TMD J/ψ Production in NRQCD

MARSTON COPELAND, REED HODGES, THOMAS MEHEN, SEAN FLEMING

Parton Model

- In hadrons, the strong dynamics of QCD take place at the scale Λ_{QCD} ~ 200 MeV.

- Perturbation theory is not applicable, so other strategies to predict observables are necessary.

- The parton model embeds the non-perturbative behavior in parton distribution functions (PDFs) and fragmentation functions (FFs).

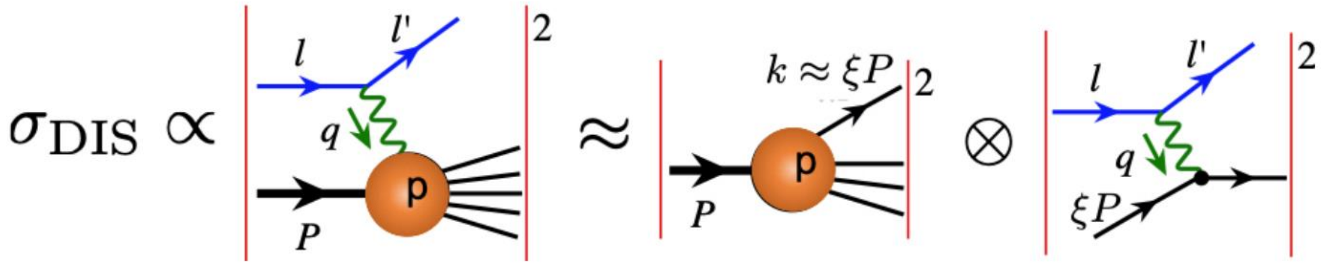
- In QCD, partons are identified with the quarks, antiquarks, and gluons inside of hadrons.

QCD Factorization

- The parton model allows for “factorization” of cross sections.

- . Separate physics taking place at different scales.
  - (a) Process independent non-perturbative contributions given by PDFs and FFs.
  - (b) Perturbatively calculable short-distance partonic cross sections ($\hat{\sigma}$).

- (a) can be extracted from experimental data, calculated on the lattice, or computed using effective field theories.

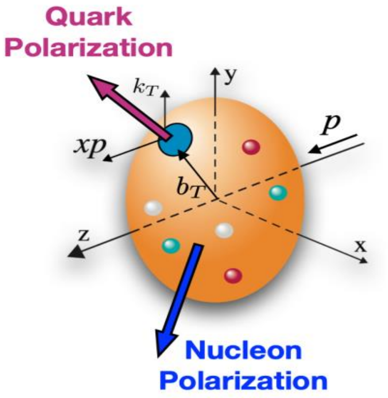


TMDs

- Transverse Momentum Dependent (TMD) PDFs and FFs probe the 3D structure of hadrons.

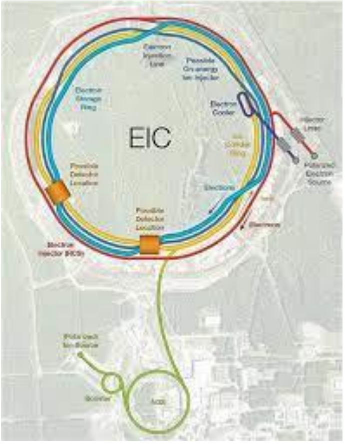
- TMDs are distribution densities to find a quark or a gluon carrying longitudinal momentum fraction (z) and transverse momentum ($\mathbf{k}_T$) with respect to their bound state.

- They provide correlations between hadron spin and parton polarization, in addition to the motion of the parton.



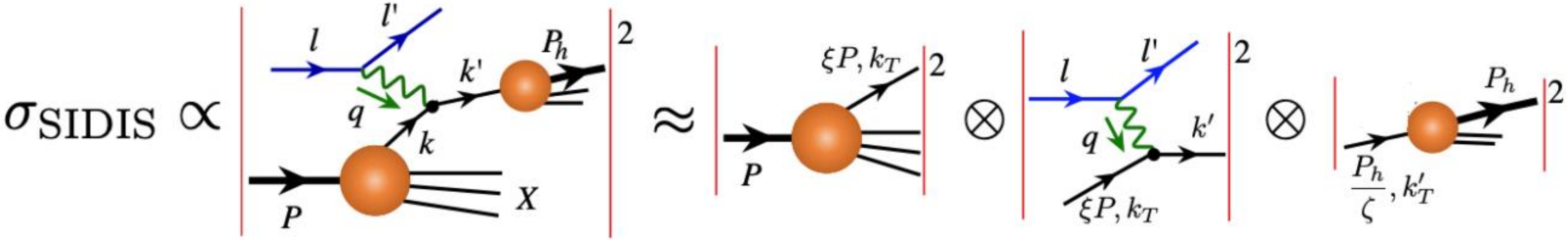
SIDIS

- Semi-Inclusive Deep Inelastic Scattering (SIDIS) will be a major focus of the upcoming Electron-Ion Collider.



$$e + p \rightarrow e + H + X$$

- Electron scatters off a proton and produces hadrons which are detected in the final state.



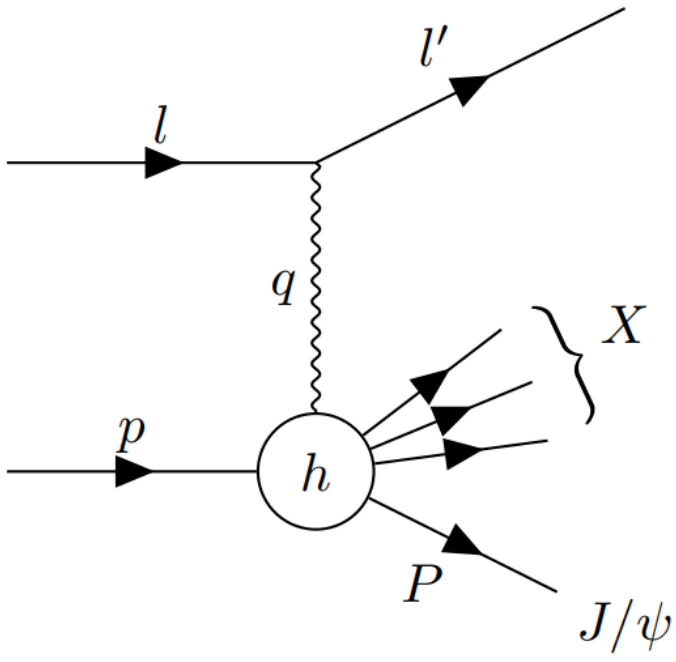
Kinematics

- Process can be written in terms of standard DIS variables.

$$x_B = \frac{Q^2}{2p\cdot q} \qquad q^2 = -Q^2$$

$$y = \frac{p\cdot q}{p\cdot l} \qquad z = \frac{p\cdot P_{J/\psi}}{p\cdot q}$$

$$l + p \to l' + P + X$$

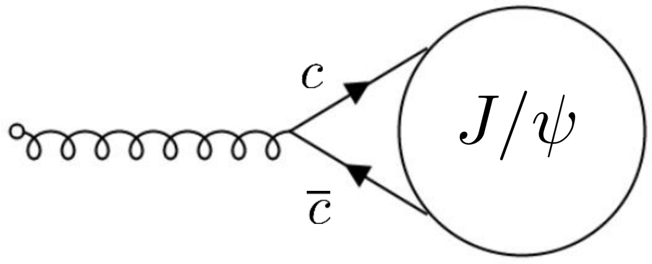


Why J/ψ Production?

- The large masses the heavy quarks allow for the non-perturbative dynamics to be studied using Non-Relativistic QCD (NRQCD).
  - Something we can actually calculate!

- J/ψ production can be identified with the production of a $c\bar{c}$.
  - Non-perturbative (hadronization) effects happen at longer distances.

- Offers one of the few direct probes of the gluon content in the proton



NRQCD

- Non-Relativistic QCD is an effective field theory of QCD where heavy quarks are treated as non-relativistic, but gluons and light-quarks are left as the fully relativistic fields.

- Can be derived by making a non-relativistic expansion of the spinors in powers of small relative velocity, of $Q\overline{Q}$ pair, v.

- Calculation involves a double expansion in α_S and in v.

$$\mathcal{L}_{NRQCD} = \sum \overline{q} i \gamma^{\mu} D_{\mu} q - \frac{1}{2} \mathrm{Tr}\, \mathrm{G}_{\mu\nu} \mathrm{G}^{\mu\nu} + \psi^{\dagger} \left(\mathrm{iD_t} + \frac{\mathbf{D}}{2\mathrm{M}} \right) \psi + \chi^{\dagger} \left(\mathrm{iD_t} - \frac{\mathbf{D}}{2\mathrm{M}} \right) \chi$$

$$D^{\mu} = \partial^{\mu} + igA^{\mu}$$

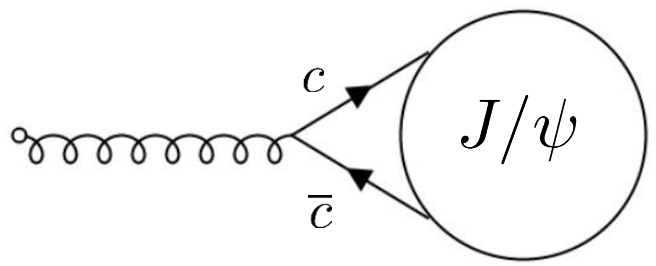
NRQCD Factorization (collinear)

- NRQCD factorization theorem separates quarkonium TMDFF into short distance coefficients ($d_{i\to c\bar{c}}$) and NRQCD long distance matrix elements ($\langle \mathcal{O}^{J/\psi}\rangle$).

- Short distance coefficients $d_{i\to c\bar{c}}$ describe production of $c\bar{c}$ from a parton, "i."
  - Perturbatively calculable though NRQCD matching.

- NRQCD LDMEs describe the hadronization of a $c\bar{c}$ with specific quantum numbers into a J/ψ .
  - Formally, a NRQCD double parton fragmentation function.
  - In practice, a constant extracted from experiment.

$$\Delta_{i\to J/\psi}(z) \to \sum_{L,s,c} d^{L,s,c}_{i\to c\bar{c}}(z)\langle \mathcal{O}^{J/\psi}(^{2s+1}L^{[c]}_J)\rangle$$



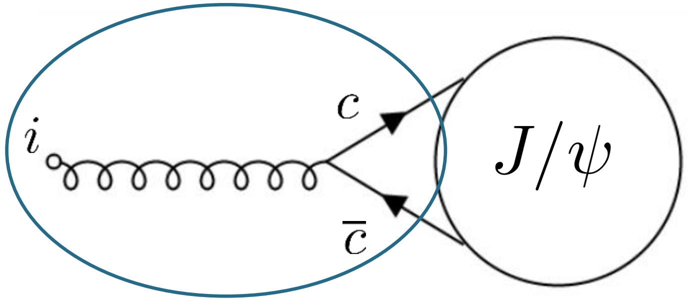
NRQCD Factorization (collinear)

- NRQCD factorization theorem separates quarkonium TMDFF into short distance coefficients ($d_{i\to c\bar{c}}$) and NRQCD long distance matrix elements ($\langle\mathcal{O}^{J/\psi}\rangle$).

- Short distance coefficients $d_{i\to c\bar{c}}$ describe production of $c\bar{c}$ from a parton, "i."
  - Perturbatively calculable though NRQCD matching.

- NRQCD LDMEs describe the hadronization of a $c\bar{c}$ with specific quantum numbers into a J/ψ .
  - Formally, a NRQCD double parton fragmentation function.
  - In practice, a constant extracted from experiment.

$$\Delta_{i\to J/\psi}(z) \to \sum_{L,s,c} d^{L,s,c}_{i\to c\bar{c}}(z)\langle\mathcal{O}^{J/\psi}(^{2s+1}L^{[c]}_J)\rangle$$



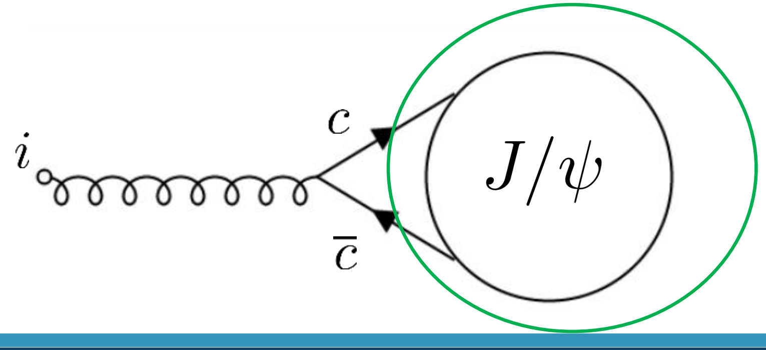
NRQCD Factorization (collinear)

- NRQCD factorization theorem separates quarkonium TMDFF into short distance coefficients ($d_{i\to c\bar{c}}$) and NRQCD long distance matrix elements ($\langle \mathcal{O}^{J/\psi} \rangle$).

- Short distance coefficients $d_{i\to c\bar{c}}$ describe production of $c\bar{c}$ from a parton, "i."
  - Perturbatively calculable though NRQCD matching.

- NRQCD LDMEs describe the hadronization of a $c\bar{c}$ with specific quantum numbers into a J/ψ .
  - Formally, a NRQCD double parton fragmentation function.
  - In practice, a constant extracted from experiment.

$$\Delta_{i\to J/\psi}(z) \to \sum_{L,s,c} d_{i\to c\bar{c}}^{L,s,c}(z) \langle \mathcal{O}^{J/\psi}({}^{2s+1}L_J^{[c]}) \rangle$$



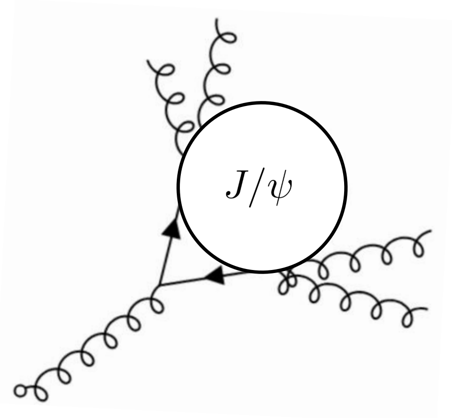
NRQCD Factorization (TMD)

- TMD NRQCD factorization theorem applies the same principles, but now both $d_{i\to c\bar{c}}$ and $\langle \mathcal{O}^{J/\psi} \rangle$ have transverse momentum dependence.

- We calculate the TMD short distance matching coefficients, $d_{i\to c\bar{c}}(\mathbf{k}_\perp)$.

- NRQCD TMDFF $(D_{c\bar{c}\to J/\psi}(\mathbf{p}_\perp))$ can have additional transverse momentum dependence due to soft gluon radiation.

$$\Delta_{i\to J/\psi}(z, \boldsymbol{k}_\perp) \to \int d^2\boldsymbol{p}_\perp d^2\boldsymbol{q}_\perp d_{i\to c\bar{c}}(z, \boldsymbol{q}_\perp) D_{c\bar{c}\to J/\psi}(\boldsymbol{p}_\perp) \delta^{(2)}(\mathbf{q}_\perp - \mathbf{k}_\perp + \mathbf{p}_\perp)$$



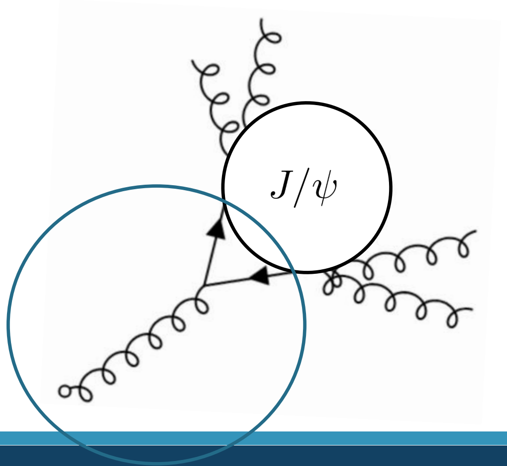
NRQCD Factorization (TMD)

- TMD NRQCD factorization theorem applies the same principles, but now both $d_{i\to c\bar{c}}$ and $\langle \mathcal{O}^{J/\psi} \rangle$ have transverse momentum dependence.

- We calculate the TMD short distance matching coefficients, $d_{i\to c\bar{c}}(\mathbf{k}_\perp)$.

- NRQCD TMDFF $(D_{c\bar{c}\to J/\psi}(\mathbf{p}_\perp))$ can have additional transverse momentum dependence due to soft gluon radiation.

$$\Delta_{i\to J/\psi}(z,\boldsymbol{k}_\perp) \to \int d^2\boldsymbol{p}_\perp d^2\boldsymbol{q}_\perp d_{i\to c\bar{c}}(z,\boldsymbol{q}_\perp) D_{c\bar{c}\to J/\psi}(\boldsymbol{p}_\perp)\delta^{(2)}(\mathbf{q}_\perp - \mathbf{k}_\perp + \mathbf{p}_\perp)$$



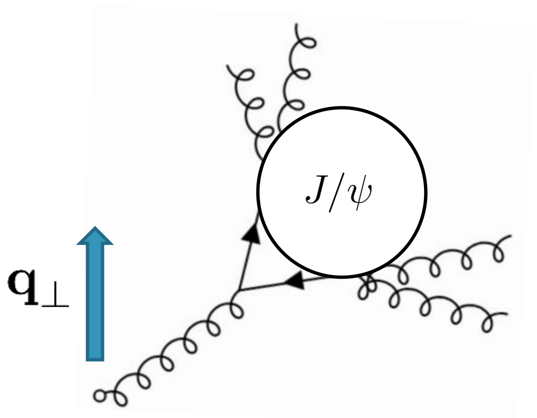
NRQCD Factorization (TMD)

- TMD NRQCD factorization theorem applies the same principles, but now both $d_{i\to c\bar{c}}$ and $\langle \mathcal{O}^{J/\psi} \rangle$ have transverse momentum dependence.

- We calculate the TMD short distance matching coefficients, $d_{i\to c\bar{c}}(\mathbf{k}_\perp)$.

- NRQCD TMDFF $(D_{c\bar{c}\to J/\psi}(\mathbf{p}_\perp))$ can have additional transverse momentum dependence due to soft gluon radiation.

$$\Delta_{i\to J/\psi}(z, \boldsymbol{k}_\perp) \to \int d^2\boldsymbol{p}_\perp d^2\boldsymbol{q}_\perp d_{i\to c\bar{c}}(z, \boldsymbol{q}_\perp) D_{c\bar{c}\to J/\psi}(\boldsymbol{p}_\perp) \delta^{(2)}(\mathbf{q}_\perp - \mathbf{k}_\perp + \mathbf{p}_\perp)$$

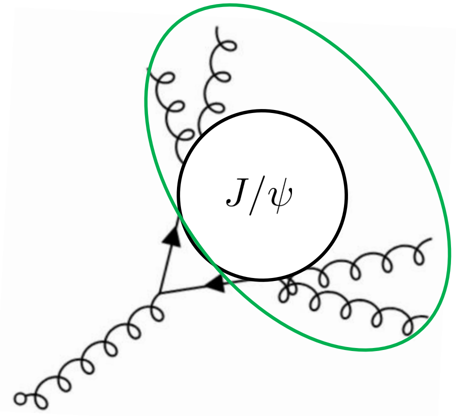


NRQCD Factorization (TMD)

- TMD NRQCD factorization theorem applies the same principles, but now both $d_{i\to c\bar{c}}$ and $\langle \mathcal{O}^{J/\psi} \rangle$ have transverse momentum dependence.

- We calculate the TMD short distance matching coefficients, $d_{i\to c\bar{c}}(\mathbf{k}_\perp)$.

- NRQCD TMDFF $(D_{c\bar{c}\to J/\psi}(\mathbf{p}_\perp))$ can have additional transverse momentum dependence due to soft gluon radiation.



$$\Delta_{i\to J/\psi}(z,\boldsymbol{k}_\perp) \to \int d^2\boldsymbol{p}_\perp d^2\boldsymbol{q}_\perp d_{i\to c\bar{c}}(z,\boldsymbol{q}_\perp) D_{c\bar{c}\to J/\psi}(\boldsymbol{p}_\perp)\delta^{(2)}(\mathbf{q}_\perp - \mathbf{k}_\perp + \mathbf{p}_\perp)$$

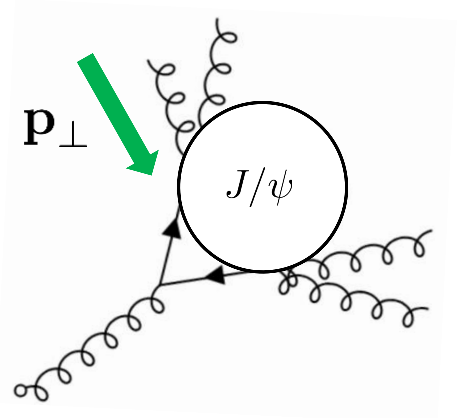
NRQCD Factorization (TMD)

- TMD NRQCD factorization theorem applies the same principles, but now both $d_{i\to c\bar{c}}$ and $\langle \mathcal{O}^{J/\psi} \rangle$ have transverse momentum dependence.

- We calculate the TMD short distance matching coefficients, $d_{i\to c\bar{c}}(\mathbf{k}_\perp)$.

- NRQCD TMDFF $(D_{c\bar{c}\to J/\psi}(\mathbf{p}_\perp))$ can have additional transverse momentum dependence due to soft gluon radiation.

$$\Delta_{i\to J/\psi}(z, \boldsymbol{k}_\perp) \to \int d^2\boldsymbol{p}_\perp d^2\boldsymbol{q}_\perp d_{i\to c\bar{c}}(z, \boldsymbol{q}_\perp) D_{c\bar{c}\to J/\psi}(\boldsymbol{p}_\perp) \delta^{(2)}(\mathbf{q}_\perp - \mathbf{k}_\perp + \mathbf{p}_\perp)$$



NRQCD Factorization (TMD)

- If the transverse momentum is sufficiently greater than $\Lambda_s \sim m_c v^2$, then we can expand the NRQCD TMDFF.

$$D_{c\bar{c}\to J/\psi}(\boldsymbol{p}_\perp) \sim \sum_{L,s,c} \langle \mathcal{O}^{J/\psi}({}^{2s+1}L_J^{[c]})\rangle H_{(L,s;c)}(\boldsymbol{p}_\perp),$$

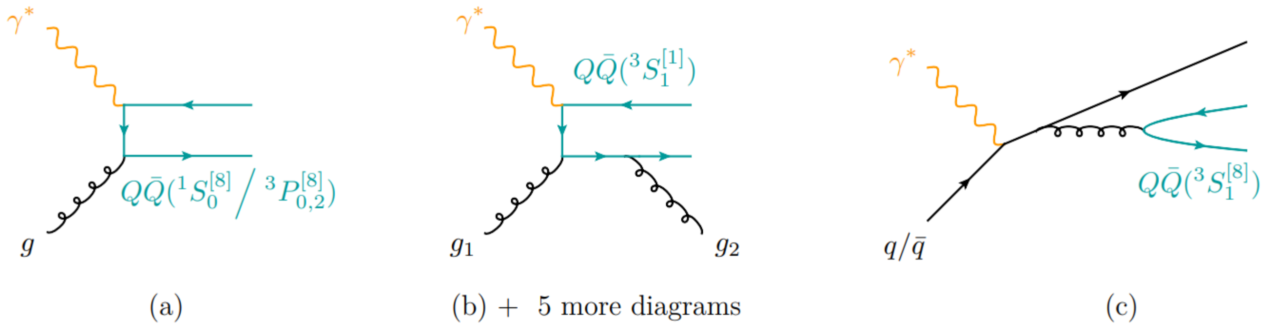
$$H_{(L,s;c)}(\boldsymbol{p}_\perp) = \delta^{(2)}(\boldsymbol{p}_\perp) + \sum_{n=1}^{\infty} \left(\frac{\alpha_s(\boldsymbol{p}_\perp)}{4\pi}\right)^n h^{(n)}_{(L,s;c)}(\boldsymbol{p}_\perp)$$

- This puts all of the transverse momentum dependence in the perturbative matching coefficient!

$$\Delta_{i\to J/\psi}(z, \boldsymbol{k}_\perp) \to \sum_{L,s,c} d_{i\to c\bar{c}}(z, \boldsymbol{k}_\perp)\langle \mathcal{O}^{J/\psi}({}^{2s+1}L_J^{[c]})\rangle$$

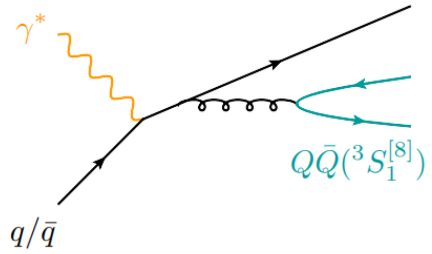
Production Mechanisms

- In SIDIS there are two ways to produce a J/ψ at leading order.
- The incoming electron can hit a quark which then fragments into a J/ψ .
  - Or the electron can probe a gluon – the emitted virtual photon fuses with the gluon to create a $c\bar{c}$ pair.
- The latter is a probe of gluon structure at leading twist!



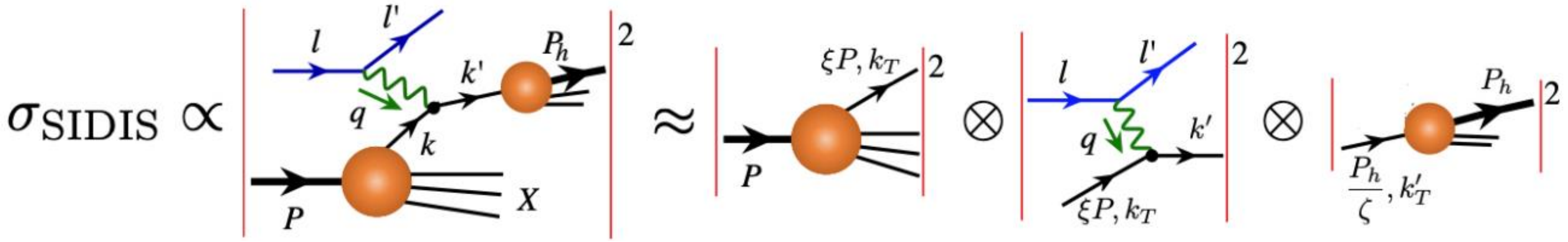
Fragmentation Functions

- Fragmentation functions tell us the probability that a parton “i” will hadronize into a hadron “H”.



- Provide information on how hadrons emerge from energetic quarks and gluons.

- Like PDFs, defined by non-perturbative matrix elements and are used in factorization theorems.

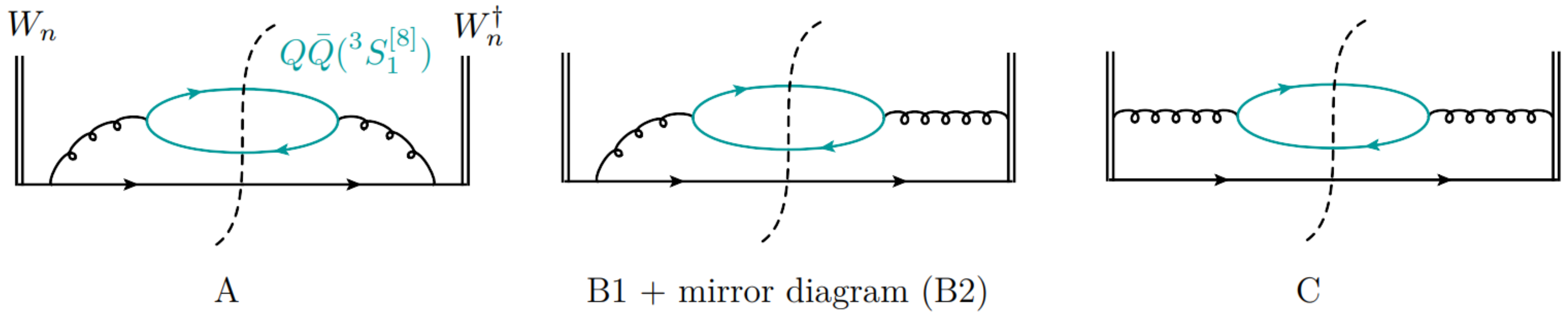


Quark Fragmentation

- Production of a $c\bar{c}$ pair fragmenting from a light quark gives 3 possible diagrams (plus mirrors) at lowest order.

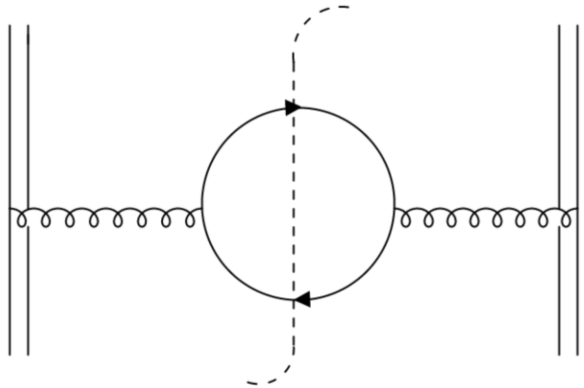
$$\Delta_{q\to J/\Psi} = \frac{1}{2N_C z}\mathrm{Tr}\left[\int \frac{\mathrm{db}^-}{2\pi}\mathrm{e}^{\mathrm{ib}^-\mathrm{P}^+/\mathrm{z}}\sum_{\mathrm{X}} \Gamma^{\alpha\alpha'}\langle 0|\mathrm{W}_\mathrm{n}^\dagger(\mathrm{b})\psi_\mathrm{i}^{\alpha 0}|\mathrm{J}/\psi,\mathrm{X}\rangle\langle \mathrm{J}/\psi,\mathrm{X}|\overline{\psi}_\mathrm{i}^{0\alpha'}\mathrm{W}_\mathrm{n}(0)|0\rangle\right]$$

- Only unpolarized quark to unpolarized J/ψ TMD FF has been studied before.



Gluon Fragmentation (Leading Order)

- TMD Gluon to J/Psi FF has not been studied in literature.



- Does not show up at leading twist in SIDIS but is relevant for quarkonium production in jets.

- Definition of FF is different; however calculation is similar.

$$\Delta^{\alpha\alpha'}_{g\to J/\psi} = \frac{1}{2z^2 P^+(N_c^2-1)} \sum_X \int \frac{db^-}{2\pi} e^{ib^- P^+ z} \langle 0|W_n^\dagger(b) G_a^{\alpha+}|J/\psi, X\rangle \left\langle J/\psi, X|G_a^{\alpha'+} W_n(0)|0\right\rangle$$

Quark Polarizations

- Quark can be unpolarized ($\frac{\gamma^+}{2}$), longitudinally polarized ($\frac{\gamma^+\gamma_5}{2}$), or and transversely polarized ($\frac{1}{2}\sigma^{\alpha+}\gamma_5$).

- Project out these states by completing spin trace in definition.

$$\Delta_{q\to J/\Psi} = \frac{1}{2N_C z}\mathrm{Tr}\left[\int \frac{\mathrm{db}^-}{2\pi}\mathrm{e}^{\mathrm{ib}^-\mathrm{P}^+/\mathrm{z}}\sum_{\mathrm{X}} \Gamma^{\alpha\alpha'}\left\langle 0|\mathrm{W}_\mathrm{n}^\dagger(\mathrm{b})\psi_\mathrm{i}^{\alpha 0}|\mathrm{J}/\psi,\mathrm{X}\right\rangle \left\langle \mathrm{J}/\psi,\mathrm{X}|\overline{\psi}_\mathrm{i}^{0\alpha'}\mathrm{W}_\mathrm{n}(0)|0\right\rangle\right]$$

$$\Gamma \in \frac{\gamma^+}{2}, \frac{\gamma^+\gamma_5}{2}, \frac{1}{2}\sigma^{\alpha+}\gamma_5$$

J/ψ Polarizations

- Project out J/ψ polarization by replacing polarization vectors (like tensor decomposition).

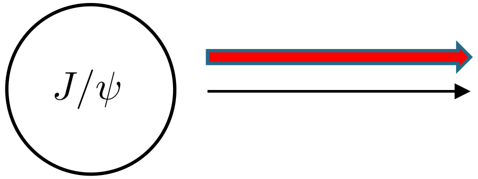
$$\epsilon^i \epsilon^{*j} = \frac{1}{3}\delta^{ij} - \frac{i}{2}\epsilon^{ijk} S_k - T^{ij}$$

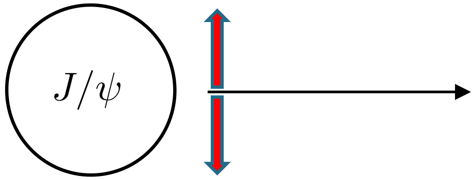
$$\Lambda^\mu_i \Lambda^\nu_j \epsilon^i \epsilon^{*j} \to \frac{1}{3}(-g^{\mu\nu} + \frac{P^\mu P^\nu}{M^2}) - \frac{i}{2M}\epsilon^{\mu\nu\alpha\beta} S_\alpha P_\beta - T^{\mu\nu}$$

- J/ψ can be unpolarized, longitudinally, or transversely polarized.
  - Determined by values for spin parameters.

$$\vec{S} = (S_T^x, S_T^y, S_L) \qquad T_{ij} = \frac{1}{2}\begin{pmatrix} -\frac{2}{3}S_{LL} + S_{TT}^{xx} & S_{TT}^{xy} & S_{LT}^x \\ S_{TT}^{yx} & -\frac{2}{3}S_{LL} - S_{TT}^{xx} & S_{LT}^y \\ S_{LT}^x & S_{LT}^y & \frac{4}{3}S_{LL} \end{pmatrix}.$$

- Polarized fragmentation functions are defined as the objects proportional to these spin parameters.





Quark Fragmentation Functions

- There are 18 polarized quark to J/ψ TMD fragmentation functions.
  - At leading order in the strong coupling, only seven FFs survive!

- Unpolarized quark:

$$D_1(z,\mathbf{k}_T;\mu) = \frac{2\alpha_s^2(\mu)}{9\pi N_c M^3 z}\frac{\mathbf{k}_T^2 z^2(z^2-2z+2)+2M^2(z-1)^2}{[z^2\mathbf{k}_T^2+M^2(1-z)]^2}\left\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]})\right\rangle ,$$

$$D_{1LL}(z,\mathbf{k}_T;\mu) = \frac{2\alpha_s^2(\mu)}{9\pi N_c M^3 z}\frac{\mathbf{k}_T^2 z^2(z^2-2z+2)-4M^2(z-1)^2}{[z^2\mathbf{k}_T^2+M^2(1-z)]^2}\left\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]})\right\rangle ,$$

$$D_{1LT}(z,\mathbf{k}_T;\mu) = \frac{2\alpha_s^2(\mu)}{3\pi N_c M}\frac{(2-z)(1-z)}{[z^2\mathbf{k}_T^2+M^2(1-z)]^2}\left\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]})\right\rangle ,$$

$$D_{1TT}(z,\mathbf{k}_T;\mu) = \frac{2\alpha_s^2(\mu)}{3\pi N_c M}\frac{z(z-1)}{[z^2\mathbf{k}_T^2+M^2(1-z)]^2}\left\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]})\right\rangle .$$

Hadron polarization	Quark polarization: Unpolarized	Quark polarization: Longitudinal	Quark polarization: Transverse
Unpolarized	D_1		$H_1^\perp$
Longitudinal		G_1	$H_{1L}^\perp$
Transverse	$D_{1T}^\perp$	$G_{1T}^\perp$	H_1, $H_{1T}^\perp$
LL	D_{1LL}		$H_{1LL}^\perp$
LT	D_{1LT}	G_{1LT}	$H_{1LT}^\perp$, H_{1LT}'
TT	D_{1TT}	G_{1TT}	$H_{1TT}^\perp$, H_{1TT}'

Quark Fragmentation Functions

- There are 18 polarized quark to J/ψ TMD fragmentation functions.
  - At leading order in the strong coupling, only six FFs survive!

- Unpolarized quark:

$$D_1(z,\mathbf{k}_T;\mu) = \frac{2\alpha_s^2(\mu)}{9\pi N_c M^3 z}\frac{\mathbf{k}_T^2 z^2(z^2-2z+2)+2M^2(z-1)^2}{[z^2\mathbf{k}_T^2+M^2(1-z)]^2}\left\langle \mathcal{O}^{J/\psi}({}^3S_1^{[8]})\right\rangle ,$$

$$D_{1LL}(z,\mathbf{k}_T;\mu) = \frac{2\alpha_s^2(\mu)}{9\pi N_c M^3 z}\frac{\mathbf{k}_T^2 z^2(z^2-2z+2)-4M^2(z-1)^2}{[z^2\mathbf{k}_T^2+M^2(1-z)]^2}\left\langle \mathcal{O}^{J/\psi}({}^3S_1^{[8]})\right\rangle ,$$

$$D_{1LT}(z,\mathbf{k}_T;\mu) = \frac{2\alpha_s^2(\mu)}{3\pi N_c M}\frac{(2-z)(1-z)}{[z^2\mathbf{k}_T^2+M^2(1-z)]^2}\left\langle \mathcal{O}^{J/\psi}({}^3S_1^{[8]})\right\rangle ,$$

$$D_{1TT}(z,\mathbf{k}_T;\mu) = \frac{2\alpha_s^2(\mu)}{3\pi N_c M}\frac{z(z-1)}{[z^2\mathbf{k}_T^2+M^2(1-z)]^2}\left\langle \mathcal{O}^{J/\psi}({}^3S_1^{[8]})\right\rangle .$$

		Quark polarization		
		Unpolarized	Longitudinal	Transverse
Hadron polarization	Unpolarized	D_1		$H_1^\perp$
	Longitudinal		G_1	$H_{1L}^\perp$
	Transverse	$D_{1T}^\perp$	$G_{1T}^\perp$	H_1, $H_{1T}^\perp$
	LL	D_{1LL}		$H_{1LL}^\perp$
	LT	D_{1LT}	G_{1LT}	$H_{1LT}^\perp$, H_{1LT}'
	TT	D_{1TT}	G_{1TT}	$H_{1TT}^\perp$, H_{1TT}'

Quark Fragmentation Functions

- Longitudinally polarized quark:

$$G_{1L}(z, \mathbf{k}_T; \mu) = \frac{\alpha_s^2(\mu)}{3\pi N_c M^3} \frac{\mathbf{k}_T^2 z^2 (2-z)}{[z^2\mathbf{k}_T^2 + M^2(1-z)]^2} \left\langle \mathcal{O}^{J/\psi}({}^3S_1^{[8]}) \right\rangle ,$$

$$G_{1T}^{\perp}(z, \mathbf{k}_T; \mu) = \frac{2\alpha_s^2(\mu)}{3\pi N_c M} \frac{z(z-1)}{[z^2\mathbf{k}_T^2 + M^2(1-z)]^2} \left\langle \mathcal{O}^{J/\psi}({}^3S_1^{[8]}) \right\rangle .$$

		Quark polarization		
		Unpolarized	Longitudinal	Transverse
Hadron polarization	Unpolarized	D_1		$H_1^{\perp}$
	Longitudinal		G_1	$H_{1L}^{\perp}$
	Transverse	$D_{1T}^{\perp}$	$G_{1T}^{\perp}$	$H_1,\ H_{1T}^{\perp}$
	LL	D_{1LL}		$H_{1LL}^{\perp}$
	LT	D_{1LT}	G_{1LT}	$H_{1LT}^{\perp},\ H_{1LT}'$
	TT	D_{1TT}	G_{1TT}	$H_{1TT}^{\perp},\ H_{1TT}'$

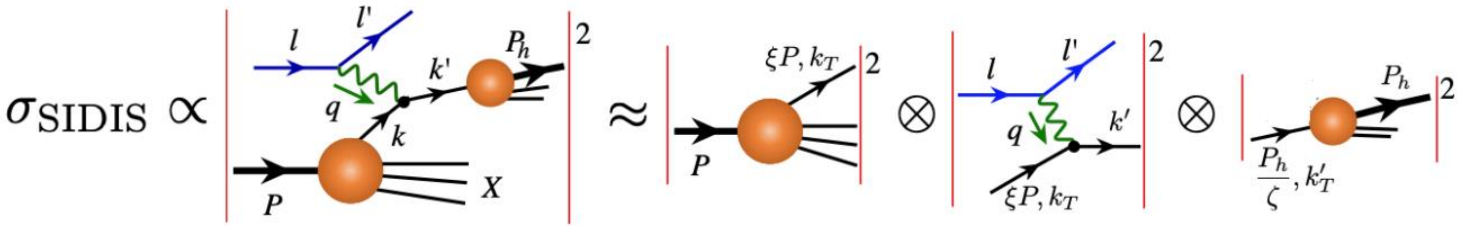
Quark SIDIS Cross Sections

- With unpolarized J/ψ, unpolarized beam, and unpolarized target, there is only one contribution to the cross section at leading twist.

$$\frac{d\sigma_{UU}(l + H \to l' + J/\psi + X)}{dx\ dz\ dy\ d^2\mathbf{P}_\perp} = \frac{4\pi\alpha^2 s}{Q^4}\left(1 - y + \frac{y^2}{2}\right)\mathbf{I}[f_1(3D_1)]$$

where

$$\mathbf{I}[f_1 D_1] = 2z \int d^2\mathbf{k_T} d^2\mathbf{p_T} f_1(x, \mathbf{p_T}) D_1(z, \mathbf{k_T}) \delta^{(2)}(\mathbf{k_T} - \mathbf{p_T} - \mathbf{q_T})$$



Polarized SIDIS cross section

- Polarized J/ψ production is a much richer test of QCD!

$$\frac{d\sigma_{UU}(l+H\to l'+J/\psi+X)}{dx\;dz\;dy\;d^2\mathbf{P}_\perp}=\frac{4\pi\alpha^2 s}{Q^4}\left(1-y+\frac{y^2}{2}\right)\left\{\mathbf{I}[f_1D_1]+S_{LL}\mathbf{I}[f_1D_{LL}]\right\}$$

$$\frac{d\sigma_{LL}(l+H\to l'+J/\psi+X)}{dx\;dz\;dy\;d^2\mathbf{P}_\perp}=\frac{4\pi\alpha^2 s}{Q^4}2\lambda_c S_{qL}\,y\left(1-\frac{y}{2}\right)x\left\{\mathbf{I}[g_{1L}D_1]+S_{LL}\mathbf{I}[g_{1L}D_{1LL}]\right\}.$$

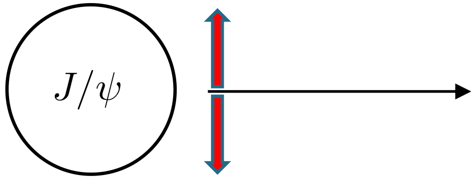
$$\frac{2\pi d\sigma_{UT}(l+H\to l'+J/\psi+X)}{d\phi\;dx\;dz\;dy\;d^2\mathbf{P}_\perp}=$$
$$\frac{4\pi\alpha^2 s}{Q^4}\left(1-y+\frac{y^2}{2}\right)x\left\{|S_{hTT}|\cos(\phi_S^l-\phi_h^l)\sin(\phi_{hTT}^l-\phi_h^l)\mathbf{I}\left[\frac{p^x[(k^x)^2-(k^y)^2]}{MM_h}g_{1T}G_{1TT}\right]\right.$$
$$\left.+|S_{hTT}|\sin(\phi_S^l-\phi_h^l)\cos(\phi_{hTT}^l-\phi_h^l)\mathbf{I}\left[\frac{2p^xk^xk^y}{MM_h}g_{1T}G_{1TT}\right]\right\}.$$

Polarized SIDIS cross sections

$$
\begin{aligned}
&\frac{2\pi d\sigma_{LT}(l + H \to l' + J/\psi + X)}{d\phi dx\ dz\ dy\ d^2\mathbf{P}_\perp} = \\
&\frac{4\pi\alpha^2 s}{Q^4}\,(1-y)\,x|S_{qT}|\Big\{|S_{hLL}|\cos(\phi_S^l - \phi_h^l)\mathbf{I}\left[\frac{p^x}{M}g_{1T}D_{1LL}\right] \\
&+ |S_{hLT}|\cos(\phi_s^h)\cos(\phi_{hLT}^l - \phi_h^l)\mathbf{I}\left[\frac{p^x k^x}{MM_h}g_{1T}D_{1LT}\right] \\
&+ |S_{hTT}|\cos(\phi_s^h)\cos(2\phi_{hTT}^h)\mathbf{I}\left[\frac{p^x[(k^x)^2 - (k^y)^2]}{MM_h^2}g_{1T}D_{1TT}\right] \\
&+ |S_{hLT}|\sin(\phi_s^h)\sin(\phi_{hLT}^l)\mathbf{I}\left[\frac{p^x k^x}{MM_h}g_{1T}D_{1LT}\right] \\
&+ |S_{hTT}|\sin(\phi_s^h)\sin(2\phi_{hTT}^h)\mathbf{I}\left[\frac{2p^x k^x k^y}{MM_h^2}g_{1T}D_{1TT}\right]\Big\}.
\end{aligned}
$$

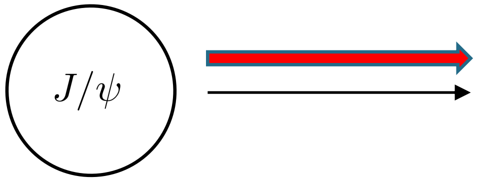
Polarized TMD Fragmentation in SIDIS

- To produce transversely polarized J/ψ, plug in $S_{LL} = 1/2$ and sum over both transverse polarizations (m = +/- 1).



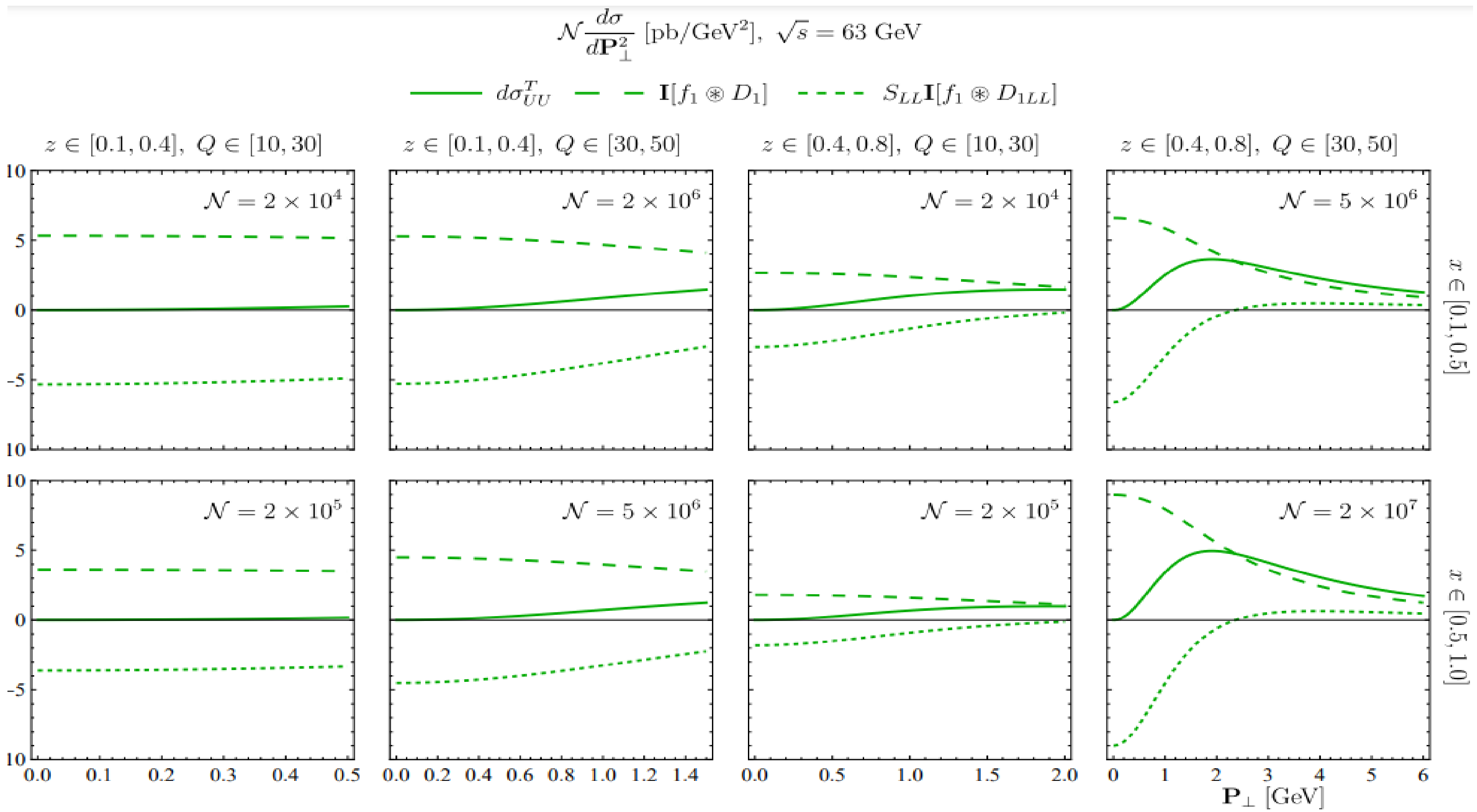
$$\epsilon^{\pm 1} = \frac{1}{\sqrt{2}}(1, \pm i, 0)$$

- To produce longitudinally polarized J/ψ, plug in $S_{LL} = -1$.

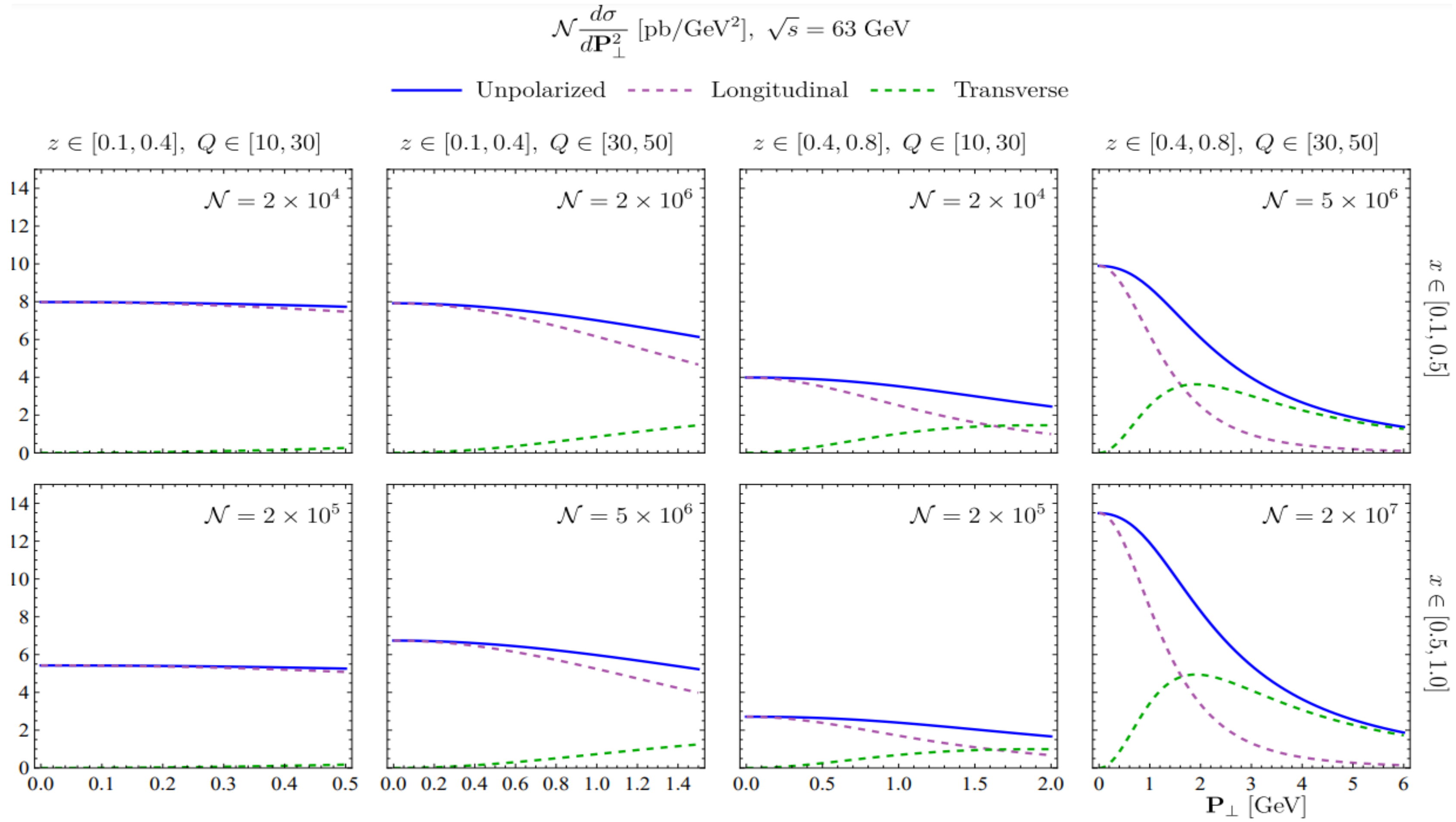


$$\epsilon^{0} = (0, 0, 1)$$

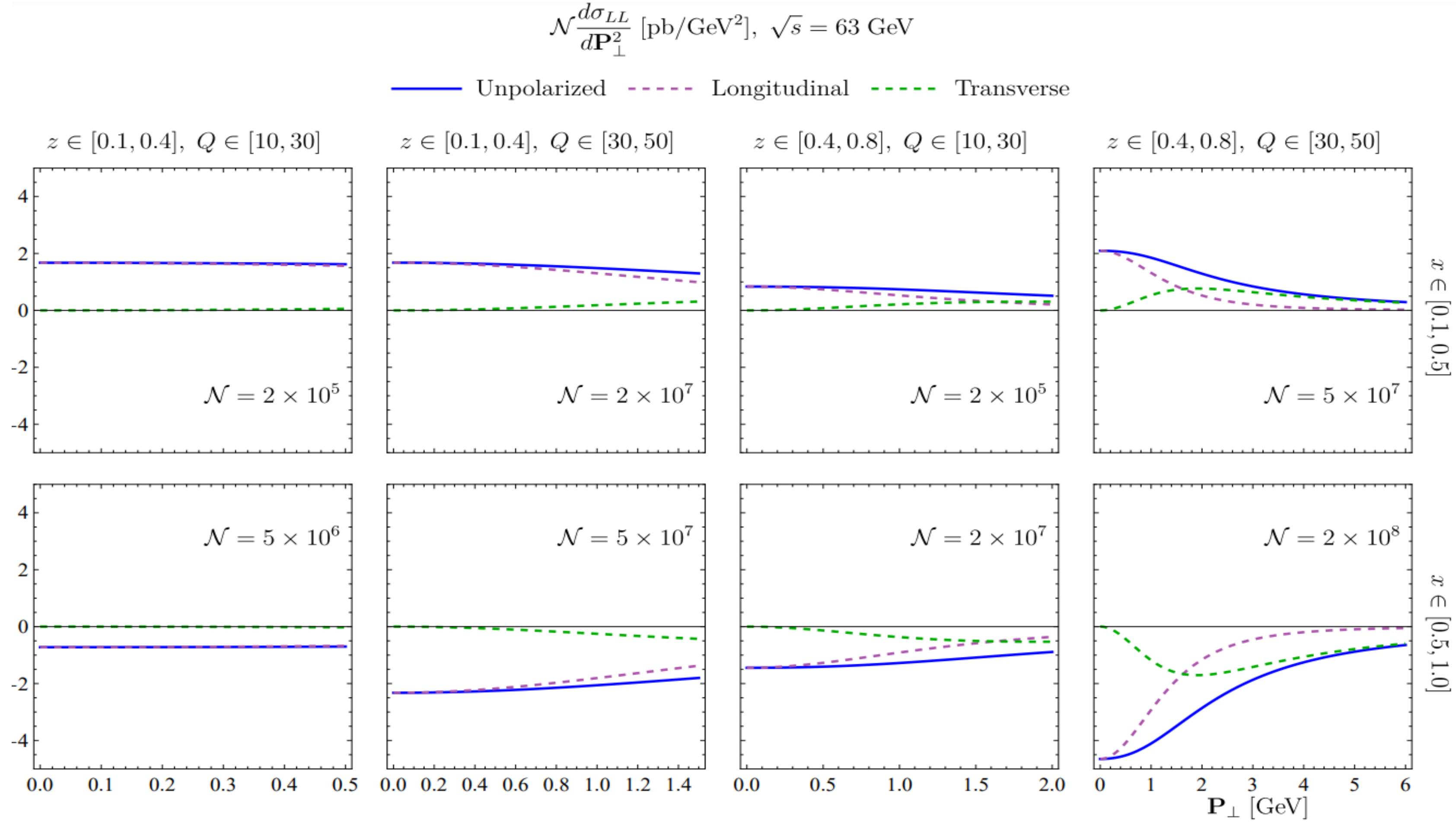
Transversely Polarized J/ψ production



Fragmentation contribution to $d\sigma_{UU}$



Fragmentation contribution to $d\sigma_{LL}$



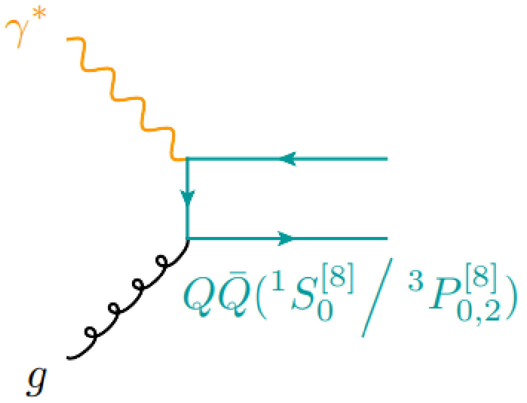
Direct Production Mechanisms

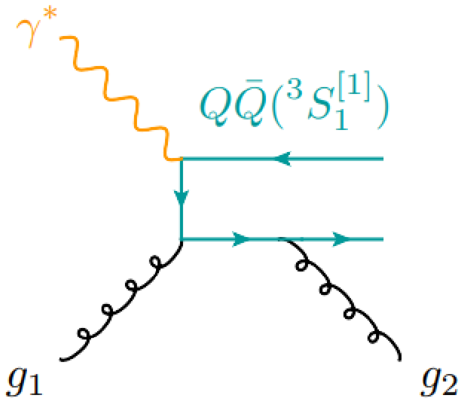
- The electron can also "strike" a gluon in the proton. This process competes with fragmentation.

- There are two ways J/ψ can be produced through photon gluon fusion.

- J/ψ can either be in a color singlet or a color octet.
  - Color octet is leading order in α_s
  - Color singlet is leading order in "v".

- Introduce TMD shape functions.





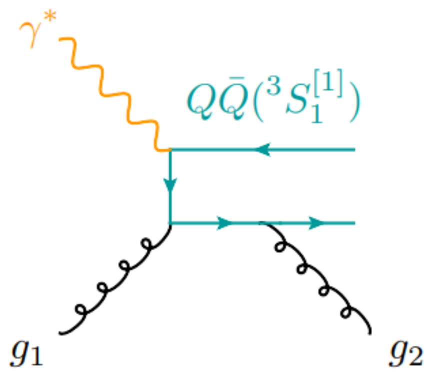
Color Singlet Photon-Gluon Fusion

- 6 diagrams – J/ψ is in the ${}^3S_1^{[1]}$ state.

$$z = \frac{p_{g_1} \cdot P_{J/\psi}}{p_{g_1} \cdot q}$$

- Expected to contribute more for z << 1.
  - Radiated gluon steals momentum from the initial parton.

$$\frac{d\sigma}{dx\,dz\,dQ\,d\mathbf{P_T}^2\,d\phi} \propto \left(F_T + \epsilon F_L + \sqrt{\epsilon(1+\epsilon)}\cos\phi F_\phi + \epsilon\cos 2\phi F_{2\phi}\right) \times f_g(x,\mu^2)$$

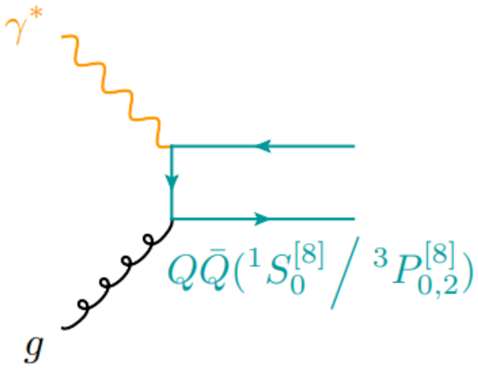


Color Octet Photon-Gluon Fusion

- J/ψ is either in ${}^1S_0^{[8]}$ or ${}^3P_J^{[8]}$ since there is no additional gluon to make it a color singlet.

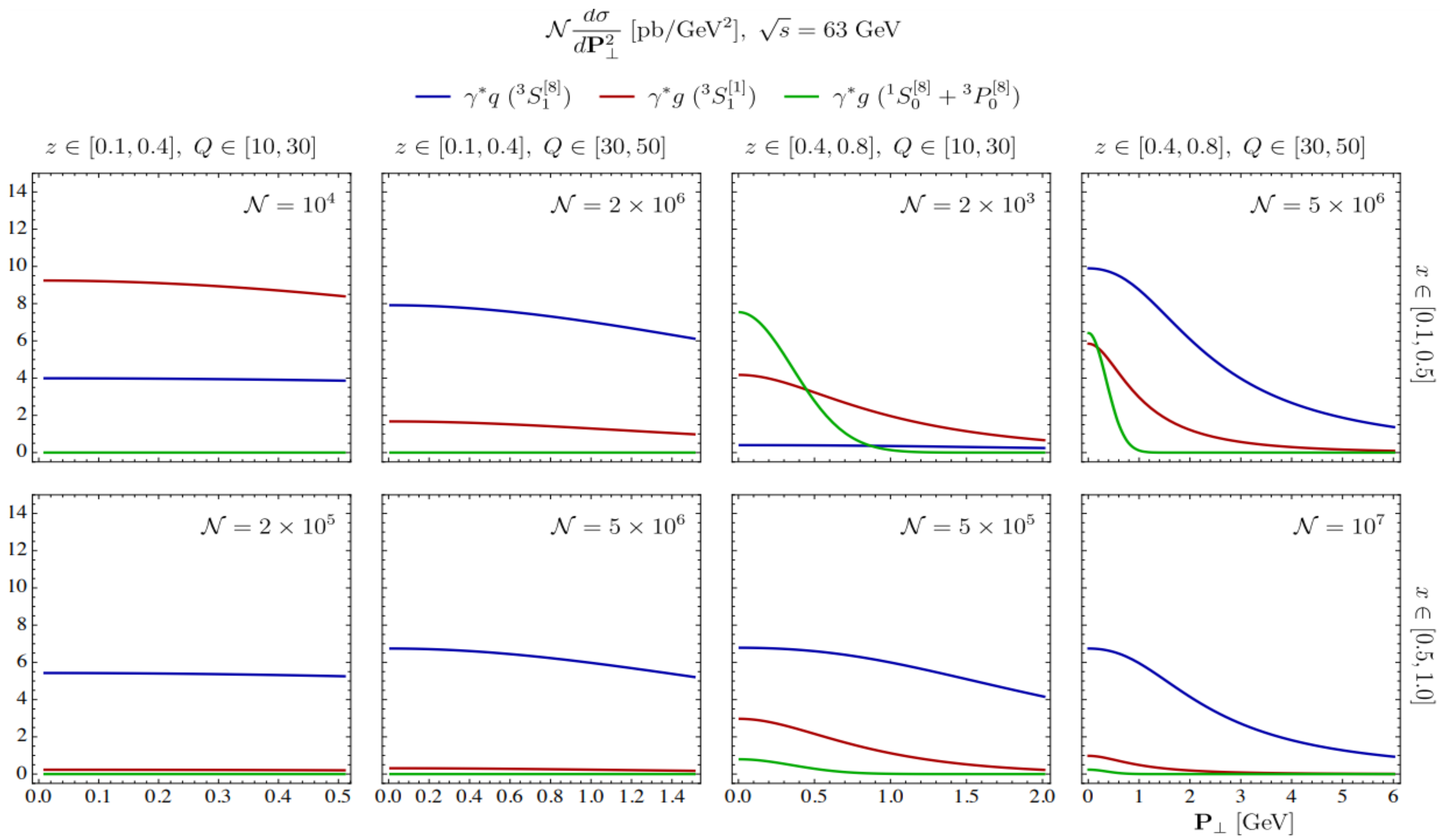
- No gluon is radiated so the J/ψ takes carries away all of the initial parton momentum.
  - This process dominates as z -> 1.

$$z = \frac{p_{g_1} \cdot P_{J/\psi}}{p_{g_1} \cdot q}$$

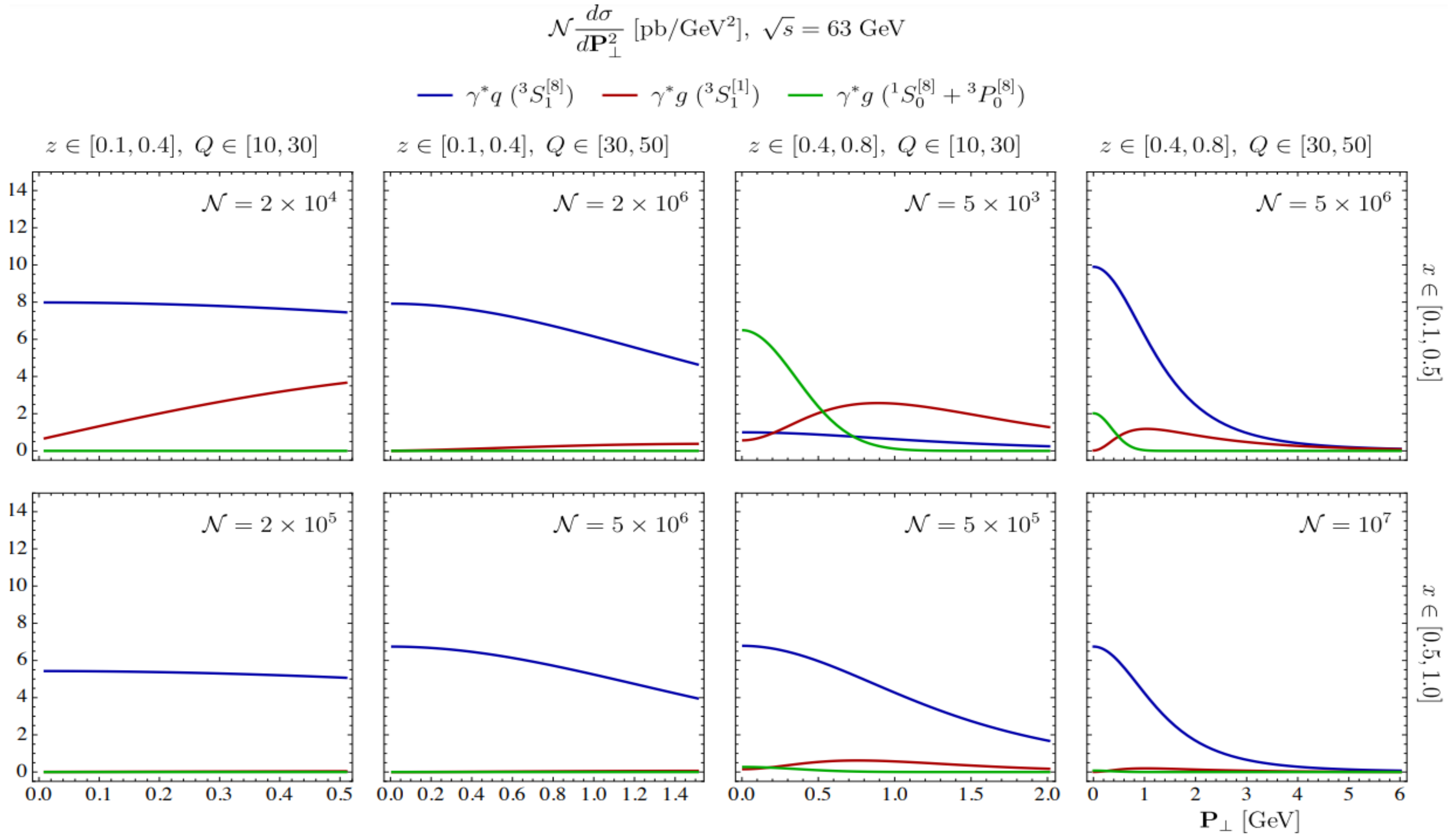


$$d\sigma \propto \delta(1-z)\delta^{(2)}(\mathbf{P}_\perp) \sim \frac{1}{\sqrt{\pi\langle z_0\rangle}} e^{-(1-z)^2/\langle z_0\rangle} \frac{1}{\pi\langle \mathbf{P}_\perp\rangle} e^{-\mathbf{P}_\perp^2/\langle \mathbf{P}_\perp\rangle}$$

Comparing production mechanisms



Comparing production mechanisms (L)



Future Work

- Calculate soft gluon corrections to NRQCD factorization theorems.
  - Important for very small transverse momenta.
- More phenomenological studies (beam asymmetries, identify other interesting observables, etc.)
  - Extract NRQCD long distance matrix elements from fitting to world data.
- Calculate polarized gluon fragmentation at NLO and study quarkonium production in jets.
- Calculate TMD fragmentation functions for other hadrons.
  - Other quarkonia
  - Kaons and pions.