TMDs at high energy

Patrick Barry

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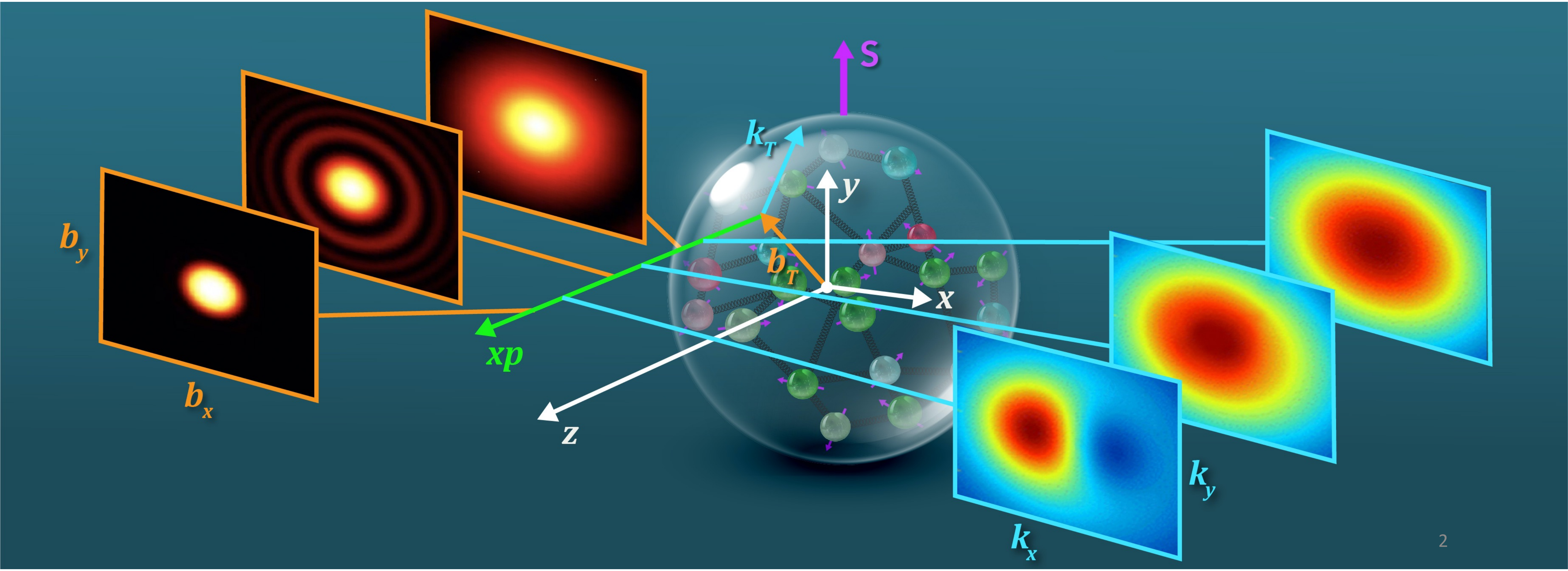
3D structures of hadrons

- Even more challenging is the 3d structure through GPDs and TMDs



Unpolarized TMD PDF

$$\tilde{f}_{q/\mathcal{N}}(x, b_T) = \int \frac{\mathrm{d}b^-}{4\pi} \, e^{-ixP^+b^-} \, \mathrm{Tr}\big[\langle \mathcal{N} \,|\, \bar{\psi}_q(b)\gamma^+ \mathcal{W}(b,0)\psi_q(0) \,|\, \mathcal{N} \rangle\big]$$

$$b \equiv (b^-, 0^+, \boldsymbol{b}_T)$$

- $\boldsymbol{b_T}$ is the Fourier conjugate to the intrinsic transverse momentum of quarks in the hadron, $\boldsymbol{k_T}$
- We can learn about the coordinate space correlations of quark fields in hadrons
- Modification needed for UV and rapidity divergences; acquire regulators: $\tilde{f}_{q/\mathcal{N}}(x, b_T) \to \tilde{f}_{q/\mathcal{N}}(x, b_T; \mu, \zeta)$

Factorization for low-q_T Drell-Yan

- Like collinear observable, a hard part with two functions that describe structure of beam and target
- So called "W"-term, valid only at low-q_T

$$\frac{d^3\sigma}{d\tau dY dq_T^2} = \frac{4\pi^2\alpha^2}{9\tau S^2} \sum_q H_{q\bar{q}}(Q^2, \mu) \int d^2 b_T \, e^{i b_T \cdot q_T} \times \tilde{f}_{q/\pi}(x_\pi, b_T, \mu, Q^2) \, \tilde{f}_{\bar{q}/A}(x_A, b_T, \mu, Q^2) \, ,$$

Small b_T operator product expansion

- At small b_T, the TMD PDF can be described in terms of its OPE:

$$\tilde{f}_{q/\mathcal{N}}(x, b_T; \mu, \zeta_F) = \sum_j \int_x^1 \frac{d\xi}{\xi} \tilde{\mathcal{C}}_{q/j}(x/\xi, b_T; \mu, \zeta_F) f_{q/\mathcal{N}}(\xi; \mu) + \mathcal{O}((\Lambda_{\mathrm{QCD}} b_T)^a)$$

- where $\tilde{C}$ are the Wilson coefficients, and $f_{q/\mathcal{N}}$ is the collinear PDF
- Breaks down when b_T gets large

b_* prescription

- A common approach to regulating large b_T behavior

$$\mathbf{b}_*(\mathbf{b}_T) \equiv \frac{\mathbf{b}_T}{\sqrt{1 + b_T^2/b_{\mathrm{max}}^2}}.$$

Must choose an appropriate value; a transition from perturbative to non-perturbative physics

- At small b_T, $b_*(b_T) = b_T$
- At large $b_T, b_*(b_T) = b_{\max}$

Introduction of non-perturbative functions

- Because $b_* \neq b_T$, have to non-perturbatively describe large b_T behavior

Completely general – independent of quark, hadron, PDF or FF

$$g_K(b_T; b_{\max}) = -\tilde{K}(b_T, \mu) + \tilde{K}(b_*, \mu)$$

Non-perturbative function dependent in principle on flavor, hadron, etc.

$$e^{-g_{q/\mathcal{N}(A)}(x, b_T)} = \frac{\tilde{f}_{q/\mathcal{N}(A)}(x, b_T; \mu, \zeta)}{\tilde{f}_{q/\mathcal{N}(A)}(x, b_*; \mu, \zeta)} e^{g_K(b_T; b_{\max}) \log\left(\sqrt{\zeta}/Q_0\right)}$$

TMD PDF within the b_* prescription

$$\mathbf{b}_*(\mathbf{b}_T) \equiv \frac{\mathbf{b}_T}{\sqrt{1 + b_T^2/b_{\max}^2}}.$$

Low-b_T: perturbative
high-b_T: non-perturbative

$$\tilde{f}_{q/\mathcal{N}(A)}(x, b_T, \mu_Q, Q^2) = \boxed{(C \otimes f)_{q/\mathcal{N}(A)}(x; b_*)}$$
$$\times \exp\left\{\boxed{-g_{q/\mathcal{N}(A)}(x, b_T) - g_K(b_T) \ln \frac{Q}{Q_0}} - \boxed{S(b_*, Q_0, Q, \mu_Q)}\right\}$$

Relates the TMD at small-b_T to the **collinear** PDF
$\Rightarrow$ TMD is sensitive to collinear PDFs

$g_{q/\mathcal{N}(A)}$: intrinsic non-perturbative structure of the TMD
g_K: universal non-perturbative Collins-Soper kernel

Controls the perturbative evolution of the TMD

Collins, Soper, Sterman, NPB **250**, 199 (1985).

MAP parametrization

- A recent work from the MAP collaboration (Phys. Rev. D **107**, 014014 (2023).) used a complicated form for the non-perturbative function

$$f_{1\,NP}(x, \boldsymbol{b}_T^2; \zeta, Q_0) = \frac{g_1(x)\, e^{-g_1(x)\frac{\boldsymbol{b}_T^2}{4}} + \lambda^2\, g_{1B}^2(x) \left[1 - g_{1B}(x)\frac{\boldsymbol{b}_T^2}{4}\right] e^{-g_{1B}(x)\frac{\boldsymbol{b}_T^2}{4}} + \lambda_2^2\, g_{1C}(x)\, e^{-g_{1C}(x)\frac{\boldsymbol{b}_T^2}{4}}}{g_1(x) + \lambda^2\, g_{1B}^2(x) + \lambda_2^2\, g_{1C}(x)} \left[\frac{\zeta}{Q_0^2}\right]^{\boxed{g_K(\boldsymbol{b}_T^2)/2}}, \tag{38}$$

$$g_{\{1,1B,1C\}}(x) = N_{\{1,1B,1C\}} \frac{x^{\sigma_{\{1,2,3\}}} (1-x)^{\alpha^2_{\{1,2,3\}}}}{\hat{x}^{\sigma_{\{1,2,3\}}} (1-\hat{x})^{\alpha^2_{\{1,2,3\}}}},$$

$$\boxed{g_K(\boldsymbol{b}_T^2) = -g_2^2 \frac{\boldsymbol{b}_T^2}{2}}$$

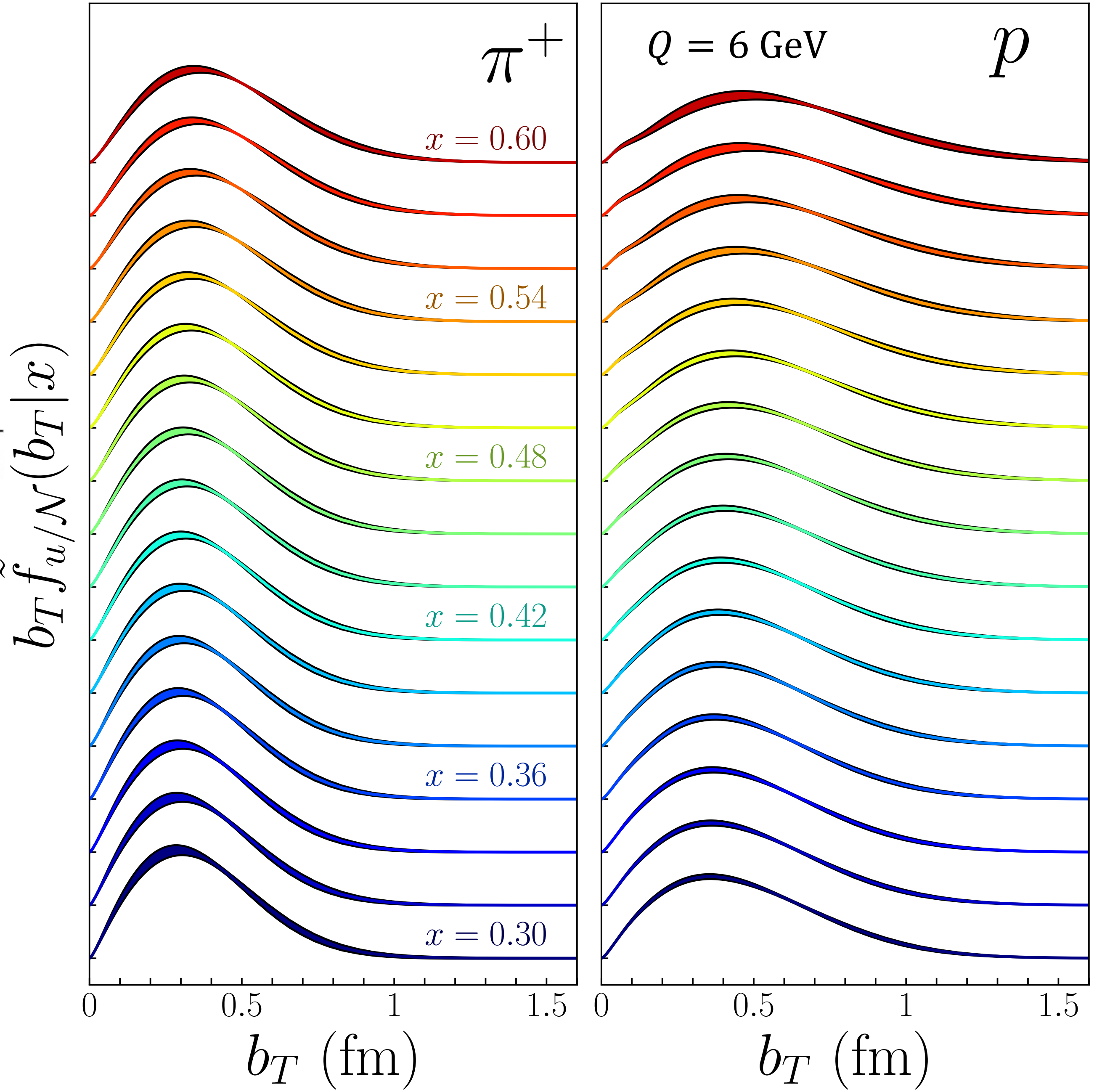
Universal CS kernel

- 11 free parameters for each hadron! (flavor dependence not necessary) (12 if we include the nuclear TMD parameter)

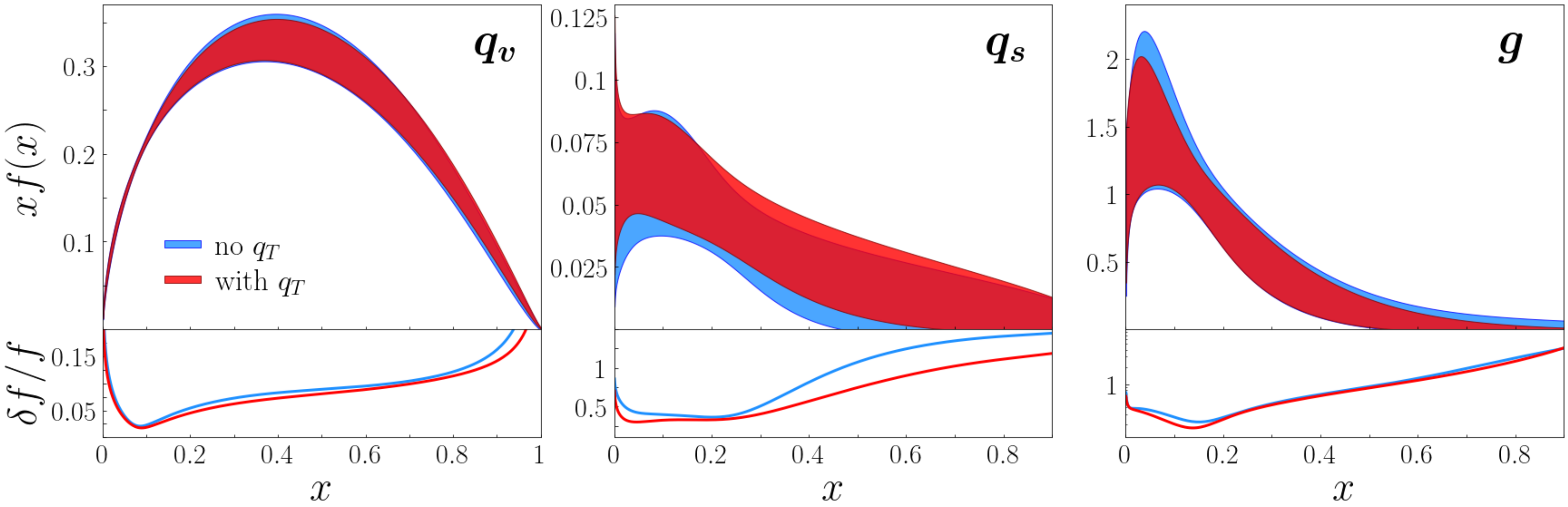
Resulting TMD PDFs of proton and pion

$$\tilde{f}_{q/\mathcal{N}}(b_T|x;Q,Q^2) \equiv \frac{\tilde{f}_{q/\mathcal{N}}(x,b_T;Q,Q^2)}{\int \mathrm{d}^2\boldsymbol{b}_T \tilde{f}_{q/\mathcal{N}}(x,b_T;Q,Q^2)}$$

- Broadening appearing as x increases
- Up quark in pion is narrower than up quark in proton



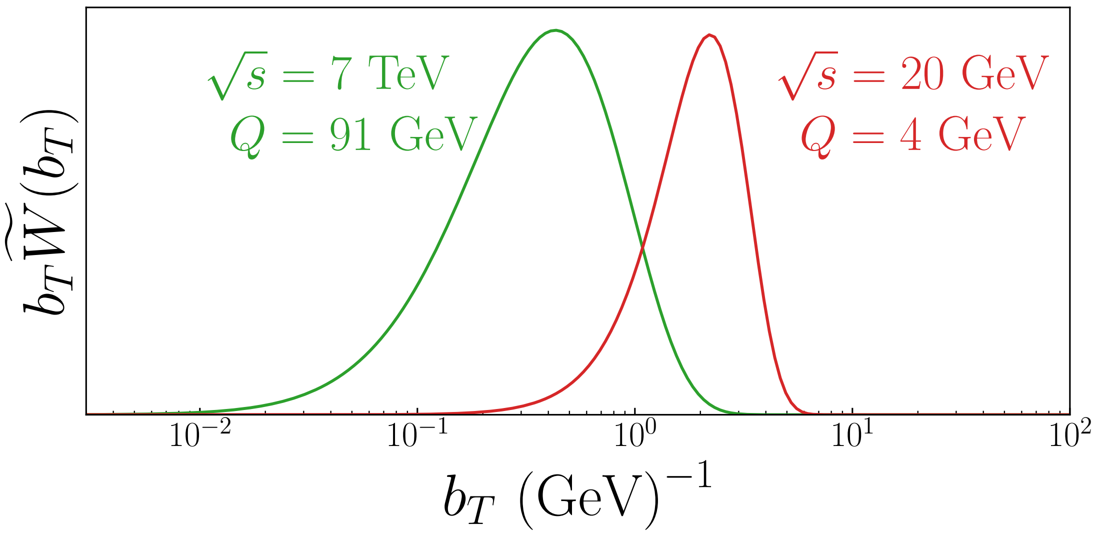
Extracted pion PDFs



- The small-q_T data do not constrain much the PDFs

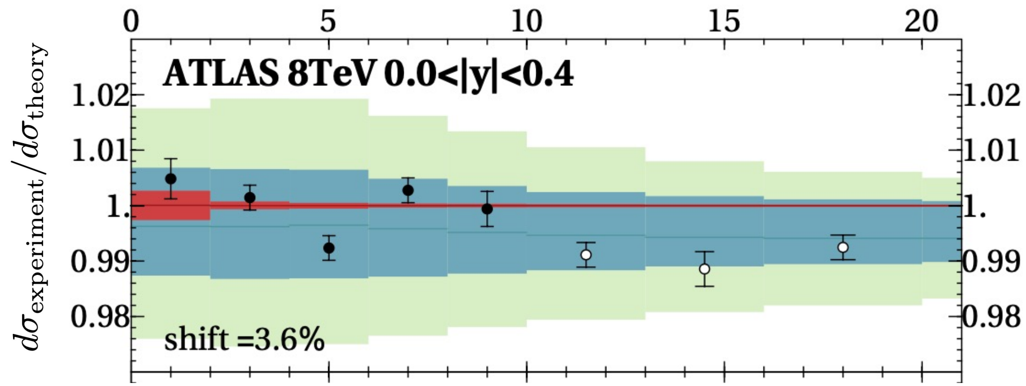
What about LHC energies?

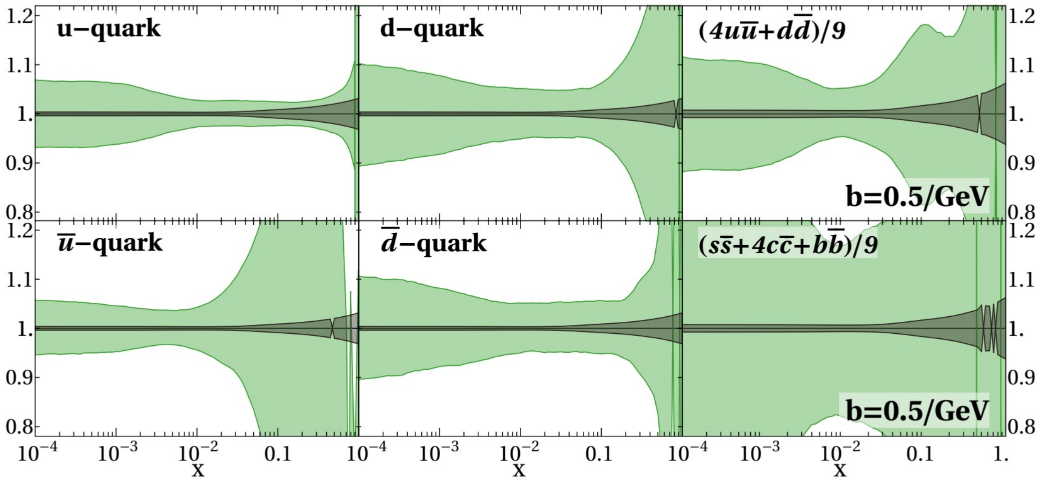
- Many studies have extracted TMDs from these data: Bertone, Scimemi, Vladimirov, JHEP **06** (2019). Bacchetta, et al. JHEP **10** (2022). etc.
- Fixed-target energies: sensitive to non-perturbative TMD structures
  - Large portion of $\widetilde{W}$ spectrum in large-b_T region
- LHC energies: sensitive to perturbative calculations
  - Have opportunity to study collinear distributions



High energy PDF uncertainties

- From Bury, et al. JHEP **118** (2022).





Moos, Scimemi, Vladimirov, Zurita, arXiv:2305.07473

- Studies about the uncertainties of the PDFs relative to data

Trust perturbative region

- Method to keep the $\widetilde{W}$ term unaltered by b_* mechanism up to a certain $b_{\max}$
- Non-perturbative effects kick in at $b_{\max}$
- Smooth function as 1st and 2nd derivatives are continuous at $b_{\max}$

$$\widetilde{W}(b_T, x_a, x_b, Q) = \widetilde{W}_{\rm pert}(b_T, x_a, x_b, Q) \quad \text{for} \quad b_T < b_{\max}$$
$$= \widetilde{W}_{\rm pert}(b_{\max}, x_a, x_b, Q) f_{\rm NP}(b_T, b_{\max}, x_a, x_b) \quad \text{for} \quad b_T > b_{\max}$$

Qiu, Zhang, PRD **63**, 114011 (2001).

Nonperturbative form

- Instead of 11 free parameters for the MAP parametrization, we have a few

$$f_{\mathrm{NP}}(b_T, b_{\max}, x_a, x_b) = \exp\left\{-\log\left(\frac{Q^2 b_{\max}^2}{C_1^2}\right)\left[g_1\left((b_T^2)^\alpha - (b_{\max}^2)^\alpha\right) + g_2(b_T^2 - b_{\max}^2)\right] - \bar{g}_2(b_T^2 - b_{\max}^2)\right\}$$

- Structure such that the g_1 term is reminiscent of a logarithm, which is predicted from perturbative calculations
- $\bar{g}_2$ term is the “intrinsic” transverse momentum component (in principle, some flexibility can be had here)

Solve for g_1 and α by differentiating

- Take first and second derivatives and solve analytically on the RHS and numerically on the LHS
- F_a^{OPE} is like $\widetilde{W}$
- R_a^{NP} is like f_{NP}

$$\left.\frac{\partial F_a^{OPE}(b_T)}{\partial b_T}\right|_{b_{max}} = \left.\frac{\partial\left(F_a^{OPE}(b_{max})\, R_a^{NP}(b_T)\right)}{\partial b_T}\right|_{b_{max}}$$

$$= F_a^{OPE}(b_{max}) \left.\frac{\partial R_a^{NP}(b_T)}{\partial b_T}\right|_{b_{max}}.$$

$$\frac{\partial R_a^{NP}(b_T)}{\partial b_T} = R_a^{NP}(b_T)\left\{-\log\left(\frac{Q^2}{\mu_{b_{max}}^2}\right)\left[2\alpha g_1 b_T^{2\alpha-1} + 2 g_2 b_T\right] - g_J^{(1)}\right\}.$$

Solve for g_1 and α by differentiating

$$\boxed{g_1 = \frac{\dfrac{-\left[\dfrac{F^{(1)}(b_{max})}{F(b_{max})} + g_J^{(1)}(b_{max})\right]}{\log\left(\dfrac{Q^2}{\mu_{b_{max}}^2}\right)} - 2 g_2 b_{max}}{2\alpha\, b_{max}^{2\alpha-1}}}$$

- Result from taking first derivative ($F^{(1)}$)

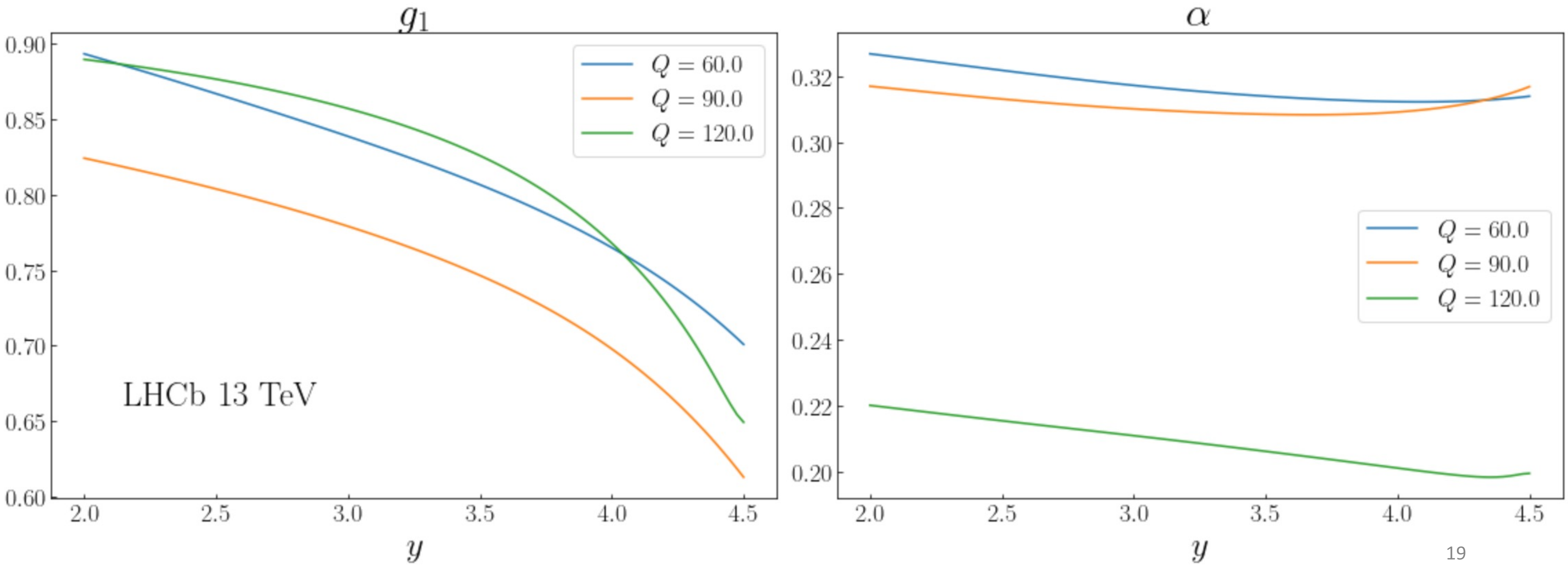
Solve for g_1 and α by differentiating

$$\alpha = \frac{1}{2}\,\frac{b_{max}\left(\frac{F^{(2)}}{F} - \left(\frac{F^{(1)}}{F}\right)^2 + g_J^{(2)} + 4 g_2 \log\left(\frac{Q^2}{\mu_{max}^2}\right)\right) + \left(\frac{F^{(1)}}{F} + g_J^{(1)}\right)}{\frac{F^{(1)}}{F} + g_J^{(1)} + 2 g_2 b_{max} \log \frac{Q^2}{\mu_{max}^2}}$$

- Result from taking second derivative ($F^{(2)}$)

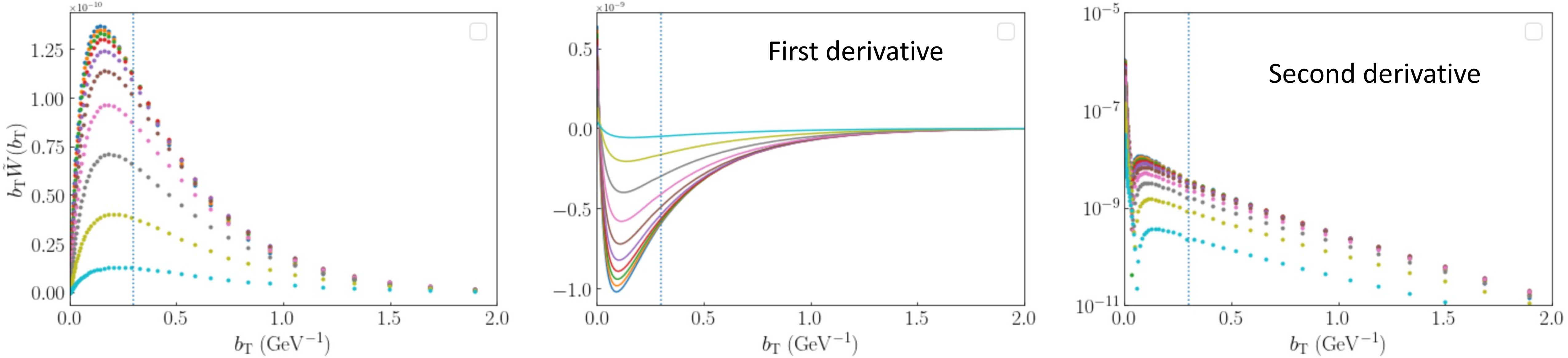
Examples of g_1 and α

- Solving for them in the codes for various Q values as a function of y



Examples of continuity

- Different rapidity values for different curves



Observables

- The factorization goes as follows

$$\frac{d\sigma}{dy dQ^2 dq_T^2} = \frac{4\pi^2\alpha_{\rm em}^2}{9Q^2 s}\mathcal{P}\sum_q c_q(Q) H_q(Q,\mu_Q) \int \frac{d^2\mathbf{b}_T}{(2\pi)^2} e^{i\mathbf{q}_T\cdot\mathbf{b}_T} \times \tilde{f}_{q/p}(x_a, b_T; Q, Q^2)\tilde{f}_{\bar{q}/\bar{p}}(x_b, b_T; Q, Q^2),$$

- $\mathcal{P}$ is a fiducial factor, which limits the phase space of the detected leptons

Data observable - techniques

Fourier transform from previous slide

- Highly integrated observable

$$\frac{d\sigma}{dp_T} = \frac{1}{\Delta p_T} \int dp_T \int dQ \int dy \mathcal{P}(Q, y, p_T) \boxed{\int db_T b_T J_0(b_T p_T) \widetilde{W}(Q, y, b_T)}$$

- 2 steps of parallelization:
  1. Compute $\widetilde{W}$ as a function of predetermined Q, y, b_T, where b_T is interpolated
  2. Perform the Fourier transform for predetermined Q, y

Data observable - techniques

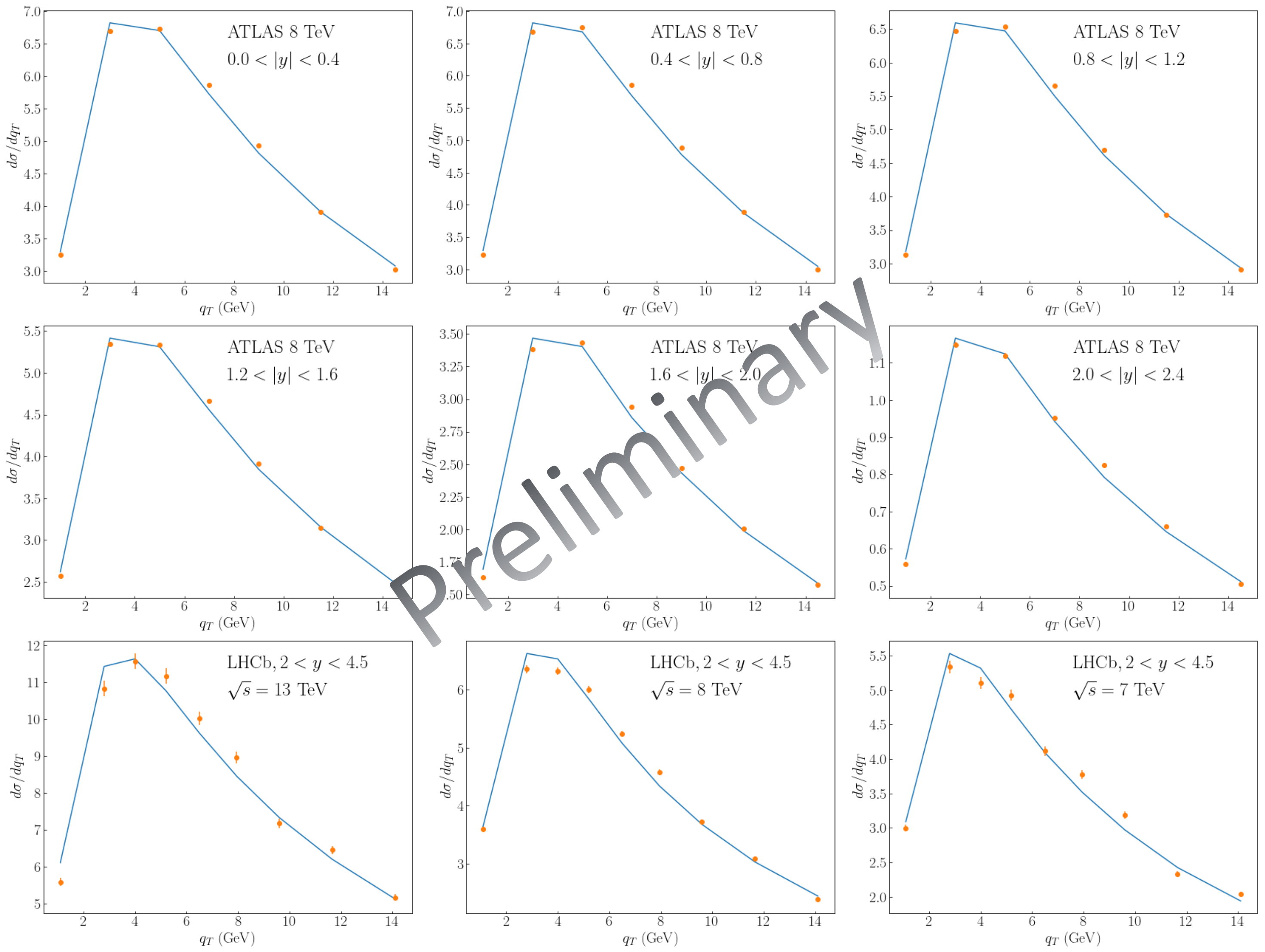
- Highly integrated observable

$$\frac{d\sigma}{dp_T} = \frac{1}{\Delta p_T} \int dp_T \int dQ \int dy \mathcal{P}(Q, y, p_T) \int db_T b_T J_0(b_T p_T) \widetilde{W}(Q, y, b_T)$$

- Have precomputed fiducial factors that are functions of Q, y, p_T
- Interpolate over a 2d grid in (Q, y) for each p_T
- Perform (very quickly) Gaussian quadrature over the interpolated sheet
- Pick a few p_T points for bin averaging integrand

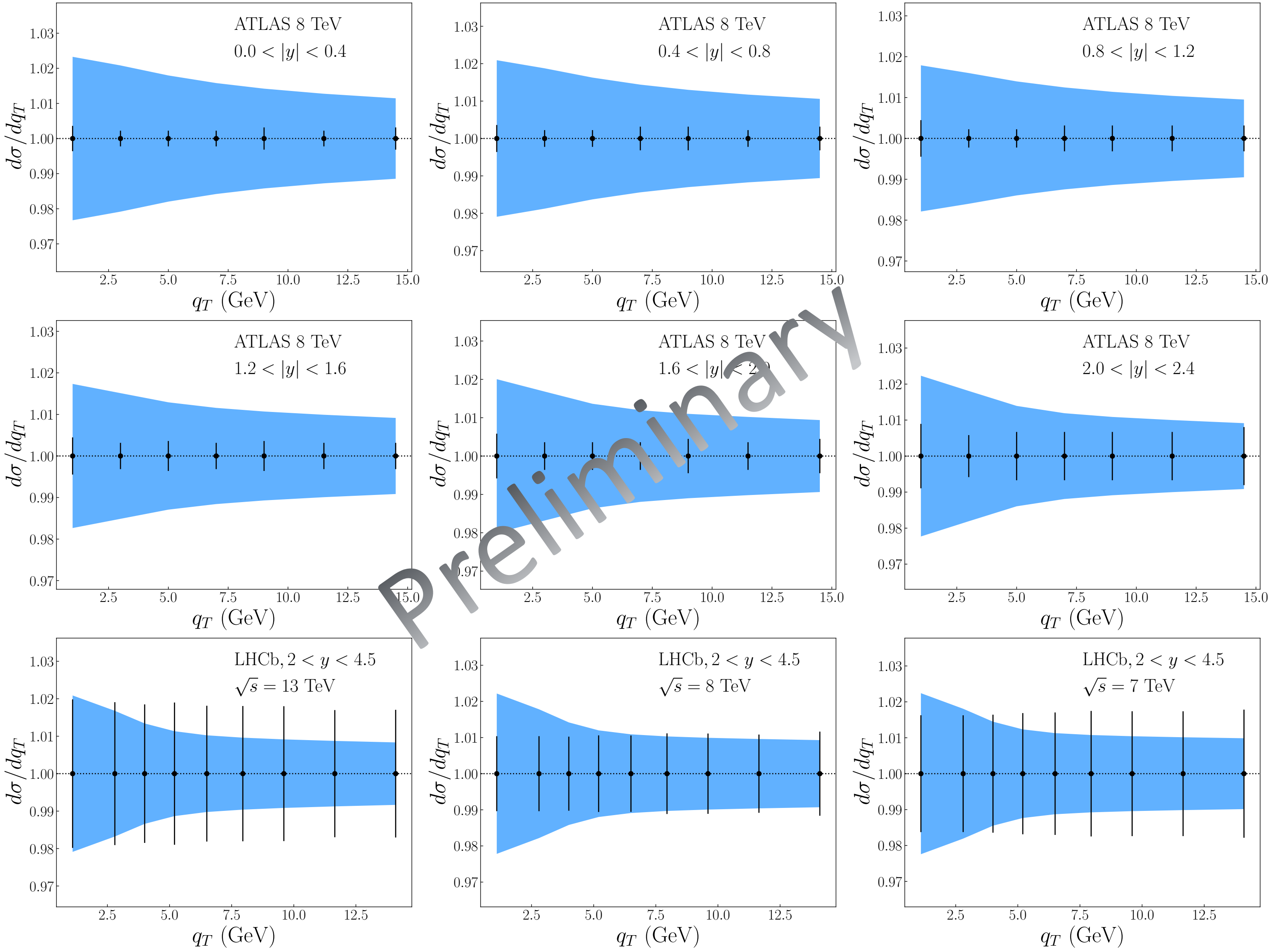
Preliminary fit to ATLAS and LHCb

- Two free non-perturbative TMD parameters from fit
- Fix the collinear PDF
- $b_{\max} = 0.3\ \mathrm{GeV}^{-1}$



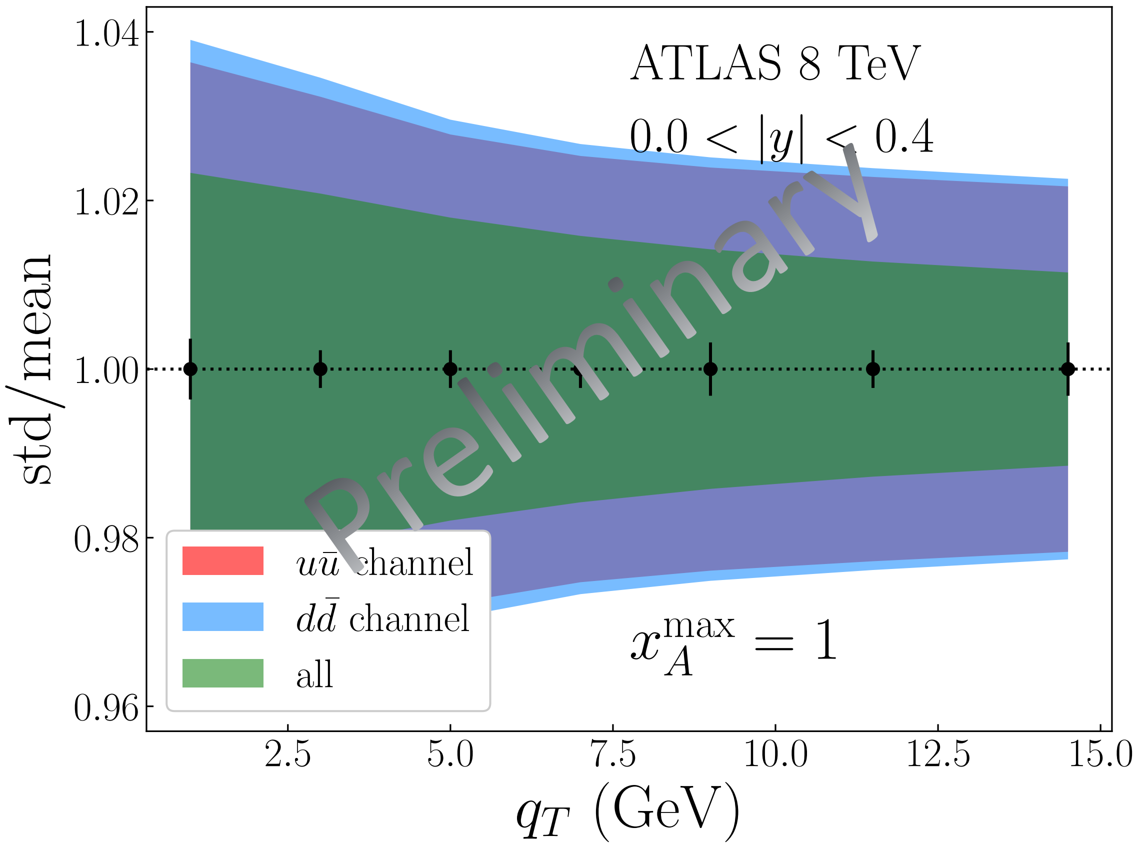
Uncertainties from JAM PDFs only

- Bands come from varying only the collinear PDFs
- High precision in ATLAS and LHCb data indicate potential constraining power



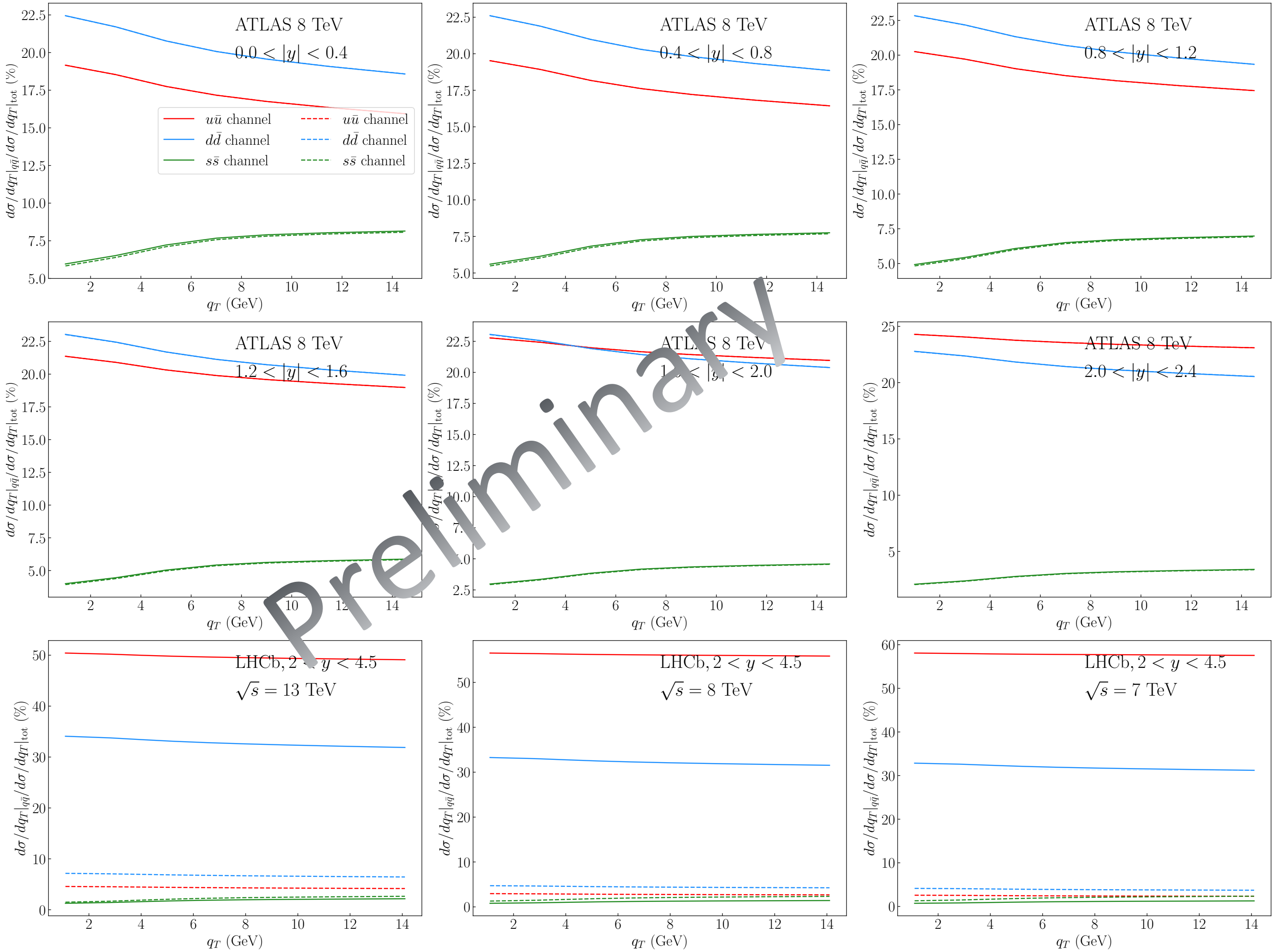
Individual quarks

- Green: full contributions
- Red (looks purple): contribution when u in beam PDF and $\bar{u}$ in target
- Blue: corresponding $d\bar{d}$



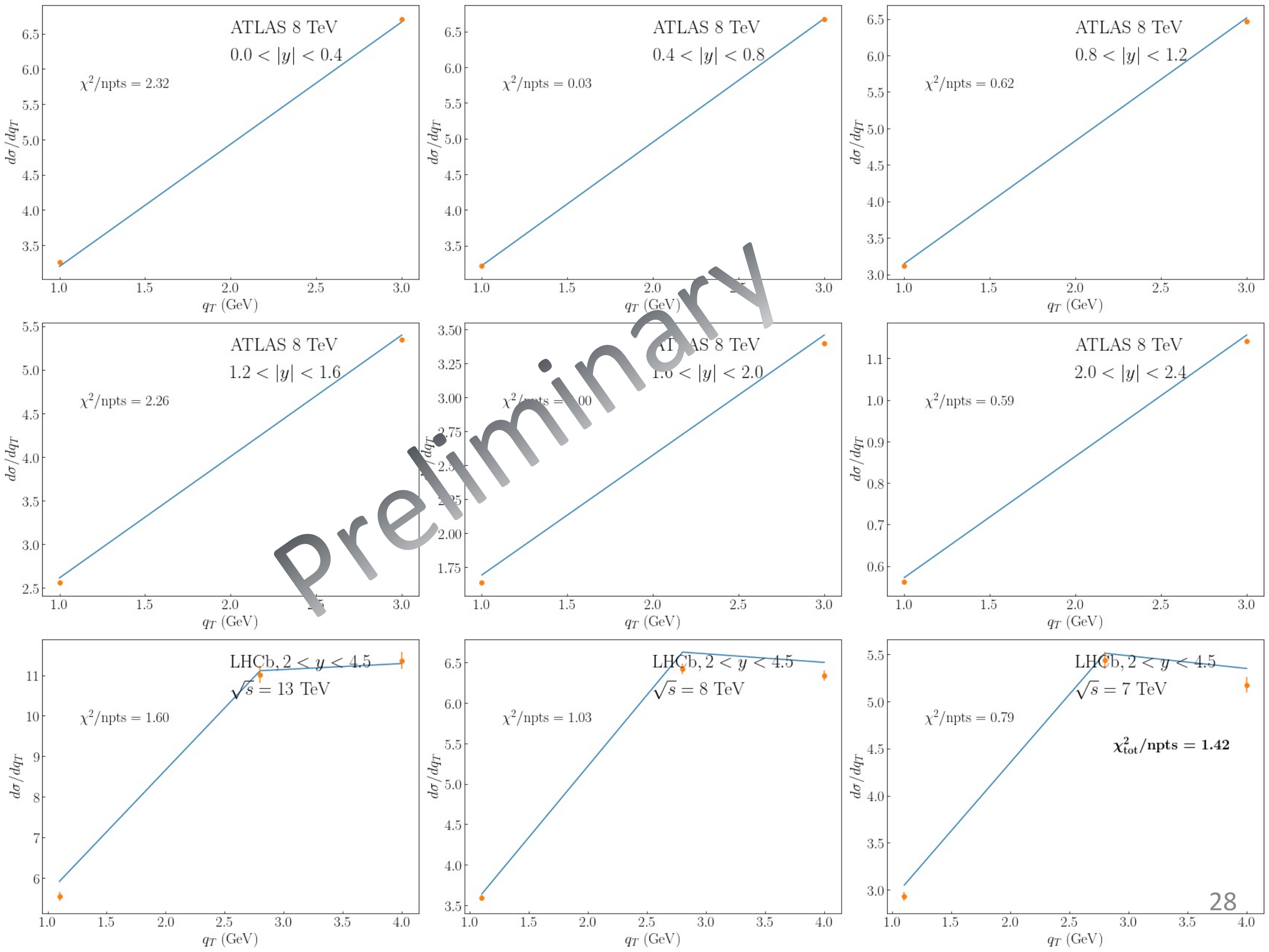
Contributions from each experiments

- Looking at percentage coming from specific quark channels



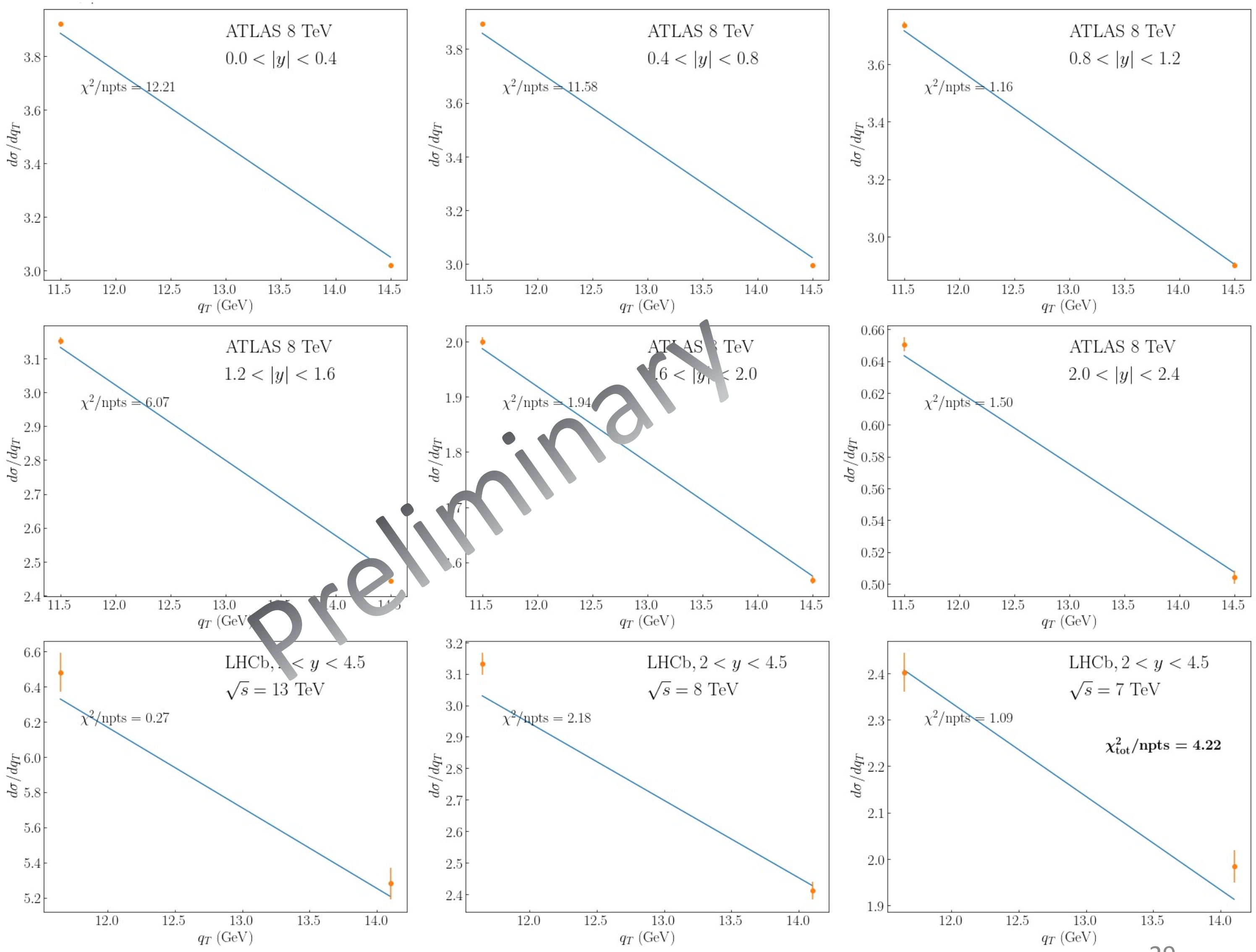
Small q_T

- Quite successful at small-q_T!



Large q_T

- Still in the “W” region, but not as good



Outlook

- Apply this matching on the TMDs themselves
- Finalize fits and perform simultaneous extractions of collinear and TMD PDFs
  - Examine the impact the low-q_T data have on the collinear PDFs
- Perform simultaneous analysis including W-boson production and analyze its mass