Pion PDFs accessible in TDIS

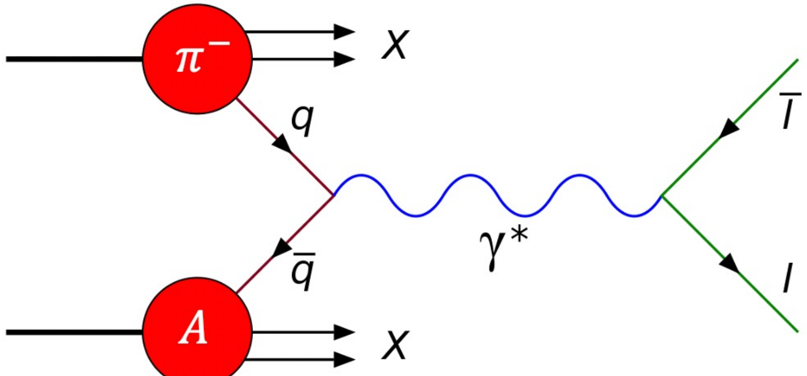
Patrick Barry, Jefferson Lab

December 2^{nd}, 2022

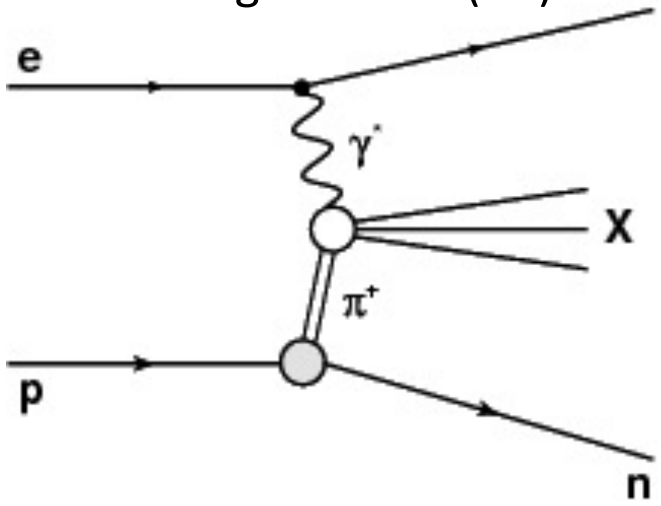
In collaboration with Chueng-Ryong Ji, Wally Melnitchouk, and Nobuo Sato

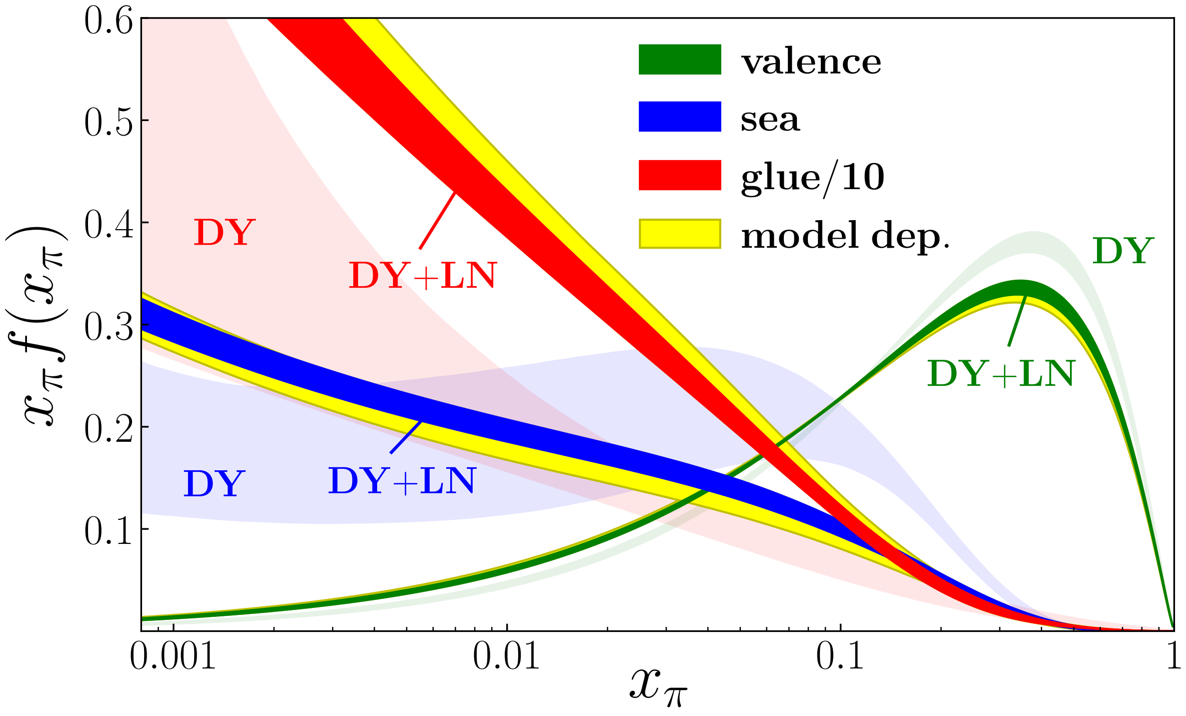
Pion PDFs in JAM

Drell-Yan (DY)



Leading Neutron (LN)





PHYSICAL REVIEW LETTERS **121,** 152001 (2018)

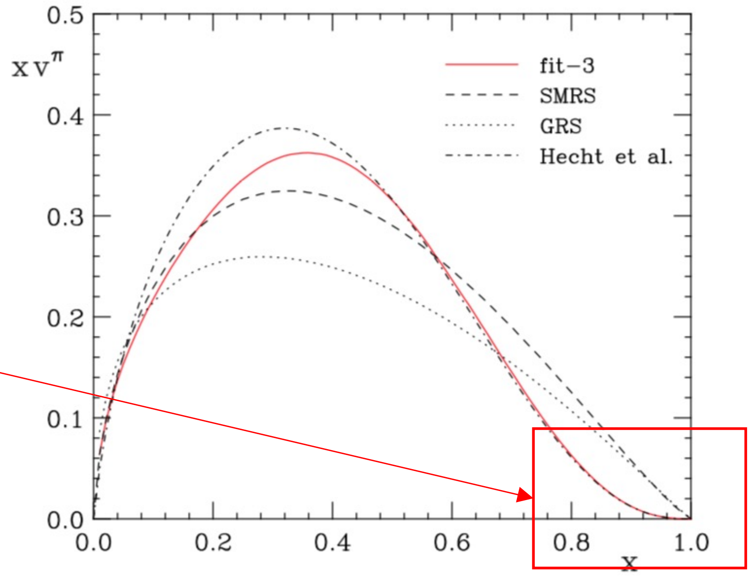
Featured in Physics

First Monte Carlo Global QCD Analysis of Pion Parton Distributions

P. C. Barry,[1] N. Sato,[2] W. Melnitchouk,[3] and Chueng-Ryong Ji[1]

Large-x_π behavior

- Generally, the parametrization lends a behavior as $x \to 1$ of the valence quark PDF of $q_v(x) \propto (1-x)^\beta$
- For a fixed order analysis, analyses find $\beta \approx 1$
- Aicher, Schaefer Vogelsang (ASV) found $\beta = 2$ with threshold resummation



ASV valence PDF

Phys. Rev. Lett. **105**, 114023 (2011).

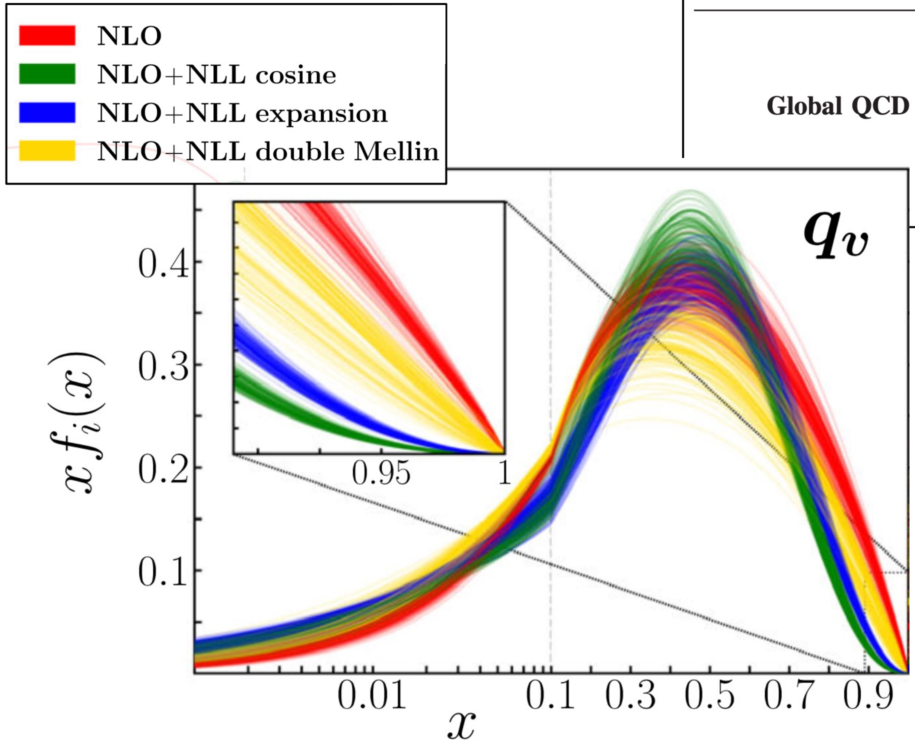
JAM analysis with threshold resummation

PHYSICAL REVIEW LETTERS **127,** 232001 (2021)

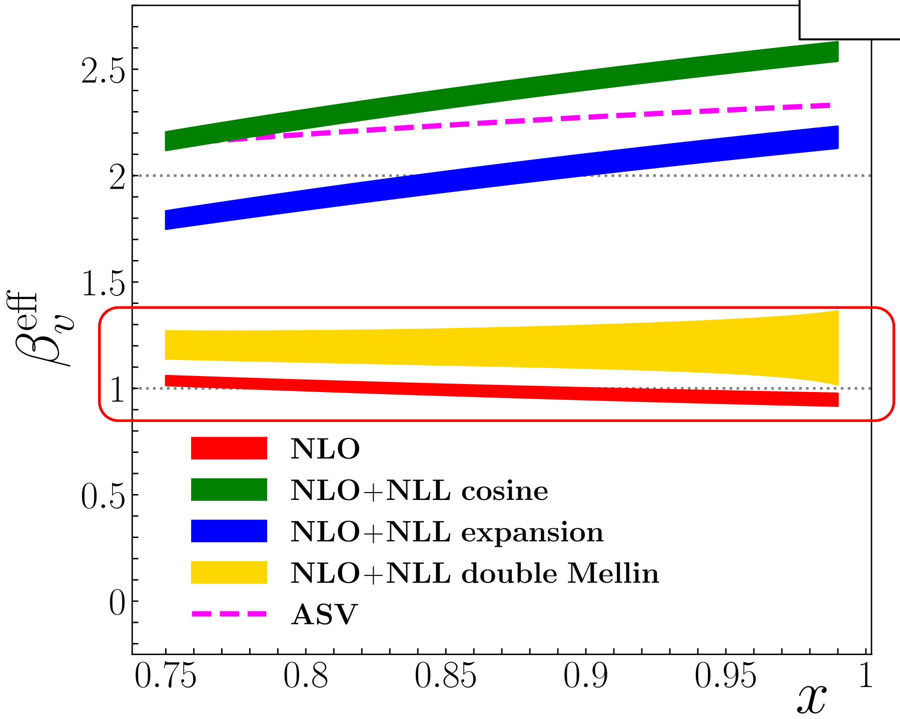
Global QCD Analysis of Pion Parton Distributions with Threshold Resummation

P. C. Barry,[1] Chueng-Ryong Ji,[2] N. Sato,[1] and W. Melnitchouk[1]

(JAM Collaboration)



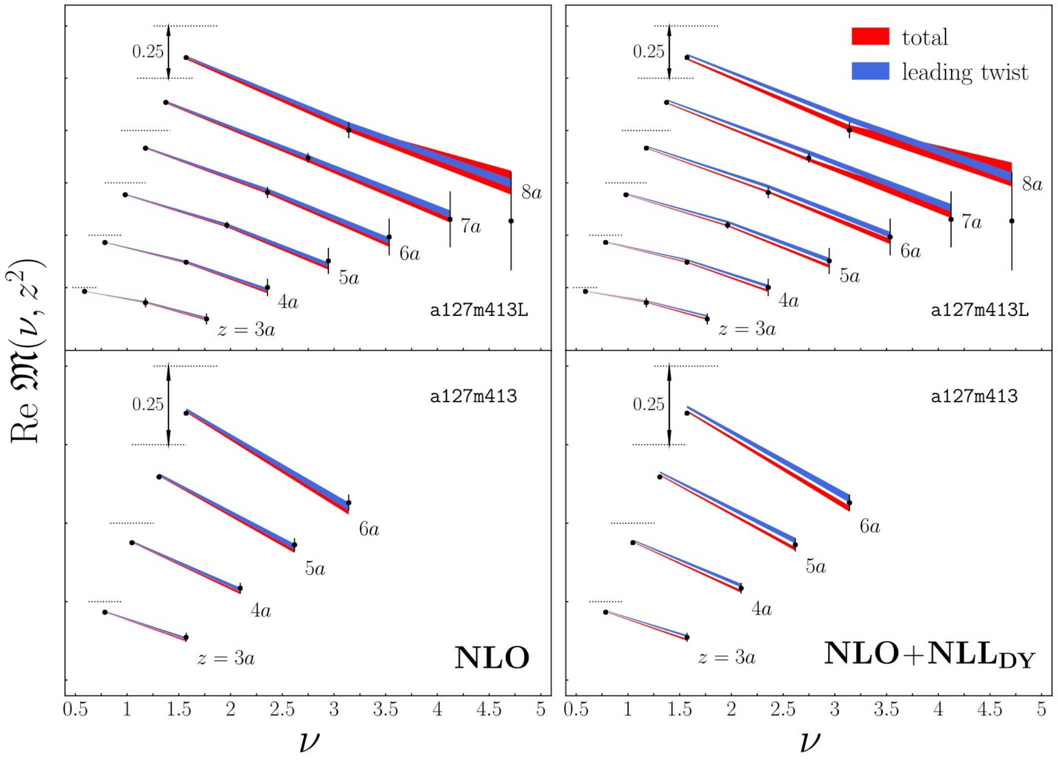
$$\beta_{\rm eff}(x,\mu) = \frac{\partial \log|q_v(x,\mu)|}{\partial \log(1-x)}$$



- Highly dependent on perturbative approach
- NLO and NLO+NLL double Mellin methods better on theoretical grounds

Introduction of lattice QCD data

- JAM has also included recent simulations on the lattice to constrain pion PDFs

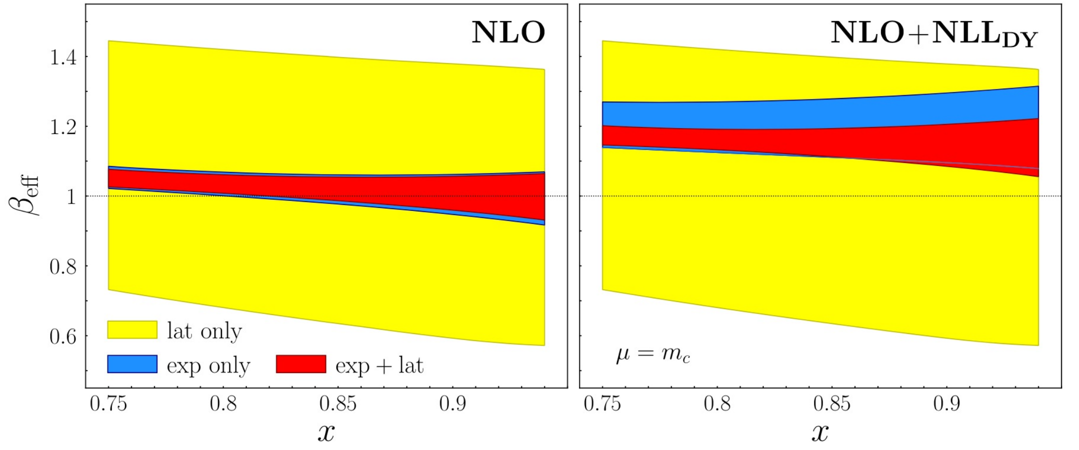


PHYSICAL REVIEW D **105,** 114051 (2022)

Complementarity of experimental and lattice QCD data on pion parton distributions

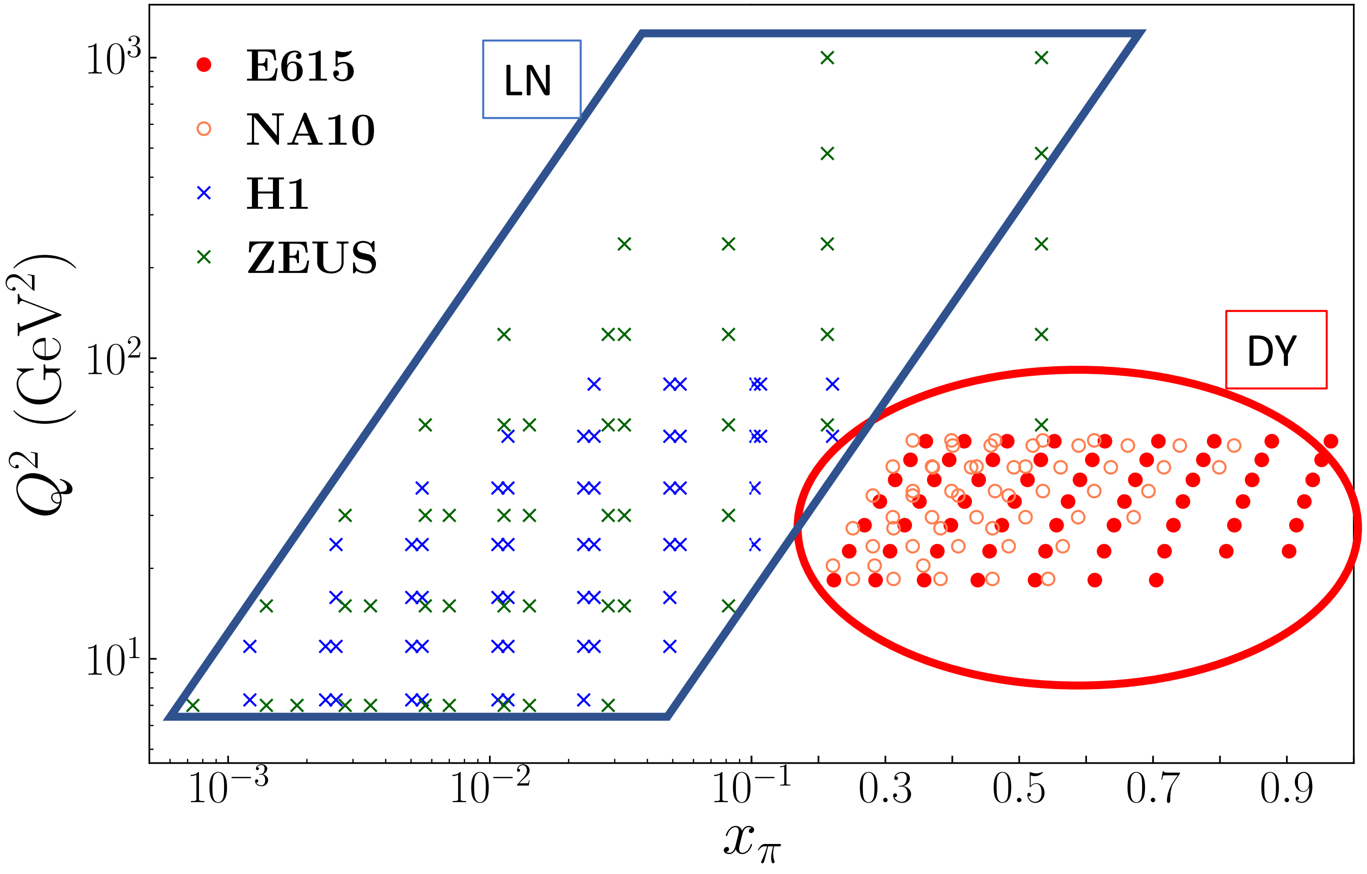
P. C. Barry,[1] C. Egerer,[1] J. Karpie,[2] W. Melnitchouk,[1] C. Monahan,[1,3] K. Orginos,[1,3] Jian-Wei Qiu,[1,3] D. Richards,[1] N. Sato,[1] R. S. Sufian,[1,3] and S. Zafeiropoulos[4]

(Jefferson Lab Angular Momentum (JAM) and HadStruc Collaborations)



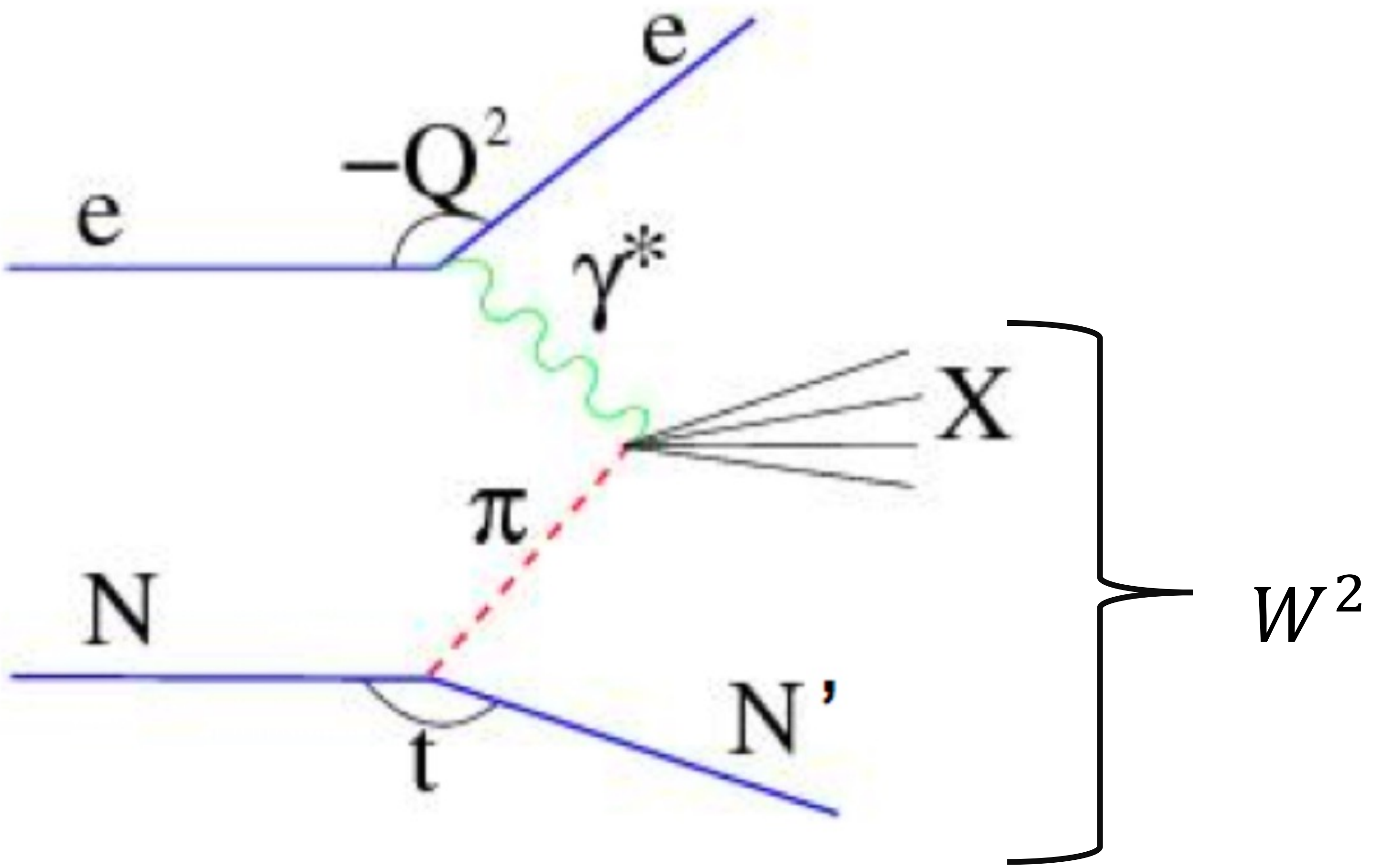
Datasets -- Kinematics

- Current experimental data is limited kinematically with little overlap
- Can JLab TDIS help us learn more about pion PDFs?



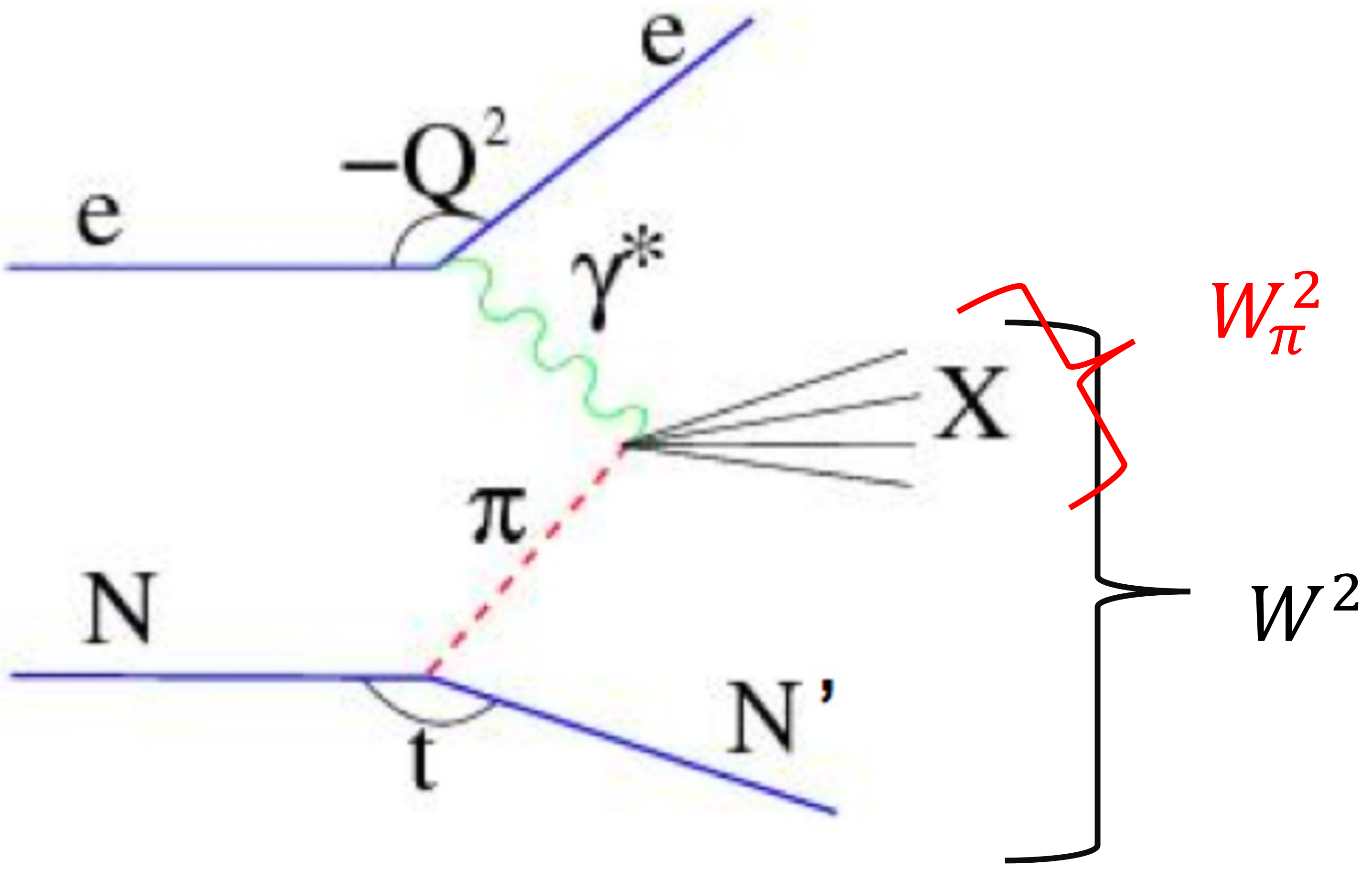
Sullivan process

- Impose kinematic cuts on experimental data
- Such as lower limit on the totally *inclusive* W^2



Sullivan process and W_π^2

- Impose kinematic cuts on experimental data
- Such as lower limit on the totally *inclusive* W^2
- What about the W_π^2?



Check the resonance regions

Baseline ingredients:

$\pi^{\pm}$ $I^G(J^P) = 1^-(0^-)$

Mass $m = 139.57039 \pm 0.00018$ MeV (S = 1.8)
Mean life $\tau = (2.6033 \pm 0.0005) \times 10^{-8}$ s (S = 1.2)
$c\tau = 7.8045$ m

γ (photon) $I(J^{PC}) = 0,1(1^{--})$

Mass $m < 1 \times 10^{-18}$ eV
Charge $q < 1 \times 10^{-46}$ e (mixed charge)
Charge $q < 1 \times 10^{-35}$ e (single charge)
Mean life τ = Stable

The quantum numbers of a charged π and photon result in specific outgoing mesons

Potential resonances in π DIS

$\rho(770)$ $I^G(J^{PC}) = 1^+(1^{--})$

See the note in $\rho(770)$ Particle Listings.
Mass $m = 775.26 \pm 0.25$ MeV
Full width $\Gamma = 149.1 \pm 0.8$ MeV
$\Gamma_{ee} = 7.04 \pm 0.06$ keV

$b_1(1235)$ $I^G(J^{PC}) = 1^+(1^{+-})$

Mass $m = 1229.5 \pm 3.2$ MeV (S = 1.6)
Full width $\Gamma = 142 \pm 9$ MeV (S = 1.2)

$a_1(1260)$ [i] $I^G(J^{PC}) = 1^-(1^{++})$

Mass $m = 1230 \pm 40$ MeV [j]
Full width $\Gamma = 250$ to 600 MeV

$\pi_1(1400)$ [k] $I^G(J^{PC}) = 1^-(1^{-+})$

See the review on "Non-$q\overline{q}$ Mesons."
Mass $m = 1354 \pm 25$ MeV (S = 1.8)
Full width $\Gamma = 330 \pm 35$ MeV

$a_0(1450)$ $I^G(J^{PC}) = 1^-(0^{++})$

See the review on "Scalar Mesons below 2 GeV."
Mass $m = 1474 \pm 19$ MeV
Full width $\Gamma = 265 \pm 13$ MeV

$\rho(1450)$ $I^G(J^{PC}) = 1^+(1^{--})$

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Mass $m = 1465 \pm 25$ MeV [j]
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$\pi_1(1600)$ $I^G(J^{PC}) = 1^-(1^{-+})$

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$\pi(1800)$ $I^G(J^{PC}) = 1^-(0^{-+})$

Mass $m = 1810^{+9}_{-11}$ MeV (S = 2.2)
Full width $\Gamma = 215^{+7}_{-8}$ MeV

Knowledge of π resonances

- In principle, we know very little
- HERA did not measure the low-W_π^2 region, so the strength of various resonances in this process in unknown
- Kinematic coverage in TDIS will be a **great use** to measure this resonance region and fill in gaps

What do we do now?

- We can look to models for guidance
- The $\pi\rho\gamma$ coupling has been studied in models (not much else)
- Dyson-Schwinger results: Maris and Tandy PRC **65**, 045211 (2002)
- They calculated the decay of a rho meson into photon and pion
- This process will share the same coupling!

Resonances in π DIS

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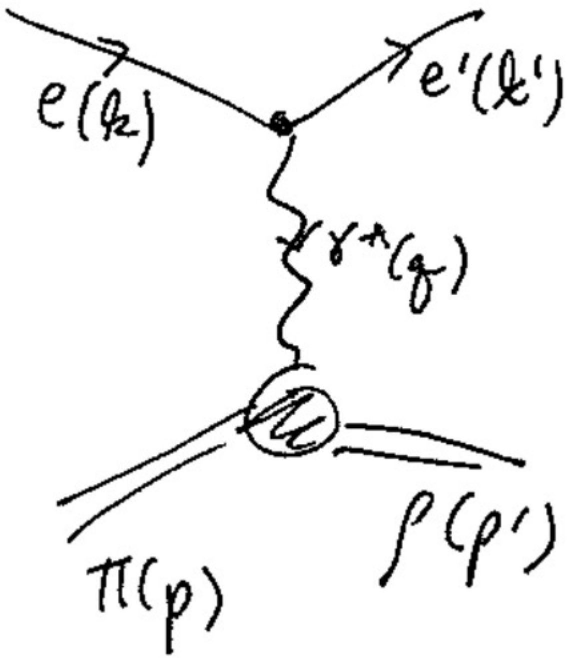
ρ-contribution to the F_2^{π}



- First, we can calculate the hadronic tensor according to this diagram
- Take the vertex and transition form factor from the $\rho \to \pi\gamma$ decay studies

$$\Lambda_{\mu\nu}^{\rho\pi\gamma}(P';Q) = \frac{g_{\rho\pi\gamma}}{m_\rho}\epsilon_{\mu\nu\alpha\beta}P'_\alpha Q_\beta F_{\rho\pi\gamma}(Q^2)$$

$$F_{\rho\pi\gamma}(Q^2) = \frac{1.0 + Q^2}{1.0 + 3.04Q^2 + 2.42Q^4 + 0.36Q^6}$$

Calculate the $W_{\mu\nu}$

- Calculate the square of the current matrix element of $\gamma^* \pi \to \rho$

$$W_{\mu\nu}^{\pi\to\rho} = \frac{g^2}{2m_\rho^2}\left(F_{\pi\rho\gamma}(Q^2)\right)^2 \left\{ g_{\mu\nu}\left[m_\pi^2 q^2 - (q\cdot P)^2\right] + (P\cdot q)\left[P_\mu q_\nu + q_\mu P_\nu\right] - q^2 P_\mu P_\nu - m_\pi^2 q_\mu q_\nu \right\} \delta(W_\pi^2 - m_\rho^2)$$

- The g/m_ρ coupling comes from experimental determinations of the decay channel (calculate the decay and compare with exp. value)
- Then project onto F_2^π using projection operators

Projection operators

- The hadronic tensor has a generic structure

$$W^{\mu\nu} =(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2})F_1(x,Q^2)$$
$$+ (p^\mu - \frac{p\cdot q}{q^2}q^\mu)(p^\nu - \frac{p\cdot q}{q^2}q^\nu)\frac{F_2(x,Q^2)}{p\cdot q}$$

- And projectors exist such that $P_1^{\mu\nu}W_{\mu\nu} = F_1(x,Q^2) \qquad P_2^{\mu\nu}W_{\mu\nu} = F_2(x,Q^2)$

- which have a generic structure: $P_i^{\mu\nu} = ag^{\mu\nu} + bp^\mu p^\nu$
- So we solve for a and b for each

Projection operators

- Solving them, we get

$$P_1^{\mu\nu}(x,Q^2) = -\frac{1}{2}g^{\mu\nu} + \frac{2x^2}{4m_\pi^2 x^2 + Q^2}p^\mu p^\nu$$

$$P_2^{\mu\nu}(x,Q^2) = -\frac{Q^2 x}{4m_\pi^2 x^2 + Q^2}g^{\mu\nu} + \frac{12Q^2 x^3}{16m_\pi^4 x^4 + 8m_\pi^2 Q^2 x^2 + Q^4}p^\mu p^\nu$$

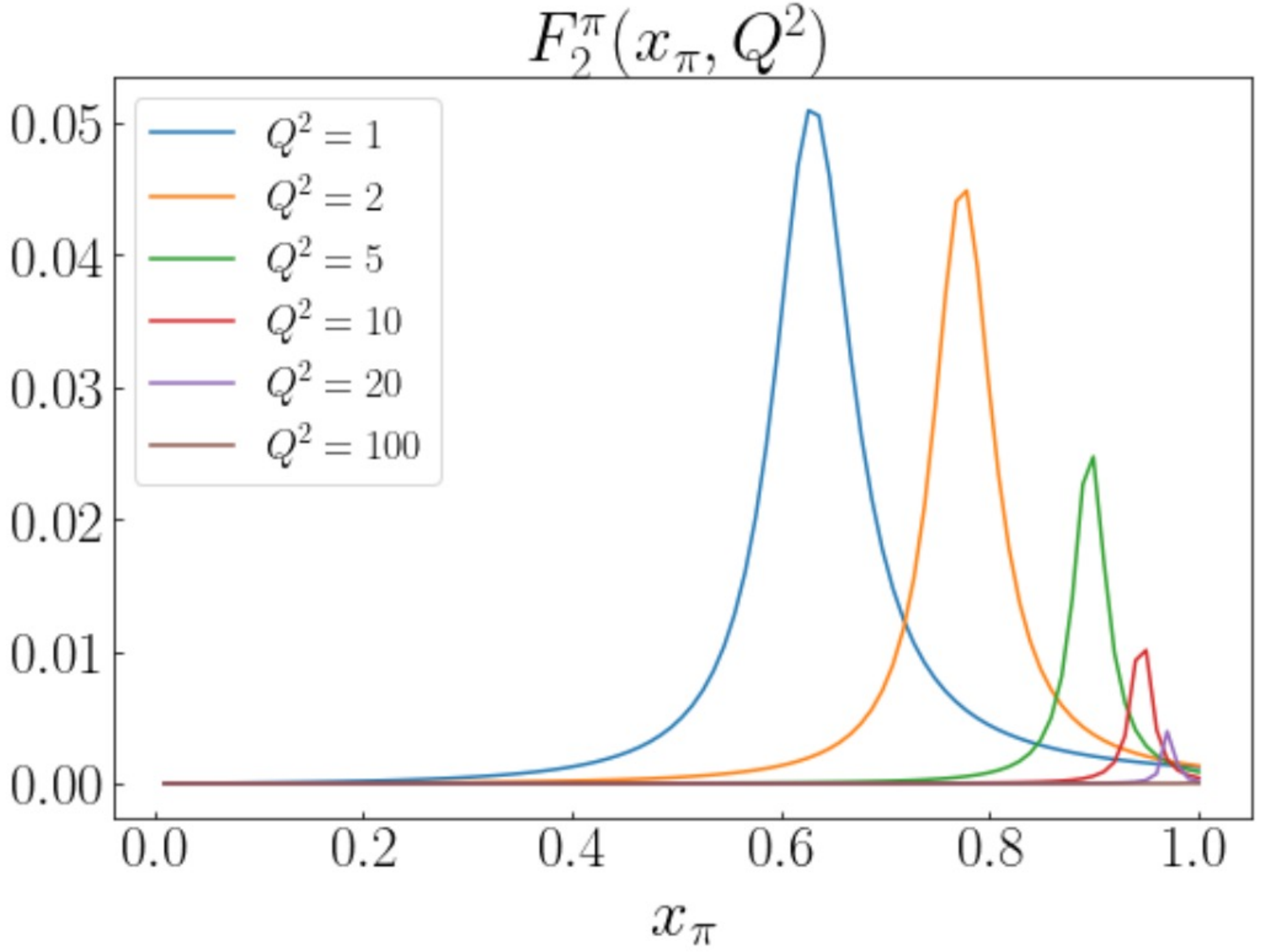
- These should work for any DIS structures on a pion target

A word on the δ-function

- From the previous slide, we saw momentum conserving $\delta\left(W_\pi^2 - m_\rho^2\right)$
- For a realistic resonance, this will have some spread
- We modify the δ-function according to the full width of the ρ peak
- From PDG: $\Gamma_\rho = 150\ \mathrm{MeV}$

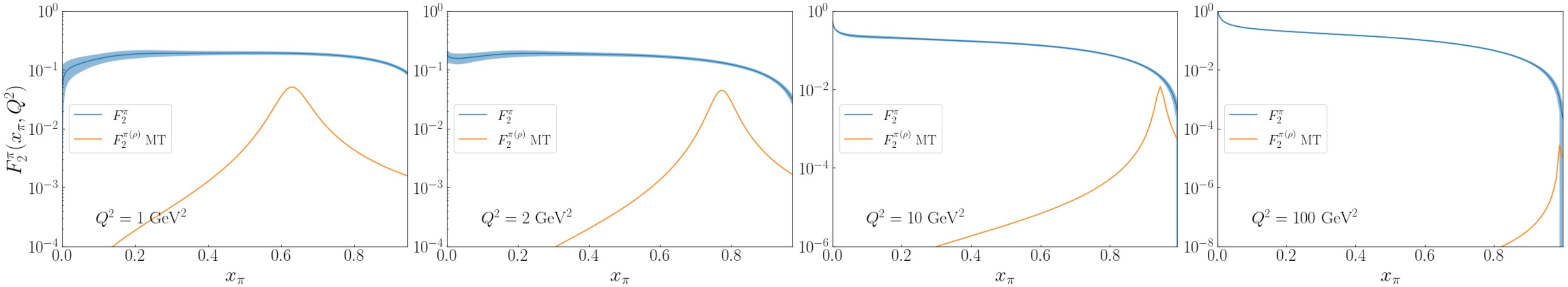
$$\delta\left(W_\pi^2 - m_\rho^2\right) \to \frac{1}{\pi} \frac{m_\rho \Gamma_\rho}{m_\rho^2 \Gamma_\rho^2 + \left(W_\pi^2 - m_\rho^2\right)^2}$$

What does $F_2^{\pi(\rho)}$ look like?



- The ρ to the F_2^π decreases in magnitude with increasing Q^2
- Peaks appearing at larger x_π with increasing Q^2

Comparison with F_2^{π} from JAM



- Use our JAM pion PDFs extracted from data comparing with the model-dependent $F_2^{\pi(\rho)}$
- At low-Q^2 and intermediate-to-large x_π, the peak is about 2 times smaller than the partonic version
- May see some influence in the data – bumps as a function of x_π or W_π

Can we say anything about other resonances?

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a_2 meson (tensor)

$a_2(1320)$ $I^G(J^{PC}) = 1^-(2^{++})$

Mass $m = 1316.9 \pm 0.9$ MeV (S = 1.9)
Full width $\Gamma = 107 \pm 5$ MeV [j]

$a_2(1320)$ DECAY MODES	Fraction (Γ_i/Γ)	Scale factor/ Confidence level	p (MeV/c)
3π	(70.1 ± 2.7) %	S=1.2	623
$\eta\pi$	(14.5 ± 1.2) %		535
$\omega\pi\pi$	(10.6 ± 3.2) %	S=1.3	364
$K\overline{K}$	(4.9 ± 0.8) %		436
$\eta'(958)\pi$	$(5.5 \pm 0.9) \times 10^{-3}$		287
$\pi^\pm\gamma$	$(2.91 \pm 0.27) \times 10^{-3}$		651

$a_1(1260)$ [i] $I^G(J^{PC}) = 1^-(1^{++})$

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Full width $\Gamma = 250$ to 600 MeV

$$\mathscr{L}_{a_2\gamma\pi} = \frac{g_{a_2\gamma\pi}}{M_a^2}\, \varepsilon^{\mu\nu\alpha\beta}\partial_\mu A_\nu a^\pm_{\alpha\lambda}(\partial^\lambda\partial_\beta\pi^\mp)\ .$$

- This interaction is pretty complicated….
- Save for a future work ☺

b_1 Meson (axial-vector)

- Any information?

$b_1(1235)$ $\quad I^G(J^{PC}) = 1^+(1^{+-})$

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Full width $\Gamma = 142 \pm 9$ MeV (S = 1.2)

b_1(1235) DECAY MODES	Fraction (Γ_i/Γ)	Confidence level	p (MeV/c)
$\omega\pi$	seen		348
[D/S amplitude ratio = 0.277 ± 0.027]			
$\pi^\pm\gamma$	$(1.6\pm0.4)\times10^{-3}$		607

- There is a measurement of this decay rate – hope for computing its contribution to F_2^π

Pieces for $b_1\pi\gamma$ interaction

- Vertex

$$\Gamma^{\mu\nu}\left(1^{+} \rightarrow 1^{-}0^{-}\right) = f_1 g^{\mu\nu} + f_2\left(p_V - p_P\right)^{\mu}\left(p_A + p_P\right)^{\nu} + f_3\left(p_V + p_P\right)^{\mu}\left(p_A + p_P\right)^{\nu} + f_4\left(p_V - p_P\right)^{\mu}\left(p_A - p_P\right)^{\nu} + f_5\left(p_V + p_P\right)^{\mu}\left(p_A - p_P\right)^{\nu},$$

- When computing the amplitude, we contract with the polarization vectors of the b_1 and the γ, which will leave us with only 2 of the above terms

$$\Gamma^{\mu\nu} = F_{b_1\pi\gamma} g^{\mu\nu} + G_{b_1\pi\gamma} k^{\mu} q^{\nu}$$

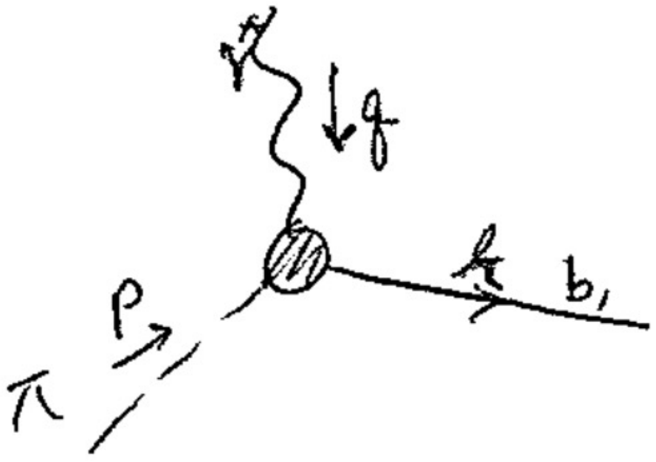
Here, q is the momentum of the b_1 and k is the photon momentum

Decay width

- First, we should calculate the decay width, to get the strength of a coupling
- See note for details

Hadronic tensor

- See note for details

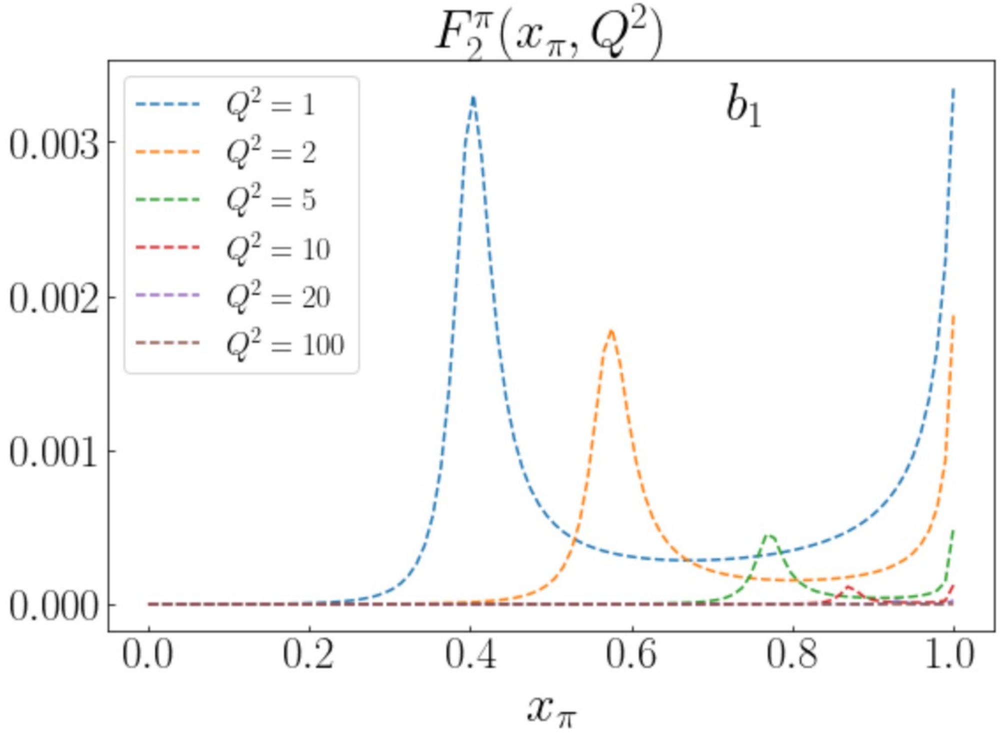


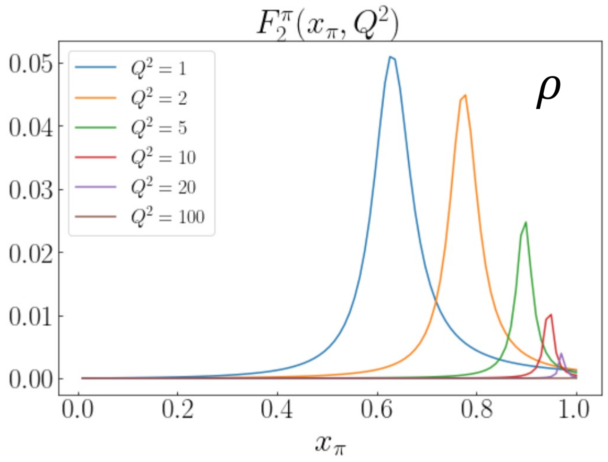
Takeaways

- The decay width and hadronic tensor are independent of this G form factor -- can determine this F from the decay width that we know
- Even though we don't know G, it does not play a role for us in the Coulomb gauge

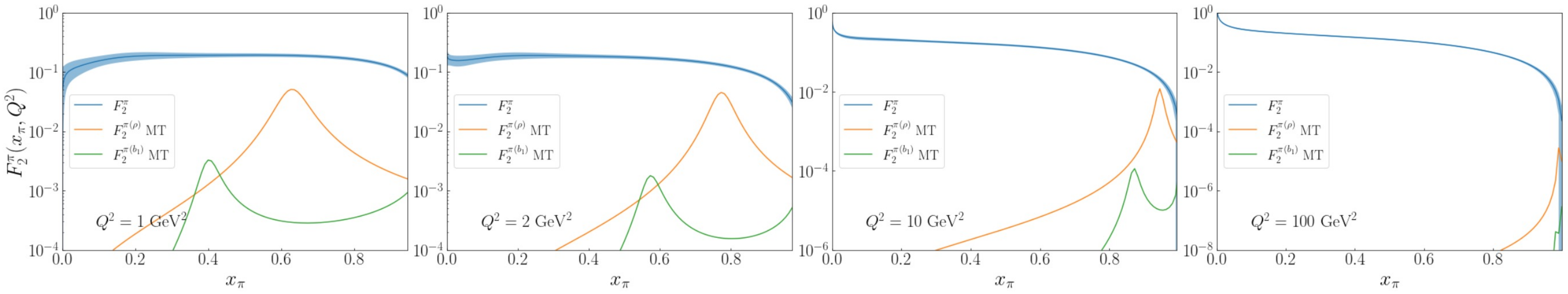
Contribution of b_1 to F_2^π

- Bit of a strange behavior of an increase as $x \to 1$; not sure what's going on
- Order of magnitude smaller than from the ρ





Comparison to F_2^{π} from PDFs



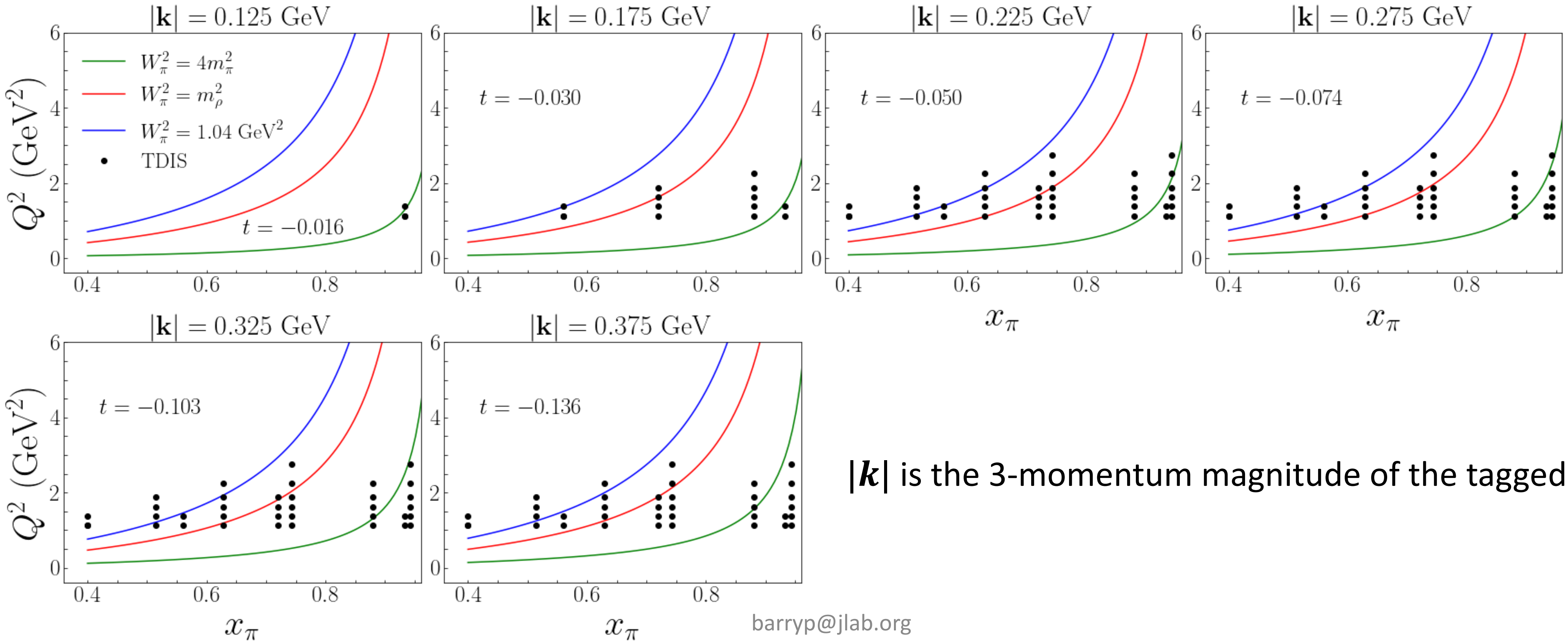
- Almost definitely the b_1 contribution will be suppressed and indistinguishable in the data within errors!

Impact study

Let’s avoid the ρ peak

Current 11 GeV TDIS kinematics

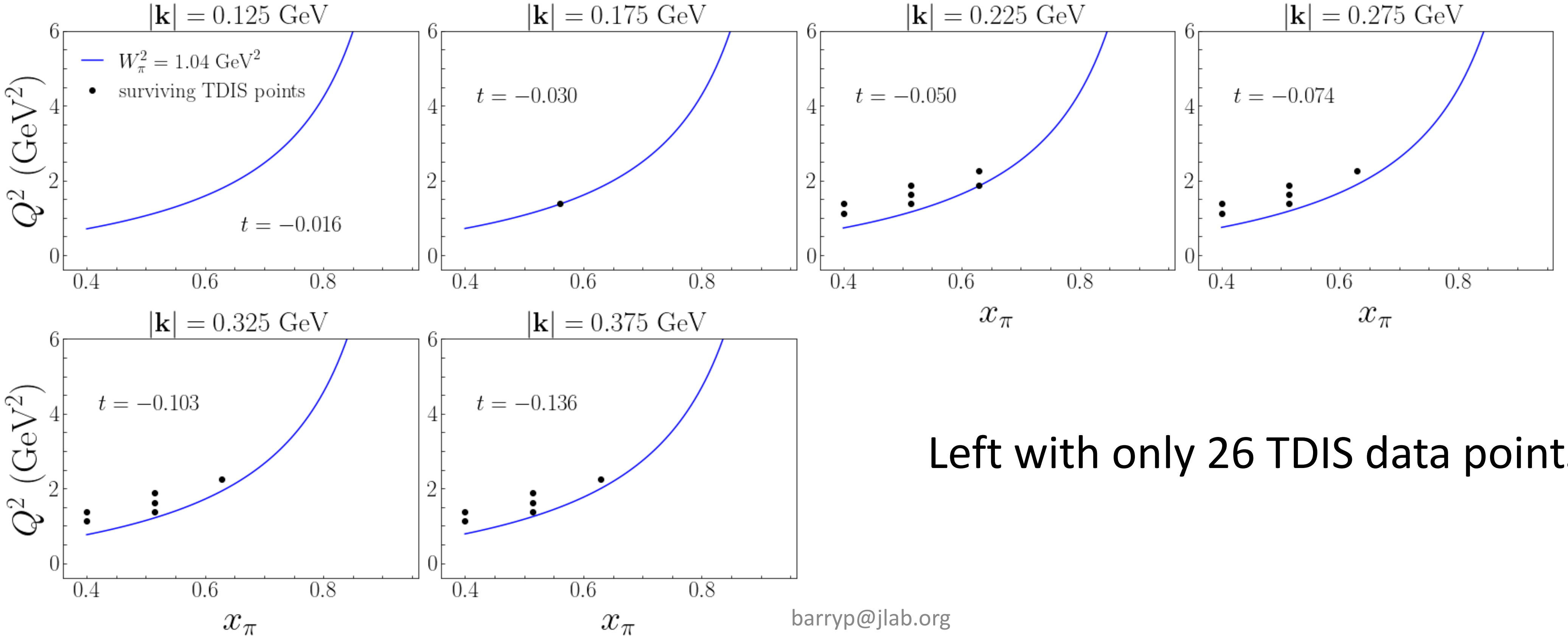
- Plotting available 11 GeV TDIS kinematics with a few representative W_π^2 curves



$|\boldsymbol{k}|$ is the 3-momentum magnitude of the tagged nucleon

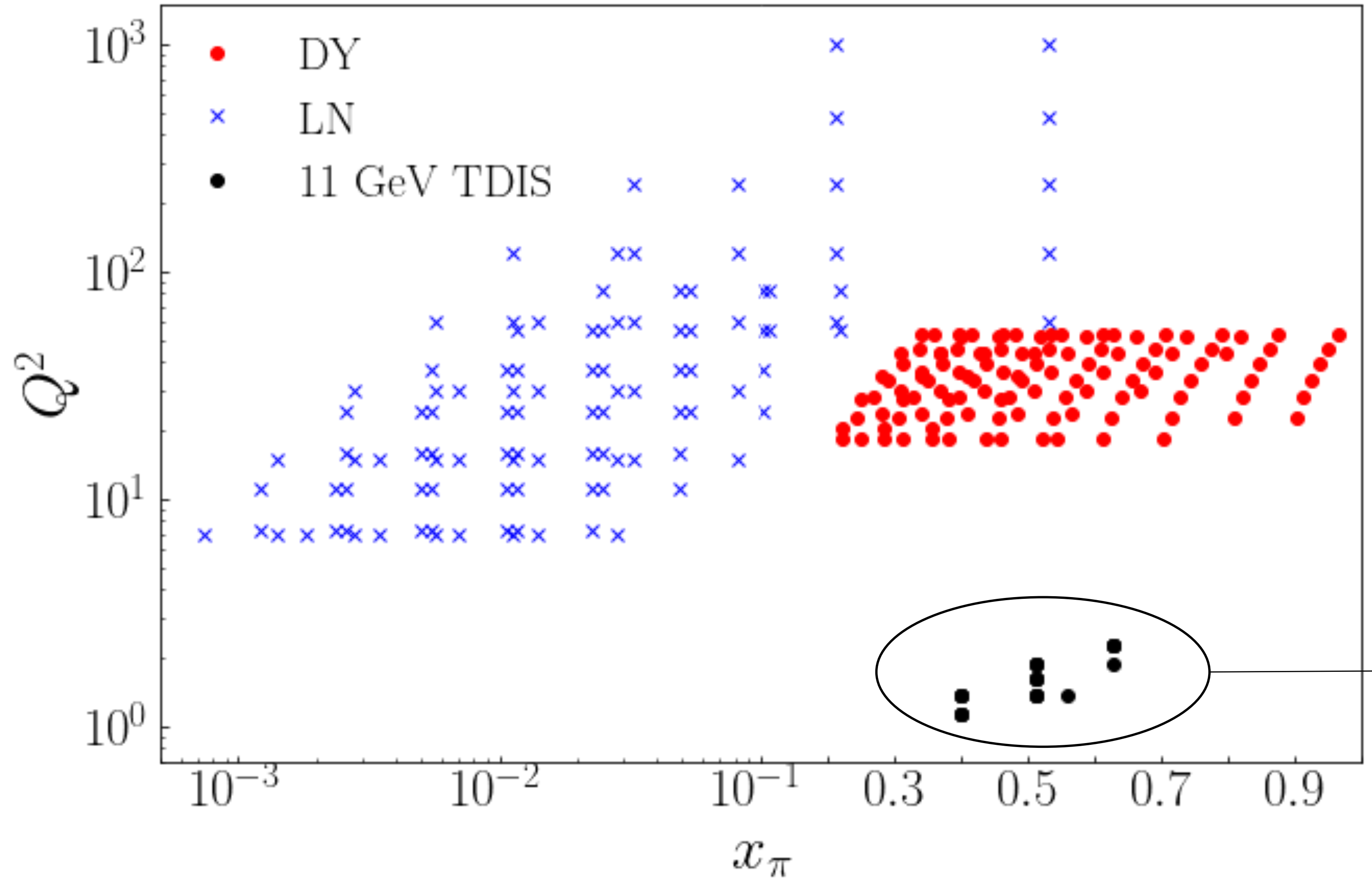
Choosing $W^2_{\pi,\max} = 1.04\ \mathrm{GeV}^2$

- Removing all data points that could be contaminated by resonance regions



Left with only 26 TDIS data points

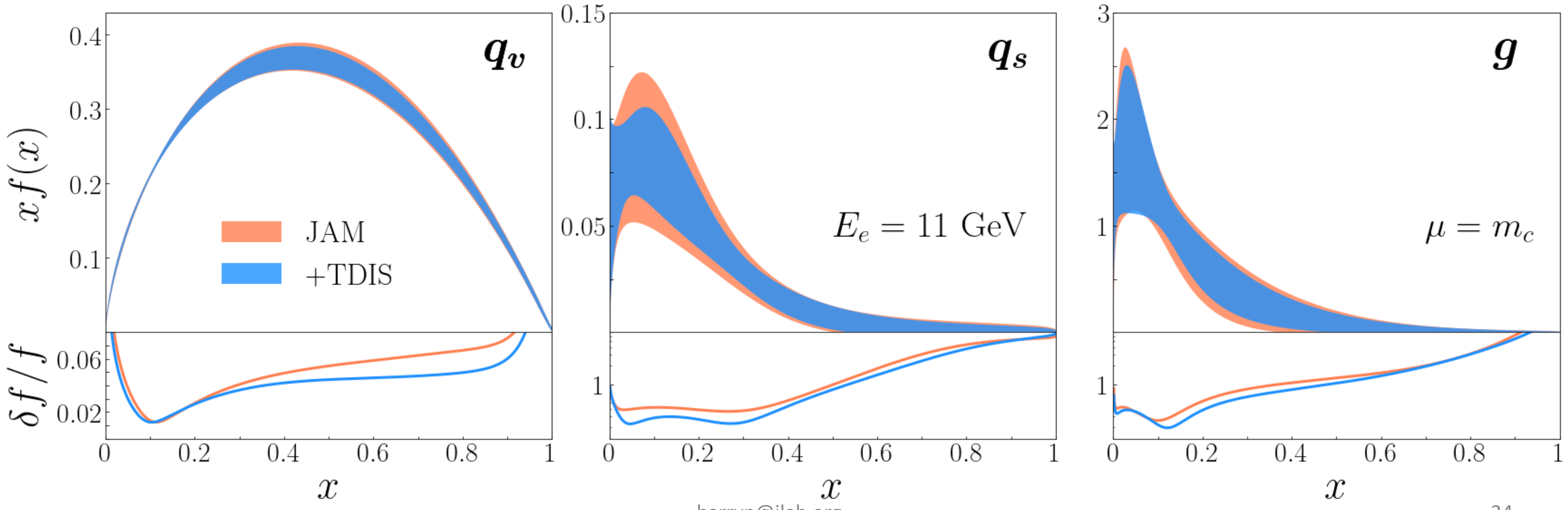
Total pion kinematics



Higher twist effects and potentially non-perturbative effects could be relevant

Performing impact study with 11 GeV

- Create pseudodata from these points and perform global analysis with available experimental data



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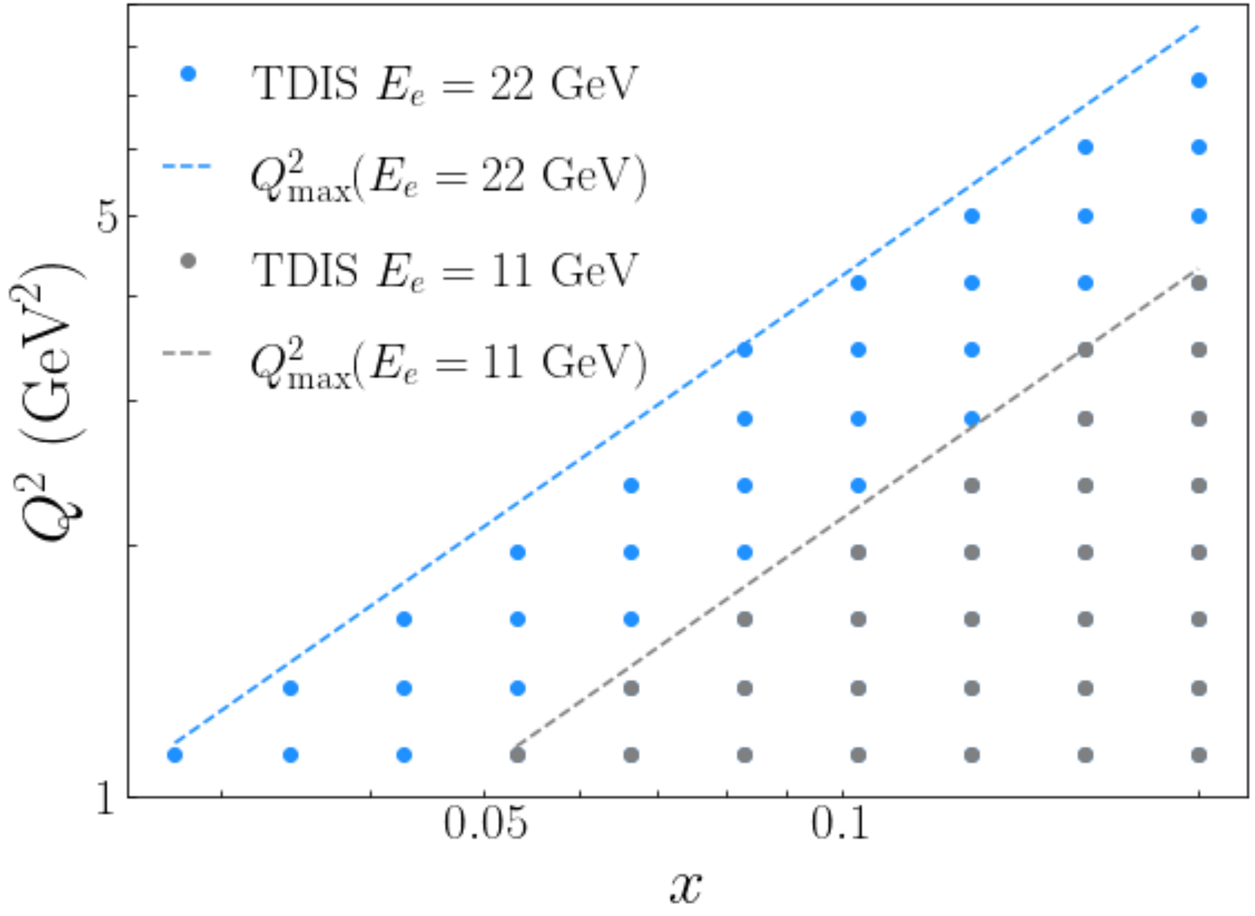
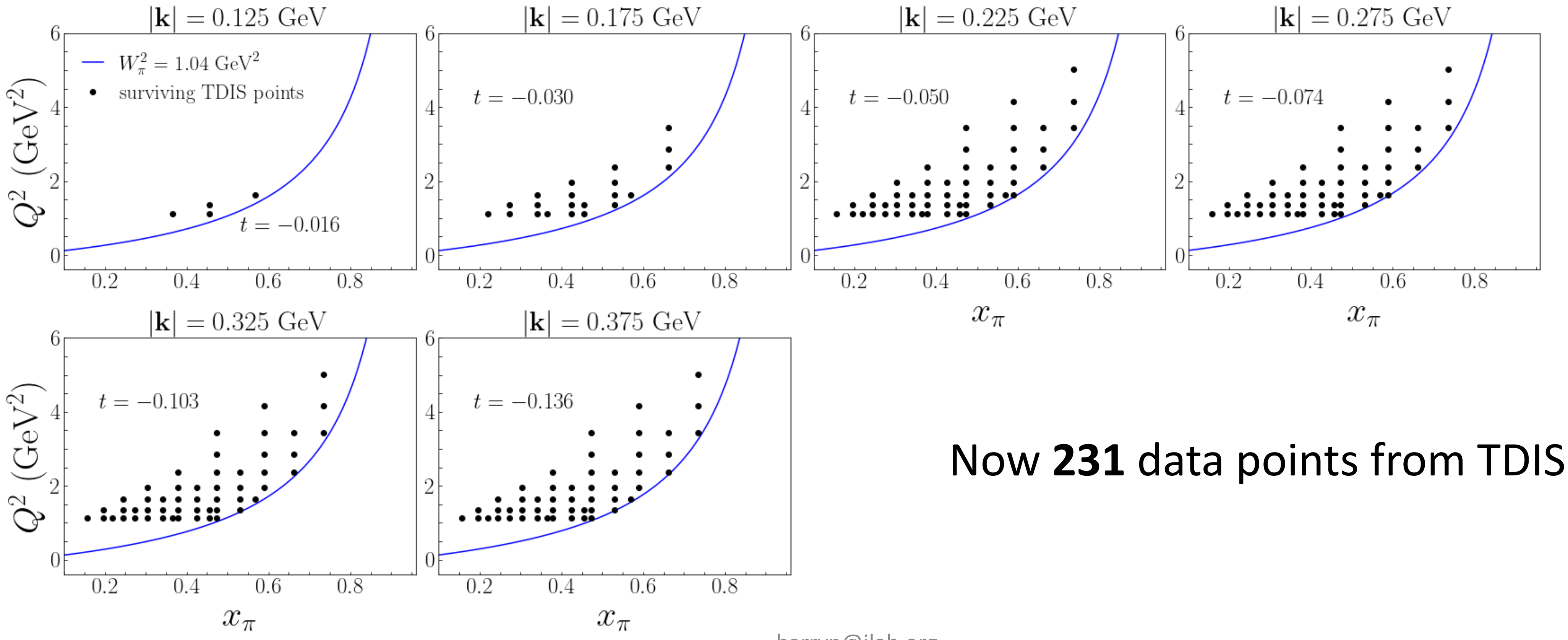
Upgrade to 22 GeV

- Much more available kinematic range in (x, Q^2)
- Recall the W_π^2 cut removed large x_π and small Q^2 data
- New blue points will survive the cut



Kinematics with 22 GeV

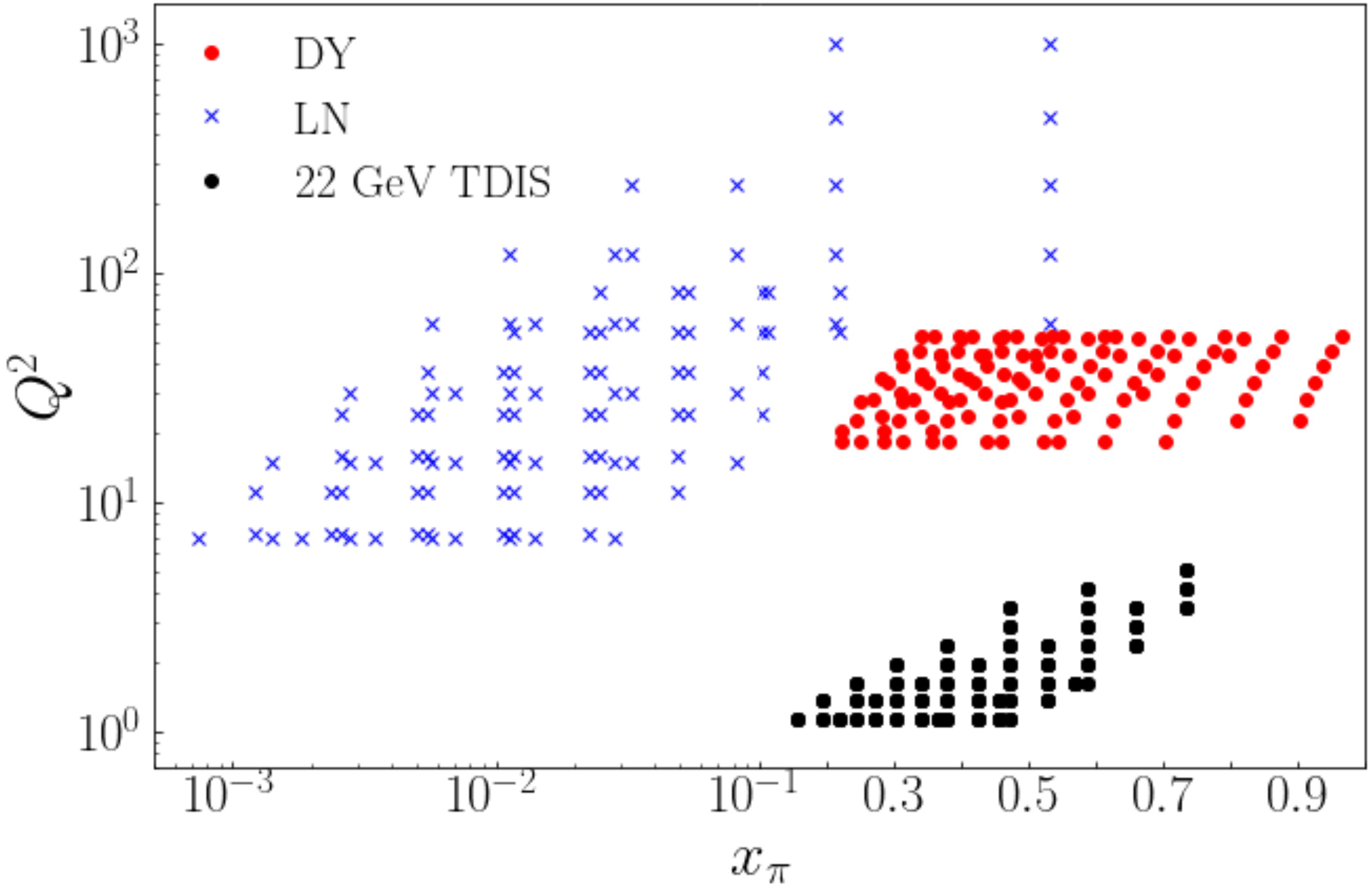
- MASSIVE increase in available data points



Now **231** data points from TDIS

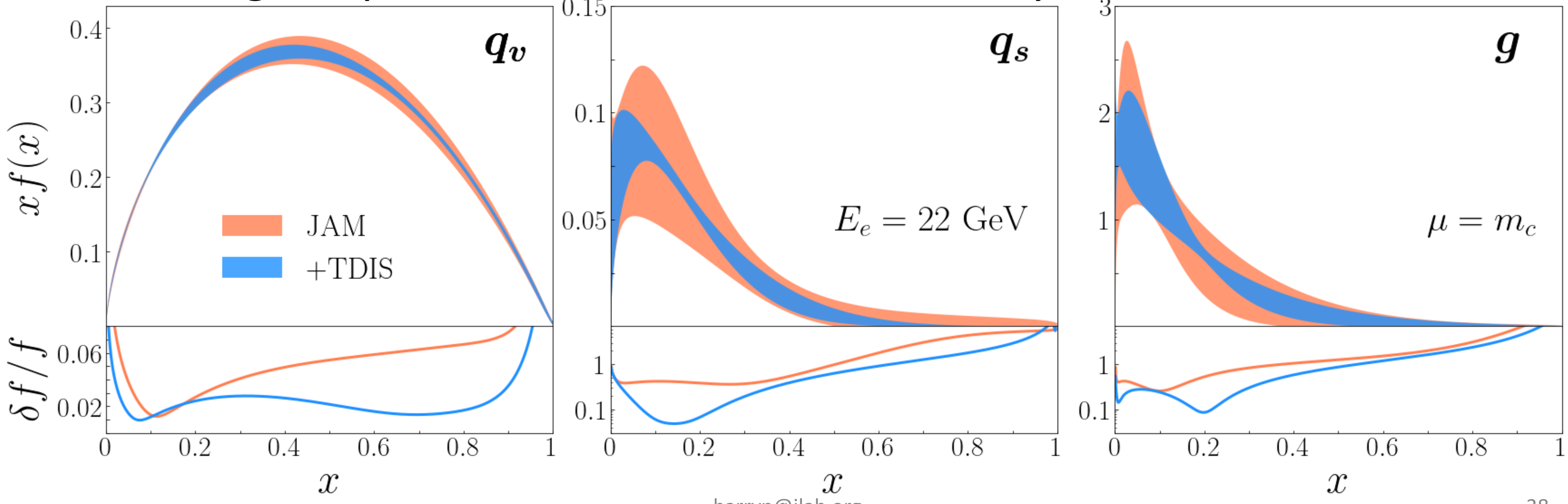
Total kinematics

- Much larger range in x_π and Q^2



Impact on pion PDFs with 22 GeV

- Sizable impact on pion PDFs, especially compared with the 11 GeV beam
- Knowledge of pion PDFs increases dramatically with 22 GeV beam

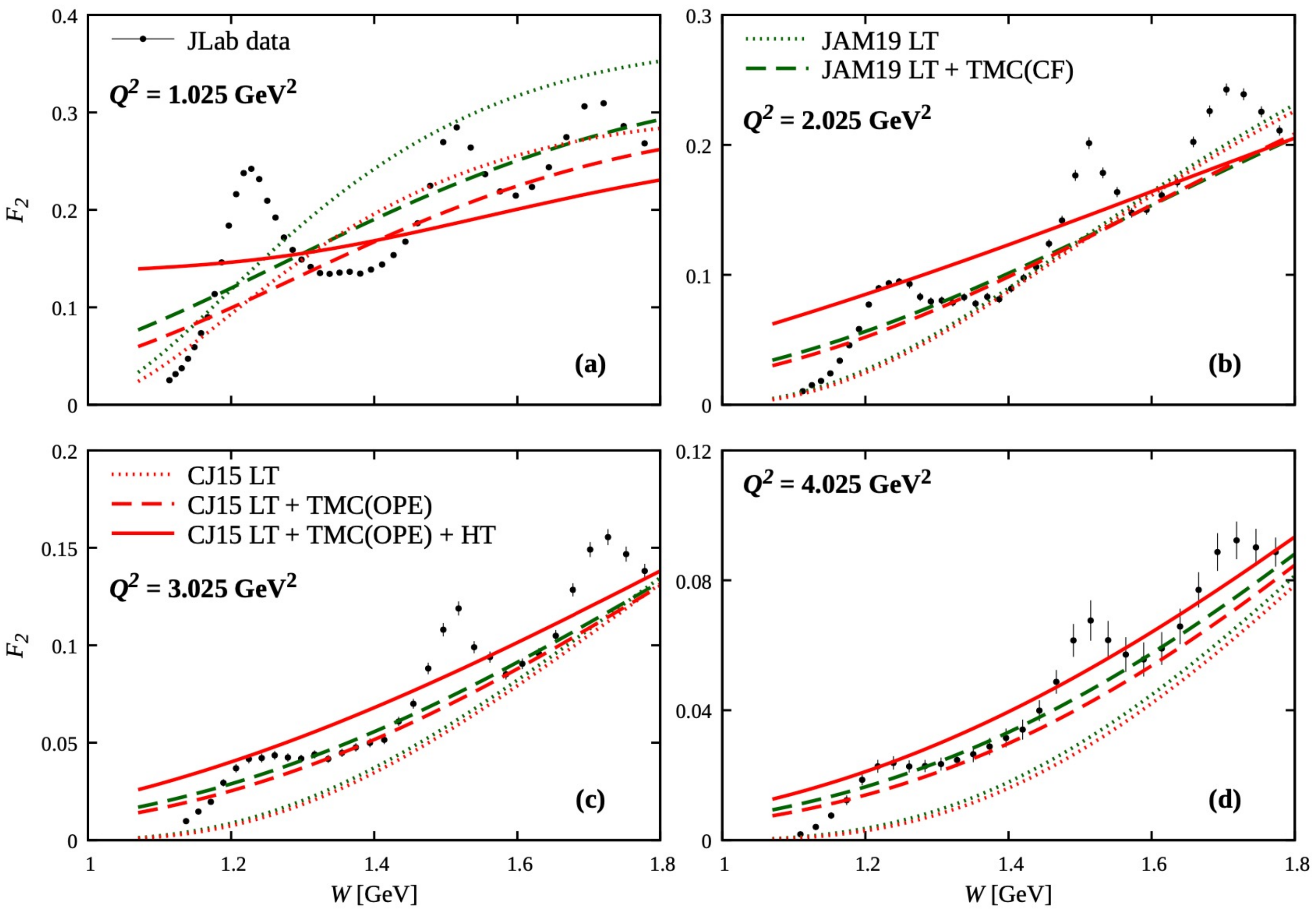


Future work

- Different extrapolations of PDFs to low-Q^2 within reason
- We can look at various models to describe the ρ-resonance
  - Also different strengths of the coupling
- Compare the proton F_2 with the resonances in the low-W^2 region
  - Why are pions and protons different?
- Can test quark-hadron duality by investigating moments of data

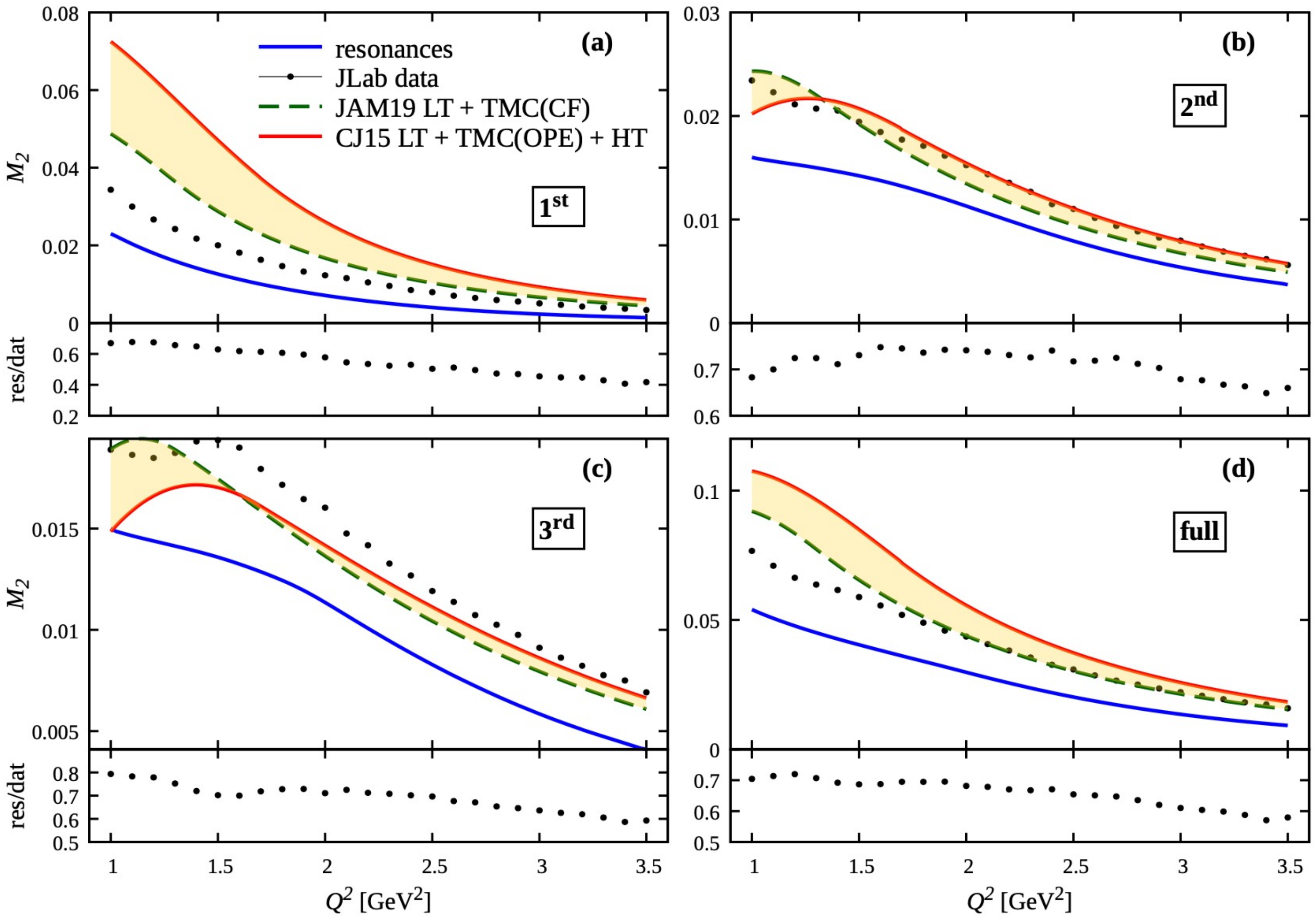
Proton example

- The expected PDFs extracted from larger-W^2 data are in the middle of the peaks
- For pions, we *think* peaks would not appreciably show up in the data



Proton example

- Even the moments of these peaks (integrated over the peak) do not agree with the same from PDFs



Conclusion

- Estimation from models for the strength of the exclusive ρ contribution to F_2^π
- Impacts from the 11 GeV TDIS experiment on pion PDFs will be limited
- The 11 GeV TDIS can measure the low-W_π pion structure function
  - We would learn much more about resonance region here!
- **Much** more constraints will come from larger 22 GeV upgrade

Backup slides

Formula for W_π^2

- Dependent on the external tagged kinematics

$$W_\pi^2 = t - Q^2 \left(1 - \frac{\bar{x}_L}{x}\right)$$

What to choose for W_π^2

- HERA did not measure the low-W_π^2 region
- Potentially largest resonance comes from the ρ-meson
- Must be well above the peak of the resonance
- Estimating the safe region to be an energy above 95% of the area under the curve