Calculating PDF with NJL model, part one: model parameter determination

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Group meeting at Dr. Ji's

Outline

1. Overview of the Nambu-Jona-Lasinio model
2. Solving the model
3. Model parameters

Overview of the Nambu-Jona-Lasinio model

- NJL model is a low energy effective theory of the strong interaction, that mimics many key features of QCD. Thus, it is a useful tool to help understand non-perturbative phenomena in low energy QCD.
- Only quarks as the explicit degrees of freedom, no gluons.
- Dynamics due to gluon-quark interaction and gluon self-couplings are absorbed into the four-fermion contact interaction.
- Local chiral symmetry is explicitly broken by non-vanishing current quark mass.
- Chiral symmetry is also dynamically broken, generating a mass gap.
- Axial anomaly can be incorporated by introducing a determinant term, leading to six-point interaction.

Overview of the Nambu-Jona-Lasinio model

The NJL Lagrangian shares the same symmetry as QCD

should share the same classical symmetries are QCD, namely[1]

$$\mathcal{S} = SU(3)_c \otimes U(3)_V \otimes U(3)_A \otimes \mathcal{C} \otimes \mathcal{P} \otimes \mathcal{T} = SU(3)_c \otimes SU(3)_V \otimes SU(3)_A \otimes U(1)_V \otimes U(1)_A \otimes \mathcal{C} \otimes \mathcal{P} \otimes \mathcal{T}. \tag{4}$$

The transformations that represent the continuous symmetries for 3-flavor QCD are

$$SU(3)_c: \quad q \longrightarrow e^{-i\,\omega_a(x)\,T^a}\,q, \qquad \bar{q} \longrightarrow \bar{q}\,e^{i\,\omega_a(x)\,T^a}, \tag{5}$$

$$SU(3)_V: \quad q \longrightarrow e^{-i\,\boldsymbol{t}\cdot\boldsymbol{\theta}_V}\,q, \qquad \bar{q} \longrightarrow \bar{q}\,e^{i\,\boldsymbol{t}\cdot\boldsymbol{\theta}_V}, \tag{6}$$

$$SU(3)_A: \quad q \longrightarrow e^{-i\gamma_5\,\boldsymbol{t}\cdot\boldsymbol{\theta}_A}\,q, \qquad \bar{q} \longrightarrow \bar{q}\,e^{-i\gamma_5\,\boldsymbol{t}\cdot\boldsymbol{\theta}_A}, \tag{7}$$

$$U(1)_V: \quad q \longrightarrow e^{-i\,\theta}\,q, \qquad \bar{q} \longrightarrow \bar{q}\,e^{i\,\theta}, \tag{8}$$

$$U(1)_A: \quad q \longrightarrow e^{-i\gamma_5\,\theta}\,q, \qquad \bar{q} \longrightarrow \bar{q}\,e^{-i\gamma_5\,\theta}, \tag{9}$$

where $t_i = \frac{\lambda_i}{2}$ $(i = 1, \ldots, 8)$ are SU(3) flavor generators. The QCD Lagrangian is also invariant under scale transformations

$$q(x) \rightarrow \lambda^{3/2}\,q(\lambda\,x), \qquad A^\mu(x) \rightarrow \lambda\,A^\mu(\lambda\,x), \tag{10}$$

which represent dilations of the coordinates with the simultaneous rescaling of the quark and gluon fields in accordance with their normal dimension. At the quantum level the symmetries $U(1)_V$ and $SU(3)_V$ remain valid, $SU(3)_A$ is dynamically broken, and $U(1)_A$ and scale symmetries are broken by anomalies.

Overview of the Nambu-Jona-Lasinio model

- original NJL
  - Nambu and Jona-Lasinio, Phys. Rev. 122, 345; Phys. Rev. 124, 246.
    - Nucleons are the fundamental degrees of freedom.
    - Is formulated in analogy to superconductivity.
    - Nucleon-antinucleon form Cooper-like pair.
    - Goldstone boson occurring in the theory is identified as pion.

Overview of the Nambu-Jona-Lasinio model

- subsequent NJL
  - e.g., S. P. Klevansky, Rev. Mod. Phys. 64, 649
    - Quarks as the fundamental d.o.f.
    - Many successes in the study of meson and baryon properties.
    - Did not have confinement.

Overview of the Nambu-Jona-Lasinio model

- confining NJL
  - H. Mineo et. al, Nucl. Phys. A 735, 482
    - Confinement is simulated by the introduction of an infrared cutoff in the proper-time regularization scheme.
    - Doing this eliminates free quark propagation (it gets rid of the imaginary part of hadron decaying into quarks).
    - In a way that maintains covariance.
    - We calculate PDF, FF, TMD, GPD, etc. with this model, as well as study nucleon in-medium modification, and the binding of atomic nuclei.

Proper-time regularization scheme

As an effective theory, NJL model is non-renormalizable, thus it needs a regularization prescription in order to be well-defined. We use the proper-time regularization scheme

$$\begin{aligned}\frac{1}{X} &= \frac{1}{(n-1)!}\int_0^\infty d\tau\ \tau^{n-1}e^{-\tau X}\\ &\longrightarrow \frac{1}{(n-1)!}\int_{1/\Lambda_{UV}^2}^{1/\Lambda_{IR}^2} d\tau\ \tau^{n-1}e^{-\tau X},\end{aligned}$$

where X represents a product of propagators that have been combined using Feynman parametrization. Only the ultraviolet cutoff Λ_{UV} is needed to render the theory finite, while Λ_{IR} is introduced to mimic confinement.

NJL Lagrangian

The SU(2) flavor NJL Lagrangian relevant to this study, in the $\bar{q}q$ interaction channel, reads[2]

$$\begin{aligned}\mathcal{L} &= \bar{\psi}(i\partial\!\!\!/ - \hat{m})\psi \\ &\quad + \tfrac{1}{2}\, G_\pi[(\bar{\psi}\psi)^2 - (\bar{\psi}\,\gamma_5\vec{\tau}\,\psi)^2] - \tfrac{1}{2}\, G_\omega(\bar{\psi}\,\gamma^\mu\,\psi)^2 \\ &\quad - \tfrac{1}{2}\, G_\rho[(\bar{\psi}\,\gamma^\mu\vec{\tau}\,\psi)^2 + (\bar{\psi}\,\gamma^\mu\gamma_5\vec{\tau}\,\psi)^2], \qquad (1)\end{aligned}$$

where $\hat{m} \equiv \mathrm{diag}[m_u, m_d]$ is the current quark mass matrix and the 4-fermion coupling constants in each chiral channel are labeled by G_π, G_ω, and G_ρ. Throughout this paper we take $m_u = m_d = m$. The interaction Lagrangian can be Fierz symmetrized, with the consequence that after a redefinition of the 4-fermion couplings one need only consider direct terms in the elementary interaction [28].

Mass gap equation

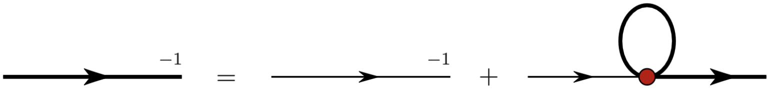


FIG. 1. (Color online) The NJL gap equation in the Hartree-Fock approximation, where the thin line represents the elementary quark propagator, $S_0^{-1}(k) = \not{k} - m + i\varepsilon$, and the shaded circle represents the $\bar{q}q$ interaction kernel given in Eq. (2). Higher-order terms, attributed to meson loops, for example, are not included in the gap equation kernel.

$$i\,S^{-1}(k) = i\,S_0^{-1}(k) - \sum_{\Omega} K_{\Omega}\,\Omega \int \frac{d^4\ell}{(2\pi)^4}\,\mathrm{Tr}[\bar{\Omega}\,i\,S(\ell)],$$

The only piece of the $\bar{q}q$ interaction kernel given in Eq. (2) that contributes to the gap equation expressed in Eq. (3) is the isoscalar-scalar interaction $2i\,G_{\pi}(\mathbb{1})_{\alpha\beta}(\mathbb{1})_{\gamma\delta}$.

Mass gap equation

The interaction kernel in the gap equation of Fig. 1 is local and therefore the dressed quark mass, M, is a constant and satisfies

$$M = m + 12\, i\, G_\pi \int \frac{d^4\ell}{(2\pi)^4}\, \mathrm{Tr}_D[S(\ell)], \tag{5}$$

where the remaining trace is over Dirac indices. For sufficiently strong coupling, $G_\pi > G_{\rm critical}$, Eq. (5) supports a nontrivial solution with $M > m$, which survives even in the chiral limit ($m = 0$).[4] This solution is a consequence of dynamical chiral symmetry breaking (DCSB) in the Nambu-Goldstone mode and it is readily demonstrated, by calculating the total energy [39], that this phase corresponds to the ground state of the vacuum.

[4] In the proper-time regularization scheme defined in Eq. (6) the critical coupling in the chiral limit has the value $G_{\rm critial} = \frac{\pi^2}{3}(\Lambda_{\rm UV}^2 - \Lambda_{\rm IR}^2)^{-1}$.

The dressed quark propagator thus has the solution

$$S(k) = \frac{1}{\not{k} - M + i\varepsilon}.$$

Bethe-Salpeter equation

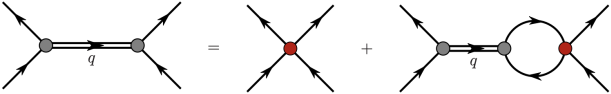


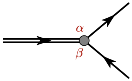
FIG. 2. (Color online) NJL Bethe-Salpeter equation for the quark-antiquark t matrix, represented as the double line with the vertices. The single line corresponds to the dressed quark propagator and the BSE $\bar{q}q$ interaction kernel, consistent with the gap equation kernel used in Eq. (5), is given by Eq. (2).

The NJL BSE, consistent with the gap equation of Fig. 1, is illustrated in Fig. 2 and reads

$$\mathcal{T}(q) = \mathcal{K} + \int \frac{d^4k}{(2\pi)^4} \mathcal{K}\, S(k+q)\, S(k)\, \mathcal{T}(q), \tag{7}$$

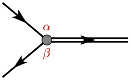
This works for both meson and diquark very similarly.

Bethe-Salpeter vertices



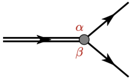
$$\Omega^{\sigma}_{\alpha\beta} = \sqrt{-Z_\sigma}\,(\mathbb{1})_{\alpha\beta}\,, \quad \Omega^{\eta}_{\alpha\beta} = \sqrt{Z_\eta}\,(\gamma_5)_{\alpha\beta}\,, \quad \Omega^{\omega}_{\alpha\beta} = \sqrt{-Z_\omega}\,(\gamma^\mu)_{\alpha\beta}\,, \quad \Omega^{f_1}_{\alpha\beta} = \sqrt{-Z_5}\,(\gamma^\mu\gamma_5)_{\alpha\beta}$$

$$\Omega^{a_0}_{\alpha\beta} = \sqrt{-Z_{a_0}}\,(\tau_i)_{\alpha\beta}\,, \quad \Omega^{\pi}_{\alpha\beta} = \sqrt{Z_\pi}\,(\gamma_5\,\tau_i)_{\alpha\beta}\,, \quad \Omega^{\rho}_{\alpha\beta} = \sqrt{-Z_\rho}\,(\gamma^\mu\,\tau_i)_{\alpha\beta}\,, \quad \Omega^{a_1}_{\alpha\beta} = \sqrt{-Z_{a_1}}\,(\gamma^\mu\gamma_5\tau_i)_{\alpha\beta}$$



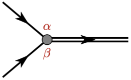
$$\overline{\Omega}^{\sigma}_{\beta\alpha} = \sqrt{-Z_\sigma}\,(\mathbb{1})_{\beta\alpha}\,, \quad \overline{\Omega}^{\eta}_{\beta\alpha} = \sqrt{Z_\eta}\,(\gamma_5)_{\beta\alpha}\,, \quad \overline{\Omega}^{\omega}_{\beta\alpha} = \sqrt{-Z_\omega}\,(\gamma^\nu)_{\beta\alpha}\,, \quad \overline{\Omega}^{f_1}_{\alpha\beta} = \sqrt{-Z_5}\,(\gamma^\mu\gamma_5)_{\alpha\beta}$$

$$\overline{\Omega}^{a_0}_{\beta\alpha} = \sqrt{-Z_{a_0}}\,(\tau_i^\dagger)_{\beta\alpha}, \quad \overline{\Omega}^{\pi}_{\beta\alpha} = \sqrt{Z_\pi}\,(\gamma_5\,\tau_i^\dagger)_{\beta\alpha}, \quad \overline{\Omega}^{\rho}_{\beta\alpha} = \sqrt{-Z_\rho}\,(\gamma^\nu\,\tau_i^\dagger)_{\beta\alpha}, \quad \overline{\Omega}^{a_1}_{\alpha\beta} = \sqrt{-Z_{a_1}}\,(\gamma^\mu\gamma_5\tau_i^\dagger)_{\alpha\beta}$$



$$\Omega^{s}_{\alpha\beta} = \sqrt{Z_s}\,(\gamma_5\,C\,\tau_2\,\beta_A)_{\alpha\beta}\,, \qquad \Omega^{p}_{\alpha\beta} = \sqrt{-Z_p}\,(C\,\tau_2\,\beta_A)_{\alpha\beta}\,,$$

$$\Omega^{a}_{\alpha\beta} = \sqrt{-Z_a}\,(\gamma^\mu\,C\,\tau_i\tau_2\,\beta_A)_{\alpha\beta}\,, \qquad \Omega^{v}_{\alpha\beta} = \sqrt{-Z_v}\,(\gamma^\mu\gamma_5\,C\,\tau_2\,\beta_A)_{\alpha\beta}$$



$$\overline{\Omega}^{s}_{\alpha\beta} = \sqrt{Z_s}\left(C^{-1}\,\gamma_5\,\tau_2\,\beta_A\right)_{\alpha\beta}, \qquad \overline{\Omega}^{p}_{\alpha\beta} = \sqrt{-Z_p}\left(C^{-1}\,\tau_2\,\beta_A\right)_{\alpha\beta},$$

$$\overline{\Omega}^{a}_{\alpha\beta} = \sqrt{-Z_a}\left(C^{-1}\,\gamma^\nu\,\tau_2\tau_i^\dagger\,\beta_A\right)_{\alpha\beta}, \qquad \overline{\Omega}^{v}_{\alpha\beta} = \sqrt{-Z_v}\left(C^{-1}\,\gamma^\nu\gamma_5\,\tau_2\,\beta_A\right)_{\alpha\beta}$$

Approximated diquark propagator

In the following I use scalar diquark as an example.

$$\tau_s(q) = 4i\,G_s - 4i\,N_c\,G_s \int \frac{d^4k}{(2\pi)^4}\,\mathrm{Tr}\left[\gamma_5\,S(k)\,\gamma_5\,S(q+k)\right]\tau_s(q) = 4i\,G_s - 2\,G_s\,\Pi_{PP}(q^2)\,\tau_s(q).$$

Therefore

$$\tau_s(q) = \frac{4i\,G_s}{1+2\,G_s\,\Pi_{PP}(q^2)} \qquad \text{where} \qquad \Pi_{PP}(q^2) = 2i\,N_c \int \frac{d^4k}{(2\pi)^4}\,\mathrm{Tr}\left[\gamma_5\,S(k)\,\gamma_5\,S(q+k)\right].$$

Expanding the bubble diagrams about the pole mass, where

$$\Pi(q^2) = \Pi(m^2) + (q^2-m^2)\,\frac{\partial}{\partial q^2}\Pi(q^2)\Big|_{q^2=m^2} + \ldots,$$

and using the pole condition $1+2\,G_s\,\Pi_{PP}(m_s^2)=0$, gives

$$\tau_s(q) = \frac{4i\,G_s}{1+2\,G_s\,\Pi_{PP}(M_s^2)+(q^2-M_s^2)\,2\,G_s\,\frac{\partial}{\partial q^2}\Pi_{PP}(q^2)\Big|_{q^2=M_s^2}}.$$

Therefore the pole form of the propagator reads

$$\tau_s(q) = -\frac{i\,Z_s}{q^2-m_s^2+i\varepsilon} \qquad \text{where} \qquad Z_s^{-1} = -\frac{1}{2}\,\frac{\partial}{\partial q^2}\Pi_{PP}(q^2)\Big|_{q^2=m_s^2},$$

where Z_s is defined to be positive.

Approximated diquark propagator

We use "contact+pole" approximation for the meson/diquark propagator, namely

For the diquarks we have

$$\tau_s(q) = \frac{4i\,G_s}{1+2\,G_s\,\Pi_{PP}(q^2)} \longrightarrow 4i\,G_s - \frac{i\,Z_s}{q^2 - M_s^2 + i\,\varepsilon},$$

$$\tau_p(q) = \frac{-4i\,G_p}{1-2\,G_p\,\Pi_{SS}(q^2)} \longrightarrow -4i\,G_p + \frac{i\,Z_p}{q^2 - M_p^2 + i\,\varepsilon},$$

$$\tau_a^{\mu\nu}(q) = \frac{4i\,G_a}{1+2\,G_a\,\Pi_{VV}(q^2)}\left[g^{\mu\nu} + 2\,G_a\,\Pi_{VV}(q^2)\,\frac{q^\mu q^\nu}{q^2}\right] \longrightarrow 4i\,G_a\,g^{\mu\nu} - \frac{i\,Z_a}{q^2 - M_a^2 + i\,\varepsilon}\left(g^{\mu\nu} - \frac{q^\mu\,q^\nu}{M_a^2}\right),$$

$$\tau_v^{\mu\nu}(q) = \frac{4i\,G_a}{1+2\,G_a\,\Pi_{AA}^{(T)}(q^2)}\left[g^{\mu\nu} + \frac{2\,G_a\,[\Pi_{AA}^{(T)}(q^2) - \Pi_{AA}^{(L)}(q^2)]}{1+2\,G_a\,\Pi_{AA}^{(L)}(q^2)}\,\frac{q^\mu q^\nu}{q^2}\right] \longrightarrow 4i\,G_a\,g^{\mu\nu} - \frac{i\,Z_v}{q^2 - M_v^2 + i\,\varepsilon}\left(g^{\mu\nu} - \frac{q^\mu\,q^\nu}{M_v^2}\right),$$

$$Z_s^{-1} = -\frac{1}{2}\,\hat{\Pi}_{PP}(m_s^2), \qquad Z_p^{-1} = \frac{1}{2}\,\hat{\Pi}_{SS}(m_p^2), \qquad Z_a^{-1} = -\frac{1}{2}\,\hat{\Pi}_{VV}(m_a^2), \qquad Z_v^{-1} = -\frac{1}{2}\,\hat{\Pi}_{AA}^{(T)}(m_v^2),$$

where $\hat{\Pi}_i(m_j^2) = \frac{\partial}{\partial p^2}\,\Pi_i(p^2)\big|_{p^2=m_j^2}$.

Faddeev equation

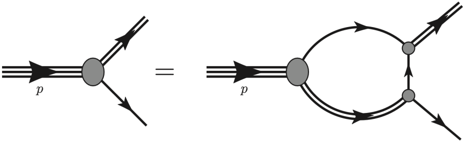


FIG. 3. Homogeneous Faddeev equation for the nucleon in the NJL model. The single lines represent the quark propagator and the double lines the diquark propagators.

$$X^{a}_{\alpha,i}(P,p) = \int \frac{d^4\ell}{(2\pi)^4}\, Z^{ab,ij}_{\alpha\beta}(p,\ell)\, S_j(\tfrac{1}{2}P+\ell)_{\beta\gamma}\, \tau^{ki}_{bc}(\tfrac{1}{2}P-\ell)\, X^{c}_{\gamma,j}(P,p).$$

where

$$Z^{ab,ij}_{\alpha\beta}(p,\ell) = \left[\Omega^{b,ik}\, S^T_k(-\ell-p)\, \overline{\Omega}^{a,kj}\right]_{\alpha\beta}$$

Faddeev equation

Therefore, the quark exchange kernel is given by

$$Z_{\alpha\beta}(p,\ell) = -3 \begin{pmatrix} \gamma_5\, S(p+\ell)\,\gamma_5 & \gamma^\sigma\, S(p+\ell)\,\gamma_5\,\tau_n & S(p+\ell)\,\gamma_5 & \gamma^\sigma\gamma_5\, S(p+\ell)\,\gamma_5 \\ \gamma_5\, S(p+\ell)\,\gamma^\mu\,\tau_m^\dagger & \gamma^\sigma\, S(p+\ell)\,\gamma^\mu\,\tau_n\tau_m^\dagger & S(p+\ell)\,\gamma^\mu\,\tau_m^\dagger & \gamma^\sigma\gamma_5\, S(p+\ell)\,\gamma^\mu\,\tau_m^\dagger \\ \gamma_5\, S(p+\ell) & \gamma^\sigma\, S(p+\ell)\,\tau_n & S(p+\ell) & \gamma^\sigma\gamma_5\, S(p+\ell) \\ \gamma_5\, S(p+\ell)\,\gamma^\mu\gamma_5 & \gamma^\sigma\, S(p+\ell)\,\gamma^\mu\gamma_5\,\tau_n & S(p+\ell)\,\gamma^\mu\gamma_5 & \gamma^\sigma\gamma_5\, S(p+\ell)\,\gamma^\mu\gamma_5 \end{pmatrix}_{\alpha\beta}, \tag{223}$$

where we have used $(\tau_i\tau_2)(\tau_2\tau_i) = (\tau_m\tau_2)(\tau_2\tau_m^\dagger)$. In the static approximation, $S(p+\ell) \to -\frac{1}{M}$, the quark exchange kernel becomes

$$Z_{\alpha\beta} = \frac{3}{M} \begin{pmatrix} 1 & \gamma^\sigma\,\gamma_5\,\tau_n & \gamma_5 & \gamma^\sigma \\ \gamma_5\,\gamma^\mu\,\tau_m^\dagger & \gamma^\sigma\,\gamma^\mu\,\tau_n\tau_m^\dagger & \gamma^\mu\,\tau_m^\dagger & \gamma^\sigma\gamma_5\,\gamma^\mu\,\tau_m^\dagger \\ \gamma_5 & \gamma^\sigma\,\tau_n & 1 & \gamma^\sigma\gamma_5 \\ \gamma_5\,\gamma^\mu\gamma_5 & \gamma^\sigma\,\gamma^\mu\gamma_5\,\tau_n & \gamma^\mu\gamma_5 & -\gamma^\sigma\gamma^\mu \end{pmatrix}_{\alpha\beta}. \tag{224}$$

Projecting the kernel onto isospin one-half gives

$$Z_{\alpha\beta} = \frac{3}{M} \begin{pmatrix} 1 & \sqrt{3}\,\gamma^\sigma\,\gamma_5 & \gamma_5 & \gamma^\sigma \\ \sqrt{3}\,\gamma_5\,\gamma^\mu & -\gamma^\sigma\,\gamma^\mu & \sqrt{3}\,\gamma^\mu & -\sqrt{3}\,\gamma^\sigma\gamma^\mu\gamma_5 \\ \gamma_5 & \sqrt{3}\,\gamma^\sigma & 1 & \gamma^\sigma\gamma_5 \\ -\gamma^\mu & \sqrt{3}\,\gamma^\sigma\,\gamma^\mu\gamma_5 & \gamma^\mu\gamma_5 & -\gamma^\sigma\gamma^\mu \end{pmatrix}_{\alpha\beta}. \tag{225}$$

The Faddeev equation reads

$$X^a_\alpha(P) = \int \frac{d^4\ell}{(2\pi)^4}\, Z_{\alpha\beta}\, S(\tfrac{1}{2}P+\ell)_{\beta\gamma}\, \tau_{bc}(\tfrac{1}{2}P-\ell)\, X^\gamma_c(P) \equiv Z^{ab,ij}_{\alpha\beta}\, \Pi^{bc,ki}_{\beta\gamma,j}(P)\, X^c_{\gamma,j}(P). \tag{226}$$

Faddeev equation

4.2 Quark–Scalar-Diquark Bubble Diagram

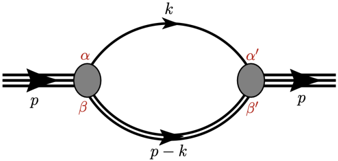


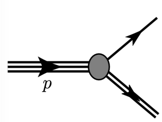
Figure 2: Quark–scalar-diquark diagram contribution to a baryon Faddeev vertex.

The baryon quark–scalar-diquark bubble diagram has the form

$$\Pi_N^s(p) = -i \int \frac{d^4k}{(2\pi)^4}\, \tau_s(p-k)\, iS(k) = \int \frac{d^4k}{(2\pi)^4} \left[4i\,G_s - \frac{i\,Z_s}{(p-k)^2 - m_s^2 + i\varepsilon}\right] \frac{\not{k} + M}{k^2 - M^2 + i\varepsilon}.$$

Faddeev equation

The nucleon vertex function is parametrized by



$$\Gamma_s = \sqrt{-Z_N}\,\alpha_1\,\chi_t\,u(p,s), \qquad \Gamma_a = \sqrt{-Z_N}\left[\alpha_2\,\frac{p^\mu}{M_N}\,\gamma_5 + \alpha_3\,\gamma^\mu\gamma_5\right]\frac{\tau_i}{\sqrt{3}}\,\chi_t\,u(p,s),$$

$$\Gamma_p = \sqrt{-Z_N}\,\alpha_4\,\gamma_5\,\chi_t\,u(p,s), \qquad \Gamma_v = \sqrt{-Z_N}\left[\alpha_5\,\frac{p^\mu}{M_N} + \alpha_6\,\gamma^\mu\right]\chi_t\,u(p,s),$$

Therefore the Faddeev equation reads

$$\begin{bmatrix} \alpha_1 \\ \alpha_2\,\frac{p^\mu}{M_N}\gamma_5 + \alpha_3\,\gamma^\mu\gamma_5 \\ \alpha_4\,\gamma_5 \\ \alpha_5\,\frac{p^\mu}{M_N} + \alpha_6\,\gamma^\mu \end{bmatrix} u(p,\lambda)$$
$$= \frac{3}{M}\begin{pmatrix} 1 & \sqrt{3}\,\gamma^\sigma\,\gamma_5 & \gamma_5 & \gamma^\sigma \\ \sqrt{3}\,\gamma_5\,\gamma^\mu & -\gamma^\sigma\,\gamma^\mu & \sqrt{3}\,\gamma^\mu & -\sqrt{3}\,\gamma^\sigma\gamma^\mu\gamma_5 \\ \gamma_5 & \sqrt{3}\,\gamma^\sigma & 1 & \gamma^\sigma\gamma_5 \\ -\gamma^\mu & \sqrt{3}\,\gamma^\sigma\,\gamma^\mu\gamma_5 & \gamma^\mu\gamma_5 & -\gamma^\sigma\gamma^\mu \end{pmatrix}\begin{pmatrix} \Pi_{Ns} & 0 & 0 & 0 \\ 0 & \Pi^{Na}_{\sigma\nu} & 0 & 0 \\ 0 & 0 & \Pi_{Np} & 0 \\ 0 & 0 & 0 & \Pi^{Nv}_{\sigma\nu} \end{pmatrix}\begin{bmatrix} \alpha_1 \\ \alpha_2\,\frac{p^\nu}{M_N}\gamma_5 + \alpha_3\,\gamma^\nu\gamma_5 \\ \alpha_4\,\gamma_5 \\ \alpha_5\,\frac{p^\nu}{M_N} + \alpha_6\,\gamma^\nu \end{bmatrix} u(p,\lambda)$$

Model parameters

The two-flavor NJL has the following parameters:

$\bar{q}q$ couplings:	G_π, G_ρ, G_ω
qq couplings:	G_s, G_a
masses:	$m_u = m_d$
regularization:	Λ_{IR}, Λ_{UV}

We assign values *a priori* to the following parameters:

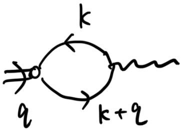
Λ_{IR} and M

The remaining parameters can then be fixed by

$\Lambda_{UV} \leftrightarrow f_\pi$
$G_{\pi,\rho,\omega} \leftrightarrow m_{\pi,\rho,\omega}$
G_s, $G_a \leftrightarrow M_N$, M_Δ

Model parameters

The Λ_{UV} and G_π together is determined by the pion decay constant and pion mass.



$$\langle 0|\bar{\psi}\gamma_\mu\gamma_5\psi|\pi(q)\rangle = 2i f_\pi q_\mu$$

From this we can obtain

$$f_\pi = -12i\sqrt{Z_\pi}M\int\frac{d^4k}{(2\pi)^4}\frac{1}{(k^2-M^2)\left((k+q)^2-M^2\right)}\Big|_{q^2=m_\pi^2}$$

Model parameters

Solving the Bethe-Salpeter equation gives

$$\begin{tikzcd} & M_i \\ G_i \arrow[ur] \arrow[dr] & \\ & Z_i \end{tikzcd}$$

where i means various either $\bar{q}q$ or qq channels,
$i = \pi,\ \rho,\ \omega;\ s,\ a$, etc.
Also in this way, m_ρ and m_ω determine G_ρ and G_ω, respectively.

Model parameters

G_s and G_a are determined by solving the two Faddeev equations for the nucleon and the delta baryon.

6×6

$$\boxed{\quad\quad} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \end{bmatrix}$$

G_s, G_a, M_N
(M_Δ)

two equations of

" eigenvalue = 1 "

fix G_s & G_a two unknowns.