac{1}{q_1^2 - m^2}$$

$$\Sigma_a^\delta = \frac{C}{2Q^{\hat{\mp}}\,(q^{\hat{\mp}} - Q^{\hat{\mp}})}$$

$$\Sigma_b^\delta = -\frac{C}{2Q^{\hat{\mp}}\,(q^{\hat{\mp}} + Q^{\hat{\mp}})}$$

Where: $C = Cos(2\delta)$, $S = Sin(2\delta)$, $q^{\hat{\mp}} = p_2^{\hat{\mp}} - p_4^{\hat{\mp}}$, $Q^{\hat{\mp}} = \pm\sqrt{Q_{\hat{=}}^2 + C(\vec{q}^2{}_{\perp} + m^2)}$

Observe symmetries between covariant Helicity amplitudes

Mt + +	=	Mt - -	Mt 0 +	=	-(Mt 0 -)	Mt 00
Mu + +	=	Mu - -	Mu 0 +	=	-(Mu 0 -)	Mu 00
Mse + +	=	Mse - -	Mse 0 +	=	-(Mse 0 -)	Mse 00
Mt + -	=	Mt - +	Mt + 0	=	-(Mt – 0)	
Mu + -	=	Mu - +	Mu + 0	=	-(Mu – 0)	
Mse + -	=	Mse - +	Mse + 0	=	-(Mse – 0)	

Mt ++ = -(Mt + -)

Mu ++ = -(Mu + -)

Mt + + (θ)	=	Mu ++ $(\pi - \theta)$
Mt + - (θ)	=	Mu - +$(\pi - \theta)$
Mt 0 + (θ)	=	Mu + 0 $(\pi - \theta)$
Mu 0 + (θ)	=	Mt + 0 $(\pi - \theta)$
Mse 0 + (θ)	=	Mse+ 0 $(\pi - \theta)$
Mt 00 (θ)	=	Mu 00 $(\pi - \theta)$

- Helicity amplitudes satisfy symmetry based on parity conservation

$$H(-s', -h', -s, -h) = (-1)^{s'+h'-s-h} H(s', h', s, h)$$

$$H(-h', -h) = (-1)^{h'-h} H(h', h)$$

- Corresponding time order amplitudes also satisfy these symmetries

C-R . Ji, B.L.G.Bakker , International Journal of Modern Physics,(2013)

Critical annihilation angle

$$q_1^+ = \frac{-\frac{pepz}{2E0}+\frac{pzp\gamma Cos[\theta]}{2E0}}{\sqrt{2}}+\frac{-\frac{pe\sqrt{4E0^2+pz^2}}{2E0}+\frac{\sqrt{4E0^2+pz^2}p\gamma Cos[\theta]}{2E0}}{\sqrt{2}}$$

$$q_2^+ = \frac{-\frac{pepz}{2E0}-\frac{pzp\gamma Cos[\theta]}{2E0}}{\sqrt{2}}+\frac{-\frac{pe\sqrt{4E0^2+pz^2}}{2E0}-\frac{\sqrt{4E0^2+pz^2}p\gamma Cos[\theta]}{2E0}}{\sqrt{2}}$$

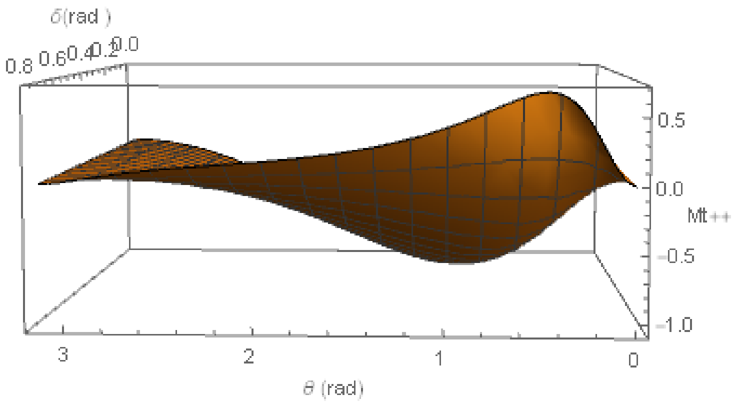
$q_1^+ > 0$ →Forward

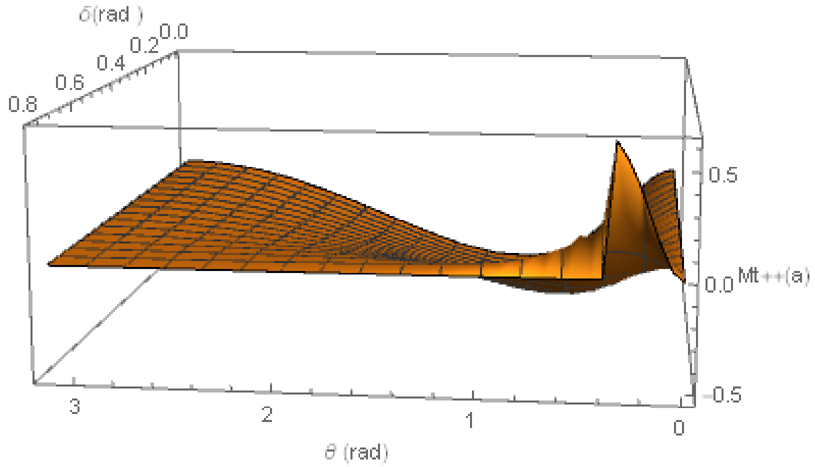
$q_1^+ < 0$ → Backward

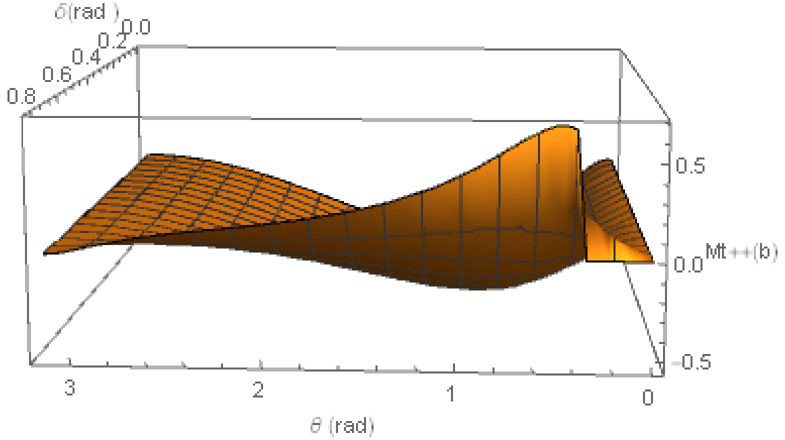
$q_1^+ = 0 \quad \rightarrow \quad \theta_{c,t} = ArcCos\left(\frac{P_e}{P_\gamma}\right)$

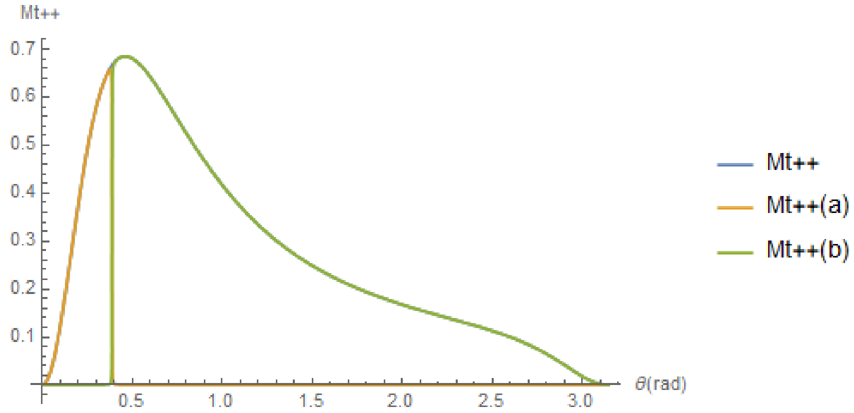
$q_2^+ = 0 \quad \rightarrow \quad \theta_{c,u} = -ArcCos\left(\frac{P_e}{P_\gamma}\right)$

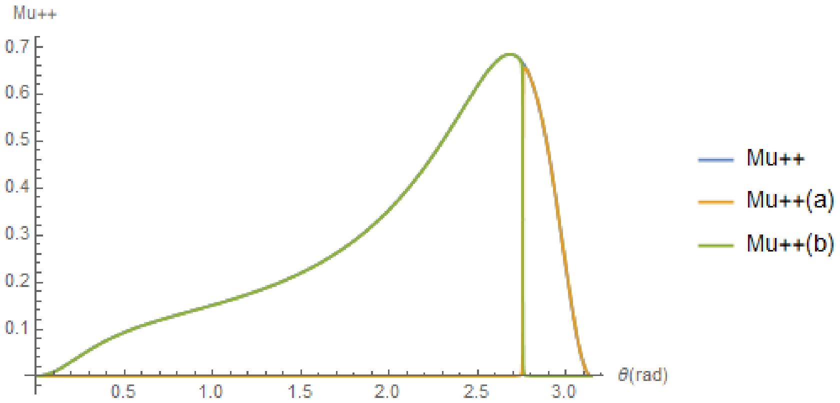
$\rightarrow \exists\ Cos(\theta_c) \leftrightarrow P_e < P_\gamma$











$$E_0 = 2m_e$$

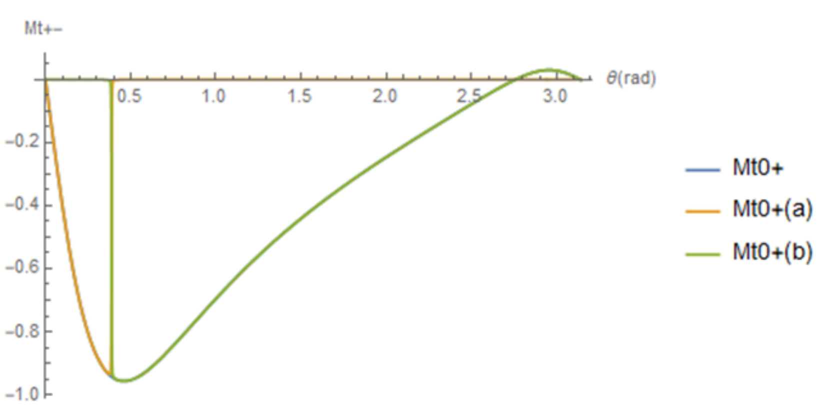
$$P_e = \sqrt{3}m_e$$

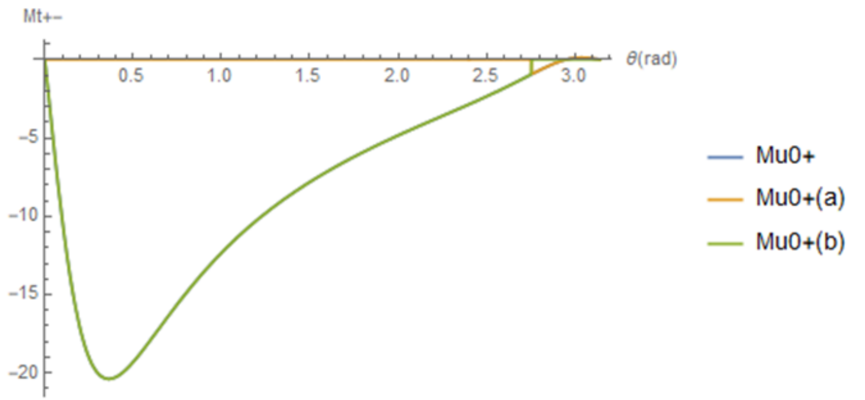
$$P_\gamma = \sqrt{3.5}m_e$$

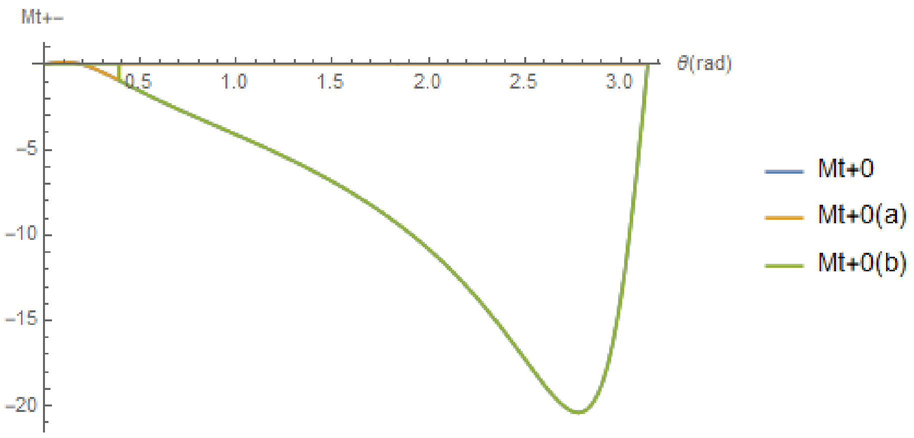
$$\delta = 0.785398 \sim \frac{\pi}{4}$$

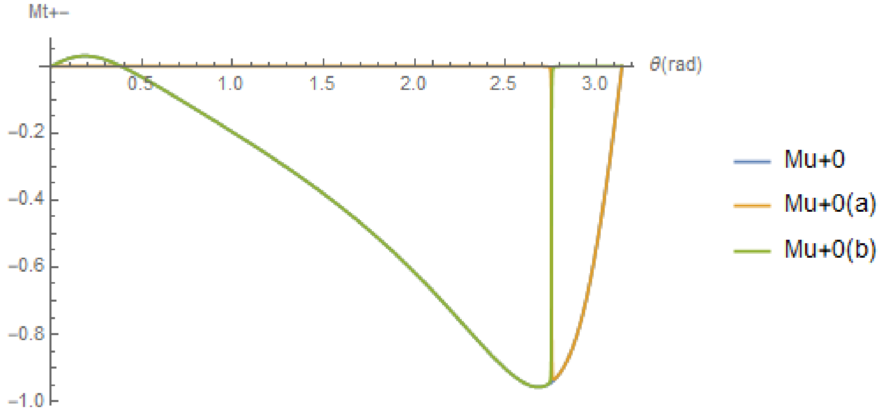
$$\theta_{c,t} = ArcCos\left(\sqrt{\frac{3}{3.5}}\right) = 0.387597$$

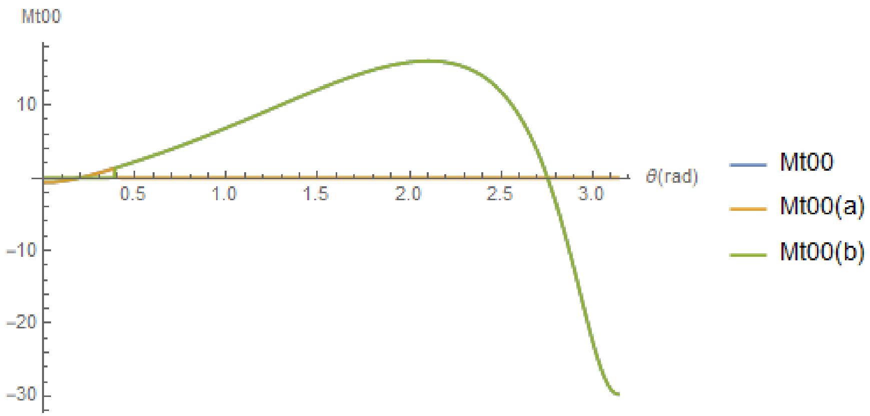
$$\theta_{c,u} = -ArcCos\left(\sqrt{\frac{3}{3.5}}\right) = 2.754$$

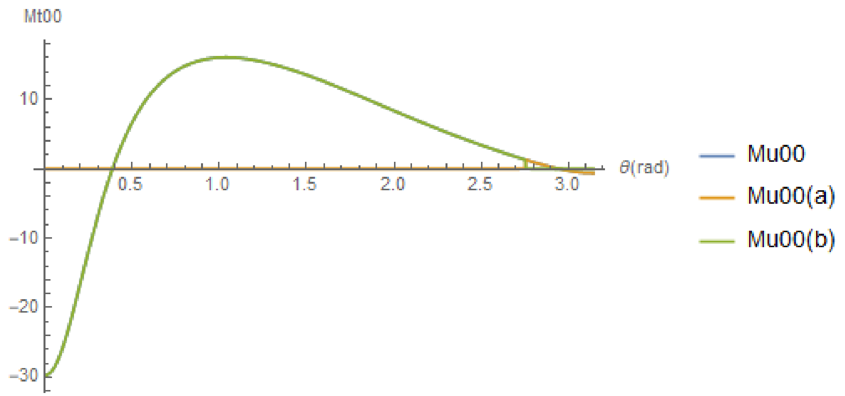

Mt+−
θ(rad)
Mt0+
Mt0+(a)
Mt0+(b)


Mt+−
θ(rad)
Mu0+
Mu0+(a)
Mu0+(b)


Mt+−
θ(rad)
Mt+0
Mt+0(a)
Mt+0(b)

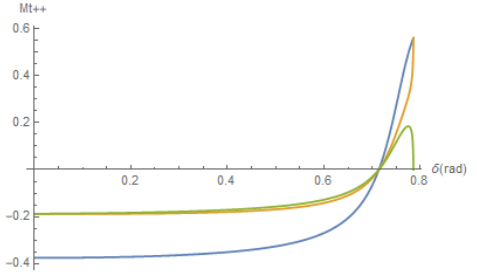

Mt+−
θ(rad)
Mu+0
Mu+0(a)
Mu+0(b)


Mt00
θ(rad)
Mt00
Mt00(a)
Mt00(b)

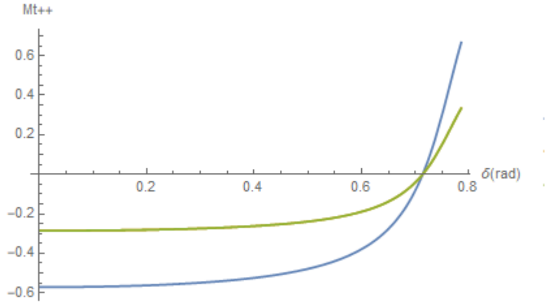

Mt00
θ(rad)
Mu00
Mu00(a)
Mu00(b)

Critical interpolation angles

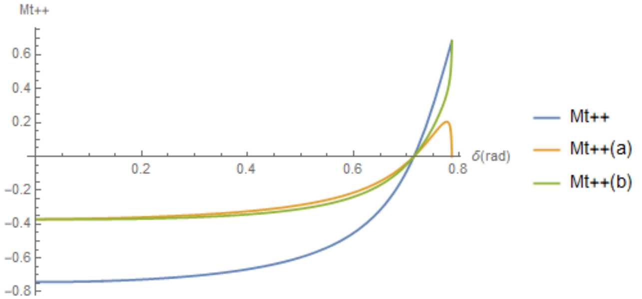
$\theta = \theta_{c,t} - 0.1$



$\theta = \theta_{c,t}$

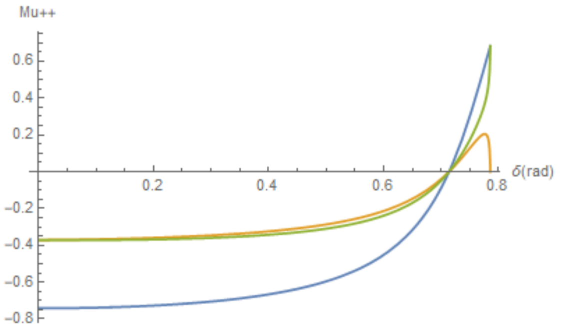


$\theta = \theta_{c,t} + 0.1$

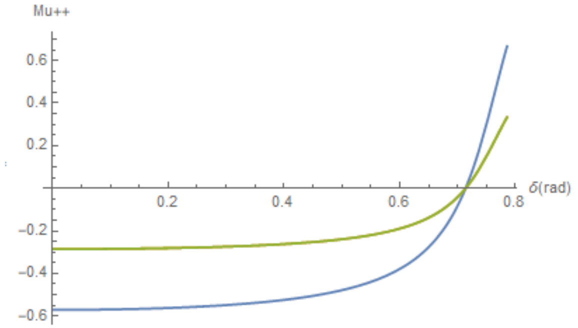


$$E_0 = 2m_e$$
$$P_\gamma = \sqrt{3.5}m_e$$
$$P_e = \sqrt{3}m_e$$

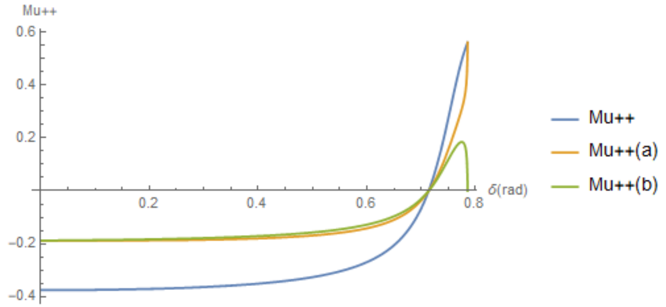
$\theta = \theta_{c,u} - 0.1$



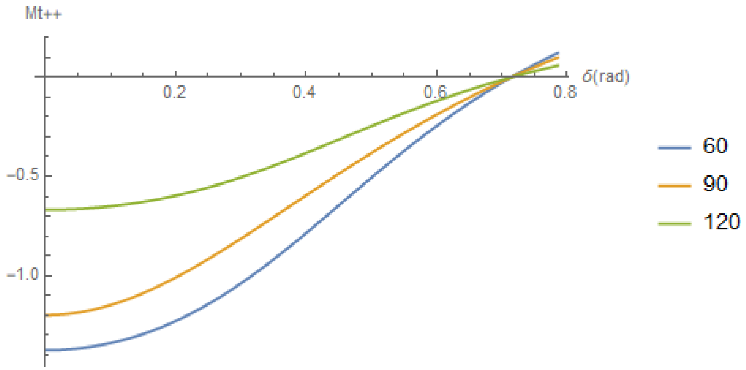
$\theta = \theta_{c,u}$

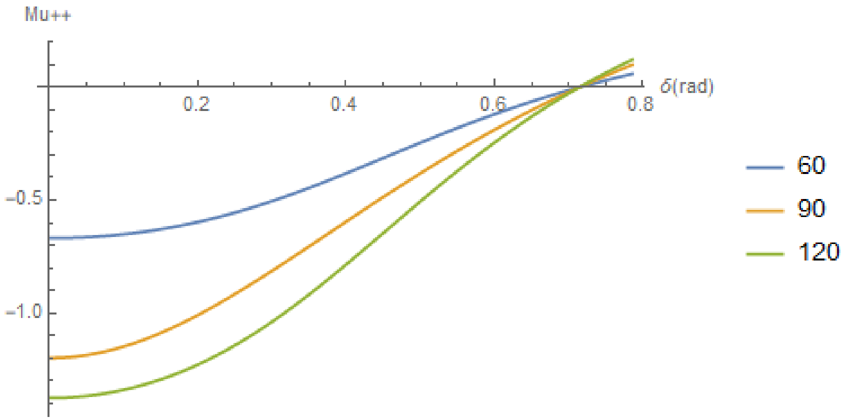


$\theta = \theta_{c,u} + 0.1$



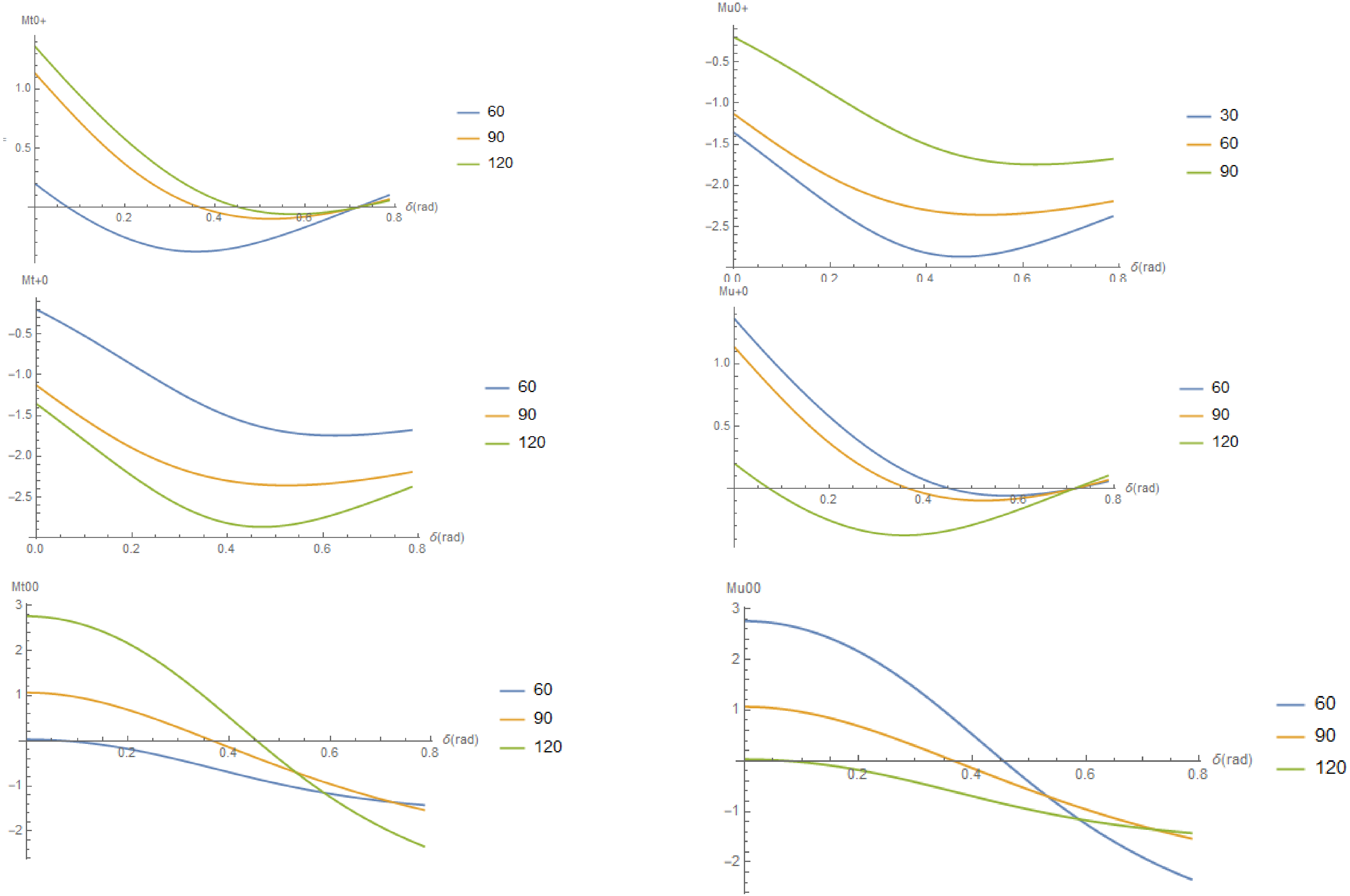
- When rho meson polarization vectors are transverse ,all δ_c values are independent of P_γ and θ .





$P_\gamma = m_e$

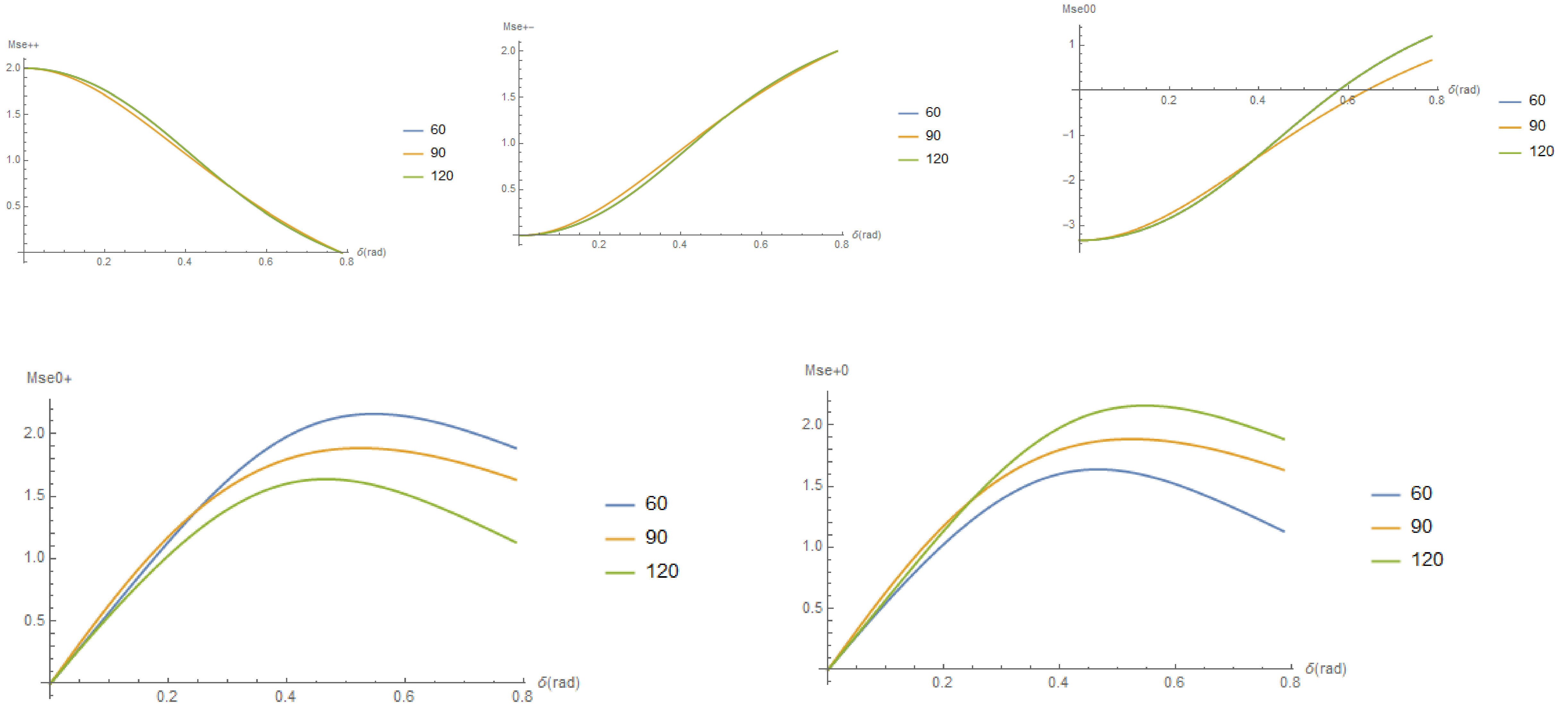
$$\delta_c = ArcTan\left(\frac{P_e}{E_0}\right)$$
$$= 0.713724$$



$$E_0 = 2m_e$$

$$P_e = \sqrt{3}m_e$$

$$P_\gamma = m_e$$



Mse++
60
90
120
δ(rad)
Mse+−
60
90
120
δ(rad)
Mse00
60
90
120
δ(rad)
Mse0+
60
90
120
δ(rad)
Mse+0
60
90
120
δ(rad)

Critical interpolation angles (Pz$\neq$ 0)

$$\delta_p^{\pm} = -ArcTan\left[\frac{\left(E0 * pz + pe * \sqrt{4E0^2 + pz^2}\right)}{\left(pe * pz + E0 * \sqrt{4E0^2 + pz^2}\right)}\right]$$

$$\delta_e^{\pm} = -ArcTan\left[\frac{\left(E0 * pz - pe * \sqrt{4E0^2 + pz^2}\right)}{\left(-pe * pz + E0 * \sqrt{4E0^2 + pz^2}\right)}\right]$$

- Pt and Pu play the role of Pe
- Et and Eu play the role of E0

$$\delta_{et}^{00}(\theta) = \delta_{eu}^{00}(\pi - \theta)$$

$$\delta_{pt}^{00}(\theta) = \delta_{pu}^{00}(\pi - \theta)$$

$$Et = E0^2 p\gamma Cos[\theta] - pe(E0^2 - p\gamma^2 Sin[\theta]^2)$$

$$Pt = E0 p\gamma(p\gamma - peCos[\theta])$$

$$Eu = E0^2 p\gamma Cos[\theta] + pe(E0^2 - p\gamma^2 Sin[\theta]^2)$$

$$Pu = -E0 p\gamma(p\gamma + peCos[\theta])$$

$$\delta_{pt}^{00} = -ArcTan\left[\frac{\left(Et * pz + pt * \sqrt{4E0^2 + pz^2}\right)}{\left(pt * pz + Et * \sqrt{4E0^2 + pz^2}\right)}\right]$$

$$\delta_{et}^{00} = -ArcTan\left[\frac{\left(Et * pz - pt * \sqrt{4E0^2 + pz^2}\right)}{\left(-pt * pz + Et * \sqrt{4E0^2 + pz^2}\right)}\right]$$

$$\delta_{pu}^{00} = -ArcTan\left[\frac{\left(Eu * pz + pu * \sqrt{4E0^2 + pz^2}\right)}{\left(pu * pz + Eu * \sqrt{4E0^2 + pz^2}\right)}\right]$$

$$\delta_{eu}^{00} = -ArcTan\left[\frac{\left(Eu * pz - pu * \sqrt{4E0^2 + pz^2}\right)}{\left(-pu * pz + Eu * \sqrt{4E0^2 + pz^2}\right)}\right]$$

$$M_t = (-p_1 + q_1)^\mu \varepsilon_\mu^*(p_3,\lambda_3) \frac{1}{q_1^2 - m^2} (p_2 + q_1)^\nu \varepsilon_\nu^*(p_4,\lambda_4)$$

$$(-p_1 + q_1)^\mu \varepsilon_\mu^*(p_3,\lambda_3) = -2\, (p_1)^\mu \varepsilon_\mu^*(p_3,\lambda_3) \quad \rightarrow (\delta_p^\pm, \delta_{pt}^{00})$$

$$(p_2 + q_1)^\nu \varepsilon_\nu^*(p_4,\lambda_4) = 2 (p_2)^\nu \varepsilon_\nu^*(p_4,\lambda_4) \quad \rightarrow (\delta_e^\pm, \delta_{et}^{00})$$

$$M_u = (-p_1 + q_2)^\nu \varepsilon_\nu^*(p_4,\lambda_4) \frac{1}{q_2^2 - m^2} (p_2 + q_2)^\mu\, \varepsilon_\mu^*(p_3,\lambda_3)$$

$$(-p_1 + q_2)^\nu \varepsilon_\nu^*(p_4,\lambda_4) = -2 (p_1)^\nu \varepsilon_\nu^*(p_4,\lambda_4) \quad \rightarrow (\delta_p^\pm, \delta_{pu}^{00})$$

$$(p_2 + q_2)^\mu \varepsilon_\mu^*(p_3,\lambda_3) = 2\, (p_2)^\mu \varepsilon_\mu^*(p_3,\lambda_3) \quad \rightarrow (\delta_e^\pm, \delta_{eu}^{00})$$

Ex

- A positive scalar meson projection to the polarization vector of the rho meson gives $\delta_p^\pm$ or δ_{pt}^{00} critical angles depending on whether polarization vectors are transverse or longitudinal respectively.

Summary

Covariant Amplitudes					δ_c	
Mt ++ =	Mt - - =	-(Mt + -) =	-(Mt + -)		$\delta_p^{\pm}$	$\delta_e^{\pm}$
Mu ++ =	Mu - - =	-(Mu + -) =	-(Mu + -)		$\delta_p^{\pm}$	$\delta_e^{\pm}$
Mt 0 + =	-(Mt 0 -)				$\delta_e^{\pm}$	δ_{pt}^{00}
Mu 0 + =	-(Mu 0 -)				$\delta_p^{\pm}$	δ_{eu}^{00}
Mt + 0 =	-(Mt – 0)				$\delta_p^{\pm}$	δ_{et}^{00}
Mu + 0 =	-(Mu – 0)				$\delta_e^{\pm}$	δ_{pu}^{00}
Mt 00					δ_{pt}^{00}	δ_{et}^{00}
Mu 00					δ_{pu}^{00}	δ_{eu}^{00}

δ_c= Critical interpolation angles in which covariant amplitudes are equal to zero

- Corresponding time order-amplitude also vanishes when δ->δ_c

Limitation

IFD ⟷ Interpolating Dynamic ⟷ LFD

0 $\qquad \delta \qquad$ $\frac{\pi}{4}$

$0 \leq Tan(\delta) \leq 1$

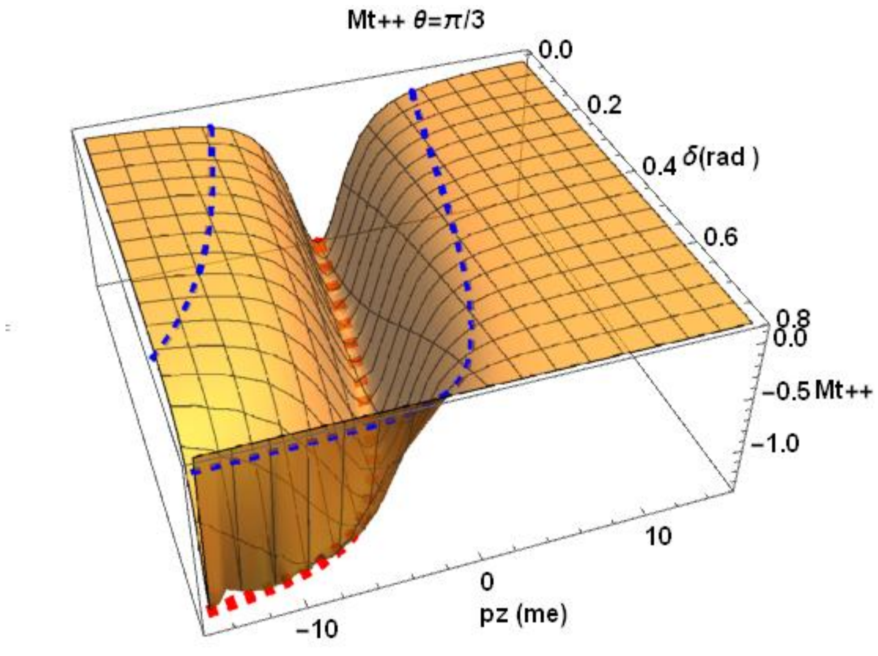
	$0 \leq Tan(\delta)$	$Tan(\delta) \leq 1$
$\delta_p^{\pm}$	-1≤R	-Rz ≥R
$\delta_e^{\pm}$	1≥R	Rz≤R
δ_{pt}^{00}	-1≤Rt	-Rz ≥Rt
δ_{et}^{00}	1≥Rt	Rz≤Rt
δ_{pu}^{00}	-1≤Ru	-Rz ≥Ru
δ_{eu}^{00}	1≥Ru	Rz≤Ru

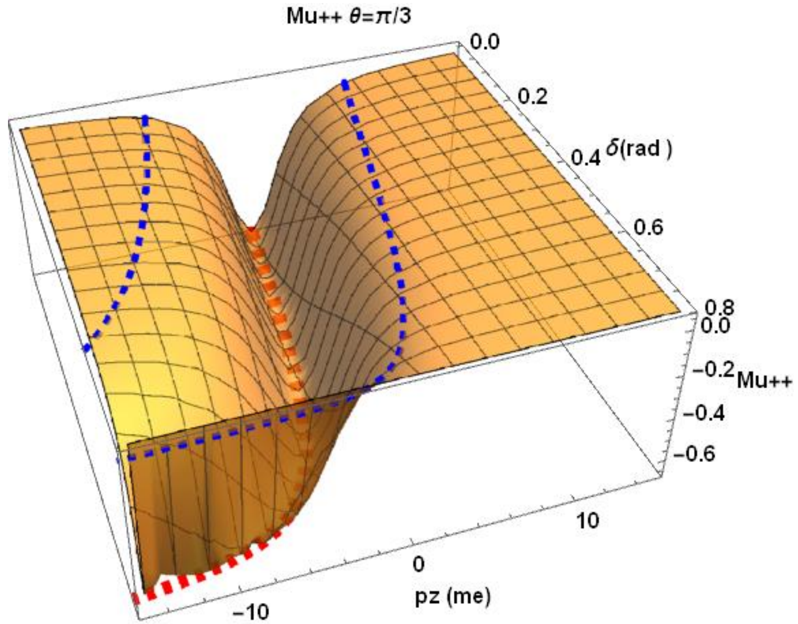
$$R = \frac{P_e}{E0} \qquad Rz = \frac{P_z}{E}$$

$$Rt = \frac{Pt}{Et} \qquad Ru = \frac{Pu}{Eu}$$

Total energy of the boosted frame =

$$E = \sqrt{4E_0^2 + P_z^2}$$



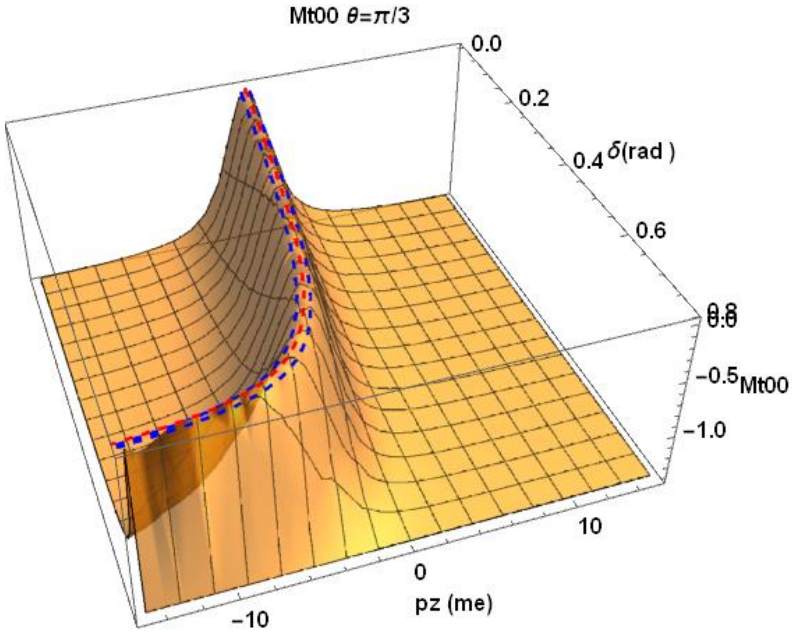


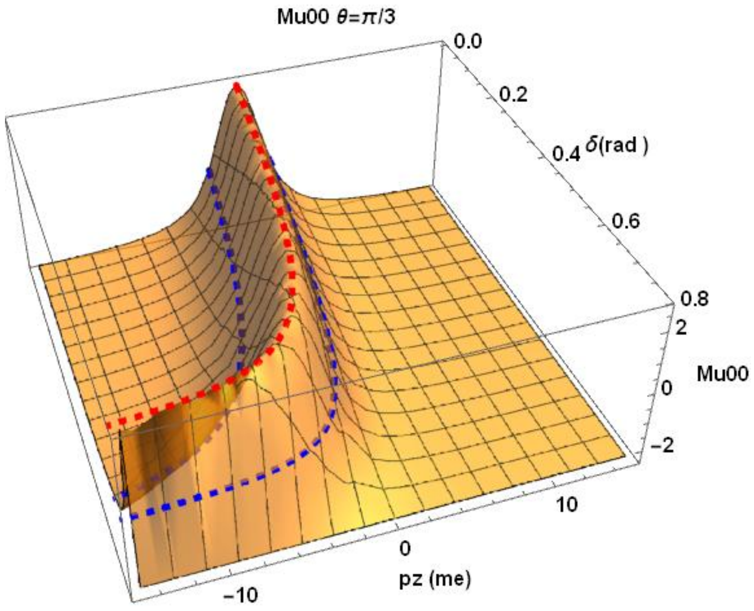
$$E_0 = 2m_e$$

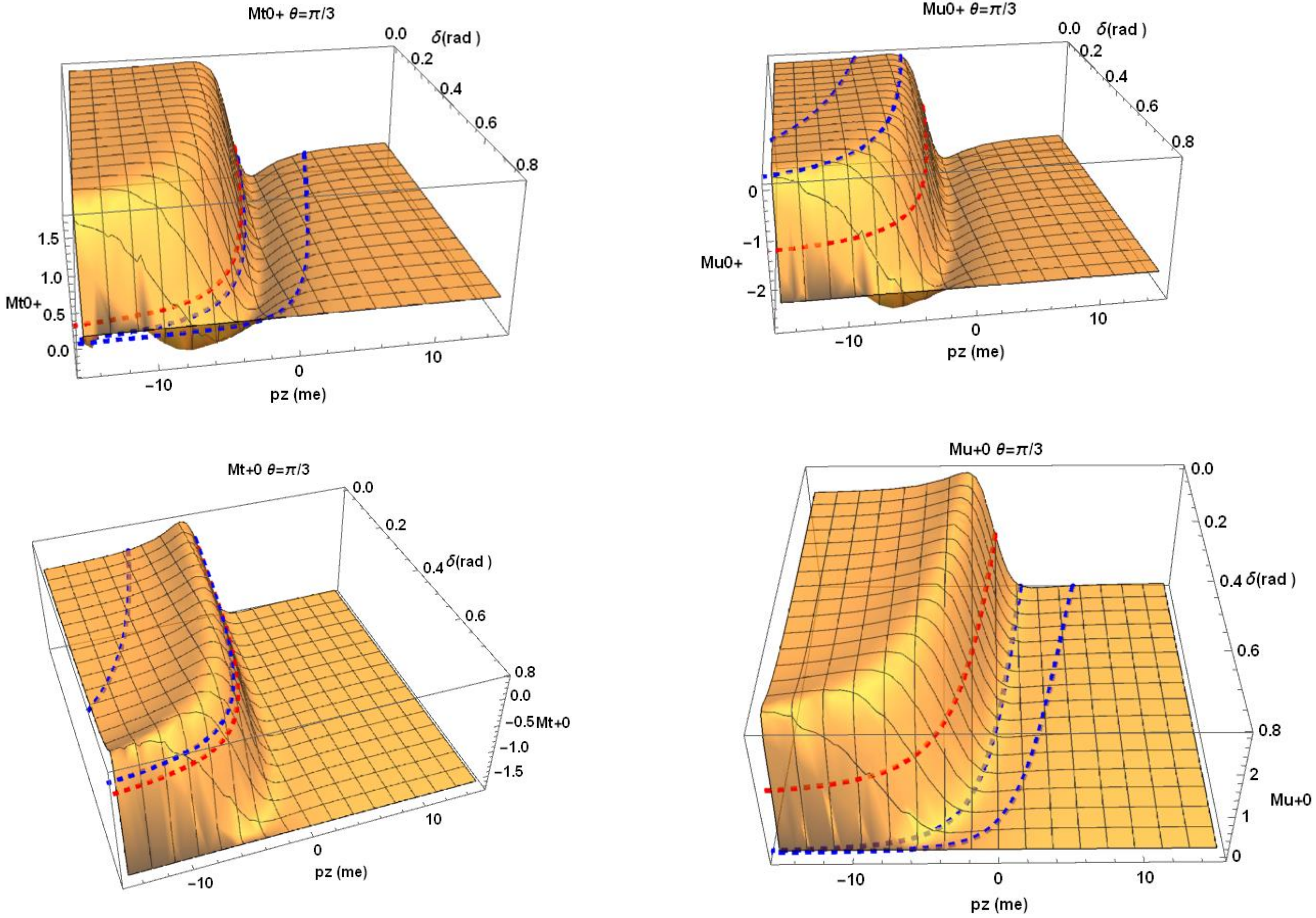
$$P_e = \sqrt{3}m_e$$

$$P_\gamma = m_e$$

$$\mathrm{Jc} = -\sqrt{\frac{(2\mathrm{E0})^2(1 - \mathrm{Cos}[2\delta])}{2\mathrm{Cos}[2\delta]}}$$







Mt0+ θ=π/3
δ(rad)
0.0
0.2
0.4
0.6
0.8
1.5
1.0
Mt0+
0.5
0.0
−10
0
10
pz (me)
Mu0+ θ=π/3
δ(rad)
0.0
0.2
0.4
0.6
0.8
0
−1
Mu0+
−2
−10
0
10
pz (me)
Mt+0 θ=π/3
0.0
0.2
0.4
δ(rad)
0.6
0.8
0.0
−0.5
Mt+0
−1.0
−1.5
10
0
−10
pz (me)
Mu+0 θ=π/3
0.0
0.2
0.4
δ(rad)
0.6
0.8
2
Mu+0
1
0
−10
0
10
pz (me)

Cross-Section of the processes using Mandelstam variable.

$$|M|^2 = \sum_{\lambda_1,\lambda_2} |M_t^{\lambda_1,\lambda_2} + M_u^{\lambda_1,\lambda_2} + M_{se}^{\lambda_1,\lambda_2}|^2$$

- Two Scalar mesons annihilation in to two rho mesons

$$|M_\rho^2| = \left[\left[\frac{2t + 2m_e^2 - m_\gamma^2}{(t - m_e^2)}\right]^2 + \left[\frac{2u + 2m_e^2 - m_\gamma^2}{(u - m_e^2)}\right]^2 + 2\left[\frac{\left[t + u + 2m_e^2 - 3m_\gamma^2\right]^2}{(t - m_e^2)(u - m_e^2)}\right]\right]$$

$$-4\left[\left[\frac{5t + u + 2m_e^2 - 4m_\gamma^2}{(t - m_e^2)}\right] + \left[\frac{5u + t + 2m_e^2 - 4m_\gamma^2}{(u - m_e^2)}\right]\right] + 16$$

- Scaler electron and proton annihilation in to two photons

$$|M|^2 = 4\left[\left[\frac{t + m_e^2}{t - m_e^2}\right]^2 + \left[\frac{u + m_e^2}{u - m_e^2}\right]^2 + 4\left[\frac{(t + m_e^2)(u + m_e^2) - 2tu}{(t - m_e^2)(u - m_e^2)}\right] + 4\right]$$

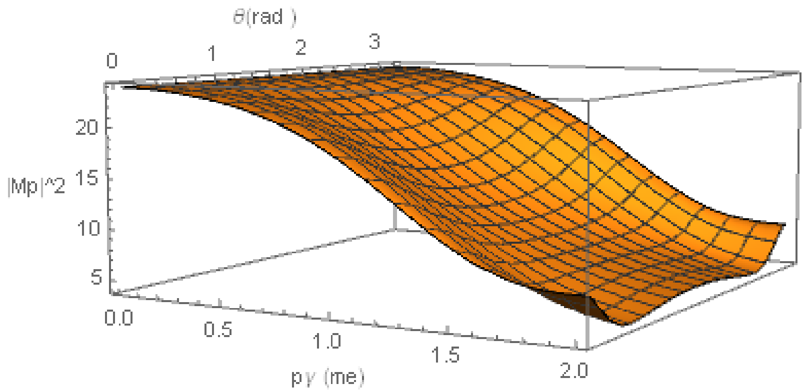
- Two Scalar mesons annihilation in to two rho mesons

$$|M_\rho^2| = 4[\left(1+\left(1-\frac{4\mathrm{pe}^2(\mathrm{E0}^2+\mathrm{p}\gamma^2)\mathrm{Sin}[\theta]^2}{(\mathrm{E0}^2+\mathrm{p}\gamma^2)^2-4\mathrm{pe}^2\mathrm{p}\gamma^2\mathrm{Cos}[\theta]^2}\right)^2\right)+$$

$$\frac{(\mathrm{E0}^2-\mathrm{p}\gamma^2)^2(\mathrm{E0}^2+\mathrm{p}\gamma^2-4\mathrm{pe}^2\mathrm{Cos}[\theta]^2)^2+32\mathrm{E0}^2\mathrm{pe}^4(\mathrm{E0}-\mathrm{p}\gamma)(\mathrm{E0}+\mathrm{p}\gamma)\mathrm{Sin}[2\theta]^2}{\left((\mathrm{E0}^2+\mathrm{p}\gamma^2)^2-4\mathrm{pe}^2\mathrm{p}\gamma^2\mathrm{Cos}[\theta]^2\right)^2}]$$

- Scaler electron and proton annihilation in to two photons

$$|M|^2 = 4\left[1+\left[1-\frac{2p_e^2 Sin^2(\theta)}{E_0^2-p_e^2 Cos^2(\theta)}\right]^2\right]$$



- Two Scalar mesons annihilation in to two rho mesons

$$|M_\rho^2| = 4\left[\left(1 - \frac{2pe^2(E0^2 + p\gamma^2)Sin[\theta]^2}{(E0^2 + p\gamma^2)^2 - 4pe^2p\gamma^2Cos[\theta]^2}\right)^2 + \frac{8pe^4(E0^2 + p\gamma^2)^2Sin[\theta]^4}{\left((E0^2 + p\gamma^2)^2 - 4pe^2p\gamma^2Cos[\theta]^2\right)^2} + \right]$$

$$4\left[\frac{(E0^2 - p\gamma^2)^2(E0^2 + p\gamma^2 - 4pe^2Cos[\theta]^2)^2}{\left((E0^2 + p\gamma^2)^2 - 4pe^2p\gamma^2Cos[\theta]^2\right)^2} + \frac{8E0^2pe^4(E0^2 - p\gamma^2)Sin[2\theta]^2}{\left((E0^2 + p\gamma^2)^2 - 4pe^2p\gamma^2Cos[\theta]^2\right)^2}\right]$$

- Scaler electron and proton annihilation in to two photons

$$|M|^2 = 4\left[1 + \left[1 - \frac{2p_e^2Sin^2(\theta)}{E_0^2 - p_e^2Cos^2(\theta)}\right]^2\right]$$

$$4\left[\frac{(E0^2 - p\gamma^2)^2(E0^2 + p\gamma^2 - 4pe^2Cos[\theta]^2)^2}{\left((E0^2 + p\gamma^2)^2 - 4pe^2p\gamma^2Cos[\theta]^2\right)^2} + \frac{8E0^2pe^4(E0^2 - p\gamma^2)Sin[2\theta]^2}{\left((E0^2 + p\gamma^2)^2 - 4pe^2p\gamma^2Cos[\theta]^2\right)^2}\right]$$

Thank You