Contraction of De Sitter and Anti De Sitter groups in the matrix representation

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Outline

- Matrix representation of the three-dimensional space
- Matrix representation of the four-dimensional space-time
- Matrix representation of the five-dimensional space-time
  - -Operators of De Sitter space
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- Contraction of five-dimensional space-time into four-dimensional space-time in the matrix representation
- Conclusion and future work

Matrix representation of the three-dimensional space

- Pauli Matrices

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad , \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad , \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- Coordinates

$$x_{m,3d} = x^i_{v,3d}\sigma_i = \begin{pmatrix} \mathrm{x3} & \mathrm{x1} - \mathrm{ix2} \\ \mathrm{x1} + \mathrm{ix2} & -\mathrm{x3} \end{pmatrix}$$

Det$[x_{m,3d}]$= $x^i x_i = -(x^1)^2 - (x^2)^2 - (x^3)^2$

- Infinitesimal transformation matrices for rotation operators in 3d

$$J^i = \frac{1}{2}\sigma^i$$

- Commutation relations

$$[J^i, J^j] = i\epsilon^{ijk}J^k$$

- Transformation

$$x'_{m,3d} = x'^{i}_{v,3d}\sigma_i = U_{m,3d}x_{m,3d}U^{-1}_{m,3d}$$

- Example: Rotation around x- axis

$$x'_{\mathrm{v,3d}} = e^{-i\theta_x J^1_{\mathrm{v,3d}}}\, x_{\mathrm{v,3d}} = \begin{pmatrix} \mathrm{x1} \\ \mathrm{x2\,Cos[\theta x] - x3\,Sin[\theta x]} \\ \mathrm{x3\,Cos[\theta x] + x2\,Sin[\theta x]} \end{pmatrix}$$

$$x_{m,3d} = \begin{pmatrix} \mathrm{x3} & \mathrm{x1 - ix2} \\ \mathrm{x1 + ix2} & \mathrm{-x3} \end{pmatrix}$$

$$x'_{m,3d} = x'^{1}_{v,3d}\sigma_1 = e^{-i\theta_x J^1_{\mathrm{m,3d}}}\, x_{\mathrm{m,3d}} e^{i\theta_x J^1_{\mathrm{m,3d}}}$$

$$= \begin{pmatrix} \mathrm{x3Cos[\theta x] + x2Sin[\theta x]} & \mathrm{x1 - ix2Cos[\theta x] + ix3Sin[\theta x]} \\ \mathrm{x1 + ix2Cos[\theta x] - ix3Sin[\theta x]} & \mathrm{-x3Cos[\theta x] - x2Sin[\theta x]} \end{pmatrix}$$

- 2x2 matrix

Matrix representation of four-dimensional space-time

- Gamma Matrices in the chiral basis

$$\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & -\sigma^i \\ \sigma^i & 0 \end{pmatrix},$$

- Clifford algebra

$$\{\gamma^\mu, \gamma^\nu\} \equiv \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2\eta^{\mu\nu}1$$

$$\gamma^\mu\gamma^\nu = -\gamma^\nu\gamma^\mu \quad \text{when } \mu \neq \nu$$

$$(\gamma^0)^2 = 1 \qquad (\gamma^i)^2 = -1 \qquad i = 1, 2, 3$$

$$\gamma_\mu = \eta_{\mu\nu}\gamma^\nu = \{\gamma^0, -\gamma^1, -\gamma^2, -\gamma^3\}$$

- Coordinates

A2

$$x_{\mathrm{m,4d}} = x^\mu_{\mathrm{v,4d}}\gamma_\mu = \begin{pmatrix} 0 & 0 & \mathrm{x0} + \mathrm{x3} & \mathrm{x1} - \mathrm{ix2} \\ 0 & 0 & \mathrm{x1} + \mathrm{ix2} & \mathrm{x0} - \mathrm{x3} \\ \mathrm{x0} - \mathrm{x3} & -\mathrm{x1} + \mathrm{ix2} & 0 & 0 \\ -\mathrm{x1} - \mathrm{ix2} & \mathrm{x0} + \mathrm{x3} & 0 & 0 \end{pmatrix}$$

A1

Space-time invariant

Det[A1]=Det[A2]= $x^\mu x_\mu = (x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2$

Det[$x_{\mathrm{m,4d}}$] = $((x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2)^2$

- Infinitesimal transformation matrices in matrix representation

$$K^1 = \frac{i}{2}(\gamma^0.\gamma^1) \qquad J^1 = \frac{i}{2}(\gamma^2.\gamma^3)$$

$$K^2 = \frac{i}{2}(\gamma^0.\gamma^2) \qquad J^2 = \frac{i}{2}(\gamma^3.\gamma^1)$$

$$K^3 = \frac{i}{2}(\gamma^0.\gamma^3) \qquad J^3 = \frac{i}{2}(\gamma^1.\gamma^2)$$

$$M^{\mu\nu} = \begin{pmatrix} 0 & K^1 & K^2 & K^3 \\ -K^1 & 0 & J^3 & -J^2 \\ -K^2 & -J^3 & 0 & J^1 \\ -K^3 & J^2 & -J^1 & 0 \end{pmatrix}$$

$$M^{\mu v} = \frac{i}{2}\begin{pmatrix} 0 & \gamma^0\gamma^1 & \gamma^0\gamma^2 & \gamma^0\gamma^3 \\ \gamma^1\gamma^0 & 0 & \gamma^1\gamma^2 & \gamma^1\gamma^3 \\ \gamma^2\gamma^0 & \gamma^2\gamma^1 & 0 & \gamma^2\gamma^3 \\ \gamma^3\gamma^0 & \gamma^3\gamma^1 & \gamma^3\gamma^2 & 0 \end{pmatrix}$$

Boost (K) and rotation(J) operators in the explicit 4-vector representation

$$K_1 = \begin{pmatrix} 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad K_2 = \begin{pmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad K_3 = \begin{pmatrix} 0 & 0 & 0 & i \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}$$

$$J_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix}, \quad J_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & i \\ 0 & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{pmatrix}, \quad J_3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- Commutation Relation $\quad [M^{\mu\nu}, M^{\rho\lambda}] = i(g^{\nu\rho}M^{\mu\lambda} - g^{\nu\lambda}M^{\mu\rho} - g^{\mu\rho}M^{\nu\lambda} + g^{\mu\lambda}M^{\nu\rho})$

- Transformation

$$x'_{m,4d} = x'^{\mu}_{v,4d}\gamma_\mu = T_{m,4d}x_{m,4d}T^{-1}_{m,4d}$$

- Example: Boost in the x- direction

$$x'_{\mathrm{v,4d}} = e^{-i\beta_x K^1_{\mathrm{v,4d}}}\, x_{\mathrm{v,4d}} = \begin{pmatrix} x0\,\mathrm{Cosh}[\beta x] + x1\,\mathrm{Sinh}[\beta x] \\ x1\,\mathrm{Cosh}[\beta x] + x0\,\mathrm{Sinh}[\beta x] \\ x2 \\ x3 \end{pmatrix}$$

$$x_{m,4d} = \begin{pmatrix} 0 & 0 & x0+x3 & x1-ix2 \\ 0 & 0 & x1+ix2 & x0-x3 \\ x0-x3 & -x1+ix2 & 0 & 0 \\ -x1-ix2 & x0+x3 & 0 & 0 \end{pmatrix}$$

$$x'_{m,4d} = x'^{\mu}_{v,4d}\gamma_\mu = e^{-i\beta_x K^1_{\mathrm{m,4d}}}\, x_{\mathrm{m,4d}} e^{i\beta_x K^1_{\mathrm{m,4d}}}$$

$$= \begin{pmatrix} 0 & 0 & x3 + x0\mathrm{Cosh}[\beta x] + x1\mathrm{Sinh}[\beta x] & -ix2 + x1\mathrm{Cosh}[\beta x] + x0\mathrm{Sinh}[\beta x] \\ 0 & 0 & ix2 + x1\mathrm{Cosh}[\beta x] + x0\mathrm{Sinh}[\beta x] & -x3 + x0\mathrm{Cosh}[\beta x] + x1\mathrm{Sinh}[\beta x] \\ -x3 + x0\mathrm{Cosh}[\beta x] + x1\mathrm{Sinh}[\beta x] & ix2 - x1\mathrm{Cosh}[\beta x] - x0\mathrm{Sinh}[\beta x] & 0 & 0 \\ -ix2 - x1\mathrm{Cosh}[\beta x] - x0\mathrm{Sinh}[\beta x] & x3 + x0\mathrm{Cosh}[\beta x] + x1\mathrm{Sinh}[\beta x] & 0 & 0 \end{pmatrix}$$

Five-Dimensional Spaces

- Einstein Field Equation

$$\underbrace{R_{ab} - \frac{1}{2} R g_{ab}}_{G_{\mu\nu}} + \Lambda\, g_{ab} = \frac{8\pi G}{c^4} T_{ab}$$

Curvature of the space time

k

$$G_{ab} + \Lambda\, g_{ab} = k\, T_{ab}$$

R_{ab} =Ricci curvature tensor , R = Scalar Curvature

Λ =Cosmological constant

T_{ab} = Stress- energy tensor (Matter tensor)

G = Newton's constant

k = Einstein gravitational constant

G_{ab} = Einstein Tensor

- Minkowski Space (Flat space)$\Rightarrow$ $G_{ab} = 0$ $\longrightarrow$ ISO (3,1)

  $T_{ab} = 0$, and Λ=0

- De sitter Space-Time : $T_{ab} = 0$ and $\Lambda > 0$ (Future space) $\longrightarrow$ SO(4,1)

- Anti-De sitter Space-time: $T_{ab} = 0$ and $\Lambda < 0$ $\longrightarrow$ SO(3,2)

- De Sitter Space- Time , S0(4,1)

:This also can be visualized as the hyperboloid in flat five-dimensional space

$$y_0 = l\ Sinh\left(\frac{t}{l}\right)$$

$$y_1 = l\ Cosh\left(\frac{t}{l}\right)\ Sin(\chi)Sin(\theta)Cos(\varphi)$$

$$y_2 = l\ Cosh\left(\frac{t}{l}\right)\ Sin(\chi)Sin(\theta)Sin(\varphi)$$

$$y_3 = l\ Cosh\left(\frac{t}{l}\right)\ Sin(\chi)Cos(\theta)$$

$$y_4 = l\ Cosh\left(\frac{t}{l}\right)\ Cos(\chi)$$

Space-time invariant

$$-y_0^2 + y_1^2 + y_2^2 + y_3^2 + y_4^2 = l^2$$

$$l = \sqrt{\frac{3}{\Lambda}}$$

- Infinitesimal operators of S0(4,1) in four vector representation

$$K^1 = \begin{pmatrix} 0 & i & 0 & 0 & 0 \\ i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad K^2 = \begin{pmatrix} 0 & 0 & i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad K^3 = \begin{pmatrix} 0 & 0 & 0 & i & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$J^1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i & 0 \\ 0 & 0 & i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad J^2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & i & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad J^3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 & 0 \\ 0 & i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\Gamma^0 = \begin{pmatrix} 0 & 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -i & 0 & 0 & 0 & 0 \end{pmatrix}, \quad \Gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & i \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -i & 0 & 0 & 0 \end{pmatrix}, \quad \Gamma^2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & i \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 & 0 \end{pmatrix}, \quad \Gamma^3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & i \\ 0 & 0 & 0 & -i & 0 \end{pmatrix}$$

What are the Infinitesimal operators of S0(4,1) in matrix representation ?

$$R^{\alpha\beta} = \begin{pmatrix} 0 & K^1 & K^2 & K^3 & -\Gamma^0 \\ -K^1 & 0 & J^3 & -J^2 & -\Gamma^1 \\ -K^2 & -J^3 & 0 & J^1 & -\Gamma^2 \\ -K^3 & J^2 & -J^1 & 0 & -\Gamma^3 \\ \Gamma^0 & \Gamma^1 & \Gamma^2 & \Gamma^3 & 0 \end{pmatrix} \quad \eta^{\alpha\beta} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix} \quad R_{\alpha\beta} = \begin{pmatrix} 0 & -K^1 & -K^2 & -K^3 & \Gamma^0 \\ K^1 & 0 & J^3 & -J^2 & -\Gamma^1 \\ K^2 & -J^3 & 0 & J^1 & -\Gamma 2 \\ K^3 & J^2 & -J^1 & 0 & -\Gamma^3 \\ -\Gamma^0 & \Gamma^1 & \Gamma^2 & \Gamma^3 & 0 \end{pmatrix}$$

They all satisfy Lie algebra given below.

$$[R^{\alpha\beta}, R^{\gamma\delta}] = i(\eta^{\beta\gamma} R^{\alpha\delta} - \eta^{\beta\delta} R^{\alpha\gamma} - \eta^{\alpha\gamma} R^{\beta\delta} + \eta^{\alpha\delta} R^{\beta\gamma})$$

Where $\alpha = \beta = \gamma = \delta = 0, 1, 2, 3, 4$

$$\Gamma^\mu = J^{4\mu}, \qquad \mu = 0, 1, 2, 3$$

$$\begin{aligned} [R^{\mu\nu}, R^{\rho\lambda}] =& i(\eta^{\nu\rho} R^{\mu\lambda} - \eta^{\nu\lambda} R^{\mu\rho} - \eta^{\mu\rho} R^{\nu\lambda} + \eta^{\mu\lambda} R^{\nu\rho}) \\ [R^{\mu\nu}, \Gamma^\rho] =& i(\eta^{\nu\rho} \Gamma^\mu - \eta^{\mu\rho} \Gamma^\nu) \\ [\Gamma^\mu, \Gamma^\nu] =& i R^{\mu\nu} \end{aligned}$$

Where the range of the indices μ ,ν,γ and δ is 0,1,2,3.

Linearly and nonlinearly transforming fields on homogeneous spaces of the (4,1)-de Sitter group

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- Contraction of de Sitter space into Minkowski Space in vector representation

Translation operator acting on the de Sitter space

$$\boldsymbol{\Gamma} = e^{i\, \eta_{\mu\nu} \frac{a^{\mu}}{l} \boldsymbol{\Gamma}^{\nu}}$$

$$\mathrm{a0}^2 - \mathrm{a1}^2 - \mathrm{a2}^2 - \mathrm{a3}^2 = \mathrm{a}^2$$

$$\begin{pmatrix} y^{0\prime} \\ y^{1\prime} \\ y^{2\prime} \\ y^{3\prime} \\ y^{4\prime} \end{pmatrix} = \begin{pmatrix} \frac{a^2 - \mathrm{a0}^2 + \mathrm{a0}^2 \mathrm{Cosh}\left[\frac{a}{l}\right]}{a^2} & -\frac{\mathrm{a0a1}\left(-1 + \mathrm{Cosh}\left[\frac{a}{l}\right]\right)}{a^2} & -\frac{\mathrm{a0a2}\left(-1 + \mathrm{Cosh}\left[\frac{a}{l}\right]\right)}{a^2} & -\frac{\mathrm{a0a3}\left(-1 + \mathrm{Cosh}\left[\frac{a}{l}\right]\right)}{a^2} & \frac{\mathrm{a0Sinh}\left[\frac{a}{l}\right]}{a} \\ \frac{\mathrm{a0a1}\left(-1 + \mathrm{Cosh}\left[\frac{a}{l}\right]\right)}{a^2} & \frac{a^2 + \mathrm{a1}^2 - \mathrm{a1}^2 \mathrm{Cosh}\left[\frac{a}{l}\right]}{a^2} & -\frac{\mathrm{a1a2}\left(-1 + \mathrm{Cosh}\left[\frac{a}{l}\right]\right)}{a^2} & -\frac{\mathrm{a1a3}\left(-1 + \mathrm{Cosh}\left[\frac{a}{l}\right]\right)}{a^2} & \frac{\mathrm{a1Sinh}\left[\frac{a}{l}\right]}{a} \\ \frac{\mathrm{a0a2}\left(-1 + \mathrm{Cosh}\left[\frac{a}{l}\right]\right)}{a^2} & -\frac{\mathrm{a1a2}\left(-1 + \mathrm{Cosh}\left[\frac{a}{l}\right]\right)}{a^2} & \frac{a^2 + \mathrm{a2}^2 - \mathrm{a2}^2 \mathrm{Cosh}\left[\frac{a}{l}\right]}{a^2} & -\frac{\mathrm{a2a3}\left(-1 + \mathrm{Cosh}\left[\frac{a}{l}\right]\right)}{a^2} & \frac{\mathrm{a2Sinh}\left[\frac{a}{l}\right]}{a} \\ \frac{\mathrm{a0a3}\left(-1 + \mathrm{Cosh}\left[\frac{a}{l}\right]\right)}{a^2} & -\frac{\mathrm{a1a3}\left(-1 + \mathrm{Cosh}\left[\frac{a}{l}\right]\right)}{a^2} & -\frac{\mathrm{a2a3}\left(-1 + \mathrm{Cosh}\left[\frac{a}{l}\right]\right)}{a^2} & \frac{a^2 + \mathrm{a3}^2 - \mathrm{a3}^2 \mathrm{Cosh}\left[\frac{a}{l}\right]}{a^2} & \frac{\mathrm{a3Sinh}\left[\frac{a}{l}\right]}{a} \\ \frac{\mathrm{a0Sinh}\left[\frac{a}{l}\right]}{a} & -\frac{\mathrm{a1Sinh}\left[\frac{a}{l}\right]}{a} & -\frac{\mathrm{a2Sinh}\left[\frac{a}{l}\right]}{a} & -\frac{\mathrm{a3Sinh}\left[\frac{a}{l}\right]}{a} & \mathrm{Cosh}\left[\frac{a}{l}\right] \end{pmatrix} \begin{pmatrix} y^{0} \\ y^{1} \\ y^{2} \\ y^{3} \\ y^{4} \end{pmatrix}$$

$$\Lambda \to 0\,, \Rightarrow l \to \infty$$

$$y^4 \to \infty, \Rightarrow y^{4\prime} \to \infty$$

Similar to the Inonu-Wigner contraction c→ ∞

$$P = e^{i\, \eta_{\mu\nu} a^{\mu} P^{\nu}}$$

$$\begin{pmatrix} \mathrm{t} \\ \mathrm{r\ Sin}[\theta]\ \mathrm{Cos}[\varphi] \\ \mathrm{r\ Sin}[\theta]\ \mathrm{Sin}[\varphi] \\ \mathrm{r\ Cos}[\theta] \\ 1 \end{pmatrix} = \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x^{0\prime} \\ x^{1\prime} \\ x^{2\prime} \\ x^{3\prime} \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & a0 \\ 0 & 1 & 0 & 0 & a1 \\ 0 & 0 & 1 & 0 & a2 \\ 0 & 0 & 0 & 1 & a3 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x^{0} \\ x^{1} \\ x^{2} \\ x^{3} \\ 1 \end{pmatrix}$$

- How can we contract de Sitter space into Minkowski Space in matrix representation ?

- Ant-De Sitter Space- Time , S0(3,2)

:This also can be visualized as the hyperboloid in flat five-dimensional space

$$z_0 = l\, Cosh(Ӽ) Sin\left(\frac{T}{l}\right)$$

$$z_1 = l\, Sinh(Ӽ)\, Sin(\theta) Cos(\varphi)$$

$$z_2 = l Sinh(Ӽ) Sin(\theta) Sin(\varphi)$$

$$z_3 = l\, Sinh(Ӽ)\, Cos(\theta)$$

$$z_4 = l\, Cosh(Ӽ) Cos\left(\frac{T}{l}\right)$$

Space-time invariant

$$-z_0^2 + z_1^2 + z_2^2 + z_3^2 - z_4^2 = -l^2$$

$$l = \sqrt{\frac{-3}{\Lambda}}$$

- Infinitesimal operators of S0(3,2) in four vector representation

$$K^1 = \begin{pmatrix} 0 & i & 0 & 0 & 0 \\ i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad K^2 = \begin{pmatrix} 0 & 0 & i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad K^3 = \begin{pmatrix} 0 & 0 & 0 & i & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$J^1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i & 0 \\ 0 & 0 & i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad J^2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & i & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad J^3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 & 0 \\ 0 & i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\Pi^0 = \begin{pmatrix} 0 & 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 & 0 \end{pmatrix}, \quad \Pi^1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & i \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & i & 0 & 0 & 0 \end{pmatrix}, \quad \Pi^2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & i \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & i & 0 & 0 \end{pmatrix}, \quad \Pi^3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & i \\ 0 & 0 & 0 & i & 0 \end{pmatrix}$$

What are the Infinitesimal operators of S0(3,2) in matrix representation ?

$$J^{\alpha\beta} = \begin{pmatrix} 0 & K^1 & K^2 & K^3 & -\Pi^0 \\ -K^1 & 0 & J^3 & -J^2 & -\Pi^1 \\ -K^2 & -J^3 & 0 & J^1 & -\Pi^2 \\ -K^3 & J^2 & -J^1 & 0 & -\Pi^3 \\ \Pi^0 & \Pi^1 & \Pi^2 & \Pi^3 & 0 \end{pmatrix} \qquad g^{\alpha\beta} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad J_{\alpha\beta} = \begin{pmatrix} 0 & -K^1 & -K^2 & -K^3 & -\Pi^0 \\ K^1 & 0 & J^3 & -J^2 & \Pi^1 \\ K^2 & -J^3 & 0 & J^1 & \Pi^2 \\ K^3 & J^2 & -J^1 & 0 & \Pi^3 \\ \Pi^0 & -\Pi^1 & -\Pi^2 & -\Pi^3 & 0 \end{pmatrix}$$

They all satisfy Lie algebra given below.

$$[J^{\alpha\beta}, J^{\gamma\delta}] = i(g^{\beta\gamma}J^{\alpha\delta} - g^{\beta\delta}J^{\alpha\gamma} - g^{\alpha\gamma}J^{\beta\delta} + g^{\alpha\delta}J^{\beta\gamma})$$

Where $\alpha = \beta = \gamma = \delta = 0, 1, 2, 3, 4$

$$\Pi^\mu = J^{4\mu}, \qquad \mu = 0, 1, 2, 3$$

$$\begin{aligned} [J^{\mu\nu}, J^{\rho\lambda}] =& i(g^{\nu\rho}J^{\mu\lambda} - g^{\nu\lambda}J^{\mu\rho} - g^{\mu\rho}J^{\nu\lambda} + g^{\mu\lambda}J^{\nu\rho}) \\ [J^{\mu\nu}, \Pi^\rho] =& i(g^{\nu\rho}\Pi^\mu - g^{\mu\rho}\Pi^\nu) \\ [\Pi^\mu, \Pi^\nu] =& -iJ^{\mu\nu} \end{aligned}$$

Where the range of the indices μ ,ν,γ and δ is 0,1,2,3.

- Contraction of Anti-de Sitter space into Minkowski Space in vector representation

Translation operator acting on the Anti- de Sitter space

$$\Pi = e^{i\, g_{\mu\nu}\frac{a^{\mu}}{l}\Pi^{\nu}}$$

$$\mathrm{a0}^2 - \mathrm{a1}^2 - \mathrm{a2}^2 - \mathrm{a3}^2 = \mathrm{a}^2$$

$$\begin{pmatrix} z^{0\prime} \\ z^{1\prime} \\ z^{2\prime} \\ z^{3\prime} \\ z^{4\prime} \end{pmatrix} = \begin{pmatrix} \frac{a^2 - \mathrm{a0}^2 + \mathrm{a0}^2\mathrm{Cos}\left[\frac{a}{l}\right]}{a^2} & -\frac{\mathrm{a0a1}\left(-1+\mathrm{Cos}\left[\frac{a}{l}\right]\right)}{a^2} & -\frac{\mathrm{a0a2}\left(-1+\mathrm{Cos}\left[\frac{a}{l}\right]\right)}{a^2} & -\frac{\mathrm{a0a3}\left(-1+\mathrm{Cos}\left[\frac{a}{l}\right]\right)}{a^2} & \frac{\mathrm{a0Sin}\left[\frac{a}{l}\right]}{a} \\ \frac{\mathrm{a0a1}\left(-1+\mathrm{Cos}\left[\frac{a}{l}\right]\right)}{a^2} & \frac{a^2 + \mathrm{a1}^2 - \mathrm{a1}^2\mathrm{Cos}\left[\frac{a}{l}\right]}{a^2} & -\frac{\mathrm{a1a2}\left(-1+\mathrm{Cos}\left[\frac{a}{l}\right]\right)}{a^2} & -\frac{\mathrm{a1a3}\left(-1+\mathrm{Cos}\left[\frac{a}{l}\right]\right)}{a^2} & \frac{\mathrm{a1Sin}\left[\frac{a}{l}\right]}{a} \\ \frac{\mathrm{a0a2}\left(-1+\mathrm{Cos}\left[\frac{a}{l}\right]\right)}{a^2} & -\frac{\mathrm{a1a2}\left(-1+\mathrm{Cos}\left[\frac{a}{l}\right]\right)}{a^2} & \frac{a^2 + \mathrm{a2}^2 - \mathrm{a2}^2\mathrm{Cos}\left[\frac{a}{l}\right]}{a^2} & -\frac{\mathrm{a2a3}\left(-1+\mathrm{Cos}\left[\frac{a}{l}\right]\right)}{a^2} & \frac{\mathrm{a2Sin}\left[\frac{a}{l}\right]}{a} \\ \frac{\mathrm{a0a3}\left(-1+\mathrm{Cos}\left[\frac{a}{l}\right]\right)}{a^2} & -\frac{\mathrm{a1a3}\left(-1+\mathrm{Cos}\left[\frac{a}{l}\right]\right)}{a^2} & -\frac{\mathrm{a2a3}\left(-1+\mathrm{Cos}\left[\frac{a}{l}\right]\right)}{a^2} & \frac{a^2 + \mathrm{a3}^2 - \mathrm{a3}^2\mathrm{Cos}\left[\frac{a}{l}\right]}{a^2} & \frac{\mathrm{a3Sin}\left[\frac{a}{l}\right]}{a} \\ -\frac{\mathrm{a0Sin}\left[\frac{a}{l}\right]}{a} & \frac{\mathrm{a1Sin}\left[\frac{a}{l}\right]}{a} & \frac{\mathrm{a2Sin}\left[\frac{a}{l}\right]}{a} & \frac{\mathrm{a3Sin}\left[\frac{a}{l}\right]}{a} & \mathrm{Cos}\left[\frac{a}{l}\right] \end{pmatrix} \begin{pmatrix} z^{0} \\ z^{1} \\ z^{2} \\ z^{3} \\ z^{4} \end{pmatrix}$$

$$\Lambda \to 0\,, \Rightarrow l \to \infty$$

$$P = e^{i\,\eta_{\mu\nu} a^{\mu} P^{\nu}}$$

$$\begin{pmatrix} \mathrm{t} \\ \mathrm{r\,Sin}[\theta]\,\mathrm{Cos}[\varphi] \\ \mathrm{r\,Sin}[\theta]\,\mathrm{Sin}[\varphi] \\ \mathrm{r\,Cos}[\theta] \\ 1 \end{pmatrix} = \begin{pmatrix} x^{0} \\ x^{1} \\ x^{2} \\ x^{3} \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x^{0\prime} \\ x^{1\prime} \\ x^{2\prime} \\ x^{3\prime} \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & a0 \\ 0 & 1 & 0 & 0 & a1 \\ 0 & 0 & 1 & 0 & a2 \\ 0 & 0 & 0 & 1 & a3 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x^{0} \\ x^{1} \\ x^{2} \\ x^{3} \\ 1 \end{pmatrix}$$

How can we contract anti- de Sitter space into Minkowski Space in matrix representation ?

Matrix representation of the five-Dimensional Space-Time

- Gamma matrices

$$\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & -\sigma^i \\ \sigma^i & 0 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

- De sitter Space-time matrix

$$\eta^{\alpha\beta} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\gamma^4 = -i\gamma^5$$

$$(\gamma^4)^2 = -1$$

- Clifford algebra for de sitter space

$$\{\gamma^\alpha, \gamma^\beta\} = \gamma^\alpha\gamma^\beta + \gamma^\beta\,\gamma^\alpha = 2\eta^{\alpha\beta} I$$

$$(\gamma^0)^2 = 1 \qquad (\gamma^i)^2 = -1$$

$$i = 1,2,3,4$$

$$\gamma_\alpha = \eta_{\alpha\beta}\gamma^\beta = \{\gamma^0, -\gamma^1, -\gamma^2, -\gamma^3, -\gamma^4\}$$

Matrix of operators in the de Sitter space

$$R^{\alpha\beta} = \begin{pmatrix} 0 & K^1 & K^2 & K^3 & -\Gamma^0 \\ -K^1 & 0 & J^3 & -J^2 & -\Gamma^1 \\ -K^2 & -J^3 & 0 & J^1 & -\Gamma^2 \\ -K^3 & J^2 & -J^1 & 0 & -\Gamma^3 \\ \Gamma^0 & \Gamma^1 & \Gamma^2 & \Gamma^3 & 0 \end{pmatrix}$$

$$R^{\alpha\beta} = \frac{i}{2}\begin{pmatrix} 0 & \gamma^0\gamma^1 & \gamma^0\gamma^2 & \gamma^0\gamma^3 & \gamma^0\gamma^4 \\ \gamma^1\gamma^0 & 0 & \gamma^1\gamma^2 & \gamma^1\gamma^3 & \gamma^1\gamma^4 \\ \gamma^2\gamma^0 & \gamma^2\gamma^1 & 0 & \gamma^2\gamma^3 & \gamma^2\gamma^4 \\ \gamma^3\gamma^0 & \gamma^3\gamma^1 & \gamma^3\gamma^2 & 0 & \gamma^3\gamma^4 \\ \gamma^4\gamma^0 & \gamma^4\gamma^1 & \gamma^4\gamma^2 & \gamma^4\gamma^3 & 0 \end{pmatrix}$$

- Identify the infinitesimal translation operators of de sitter space in the spinor representation

They all satisfy Lie algebra given below.

$$[R^{\alpha\beta}, R^{\gamma\delta}] = i(\eta^{\beta\gamma}R^{\alpha\delta} - \eta^{\beta\delta}R^{\alpha\gamma} - \eta^{\alpha\gamma}R^{\beta\delta} + \eta^{\alpha\delta}R^{\beta\gamma})$$

Where $\alpha = \beta = \gamma = \delta = 0, 1, 2, 3, 4$

- Coordinates of de Sitter space in the matrix representation

$$y_{\mathrm{m}} = y^{\alpha}_{\mathrm{v},}\gamma_{\alpha} = \begin{pmatrix} \mathrm{iy4} & 0 & \mathrm{y0+y3} & \mathrm{y1-iy2} \\ 0 & \mathrm{iy4} & \mathrm{y1+iy2} & \mathrm{y0-y3} \\ \mathrm{y0-y3} & \mathrm{-y1+iy2} & \mathrm{-iy4} & 0 \\ \mathrm{-y1-iy2} & \mathrm{y0+y3} & 0 & \mathrm{-iy4} \end{pmatrix}$$

Space-time invariant

$$Det[y_{\mathrm{m}}] = (\mathrm{y0}^2 - \mathrm{y1}^2 - \mathrm{y2}^2 - \mathrm{y3}^2 - \mathrm{y4}^2)^2$$

- Transformations in matrix representation for de sitter space $y'_m = y'^{\alpha}_{v,}\gamma_\alpha = T_{m,ds} y_m T^{-1}_{m,ds}$

- Contraction of de Sitter space into Makowski Space in matrix representation

Consider de sitter translation in the matrix representation and take $l \to \infty$ (Infinite de-sitter radius)

$$(y_{\mathrm{m}})_{l\to\infty} = \begin{pmatrix} i\infty & 0 & t + r\mathrm{Cos}[\theta] & \mathrm{e}^{-\mathrm{i}\phi} r\mathrm{Sin}[\theta] \\ 0 & i\infty & \mathrm{e}^{\mathrm{i}\phi} r\mathrm{Sin}[\theta] & t - r\mathrm{Cos}[\theta] \\ t - r\mathrm{Cos}[\theta] & -\mathrm{e}^{-\mathrm{i}\phi} r\mathrm{Sin}[\theta] & -i\infty & 0 \\ -\mathrm{e}^{\mathrm{i}\phi} r\mathrm{Sin}[\theta] & t + r\mathrm{Cos}[\theta] & 0 & -i\infty \end{pmatrix} = \begin{pmatrix} i\infty & 0 & \mathrm{x0}+\mathrm{x3} & \mathrm{x1}-\mathrm{ix2} \\ 0 & i\infty & \mathrm{x1}+\mathrm{ix2} & \mathrm{x0}-\mathrm{x3} \\ \mathrm{x0}-\mathrm{x3} & -\mathrm{x1}+\mathrm{ix2} & -i\infty & 0 \\ -\mathrm{x1}-\mathrm{ix2} & \mathrm{x0}+\mathrm{x3} & 0 & -i\infty \end{pmatrix}$$

$$(y'_{\mathrm{m}})_{l\to\infty} = (\boldsymbol{\Gamma}_m y_m \boldsymbol{\Gamma}_m^{-1})_{l\to\infty}$$

$$= \begin{pmatrix} i\infty & 0 & \mathrm{x0}+\mathrm{a0}+\mathrm{x3}+\mathrm{a3} & \mathrm{x1}+\mathrm{a1}-\mathrm{ix2}-\mathrm{ia2} \\ 0 & i\infty & \mathrm{x1}+\mathrm{a1}+\mathrm{ix2}+ia2 & \mathrm{x0}+\mathrm{a0}-\mathrm{x3}-\mathrm{a3} \\ \mathrm{x0}+\mathrm{a0}-\mathrm{x3}-\mathrm{a3} & -\mathrm{x1}-\mathrm{a1}+\mathrm{ix2}+\mathrm{ia2} & -i\infty & 0 \\ -\mathrm{x1}-\mathrm{a1}-\mathrm{ix2}-ia2 & \mathrm{x0}+\mathrm{a0}+\mathrm{x3}+a3 & 0 & -i\infty \end{pmatrix}$$

- Anti-De sitter Space-time matrix

$$g^{\alpha\beta} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\gamma^{\alpha} = \{\gamma^0, \gamma^1, \gamma^2, \gamma^3, \gamma^5\}$$

$$\gamma_{\alpha} = g_{\alpha\beta}\gamma^{\beta} = \{\gamma^0, -\gamma^1, -\gamma^2, -\gamma^3, \gamma^5\}$$

- Clifford algebra for Anti- de sitter space

$$\{\gamma^{\alpha}, \gamma^{\beta}\} = \gamma^{\alpha}\gamma^{\beta} + \gamma^{\beta}\gamma^{\alpha} = 2g^{\alpha\beta}I$$

$$(\gamma^0)^2 = 1 \qquad (\gamma^i)^2 = -1 \qquad (\gamma^5)^2 = 1$$

$$i = 1,2,3,$$

- Coordinates of Anti-de Sitter space in the matrix representation

$$z_{\mathrm{m}} = z_{\mathrm{v},}^{\alpha}\gamma_{\alpha} = \begin{pmatrix} \mathrm{z4} & 0 & \mathrm{z0} + \mathrm{z3} & \mathrm{z1} - \mathrm{iz2} \\ 0 & \mathrm{z4} & \mathrm{z1} + \mathrm{iz2} & \mathrm{y0} - \mathrm{z3} \\ \mathrm{z0} - \mathrm{z3} & -\mathrm{z1} + \mathrm{iz2} & -\mathrm{z4} & 0 \\ -\mathrm{z1} - \mathrm{iz2} & \mathrm{z0} + \mathrm{z3} & 0 & -\mathrm{z4} \end{pmatrix}$$

Space-time invariant

$$Det[z_{\mathrm{m}}] = (\mathrm{z0}^2 - \mathrm{z1}^2 - \mathrm{z2}^2 - \mathrm{z3}^2 + \mathrm{z4}^2)^2$$

Matrix of operators in the Anti-de Sitter space

$$J^{\alpha\beta} = \begin{pmatrix} 0 & K^1 & K^2 & K^3 & -\Pi^0 \\ -K^1 & 0 & J^3 & -J^2 & -\Pi^1 \\ -K^2 & -J^3 & 0 & J^1 & -\Pi^2 \\ -K^3 & J^2 & -J^1 & 0 & -\Pi^3 \\ \Pi^0 & \Pi^1 & \Pi^2 & \Pi^3 & 0 \end{pmatrix}$$

$$J^{\alpha\beta} = \frac{i}{2}\begin{pmatrix} 0 & \gamma^0\gamma^1 & \gamma^0\gamma^2 & \gamma^0\gamma^3 & \gamma^0\gamma^5 \\ \gamma^1\gamma^0 & 0 & \gamma^1\gamma^2 & \gamma^1\gamma^3 & \gamma^1\gamma^5 \\ \gamma^2\gamma^0 & \gamma^2\gamma^1 & 0 & \gamma^2\gamma^3 & \gamma^2\gamma^5 \\ \gamma^3\gamma^0 & \gamma^3\gamma^1 & \gamma^3\gamma^2 & 0 & \gamma^3\gamma^5 \\ \gamma^5\gamma^0 & \gamma^5\gamma^1 & \gamma^5\gamma^2 & \gamma^5\gamma^3 & 0 \end{pmatrix}$$

- Identify the infinitesimal translation operators of Anti-de sitter space in the spinor representation

They all satisfy Lie algebra given below.

$$[J^{\alpha\beta}, J^{\gamma\delta}] = i(g^{\beta\gamma}J^{\alpha\delta} - g^{\beta\delta}J^{\alpha\gamma} - g^{\alpha\gamma}J^{\beta\delta} + g^{\alpha\delta}J^{\beta\gamma})$$

Where $\alpha = \beta = \gamma = \delta = 0, 1, 2, 3, 4$

- Transformations in matrix representation for de sitter space

$$z'_m = z'^{\alpha}_{v,}\gamma_\alpha = T_{m,Ads} z_m T^{-1}_{m,Ads}$$

$$\gamma_\alpha = g_{\alpha\beta}\gamma^\beta = \{\gamma^0, -\gamma^1, -\gamma^2, -\gamma^3, \gamma^5\}$$

- Contraction of Anti-de Sitter space into Makowski Space in matrix representation

Consider Anti-de sitter translation in the matrix representation and take $l \to \infty$ (Infinite Anti-de-sitter radius)

$$(z_{\mathrm{m}})_{l\to\infty} = \begin{pmatrix} \infty & 0 & t + r\mathrm{Cos}[\theta] & \mathrm{e}^{-\mathrm{i}\phi} r\mathrm{Sin}[\theta] \\ 0 & \infty & \mathrm{e}^{\mathrm{i}\phi} r\mathrm{Sin}[\theta] & t - r\mathrm{Cos}[\theta] \\ t - r\mathrm{Cos}[\theta] & -\mathrm{e}^{-\mathrm{i}\phi} r\mathrm{Sin}[\theta] & -\infty & 0 \\ -\mathrm{e}^{\mathrm{i}\phi} r\mathrm{Sin}[\theta] & t + r\mathrm{Cos}[\theta] & 0 & -\infty \end{pmatrix} = \begin{pmatrix} \infty & 0 & \mathrm{x0} + \mathrm{x3} & \mathrm{x1} - \mathrm{ix2} \\ 0 & \infty & \mathrm{x1} + \mathrm{ix2} & \mathrm{x0} - \mathrm{x3} \\ \mathrm{x0} - \mathrm{x3} & -\mathrm{x1} + \mathrm{ix2} & -\infty & 0 \\ -\mathrm{x1} - \mathrm{ix2} & \mathrm{x0} + \mathrm{x3} & 0 & -\infty \end{pmatrix}$$

$$(z'_{\mathrm{m}})_{l\to\infty} = (\Pi_m z_m \Pi_m^{-1})_{l\to\infty}$$

$$= \begin{pmatrix} \infty & 0 & \mathrm{x0} + \mathrm{a0} + \mathrm{x3} + \mathrm{a3} & \mathrm{x1} + \mathrm{a1} - \mathrm{ix2} - \mathrm{ia2} \\ 0 & \infty & \mathrm{x1} + \mathrm{a1} + \mathrm{ix2} + ia2 & \mathrm{x0} + \mathrm{a0} - \mathrm{x3} - \mathrm{a3} \\ \mathrm{x0} + \mathrm{a0} - \mathrm{x3} - \mathrm{a3} & -\mathrm{x1} - \mathrm{a1} + \mathrm{ix2} + \mathrm{ia2} & -\infty & 0 \\ -\mathrm{x1} - \mathrm{a1} - \mathrm{ix2} - ia2 & \mathrm{x0} + \mathrm{a0} + \mathrm{x3} + a3 & 0 & -\infty \end{pmatrix}$$

$$(y'_{\mathrm{m}})_{l\to\infty} = (\mathbf{\Gamma}_m y_m \mathbf{\Gamma}_m^{-1})_{l\to\infty}$$

$$= \begin{pmatrix} i\infty & 0 & \mathrm{x0} + \mathrm{a0} + \mathrm{x3} + \mathrm{a3} & \mathrm{x1} + \mathrm{a1} - \mathrm{ix2} - \mathrm{ia2} \\ 0 & i\infty & \mathrm{x1} + \mathrm{a1} + \mathrm{ix2} + ia2 & \mathrm{x0} + \mathrm{a0} - \mathrm{x3} - \mathrm{a3} \\ \mathrm{x0} + \mathrm{a0} - \mathrm{x3} - \mathrm{a3} & -\mathrm{x1} - \mathrm{a1} + \mathrm{ix2} + \mathrm{ia2} & -i\infty & 0 \\ -\mathrm{x1} - \mathrm{a1} - \mathrm{ix2} - ia2 & \mathrm{x0} + \mathrm{a0} + \mathrm{x3} + a3 & 0 & -i\infty \end{pmatrix}$$

Conclusion

- Each gamma matrix has its own physical meaning that connect with the dimension and probably with the geometry of the space.

- That dimensional analogue help us to find the relationship between the vector representation and matrix representation for 3D , 4D as well as the 5D.

- We show the contraction of De Sitter and Anti-de Sitter space into the Minkowski space using the vanishing cosmological method in even in the matrix representation. But we should handle infinities more carefully.

Future work

- Find the interpolating transformations between de-Sitter and anti-de Sitter spaces' instant form and their respective horizons.
- Find the analogue between the extended Poincare matrix with translation operator and electromagnetic field strength tensor with gravity as a scalar field. (Related to the Kaluza -Klein theory)