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Manifestation of quantum correlation in the interpolating helicity amplitudes

Deepasika Dayananda[1] , Chueng-Ryong Ji[2]
North Carolina State University

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e-HUGS

[1]isamara@ncsu.edu
[2]crji@ncsu.edu

Quantum Correlation due to orientation entanglement of spins

- The angle should be rotated to get the same initial configuration.
  - Spin- 0 → Any angle
  - Spin-1/2 → 720^0
  - Spin-1 → 360^0

$$d^{(j)}_{m',m}(\beta) = < j, m'|exp(\frac{-ij_y\beta}{\hbar})|j, m >$$

(Wigner-d function)

Rotation by 180^0

Spin-0 particles

$$|0,0> \longrightarrow |0,0>$$

Spin-1/2 particles

$$|1/2, 1/2> \longrightarrow |1/2, -1/2>$$
$$|1/2, -1/2> \longrightarrow -|1/2, 1/2>$$

Spin-1 particles

$$|1,1> \longrightarrow |1,-1>$$
$$|1,0> \longrightarrow -|1,0>$$
$$|1,-1> \longrightarrow |1,1>$$

Quantum –orientation entangled states

- Spinors and polarization vectors
  - Spin orientation
  - Momentum of the particle

Quantum correlation in the interpolating helicity amplitudes ?

Interpolating Dynamic

Interpolation space time transformation

$$\begin{pmatrix} x^{\hat{+}} \\ x^{\hat{1}} \\ x^{\hat{2}} \\ x^{\hat{-}} \end{pmatrix} = \begin{pmatrix} \cos(\delta) & 0 & 0 & \sin(\delta) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \sin(\delta) & 0 & 0 & -\cos(\delta) \end{pmatrix} \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}$$

- We connect two relativistic dynamics, proposed by Dirac

Helicity is not invariant under the boost $\quad \mathfrak{J}_3 \rightarrow \frac{\boldsymbol{P}.\boldsymbol{J}}{|\boldsymbol{P}|}$

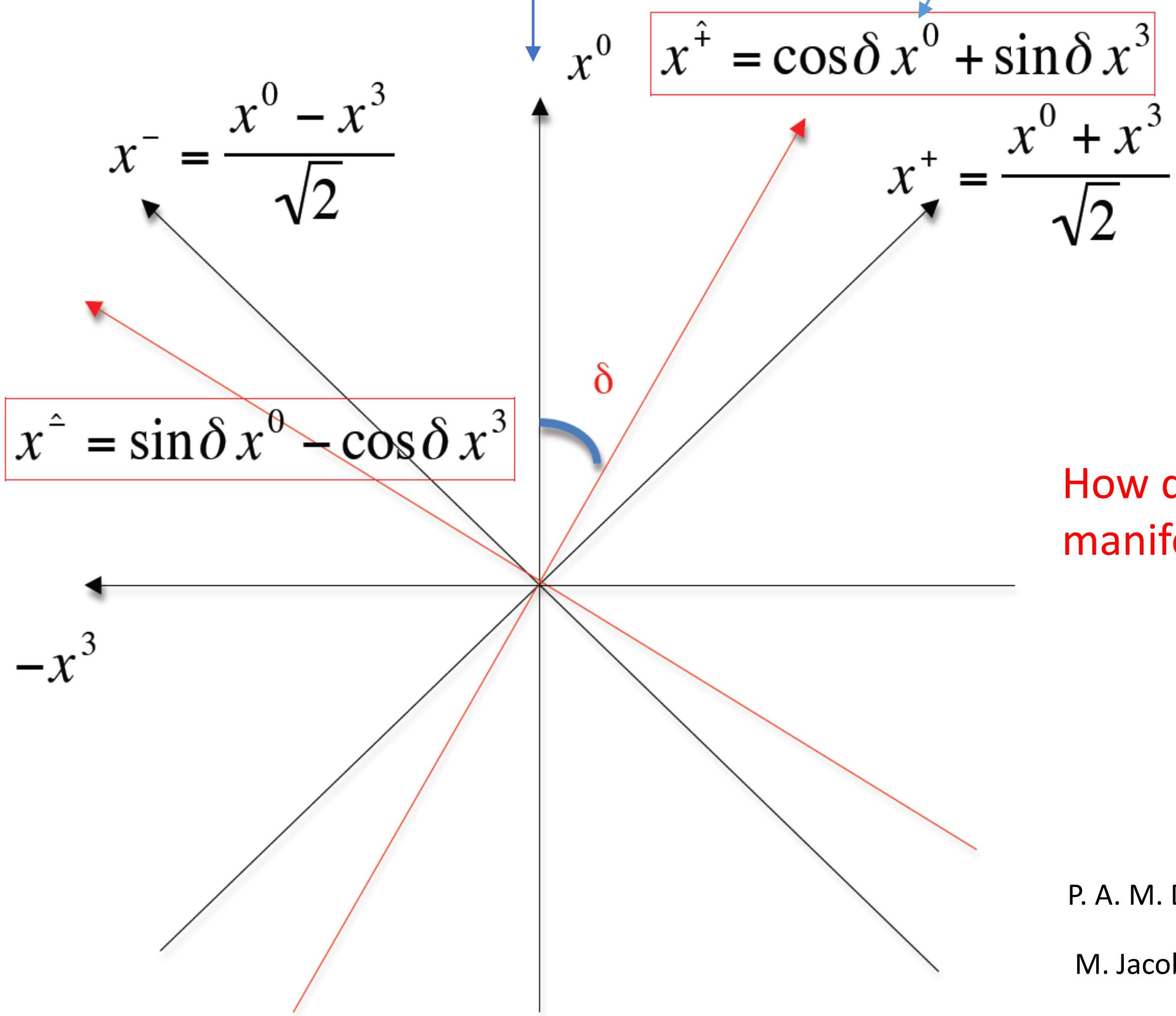
IFD ($\delta = 0$)

δ_c

LFD ($\delta = \frac{\pi}{4}$)

Helicity is invariant under the boost

$$\mathfrak{J}_3 \rightarrow J_3 + \frac{(P^1 E_2 - P^2 E_1)}{P^+}$$



How quantum correlation manifests in different dynamic ?

P. A. M. Dirac, Rev. Mod. Phys. **21**, 392 (1949)

M. Jacob and G. Wick, Ann. Phys.**7**, 404 (1959)

C.-R. Ji, Z. Li, and B. Ma Phys. Rev. D **98**, 036017 (2018)

Spin orientation for generalize helicity spinors and vectors

Helicity transformation matrix

$$T = T_{12}T_3 = e^{i\beta_1\mathcal{K}^{\hat{1}}+i\beta_2\mathcal{K}^{\hat{2}}}e^{-i\beta_3 K^3}$$

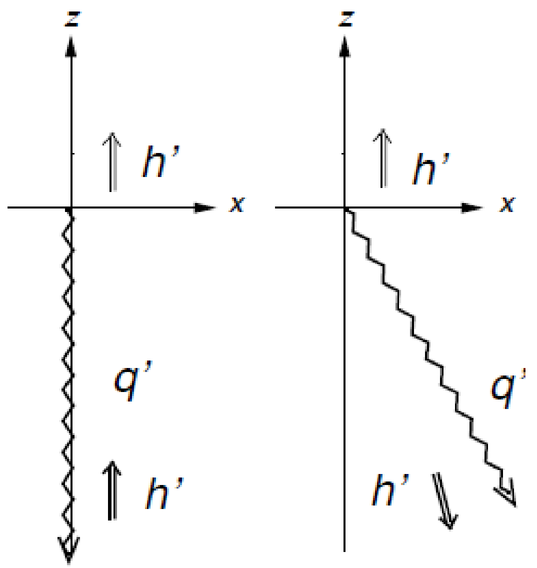
$$\mathcal{K}^{\hat{1}} = -K^1\sin\delta - J^2\cos\delta,$$
$$\mathcal{K}^{\hat{2}} = J^1\cos\delta - K^2\sin\delta,$$

$$(\delta\to\pi/4),\ \mathcal{K}^{\hat{1}}\to -E_1,\ \mathcal{K}^{\hat{2}}\to -E_2$$
$$(\delta\to 0),\ \mathcal{K}^{\hat{1}}\to -J^2,\ \mathcal{K}^{\hat{2}}\to J^1$$

$$T = B(\boldsymbol{\eta})\mathcal{D}(\hat{\mathbf{m}},\theta_s) = e^{-i\boldsymbol{\eta}\cdot\mathbf{K}}e^{-i\hat{\mathbf{m}}\cdot\mathbf{J}\theta_s},$$

Boosts to momentum **P**
$\hat{n} = (\ sin\theta cos\varphi, sin\theta sin\varphi, cos\theta)$

Rotates the spin around the axis by a unit vector $\hat{m}$ = $(\ -sin\varphi_s, cos\varphi_s, 0)$ by angle θ_s .



Spin-direction corresponding to the LF boosts in the z-direction only (LHS) and one including transverse momentum (RHS)

$$\mathrm{Cos}[\theta s] = \frac{\mathrm{Cos}[\alpha] + \mathrm{Cosh}[\beta 3] + \mathrm{Cos}[\alpha]\mathrm{Cosh}[\beta 3] - \mathrm{Cosh}[\eta]}{1 + \mathrm{Cosh}[\eta]}$$

$$\cos\alpha = \frac{P_{\hat{-}}}{\mathbb{P}} \qquad e^{-\beta_3} = \frac{P^{\hat{+}} - \mathbb{P}}{M(\cos\delta - \sin\delta)}$$

$$\mathbb{P} = \sqrt{P_{\hat{-}}^2 + \mathbf{P}_\perp^2 \mathbb{C}} \qquad e^{\beta_3} = \frac{P^{\hat{+}} + \mathbb{P}}{M(\sin\delta + \cos\delta)}$$

Since $\Phi s = \phi$, without loss and generality we can make $\phi = \phi s = 0$

When, we fix the particle's initial momentum direction as +z ,

We can simplify $\theta_s = 0\, , cos\alpha \to 1$ $P_{\hat{-}} > 0$ & $\theta_s = \pi\, , cos\alpha \to -1$ $P_{\hat{-}} < 0$

$\Rrightarrow$ Indicates sign change of $P_{\hat{-}}$

<u>QC in Spin-1/2 spinors</u>

$$U^{+1/2}(P_{\hat{-}} > 0) \Rrightarrow U^{-1/2}(P_{\hat{-}} > 0)$$
$$U^{-1/2}(P_{\hat{-}} > 0) \Rrightarrow -U^{+1/2}(P_{\hat{-}} > 0)$$

<u>QC in spin-1 spinors</u>

$$U^{+1}(P_{\hat{-}} > 0) \Rrightarrow U^{-1}(P_{\hat{-}} > 0)$$
$$U^{0}(P_{\hat{-}} > 0) \Rrightarrow -U^{0}(P_{\hat{-}} > 0)$$
$$U^{-1}(P_{\hat{-}} > 0) \Rrightarrow U^{+1}(P_{\hat{-}} > 0)$$

<u>QC in polarization vectors</u>

$$\epsilon^{+1}(P_{\hat{-}} > 0) \Rrightarrow \epsilon^{-1}(P_{\hat{-}} > 0)$$
$$\epsilon^{0}(P_{\hat{-}} > 0) \Rrightarrow -\epsilon^{0}(P_{\hat{-}} > 0)$$
$$\epsilon^{-1}(P_{\hat{-}} > 0) \Rrightarrow \epsilon^{+1}(P_{\hat{-}} > 0)$$

Interpolating Longitudinal Momentum

$$P_1 = \{E_0, 0{,}0, P_v\}$$
$$P_2 = \{E_0, 0{,}0, -P_v\}$$

$$\bar{E} = 2E_0$$

We see $0 \le \delta < \frac{\pi}{4}$ range $P_{\hat{-}}$ can get any real value, but exactly at the LF we do not see $P_{\hat{-}} < 0$ values , since $P_{\hat{-}} \to P^+$ at $\delta = \frac{\pi}{4}$.

- Change of K^3 from "dynamic" operator to the "kinematic " operator at LF

$\Longrightarrow$

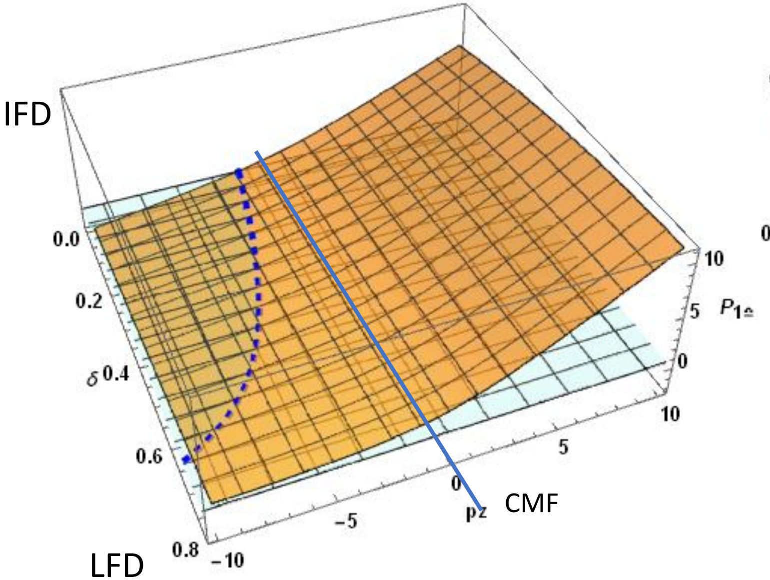
$P_{1\hat{-}} = 0$, and $P_{2\hat{-}} = 0$, $P_1^+ \to 0$ and $P_2^+ \to 0$

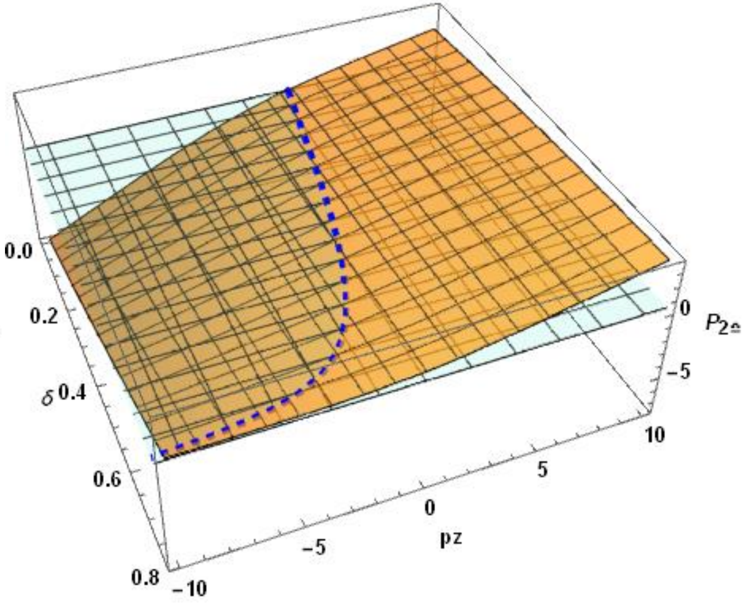
$$P^z \to -\infty$$

Helicity Boundaries

$$P_{1\hat{-}} = [(P_v P^z + E_0\sqrt{\bar{E}^2 + (P^z)^2})\sin\delta + (E_0 P^z + P_v\sqrt{\bar{E}^2 + (P^z)^2})\cos\delta]/\bar{E}$$

$$P_{2\hat{-}} = [(-P_v P^z + E_0\sqrt{\bar{E}^2 + (P^z)^2})\sin\delta + (E_0 P^z - P_v\sqrt{\bar{E}^2 + (P^z)^2})\cos\delta]/\bar{E}$$





$$\tan(\delta_{c1}) = -\frac{E_0 P^z + P_v\sqrt{\bar{E}^2 + (P^z)^2}}{P_v P^z + E_0\sqrt{\bar{E}^2 + (P^z)^2}}$$

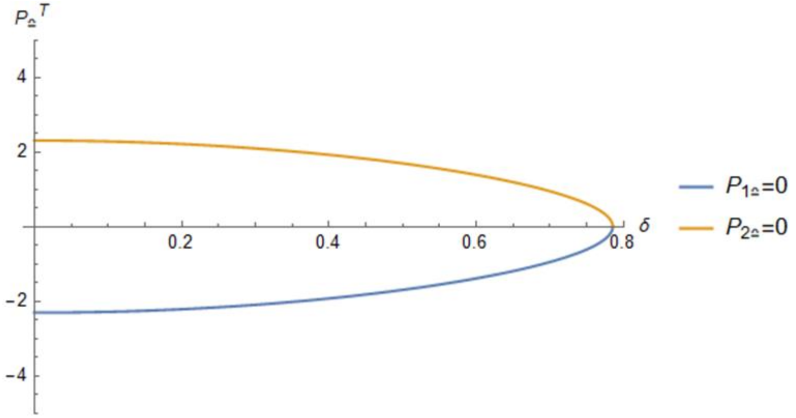
$$\tan(\delta_{c2}) = -\frac{E_0 P^z - P_v\sqrt{\bar{E}^2 + (P^z)^2}}{-P_v P^z + E_0\sqrt{\bar{E}^2 + (P^z)^2}}$$

It seems that the QC of LF is accumulated in the zero-mode

To see the QC in the zero-mode we consider total longitudinal momentum of the system

$$(P_{\hat{-}})^T = \sqrt{\bar{E}^2 + (P^z)^2}\sin\delta + P^z\cos\delta$$

$$\Rightarrow P^z = \frac{P_{\hat{-}}^T\cos\delta - \sin\delta\sqrt{(P_{\hat{-}}^T)^2 + \bar{E}^2\cos 2\delta}}{\cos 2\delta}$$



Zero-mode of the light front

$$P^z \to -\infty\,,\ (P_{\hat{-}})^T \to 0$$

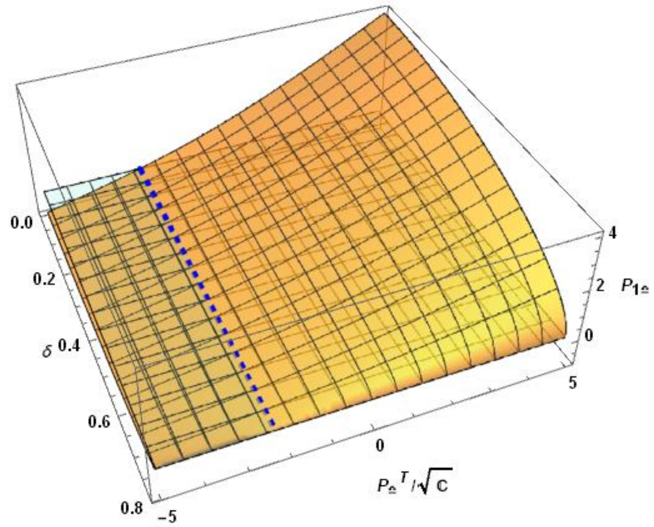
This further confirm that the QC accumulated in the zero-mode

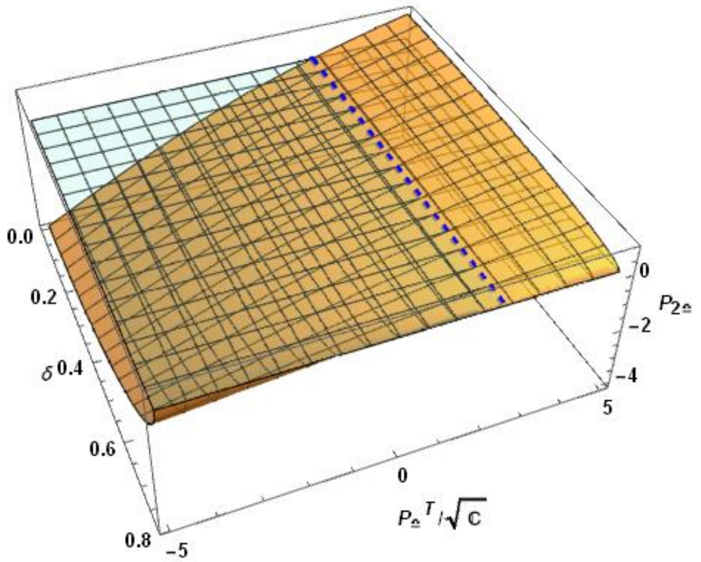
Scanning the zero-mode

At $\delta = \frac{\pi}{4}$ $\quad \frac{(P_{\hat{-}})^T}{\sqrt{\mathbb{C}}}$ $\longrightarrow$ Finite Value

$\mathbb{C} = \cos 2\delta$

QC Boundaries





$$\left(\frac{(P_{\hat{-}})^T}{\sqrt{\mathbb{C}}}\right)_{c1} = -\frac{\bar{E}P_v}{M} = -2.3094$$

$$\left(\frac{(P_{\hat{-}})^T}{\sqrt{\mathbb{C}}}\right)_{c2} = \frac{\bar{E}P_v}{M} = 2.3094$$

$$\bar{E} = 2E_0, E_0 = 2, P_v = 1, M = \sqrt{3}$$

Scalar particle and its anti-particle production by two neutral massive spin-1 particles

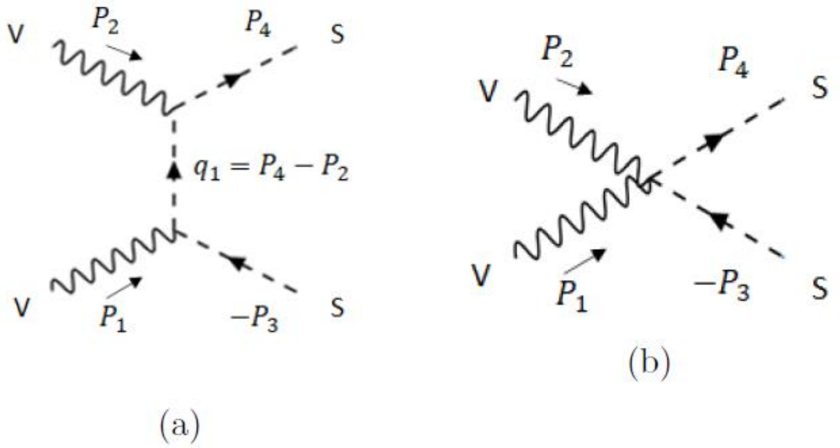


Fig. (a) t-channel Feynman diagram, the cross channel (u-channel) can be drawn by crossing the two final states' particles. Fig. (b) is drawn for the seagull channel.

v -> Vector particle (Spin-1)
s -> Scalar particle (Spin-0)

Interpolating helicity amplitudes

$$M_t^{\lambda_1\lambda_2} = (-p_3 + q_1)^{\hat{\mu}} \varepsilon_{\hat{\mu}}(p_1,\lambda_1) \frac{1}{q_1^2 - m_s{}^2} (p_4 + q_1)^{\hat{\nu}} \varepsilon_{\hat{\nu}}(p_2,\lambda_2)$$

$$M_u^{\lambda_1\lambda_2} = (-p_3 + q_2)^{\hat{\nu}} \varepsilon_{\hat{\nu}}(p_2,\lambda_2) \frac{1}{q_2^2 - m_s{}^2} (-p_4 + q_2)^{\hat{\mu}} \varepsilon_{\hat{\mu}}(p_1,\lambda_1)$$

$$M_{se}^{\lambda_1\lambda_2} = -2 g_{\hat{\mu}\hat{\nu}} \varepsilon^{\hat{\mu}} (p_1,\lambda_1)\, \varepsilon^{\hat{\nu}} (p_2,\lambda_2)$$

Where $q_2 = p_3 - p_2$

- T and U channels have time-ordered interpolating helicity amplitudes.

Seagull Channel

- Contact interaction - Angular momentum is conserved without involving orbital angular momentum

$$\epsilon^{+1}(P_{\hat{-}} > 0) \Rrightarrow \epsilon^{-1}(P_{\hat{-}} > 0)$$
$$\epsilon^{-1}(P_{\hat{-}} > 0) \Rrightarrow \epsilon^{+1}(P_{\hat{-}} > 0)$$

$$|1,1> \longrightarrow |1,-1>$$

$$|1,-1> \longrightarrow |1,1>$$

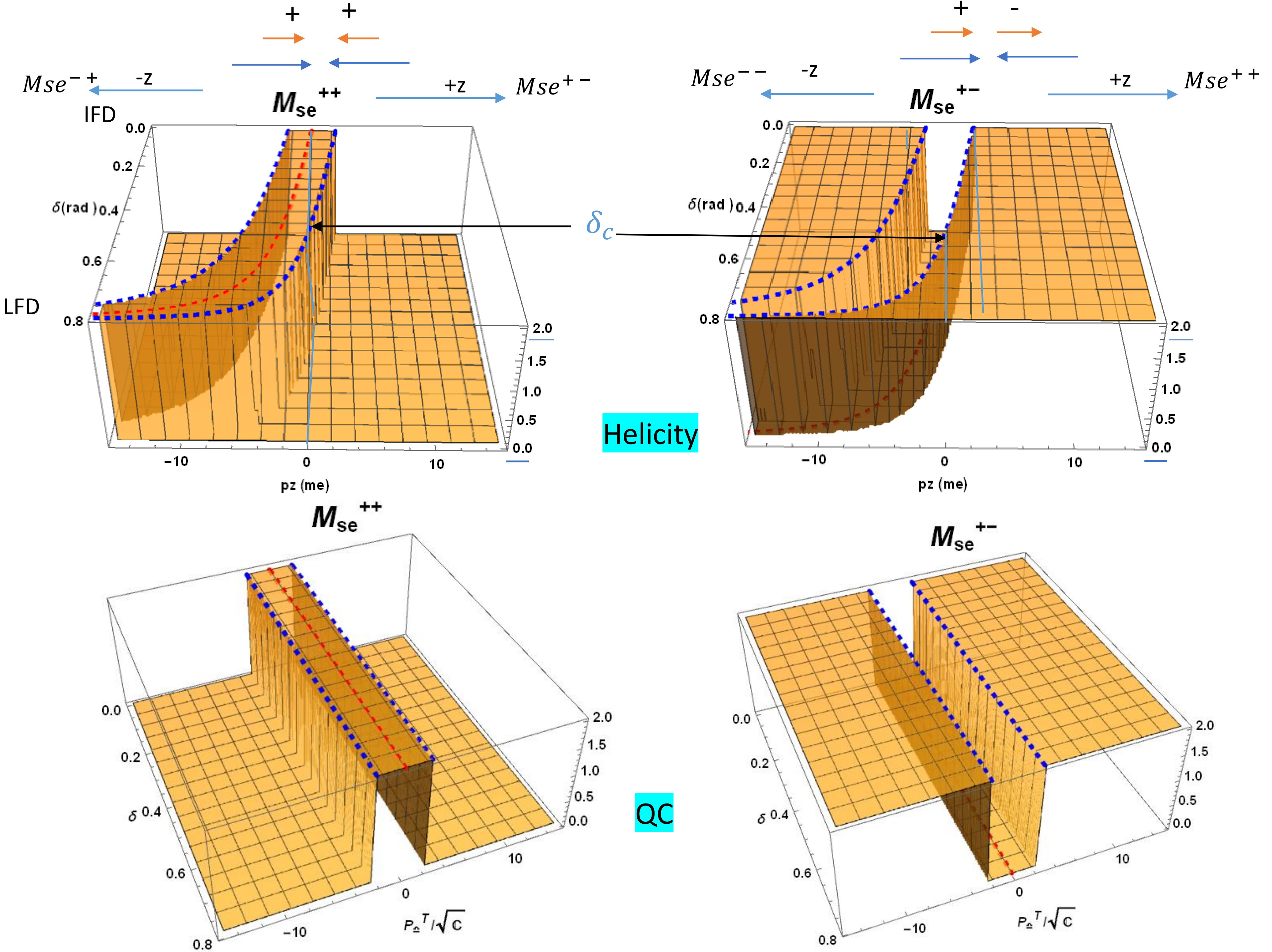
Parity conservation

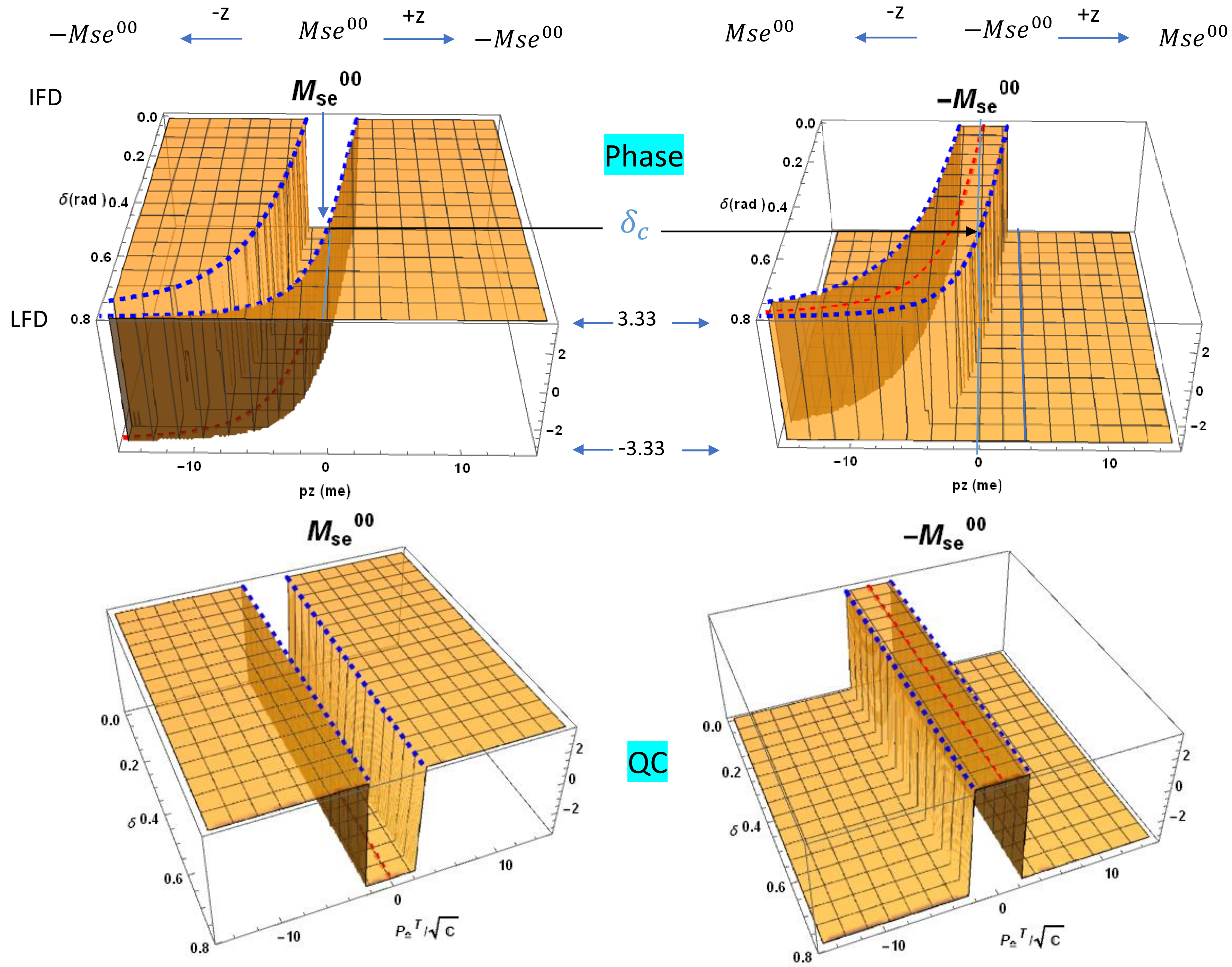
$$Mse^{--} = Mse^{++}$$

$$Mse^{-+} = Mse^{+-}$$

[C-R . Ji, B.L.G.Bakker , International Journal of Modern Physics,(2013)]

$P_s = \sqrt{3}$ $\quad P_v = 1$ $\quad E_0 = 2$





$$|1,0> \longrightarrow -|1,0>$$

$$\epsilon^0(P_{\triangle} > 0) \Rrightarrow -\epsilon^0(P_{\triangle} > 0)$$

- $Mse^{+0} = Mse^{-0} = Mse^{0+} = Mse^{0-} = 0$

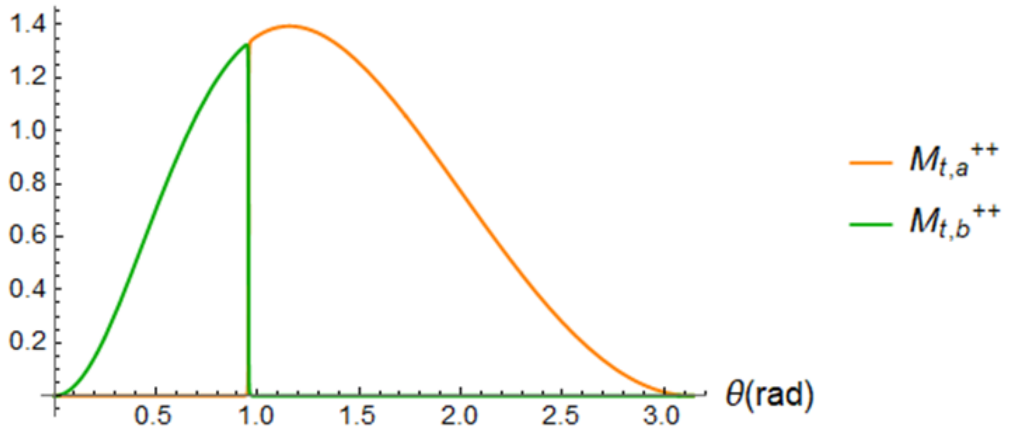
  Do not satisfy conservation of total angular momentum in any frame

- T and U channel helicity amplitudes depend on orbital angular momentum involving impact parameter. But they share the same feature

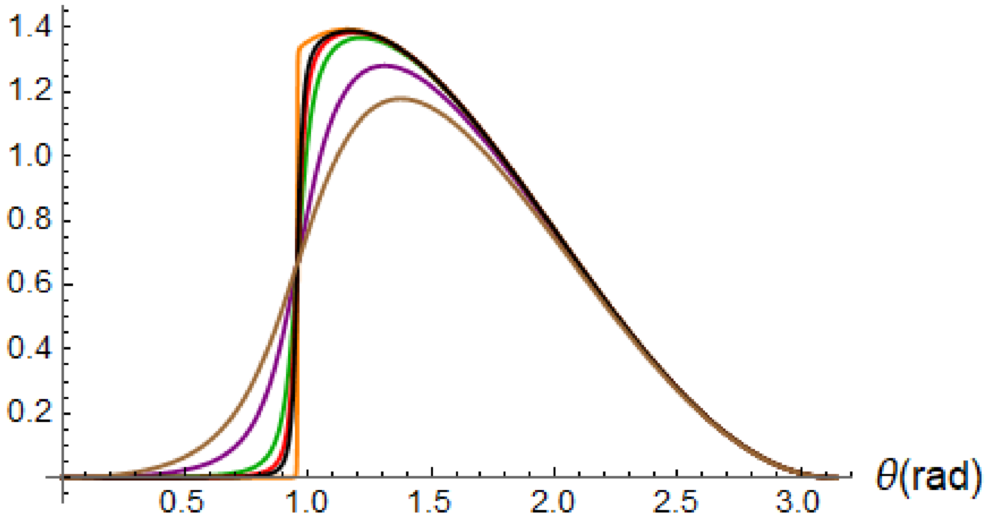
LFD and Large Momentum Frame in the IFD

- Decomposition of the covariant scalar propagator exactly at the LF

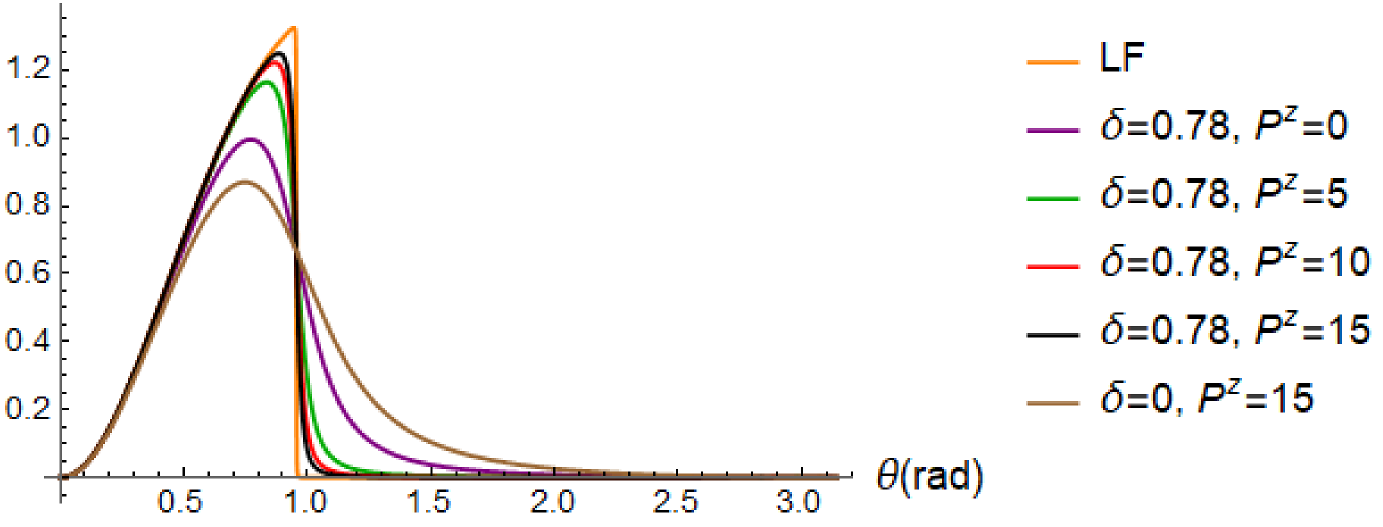
Ex: $M_t^{++} = \underbrace{M_{t,a}^{++} + M_{t,b}^{++}}_{\text{“Forward” and “Backward “ helicity amplitudes}}$



$M_{t,a}^{++}$



$M_{t,b}^{++}$



CONCLUSION

- We confirm QC in interpolating spin-1/2 spinors , interpolating spin-1 spinors and polarization vectors.
- Quantum correlation manifests itself as district boundaries in the landscape of helicity amplitudes when we change the interpolation angle and normalized total longitudinal momentum of the system.
- LF QC appears in the zero-mode (Quantum entanglement in the LF) .This hints the ‘a la Einstein’s “spooky action at a distance “ even in the LFD.
- Helicity amplitudes with higher interpolating angle values and lower momentums produce better LF results than the helicity amplitudes with large momentum values in the instant form dynamic.

THANK YOU

BACKUP SLIDES

Interpolating spin- ½ spinors

$$u_H^{(-1/2)}(P) = \begin{pmatrix} -P^L\sqrt{\frac{\cos\delta-\sin\delta}{2\mathbb{P}(\mathbb{P}+P_{\hat{-}})}}\sqrt{P^{\hat{+}}-\mathbb{P}} \\ \sqrt{\frac{P_{\hat{-}}+\mathbb{P}}{2\mathbb{P}}}\sqrt{\frac{P^{\hat{+}}-\mathbb{P}}{(\cos\delta-\sin\delta)}} \\ -P^L\sqrt{\frac{\sin\delta+\cos\delta}{2\mathbb{P}(\mathbb{P}+P_{\hat{-}})}}\sqrt{P^{\hat{+}}+\mathbb{P}} \\ \sqrt{\frac{P_{\hat{-}}+\mathbb{P}}{2\mathbb{P}}}\sqrt{\frac{P^{\hat{+}}+\mathbb{P}}{(\sin\delta+\cos\delta)}} \end{pmatrix} \quad u_H^{(1/2)}(P) = \begin{pmatrix} \sqrt{\frac{P_{\hat{-}}+\mathbb{P}}{2\mathbb{P}}}\sqrt{\frac{P^{\hat{+}}+\mathbb{P}}{(\sin\delta+\cos\delta)}} \\ P^R\sqrt{\frac{\sin\delta+\cos\delta}{2\mathbb{P}(\mathbb{P}+P_{\hat{-}})}}\sqrt{P^{\hat{+}}+\mathbb{P}} \\ \sqrt{\frac{P_{\hat{-}}+\mathbb{P}}{2\mathbb{P}}}\sqrt{\frac{P^{\hat{+}}-\mathbb{P}}{(\cos\delta-\sin\delta)}} \\ P^R\sqrt{\frac{\cos\delta-\sin\delta}{2\mathbb{P}(\mathbb{P}+P_{\hat{-}})}}\sqrt{P^{\hat{+}}-\mathbb{P}} \end{pmatrix}$$

Interpolating Spin-1 polarization vectors

$$\epsilon_{\hat{\mu}}(P,+) = -\frac{1}{\sqrt{2}\mathbb{P}}\left(\mathbb{S}P^R, \frac{(P_1P_{\hat{-}} - iP_2\mathbb{P})}{P^L}, \frac{(P_2P_{\hat{-}} + iP_1\mathbb{P})}{P^L}, -\mathbb{C}P^R\right)$$

$$\epsilon_{\hat{\mu}}(P,-) = \frac{1}{\sqrt{2}\mathbb{P}}\left(\mathbb{S}P^L, \frac{(P_1P_{\hat{-}} + iP_2\mathbb{P})}{P^R}, \frac{(P_2P_{\hat{-}} - iP_1\mathbb{P})}{P^R}, -\mathbb{C}P^L\right)$$

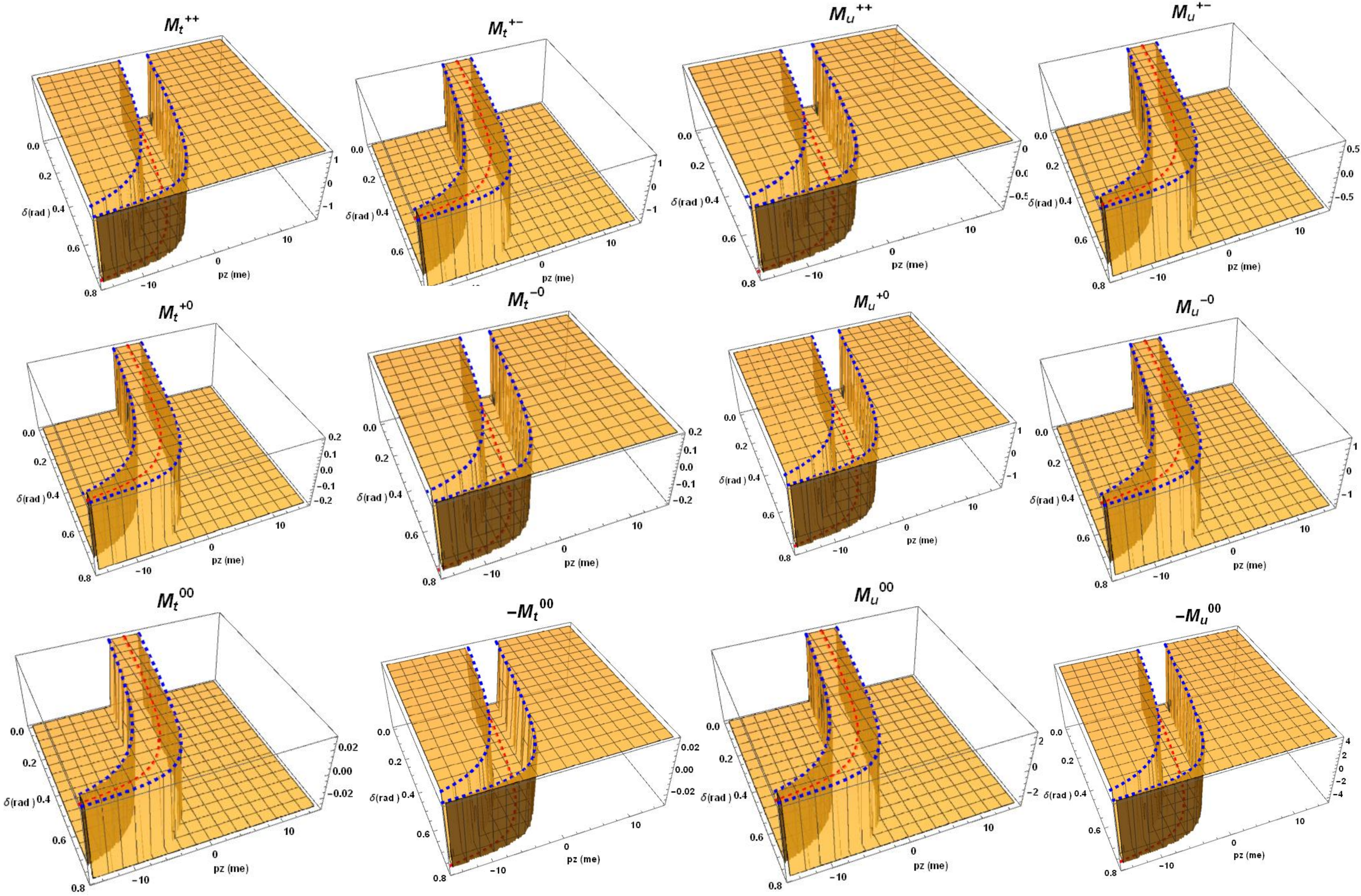
$$\epsilon_{\hat{\mu}}(P,0) = \frac{P^{\hat{+}}}{M\mathbb{P}}\left(P_{\hat{+}} - \frac{M^2}{P^{\hat{+}}}, P_1, P_2, P_{\hat{-}}\right)$$

Interpolating Spin-1 Helicity spinors

$$u_H^{(+1)} = \frac{1}{2\sqrt{M\mathbb{P}^2}}\begin{pmatrix} \frac{(P_{\hat{-}}+\mathbb{P})(P^{\hat{+}}+\mathbb{P})}{(A-B)} \\ \sqrt{2}P^R(P^{\hat{+}}+\mathbb{P}) \\ \frac{(A-B)(P^R)^2(P^{\hat{+}}+\mathbb{P})}{(P_{\hat{-}}+\mathbb{P})} \\ (A-B)(P_{\hat{-}}+\mathbb{P})\mathbb{X} \\ \sqrt{2}P^R(P^{\hat{+}}-\mathbb{P}) \\ \frac{(A+B)(P^R)^2(P^{\hat{+}}-\mathbb{P})}{(P_{\hat{-}}+\mathbb{P})} \end{pmatrix}, \quad u_H^{(-1)} = \frac{1}{2\sqrt{M\mathbb{P}^2}}\begin{pmatrix} \frac{(A+B)(P^L)^2(P^{\hat{+}}-\mathbb{P})}{(P_{\hat{-}}+\mathbb{P})} \\ -\sqrt{2}P^L(P^{\hat{+}}-\mathbb{P}) \\ (A-B)(P_{\hat{-}}+\mathbb{P})\mathbb{X} \\ \frac{(A-B)(P^L)^2(P^{\hat{+}}+\mathbb{P})}{(P_{\hat{-}}+\mathbb{P})} \\ -\sqrt{2}P^L(P^{\hat{+}}+\mathbb{P}) \\ \frac{(P_{\hat{-}}+\mathbb{P})(P^{\hat{+}}+\mathbb{P})}{(A-B)} \end{pmatrix} \quad u_H^{(0)} = \sqrt{\frac{M}{2\mathbb{P}^2}}\begin{pmatrix} -(A+B)P^L \\ \sqrt{2}P_{\hat{-}} \\ (A-B)P^R \\ (-A+B)P^L \\ \sqrt{2}P_{\hat{-}} \\ (A+B)P^R \end{pmatrix}$$

$P^L = P^1 - iP^2$, $\quad P^R = P^1 + iP^2$ $\quad A = \cos\delta,\ B = -\sin\delta$ $\quad \mathbb{X} \equiv \frac{P^{\hat{+}}-\mathbb{P}}{\mathbb{C}} = \frac{P^{\hat{+}}-\sqrt{(P^{\hat{+}})^2-M^2\mathbb{C}}}{\mathbb{C}}$

T and U channels helicity amplitudes



QC in T and U channels helicity amplitudes