# Tackling the bad current in the light front dynamics

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Dr. Ji's group meeting

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# Scalar meson $\to \gamma^*\gamma^*$ Transition Form Factor in 1+1-d scalar model: Manifestly covariant calculation

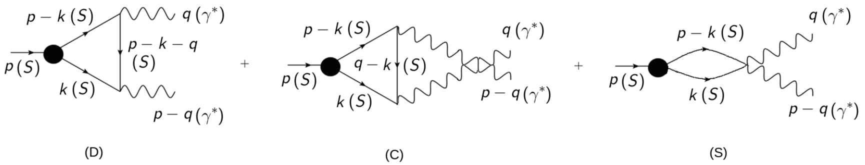



Figure: One-loop covariant Feynman Diagrams that contribute to the $S \to \gamma^*\gamma^*$ transition form factor

The total amplitude consists of these three Feynman diagrams, i.e., the direct (D), crossed (C), and the seagull (S) diagrams, where $p$ is the momentum of the incident scalar meson, while $q$ is the momentum of the emitted photon. As a result of momentum conservation, $q' = p - q$ is the momentum of the final state photon.

From gauge invariance argument, we can know that the total amplitude $\Gamma^{\mu\nu}$ is of the form

$$\Gamma^{\mu\nu} = F(q^2, q'^2) \left( g^{\mu\nu} q \cdot q' - q'^{\mu} q^{\nu} \right), \tag{1}$$

which satisfies both

$$q_{\mu} \left( g^{\mu\nu} q \cdot q' - q'^{\mu} q^{\nu} \right) = 0 \tag{2}$$

and

$$q'_{\nu} \left( g^{\mu\nu} q \cdot q' - q'^{\mu} q^{\nu} \right) = 0, \tag{3}$$

so that the form factor can be obtained by

$$F(q^2, q'^2) = \frac{\Gamma^{\mu\nu}}{g^{\mu\nu} q \cdot q' - q'^{\mu} q^{\nu}}. \tag{4}$$

The amplitude $\Gamma^{\mu\nu}$ is calculated as such, following the Feynman rules for the scalar field theory.

$$\begin{aligned}
\Gamma^{\mu\nu} =& \Gamma_D^{\mu\nu} + \Gamma_C^{\mu\nu} + \Gamma_S^{\mu\nu} \\
=& ie^2 g_s \int \frac{d^2k}{(2\pi)^2} \left\{ \frac{(2p-2k-q)^\mu (p-2k-q)^\nu}{((p-k-q)^2-m^2)((p-k)^2-m^2)(k^2-m^2)} \right. \\
&+ \frac{(q-2k)^\mu (p-2k+q)^\nu}{((p-k)^2-m^2)(k^2-m^2)((q-k)^2-m^2)} \\
&+ \left. \frac{-2g^{\mu\nu}}{((p-k)^2-m^2)(k^2-m^2)} \right\},
\end{aligned} \tag{5}$$

where the coupling constant of the simple scalar model $g_s$ is fixed from the normalization condition. For simplicity, we take all the intermediate scalar particles' mass to be $m$ and their charge to be $e$, but it can be easily generalized to unequal mass/charge cases. The initial scalar meson has mass $M$.

We finally obtain

$$F(q^2, q'^2) = \frac{e^2 g_s}{4\pi} \int_0^1 dx \int_0^{1-x} dy (1-2y) \left( \frac{1}{\Delta_1^2} + \frac{1}{\Delta_2^2} \right), \qquad (6)$$

where

$$\Delta_1 = x(x-1)q^2 + 2x(x+y-1)q \cdot q' + (x+y)(x+y-1)q'^2 + m^2, \quad (7)$$

$$\Delta_2 = x(x-1)q'^2 + 2x(x+y-1)q \cdot q' + (x+y)(x+y-1)q^2 + m^2. \quad (8)$$

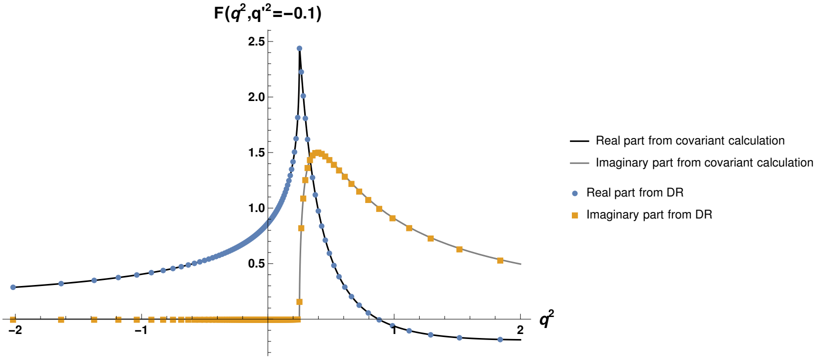
Doing the $x$ and $y$ integrations, we get the analytic formula for the transition form factor,

$$F(q^2, q'^2) = \frac{e^2 g_s}{4\pi} \times$$

$$\frac{(2-\omega-\gamma'-\gamma)\frac{\sqrt{\omega}}{\sqrt{1-\omega}}\tan^{-1}\left(\frac{\sqrt{\omega}}{\sqrt{1-\omega}}\right) + (\gamma-\gamma'-\omega)\frac{\sqrt{1-\gamma'}}{\sqrt{\gamma'}}\tan^{-1}\left(\frac{\sqrt{\gamma'}}{\sqrt{1-\gamma'}}\right) + (\gamma'-\gamma-\omega)\frac{\sqrt{1-\gamma}}{\sqrt{\gamma}}\tan^{-1}\left(\frac{\sqrt{\gamma}}{\sqrt{1-\gamma}}\right)}{m^4\left[4\omega\gamma'\gamma + \omega^2 + (\gamma'-\gamma)^2 - 2\omega(\gamma'+\gamma)\right]}, \qquad (9)$$

where $\gamma = \frac{q^2}{4m^2}$, $\gamma' = \frac{q'^2}{4m^2}$, and $\omega = \frac{M^2}{4m^2}$.

Now, taking $m = 0.25\ GeV$, $M = 0.14\ GeV$, and normalizing the form factor so that $F(q^2 = 0, q'^2 = 0) = 1$ (thus fixing $g_s$), and taking the value of $q'^2 = -0.1\ GeV^2$, we show below the numerical results of the form factor as a function of $q^2$. The agreement of the lines with the dots show the agreement of our result with the Dispersion Relation (DR)



# Scalar meson $\to \gamma^*\gamma^*$ Transition Form Factor in 1+1-d scalar model: LFTO calculation

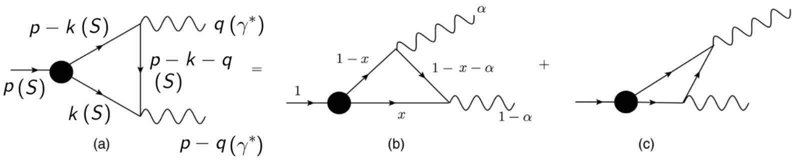



Figure: (Take the direct diagram as an example). The covariant diagram (a) is sum of the two LF $x^+$-ordered diagrams (b) and (c).

If one assumes each individual LFTO diagram contribution is of the gauge invariant form, i.e. $\Gamma_i^{\mu\nu} = f_i(q^2, q'^2)\,(g^{\mu\nu} q\cdot q' - q'^{\mu} q^{\nu})$, one can obtain the LFTO contributions by calculating just the plus-plus current:
$f_{(b)} = \frac{\Gamma^{++}_{(b)}}{g^{++} q\cdot q' - q'^{+} q^{+}}$, $f_{(c)} = \frac{\Gamma^{++}_{(c)}}{g^{++} q\cdot q' - q'^{+} q^{+}}$.

However, this is not a meaningful representation of how much each LFTO diagram contributes to the total form factor, since it depends on the component. So actually,

$$f_{(b)}^{(++)} = \frac{\Gamma_{(b)}^{++}}{g^{++} q \cdot q' - q'^{+} q^{+}}, \quad f_{(c)}^{(++)} = \frac{\Gamma_{(c)}^{++}}{g^{++} q \cdot q' - q'^{+} q^{+}}; \tag{10}$$

$$f_{(b)}^{(+-)} = \frac{\Gamma_{(b)}^{+-}}{g^{+-} q \cdot q' - q'^{+} q^{-}}, \quad f_{(c)}^{(+-)} = \frac{\Gamma_{(c)}^{+-}}{g^{+-} q \cdot q' - q'^{+} q^{-}}; \tag{11}$$

$$f_{(b)}^{(-+)} = \frac{\Gamma_{(b)}^{-+}}{g^{-+} q \cdot q' - q'^{-} q^{+}}, \quad f_{(c)}^{(-+)} = \frac{\Gamma_{(c)}^{-+}}{g^{-+} q \cdot q' - q'^{-} q^{+}}; \tag{12}$$

$$f_{(b)}^{(--)} = \frac{\Gamma_{(b)}^{--}}{g^{--} q \cdot q' - q'^{-} q^{-}}, \quad f_{(c)}^{(--)} = \frac{\Gamma_{(c)}^{--}}{g^{--} q \cdot q' - q'^{-} q^{-}}; \tag{13}$$

and each of these are different.

Usually people just look at the “good current” only, and call $f_{(b)}^{(++)}$ and $f_{(c)}^{(++)}$ the LFTO contributions to the form factor, or similarly, the DGLAP and ERBL regions of GPD. So that is the motivation for us to look at other components. We do this so-called “theoretical simulation” in this simple model.

Last time I showed a lot of failed attempts to get the “$--$” component, including the method which I thought it worked, but turned out that also failed. But now we finally obtained it. For real this time.

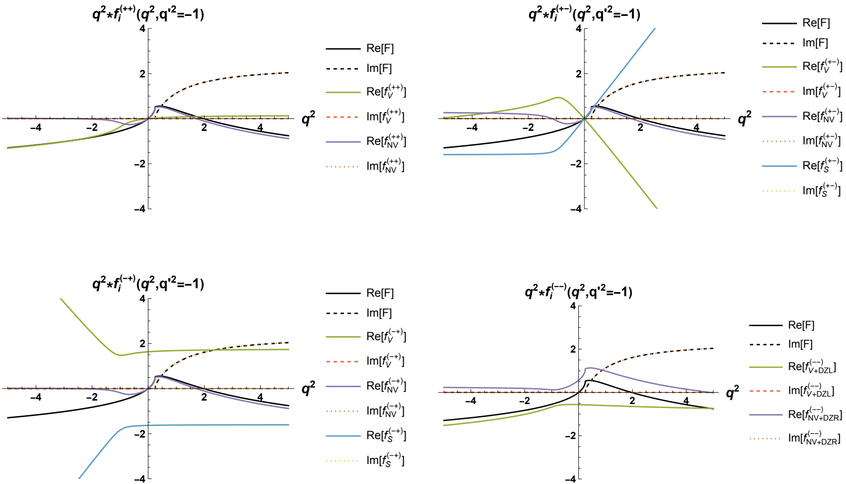



Figure: LFTO diagram contributions to the transition form factor for all 4 components.

# Scalar meson $\to \gamma^*\gamma^*$ Transition Form Factor in 1+1-d scalar model: "$--$" component of LFTO calculation

The minus minus component of the transition amplitude is

$$\begin{aligned}\Gamma^{--} &= \Gamma_D^{--} + \Gamma_C^{--} \\ &= ie^2 g_s \int \frac{d^2k}{(2\pi)^2} \frac{(2p-2k-q)^-\,(p-2k-q)^-}{((p-k-q)^2-m^2)\,((p-k)^2-m^2)\,(k^2-m^2)} \\ &+ ie^2 g_s \int \frac{d^2k}{(2\pi)^2} \frac{(q-2k)^-\,(p-2k+q)^-}{((p-k)^2-m^2)\,(k^2-m^2)\,((q-k)^2-m^2)}. \end{aligned} \tag{14}$$

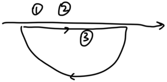
There is no seagull term contribution for the $--$ current. Now plugging in the kinematics, we get ( we just show the direct diagram calculation, the crossed diagram is very much same calculation)

$$\begin{aligned}\Gamma_D^{--} = \frac{ie^2 g_s}{4\pi^2} \int dk^+ \int dk^- &\left(\frac{M^2}{2p^+} - 2k^- + \frac{q'^2}{2(1-\alpha)p^+}\right)\left(-2k^- + \frac{q'^2}{2(1-\alpha)p^+}\right) \\ &\cdot \left(2(p-k-q)^+(p-k-q)^- - m^2\right)^{-1}\left(2(p-k)^+(p-k)^- - m^2\right)^{-1} \\ &\cdot \left(2k^+k^- - m^2\right)^{-1}. \end{aligned} \tag{15}$$

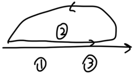
Now let us do the integration $\int dk^-$. There are 3 poles

$$
\begin{aligned}
k_1^- &= p^- - q^- - \frac{m^2 - i\varepsilon}{2(p-k-q)^+} \\
k_2^- &= p^- - \frac{m^2 - i\varepsilon}{2(p-k)^+} \\
k_3^- &= \frac{m^2 - i\varepsilon}{2k^+}
\end{aligned}
\tag{16}
$$

Region (b): $0 < x < 1-\alpha < 1$




Region (c): $0 < 1-\alpha < x < 1$




where the $k_2^-$ pole is located at the upper half plane, $k_3^-$ pole is at lower half plane, while $k_1^-$ pole depending on the sign of $1-x-\alpha$, when $1-x-\alpha > 0$, it is at upper plane, and we call this region (b). For region (b), we enclose the contour for lower half plane and catch pole 3. When $1-x-\alpha < 0$, it is at lower plane, and we call this region (c). For region (c), we enclose the contour for upper half plane and catch pole 2.

Calculating the pole residue, we get

$$\begin{aligned}
\Gamma^{--}_{D(b)} &= \frac{ie^2 g_s}{4\pi^2}(-2\pi i)\int dk^+ \left(\frac{M^2}{2p^+} + \frac{q'^2}{2(1-\alpha)p^+} - 2k_3^-\right)\left(\frac{q'^2}{2(1-\alpha)p^+} - 2k_3^-\right) \\
&\cdot \left[2\left(p-k-q\right)^+ 2\left(p-k\right)^+ 2k^+ \left(k_3^- - k_1^-\right)\left(k_3^- - k_2^-\right)\right]^{-1} \\
&= \frac{e^2 g_s}{2\pi} p^+ \int_0^{1-\alpha} dx \left(\frac{M^2}{2p^+} + \frac{q'^2}{2(1-\alpha)p^+} - \frac{m^2}{xp^+}\right)\left(\frac{q'^2}{2(1-\alpha)p^+} - \frac{m^2}{xp^+}\right) \\
&\cdot \Big[2p^+\left(1-x-\alpha\right)2p^+\left(1-x\right)2p^+ x \\
&\cdot \frac{1}{2p^+}\left(\frac{m^2}{x} - \frac{q'^2}{1-\alpha} + \frac{m^2}{1-x-\alpha}\right)\frac{1}{2p^+}\left(\frac{m^2}{x} - M^2 + \frac{m^2}{1-x}\right)\Big]^{-1} \\
&= \frac{e^2 g_s}{4\pi p^+ p^+}\int_0^{1-\alpha} dx \left(\frac{M^2}{2} + \frac{q'^2}{2(1-\alpha)} - \frac{m^2}{x}\right)\left(\frac{q'^2}{2(1-\alpha)} - \frac{m^2}{x}\right) \\
&\cdot \left[(1-x-\alpha)(1-x)x\left(\frac{m^2}{x} + \frac{m^2}{1-x-\alpha} - \frac{q'^2}{1-\alpha}\right)\left(\frac{m^2}{x} + \frac{m^2}{1-x} - M^2\right)\right]^{-1}
\end{aligned} \tag{17}$$

and

$$
\begin{aligned}
\Gamma_{D(c)}^{--} &= \frac{ie^2 g_s}{4\pi^2}(2\pi i)\int dk^+ \left(\frac{M^2}{2p^+} + \frac{q'^2}{2(1-\alpha)p^+} - 2k_2^-\right)\left(\frac{q'^2}{2(1-\alpha)p^+} - 2k_2^-\right) \\
&\cdot \left[2\left(p-k-q\right)^+ 2\left(p-k\right)^+ 2k^+ \left(k_2^- - k_1^-\right)\left(k_2^- - k_3^-\right)\right]^{-1} \\
&= \frac{-e^2 g_s}{2\pi} p^+ \int_{1-\alpha}^{1} dx \left(\frac{M^2}{2p^+} + \frac{q'^2}{2(1-\alpha)p^+} - \frac{M^2}{p^+} + \frac{m^2}{(1-x)p^+}\right) \\
&\cdot \left(\frac{q'^2}{2(1-\alpha)p^+} - \frac{M^2}{p^+} + \frac{m^2}{(1-x)p^+}\right) \\
&\cdot \Bigg[2p^+\left(1-x-\alpha\right) 2p^+\left(1-x\right) 2p^+ x \\
&\cdot \frac{1}{2p^+}\left(-\frac{m^2}{1-x} + M^2 - \frac{q'^2}{1-\alpha} + \frac{m^2}{1-x-\alpha}\right)\frac{1}{2p^+}\left(M^2 - \frac{m^2}{1-x} - \frac{m^2}{x}\right)\Bigg]^{-1} \\
&= \frac{e^2 g_s}{4\pi p^+ p^+}\int_{1-\alpha}^{1} dx \left(\frac{q'^2}{2(1-\alpha)} - \frac{M^2}{2} + \frac{m^2}{1-x}\right)\left(\frac{q'^2}{2(1-\alpha)} - M^2 + \frac{m^2}{1-x}\right) \\
&\cdot \left[\left(1-x-\alpha\right)\left(1-x\right)x\left(\frac{m^2}{1-x-\alpha} - \frac{m^2}{1-x} + M^2 - \frac{q'^2}{1-\alpha}\right)\left(\frac{m^2}{1-x} + \frac{m^2}{x} - M^2\right)\right]
\end{aligned}
\tag{18}
$$

However, if one adds all the LFTO contributions, one finds that it does not agree with the covariant result.

Upon inspection, we realize that for this "minus-minus" component case, there is enough power of $k^-$ on the numerator for the contour integration to have contribution from the arc. Subtracting the arc from the residue gives

$$\Gamma^{--}_{D(b)} = \frac{e^2 g_s}{4\pi p^+ p^+} \int_0^{1-\alpha} dx \left\{ \left( \frac{M^2}{2} + \frac{q'^2}{2(1-\alpha)} - \frac{m^2}{x} \right) \left( \frac{q'^2}{2(1-\alpha)} - \frac{m^2}{x} \right) \right.$$
$$\cdot \left[ (1-x-\alpha)(1-x)x \left( \frac{m^2}{x} + \frac{m^2}{1-x-\alpha} - \frac{q'^2}{1-\alpha} \right) \left( \frac{m^2}{x} + \frac{m^2}{1-x} - M^2 \right) \right]^{-1}$$
$$\left. - \frac{1}{2(1-x-\alpha)(1-x)x} \right\}, \tag{19}$$

and

$$\Gamma^{--}_{D(c)} = \frac{e^2 g_s}{4\pi p^+ p^+} \int_{1-\alpha}^{1} dx \left\{ \left( \frac{q'^2}{2(1-\alpha)} - \frac{M^2}{2} + \frac{m^2}{1-x} \right) \left( \frac{q'^2}{2(1-\alpha)} - M^2 + \frac{m^2}{1-x} \right) \right.$$
$$\cdot \left[ (1-x-\alpha)\,(1-x)\,x \left( \frac{m^2}{1-x-\alpha} - \frac{m^2}{1-x} + M^2 - \frac{q'^2}{1-\alpha} \right) \left( \frac{m^2}{1-x} + \frac{m^2}{x} - M^2 \right) \right]$$
$$\left. + \frac{1}{2(1-x-\alpha)(1-x)x} \right\}. \tag{20}$$

But, this still results in disagreement between the "minus-minus" component calculation and the covariant one.

# Interpolating between the IFD and LFD

To quantify the contributions at $k^- \to \infty$ accurately, we introduce the interpolation method.

$$\begin{bmatrix} x^{\hat{+}} \\ x^{\hat{-}} \end{bmatrix} = \begin{bmatrix} \cos\delta & \sin\delta \\ \sin\delta & -\cos\delta \end{bmatrix} \begin{bmatrix} x^0 \\ x^1 \end{bmatrix}, \tag{21}$$

in which the interpolation angle is allowed to run from 0 through 45°, $0 \le \delta \le \frac{\pi}{4}$. The lower index variables $x_{\hat{+}}$ and $x_{\hat{-}}$ are related to the upper index variables as $x_{\hat{+}} = g_{\hat{+}\hat{\mu}} x^{\hat{\mu}} = \mathbb{C} x^{\hat{+}} + \mathbb{S} x^{\hat{-}}$ and $x_{\hat{-}} = g_{\hat{-}\hat{\mu}} x^{\hat{\mu}} = -\mathbb{C} x^{\hat{-}} + \mathbb{S} x^{\hat{+}}$, denoting $\mathbb{C} = \cos 2\delta$ and $\mathbb{S} = \sin 2\delta$ and realizing $g_{\hat{+}\hat{+}} = -g_{\hat{-}\hat{-}} = \cos 2\delta = \mathbb{C}$ and $g_{\hat{+}\hat{-}} = g_{\hat{-}\hat{+}} = \sin 2\delta = \mathbb{S}$. All the indices with the hat notation signify the variables with the interpolation angle $\delta$. For the limit $\delta \to 0$ we have $x^{\hat{+}} = x^0$ and $x^{\hat{-}} = -x^1$ so that we recover usual space-time coordinates although the $z$-axis is inverted while for the other extreme limit, $\delta \to \frac{\pi}{4}$, we have $x^{\hat{\pm}} = (x^0 \pm x^1)/\sqrt{2} = x^{\pm}$ which leads to the standard light-front coordinates.

In the interpolation form, the 3 denominators of Eq. (15) can be rewritten as

$$D_1 = \mathbb{C}\left(p_{\hat{+}} - k_{\hat{+}} - q_{\hat{+}}\right)^2 + 2\mathbb{S}\left(p_{\hat{+}} - k_{\hat{+}} - q_{\hat{+}}\right)\left(p_{\hat{-}} - k_{\hat{-}} - q_{\hat{-}}\right) - \mathbb{C}\left(p_{\hat{-}} - k_{\hat{-}} - q_{\hat{-}}\right)^2 - m^2 + i\varepsilon, \tag{22}$$

$$D_2 = \mathbb{C}\left(p_{\hat{+}} - k_{\hat{+}}\right)^2 + 2\mathbb{S}\left(p_{\hat{+}} - k_{\hat{+}}\right)\left(p_{\hat{-}} - k_{\hat{-}}\right) - \mathbb{C}\left(p_{\hat{-}} - k_{\hat{-}}\right)^2 - m^2 + i\varepsilon, \tag{23}$$

and

$$D_3 = \mathbb{C}k_{\hat{+}}^2 + 2\mathbb{S}k_{\hat{+}}k_{\hat{-}} - \mathbb{C}k_{\hat{-}}^2 - m^2 + i\varepsilon. \tag{24}$$

There are 6 poles in total

$$k_{\hat{+}1,1'} = p_{\hat{+}} - q_{\hat{+}} + \frac{\mathbb{S}}{\mathbb{C}}(p_{\hat{-}} - k_{\hat{-}} - q_{\hat{-}}) \mp \frac{\omega_1}{\mathbb{C}} \pm i\varepsilon \tag{25}$$

$$k_{\hat{+}2,2'} = p_{\hat{+}} + \frac{\mathbb{S}}{\mathbb{C}}(p_{\hat{-}} - k_{\hat{-}}) \mp \frac{\omega_2}{\mathbb{C}} \pm i\varepsilon \tag{26}$$

$$k_{\hat{+}3,3'} = -\frac{\mathbb{S}}{\mathbb{C}}k_{\hat{-}} \pm \frac{\omega_3}{\mathbb{C}} \mp i\varepsilon \tag{27}$$

where

$$\omega_1 = \sqrt{(p_{\hat{-}} - k_{\hat{-}} - q_{\hat{-}})^2 + \mathbb{C}m^2} \tag{28}$$

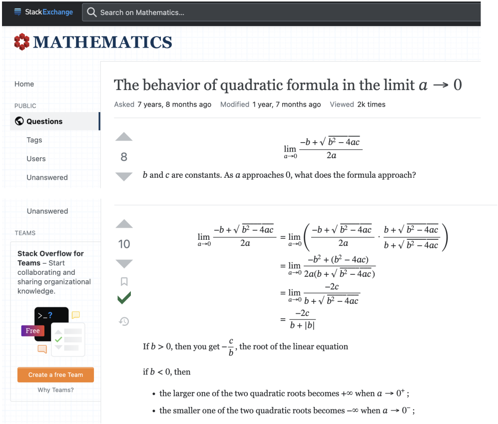
$$\omega_2 = \sqrt{(p_{\hat{-}} - k_{\hat{-}})^2 + \mathbb{C}m^2} \tag{29}$$

$$\omega_3 = \sqrt{k_{\hat{-}}^2 + \mathbb{C}m^2} \tag{30}$$

In the $\mathbb{C} \to 0$ limit, in each pair of the poles, one of them goes to infinity, the other goes to the light-front poles, Eq. (16).
We know the behavior of solutions to the quadratic equation

$$ax^2 + bx + c = 0 \tag{31}$$

depends on the sign of $b$.



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## The behavior of quadratic formula in the limit $a \to 0$

Asked 7 years, 8 months ago Modified 1 year, 7 months ago Viewed 2k times

8

$$\lim_{a\to 0} \frac{-b+\sqrt{b^2-4ac}}{2a}$$

$b$ and $c$ are constants. As $a$ approaches 0, what does the formula approach?

10

$$\begin{aligned}\lim_{a\to 0} \frac{-b+\sqrt{b^2-4ac}}{2a} &= \lim_{a\to 0}\left(\frac{-b+\sqrt{b^2-4ac}}{2a}\cdot\frac{b+\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)\\ &= \lim_{a\to 0}\frac{-b^2+(b^2-4ac)}{2a(b+\sqrt{b^2-4ac})}\\ &= \lim_{a\to 0}\frac{-2c}{b+\sqrt{b^2-4ac}}\\ &= \frac{-2c}{b+|b|}\end{aligned}$$

If $b>0$, then you get $-\frac{c}{b}$, the root of the linear equation

if $b<0$, then

- the larger one of the two quadratic roots becomes $+\infty$ when $a \to 0^+$ ;
- the smaller one of the two quadratic roots becomes $-\infty$ when $a \to 0^-$ ;

Keeping the limit to the light-front in mind, we keep the relevant terms in the pole values at infinity taking limits such as $\mathbb{S} \to 1$, $k_{\hat{-}} \to k^+$ etc. already, while for the regular poles we substitute Eq. (16). The 6 poles become

$$k^-_{1reg} = p^- - q^- - \frac{m^2 - i\varepsilon}{2(p-k-q)^+} \tag{32}$$

$$k^-_{1inf} = \frac{p^+ - k^+ - q^+}{\mathbb{C}} \mp \frac{|p^+ - k^+ - q^+|}{\mathbb{C}} \pm i\varepsilon \tag{33}$$

$$k^-_{2reg} = p^- - \frac{m^2 - i\varepsilon}{2(p-k)^+} \tag{34}$$

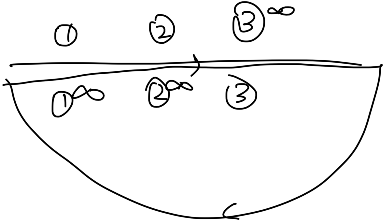
$$k^-_{2inf} = \frac{p^+ - k^+}{\mathbb{C}} \mp \frac{|p^+ - k^+|}{\mathbb{C}} \pm i\varepsilon \tag{35}$$

$$k^-_{3reg} = \frac{m^2 - i\varepsilon}{2k^+} \tag{36}$$

$$k^-_{3inf} = -\frac{k^+}{\mathbb{C}} \pm \frac{|k^+|}{\mathbb{C}} \mp i\varepsilon. \tag{37}$$

For the case of (b) time-ordering, we have $0 < x < 1 - \alpha < 1$. So, Eq. (32) is located at upper half plane, Eq. (33) takes the lower sign and is located at the lower half plane. Eq. (34) is located at upper half plane, Eq. (35) takes the lower sign and is located at the lower half plane. Eq. (36) is located at lower half plane, Eq. (37) takes the lower sign and is located at the upper half plane. Now, we actually have 3 poles located at the lower half plane, instead of just one pole in the naive light-front calculation.

Region (b) : $0 < x < 1 - \alpha < 1$



Let us now calculate the residues of $k^-_{1inf}$ and $k^-_{2inf}$. The residue of $k^-_{1inf}$ is

$$\frac{4\left(2(p^+ - k^+ - q^+) + \mathcal{O}(\mathbb{C})\right)^2}{\left(2(p^+ - k^+ - q^+) + \mathcal{O}(\mathbb{C})\right)\left(\left(2(p^+ - k^+ - q^+) + \mathcal{O}(\mathbb{C})\right)^2 + 2(p^+ - k^+)\left(-2(p^+ - k^+ - q^+)\right) + \mathcal{O}(\mathbb{C})\right)\left(\left(2(p^+ - k^+ - q^+) + \mathcal{O}(\mathbb{C})\right)^2 + 2k^+\left(2(p^+ - k^+ - q^+)\right) + \mathcal{O}(\mathbb{C})\right)} \tag{38}$$

$$= \frac{4}{2(p^+ - k^+ - q^+)(-2q^+)(2(p^+ - q^+))} \tag{39}$$

The residue of $k^-_{2inf}$ is

$$\frac{4\left(2(p^+ - k^+) + \mathcal{O}(\mathbb{C})\right)^2}{\left(2(p^+ - k^+) + \mathcal{O}(\mathbb{C})\right)\left(\left(2(p^+ - k^+) + \mathcal{O}(\mathbb{C})\right)^2 + 2(p^+ - k^+ - q^+)\left(-2(p^+ - k^+)\right) + \mathcal{O}(\mathbb{C})\right)\left(\left(2(p^+ - k^+) + \mathcal{O}(\mathbb{C})\right)^2 + 2k^+\left(2(p^+ - k^+)\right) + \mathcal{O}(\mathbb{C})\right)} \tag{40}$$

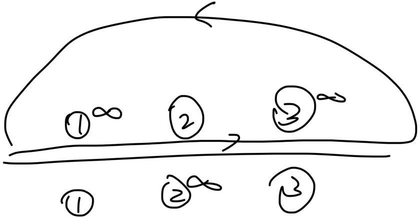
$$= \frac{4}{2(p^+ - k^+)(2q^+)(2p^+)}. \tag{41}$$

Adding the residues from all 3 poles on the lower half plane, we get

$$\Gamma^{--}_{D(b)} = \frac{e^2 g_s}{4\pi p^+ p^+} \int_0^{1-\alpha} dx \left\{ \left( \frac{M^2}{2} + \frac{q'^2}{2(1-\alpha)} - \frac{m^2}{x} \right) \left( \frac{q'^2}{2(1-\alpha)} - \frac{m^2}{x} \right) \right.$$
$$\cdot \left[ (1-x-\alpha)(1-x)x \left( \frac{m^2}{x} + \frac{m^2}{1-x-\alpha} - \frac{q'^2}{1-\alpha} \right) \left( \frac{m^2}{x} + \frac{m^2}{1-x} - M^2 \right) \right]^{-1}$$
$$\left. + \frac{1}{(1-x-\alpha)(-\alpha)(1-\alpha)} + \frac{1}{(1-x)\alpha} \right\}. \tag{42}$$

Now similarly, for the case of (c) time-ordering, we have $0 < 1 - \alpha < x < 1$. So, Eq. (32) is located at lower half plane, Eq. (33) takes the upper sign and is located at the upper half plane. Eq. (34) is still located at upper half plane, Eq. (35) still takes the lower sign and is located at the lower half plane. Eq. (36) is still located at lower half plane, Eq. (37) still takes the lower sign and is located at the upper half plane.

Region (c) : $0<1-\alpha<x<1$



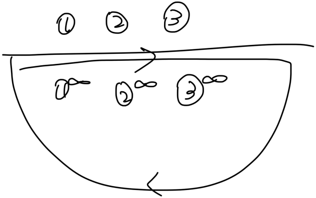
Now, in addition to the regular pole residue, we need the contributions from the 2 poles at infinity: $k^-_{1inf}$ and $k^-_{3inf}$. Similar to the (b) time-ordering case, we calculate the residues from these 2 poles at infinity and add them to the regular pole residue, which is the naive answer we already obtained on the light-front. The residues from all 3 poles on the upper half plane gives

$$\begin{aligned}\Gamma^{--}_{D(c)} = \frac{e^2 g_s}{4\pi p^+ p^+}\int_{1-\alpha}^{1} dx \Bigg\{ & \left(\frac{q'^2}{2(1-\alpha)} - \frac{M^2}{2} + \frac{m^2}{1-x}\right)\left(\frac{q'^2}{2(1-\alpha)} - M^2 + \frac{m^2}{1-x}\right) \\ & \cdot \left[(1-x-\alpha)(1-x)x\left(\frac{m^2}{1-x-\alpha} - \frac{m^2}{1-x} + M^2 - \frac{q'^2}{1-\alpha}\right)\left(\frac{m^2}{1-x} + \frac{m^2}{x} - M^2\right)\right] \\ & - \frac{1}{(1-x-\alpha)(-\alpha)(1-\alpha)} - \frac{1}{(-x)(1-\alpha)}\Bigg\}. \end{aligned} \quad (43)$$

Now, adding these two time-ordered contributions still does not give agreement to the covariant result. That is because we have forgotten the regions of $x < 0$ and $x > 1$. When $\mathbb{C} \neq 0$, $x$ is not limited to the region $[0, 1]$, and these two regions outside of $[0, 1]$ do give contributions.

When $x < 0$ (and thus necessarily $< 1 - \alpha$ because $\alpha \in [0, 1]$), Eq. (32) is located at upper half plane, Eq. (33) takes the lower sign and is located at the lower half plane. Eq. (34) is located at upper half plane, Eq. (35) takes the lower sign and is located at the lower half plane. But now Eq. (36) is located at upper half plane, Eq. (37) takes the upper sign and is located at the lower half plane. The 3 regular poles are all on the same half plane, thus in the naive limit of the light-front dynamics, one concludes that the contribution is zero, but now we see that there are 3 "poles at infinity" on the lower half plane.

region of $x \in (-\infty, 0)$



Calculating their residues, we obtain

$$\Gamma^{--}_{D(DZL)} = \frac{e^2 g_s}{4\pi p^+ p^+} \int_{-\infty}^{0} dx \left\{ \frac{1}{(1-x-\alpha)(-\alpha)(1-\alpha)} + \frac{1}{(1-x)\alpha} + \frac{1}{(-x)(1-\alpha)} \right\}. \tag{44}$$

Similarly for the region of $x > 1$.


We obtain

$$\Gamma^{--}_{D(DZR)} = \frac{e^2 g_s}{4\pi p^+ p^+} \int_{1}^{+\infty} dx \left\{ -\frac{1}{(1-x-\alpha)(-\alpha)(1-\alpha)} - \frac{1}{(1-x)\alpha} - \frac{1}{(-x)(1-\alpha)} \right\}. \tag{45}$$

Now finally, adding the 4 time-ordered diagrams altogether, we reached agreement between the "minus-minus" component form factor calculation and the covariant one.

# The double zero mode in terms of the time-ordered diagrams

In Equal-Time dynamics, the 3 denominator can be written as

covariant amplitude $\sim \dfrac{1}{[(p-k-q)^2-m^2+i\varepsilon][(p-k)^2-m^2+i\varepsilon][k^2-m^2+i\varepsilon]}$

$$= \frac{1}{-2(\omega_1-i\varepsilon)}\left(\frac{1}{k^0-p^0+q^0+\omega_1-i\varepsilon}-\frac{1}{k^0-p^0+q^0-\omega_1+i\varepsilon}\right)$$

$$\frac{1}{-2(\omega_2-i\varepsilon)}\left(\frac{1}{k^0-p^0+\omega_2-i\varepsilon}-\frac{1}{k^0-p^0-\omega_2+i\varepsilon}\right)$$

$$\frac{1}{-2(\omega_3-i\varepsilon)}\left(\frac{1}{k^0+\omega_3-i\varepsilon}-\frac{1}{k^0-\omega_3+i\varepsilon}\right)$$

where

$$\omega_1=\sqrt{m^2+(p^1-k^1-q^1)^2}$$

$$\omega_2=\sqrt{m^2+(p^1-k^1)^2}$$

$$\omega_3=\sqrt{m^2+(k^1)^2}$$

because $\omega$'s are squareroots ($>0$)

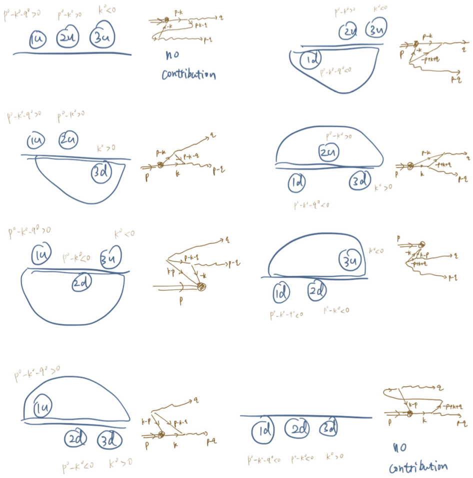
The 6 poles are

this means

| | | |
|---|---|---|
| $1u = p^0 - q^0 - \omega_1 + i\varepsilon$ | $(k^0 - p^0 + q^0 < 0)$ | ($p-k-q$ propagates forward) |
| $1d = p^0 - q^0 + \omega_1 - i\varepsilon$ | $(k^0 - p^0 + q^0 > 0)$ | ($p-k-q$ propagates backward) |
| $2u = p^0 - \omega_2 + i\varepsilon$ | $(k^0 - p^0 < 0)$ | ($p-k$ propagates forward) |
| $2d = p^0 + \omega_2 - i\varepsilon$ | $(k^0 - p^0 > 0)$ | ($p-k$ propagates backward) |
| $3u = -\omega_3 + i\varepsilon$ | $(k^0 < 0)$ | ($k$ propagates backward) |
| $3d = \omega_3 - i\varepsilon$ | $(k^0 > 0)$ | ($k$ propagates forward) |

In terms of 3 poles, each being either up or down, there are 8 cases.

Accordingly, there are 8 time-ordered diagrams in which 2 have no contribution.



covariant amplitude =

$$\frac{1}{-2(\omega_1-i\varepsilon)}\frac{1}{-2(\omega_2-i\varepsilon)}\frac{1}{-2(\omega_3-i\varepsilon)}$$

take downward plane as default sign

$$\Bigg(\frac{1}{k^0-p^0+q^0+\omega_1-i\varepsilon}\cancel{\frac{1}{k^0-p^0+\omega_2-i\varepsilon}}\cancel{\frac{1}{k^0+\omega_3-i\varepsilon}}$$

1u 2u 3u

no contribution

$$-\frac{1}{k^0-p^0+q^0+\omega_1-i\varepsilon}\frac{1}{k^0-p^0+\omega_2-i\varepsilon}\cancel{\frac{1}{k^0-\omega_3+i\varepsilon}}$$

1u 2u 3d

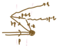
$$-\frac{1}{\omega_3-p^0+q^0+\omega_1}\frac{1}{\omega_3-p^0+\omega_2}$$ ✓

$$=\frac{1}{p^0-\omega_2-\omega_3}\frac{1}{\omega_1+\omega_3-(p^0-q^0)}$$

$$-\frac{1}{k^0-p^0+q^0+\omega_1-i\varepsilon}\cancel{\frac{1}{k^0-p^0-\omega_2+i\varepsilon}}\frac{1}{k^0+\omega_3-i\varepsilon}$$

1u 2d 3u

$$-\frac{1}{\cancel{p^0}+\omega_2-\cancel{p^0}+q^0+\omega_1}\frac{1}{p^0+\omega_2+\omega_3}$$ ✓



$$=\frac{1}{0-q^0-\omega_1-\omega_2}\frac{1}{\omega_3+\omega_2+p^0-0}$$

$$+\cancel{\frac{1}{k^0-p^0+q^0+\omega_1-i\varepsilon}}\frac{1}{k^0-p^0-\omega_2+i\varepsilon}\frac{1}{k^0-\omega_3+i\varepsilon}$$

1u 2d 3d

$$+\frac{1}{\cancel{p^0}-q^0-\omega_1-\cancel{p^0}-\omega_2}\frac{1}{p^0-q^0-\omega_1-\omega_3}\ (-)$$ ✓

$$=\frac{1}{0-q^0-\omega_1-\omega_2}\frac{1}{\omega_1+\omega_3-(p^0-q^0)}$$

$$-\cancel{\frac{1}{k^0-p^0+q^0-\omega_1+i\varepsilon}}\frac{1}{k^0-p^0+\omega_2-i\varepsilon}\frac{1}{k^0+\omega_3-i\varepsilon}$$

1d 2u 3u

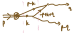
$$-\frac{1}{\cancel{p^0}-q^0+\omega_1-\cancel{p^0}+\omega_2}\frac{1}{p^0-q^0+\omega_1+\omega_3}$$ ✓

$$=\frac{1}{0-\omega_3-\omega_1-(p^0-q^0)}\frac{1}{\omega_2+\omega_1-q^0}$$

$$+\frac{1}{k^0-p^0+q^0-\omega_1+i\varepsilon}\cancel{\frac{1}{k^0-p^0+\omega_2-i\varepsilon}}\frac{1}{k^0-\omega_3+i\varepsilon}$$

1d 2u 3d

$$+\frac{1}{p^0-\omega_2-p^0+q^0-\omega_1}\frac{1}{p^0-\omega_2-\omega_3}\ (-)$$ ✓



$$=\frac{1}{p^0-\omega_2-\omega_3}\frac{1}{\omega_2+\omega_1-q^0}$$

$$+\frac{1}{k^0-p^0+q^0-\omega_1+i\varepsilon}\frac{1}{k^0-p^0-\omega_2+i\varepsilon}\cancel{\frac{1}{k^0+\omega_3-i\varepsilon}}$$

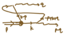
1d 2d 3u

$$+\frac{1}{-\omega_3-p^0+q^0-\omega_1}\frac{1}{-\omega_3-p^0-\omega_2}\ (-)$$ ✓

$$=\frac{1}{0-\omega_3-\omega_1-(p^0-q^0)}\frac{1}{p^0+\omega_3+\omega_2-0}$$

$$\cancel{-\frac{1}{k^0-p^0+q^0-\omega_1+i\varepsilon}\frac{1}{k^0-p^0-\omega_2+i\varepsilon}\frac{1}{k^0-\omega_3+i\varepsilon}}\Bigg)$$

1d 2d 3d



no contribution

# Interpolation between ET & LF

covariant amplitude $\sim \dfrac{1}{D_1 D_2 D_3}$

where

$$D_1 = \mathbb{C}(p_{\hat{+}} - k_{\hat{+}} - q_{\hat{+}})^2 + 2\mathbb{S}(p_{\hat{+}} - k_{\hat{+}} - q_{\hat{+}})(p_{\hat{-}} - k_{\hat{-}} - q_{\hat{-}}) - \mathbb{C}(p_{\hat{-}} - k_{\hat{-}} - q_{\hat{-}})^2 - m^2 + i\varepsilon$$

$$D_2 = \mathbb{C}(p_{\hat{+}} - k_{\hat{+}})^2 + 2\mathbb{S}(p_{\hat{+}} - k_{\hat{+}})(p_{\hat{-}} - k_{\hat{-}}) - \mathbb{C}(p_{\hat{-}} - k_{\hat{-}})^2 - m^2 + i\varepsilon$$

$$D_3 = \mathbb{C}k_{\hat{+}}^2 + 2\mathbb{S}k_{\hat{+}}k_{\hat{-}} - \mathbb{C}k_{\hat{-}}^2 - m^2 + i\varepsilon$$

There are 6 poles

$$k_{\hat{+}}\begin{Bmatrix}1u\\1d\end{Bmatrix} = p_{\hat{+}} - q_{\hat{+}} + \frac{\mathbb{S}}{\mathbb{C}}(p_{\hat{-}} - k_{\hat{-}} - q_{\hat{-}}) \mp \frac{\omega_1}{\mathbb{C}} \pm i\varepsilon$$

$$k_{\hat{+}}\begin{Bmatrix}2u\\2d\end{Bmatrix} = p_{\hat{+}} + \frac{\mathbb{S}}{\mathbb{C}}(p_{\hat{-}} - k_{\hat{-}}) \mp \frac{\omega_2}{\mathbb{C}} \pm i\varepsilon$$

$$k_{\hat{+}}\begin{Bmatrix}3d\\3u\end{Bmatrix} = -\frac{\mathbb{S}}{\mathbb{C}}k_{\hat{-}} \pm \frac{\omega_3}{\mathbb{C}} \mp i\varepsilon$$

where

$$\omega_1 = \sqrt{(p_{\hat{-}} - k_{\hat{-}} - q_{\hat{-}})^2 + \mathbb{C}m^2}$$

$$\omega_2 = \sqrt{(p_{\hat{-}} - k_{\hat{-}})^2 + \mathbb{C}m^2}$$

$$\omega_3 = \sqrt{k_{\hat{-}}^2 + \mathbb{C}m^2}$$

covariant amplitude =

$$\frac{1}{-2\omega_1}\left(\frac{1}{k_{\hat{+}} - k_{\hat{+}1u}} - \frac{1}{k_{\hat{+}} - k_{\hat{+}1d}}\right)$$

$$\frac{1}{-2\omega_2}\left(\frac{1}{k_{\hat{+}} - k_{\hat{+}2u}} - \frac{1}{k_{\hat{+}} - k_{\hat{+}2d}}\right)$$

$$\frac{1}{-2\omega_3}\left(\frac{1}{k_{\hat{+}} - k_{\hat{+}3u}} - \frac{1}{k_{\hat{+}} - k_{\hat{+}3d}}\right)$$

$$
\begin{aligned}
= \frac{1}{-2\omega_1}\frac{1}{-2\omega_2}\frac{1}{-2\omega_3}\Bigg( & \frac{1}{k_{\hat{+}}-k_{\hat{+}1u}}\frac{1}{k_{\hat{+}}-k_{\hat{+}2u}}\frac{1}{k_{\hat{+}}-k_{\hat{+}3u}} && \boxed{\begin{array}{l} k_{\hat{+}}-p_{\hat{+}}+q_{\hat{+}}-\frac{\mathbb{S}}{\mathbb{C}}(p_{\hat{-}}-k_{\hat{-}}-q_{\hat{-}})<0 \\ k_{\hat{+}}-p_{\hat{+}}-\frac{\mathbb{S}}{\mathbb{C}}(p_{\hat{-}}-k_{\hat{-}})<0 \\ k_{\hat{+}}+\frac{\mathbb{S}}{\mathbb{C}}k_{\hat{-}}<0 \end{array}} \\
- & \frac{1}{k_{\hat{+}}-k_{\hat{+}1u}}\frac{1}{k_{\hat{+}}-k_{\hat{+}2u}}\frac{1}{k_{\hat{+}}-k_{\hat{+}3d}} && \boxed{\begin{array}{l} k_{\hat{+}}-p_{\hat{+}}+q_{\hat{+}}-\frac{\mathbb{S}}{\mathbb{C}}(p_{\hat{-}}-k_{\hat{-}}-q_{\hat{-}})<0 \\ k_{\hat{+}}-p_{\hat{+}}-\frac{\mathbb{S}}{\mathbb{C}}(p_{\hat{-}}-k_{\hat{-}})<0 \\ k_{\hat{+}}+\frac{\mathbb{S}}{\mathbb{C}}k_{\hat{-}}>0 \end{array}} \\
- & \frac{1}{k_{\hat{+}}-k_{\hat{+}1u}}\frac{1}{k_{\hat{+}}-k_{\hat{+}2d}}\frac{1}{k_{\hat{+}}-k_{\hat{+}3u}} && \boxed{\begin{array}{l} k_{\hat{+}}-p_{\hat{+}}+q_{\hat{+}}-\frac{\mathbb{S}}{\mathbb{C}}(p_{\hat{-}}-k_{\hat{-}}-q_{\hat{-}})<0 \\ k_{\hat{+}}-p_{\hat{+}}-\frac{\mathbb{S}}{\mathbb{C}}(p_{\hat{-}}-k_{\hat{-}})>0 \\ k_{\hat{+}}+\frac{\mathbb{S}}{\mathbb{C}}k_{\hat{-}}<0 \end{array}} \\
+ & \frac{1}{k_{\hat{+}}-k_{\hat{+}1u}}\frac{1}{k_{\hat{+}}-k_{\hat{+}2d}}\frac{1}{k_{\hat{+}}-k_{\hat{+}3d}} && \boxed{\begin{array}{l} k_{\hat{+}}-p_{\hat{+}}+q_{\hat{+}}-\frac{\mathbb{S}}{\mathbb{C}}(p_{\hat{-}}-k_{\hat{-}}-q_{\hat{-}})<0 \\ k_{\hat{+}}-p_{\hat{+}}-\frac{\mathbb{S}}{\mathbb{C}}(p_{\hat{-}}-k_{\hat{-}})>0 \\ k_{\hat{+}}+\frac{\mathbb{S}}{\mathbb{C}}k_{\hat{-}}>0 \end{array}} \\
- & \frac{1}{k_{\hat{+}}-k_{\hat{+}1d}}\frac{1}{k_{\hat{+}}-k_{\hat{+}2u}}\frac{1}{k_{\hat{+}}-k_{\hat{+}3u}} && \boxed{\begin{array}{l} k_{\hat{+}}-p_{\hat{+}}+q_{\hat{+}}-\frac{\mathbb{S}}{\mathbb{C}}(p_{\hat{-}}-k_{\hat{-}}-q_{\hat{-}})>0 \\ k_{\hat{+}}-p_{\hat{+}}-\frac{\mathbb{S}}{\mathbb{C}}(p_{\hat{-}}-k_{\hat{-}})<0 \\ k_{\hat{+}}+\frac{\mathbb{S}}{\mathbb{C}}k_{\hat{-}}<0 \end{array}} \\
+ & \frac{1}{k_{\hat{+}}-k_{\hat{+}1d}}\frac{1}{k_{\hat{+}}-k_{\hat{+}2u}}\frac{1}{k_{\hat{+}}-k_{\hat{+}3d}} && \boxed{\begin{array}{l} k_{\hat{+}}-p_{\hat{+}}+q_{\hat{+}}-\frac{\mathbb{S}}{\mathbb{C}}(p_{\hat{-}}-k_{\hat{-}}-q_{\hat{-}})>0 \\ k_{\hat{+}}-p_{\hat{+}}-\frac{\mathbb{S}}{\mathbb{C}}(p_{\hat{-}}-k_{\hat{-}})<0 \\ k_{\hat{+}}+\frac{\mathbb{S}}{\mathbb{C}}k_{\hat{-}}>0 \end{array}} \\
+ & \frac{1}{k_{\hat{+}}-k_{\hat{+}1d}}\frac{1}{k_{\hat{+}}-k_{\hat{+}2d}}\frac{1}{k_{\hat{+}}-k_{\hat{+}3u}} && \boxed{\begin{array}{l} k_{\hat{+}}-p_{\hat{+}}+q_{\hat{+}}-\frac{\mathbb{S}}{\mathbb{C}}(p_{\hat{-}}-k_{\hat{-}}-q_{\hat{-}})>0 \\ k_{\hat{+}}-p_{\hat{+}}-\frac{\mathbb{S}}{\mathbb{C}}(p_{\hat{-}}-k_{\hat{-}})>0 \\ k_{\hat{+}}+\frac{\mathbb{S}}{\mathbb{C}}k_{\hat{-}}<0 \end{array}} \\
- & \frac{1}{k_{\hat{+}}-k_{\hat{+}1d}}\frac{1}{k_{\hat{+}}-k_{\hat{+}2d}}\frac{1}{k_{\hat{+}}-k_{\hat{+}3d}}\Bigg) && \boxed{\begin{array}{l} k_{\hat{+}}-p_{\hat{+}}+q_{\hat{+}}-\frac{\mathbb{S}}{\mathbb{C}}(p_{\hat{-}}-k_{\hat{-}}-q_{\hat{-}})>0 \\ k_{\hat{+}}-p_{\hat{+}}-\frac{\mathbb{S}}{\mathbb{C}}(p_{\hat{-}}-k_{\hat{-}})>0 \\ k_{\hat{+}}+\frac{\mathbb{S}}{\mathbb{C}}k_{\hat{-}}>0 \end{array}}
\end{aligned}
$$

In the IFD limit, these are energy ordered

In the LFD limit, these become momentum ordered

In between, it is mixture of energy and momentum.

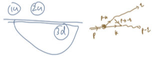
It is actually the $a^{\hat{+}}$ variable.

$p^{\hat{+}} - k^{\hat{+}} - q^{\hat{+}} > 0$
$p^{\hat{+}} - k^{\hat{+}} > 0$
$k^{\hat{+}} < 0$


In LFD limit, these conditions become

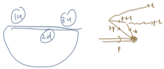
$$\left.\begin{array}{l} 1-x-\alpha > 0 \\ 1-x > 0 \\ x < 0 \end{array}\right\} \text{ok.}$$

$p^{\hat{+}} - k^{\hat{+}} - q^{\hat{+}} > 0$
$p^{\hat{+}} - k^{\hat{+}} > 0$
$k^{\hat{+}} > 0$




$$\begin{array}{l} 1-x-\alpha > 0 \\ 1-x > 0 \\ x > 0 \end{array} \Rightarrow \text{region (b)}$$

$p^{\hat{+}} - k^{\hat{+}} - q^{\hat{+}} > 0$
$p^{\hat{+}} - k^{\hat{+}} < 0$
$k^{\hat{+}} < 0$



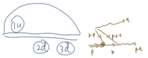

in ET, the conditions of the energy component of the p-k particle <0 and the energy component of the k particle <0 has nothing to do with each other. because energy component is not conserved in T.O. diagrams. (written all as $k^0$, but take different pole values)

$$\left.\begin{array}{l} 1-x-\alpha > 0 \\ 1-x < 0 \\ x < 0 \end{array}\right\} \text{not possible}$$

Now in LF, this becomes the momentum component.

Now this is confusing. Momentum IS conserved. x cannot be both <0 and >1.

$p^{\hat{+}} - k^{\hat{+}} - q^{\hat{+}} > 0$
$p^{\hat{+}} - k^{\hat{+}} < 0$
$k^{\hat{+}} > 0$




$$\left.\begin{array}{l} 1-x-\alpha > 0 \\ 1-x < 0 \end{array}\right\} \text{not possible}$$
$$x > 0$$

$p^{\hat{+}} - k^{\hat{+}} - q^{\hat{+}} < 0$
$p^{\hat{+}} - k^{\hat{+}} > 0$
$k^{\hat{+}} < 0$


In between, it is still ok because energy component is mixed in. As long as energy is not conserved, it is okay.

$$\left.\begin{array}{l} 1-x-\alpha < 0 \\ 1-x > 0 \\ x < 0 \end{array}\right\} \rightarrow \text{not possible}$$

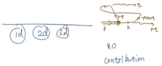
$p^{\hat{+}} - k^{\hat{+}} - q^{\hat{+}} < 0$
$p^{\hat{+}} - k^{\hat{+}} > 0$
$k^{\hat{+}} > 0$


$$\begin{array}{l} 1-x-\alpha < 0 \\ 1-x > 0 \\ x > 0 \end{array} \Rightarrow \text{region (c)}$$

$p^{\hat{+}} - k^{\hat{+}} - q^{\hat{+}} < 0$
$p^{\hat{+}} - k^{\hat{+}} < 0$
$k^{\hat{+}} < 0$


$$\begin{array}{l} 1-x-\alpha < 0 \\ \left.\begin{array}{l} 1-x < 0 \\ x < 0 \end{array}\right\} \text{not possible} \end{array}$$

$p^{\hat{+}} - k^{\hat{+}} - q^{\hat{+}} < 0$
$p^{\hat{+}} - k^{\hat{+}} < 0$
$k^{\hat{+}} > 0$




$$\left.\begin{array}{l} 1-x-\alpha < 0 \\ 1-x < 0 \\ x > 0 \end{array}\right\} \text{ok.}$$

# Light-Front limit: The rabbit and the magician's hat

- Now, according to the above analysis, one would conclude that in LFD the only possible contributions are coming from the region (b) and region (c), i.e., the 2nd and the 6th diagrams in the previous page.
- Following the time-ordered rules, or doing the pole integrations (we already see that they are the same), one obtains the sum of Eqs. (17) and (18) as the total of the minus-minus component of the transition form factor, which, as we said before, is not correct.

- The rabbit:
  - In our previous calculation in the interpolation form, we caught 2 additional poles for the region (b) of $0 < x < 1 - \alpha < 1$, 2 additional poles for the region (c) of $0 < 1 - \alpha < x < 1$. Also, we caught 3 poles for the region (DZL) of $x < 0$, and 3 for the region (DZR) of $x > 1$. And adding the contributions from all 4 time-ordered regions, we achieved agreement between the minus-minus component calculation and the covariant result.
- The hat:
  - However, in terms of the old-fashioned time-ordered diagrams, we see that among the 8 diagrams, 4 are not possible (consider $0 < \alpha < 1$ without equal signs), 2 give no contributions, so only 2 are left, and the agreement with the covariant result is lost.

- Even we disregard the contradicting conditions, i.e., we think any combinations of the following can be satisfied simultaneously,

When $1-x-\alpha>0$
1reg is 1u
1inf is 1d

When $1-x>0$
2reg is 2u
2inf is 2d

When $x<0$
3reg is 3u
3inf is 3d

When $1-x-\alpha<0$
1reg is 1d
1inf is 1u

When $1-x<0$
2reg is 2d
2inf is 2u

When $x>0$
3reg is 3d
3inf is 3u

we find

$$
\begin{aligned}
= \frac{1}{-2\omega_1}\frac{1}{-2\omega_2}\frac{1}{-2\omega_3}\Bigg( & \frac{1}{k_\phi - k_{\phi 1u}}\frac{1}{k_\phi - k_{\phi 2u}}\frac{1}{k_\phi - k_{\phi 3u}} \\
- & \frac{1}{k_\phi - k_{\phi 1u}}\frac{1}{k_\phi - k_{\phi 2u}}\frac{1}{k_\phi - k_{\phi 3d}} \\
- & \frac{1}{k_\phi - k_{\phi 1u}}\frac{1}{k_\phi - k_{\phi 2d}}\frac{1}{k_\phi - k_{\phi 3u}} \\
+ & \frac{1}{k_\phi - k_{\phi 1u}}\frac{1}{k_\phi - k_{\phi 2d}}\frac{1}{k_\phi - k_{\phi 3d}} \\
- & \frac{1}{k_\phi - k_{\phi 1d}}\frac{1}{k_\phi - k_{\phi 2u}}\frac{1}{k_\phi - k_{\phi 3u}} \\
+ & \frac{1}{k_\phi - k_{\phi 1d}}\frac{1}{k_\phi - k_{\phi 2u}}\frac{1}{k_\phi - k_{\phi 3d}} \\
+ & \frac{1}{k_\phi - k_{\phi 1d}}\frac{1}{k_\phi - k_{\phi 2d}}\frac{1}{k_\phi - k_{\phi 3u}} \\
- & \frac{1}{k_\phi - k_{\phi 1d}}\frac{1}{k_\phi - k_{\phi 2d}}\frac{1}{k_\phi - k_{\phi 3d}}\Bigg)
\end{aligned}
$$

1reg 1u 2u 2reg 3u 3reg — $p^{\hat\phi}-k^{\hat\phi}-q^{\hat\phi}>0$, $p^{\hat\phi}-k^{\hat\phi}>0$, $k^{\hat\phi}<0$

1reg 2u 2reg 3d 3reg — $p^{\hat\phi}-k^{\hat\phi}-q^{\hat\phi}>0$, $p^{\hat\phi}-k^{\hat\phi}>0$, $k^{\hat\phi}>0$

1reg 1u 2d 2reg 3u 3reg — $p^{\hat\phi}-k^{\hat\phi}-q^{\hat\phi}>0$, $p^{\hat\phi}-k^{\hat\phi}<0$, $k^{\hat\phi}<0$

1reg 2d 2reg 3d 3reg — $p^{\hat\phi}-k^{\hat\phi}-q^{\hat\phi}>0$, $p^{\hat\phi}-k^{\hat\phi}<0$, $k^{\hat\phi}>0$

1reg 1d 2u 2reg 3u 3reg — $p^{\hat\phi}-k^{\hat\phi}-q^{\hat\phi}<0$, $p^{\hat\phi}-k^{\hat\phi}>0$, $k^{\hat\phi}<0$

1reg d 2reg 3d 3reg — $p^{\hat\phi}-k^{\hat\phi}-q^{\hat\phi}<0$, $p^{\hat\phi}-k^{\hat\phi}>0$, $k^{\hat\phi}>0$

1reg d 2d 2reg 3u 3reg — $p^{\hat\phi}-k^{\hat\phi}-q^{\hat\phi}<0$, $p^{\hat\phi}-k^{\hat\phi}<0$, $k^{\hat\phi}<0$

1reg 1d 2d 2reg 3d 3reg — $p^{\hat\phi}-k^{\hat\phi}-q^{\hat\phi}<0$, $p^{\hat\phi}-k^{\hat\phi}<0$, $k^{\hat\phi}>0$

# Future work

- Find the rabbit in the hat.

Thanks for you attention!