Some applications of nonlocal EFT and nonlocal QED

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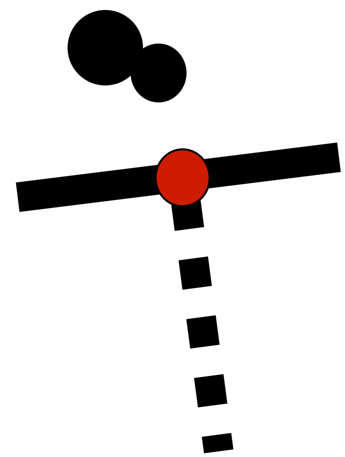
Nonlocal EFT

Local interaction including pi meson



$$\mathcal{L}_\pi{}^{local} = \int dx \frac{D+F}{\sqrt{2}f} \bar{p}(x)\gamma^\mu \gamma_5\, n(x)(\partial_\mu + ie\,\mathscr{A}_\mu(x))\pi^+(x)$$

Corresponding nonlocal Lagrangian

$$\mathcal{L}_\pi{}^{nl} = \int dx \int dy \frac{D+F}{\sqrt{2}f} \bar{p}(x)\gamma^\mu \gamma_5 n(x) F(x-y) \exp[ie \int_x^y dz_\nu \int da\, \mathscr{A}^\nu(z-a)F(a)]$$
$$\times\, (\partial_\mu + ie \int da\, \mathscr{A}_\mu(y-a)F(a))\pi^+(y),$$

Local EM interaction

$$\mathcal{L}_{EM}^{local} = -\,e\bar{p}(x)\gamma^\mu p(x)\mathscr{A}_\mu(x) + \frac{(c_1-1)e}{4m_N}\bar{p}(x)\sigma^{\mu\nu}p(x)F_{\mu\nu}(x)$$

Corresponding nonlocal EM interaction

$$\mathcal{L}_{EM}^{nl} = -e \int da \bar{p}(x)\gamma^\mu p(x)\mathscr{A}_\mu(x-a)F_1(a) + \frac{(c_1-1)e}{4m_N} \int da \bar{p}(x)\sigma^{\mu\nu}p(x)F_{\mu\nu}(x-a)F_2(a)$$

EM currents with pi meson:

$$\mathcal{L}^{nor} = ie \int dx \int dy \frac{D+F}{\sqrt{2}f} \bar{p}(x)\gamma^\mu \gamma_5 n(x) F(x-y)\pi^+(y) \int da\, \mathscr{A}_\mu(y-a)F(a)$$

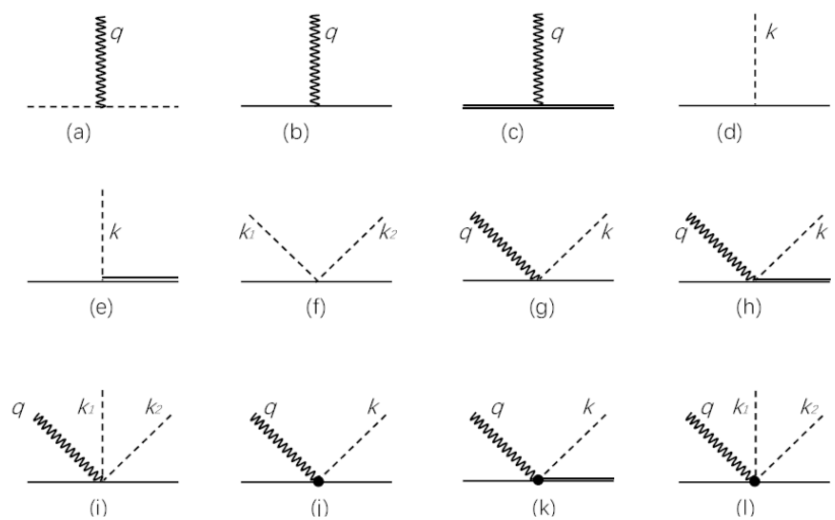
$$\mathcal{L}^{add} = ie \int dx \int dy \frac{D+F}{\sqrt{2}f} \bar{p}(x)\gamma^\mu \gamma_5 n(x) F(x-y) \int_x^y dz_\nu \int da\, \mathscr{A}^\nu(z-a)F(a)\partial_\mu \pi^+(y)$$

Nonlocal EFT

The Lagrangian is gauge invariant under the following transformation:

$$\pi^+(y) \to e^{i\alpha(y)}\pi^+(y), \quad p(x) \to e^{i\alpha(x)}p(x), \quad \mathscr{A}_\mu(x) \to \mathscr{A}_\mu(x) - \frac{1}{e}\partial_\mu \alpha'(x)$$

$$\alpha(x) = \int da \alpha'(x-a)F(a)$$



Nonlocal EFT

$$V^{\mu}_{(a)} = -ie^q_{\phi}(k_1^{\mu} + k_2^{\mu})\widetilde{F}(q), \qquad V^{\mu}_{(b)} = -ie^q_B \gamma^{\mu}\widetilde{F}(q), \qquad V^{\mu\nu\alpha}_{(c)} = -ie^q_T \gamma^{\mu\nu\alpha}\widetilde{F}(q)$$

$$V_{(d)} = \frac{C_{B\phi}}{f} \not{k}\gamma_5 \widetilde{F}(k), \quad V^{\mu}_{(e)} = \frac{C_{T\phi}}{f}\left[k^{\mu} - (Z + \tfrac{1}{2})\gamma^{\mu}\not{k}\right]\widetilde{F}(k), \quad V_{(f)} = \frac{C_{\phi\phi^{\dagger}}}{2f^2}(\not{k}_1 + \not{k}_2)\widetilde{F}(k_1)\widetilde{F}(k_2)$$

$$\begin{aligned}
V^{\mu}_{(g)} &= -\frac{e^q_{\phi} C_{B\phi}}{f}\gamma^{\mu}\gamma_5 \widetilde{F}(k)\widetilde{F}(q), \\
V^{\mu\nu}_{(h)} &= -\frac{e^q_{\phi} C_{T\phi}}{f}\left[g^{\mu\nu} - (Z + \tfrac{1}{2})\gamma^{\mu}\gamma^{\nu}\right]\widetilde{F}(k)\widetilde{F}(q) \\
V^{\mu}_{(i)} &= -\frac{e^q_{\phi} C_{\phi\phi^{\dagger}}}{2f^2}\gamma^{\mu}\widetilde{F}(k_1)\widetilde{F}(k_2)\widetilde{F}(q),
\end{aligned}$$

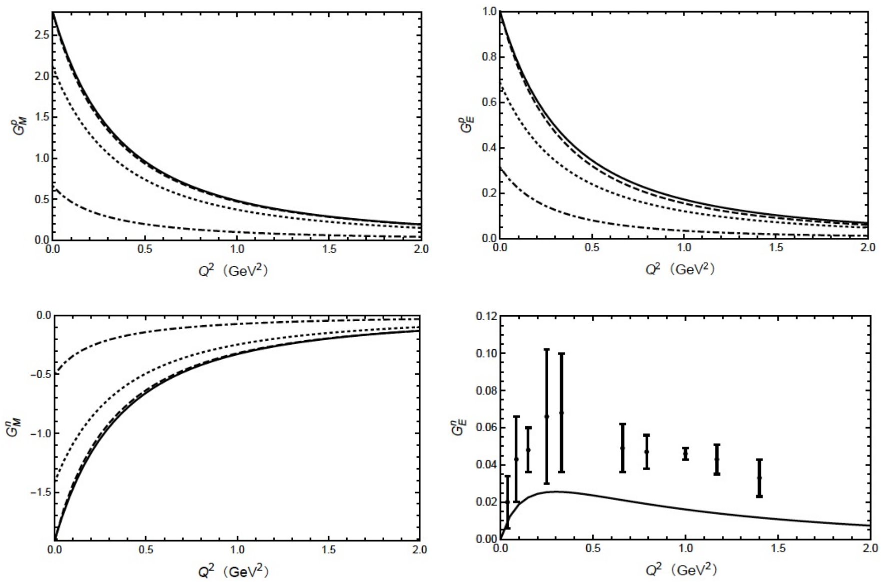
$$V^{\mu}_{(j)} = -\frac{e^q_{\phi} C_{B\phi}}{f}(\not{k} + \not{q})\gamma_5 \frac{2k^{\mu} + q^{\mu}}{2k \cdot q + q^2}\left[\widetilde{F}(k+q) - \widetilde{F}(k)\right]\widetilde{F}(q)$$

$$V^{\mu\nu}_{(k)} = -\frac{e^q_{\phi} C_{T\phi}}{f}\left[k^{\mu} + q^{\mu} - (Z - \tfrac{1}{2})\gamma^{\mu}(\not{k} + \not{q})\right]\frac{2k^{\nu} + q^{\nu}}{2k \cdot q + q^2}\left[\widetilde{F}(k+q) - \widetilde{F}(k)\right]\widetilde{F}(q)$$

$$\begin{aligned}
V^{\mu}_{(l)} = \frac{e^q_{\phi} C_{\phi\phi^{\dagger}}}{2f^2}(\not{k}_1 + \not{k}_2)\Bigg\{ & \frac{2k_1^{\mu} + q^{\mu}}{2k_1 \cdot q + q^2}\left[\widetilde{F}(k_1+q) - \widetilde{F}(k_1)\right]\widetilde{F}(k_2)\widetilde{F}(q) \\
&+ \frac{2k_2^{\mu} + q^{\mu}}{2k_2 \cdot q + q^2}\left[\widetilde{F}(k_2+q) - \widetilde{F}(k_2)\right]\widetilde{F}(k_1)\widetilde{F}(q)\Bigg\}
\end{aligned}$$

Form Factors

Nucleon electromagnetic Form Factors



<r_M>p = 0.848 (0.836) fm

<r_E>p = 0.857 (0.847) fm

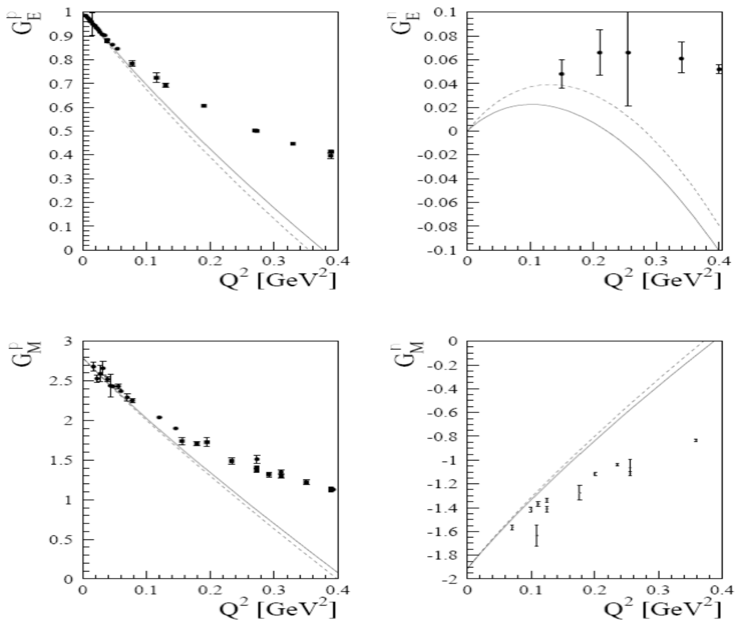
<r_M>n = 0.867 (0.889) fm

<r^2_E>n = -0.077 (-0.113) fm^2

F. C. He and P. Wang, Phys. Rev. D97 (2018) 036007

Form Factors

Nucleon form factors in traditional ChPT:



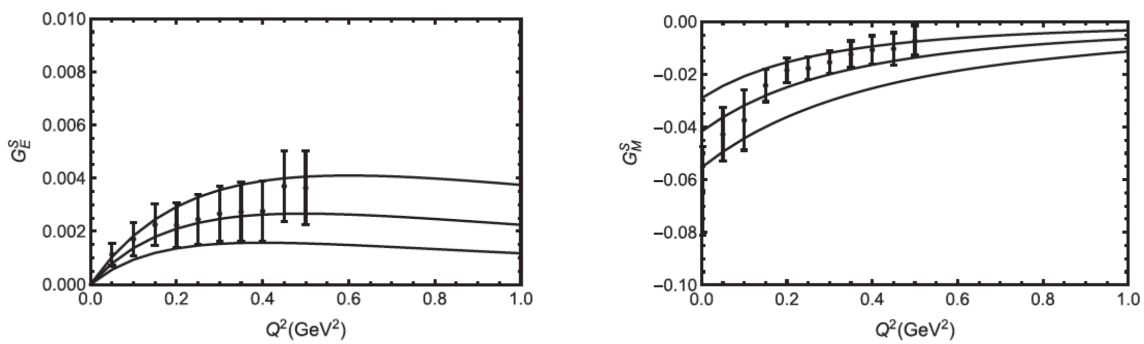
T. Fuchs, J. Gegelia, S. Scherer, J. Phys. G30 (2004) 1407

Form Factors

Strange form factors

$$< N(p')|J^s_\mu|N(p) >= \bar{u}(p')\left\{\gamma^\mu F^s_1(Q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m_N}F^s_2(Q^2)\right\}u(p)$$

$$G^s_E(Q^2) = F^s_1(Q^2) - \frac{Q^2}{4m_N^2}F^s_2(Q^2), \qquad G^s_M(Q^2) = F^s_1(Q^2) + F^s_2(Q^2)$$



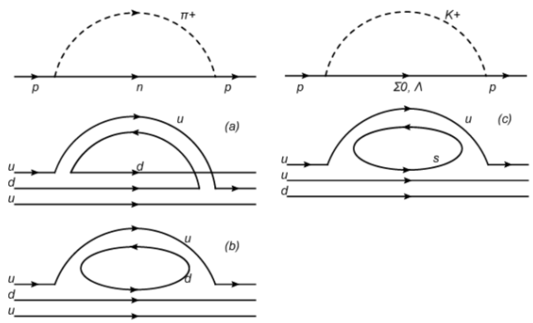
$$< (r^s_E)^2 >= -6\frac{dG^s_E(Q^2)}{dQ^2}|_{Q^2=0}, \qquad < (r^s_M)^2 >= -6\frac{dG^s_M(Q^2)}{dQ^2}|_{Q^2=0}$$

$$< (r^s_E)^2 >= -0.004 \pm 0.001 \text{ fm}^2 \text{ and } < (r^s_M)^2 >= -0.028 \pm 0.003 \text{ fm}^2$$

F. C. He and P. Wang, Phys. Rev. D98 (2018) 036007

Form Factors

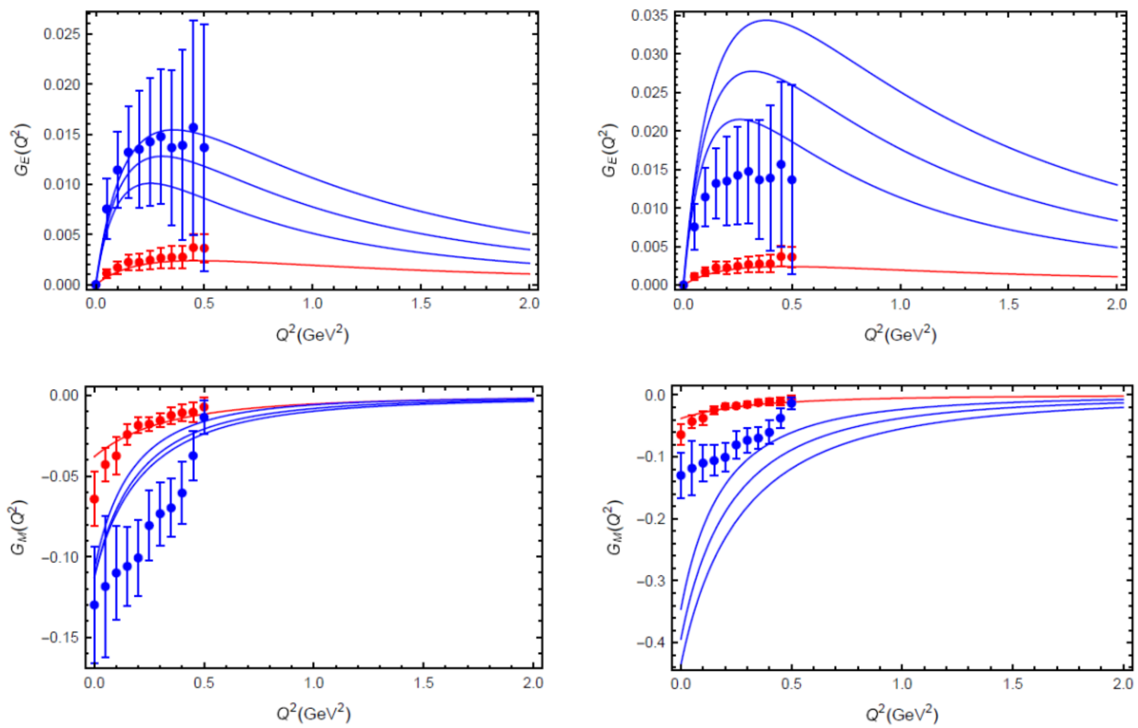
Light sea quark form factors



meson	full coefficient	quenched diagram	sea diagram
π^0	$\frac{1}{2}(D+F)^2$	$-\frac{1}{3}D^2+2DF-F^2$	$\frac{1}{3}(D^2+3F^2)$ [u] $\frac{1}{2}(D-F)^2$ [d]
π^+	$(D+F)^2$	$\frac{1}{3}(D^2+6DF-3F^2)$	$\frac{2}{3}(D^2+3F^2)$
π^-	0	$-(D-F)^2$	$(D-F)^2$
K^0	$(D-F)^2$	0	$(D-F)^2$
K^+	$\frac{2}{3}(D^2+3F^2)$	0	$\frac{2}{3}(D^2+3F^2)$

Form Factors

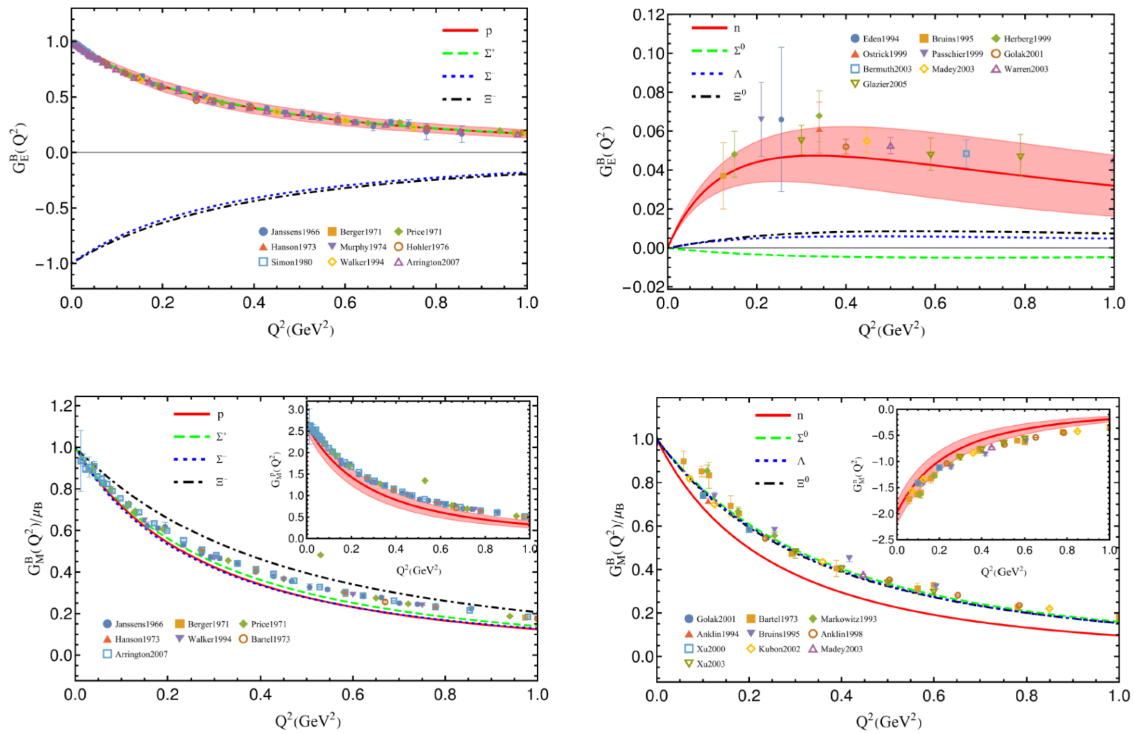
Light sea quark form factors



M. Y. Yang and P. Wang, Chin. Phys. C44 (2020) 053101

Form Factors

Octet form factors



M. Y. Yang and P. Wang, Phys. Rev. D102 (2020) 056024

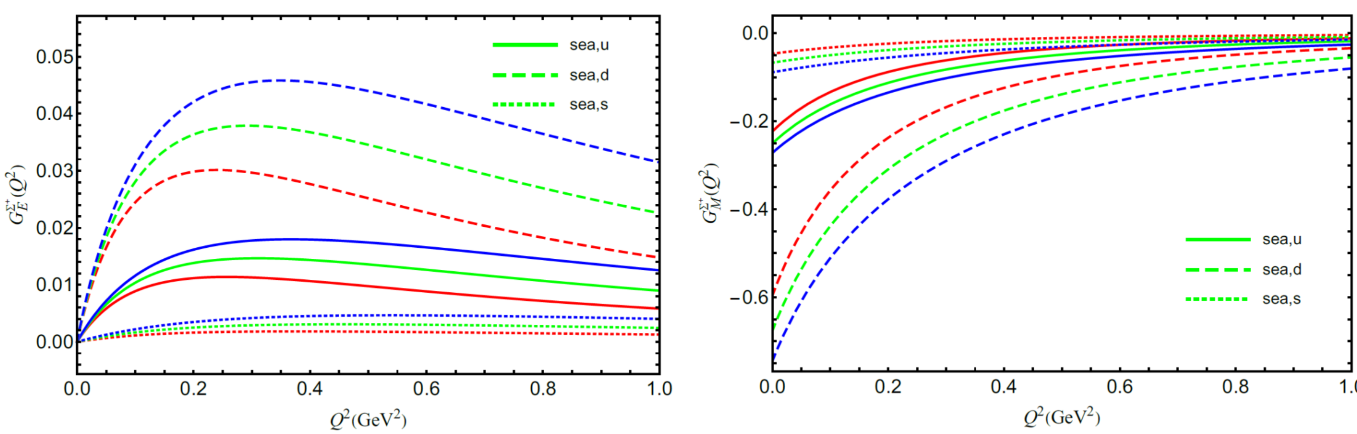
Form Factors

	Nonlocal [319]	Latt. [330]	Latt. [331]	IR [288]	EOMS [332]	NJL [202]	PCQM [333]	Exp. [39]
μ_p	2.644(159)	2.4(2)	2.3(3)	2.61	2.79	2.78	2.735(121)	2.793
μ_n	−1.984(216)	−1.59(17)	−1.45(17)	−1.69	−1.913	−1.81	−1.956(103)	−1.913
μ_{Σ^+}	2.421(147)	2.27(16)	2.12(18)	2.53	2.1(4)	2.62	2.537(201)	2.458(10)
μ_{Σ^0}	0.584(77)	–	–	0.76	0.5(2)	–	0.838(91)	–
μ_{Σ^-}	−1.253(8)	−0.88(8)	−0.85(10)	−1.00	−1.1(1)	−1.62	−0.861(40)	−1.160(25)
μ_{Λ}	−0.594(57)	–	–	−0.76	−0.5(2)	–	−0.867(74)	−0.613(4)
μ_{Ξ^0}	−1.380(169)	−1.32(4)	−1.07(7)	−1.51	−1.0(4)	−1.14	−1.690(142)	−1.250(14)
μ_{Ξ^-}	−0.725(77)	−0.71(3)	−0.57(5)	−0.93	−0.7(1)	−0.67	−0.840(87)	−0.651(80)

	Nonlocal [319]	Latt. [334]	Latt. [331]	IR [288]	EOMS [332]	NJL [202]	PCQM [333]	Exp. [39]
$\langle r_M^2\rangle_p$	0.785(132)	0.470(48)	0.71(8)	0.699	0.9(2)	0.76	0.909(84)	0.72(4)
$\langle r_M^2\rangle_n$	0.845(148)	0.478(50)	0.86(9)	0.790	0.8(2)	0.83	0.922(79)	0.75(2)
$\langle r_M^2\rangle_{\Sigma^+}$	0.765(131)	0.466(42)	0.66(5)	0.80(5)	1.2(2)	0.77	0.885(94)	–
$\langle r_M^2\rangle_{\Sigma^0}$	0.618(124)	0.432(38)	–	0.45(8)	1.1(2)	–	0.851(102)	–
$\langle r_M^2\rangle_{\Sigma^-}$	0.901(119)	0.483(49)	1.05(9)	1.20(13)	1.2(2)	0.92	0.951(83)	–
$\langle r_M^2\rangle_{\Lambda}$	0.620(126)	0.347(24)	–	0.48(9)	0.6(2)	–	0.852(103)	–
$\langle r_M^2\rangle_{\Xi^0}$	0.657(128)	0.384(22)	0.53(5)	0.61(12)	0.7(3)	0.44	0.871(99)	–
$\langle r_M^2\rangle_{\Xi^-}$	0.534(135)	0.336(18)	0.44(5)	0.50(16)	0.8(1)	0.26	0.840(109)	–

Form Factors

	Nonlocal [319]	Latt. [335]	Latt. [304]	IR [288]	EOMS [332]	NJL [202]	PCQM [333]	Exp. [39]
$\langle r_E^2 \rangle_p$	0.729(112)	0.685(66)	0.76(10)	0.717	0.878	0.76	0.767(113)	0.707(1)
$\langle r_E^2 \rangle_n$	−0.146(18)	−0.158(33)	–	−0.113	0.03(7)	−0.14	−0.014(1)	−0.116(2)
$\langle r_E^2 \rangle_{\Sigma^+}$	0.719(116)	0.749(72)	0.61(8)	0.60(2)	0.99(3)	0.92	0.781(108)	–
$\langle r_E^2 \rangle_{\Sigma^0}$	0.010(4)	–	–	−0.03(1)	0.10(2)	–	0	–
$\langle r_E^2 \rangle_{\Sigma^-}$	0.700(124)	0.657(58)	0.45(3)	0.67(3)	0.780	0.74	0.781(63)	0.61(16)
$\langle r_E^2 \rangle_{\Lambda}$	−0.015(4)	0.010(9)	–	0.11(2)	0.18(1)	–	0	–
$\langle r_E^2 \rangle_{\Xi^0}$	−0.015(7)	0.082(29)	–	0.13(3)	0.36(2)	0.24	0.014(8)	–
$\langle r_E^2 \rangle_{\Xi^-}$	0.601(127)	0.502(47)	0.37(2)	0.49(5)	0.61(1)	0.58	0.767(113)	–



M. Y. Yang and P. Wang, Phys. Rev. D105 (2022) 016006

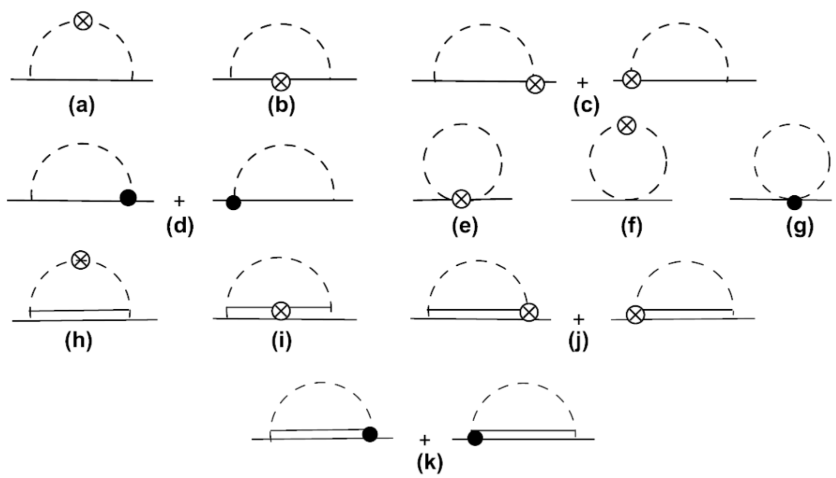
Parton Distributions Functions

Match quark operator to hadron operator: $$\mathcal{O}_q^{\mu_1\cdots\mu_n} = \sum_j c_{q/j}^{(n)}\,\mathcal{O}_j^{\mu_1\cdots\mu_n} \qquad c_{q/j}^{(n)} = \int_{-1}^{1} dx\, x^{n-1}\, q_j(x) \equiv \langle x^{n-1}\rangle_{q/j}$$

$$\langle N(p)|\mathcal{O}_q^{\mu_1\cdots\mu_n}|N(p)\rangle = 2\,\langle x^{n-1}\rangle_q\, p^{\mu_1}\cdots p^{\mu_n} \qquad \langle N(p)|\mathcal{O}_j^{\mu_1\cdots\mu_n}|N(p)\rangle = 2\, f_j^{(n)}\, p^{\mu_1}\cdots p^{\mu_n}$$

$$\int_{-1}^{1} dx\, x^{n-1} q(x) = \int_{-1}^{1} dx\, x^{n-1} \sum_j \int_0^1 dy\, f_j(y) \int_0^1 dz\, \delta(x-yz)\, q_j^v(z)$$

$$\longrightarrow\ q(x) = \sum_j \left(f_j \otimes q_j^v\right)(x) \equiv \sum_j \int_0^1 dy \int_0^1 dz\, \delta(x-yz)\, f_j(y)\, q_j^v(z)$$



Parton Distributions Functions

dbar - ubar asymmetry

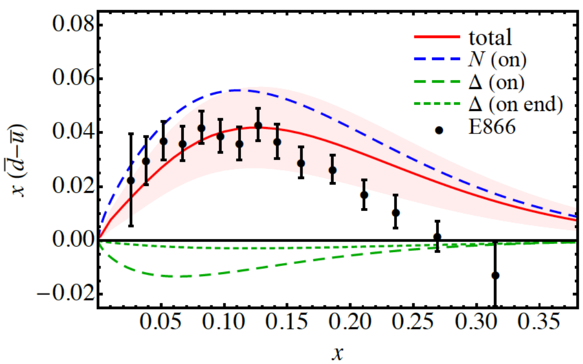
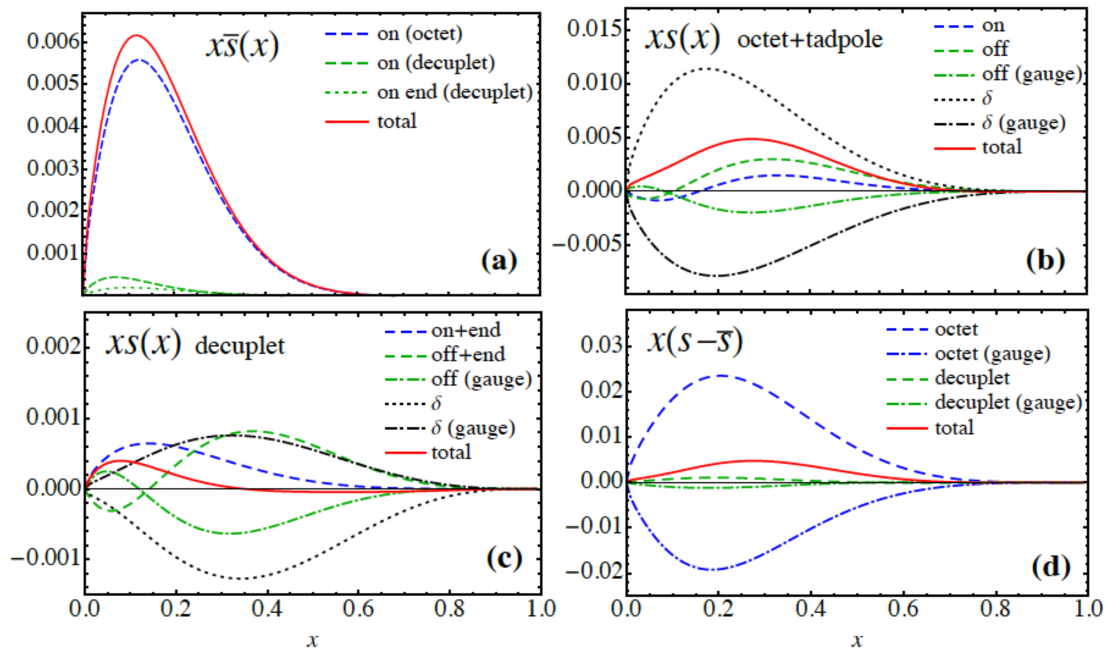


diagram		$\langle \bar{d} - \bar{u} \rangle$
πN (rbw)	$f_N^{(\text{on})}$	$0.152^{+0.032}_{-0.030}$
	$f_\pi^{(\delta)}$	$-(0.079^{+0.020}_{-0.018})$
	$\delta f_\pi^{(\delta)}$	$0.044^{+0.010}_{-0.009}$
total πN		$0.116^{+0.022}_{-0.022}$
$\pi\Delta$ (rbw)	$f_\Delta^{(\text{on})}$	$-(0.044^{+0.012}_{-0.012})$
	$f_\Delta^{(\text{on end})}$	$-(0.009^{+0.004}_{-0.003})$
	$f_\Delta^{(\delta)}$	$0.002^{+0.001}_{-0.001}$
	$f_\pi^{(\delta)}$	$0.039^{+0.010}_{-0.010}$
	$\delta f_\pi^{(\delta)}$	$-(0.022^{+0.005}_{-0.005})$
total $\pi\Delta$		$-(0.033^{+0.010}_{-0.010})$
π (bub)	$f_\pi^{(\delta)}$	$0.099^{+0.025}_{-0.022}$
	$\delta f_\pi^{(\delta)}$	$-(0.054^{+0.013}_{-0.012})$
total π bubble		$0.044^{+0.012}_{-0.010}$
total		$\mathbf{0.127^{+0.044}_{-0.042}}$
$x > 0$		$0.099^{+0.047}_{-0.046}$
$x = 0$		$0.028^{+0.008}_{-0.007}$
local		$0.159^{+0.041}_{-0.039}$
nonlocal		$-(0.032^{+0.008}_{-0.008})$

Y. Salamu, C. R. Ji, W. Melnitchouk, A. W. Thomas, P. Wang and X. G. Wang, Phys. Rev. D 100 (2019) 094026

Parton Distributions Functions

s-sbar asymmetry



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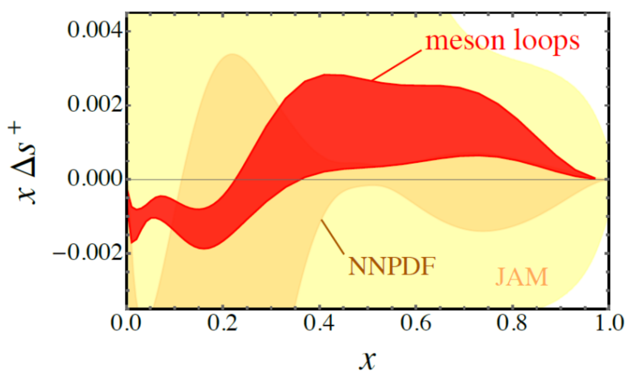
Phys. Rev. D 100 (2019) 094026

Parton Distributions Functions



F.C. He, C.R. Ji, W. Melnitchouk, Y. Salamu, A.W. Thomas, P. Wang and X.G. Wang, Phys. Rev. D 105 (2022) 094007

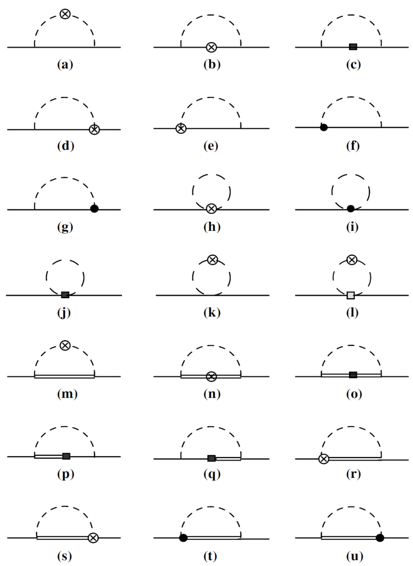
Λ_B (GeV)	$\langle\Delta s\rangle^{(\text{on})}_{B\,\text{rbw}}$	$\langle\Delta s\rangle^{(\text{off})}_{B\,\text{rbw}}$	$\langle\Delta s\rangle^{(\delta)}_{B\,\text{rbw}}$	$\langle\Delta s\rangle^{(\text{off})}_{\text{KR}}$	$\langle\Delta s\rangle^{(\delta)}_{\text{KR}}$	total octet	$\langle\Delta s\rangle^{(\delta)}_{\text{tad}}$
1.0	0.02	−0.90	0.73	0.87	−1.55	**−0.83**	**0.84**
1.1	0.00	−1.38	1.18	1.32	−2.48	**−1.36**	**1.35**
1.2	−0.04	−1.94	1.73	1.86	−3.65	**−2.04**	**1.99**
Λ_T (GeV)	$\langle\Delta s\rangle^{(\text{on})}_{T\,\text{rbw}}$	$\langle\Delta s\rangle^{(\text{off})}_{T\,\text{rbw}}$	$\langle\Delta s\rangle^{(\delta)}_{T\,\text{rbw}}$	$\langle\Delta s\rangle^{(\text{on})}_{TB\,\text{rbw}}$	$\langle\Delta s\rangle^{(\text{off})}_{TB\,\text{rbw}}$	$\langle\Delta s\rangle^{(\delta)}_{TB\,\text{rbw}}$	total decuplet
0.7	0.04	0.02	0.04	−0.45	0.42	−0.41	**−0.34**
0.8	0.14	0.08	0.18	−0.83	0.74	−0.77	**−0.46**
0.9	0.33	0.22	0.53	−1.24	1.05	−1.16	**−0.27**

$$\langle\boldsymbol{\Delta s}\rangle = [\mathbf{-0.51}, \mathbf{-0.26}] \times \mathbf{10^{-2}}$$

Parton Distributions Functions

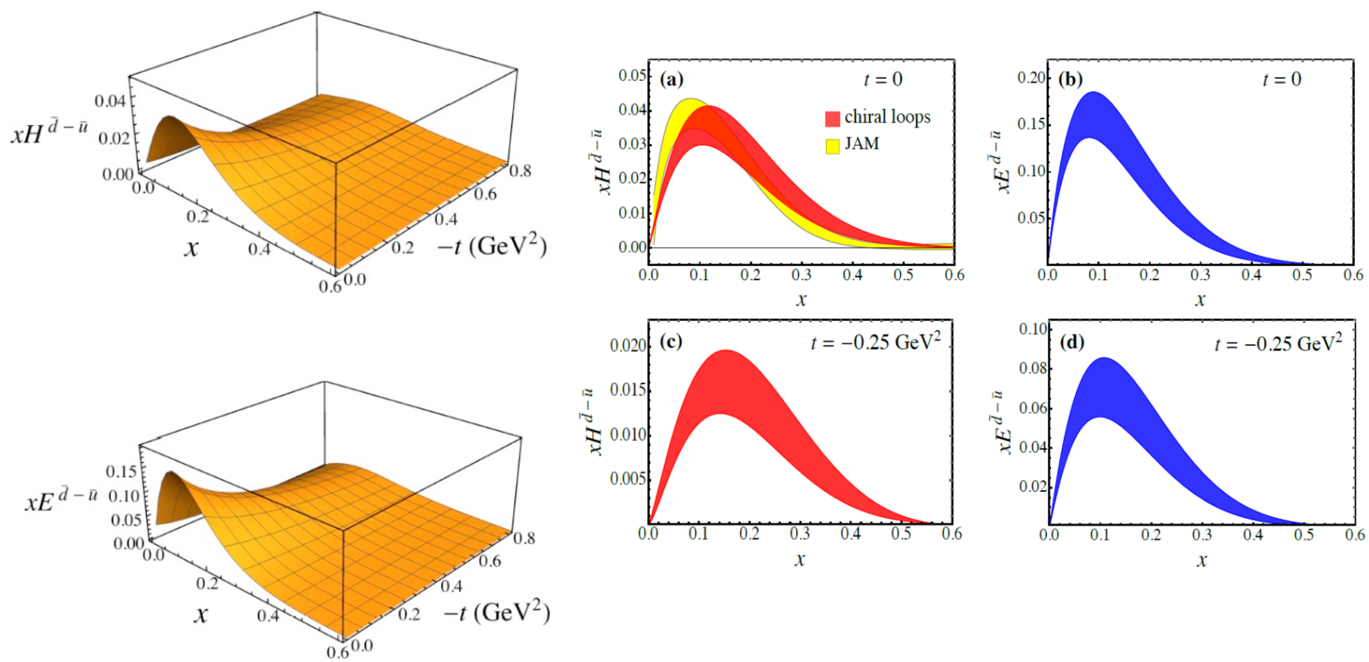
Generalized PDFs

$$\int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} e^{-ix\lambda}\langle p'|\bar{\psi}_q(\tfrac{1}{2}\lambda n)\not{n}\,\psi_q(-\tfrac{1}{2}\lambda n)|p\rangle = \bar{u}(p')\Big[\not{n} H^q(x,\xi,t) + \frac{i\sigma^{\mu\nu}n_\mu\Delta_\nu}{2M}\,E^q(x,\xi,t)\Big]u(p)$$

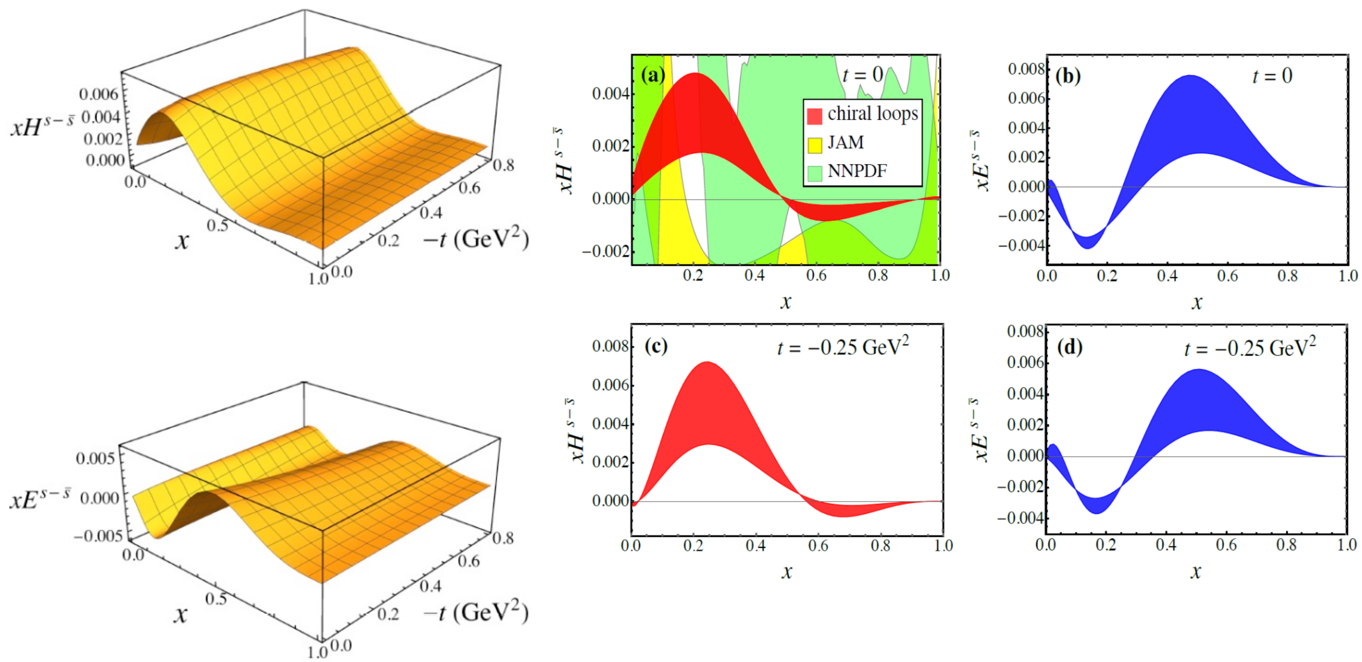


$$\begin{aligned}
H^q(x,t) = \sum_{\phi BT} \Big[& \big(f_{\phi B}^{(\mathrm{rbw})} + f_{\phi T}^{(\mathrm{rbw})} + f_{\phi}^{(\mathrm{bub})}\big) \otimes H_\phi^q \\
& + \bar{f}_{B\phi}^{(\mathrm{rbw})} \otimes H_B^q \; + \; \bar{f}_{B\phi}^{(\mathrm{KR})} \otimes H_B^{q(\mathrm{KR})} \; + \; \delta\bar{f}_B^{(\mathrm{KR})} \otimes H_B^{q(\mathrm{KR})} \\
& + \bar{f}_{T\phi}^{(\mathrm{rbw})} \otimes H_T^q \; + \; \bar{f}_{T\phi}^{(\mathrm{KR})} \otimes H_T^{q(\mathrm{KR})} \; + \; \delta\bar{f}_{T\phi}^{(\mathrm{KR})} \otimes H_T^{q(\mathrm{KR})} \\
& + \bar{f}_{B\phi}^{(\mathrm{rbw\ mag})} \otimes E_B^q \; + \; \bar{f}_{T\phi}^{(\mathrm{rbw\ mag})} \otimes E_T^q \; + \; \bar{f}_{BT}^{(\mathrm{rbw\ mag})} \otimes E_{BT}^q \\
& + \bar{f}_{\phi}^{(\mathrm{tad})} \otimes H_{\phi\phi^\dagger}^{q(\mathrm{tad})} \; + \; \delta\bar{f}_{\phi}^{(\mathrm{tad})} \otimes H_{\phi\phi^\dagger}^{q(\mathrm{tad})}\Big](x,t), \\
E^q(x,t) = \sum_{\phi BT} \Big[& \big(g_{\phi B}^{(\mathrm{rbw})} + g_{\phi T}^{(\mathrm{rbw})} + g_{\phi}^{\prime(\mathrm{bub})}\big) \otimes H_\phi^q \\
& + \bar{g}_{B\phi}^{(\mathrm{rbw})} \otimes H_B^q \; + \; \bar{g}_{B\phi}^{(\mathrm{KR})} \otimes H_B^{q(\mathrm{KR})} \; + \; \delta\bar{g}_B^{(\mathrm{KR})} \otimes H_B^{q(\mathrm{KR})} \\
& + \bar{g}_{T\phi}^{(\mathrm{rbw})} \otimes H_T^q \; + \; \bar{g}_{T\phi}^{(\mathrm{KR})} \otimes H_T^{q(\mathrm{KR})} \; + \; \delta\bar{g}_{T\phi}^{(\mathrm{KR})} \otimes H_T^{q(\mathrm{KR})} \\
& + \bar{g}_{B\phi}^{(\mathrm{rbw\ mag})} \otimes E_B^q \; + \; \bar{g}_{T\phi}^{(\mathrm{rbw\ mag})} \otimes E_T^q \; + \; \bar{g}_{BT}^{(\mathrm{rbw\ mag})} \otimes E_{BT}^q \\
& + \bar{g}_{\phi}^{(\mathrm{tad\ mag})} \otimes E_{\phi\phi^\dagger}^{q(\mathrm{tad})}\Big](x,t), \qquad (
\end{aligned}$$

Parton Distributions Functions



Parton Distributions Functions



F.C. He, C.R. Ji, W. Melnitchouk, A.W. Thomas and P. Wang, Phys. Rev. D 106 (2022) 054006

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Progress in Particle and Nuclear Physics

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Review

Nucleon form factors and parton distributions in nonlocal chiral effective theory

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ARTICLE INFO

Article history:
Available online xxxx

Keywords:
Nucleon form factors
Parton distribution functions
GPDs
Chiral effective theory
Nonlocal Lagrangian

ABSTRACT

We present a review of recent applications of nonlocal chiral effective theory to hadron structure studies. Starting from a nonlocal meson–baryon effective chiral Lagrangian, we show how the introduction of a correlation function representing the finite extent of hadrons regularizes the meson loop integrals and introduces momentum dependence in vertex form factors in a gauge invariant manner. We apply the framework to the calculation of nucleon electromagnetic form factors, unpolarized and polarized parton distributions, as well as transverse momentum dependent distributions and generalized parton distributions.

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Nonzero skewness GPDs:

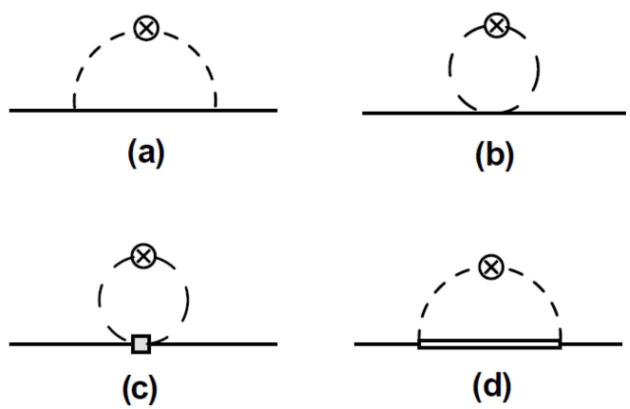
$$\int_{-\infty}^{\infty} \frac{\mathrm{d}\lambda}{2\pi} e^{-ix\lambda} \langle p' | \bar{\psi}_q(\tfrac{1}{2}\lambda n) \not{n}\, \psi_q(-\tfrac{1}{2}\lambda n) | p \rangle \;=\; \bar{u}(p') \Big[\not{n} H^q(x,\xi,t) + \frac{i\sigma^{\mu\nu} n_\mu \Delta_\nu}{2M} E^q(x,\xi,t) \Big] u(p)$$

$$x \;\equiv\; \frac{k_q^+}{P^+}, \qquad \xi \;\equiv\; -\frac{\Delta^+}{2P^+},$$

$$P \;=\; \frac{1}{2}(p+p'), \qquad \Delta \;=\; p' - p$$

$$\langle N(p') | J^\mu | N(p) \rangle \;=\; \bar{u}(p') \Big[\gamma^\mu F_1^N(t) + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2M} F_2^N(t) \Big] u(p) \;\equiv\; \int \mathrm{d}^4 k\, \widetilde{\Gamma}^\mu(k)$$

$$\bar{u}(p') \Big[\gamma^+ f(y,\xi,t) + \frac{i\sigma^{+\nu}\Delta_\nu}{2M} g(y,\xi,t) \Big] u(p) = \int \mathrm{d}^4 k\, \widetilde{\Gamma}^+(k)\, \delta\Big(y + \xi - \frac{k^+}{P^+} \Big) \;\equiv\; \Gamma^+$$



$$H_q(x,\xi,t) = \begin{cases} \int_x^1 \frac{d\,y}{y} f(y,\xi,t) H_{q/\pi}(x/y,\xi/y,t) & y > x > \xi \\ \int_{|\xi|}^1 \frac{d\,y}{y} f(y,\xi,t) H_{q/\pi}(x/y,\xi/y,t) & y > \xi > x \\ \int_{-\xi}^{\xi} \frac{d\,y}{2\xi} f(y,\xi,t) \frac{1}{\pi}\frac{\xi}{y} \int_{s_0}^{\infty} \frac{\mathrm{Im}\Phi_{q/\pi}(\frac{\frac{x}{\xi}+1}{2}, \frac{\frac{y}{\xi}+1}{2}, s)}{s-t+i\epsilon} & \xi > \{|y|,|x|\} \\ \int_{-x}^1 \frac{d\,y}{y} f(y,\xi,t) H_{q/\pi}(x/y,\xi/y,t) & -\xi > x > -1, \end{cases}$$

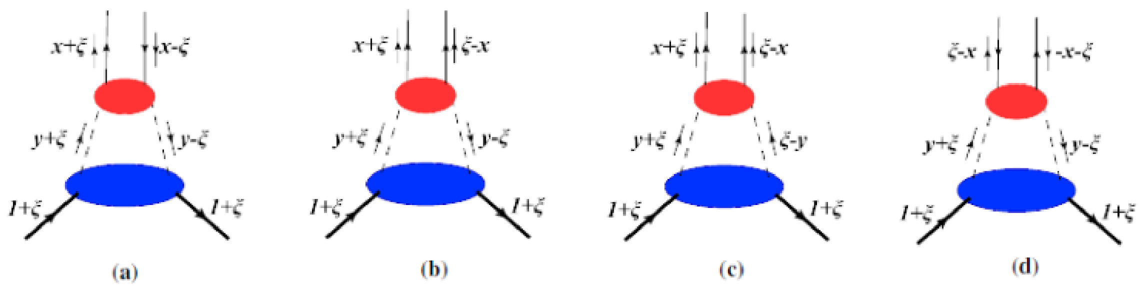
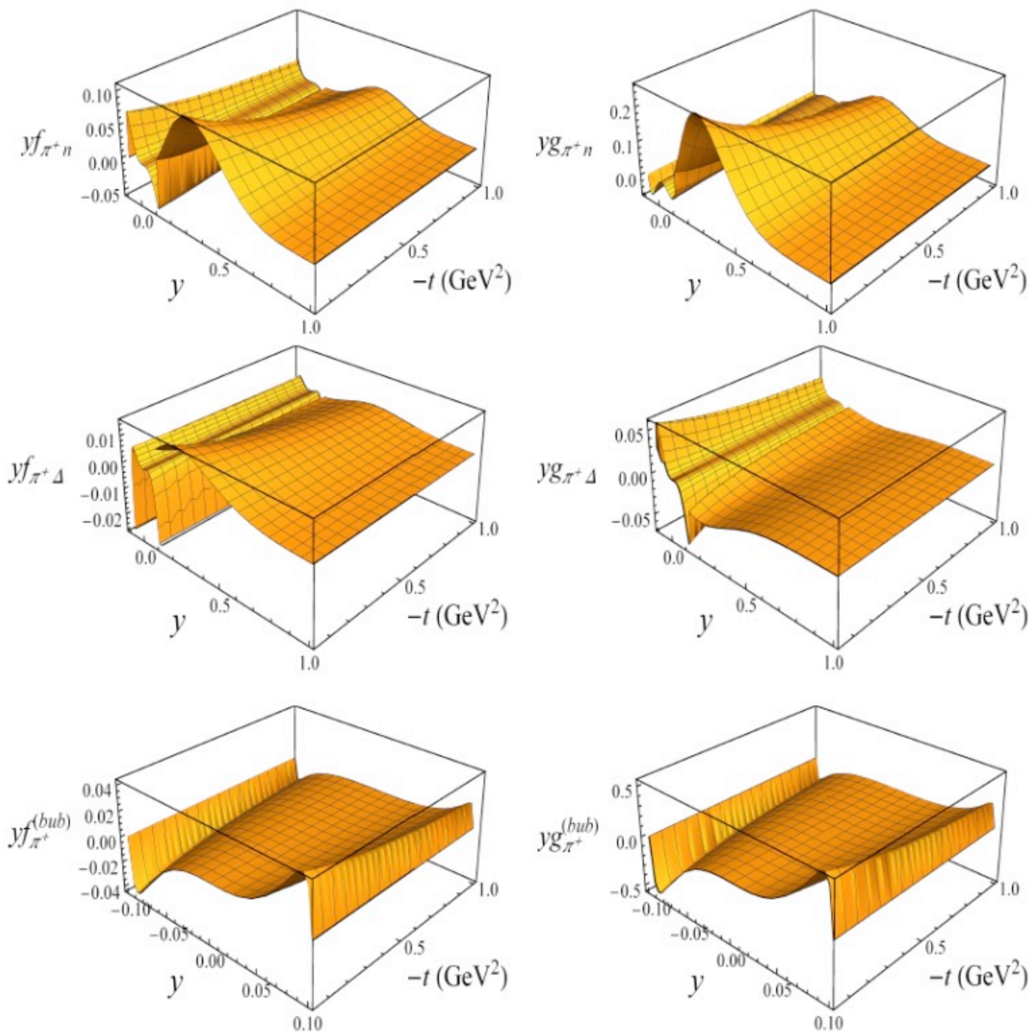
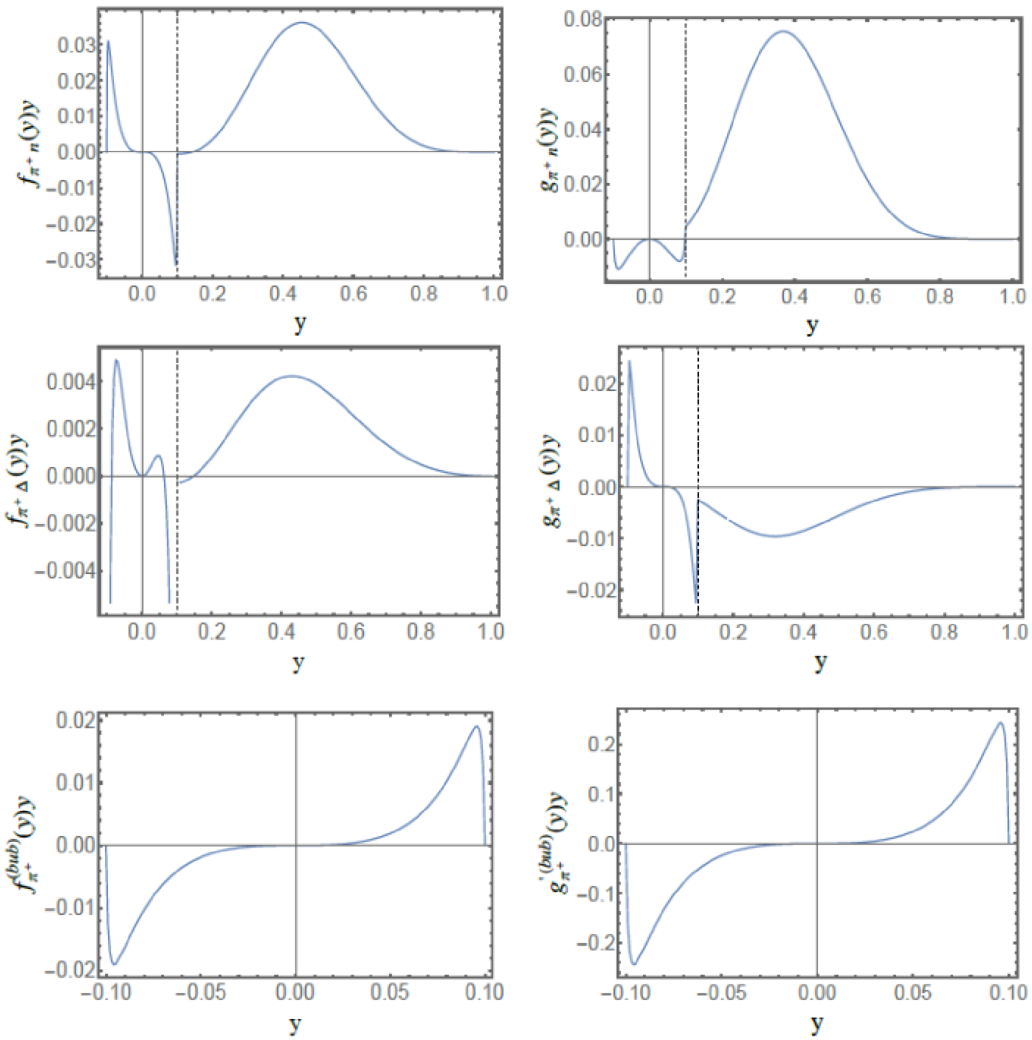
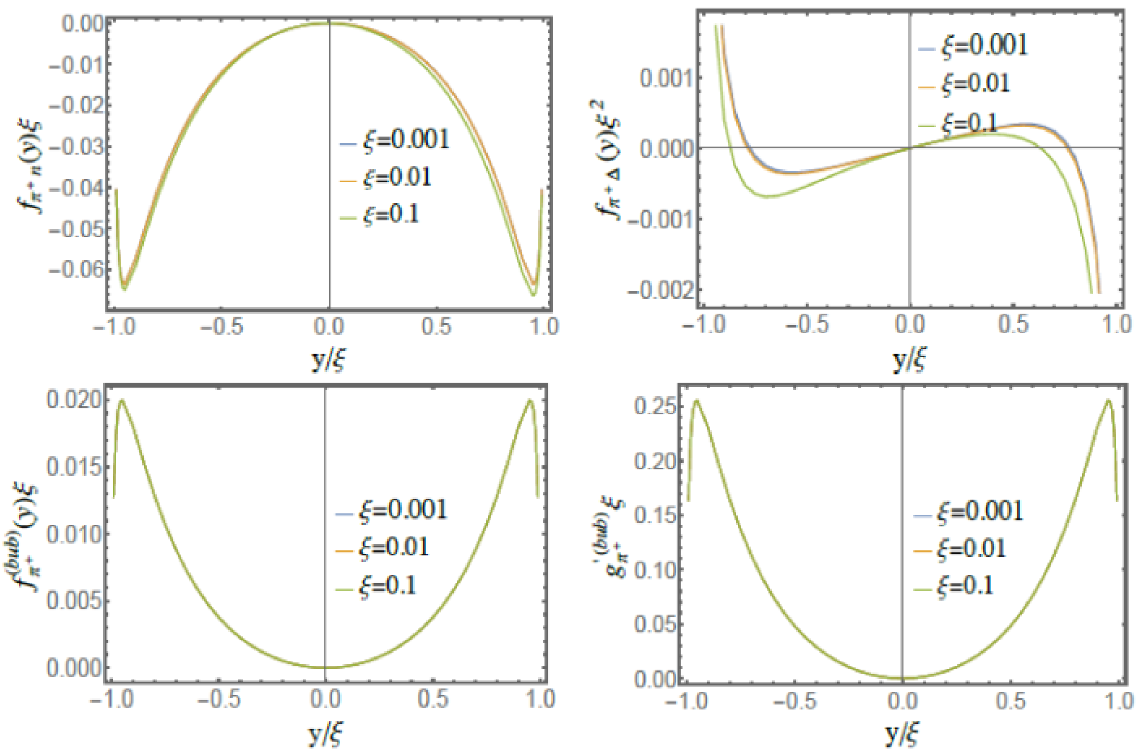


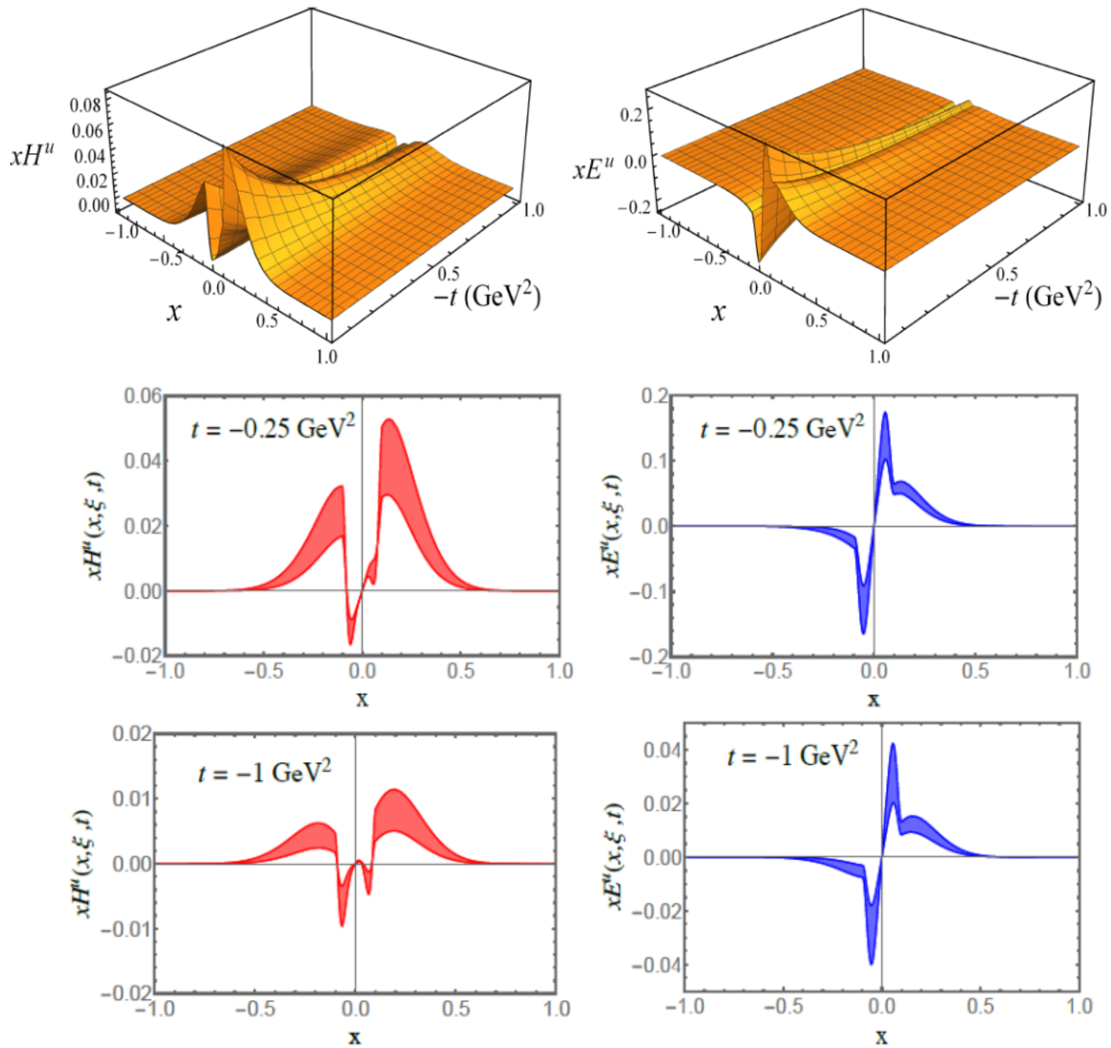
FIG. 2. The illustration for the convolution formula in Eq. (18). The dashed, thick solid and thin lines represent the pseudoscalar meson, proton and quark respectively. The process in Fig. (a) and Fig. (d) represent the DGLAP region for quark and antiquark, respcetivtly. The process shown in Fig. (b) and Fig. (c) represent the contribution to the ERBL region, the inputs are pion GPD and GDA, and correspond to the second and third lines of the convolution formula, respectively.



0.10
0.05
0.00
−0.05
$yf_{\pi^+ n}$
0.0
0.5
1.0
y
0.5
1.0
$-t$ (GeV^2)
0.2
0.1
0.0
$yg_{\pi^+ n}$
0.0
0.5
1.0
y
0.5
1.0
$-t$ (GeV^2)
0.01
0.00
−0.01
−0.02
$yf_{\pi^+ \Delta}$
0.0
0.5
1.0
y
0.5
1.0
$-t$ (GeV^2)
0.05
0.00
−0.05
$yg_{\pi^+ \Delta}$
0.0
0.5
1.0
y
0.5
1.0
$-t$ (GeV^2)
0.04
0.02
0.00
−0.02
−0.04
$yf^{(bub)}_{\pi^+}$
−0.10
−0.05
0.00
0.05
0.10
y
0.5
1.0
$-t$ (GeV^2)
0.5
0.0
−0.5
$yg^{(bub)}_{\pi^+}$
−0.10
−0.05
0.00
0.05
0.10
y
0.5
1.0
$-t$ (GeV^2)

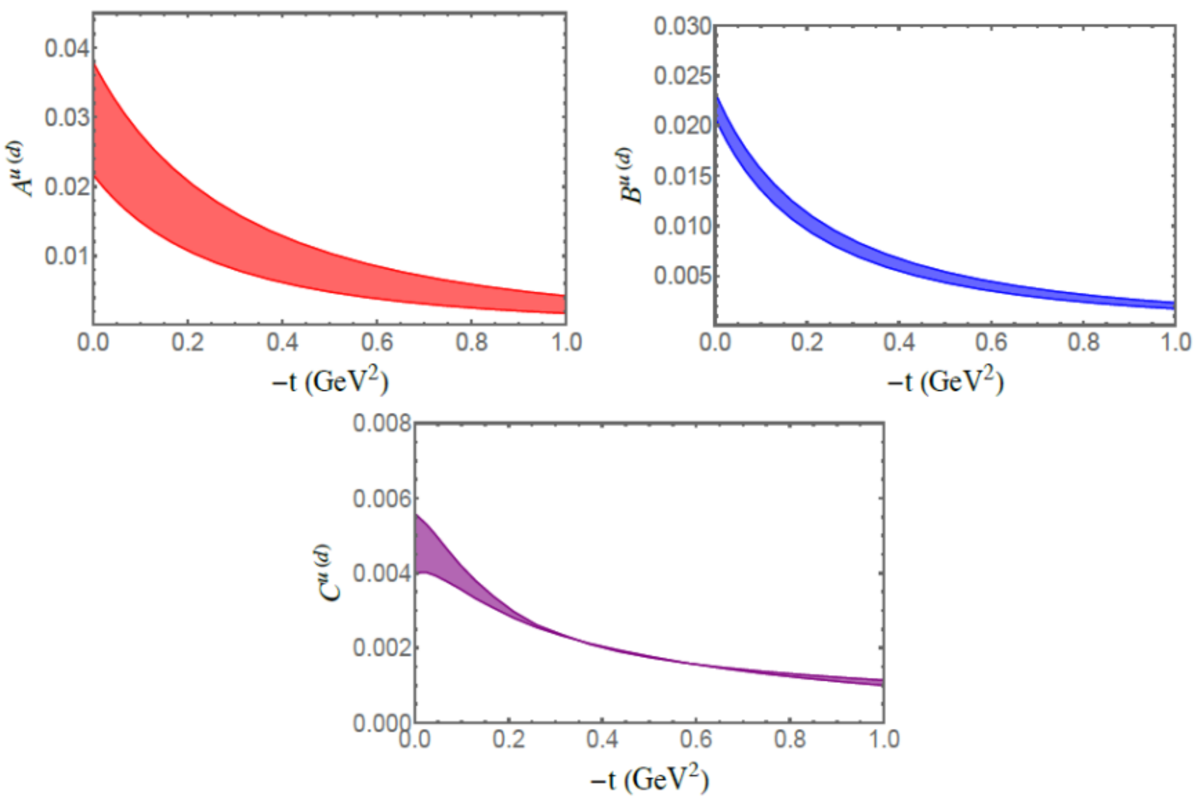

$f_{\pi^+ n}(y)y$
0.03
0.02
0.01
0.00
−0.01
−0.02
−0.03
0.0
0.2
0.4
0.6
0.8
1.0
y
$g_{\pi^+ n}(y)y$
0.08
0.06
0.04
0.02
0.00
0.0
0.2
0.4
0.6
0.8
1.0
y
$f_{\pi^+ \Delta}(y)y$
0.004
0.002
0.000
−0.002
−0.004
0.0
0.2
0.4
0.6
0.8
1.0
y
$g_{\pi^+ \Delta}(y)y$
0.02
0.01
0.00
−0.01
−0.02
0.0
0.2
0.4
0.6
0.8
1.0
y
$f^{(bub)}_{\pi^+}(y)y$
0.02
0.01
0.00
−0.01
−0.02
−0.10
−0.05
0.00
0.05
0.10
y
$g'^{(bub)}_{\pi^+}(y)y$
0.2
0.1
0.0
−0.1
−0.2
−0.10
−0.05
0.00
0.05
0.10
y

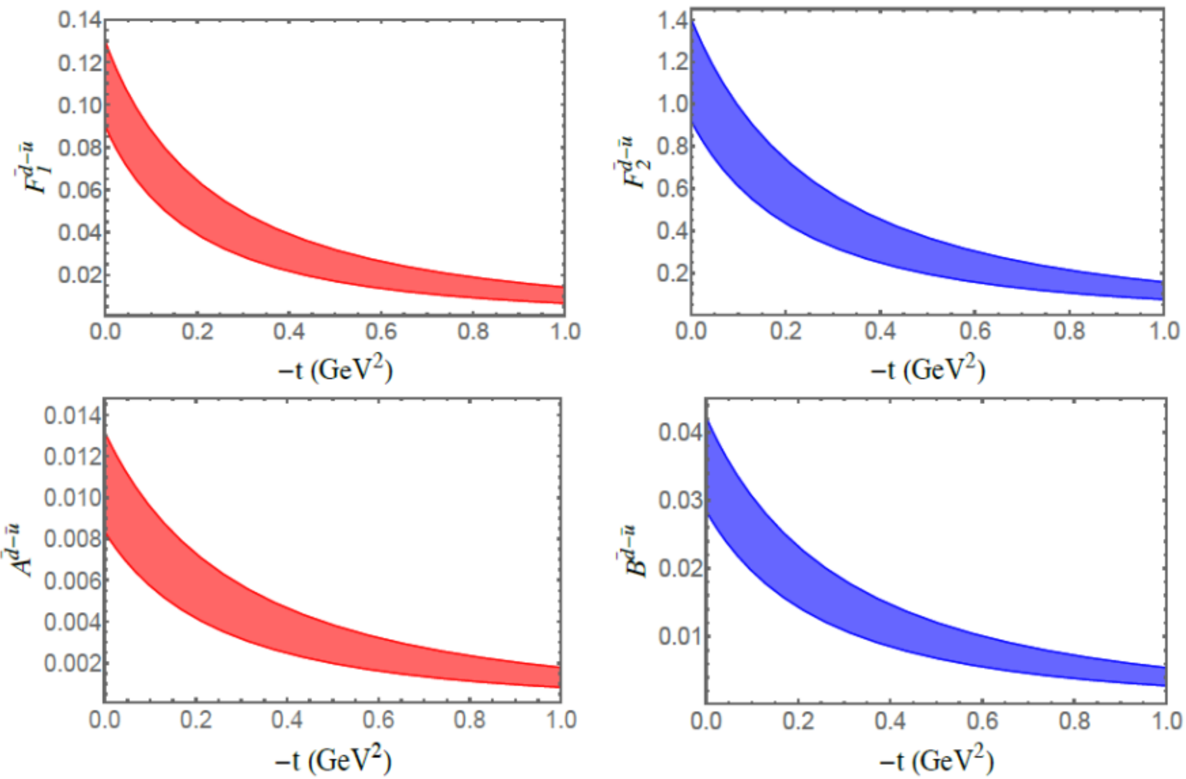

0.00
−0.01
−0.02
−0.03
−0.04
−0.05
−0.06
$f_{\pi^+ n}(y)\xi$
ξ=0.001
ξ=0.01
ξ=0.1
−1.0
−0.5
0.0
0.5
1.0
y/ξ
0.001
0.000
−0.001
−0.002
$f_{\pi^+ \Delta}(y)\xi^2$
ξ=0.001
ξ=0.01
ξ=0.1
−1.0
−0.5
0.0
0.5
1.0
y/ξ
0.020
0.015
0.010
0.005
0.000
$f^{(bub)}_{\pi^+}(y)\xi$
ξ=0.001
ξ=0.01
ξ=0.1
−1.0
−0.5
0.0
0.5
1.0
y/ξ
0.25
0.20
0.15
0.10
0.05
0.00
$g'^{(bub)}_{\pi^+}\xi$
ξ=0.001
ξ=0.01
ξ=0.1
−1.0
−0.5
0.0
0.5
1.0
y/ξ



xH^u
0.08
0.06
0.04
0.02
0.00
−1.0
−0.5
0.0
0.5
1.0
x
0.5
1.0
−t (GeV²)
xE^u
0.2
0.0
−0.2
t = −0.25 GeV²
xH^u(x,ξ,t)
xE^u(x,ξ,t)
x
t = −1 GeV²

$$\int_{-1}^{1} \mathrm{d}x\, x\, H^q(x,\xi,t) = A^q(t) + (2\xi)^2 C^q(t), \qquad \int_{-1}^{1} \mathrm{d}x\, x\, E^q(x,\xi,t) = B^q(t) - (2\xi)^2 C^q(t)$$





$F_1^{\bar{d}-\bar{u}}$
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.0 0.2 0.4 0.6 0.8 1.0
−t (GeV^2)
$F_2^{\bar{d}-\bar{u}}$
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0 0.2 0.4 0.6 0.8 1.0
−t (GeV^2)
$A^{\bar{d}-\bar{u}}$
0.014
0.012
0.010
0.008
0.006
0.004
0.002
0.0 0.2 0.4 0.6 0.8 1.0
−t (GeV^2)
$B^{\bar{d}-\bar{u}}$
0.04
0.03
0.02
0.01
0.0 0.2 0.4 0.6 0.8 1.0
−t (GeV^2)

Gravitational Form Factors

$$\langle p'|T_{\mu\nu}|p\rangle = \bar{u}(p', m_N)[\frac{A(t)}{2}(\gamma_\mu P_\nu + \gamma_\nu P_\mu) + iB(t)\frac{P_\mu \sigma_{\nu\alpha} q^\alpha + P_\nu \sigma_{\mu\alpha} q^\alpha}{4m_N} + D(t)\frac{q_\mu q_\nu - \eta_{\mu\nu} q^2}{4m_N}]u(p, m_N)$$

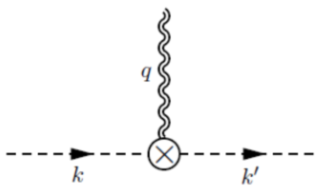
$$S^{(2)}_{\pi\pi} = \int d^4x \sqrt{-g} \left\{ \frac{F^2}{4} g^{\mu\nu} \, \mathrm{Tr}(D_\mu U (D_\nu U)^\dagger) + \frac{F^2}{4} \mathrm{Tr}(\chi U^\dagger + U \chi^\dagger) \right\}$$

$$T^{(\pi\pi,2)}_{\mu\nu}(x) \;=\; \frac{2}{\sqrt{-g(x)}} \frac{\delta S^{(\pi\pi,2)}_{\mathrm{m}}}{\delta g^{\mu\nu}(x)}|_{g=\eta}$$

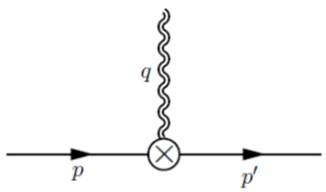
$$T^{(\pi\pi,2)}_{\mu\nu}(x) = \frac{F^2}{4} \mathrm{Tr}(D_\mu U (D_\nu U)^\dagger + D_\nu U (D_\mu U)^\dagger) - \eta_{\mu\nu} \left\{ \frac{F^2}{4} \mathrm{Tr}(D^\alpha U (D_\alpha U)^\dagger) + \frac{F^2}{4} \mathrm{Tr}(\chi U^\dagger + U \chi^\dagger) \right\}$$

$$\begin{aligned} T^{(\pi N,1)}_{\mu\nu} \;=\; & \frac{i}{4} \left(\bar\Psi \gamma_\mu D_\nu \Psi + \bar\Psi \gamma_\nu D_\mu \Psi - D_\mu \bar\Psi \gamma_\nu \Psi - D_\nu \bar\Psi \gamma_\mu \Psi \right) + \frac{g_A}{4} \left(\bar\Psi \gamma_\mu \gamma_5 u_\nu \Psi + \bar\Psi \gamma_\nu \gamma_5 u_\mu \Psi \right) \\ & - \eta_{\mu\nu} \left[\frac{1}{2} \bar\Psi i\gamma^\alpha D_\alpha \Psi - \frac{1}{2} D_\alpha \bar\Psi i\gamma^\alpha \Psi - m \bar\Psi \Psi + \frac{g_A}{2} \bar\Psi \gamma^\alpha \gamma_5 u_\alpha \Psi \right], \qquad (19) \end{aligned}$$

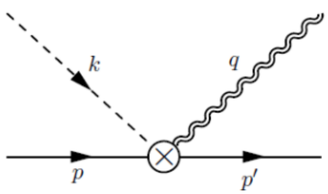
$$
\begin{aligned}
T_{\mu\nu}^{(\pi N,2)} &= -\frac{c_2}{8m^2}\left[\langle u_\mu u^\beta\rangle\left(\bar\Psi\left\{D_\nu, D_\beta\right\}\Psi + \left\{D_\nu, D_\beta\right\}\bar\Psi\Psi\right) + \langle u^\alpha u_\mu\rangle\left(\bar\Psi\left\{D_\alpha, D_\nu\right\}\Psi + \left\{D_\alpha, D_\nu\right.\right.\right. \\
&+ \langle u_\nu u^\beta\rangle\left(\bar\Psi\left\{D_\mu, D_\beta\right\}\Psi + \left\{D_\mu, D_\beta\right\}\bar\Psi\Psi\right) + \langle u^\alpha u_\nu\rangle\left(\bar\Psi\left\{D_\alpha, D_\mu\right\}\Psi + \left\{D_\alpha, D_\mu\right\}\bar\Psi\Psi\right)\Big] \\
&+ \frac{ic_2}{16m^2}\partial^\rho\left\{\langle u^\alpha u^\beta\rangle\left[D_\alpha\bar\Psi\left(\eta_{\beta\nu}\sigma_{\rho\mu} + \eta_{\beta\mu}\sigma_{\rho\nu}\right)\Psi + D_\beta\bar\Psi\left(\eta_{\alpha\nu}\sigma_{\rho\mu} + \eta_{\alpha\mu}\sigma_{\rho\nu}\right)\Psi\right]\right. \\
&- \left.\langle u^\alpha u^\beta\rangle\left[\bar\Psi\left(\eta_{\beta\nu}\sigma_{\rho\mu} + \eta_{\beta\mu}\sigma_{\rho\nu}\right)D_\alpha\Psi + \bar\Psi\left(\eta_{\alpha\nu}\sigma_{\rho\mu} + \eta_{\alpha\mu}\sigma_{\rho\nu}\right)D_\beta\Psi\right]\right\} \\
&+ \frac{c_2}{4m^2}\left\{\partial^\alpha\left[\langle u_\alpha u_\mu\rangle D_\nu(\bar\Psi\Psi) + \langle u_\alpha u_\nu\rangle D_\mu(\bar\Psi\Psi) - \langle u_\mu u_\nu\rangle D_\alpha(\bar\Psi\Psi)\right]\right\} \\
&+ c_3\,\bar\Psi\langle u_\mu u_\nu\rangle\Psi + \frac{ic_4}{8}\bar\Psi\left(\sigma_{\nu\beta}\left[u_\mu, u^\beta\right] + \sigma_{\alpha\nu}\left[u^\alpha, u_\mu\right] + \sigma_{\mu\beta}\left[u_\nu, u^\beta\right] + \sigma_{\alpha\mu}\left[u^\alpha, u_\nu\right]\right)\Psi \\
&+ \frac{c_8}{4}(\eta_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu)\bar\Psi\Psi + \frac{ic_9}{2m}\left(\eta_{\mu\alpha}\eta_{\nu\beta}\partial^2 + \eta_{\mu\nu}\partial_\alpha\partial_\beta - \eta_{\mu\alpha}\partial_\nu\partial_\beta - \eta_{\nu\alpha}\partial_\mu\partial_\beta\right) \\
&\times \left(\bar\Psi\gamma^\alpha D^\beta\Psi - D^\beta\bar\Psi\gamma^\alpha\Psi + \bar\Psi\gamma^\beta D^\alpha\Psi - D^\alpha\bar\Psi\gamma^\beta\Psi\right) \\
&- \eta_{\mu\nu}\left[\frac{1}{2}\,\bar\Psi\,i\gamma^\alpha D_\alpha\Psi - \frac{1}{2}\,D_\alpha\bar\Psi\,i\gamma^\alpha\Psi - m\bar\Psi\Psi + \frac{g_A}{2}\,\bar\Psi\gamma^\alpha\gamma_5 u_\alpha\Psi\right. \\
&+ c_1\langle\chi_+\rangle\bar\Psi\Psi - \frac{c_2}{8m^2}\langle u_\alpha u_\beta\rangle\left(\bar\Psi\left\{D^\alpha, D^\beta\right\}\Psi + \left\{D^\alpha, D^\beta\right\}\bar\Psi\Psi\right) + \frac{c_3}{2}\,\langle u_\alpha u^\alpha\rangle\bar\Psi\Psi + \frac{ic_4}{4}\,\bar\Psi\sigma^{\alpha\beta}\left[u_\alpha, u_\beta\right]\Psi \\
&+ \left. c_5\bar\Psi\hat\chi_+\Psi + \frac{c_6}{8m}\bar\Psi\sigma^{\alpha\beta}F^+_{\alpha\beta}\Psi + \frac{c_7}{8m}\bar\Psi\sigma^{\alpha\beta}\langle F^+_{\alpha\beta}\rangle\Psi\right],
\end{aligned}
$$



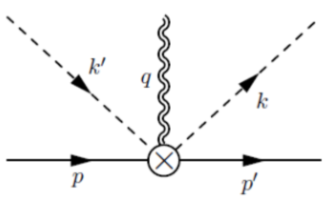
$$i[(k_\mu k'_\nu + k'_\mu k_\nu) - g_{\mu\nu}(k.k' - m_\pi^2)]$$



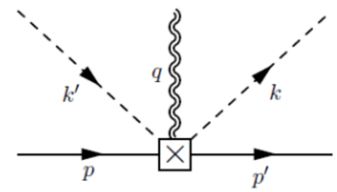
$$\tfrac{i}{4}[(p'_\mu + p_\mu)\gamma_\nu + (p'_\nu + p_\nu)\gamma_\mu] - g_{\mu\nu}[\tfrac{1}{2}(\not p' + \not p) - m]$$



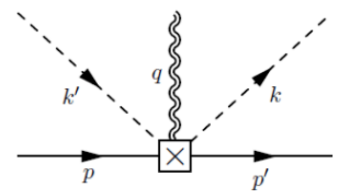
$$\tfrac{g_{NN\pi}}{2f_\pi}(\gamma_\mu\gamma^5 k_\nu + \gamma_\nu\gamma^5 k_\nu - 2g_{\mu\nu} \not k\gamma^5)$$



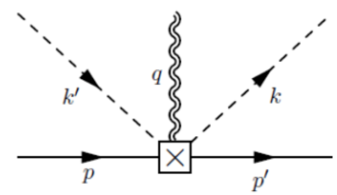
$$i\tfrac{g_{NN\pi\pi}}{f_\pi^2}[(k'_\mu + k_\mu)\gamma_\nu + (k'_\nu + k_\nu)\gamma_\mu - \tfrac{1}{2}g_{\mu\nu}(\not k + \not k')]$$



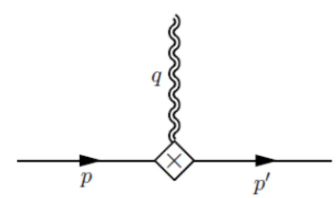
$$ic_1\tfrac{m_\pi^2 C_{NN\pi\pi}}{f_\pi^2}g_{\mu\nu}$$



$$ic_2\tfrac{C_{NN\pi\pi}}{f_\pi^2}(k.pk_\mu p'_\nu + k.pk_\nu p'_\mu$$



$$ic_3\tfrac{C_{NN\pi\pi}}{f_\pi^2}(k_\mu k'_\nu + k_\nu k_\mu - g_{\mu\nu}k'.k)$$



$$iic_8[(k_\mu k'_\nu + k'_\mu k_\nu) - g_{\mu\nu}(k.k' - m_\pi^2)]$$

How to calculate the gravitational form factors with nonlocal interaction?

Electromagnetic interaction:
local U(1) symmetry

→

Gravitational Interaction:
Local translation invarance

$$\mathcal{L}_{\rm F}^0 = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi \longrightarrow J^\mu(x) = e\bar{\psi}(x)\gamma^\mu\psi(x) \longrightarrow J^\mu(x)A_\mu(x)$$

Local translation: $\psi(x) \to \psi'(x) = \psi(x')$ $\quad x'^\mu = x^\mu + \theta^\mu(x)$

$$\begin{aligned}
\mathcal{L}_{\rm F}^0(x) \;\to\; & \frac{i}{2}\left[\bar{\psi}(x')\gamma^\mu\partial_\mu\psi(x') - (\partial_\mu\bar{\psi}(x'))\gamma^\mu\psi(x')\right] - m\bar{\psi}(x')\psi(x') \\
= \; & \frac{i}{2}\left[\bar{\psi}(x')\gamma^\mu\partial'_\mu\psi(x') - (\partial'_\mu\bar{\psi}(x'))\gamma^\mu\psi(x')\right] - m\bar{\psi}(x')\psi(x') \\
& +\frac{i}{2}\left[\bar{\psi}(x')\gamma^\mu\partial'^\nu\psi(x') - (\partial'^\nu\bar{\psi}(x'))\gamma^\mu\psi(x')\right]\partial_\mu\theta_\nu(x),
\end{aligned}$$

To cancel the second line: $g\bar{\psi}(x)i\gamma^\mu h_{\mu\nu}(x)\partial^\nu\psi(x) - g(h_{\mu\nu}(x)\partial^\nu\bar{\psi}(x))\gamma^\mu\psi(x)$

Transformation of *h*: $h_{\mu\nu}(x)\partial_\rho \to \left[h_{\mu\nu}(x') - \frac{1}{g}\partial_\mu\theta_\nu(x)\right]\partial'_\rho$

$$
\begin{aligned}
\mathcal{L}_{\mathrm{F}}(x) &= \frac{i}{2}(\eta_{\mu\nu} + gh_{\mu\nu}(x))\left[\bar{\psi}(x)\gamma^{\mu}\partial^{\nu}\psi(x) - (\partial^{\nu}\bar{\psi}(x))\gamma^{\mu}\psi(x)\right] - m\bar{\psi}(x)\psi(x) \\
&= \mathcal{L}_{\mathrm{F}}^{0}(x) + gh_{\mu\nu}(x)\tilde{T}_{\mathrm{F}}^{\mu\nu}(x) \\
&= \frac{i}{2}\left[\bar{\psi}(x)\gamma^{\mu}D_{\mu}\psi(x) - (D_{\mu}\bar{\psi}(x))\gamma^{\mu}\psi(x)\right] - m\bar{\psi}(x)\psi(x),
\end{aligned}
$$

$$
\tilde{T}_{\mathrm{F}}^{\mu\nu}(x) = \frac{i}{2}\left[\bar{\psi}(x)\gamma^{\mu}\partial^{\nu}\psi(x) - (\partial^{\nu}\bar{\psi}(x))\gamma^{\mu}\psi(x)\right] \qquad D_{\mu} = \partial_{\mu} + gh_{\mu\nu}(x)\partial^{\nu}
$$

Leading order: $h_{\mu\nu}(x) \to h_{\mu\nu}(x') - \frac{1}{g}\partial_{\mu}\theta_{\nu}(x)$

Next-to leading order: $h_{\mu\nu}(x) \to h_{\mu\nu}(x') - \frac{1}{g}\partial_{\mu}\theta_{\nu}(x) - \left[h_{\mu\rho}(x') - \frac{1}{g}\partial_{\mu}\theta_{\rho}(x)\right]\partial^{\rho}\theta_{\nu}(x)$

It is interesting that the tensor $\tilde{T}_{\mathrm{F}}^{\mu\nu}(x)$ is not exactly the same as the energy-momentum tensor obtained by the Nöther theory from the global translation symmetry. This is different from the electromagnetic case, where the electromagnetic current in the interaction $A_{\mu}(x)J^{\mu}(x)$ obtained from the local $U(1)$ symmetry is the same as that obtained from the global $U(1)$ symmetry.

$$\tilde{T}_{\rm F}^{\mu\nu}(x) \to \tilde{T}_{\rm F}^{\mu\rho}(x')(\delta^\nu_\rho + \partial^\nu\theta_\rho(x)) \qquad h_{\mu\nu}(x)\tilde{T}_{\rm F}^{\mu\nu}(x) \to \left[h_{\mu\nu}(x') - \frac{1}{g}\partial_\mu\theta_\nu(x)\right]\tilde{T}_{\rm F}^{\mu\nu}(x')$$

$$A_\mu(x)J^\mu(x) \to \left[A_\mu(x) - \frac{1}{e}\partial_\mu\theta(x)\right]J^\mu(x)$$

$$h_{\mu\nu}(x) = h_{\nu\mu}(x), \quad \eta^{\mu\nu}h_{\mu\nu}(x) = 0, \quad \partial^\mu h_{\mu\nu}(x) = 0$$

$$\mathcal{L}_{\rm F}(x) = \mathcal{L}^0_{\rm F}(x) + g h_{\mu\nu}(x) T_{\rm F}^{\mu\nu}(x)$$

Belinfante-Rosenfeld
EM tensor:

$$T_{\rm F}^{\mu\nu}(x) = \frac{i}{4}(\bar\psi\gamma^\mu\partial^\nu\psi + \bar\psi\gamma^\nu\partial^\mu\psi) - \frac{i}{4}((\partial^\mu\bar\psi)\gamma^\nu\psi + (\partial^\nu\bar\psi)\gamma^\mu\psi) - \eta^{\mu\nu}\mathcal{L}^0_{\rm F}$$

$$\begin{aligned}\mathcal{L}_{\rm F}(x) \;\to\; & \mathcal{L}_{\rm F}(x') + \frac{i}{4}(\partial_\mu\theta_\nu(x) - \partial_\nu\theta_\mu(x))\left[\bar\psi(x')\gamma^\mu\partial'^\nu\psi(x') - (\partial'^\nu\bar\psi(x'))\gamma^\mu\psi(x')\right] \\ & + (\partial_\mu\theta^\mu(x))\left\{\frac{i}{2}\left[\bar\psi(x')\gamma^\nu\partial'_\nu\psi(x') - (\partial'_\nu\bar\psi(x'))\gamma^\nu\psi(x')\right] - m\bar\psi(x')\psi(x')\right\}\end{aligned}$$

Invariance under the symmetric and traceless
transformation:

$$\partial_\mu\theta_\nu(x) = \partial_\nu\theta_\mu(x), \quad \partial_\mu\theta^\mu(x) = 0$$

$$\mathcal{L}_{\mathrm{S}}^0 = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2$$

$$\mathcal{L}_{\mathrm{S}} = \frac{1}{2}D_\mu\phi D^\mu\phi - \frac{1}{2}m^2\phi^2 = \mathcal{L}_{\mathrm{S}}^0 + gh_{\mu\nu}\tilde{T}_{\mathrm{S}}^{\mu\nu} + \frac{1}{2}g^2 h_{\mu\rho}h^{\mu\sigma}\partial^\rho\phi\partial_\sigma\phi$$

$$\tilde{T}_{\mathrm{S}}^{\mu\nu} = \partial^\mu\phi\partial^\nu\phi \qquad T_{\mathrm{S}}^{\mu\nu} = \partial^\mu\phi\partial^\nu\phi - \eta^{\mu\nu}\mathcal{L}_{\mathrm{S}}^0$$

Leading order: $gh_{\mu\nu}\tilde{T}_{\mathrm{S}}^{\mu\nu}$ Next-to-Leading order: $\frac{1}{2}g^2 h_{\mu\rho}h^{\mu\sigma}\partial^\rho\phi\partial_\sigma\phi$

$$\mathcal{L}_{\mathrm{Naive}}^0 = -\frac{1}{2}\partial_\mu A_\nu\partial^\mu A^\nu + \frac{m^2}{2}A_\mu A^\mu$$

$$\mathcal{L}_{\mathrm{Naive}} = -\frac{1}{2}D_\mu A_\nu D^\mu A^\nu + \frac{m^2}{2}A_\mu A^\mu = \mathcal{L}_{\mathrm{Naive}}^0 + gh_{\mu\nu}\tilde{T}_{\mathrm{Naive}}^{\mu\nu} - \frac{1}{2}g^2 h_{\mu\rho}h^{\mu\sigma}\partial^\rho A_\nu\partial_\sigma A^\nu$$

$$\tilde{T}_{\mathrm{Naive}}^{\mu\nu} = -\partial^\mu A^\rho\partial^\nu A_\rho$$

Proca Lagrangian: $$\mathcal{L}_{\mathrm{V}}^0 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$\mathcal{L}_{\mathrm{V}} = -\frac{1}{4}(D_\mu A_\nu - D_\nu A_\mu)(D^\mu A^\nu - D^\nu A^\mu)$$

$$\tilde{T}_{\mathrm{V}}^{\mu\nu} = -F^{\mu\rho}\partial^\nu A_\rho$$

$$= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + gh_{\mu\nu}\tilde{T}_{\mathrm{V}}^{\mu\nu} - \frac{1}{2}g^2(h_{\mu\rho}h^{\mu\sigma}\partial^\rho A^\nu\partial_\sigma A_\nu - h_{\mu\rho}h^{\nu\sigma}\partial^\rho A_\nu\partial_\sigma A^\mu)$$

If replacing $F_{\mu\nu}$ with $F_{\mu\nu} + gh_{\mu\rho}F^{\rho}_{\nu} - gh_{\nu\rho}F^{\rho}_{\mu}$

$$\begin{aligned}\mathcal{L}_{\rm V} &= -\frac{1}{4}(F_{\mu\nu} + gh_{\mu\rho}F^{\rho}_{\nu} - gh_{\nu\rho}F^{\rho}_{\mu})(F^{\mu\nu} + gh^{\mu\sigma}F^{\nu}_{\sigma} - gh^{\nu\sigma}F^{\mu}_{\sigma}) \\ &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + gh_{\mu\nu}T^{\mu\nu}_{\rm EM} - \frac{1}{2}g^2(h_{\mu\rho}h^{\mu\sigma}F^{\rho\nu}F_{\sigma\nu} - h_{\nu\rho}h^{\mu\sigma}F^{\rho}_{\mu}F^{\nu}_{\sigma})\end{aligned}$$

$$T^{\mu\nu}_{\rm EM} = F^{\mu\rho}F^{\nu}_{\rho} + \frac{1}{4}\eta^{\mu\nu}F_{\rho\sigma}F^{\rho\sigma} = T^{\mu\nu}_{\rm V} + \partial_\rho(F^{\mu\rho}A^{\nu})$$

$$\mathcal{L}'_{\rm F} = \bar{\psi}\gamma^{\mu}(\eta_{\mu\nu} + gh_{\mu\nu})(\partial^{\nu} + ieA^{\nu})\psi - m\bar{\psi}\psi$$

$$\mathcal{L}^0_{\rm G} = \frac{1}{2}\partial_\mu h_{\rho\sigma}\partial^\mu h^{\rho\sigma} - \partial_\mu h^{\mu\nu}\partial^\rho h_{\rho\nu} + \partial_\mu h\partial_\nu h^{\mu\nu} - \frac{1}{2}\partial_\mu h\partial^\mu h$$

The above Lagrangian is the same as the linear approximation of general relativity where hμν is a weak field. It is only invariant under the infinitesimal local translation.

$$\begin{aligned}\mathcal{L}_{\rm G} &= \frac{1}{2}D_\mu h_{\rho\sigma}D^\mu h^{\rho\sigma} - D_\mu h^{\mu\nu}D^\rho h_{\rho\nu} + D_\mu hD_\nu h^{\mu\nu} - \frac{1}{2}D_\mu hD^\mu h \\ &= \mathcal{L}^0_{\rm G} + gh_{\mu\nu}\tilde{T}^{\mu\nu}_{\rm G} + \frac{g^2}{2}\big(h_{\alpha\mu}h^{\alpha\nu}\partial^\mu h^{\rho\sigma}\partial_\nu h_{\rho\sigma} - h_{\alpha\mu}h^{\alpha\nu}\partial^\mu h\partial_\nu h \\ &\quad -2h^{\mu\rho}h_{\nu\sigma}\partial_\rho h_{\mu\alpha}\partial^\sigma h^{\nu\alpha} + 2h_{\mu\rho}h_{\nu\sigma}\partial^\rho h^{\mu\nu}\partial^\sigma h\big),\end{aligned}$$

$$\tilde{T}^{\mu\nu}_{\rm G} = \partial^\mu h_{\rho\sigma}\partial^\nu h^{\rho\sigma} - \partial^\mu h\partial^\nu h - 2\partial^\nu h^{\mu\rho}\partial^\sigma h_{\sigma\rho} + \partial^\nu h^{\mu\rho}\partial_\rho h + \partial^\mu h\partial_\rho h^{\rho\nu}$$

$$
\begin{aligned}
\mathcal{L}_{\mathrm{TOT}} \;=\; & \bar{\psi}(x)\gamma^{\mu}(\partial_{\mu} + ieA_{\mu}(x))\psi(x) + gh_{\mu\nu}(x)\tilde{T}^{\mu\nu}_{\mathrm{F}} - m\bar{\psi}(x)\psi(x) \\
& -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + gh_{\mu\nu}\tilde{T}^{\mu\nu}_{\mathrm{V}} - \frac{g^2}{2}(h_{\mu\rho}h_{\mu\sigma}\partial^{\rho}A^{\nu}\partial^{\sigma}A^{\nu} - h_{\mu\rho}h_{\nu\sigma}\partial^{\rho}A^{\nu}\partial^{\sigma}A^{\mu}) \\
& +\frac{1}{2}\partial_{\mu}h_{\rho\sigma}\partial^{\mu}h^{\rho\sigma} - \partial_{\mu}h^{\mu\nu}\partial^{\rho}h_{\rho\nu} + \partial_{\mu}h\partial_{\nu}h^{\mu\nu} - \frac{1}{2}\partial_{\mu}h\partial^{\mu}h + gh_{\mu\nu}\tilde{T}^{\mu\nu}_{\mathrm{G}} \\
& +\frac{g^2}{2}\big(h_{\alpha\mu}h^{\alpha\nu}\partial^{\mu}h^{\rho\sigma}\partial_{\nu}h_{\rho\sigma} - h_{\alpha\mu}h^{\alpha\nu}\partial^{\mu}h\partial_{\nu}h - 2h^{\mu\rho}h_{\nu\sigma}\partial_{\rho}h_{\mu\alpha}\partial^{\sigma}h^{\nu\alpha} \\
& +2h_{\mu\rho}h_{\nu\sigma}\partial^{\rho}h^{\mu\nu}\partial^{\sigma}h\big),
\end{aligned}
$$

Equation of motion:

$$
H_0^{\mu\nu} + gH_1^{\mu\nu} + g^2H_2^{\mu\nu} = g\left(\tilde{T}^{\mu\nu}_{\mathrm{F}} + \tilde{T}^{\mu\nu}_{\mathrm{V}}\right) - g^2(h^{\mu\rho}F_{\rho\sigma}F^{\nu\sigma} + h_{\rho\sigma}F^{\mu\rho}F^{\nu\sigma})
$$

$$
H_0^{\mu\nu} = \Box h^{\mu\nu} - \eta^{\mu\nu}\Box h - \partial^{\mu}\partial_{\rho}h^{\rho\nu} - \partial^{\nu}\partial_{\rho}h^{\rho\mu} + \partial^{\mu}\partial^{\nu}h + \eta^{\mu\nu}\partial_{\rho}\partial_{\sigma}h^{\rho\sigma},
$$

$$
\begin{aligned}
H_1^{\mu\nu} = \; & 2\partial_{\rho}h^{\rho\sigma}\partial_{\sigma}h^{\mu\nu} + 2h^{\rho\sigma}\partial_{\rho}\partial_{\sigma}h^{\mu\nu} - 2\eta^{\mu\nu}\partial_{\rho}h^{\rho\sigma}\partial_{\sigma}h - 2\eta^{\mu\nu}h^{\rho\sigma}\partial_{\rho}\partial_{\sigma}h \\
& - 2\partial_{\rho}h^{\rho\mu}\partial_{\sigma}h^{\sigma\nu} - 2h^{\rho\mu}\partial_{\rho}\partial_{\sigma}h^{\sigma\nu} - 2\partial^{\mu}h_{\rho\sigma}\partial^{\rho}h^{\sigma\nu} - 2h_{\rho\sigma}\partial^{\mu}\partial^{\rho}h^{\sigma\nu} \\
& + \partial_{\rho}h^{\rho\mu}\partial^{\nu}h + h^{\rho\mu}\partial_{\rho}\partial^{\nu}h + \eta^{\mu\nu}\partial_{\rho}h_{\alpha\beta}\partial^{\beta}h^{\alpha\rho} + \eta^{\mu\nu}h_{\alpha\beta}\partial_{\rho}\partial^{\beta}h^{\alpha\rho} \\
& + \partial^{\mu}h^{\rho\nu}\partial_{\rho}h + h^{\rho\nu}\partial^{\mu}\partial_{\rho}h + \eta^{\mu\nu}\partial_{\rho}h^{\rho\sigma}\partial^{\alpha}h_{\alpha\sigma} + \eta^{\mu\nu}h^{\rho\sigma}\partial_{\rho}\partial^{\alpha}h_{\alpha\sigma} - \tilde{T}^{\mu\nu}_{\mathrm{G}},
\end{aligned}
$$

$$
\begin{aligned}
H_2^{\mu\nu} = \; & h^{\alpha\sigma}\partial^{\rho}h_{\alpha\rho}\partial_{\sigma}(h^{\mu\nu} - \eta^{\mu\nu}h) + h_{\alpha\rho}\partial^{\rho}h^{\alpha\sigma}\partial_{\sigma}(h^{\mu\nu} - \eta^{\mu\nu}h) + h_{\alpha\rho}h^{\alpha\sigma}\partial^{\rho}\partial_{\sigma}(h^{\mu\nu} - \eta^{\mu\nu}h) \\
& - 2\partial_{\rho}(h^{\rho\mu}h_{\sigma\alpha}\partial^{\sigma}h^{\alpha\nu}) + \partial_{\rho}(h^{\rho\mu}h^{\sigma\nu}\partial_{\sigma}h) + \eta^{\mu\nu}\partial^{\rho}(h_{\rho\alpha}h_{\sigma\beta}\partial^{\sigma}h^{\alpha\beta}) - h^{\mu\rho}\partial^{\nu}h^{\alpha\beta}\partial_{\rho}h_{\alpha\beta} \\
& + h^{\mu\rho}\partial^{\nu}h\partial_{\rho}h + 2h^{\alpha\sigma}\partial^{\nu}h^{\mu\beta}\partial_{\sigma}h_{\alpha\beta} - h_{\rho\sigma}\partial^{\nu}h^{\mu\rho}\partial^{\sigma}h - h_{\rho\sigma}\partial^{\rho}h^{\sigma\nu}\partial^{\mu}h.
\end{aligned}
$$

$$\Box h^{\mu\nu}_{(0)} - \eta^{\mu\nu}\Box h_{(0)} - \partial^\mu\partial_\rho h^{\rho\nu}_{(0)} - \partial^\nu\partial_\rho h^{\rho\mu}_{(0)} + \partial^\mu\partial^\nu h_{(0)} + \eta^{\mu\nu}\partial_\rho\partial_\sigma h^{\rho\sigma}_{(0)} = gT^{\mu\nu}_{\text{sun}}$$

$$\tilde{T}^{\mu\nu}_{\text{F}} - T^{\mu\nu}_{\text{F}} = -\tfrac{i}{16}\partial_\rho\left(\bar{\psi}\{\gamma^\mu,[\gamma^\nu,\gamma^\rho]\}\psi\right) + \eta^{\mu\nu}\mathcal{L}^0_{\text{F}} \qquad g^2 = 16\pi G$$

$$T^{\mu\nu}_{\text{cl}}(x) = \int m\frac{dz^\mu}{d\tau}\frac{dz^\nu}{d\tau}\delta^4(\mathbf{x}-\mathbf{z}(\tau))d\tau \qquad T^{00}_{\text{sun}}(x) = M\delta^{(3)}(\mathbf{x})$$

The equation is comparable with that for the metric hμν in the linear approximation of general relativity, where hμν = gμν − ημν and gμν is the metric tensor of curved spacetime.

$$h_{\mu\nu} = h_{\nu\mu}, \quad \partial^\mu h_{\mu\nu} = 0$$

$$-\nabla^2 h^{00}_{(0)} + \nabla^2 h_{(0)} = 4\sqrt{\pi G}M\delta^{(3)}(\mathbf{x}),$$

$$-\nabla^2 h^{ij}_{(0)} - \nabla^2 h_{(0)}\delta^{ij} + \partial^i\partial^j h_{(0)} = 0.$$

$$h_{\mu\nu} = h_{\nu\mu}, \quad 2\partial_\mu h^{\mu\nu} = \partial^\nu h$$

$$-\nabla^2\left(h^{00}_{(0)} - \frac{1}{2}h_{(0)}\right) = 4\sqrt{\pi G}M\delta^{(3)}(\mathbf{x}),$$

$$-\nabla^2\left(h^{ii}_{(0)} + \frac{1}{2}h_{(0)}\right) = 0.$$

The solutions for the above equations are

$$h^{00}_{(0)} = \frac{1}{2}\sqrt{\frac{G}{\pi}}\frac{M}{r}, \quad h^{ij}_{(0)} = \frac{1}{4}\sqrt{\frac{G}{\pi}}\left(\frac{M}{r}\delta^{ij} + \frac{Mx^ix^j}{r^3}\right).$$

$$h^{00}_{(0)} = \frac{1}{2}\sqrt{\frac{G}{\pi}}\frac{M}{r}, \quad h^{ii}_{(0)} = \frac{1}{2}\sqrt{\frac{G}{\pi}}\frac{M}{r}.$$

$$I_{\text{particle}} = -m \int (\eta_{\mu\nu} + g h_{\mu\nu}) \frac{dz^\mu}{d\tau} \frac{dz^\nu}{d\tau} d\tau$$

With variational principle, the equation of motion for the particle is expressed as

$$\frac{d^2 z^\mu}{d\tau^2} + \Gamma^\mu_{\rho\sigma} \frac{dz^\rho}{d\tau} \frac{dz^\sigma}{d\tau} = 0,$$

where

$$\Gamma^\mu_{\rho\sigma} \equiv \frac{1}{2} (\eta^{\mu\nu} + g h^{\mu\nu}) (\partial_\rho g h_{\nu\sigma} + \partial_\sigma g h_{\nu\rho} - \partial_\nu g h_{\rho\sigma}).$$

This equation is similar to the geodesic equation in general relativity case.

$$P_\mu \equiv (\eta_{\mu\nu} + g h_{\mu\nu}) p^\nu \text{ where } p^\nu = \tfrac{dz^\nu}{d\tau} \qquad dP_\mu = \frac{g}{2} \partial_\mu h_{\rho\sigma} p^\rho dz^\sigma$$

$$dP_y = -\left(\frac{3GMR}{2\sqrt{x^2 + R^2}^3} + \frac{3GMx^2 R}{2\sqrt{x^2 + R^2}^5} \right) p^x dx$$

$$P_y(x = +\infty) = -p^x \int_{-\infty}^{\infty} \left(\frac{3GMR}{2\sqrt{x^2 + R^2}^3} + \frac{3GMx^2 R}{2\sqrt{x^2 + R^2}^5} \right) dx = -\frac{4GM}{R} p^x$$

$$\Delta\phi = -\frac{p^y(x = +\infty)}{p^x} = -\frac{P_y(x = +\infty)}{p^x} = \frac{4GM}{R}$$

$$dP_y = -\frac{2GMR}{\sqrt{x^2+R^2}^3} p^x dx \qquad P_y(x=+\infty) = -p^x \int_{-\infty}^{\infty} \frac{2GMR}{\sqrt{x^2+R^2}^3} dx = -\frac{4GM}{R} p^x$$

$$-\nabla^2 \left(h_{(1)}^{00} - \frac{1}{2} h_{(1)} \right) = 4\sqrt{\pi G} M \delta^{(3)}(\mathbf{x}) - 4G\sqrt{\frac{G}{\pi}} \frac{M^2}{r^4},$$

$$-\nabla^2 \left(h_{(1)}^{ij} + \frac{1}{2} \delta^{ij} h_{(1)} \right) = 2G\sqrt{\frac{G}{\pi}} \frac{M^2}{r^6} x^i x^j.$$

$$h_{(1)}^{00} = \frac{1}{2}\sqrt{\frac{G}{\pi}} \left(\frac{M}{r} + \frac{GM^2}{r^2} \right), \quad h_{(1)}^{ij} = \frac{1}{2}\sqrt{\frac{G}{\pi}} \left(\frac{M}{r}\delta^{ij} + \frac{2GM^2}{r^2}\delta^{ij} + \frac{GM^2}{r^4} x^i x^j \right)$$

With the same method, we obtain

$$\begin{aligned} P_y(x=+\infty) &= -p^x \int_{-\infty}^{\infty} \left(\frac{2GMR}{\sqrt{x^2+R^2}^3} + \frac{6G^2M^2R}{\sqrt{x^2+R^2}^4} + \frac{4G^2M^2x^2R}{\sqrt{x^2+R^2}^6} \right) dx \\ &= -\left(\frac{4GM}{R} + \frac{7\pi}{2} \frac{G^2M^2}{R^2} \right) p^x. \end{aligned}$$

$$\Delta\phi = \frac{4GM}{R} + \frac{7\pi}{2} \frac{G^2M^2}{R^2}$$ = 1.75“ + 1.0” x 10^-5

Translation gauge field theory of gravity in Minkowski spacetime, H. Li and P. Wang, Chin. Phys. C (Accepted).

There are some major differences between ours and previous gauge gravity theories：

1) Our Lagrangian is invariant under the finite translation transformation, while the previous Lagrangian is invariant under the infinitesimal transformation. As a result, the corresponding transformation of gravitational field $h_{\mu\nu}$ in this manuscript is always associated with the derivative. The transformation of $h_{\mu\nu}$ itself can be obtained order by order.
2) Our gravitational field $h_{\mu\nu}$ has nothing to do with the metric. The metric $g_{\mu\nu}$ in our Lagrangian is always $\eta_{\mu\nu}$. The gravitational field $h_{\mu\nu}$ is an independent quantity. Due to the translation invariance of the Lagrangian, one can choose different "gauge" for $h_{\mu\nu}$. While in previous gauge theories, the gravitational field was proved to be vierbein (tetrad) field and related to the metric.
3) Based on the local translation invariance, the interactions between $h_{\mu\nu}$ and matter fields with spin 0, 1/2 and 1 are obtained. Except the interaction between $h_{\mu\nu}$ and spin-1/2 field, there exist high-order interactions. In particular, the interaction between $h_{\mu\nu}$ and electromagnetic field A_μ is not locally U(1) invariant. In addition, our Lagrangian for the gravitational field is obtained from the free Lagrangian for spin-2 field with the requirement of locally translation invariance. The obtained Lagrangian is also different for previous gauge theories.
4) We describe gravity in the same frame as that for the other interactions in the standard model. The result obtained with our Lagrangian is different from that with general relativity. While previous gauge theories of gravity lead to the so called "new general relativity", which is teleparallel equivalent to the Einstein's general relativity. The "vierbein" approach can be regarded as another formalism to derive Einstein's equation.

$$\mathcal{L} = \bar{\psi}(x)\gamma^\mu\gamma^5\psi(x)\partial_\mu\phi(x)$$

if locally gauge invariant, $\mathcal{L}$ should be: $\mathcal{L} = \bar{\psi}(x)\gamma^\mu\gamma^5\psi(x)(\partial_\mu + h_{\mu\nu}\partial^\nu)\phi(x)$

under local translation, $\psi(x) \to \psi(x') = \psi(x+\theta(x))$, $\phi(x) \to \phi(x') = \phi(x+\theta(x))$

$$h_{\mu\nu}(x)\partial^\nu \to [h_{\mu\nu}(x') - \partial_\mu\theta_\nu(x)]\partial^{\nu'}$$

$$\mathcal{L} \to \mathcal{L}' = \bar{\psi}(x')\gamma^\mu\gamma^5\psi(x')\left[\partial_\mu + (h_{\mu\nu}(x') - \partial_\mu\theta_\nu(x))\partial^{\nu'}\right]\phi(x')$$

$$= \bar{\psi}(x')\gamma^\mu\gamma^5\psi(x')\left[\partial'_\mu + h_{\mu\nu}(x')\partial^{\nu'}\right]\phi(x')$$

nonlocal interaction: $\mathcal{L}_{nl} = \bar{\psi}(x)\gamma^\mu\gamma^5\psi(x)\partial_\mu\phi(x+a)$

if locally translation invariant, $\mathcal{L}$ should be:

$$\mathcal{L}_{nl} = \bar{\psi}(x)\gamma^\mu\gamma^5\psi(x)[\partial_\mu + h_{\mu\nu}\partial^\nu] e^{i\int_x^{x+a} h_{\alpha\beta}dz^\alpha\partial^\beta}\phi(x+a)$$

$$\mathcal{L}'_{nl} = \bar{\psi}(x')\gamma^\mu\gamma^5\psi(x')\left[\partial_\mu + (h_{\mu\nu}(x') - \partial_\mu\theta_\nu(x))\partial^{\nu'}\right] e^{i\int_x^{x+a}(h_{\alpha\beta}(z') - \partial_\alpha\theta_\beta(z))dz^\alpha\partial^{\beta'}}\phi(x+a+\theta(x+a))$$

$$= \bar{\psi}(x')\gamma^\mu\gamma^5\psi(x')\left[\partial_\mu + (h_{\mu\nu}(x') - \partial_\mu\theta_\nu(x))\partial^{\nu'}\right] e^{i\int_{x'}^{x'+a} h_{\alpha\beta}(z')dz^{\alpha'}\partial^{\beta'}} e^{[i\theta_\beta(x+a) - i\theta_\beta\theta(x)]\partial^{\beta'}}\phi(x+a+\theta(x+a))$$

$$= \bar{\psi}(x')\gamma^\mu\gamma^5\psi(x')\left[\partial_\mu + (h_{\mu\nu}(x') - \partial_\mu\theta_\nu(x))\partial^{\nu'}\right] e^{i\int_{x'}^{x'+a} h_{\alpha\beta}(z')dz^{\alpha'}\partial^{\beta'}}\phi(x+a+\theta(x))$$

$$= \bar{\psi}(x')\gamma^\mu\gamma^5\psi(x')\left[\partial_\mu + (h_{\mu\nu}(x') - \partial_\mu\theta_\nu(x))\partial^{\nu'}\right] e^{iI}\phi(x'+a)$$

$$= \bar{\psi}(x')\gamma^\mu\gamma^5\psi(x')\left[\partial'_\mu + h_{\mu\nu}(x')\partial^{\nu'}\right] e^{iI}\phi(x'+a).$$

$\mathcal{L}_{nl} = \mathcal{L}'_{nl}$ except the argument $x \to x'$.

note: $\partial_\mu = (\eta_{\mu\nu} + \partial_\mu\theta_\nu(x))\partial^{\nu'}$

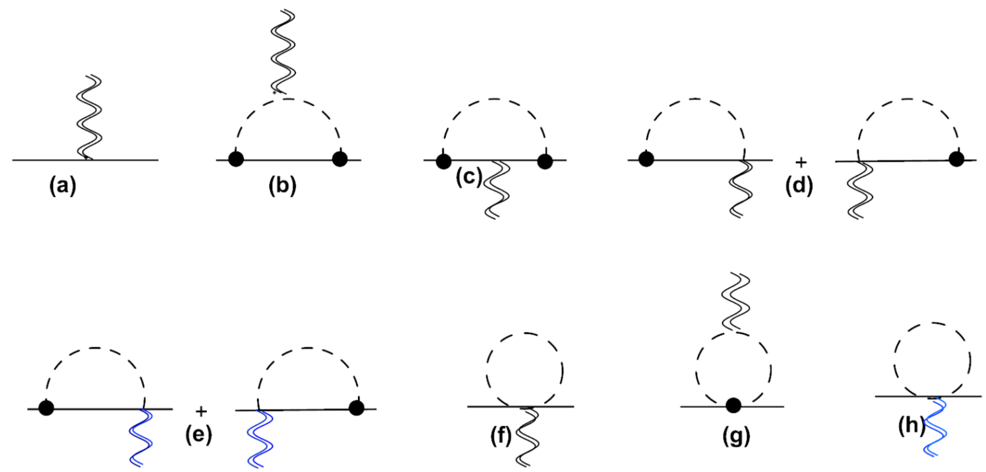
The action for delocalized meson field in curved space-time is given by

$$S^{\text{nonlocal}} = C_{\text{BB}\phi}\int F(a)da \int d^4x\sqrt{-g}\bar{\Psi}\gamma^a\gamma^5 e_a^\mu\Psi\partial_\mu[G(x, x+a)\phi(x+a)]$$

where $G(x, x+a)$ represents Gravitation Wilson line operator and is defined as

$$G(x, x+a) = \text{Exp}\{\frac{\kappa}{2}\int_x^{x+a} dz^\nu h_{\mu\nu}(z)\partial^\mu\}$$

$$h_{\mu\nu}(x)\partial_\rho \to \left[h_{\mu\nu}(x') - \frac{1}{g}\partial_\mu\theta_\nu(x)\right]\partial'_\rho$$

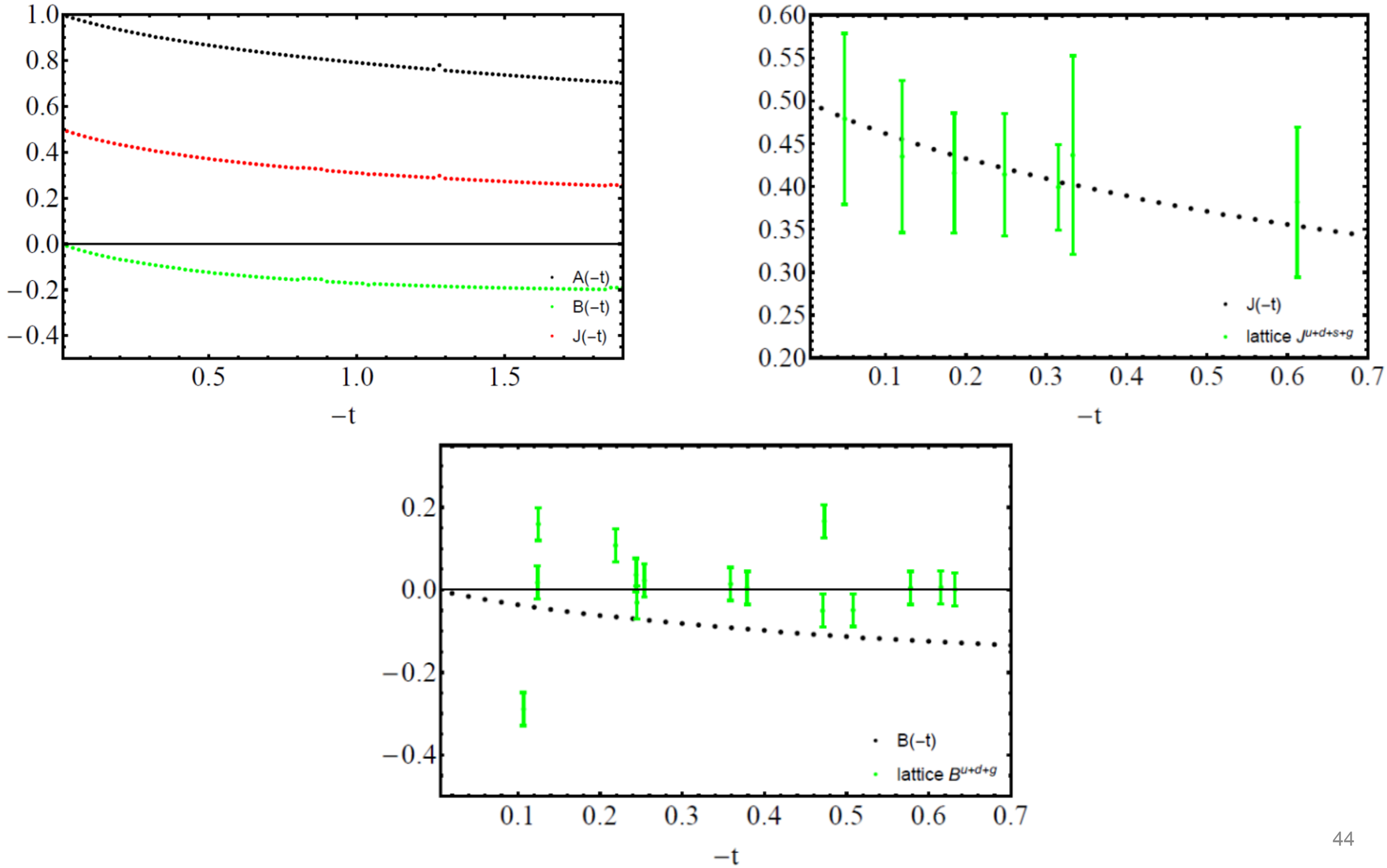


$$q_\mu\frac{2k^\nu + q^\nu}{2k\cdot q + q^2}[\tilde{F}(k+q) - \tilde{F}(k)]$$

LO contributions	Tree	Meson rainbow	Nulceon rainbow	KR	KR additional	Tadpole	Buble	Tadpole additional	Total LO
A(0)	1.16	0.07	0.41	-0.08	0.09	0	0	0	1.16+0.5
B(0)	0	0.15	-0.28	0	0.13	0	0	0	0

NLO contributions	NLO Tadpole			NLO Tadpole additional			NLO Bubble			Total NLO
Coupling constants	c_1	c_2	c_3	c_1	c_2	c_3	c_1	c_2	c_3	
A(0)	0	0.44	0	0	-1.31	0	0	0.21	0	-0.66
B(0)	0	-0.44	0	0	0.65	0	0	-0.21	0	0

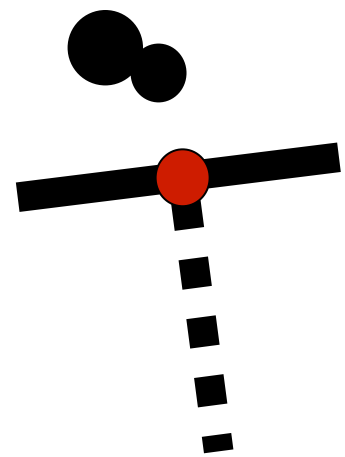
Total A(0) = 1, Total B(0) = 0

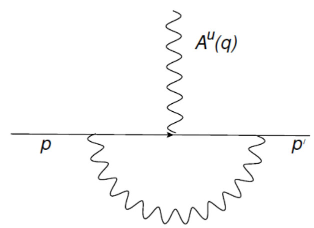

1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
0.5
1.0
1.5
−t
A(−t)
B(−t)
J(−t)
0.60
0.55
0.50
0.45
0.40
0.35
0.30
0.25
0.20
0.1
0.2
0.3
0.4
0.5
0.6
0.7
−t
J(−t)
lattice $J^{u+d+s+g}$
0.2
0.0
−0.2
−0.4
0.1
0.2
0.3
0.4
0.5
0.6
0.7
−t
B(−t)
lattice B^{u+d+g}

Nonlocal behavior is general for all the interactions?

Nonlocal EFT (Nucleon structure, FFs, PDFs)

Nonlocal QED (lepton g-2) , Nonlocal Gravity (Gravitational FFs)





$$a_\mu^{\rm SM} = 116591810(43) \times 10^{-11}$$

$$\Delta a_\mu^{\rm FNAL} = a_\mu^{\rm FNAL} - a_\mu^{\rm SM} = (230 \pm 69) \times 10^{-11} \qquad \Delta a_\mu = a_\mu^{\rm FNAL+BNL} - a_\mu^{\rm SM} = (251 \pm 59) \times 10^{-11}$$

$$a_e^{\rm SM} = 1159652182.032(720) \times 10^{-12}$$

$$\Delta a_e^{\rm B} = a_e^{\rm exp} - a_e^{\rm SM,B} = (-87 \pm 36) \times 10^{-14} \qquad \Delta a_e^{\rm LKB} = a_e^{\rm exp} - a_e^{\rm SM,LKB} = (48 \pm 30) \times 10^{-14}$$

The local QED Lagrangian is

$$\mathcal{L}^{local} = \bar{\psi}(x)\left(i\partial\!\!\!/ - m\right)\psi(x) - e\bar{\psi}(x)A\!\!\!/(x)\psi(x) - \frac{1}{4}F^{\mu\nu}(x)F_{\mu\nu}(x)$$

We start from a "minimum" extension of the standard model. The nonlocal QED Lagrangian for studying lepton anomalous magnetic moments can be written as

$$\mathcal{L}^{\rm nl}_{\rm QED} = \int d^4a\Big[\bar{\psi}(x)(iD\!\!\!\!/ - m)\psi(x) - \frac{1}{4}F^{\mu\nu}(x)F_{\mu\nu}(x+a)F_{\gamma}(a)\Big], \qquad (5)$$

where the covariant derivative $D_\mu = \partial_\mu + ieA_\mu(x+a)F_l(a)$. In the above Lagrangian, $F_l(a)$ and $F_\gamma(a)$ are the correlation functions. If they are chosen to be δ functions, the nonlocal Lagrangian will change back to the local one. The above Lagrangian is invariant under the following U(1) gauge transformation

$$\psi(x) \to e^{i\alpha(x)}\psi(x), \quad A_\mu(x) \to A_\mu(x) - \frac{1}{e}\partial_\mu\alpha'(x), \quad (6)$$

where

$$\alpha(x) = \int d^4a\alpha'(x+a)F_l(a). \qquad (7)$$

photon propagator: $$D_{\mu\nu}(k) = \frac{-ig_{\mu\nu}}{(k^2+i\epsilon)\tilde{F}_\gamma(k)}$$

$$F_1^{\rm loop}(0) = -2ie^2 \int \frac{d^4k}{(2\pi)^4} \frac{\tilde{F}_l^2(k)}{\tilde{F}_\gamma(k)} \times \frac{2m_l^4 + m_l^2 k^2 - 2m_l^2 k\cdot p - 2(k\cdot p)^2}{k^2(k^2-2k\cdot p)^2 m_l^2}$$

$$F_1^{\rm loop}(0) = -\frac{\Sigma(\not p)}{d\not p}$$

and

$$\Sigma(\not p) = -ie^2 \int \frac{d^4k}{(2\pi)^4} \gamma_\mu \frac{1}{\not p - \not k - m_l} \gamma^\mu \frac{1}{k^2} \frac{\tilde{F}_l^2(k)}{\tilde{F}_\gamma(k)}.$$

$$a_l = -2ie^2 \int \frac{d^4k}{(2\pi)^4} \frac{\tilde{F}_l^2(k)}{\tilde{F}_\gamma(k)} \left\{ \frac{(k\cdot q)^2}{q^2 k^2 (k^2 - 2k\cdot p)^2} - \frac{(k^2+2k\cdot p)m_l^2 - 3(k\cdot p)^2}{k^2(k^2-2k\cdot p)^2 m_l^2} \right\}\Bigg|_{q^2=0}.$$

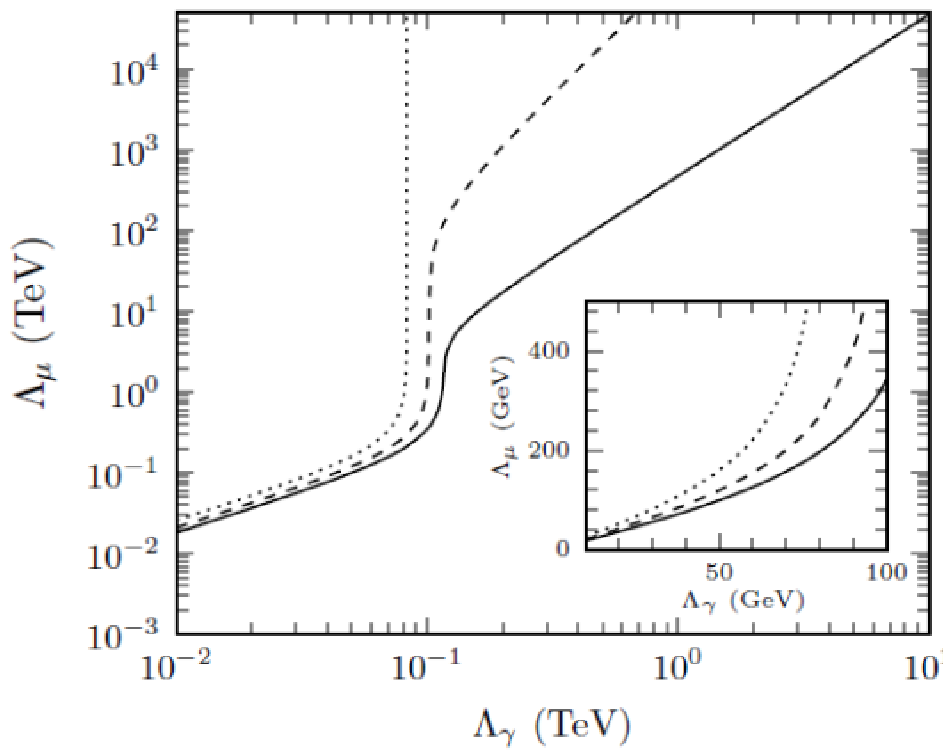
$$\tilde{F}_l(k) = \left(\frac{\Lambda_l^2 - m_l^2}{\Lambda_l^2 - k^2}\right)^2$$

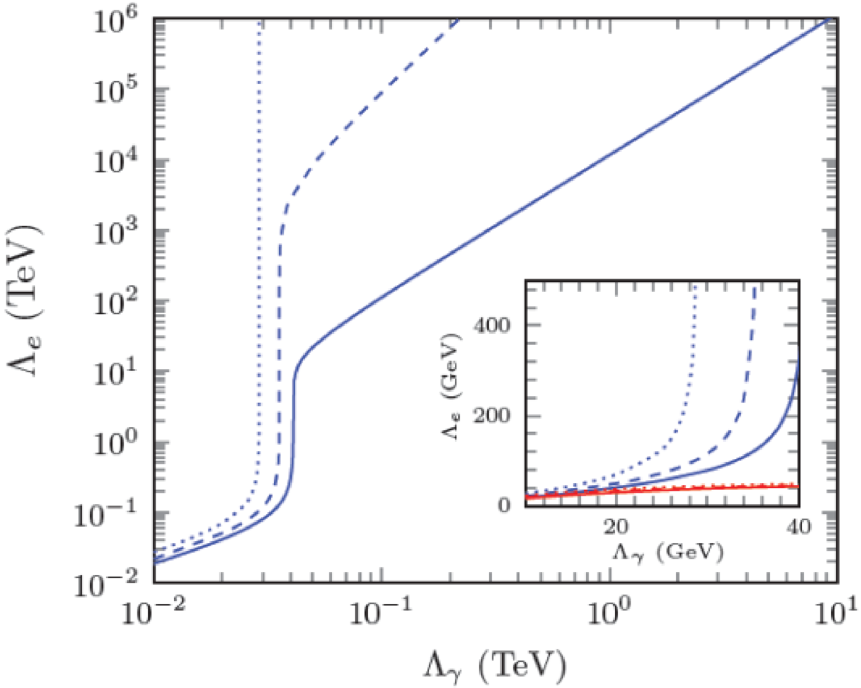
$$\tilde{F}_\gamma(k) = \left(\frac{\Lambda_\gamma^2}{\Lambda_\gamma^2 - k^2}\right)^n$$ n = 2, 3, 4

$$a_l^{2,2} = \frac{\alpha}{2\pi}\left[1 + \frac{4}{3}\left(\frac{1}{\Lambda_\gamma^2} - \frac{2}{\Lambda_l^2}\right) m_l^2 + 4\left(\frac{1}{\Lambda_\gamma^4} - \frac{8}{\Lambda_\gamma^2\Lambda_l^2} + \frac{10}{\Lambda_l^4}\right) m_l^4 \log\frac{\Lambda_l}{m_l} + \mathcal{O}(m_l^6)\right]$$

$$a_l^{2,3} = \frac{\alpha}{2\pi}\left[1 + \frac{2}{3}\left(\frac{3}{\Lambda_\gamma^2} - \frac{4}{\Lambda_l^2}\right) m_l^2 + \frac{1}{3}\left(\frac{2\Lambda_l^2}{\Lambda_\gamma^6} - \frac{141}{2\Lambda_\gamma^4} + \frac{228}{\Lambda_\gamma^2\Lambda_l^2} - \frac{172}{\Lambda_l^4}\right) m_l^4 + 4\left(\frac{3}{\Lambda_\gamma^4} - \frac{12}{\Lambda_\gamma^2\Lambda_l^2} + \frac{10}{\Lambda_l^4}\right) m_l^4 \log\frac{\Lambda_l}{m_l} + \mathcal{O}(m_l^6)\right]$$

$$a_l^{2,4} = \frac{\alpha}{2\pi}\left[1 + \frac{8}{3}\left(\frac{1}{\Lambda_\gamma^2} - \frac{1}{\Lambda_l^2}\right) m_l^2 + \frac{1}{3}\left(\frac{\Lambda_l^4}{\Lambda_\gamma^8} + \frac{8\Lambda_l^2}{\Lambda_\gamma^6} - \frac{141}{\Lambda_\gamma^4} + \frac{304}{\Lambda_\gamma^2\Lambda_l^2} - \frac{172}{\Lambda_l^4}\right) m_l^4 + 8\left(\frac{3}{\Lambda_\gamma^4} - \frac{8}{\Lambda_\gamma^2\Lambda_l^2} + \frac{5}{\Lambda_l^4}\right) m_l^4 \log\frac{\Lambda_l}{m_l} + \mathcal{O}(m_l^6)\right]$$





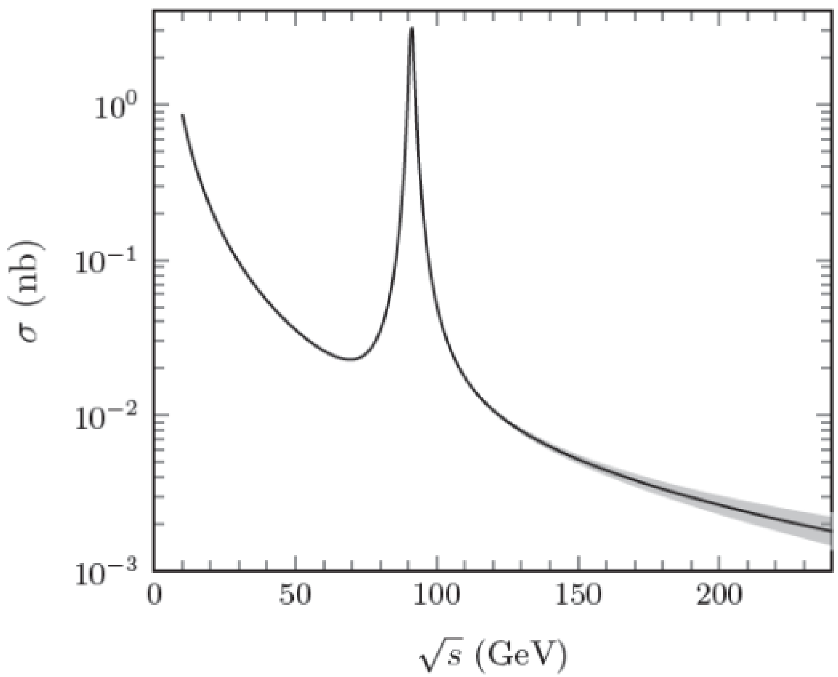


FIG. 3. The $\sqrt{s}$ dependence of the cross section of $e^+e^- \to \mu^+\mu^-$ at leading tree level with γ and Z exchanges. The solid line is the SM result. The narrow band is the result of nonlocal QED with $\Lambda_\gamma = 1$ TeV and $\Lambda_l \geq 1$ TeV.

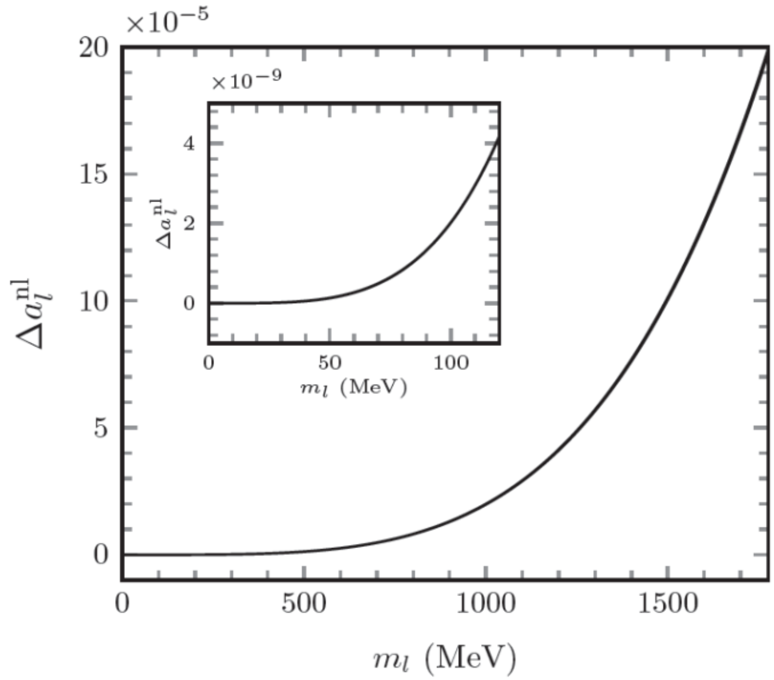


FIG. 4: The calculated discrepancy of lepton anomalous magnetic moment Δa_l^{nl} versus lepton mass m_l with $\Lambda_\gamma = 1$ TeV and $\Lambda_l = \Lambda_\mu$. The small figure at the corner is for the result at small lepton mass.

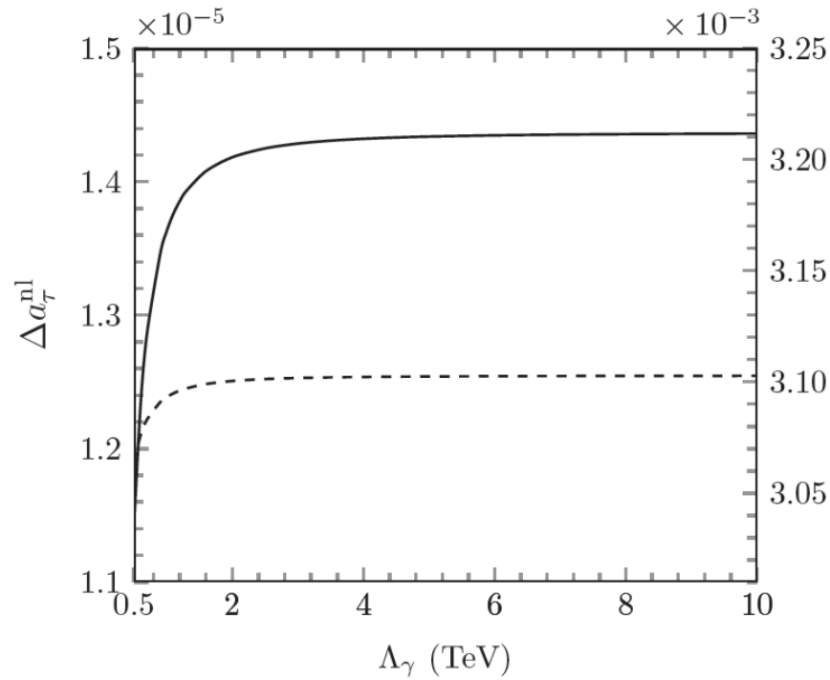


FIG. 5: The calculated discrepancy of anomalous magnetic moment Δa_τ^{nl} versus Λ_γ. The solid line is for the upper limit of Δa_τ^{nl} with $\Lambda_\tau = 2\Lambda_\mu$ (right axis), while the dashed line is for the lower limit with $\Lambda_\tau = \frac{\Lambda_\mu}{2}$ (left axis).

Solution of lepton g−2 anomalies with nonlocal QED, H. Li and P. Wang, J. Phys. G (Accepted).

$$\mathcal{L}^{nl} = \int d^4a \bar{\psi}\left(x+\frac{a}{2}\right)\bar{I}\left(x,x+\frac{a}{2}\right)(i\partial\!\!\!/ - m)\,\psi\left(x-\frac{a}{2}\right)I\left(x,x-\frac{a}{2}\right)F_1(a)$$
$$- e\int d^4a d^4b \bar{\psi}\left(x+\frac{a}{2}\right)\bar{I}\left(x,x+\frac{a}{2}\right)A\!\!\!/(x+b)\psi\left(x-\frac{a}{2}\right)I\left(x,x-\frac{a}{2}\right)F_1(a)F_2(a,b)$$
$$- \frac{1}{4}\int d^4d F^{\mu\nu}(x)F_{\mu\nu}(x+d)F_4(d)$$

where the gauge link

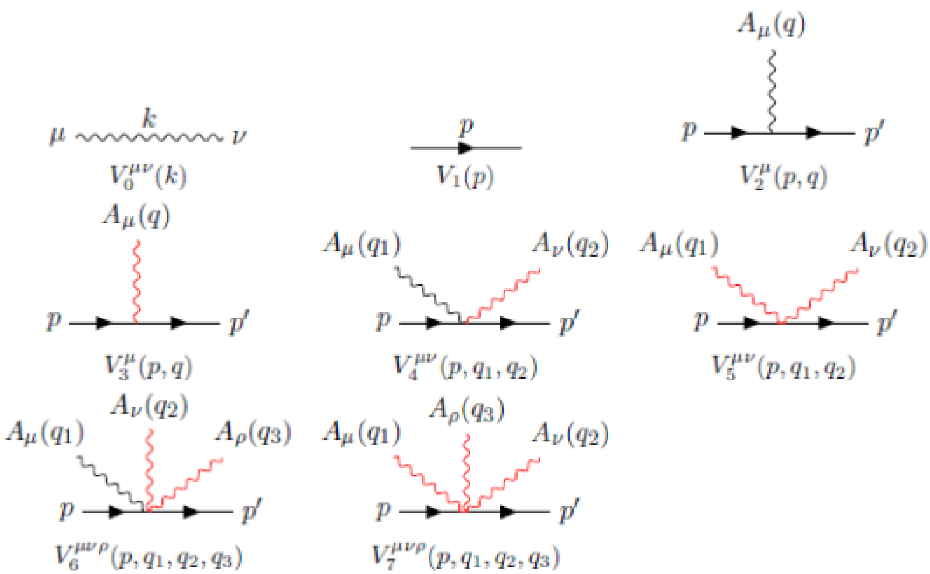
$$I(x,y) \equiv \exp\left(ie\int d^4c\int_x^y dz^\mu A_\mu(z+c)F_3(a,c)\right)$$

$$\int d^4aF_1(a) = \int d^4bF_2(a,b) = \int d^4cF_3(a,c) = \int d^4dF_4(d) = 1$$

$$\psi(x)\to e^{i\alpha(x)}\psi(x), \quad A_\mu(x)\to A_\mu(x) - \frac{1}{e}\partial_\mu\alpha'(x),$$

$$\alpha(x) = \int d^4b\alpha'(x+b)F_2(a,b) = \int d^4c\alpha'(x+c)F_3(a,c)$$

$$
\begin{aligned}
\mathcal{L}^{nl} \to & \int d^4a \bar{\psi}\left(x+\frac{a}{2}\right) e^{-i\alpha(x+\frac{a}{2})} \bar{I}\left(x, x+\frac{a}{2}\right) \exp\left(-ie\left(-\frac{1}{e}\right)\int d^4c \int_x^{x+\frac{a}{2}} dz^\mu \partial_\mu \alpha'(z+c) F_3(a,c)\right) \times \\
& (i\partial\!\!\!/ - m)\, e^{i\alpha(x-\frac{a}{2})} \psi\left(x-\frac{a}{2}\right) I\left(x, x-\frac{a}{2}\right) \exp\left(ie\left(-\frac{1}{e}\right)\int d^4c \int_x^{x-\frac{a}{2}} dz^\mu \partial_\mu \alpha'(z+c) F_3(a,c)\right) F_1(a) \\
& - e^{-i[\alpha(x+\frac{a}{2})-\alpha(x-\frac{a}{2})]} e \int d^4a d^4b \bar{\psi}\left(x+\frac{a}{2}\right) I\left(x, x+\frac{a}{2}\right) A\!\!\!/(x+b) \psi\left(x-\frac{a}{2}\right) I\left(x, x-\frac{a}{2}\right) F_1(a) F_2(a,b) \times \\
& \exp\left(-ie\left(-\frac{1}{e}\right)\int d^4c \int_x^{x+\frac{a}{2}} dz^\mu \partial_\mu \alpha'(z+c) F_3(a,c)\right) \exp\left(ie\left(-\frac{1}{e}\right)\int d^4c \int_x^{x-\frac{a}{2}} dz^\mu \partial_\mu \alpha'(z+c) F_3(a,c)\right) \\
& + e^{-i[\alpha(x+\frac{a}{2})-\alpha(x-\frac{a}{2})]} \int d^4a d^4b \bar{\psi}\left(x+\frac{a}{2}\right) \bar{I}\left(x, x+\frac{a}{2}\right) \partial\!\!\!/ \alpha'(x+b) \psi\left(x-\frac{a}{2}\right) I\left(x, x-\frac{a}{2}\right) F_1(a) F_2(a,b) \times \\
& \exp\left(-ie\left(-\frac{1}{e}\right)\int d^4c \int_x^{x+\frac{a}{2}} dz^\mu \partial_\mu \alpha'(z+c) F_3(a,c)\right) \exp\left(ie\left(-\frac{1}{e}\right)\int d^4c \int_x^{x-\frac{a}{2}} dz^\mu \partial_\mu \alpha'(z+c) F_3(a,c)\right) \\
& - \frac{1}{4}\int d^4d F^{\mu\nu}(x) F_{\mu\nu}(x+d) F_4(d) \\
= & \int d^4a \bar{\psi}\left(x+\frac{a}{2}\right) I\left(x, x+\frac{a}{2}\right) (i\partial\!\!\!/ - m)\, \psi\left(x-\frac{a}{2}\right) I\left(x, x-\frac{a}{2}\right) F_1(a) \\
& + i^2 \partial\!\!\!/ \alpha(x) \int d^4a \bar{\psi}\left(x+\frac{a}{2}\right) \bar{I}\left(x, x+\frac{a}{2}\right) \psi\left(x-\frac{a}{2}\right) I\left(x, x-\frac{a}{2}\right) F_1(a) \\
& - e\int d^4a d^4b \bar{\psi}\left(x+\frac{a}{2}\right) \bar{I}\left(x, x+\frac{a}{2}\right) A\!\!\!/(x+b) \psi\left(x-\frac{a}{2}\right) I\left(x, x-\frac{a}{2}\right) F_1(a) F_2(a,b) \\
& + \int d^4a d^4b \bar{\psi}\left(x+\frac{a}{2}\right) \bar{I}\left(x, x+\frac{a}{2}\right) \partial\!\!\!/ \alpha'(x+b) \psi\left(x-\frac{a}{2}\right) I\left(x, x-\frac{a}{2}\right) F_1(a) F_2(a,b) \\
& - \frac{1}{4} F^{\mu\nu}(x) F_{\mu\nu}(x+d) F_4(d) \\
= & \mathcal{L}^{nl} - \partial\!\!\!/ \alpha(x) \int d^4a \bar{\psi}\left(x+\frac{a}{2}\right) \bar{I}\left(x, x+\frac{a}{2}\right) \psi\left(x-\frac{a}{2}\right) I\left(x, x-\frac{a}{2}\right) F_1(a) \\
& + \partial\!\!\!/ \alpha(x) \int d^4a \bar{\psi}\left(x+\frac{a}{2}\right) \bar{I}\left(x, x+\frac{a}{2}\right) \psi\left(x-\frac{a}{2}\right) I\left(x, x-\frac{a}{2}\right) F_1(a) \\
= & \mathcal{L}^{nl}.
\end{aligned}
$$



$$V_0^{\mu\nu}(k) = \frac{-ig^{\mu\nu}}{k^2}\frac{1}{\tilde{F}_4(k)},$$

$$V_1(p) = \frac{i}{\not{p} - m}\frac{1}{\tilde{F}_1(p)}.$$

$$V_2^\mu(p,q) = \gamma^\mu \int \frac{d^4k}{(2\pi)^4}\tilde{F}_1(k)\tilde{F}_2\left(\frac{p+p'}{2} - k, q\right) = \gamma^\mu \tilde{G}_2(p,q)$$

$$V_3^\mu(p,q) = (\not{p} - m)\frac{(q+2p)^\mu}{(q+p)^2 - p^2}\left[\tilde{G}_3(q+p,q) - \tilde{G}_3(p,q)\right]$$

FIG. 1. Propagators and vertices in nonlocal QED appeared in one-loop diagrams.

$$V_4^{\mu\nu}(p,q_1,q_2) = \gamma^\mu \frac{(q_2+2p)^\nu}{(q_2+p)^2 - p^2}\left[\tilde{G}_{23}(q_2+p,q_1,q_2) - \tilde{G}_{23}(p,q_1,q_2)\right]$$

$$V_5^{\mu\nu}(p,q_1,q_2) = (\not{p} - m)\Bigg\{2g^{\mu\nu}\frac{\tilde{G}_{33}(p+q_1+q_2,q_1,q_2) - \tilde{G}_{33}(p,q_1,q_2)}{(p+q_1+q_2)^2 - p^2}$$

$$+ \frac{(2p+q_1)^\mu(2p+2q_1+q_2)^\nu}{(p+q_1+q_2)^2 - (p+q_1)^2}\left[\frac{\tilde{G}_{233}(p+q_1+q_2,q_1,q_2) - \tilde{G}_{33}(p,q_1,q_2)}{(p+q_1+q_2)^2 - p^2} - \frac{\tilde{G}_{233}(p+q_1,q_1,q_2) - \tilde{G}_{33}(p,q_1,q_2)}{(p+q_1)^2 - p^2}\right]$$

$$+ \frac{(2p+q_2)^\nu(2p+2q_2+q_1)^\mu}{(p+q_1+q_2)^2 - (p+q_2)^2}\left[\frac{\tilde{G}_{233}(p+q_1+q_2,q_1,q_2) - \tilde{G}_{33}(p,q_1,q_2)}{(p+q_1+q_2)^2 - p^2} - \frac{\tilde{G}_{33}(p+q_2,q_1,q_2) - \tilde{G}_{233}(p,q_1,q_2)}{(p+q_2)^2 - p^2}\right]\Bigg\}$$

$$V_6^{\mu\nu\rho}(p,q_1,q_2,q_3) = \gamma^\mu \left\{ 2g^{\nu\rho} \frac{\tilde{G}_{233}(p+q_2+q_3,q_1,q_2,q_3) - \tilde{G}_{233}(p,q_1,q_2,q_3)}{(p+q_2+q_3)^2 - p^2} \right.$$

$$+ \frac{(2p+q_2)^\nu(2p+2q_2+q_3)^\rho}{(p+q_2+q_3)^2-(p+q_2)^2} \left[\frac{\tilde{G}_{233}(p+q_2+q_3,q_1,q_2,q_3) - \tilde{G}_{233}(p,q_1,q_2,q_3)}{(p+q_2+q_3)^2 - p^2} - \frac{\tilde{G}_{3233}(p+q_2,q_1,q_2,q_3) - \tilde{G}_{233}(p,q_1,q_2,q_3)}{(p+q_2)^2 - p^2} \right]$$

$$\left. + \frac{(2p+q_3)^\rho(2p+2q_3+q_2)^\nu}{(p+q_2+q_3)^2-(p+q_3)^2} \left[\frac{\tilde{G}_{233}(p+q_2+q_3,q_1,q_2,q_3) - \tilde{G}_{233}(p,q_1,q_2,q_3)}{(p+q_2+q_3)^2 - p^2} - \frac{\tilde{G}_{233}(p+q_3,q_1,q_2,q_3) - \tilde{G}_{233}(p,q_1,q_2,q_3)}{(p+q_3)^2 - p^2} \right] \right\}$$

In the expression of Feynman rules above we've introduced

$$\tilde{G}_i(p,q) = \int \frac{d^4k}{(2\pi)^4} \tilde{F}_1(k) \tilde{F}_i\,(p-k,q)$$

$$\tilde{G}_{ij}(p,q_1,q_2) = \int \frac{d^4k_1 d^4k_2}{(2\pi)^8} \tilde{F}_1(k_1) \tilde{F}_i\,(k_2,q_1)\,\tilde{F}_j\,(p-k_1-k_2,q_2)$$

$$\tilde{G}_{ijk}(p,q_1,q_2,q_3) = \int \frac{d^4k_1 d^4k_2 d^4k_3}{(2\pi)^{12}} \tilde{F}_1(k_1) \tilde{F}_i\,(k_2,q_1)\,\tilde{F}_j\,(k_3,q_2)\,\tilde{F}_k\,(p-k_1-k_2-k_3,q_3)$$

$i,j,k,l \in \{2,3\}, i \leqslant j \leqslant k \leqslant l$

If we suppose

$$\forall i \in \{2,3\},\ \tilde{G}_i(p,q=0) = \tilde{F}_1(p).$$

Then $$F_2(a,b) = F_2(b),\ F_3(a,c) = F_3(c)$$

$$\begin{aligned}
\tilde{G}_{ij}(p,q_1,q_2=0) &= \int \frac{d^4k_1 d^4k_2}{(2\pi)^8} \tilde{F}_1(k_1) \tilde{F}_i\,(k_2,q_1)\,\tilde{F}_j\,(p-k_1-k_2,0) \\
&= \int \frac{d^4k_2}{(2\pi)^4} \tilde{G}_{1j}(p-k_2,0) \tilde{F}_i\,(k_2,q_1) \\
&= \int \frac{d^4k_2}{(2\pi)^4} \tilde{F}_1(p-k_2) \tilde{F}_i\,(k_2,q_1) \\
&= \tilde{G}_i(p,q_1).
\end{aligned}$$

Similarly,

$$\tilde{G}_{ijk}(p,q_1,q_2,q_3=0) = \tilde{G}_{ij}(p,q_1,q_2).$$

$$\begin{aligned}
\frac{dV_1(p)}{dp_\mu} &= i \lim_{q\to 0} V_1(p)[V_2^\mu(p,q) + V_3^\mu(p,q)]V_1(p) \\
\frac{\partial V_2^\mu(p,q_1)}{\partial p_\nu} &= i \lim_{q_2\to 0} V_4^{\mu\nu}(p,q_1,q_2) \\
\frac{\partial V_3^\mu(p,q_1)}{\partial p_\nu} &= i \lim_{q_2\to 0} [V_4^{\nu\mu}(p,q_2,q_1) + V_5^{\mu\nu}(p,q_1,q_2)] \\
\frac{\partial V_4^{\mu\nu}(p,q_1,q_2)}{\partial p_\rho} &= i \lim_{q_3\to 0} V_6^{\mu\nu\rho}(p,q_1,q_2,q_3) \\
\frac{\partial V_5^{\mu\nu}(p,q_1,q_2)}{\partial p_\rho} &= i \lim_{q_3\to 0} [V_6^{\rho\mu\nu}(p,q_3,q_1,q_2) + V_7^{\mu\nu\rho}(p,q_1,q_2,q_3)]
\end{aligned}$$

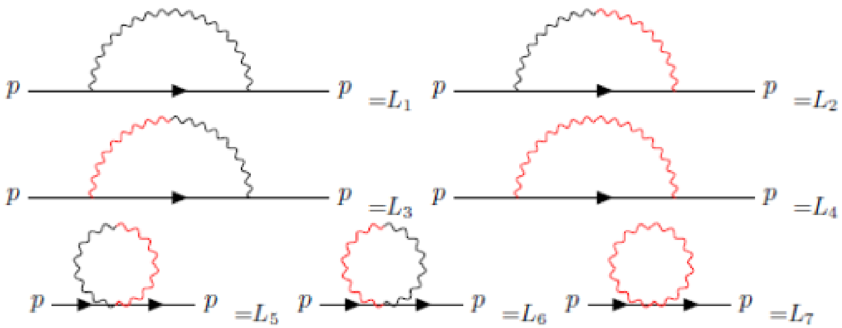


FIG. 7. Lepton's one loop self energy diagrams in nonlocal QED.

$$L_1(p) = -e^2 \int \frac{d^4k}{(2\pi)^4} V_2^{\nu}(p-k,k)\, V_1(p-k) V_2^{\mu}(p,-k)\, V_{0\mu\nu}(k),$$

$$L_2(p) = -e^2 \int \frac{d^4k}{(2\pi)^4} V_3^{\nu}(p-k,k)\, V_1(p-k) V_2^{\mu}(p,-k)\, V_{0\mu\nu}(k),$$

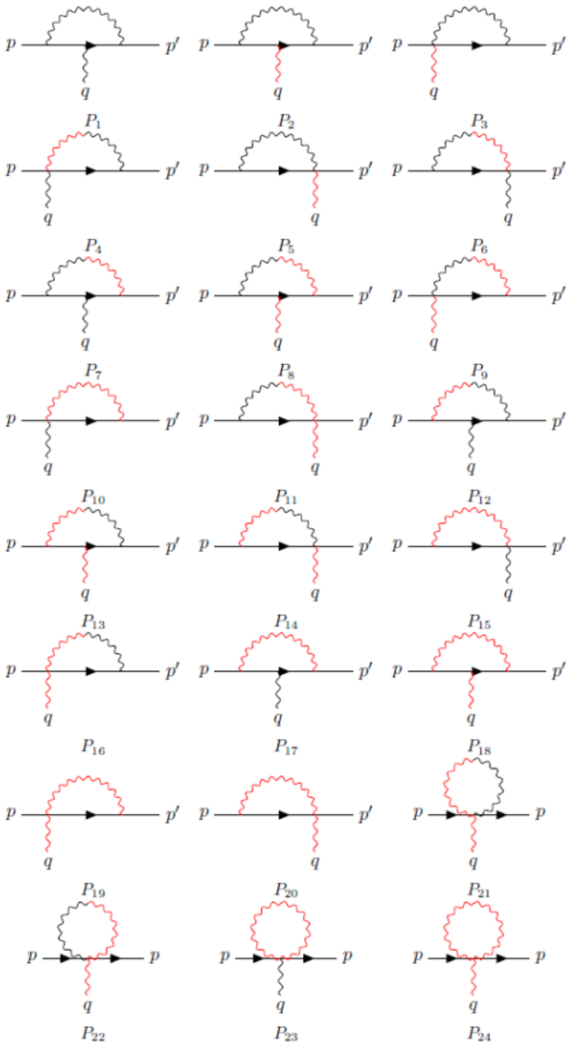
$$L_3(p) = -e^2 \int \frac{d^4k}{(2\pi)^4} V_2^{\nu}(p-k,k)\, V_1(p-k) V_3^{\mu}(p,-k)\, V_{0\mu\nu}(k),$$

$$L_4(p) = -e^2 \int \frac{d^4k}{(2\pi)^4} V_3^{\nu}(p-k,k)\, V_1(p-k) V_3^{\mu}(p,-k)\, V_{0\mu\nu}(k),$$

$$L_5(p) = -e^2 \int \frac{d^4k}{(2\pi)^4} V_4^{\mu\nu}(p,-k,k) g_{\mu\nu},$$

$$L_6(p) = -e^2 \int \frac{d^4k}{(2\pi)^4} V_4^{\nu\mu}(p,k,-k) g_{\mu\nu},$$

$$L_7(p) = -e^2 \int \frac{d^4k}{(2\pi)^4} V_5^{\mu\nu}(p,k,-k) g_{\mu\nu}.$$



$$P_1^\mu(p,q) = ie^3 \int \frac{d^4k}{(2\pi)^4} V_2^\nu\left(p'-k,k\right) V_1(p-k+q) V_2^\mu(P-k,q) V_1(p-k) V_2^\rho\left(p,-k\right) V_{0\nu\rho}(k),$$

$$P_2^\mu(p,q) = ie^3 \int \frac{d^4k}{(2\pi)^4} V_2^\nu\left(p'-k,k\right) V_1(p-k+q) V_3^\mu(p-k,q) V_1(p-k) V_2^\rho\left(p,-k\right) V_{0\nu\rho}(k),$$

$$P_3^\mu(p,q) = ie^3 \int \frac{d^4k}{(2\pi)^4} V_2^\nu\left(p'-k,k\right) V_1(p-k+q) V_{0\nu\rho}(k) V_4^{\mu\rho}\left(p,-k,q\right),$$

$$P_4^\mu(p,q) = ie^3 \int \frac{d^4k}{(2\pi)^4} V_2^\nu\left(p'-k,k\right) V_1(p-k+q) V_{0\nu\rho}(k) V_4^{\mu\rho}\left(p,q,-k\right),$$

$$P_5^\mu(p,q) = ie^3 \int \frac{d^4k}{(2\pi)^4} V_4^{\mu\nu}\left(p-k,k,q\right) V_1(p-k) V_{0\nu\rho}(k) V_2^\rho\left(p,-k\right),$$

$$P_6^\mu(p,q) = ie^3 \int \frac{d^4k}{(2\pi)^4} V_4^{\mu\nu}\left(p-k,q,k\right) V_1(p-k) V_{0\nu\rho}(k) V_2^\rho\left(p,-k\right),$$

$$P_7^\mu(p,q) = ie^3 \int \frac{d^4k}{(2\pi)^4} V_3^\nu\left(p'-k,k\right) V_1(p-k+q) V_2^\mu(P-k,q) V_1(p-k) V_2^\rho\left(p,-k\right) V_{0\nu\rho}(k),$$

$$P_8^\mu(p,q) = ie^3 \int \frac{d^4k}{(2\pi)^4} V_3^\nu\left(p'-k,k\right) V_1(p-k+q) V_3^\mu(p-k,q) V_1(p-k) V_2^\rho\left(p,-k\right) V_{0\nu\rho}(k),$$

$$P_9^\mu(p,q) = ie^3 \int \frac{d^4k}{(2\pi)^4} V_3^\nu\left(p'-k,k\right) V_1(p-k+q) V_{0\nu\rho}(k) V_4^{\mu\rho}\left(p,-k,q\right),$$

$$P_{10}^\mu(p,q) = ie^3 \int \frac{d^4k}{(2\pi)^4} V_3^\nu\left(p'-k,k\right) V_1(p-k+q) V_{0\nu\rho}(k) V_4^{\mu\rho}\left(p,q,-k\right),$$

$$P_{11}^\mu(p,q) = ie^3 \int \frac{d^4k}{(2\pi)^4} V_5^{\mu\nu}\left(p-k,q,k\right) V_1(p-k) V_{0\nu\rho}(k) V_2^\rho\left(p,-k\right),$$

$$P_{12}^\mu(p,q) = ie^3 \int \frac{d^4k}{(2\pi)^4} V_2^\nu\left(p'-k,k\right) V_1(p-k+q) V_2^\mu(P-k,q) V_1(p-k) V_3^\rho\left(p,-k\right) V_{0\nu\rho}(k),$$

$$-e\frac{dL_1(p)}{dp_\mu} = \lim_{q\to 0}[P_1^\mu(p,q) + P_2^\mu(p,q) + P_3^\mu(p,q) + P_5^\mu(p,q)],$$

$$-e\frac{dL_2(p)}{dp_\mu} = \lim_{q\to 0}[P_7^\mu(p,q) + P_8^\mu(p,q) + P_6^\mu(p,q) + P_9^\mu(p,q) + P_{11}^\mu(p,q)],$$

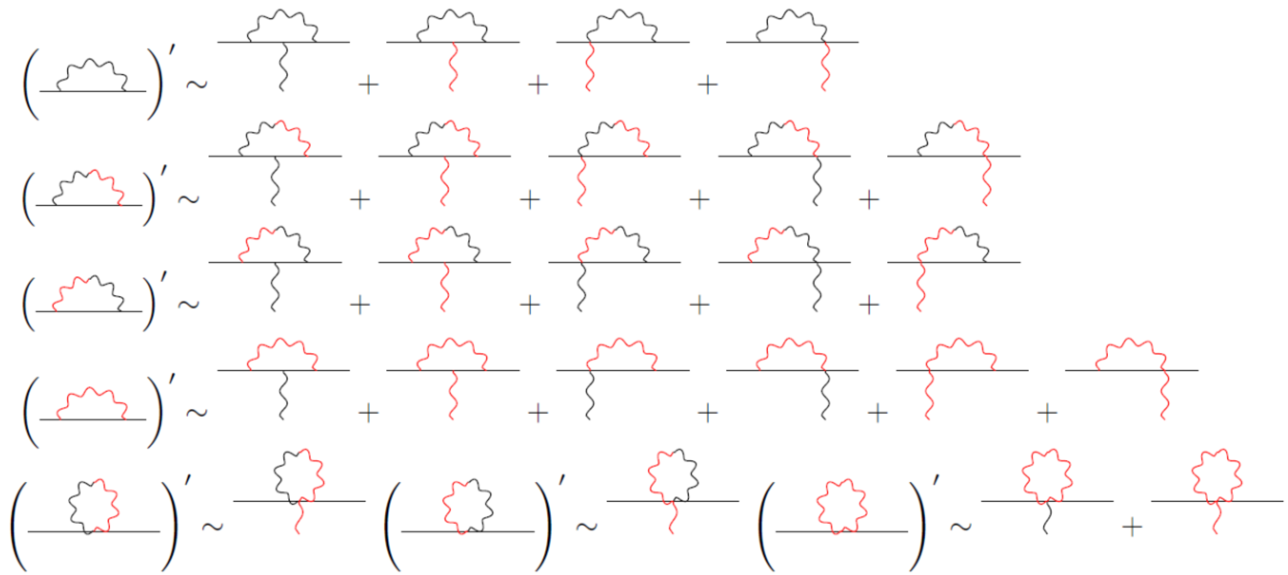
$$-e\frac{dL_3(p)}{dp_\mu} = \lim_{q\to 0}[P_{12}^\mu(p,q) + P_{13}^\mu(p,q) + P_{14}^\mu(p,q) + P_4^\mu(p,q) + P_{16}^\mu(p,q)],$$

$$-e\frac{dL_4(p)}{dp_\mu} = \lim_{q\to 0}[P_{17}^\mu(p,q) + P_{18}^\mu(p,q) + P_{10}^\mu(p,q) + P_{15}^\mu(p,q) + P_{19}^\mu(p,q) + P_{20}^\mu(p,q)],$$

$$-e\frac{dL_5(p)}{dp_\mu} = \lim_{q\to 0} P_{22}^\mu(p,q),$$

$$-e\frac{dL_6(p)}{dp_\mu} = \lim_{q\to 0} P_{21}^\mu(p,q),$$

$$-e\frac{dL_7(p)}{dp_\mu} = \lim_{q\to 0}[P_{23}^\mu(p,q) + P_{24}^\mu(p,q)].$$



$$\int d^4 a F_1(a) = \int d^4 b F_2(a,b) = \int d^4 c F_3(a,c) = \int d^4 d F_4(d) = 1.$$

$$\alpha(x) = \int d^4 b \alpha'(x+b) F_2(a,b) = \int d^4 c \alpha'(x+c) F_3(a,c).$$

$$\int d^4 x d^4 a \bar{\psi}\left(x+\frac{a}{2}\right)\psi\left(x-\frac{a}{2}\right) F_1(a)\alpha(x) = \int d^4 x d^4 a d^4 b \bar{\psi}\left(x+\frac{a}{2}\right)\psi\left(x-\frac{a}{2}\right) F_1(a) F_2(a,b)\alpha'(x+b).$$

$$\bar{u}(k_1)u(k_2)\tilde{F}_1(K)\tilde{\alpha}(k_1-k_2) = \int d^4 k_3 \bar{u}(k_1)u(k_2)\tilde{F}_1(k_3)\tilde{F}_2(K-k_3, k_2-k_1)\tilde{\alpha}'(k_1-k_2)$$

$$\int d^4 k \tilde{F}_2(k,0) e^{-ik\cdot a} = 1 \qquad \tilde{\alpha}(k) = \int d^4 k' \tilde{F}_2(k', -k)\tilde{\alpha}'(k) e^{-ik'\cdot a}$$

$$\tilde{F}_1(p) = \int d^4 k \tilde{F}_1(k)\tilde{F}_2(p-k, 0).$$

$$\tilde{F}_1(p) = \int d^4 k \tilde{F}_1(k)\tilde{F}_3(p-k, 0).$$

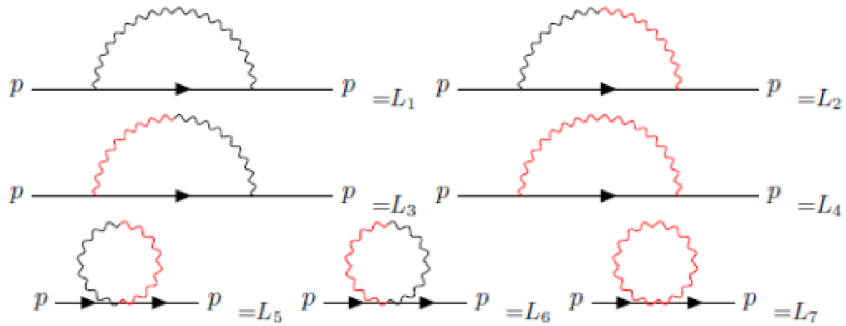


FIG. 7. Lepton's one loop self energy diagrams in nonlocal QED.

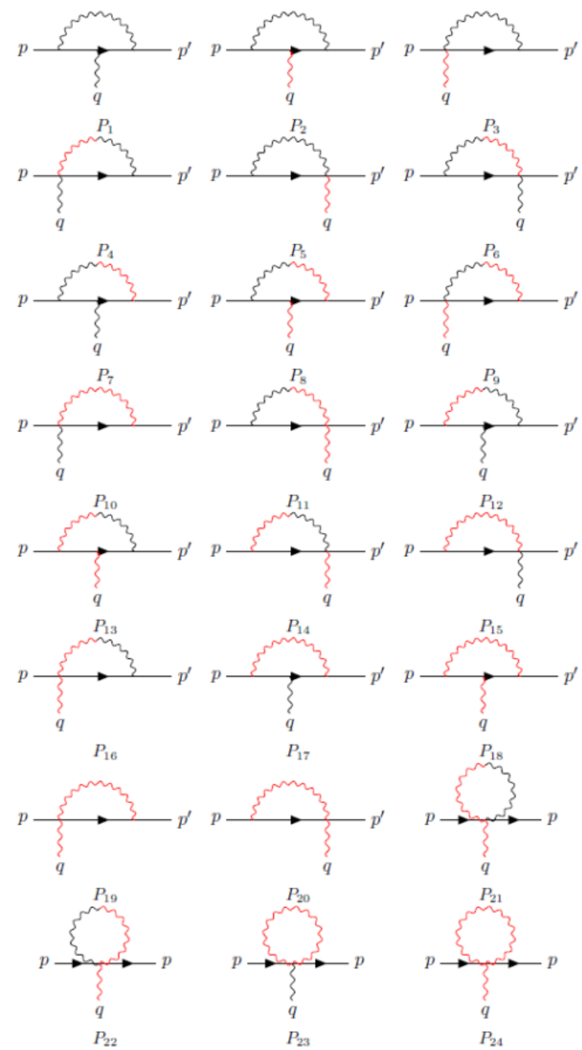
$$S(p) = V_1(p) + V_1(p) \cdot 1\text{PI} \cdot V_1(p) + V_1(p) \cdot 1\text{PI} \cdot V_1(p) \cdot 1\text{PI} \cdot V_1(p) + \cdots$$

$$\lim_{q \to 0} S(p+q)[-ie\Gamma^\mu(p+q,p)]S(p) = -e\frac{dS(p)}{dp_\mu}$$

$$\lim_{q \to 0} [-ieq_\mu \Gamma^\mu(p+q,p)] = e \lim_{q \to 0} \left[S^{-1}(p+q) - S^{-1}(p)\right]$$

$$\frac{d\Sigma(\not{p})}{d\not{p}} = -F_1^{loop}(0) \qquad Z_2 - 1 \; = \; \left.\frac{d\Sigma(\not{p})}{d\not{p}}\right|_{\not{p}=m}$$

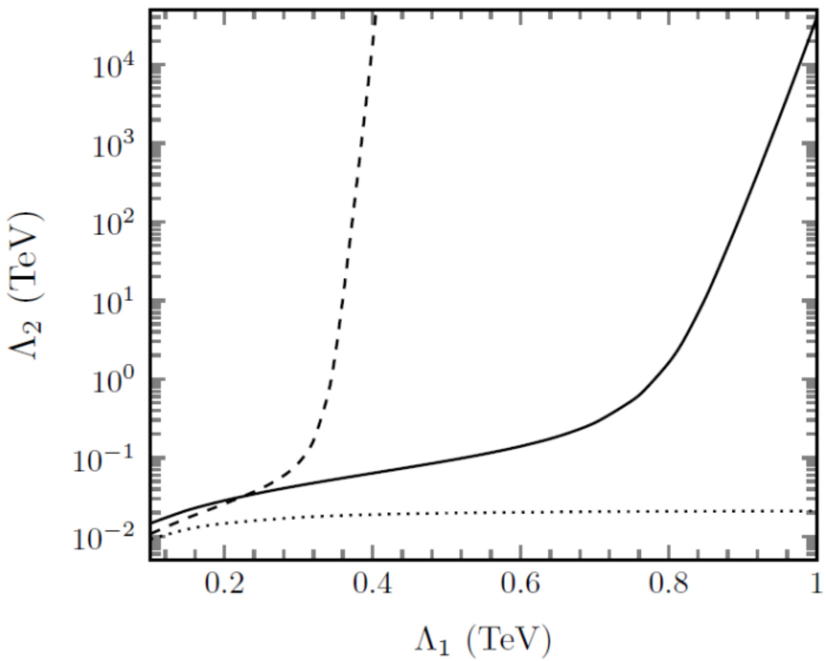
$$F_1(0) = Z_2 + F_1^{loop}(0) = 1$$

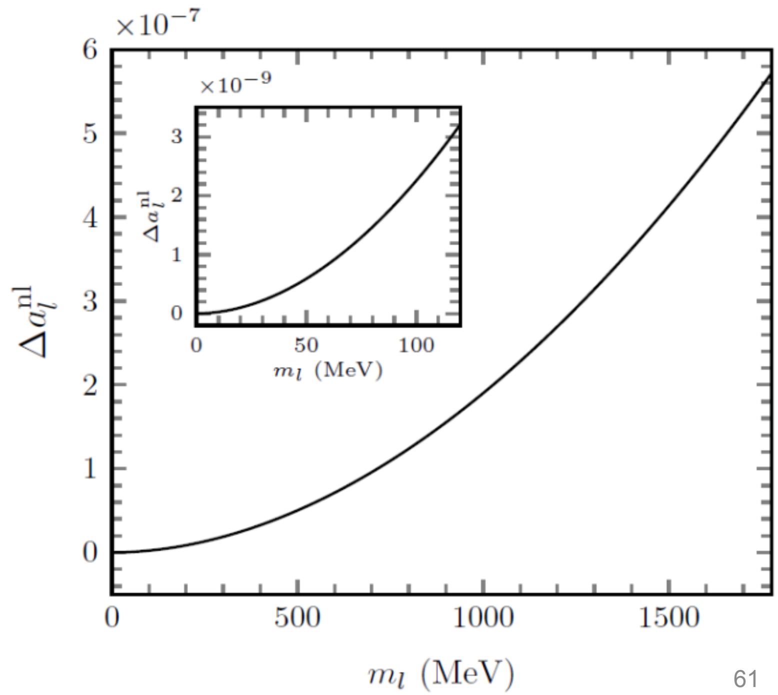


$$F_2(a,b) = F_2(b),\ F_3(a,c) = F_3(c)$$

$$\tilde{F}_1(p) = \frac{\Lambda_1^2 - p^2}{\Lambda_1^2}, \quad \tilde{F}_2(k) = \frac{\Lambda_2^2}{\Lambda_2^2 - k^2}$$

$$\tilde{G}_2(p,q) = \tilde{F}_1(P)\tilde{F}_2(q),$$
$$\tilde{G}_3(p,q) = \tilde{F}_1(p)\tilde{F}_3(q),$$
$$\tilde{G}_{ij}(p,q_1,q_2) = \tilde{F}_1(p)\tilde{F}_i(q_1)\tilde{F}_j(q_2),$$
$$\tilde{G}_{ijk}(p,q_1,q_2,q_3) = \tilde{F}_1(p)\tilde{F}_i(q_1)\tilde{F}_j(q_2)\tilde{F}_k(q_3)$$





Summary

- Nonlocal Lagrangian is constructed by introducing the correlation function and the gauge link.
- Correlation function generates the relativistic regulator and the loop integral is convergent.
- Gauge link guarantees the locally gauge invariance and generate additional diagrams.
- When Lambda goes to infinity, all the formulas are the same as those in the local case.
- The nonlocal Lagrangian can be applied to study hadron properties at relatively at large Q^2.
- EM form factors, strange form factors, light sea quark form factors, octet form factors, gravitational form factors, unpolarized PDFs, polarized PDFs, GPDs, TMDs … …
- Without fine-tuning, the agreement between the calculation and experiment is very good.
- The nonlocal behaviour could be the general property for all the interactions.
- Lepton g-2 anomaly can be explained in nonlocal QED.
- Gravity can be described in the same framework as other interactions in SM.

The End