Color confinement

The Yang-Mills theory of coloured quarks and gluons contains all of the ingredients believed to be required of a fundamental theory of the strong interactions. Furthermore it is very precisely formulated. Nevertheless it has not been possible to address the basic question: does this theory lead to the confinement of quarks?

"Quark confinement", R. L. Jaffe, Nature 268 (1977) 201.

$$R \equiv \frac{\sigma(e^+e^- \to \text{强子})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

Δ++ anti-symmetric wave function

The non-Abelian property of the strong interaction, linear potential of colored quarks simulated on lattice, vacuum structure of QCD, etc

$$V = \sigma r\,, \quad V = \frac{(1-e^{-\mu r})}{A}$$

Color confinement

$$\mathcal{L}^{\mathrm{QCD}} = \bar{q}_f^\alpha (i\not{D}_{\alpha\beta} - m_f\delta_{\alpha\beta})q_f^\beta - \frac{1}{4}\mathcal{G}^a_{\mu\nu}\mathcal{G}_a^{\mu\nu}$$

$$\begin{pmatrix} q_f^r \\ q_f^g \\ q_f^b \end{pmatrix} \to U(x)\begin{pmatrix} q_f^r \\ q_f^g \\ q_f^b \end{pmatrix}, \quad \mathcal{A}^a_\mu \frac{\lambda^a}{2} \to U(x)\mathcal{A}^a_\mu \frac{\lambda^a}{2} U^{-1}(x) + \frac{i}{g_s} U(x)\partial_\mu U^{-1}(x)$$

$$U(x) = \exp\left(i\theta^a(x)\tfrac{\lambda^a}{2}\right)$$

$$\mathcal{L}_0^{\mathrm{QCD}} = \bar{q}_f^\alpha (i\not{\partial} - m_f)q_f^\alpha - \frac{1}{4}F^a_{\mu\nu}F_a^{\mu\nu}$$

$$\mathcal{L}_0^{\mathrm{QCD}} \to \bar{q}_f U^{-1}(i\not{\partial} - m_f)Uq_f - \frac{1}{2}\mathrm{Tr}[UF_{\mu\nu}U^{-1}UF^{\mu\nu}U^{-1}] = \bar{q}_f(i\not{\partial} - m_f)q_f - \frac{1}{2}\mathrm{Tr}[U^{-1}UF_{\mu\nu}F^{\mu\nu}] = \mathcal{L}_0^{\mathrm{QCD}}$$

$$\begin{pmatrix} q_u^\alpha \\ q_d^\alpha \end{pmatrix} \to \exp\left(i\theta^a \frac{\tau^a}{2}\right)\begin{pmatrix} q_u^\alpha \\ q_d^\alpha \end{pmatrix}, \quad \mathcal{A}^a_\mu \to \mathcal{A}^a_\mu$$

Color confinement

$$\mathcal{L}_I^{\rm EW} = -\frac{g}{\sqrt{2}}\left(J_\mu^+ W^{+\mu} + J_\mu^- W^{-\mu}\right) - \frac{g}{\cos\theta_w}\left(J_\mu^3 - \sin^2\theta_w J_\mu^{\rm em}\right) Z^\mu - e J_\mu^{\rm em} A^\mu$$

$$\begin{aligned}
J_\mu^\pm &= \frac{1}{2}\bar{q}_i^{L,\alpha}\gamma_\mu(\tau_1 \pm i\tau_2)^{ij} q_j^{L,\alpha}, \\
J_\mu^3 &= \frac{1}{2}\bar{q}_i^{L,\alpha}\gamma_\mu \tau_3^{ij} q_j^{L,\alpha}, \\
J_\mu^{\rm em} &= J_\mu^3 + J_\mu^Y = \frac{1}{2}\bar{q}_i^{L,\alpha}\gamma_\mu \tau_3^{ij} q_j^{L,\alpha} + \frac{1}{2} Y \bar{q}_i^\alpha \gamma_\mu q_i^\alpha
\end{aligned}$$

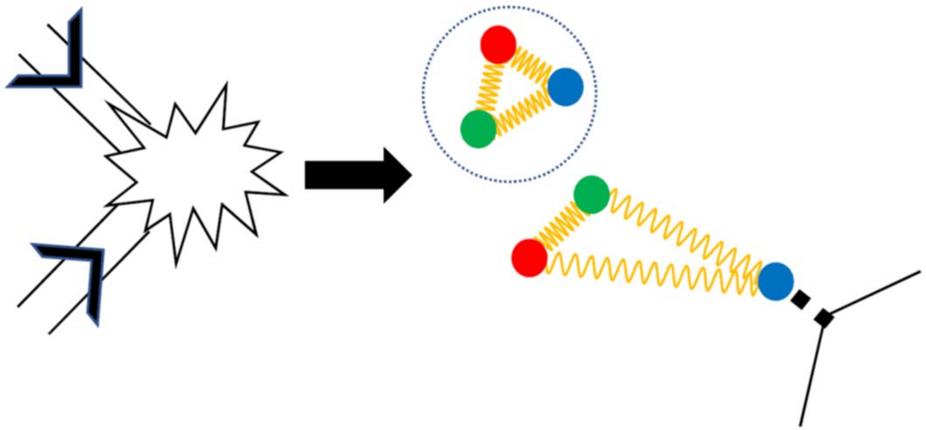
$$(\bar{q}')_i^\beta \Gamma_{ij} (q')_j^\beta = \bar{q}_i^\gamma \exp\left(-i\theta^a \frac{\lambda^a}{2}\right)_{\gamma\beta} \Gamma_{ij} \exp\left(i\theta^a \frac{\lambda^a}{2}\right)_{\beta\alpha} q_j^\alpha = \bar{q}_i^\gamma \Gamma_{ij} \delta_{\gamma\alpha} q_j^\alpha = \bar{q}_i^\alpha \Gamma_{ij} q_j^\alpha$$

All the interactions in the standard model are SU(3)_c invariant.

$$S|i\rangle = |i\rangle, \text{ the final state } |j\rangle = \exp(-i\textstyle\int d^4x \mathcal{L}_I(x))|i\rangle$$

$$S|j\rangle = S\exp(-i\int d^4x \mathcal{L}_I(x)) S^{-1} S|i\rangle = \exp(-i\int d^4x \mathcal{L}'_I(x))|i\rangle = |j\rangle$$

Color confinement

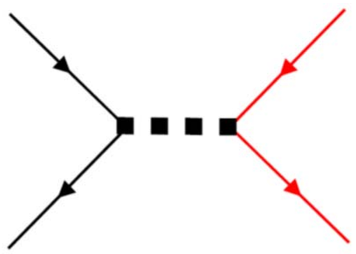


We can not detect or separate the color because the whole system has to be color singlet. Therefore, without the detailed information of the potential between colored objects, or no matter what kind of potential between colored objects, we can conclude quark or colored object can never be separated due to the common symmetry of the standard model.

$$\Psi(\vec{x}) = \int \frac{d^3\vec{p}}{(2\pi)^3}\psi(\vec{p})e^{i\vec{p}\cdot\vec{x}} = \sum_n c_n\phi_n(\vec{x}) \qquad \Psi_c = \frac{1}{\sqrt{3}}\delta_{\alpha\beta}\alpha\bar{\beta}$$

$$e^+e^- \to q\bar{q} \qquad e^+e^- \to q_r\bar{q}_r \text{ is zero instead of } \tfrac{1}{3}\sigma$$

Color confinement



$$\mathcal{L}_I^{\rm em} = e_q J^\mu(x) A_\mu(x) = e_q \bar{q}_\alpha(x)\gamma^\mu q_\beta(x)\delta_{\alpha\beta} A_\mu(x)$$

There is no interaction between color singlet states and colored quark.
The colorless photon can only be emitted from the color singlet current where the color structure can not be changed by any SU(3)c invariant interaction.
Therefore, colored objects are reasonable candidates for dark matter.

$$\mathcal{L}_{\rm QED}^{\rm nl} = \bar{q}_\alpha(x)(i\partial\!\!\!/ - m)q_\alpha(x) - \frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) - e_q \int d^4 a \bar{q}_\alpha(x)\gamma^\mu q_\alpha(x) A_\mu(x+a) F(a,\alpha)$$

$$\psi(x) \to e^{i\theta(x)}\psi(x), \qquad A_\mu \to A_\mu - \frac{1}{e_q}\partial_\mu \theta'(x) \qquad \theta(x) = \int da \theta'(x+a) F(a,\alpha)$$

$$\tilde{F}(k,\alpha) = \frac{\Lambda_\alpha^4}{(k^2 - \Lambda_\alpha^2)^2}$$

Color confinement

Colored particles are created and form into color singlet and non-singlet states after the big bang. For any color, quark and anti-quark are created in pair and the total baryon number is conserved to be 0. We know the baryon number of the color singlet universe is positive and as a result, the baryon number of the color non-singlet objects, the dark matter, has to be negative, which is the “disappeared” anti-matter.

P. Wang, J. Phys. G 48 （2021） 105002.

"It is in a way a potentially refreshing new view to possible solutions for some of the deep questions we have today in fundamental physics: dark matter and the baryon asymmetry in the Universe."
"The paper reports a very new approach to tackle standing problems in particle physics."
“The present manuscript discusses an important issue of strong interactions, the color confinement of strong interactions. Interesting explanations for dark matter and missing antimatter has been provided."