NCSU Seminar, February 16, 2024

Chiral quark-soliton approach to quark distribution functions

In collaboration with

Hyun-Chul Kim, Maxim V. Polyakov, and Asli Tandogan

Hyeon-Dong Son

Hadron Theory Group, Inha University

Contents

Introduction

Parton distribution functions (PDFs)

Longitudinal momentum distribution of quarks and gluons (k^+) in hadrons (P^+)

Probability density to find a quark 'q' with momentum fraction $x = k^+/P^+$

QCD collinear factorization and universality

eg. Deep inelastic scattering (ep), Drell-Yan process (pp), …

→Fitting model PDFs using various reactions (**Global analysis**)

Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution (1970')

Perturbative evolution of PDFs
$$\frac{dq_i(x,\mu^2)}{\partial\mu^2} = P_{qq} \otimes q_i + P_{qg} \otimes g$$

Splitting functions P_{ij}: probability of perturbative emission of i from j

Twist-2 quark distribution functions

$$f_q(x,\mu) = \int_{-\infty}^{+\infty} \frac{dz^-}{4\pi} e^{-iz^- xP^+} \langle P|\bar{\psi}_q(z)\gamma^+ W(z,0)\psi_q(0)|P\rangle\Big|_{z^+=z_\perp=0}$$

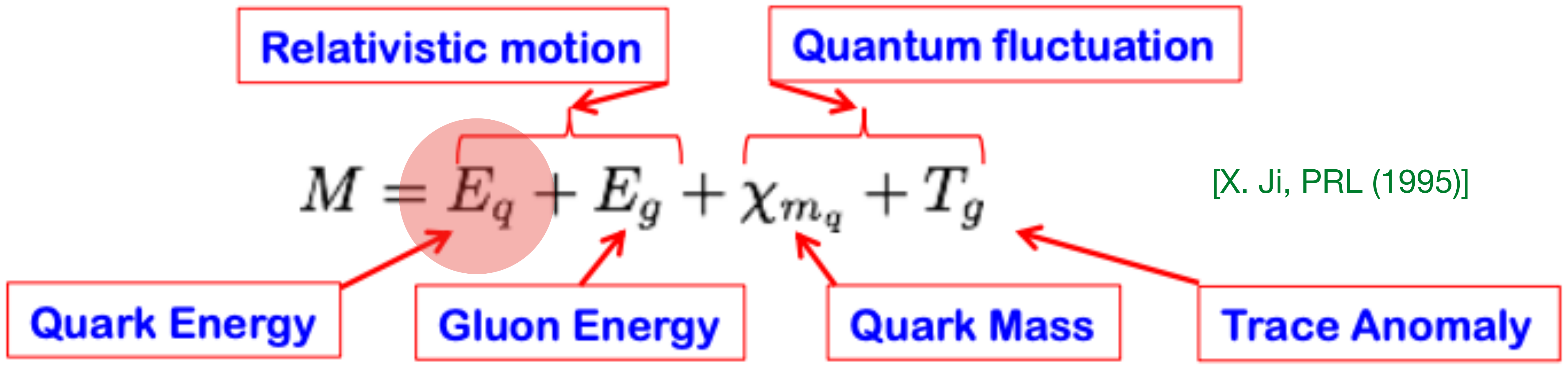
Unpolarized quark distributions

Probability to find a quark with momentum fraction x

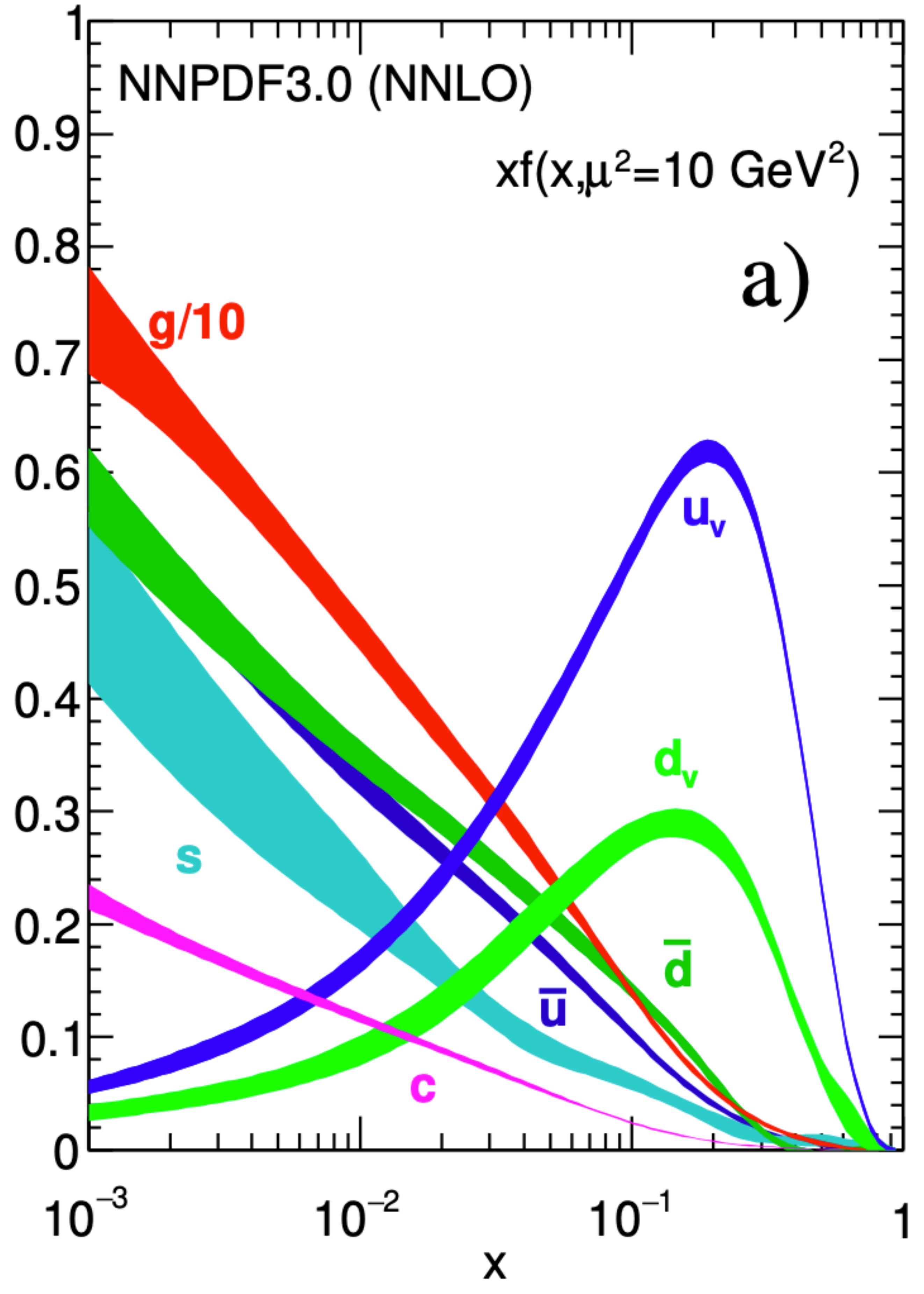
Baryon number and momentum sum rules

→ Momentum sum-rule: Mass form factor (EMT)

→ Mass decomposition



[X. Ji, PRL (1995)]

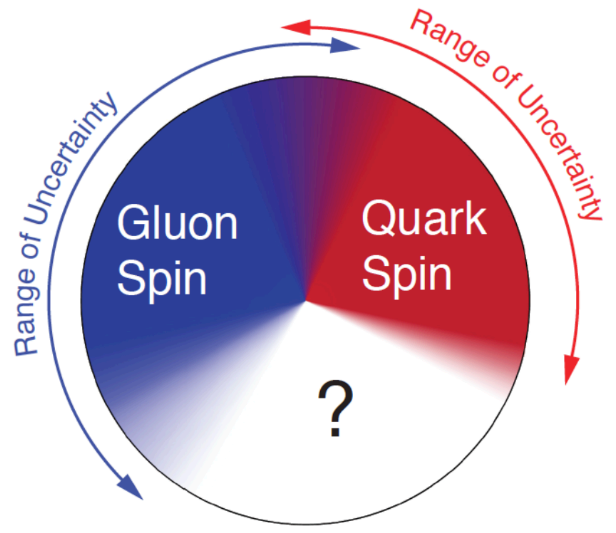


Twist-2 quark distribution functions

Longitudinally polarized quark distribution

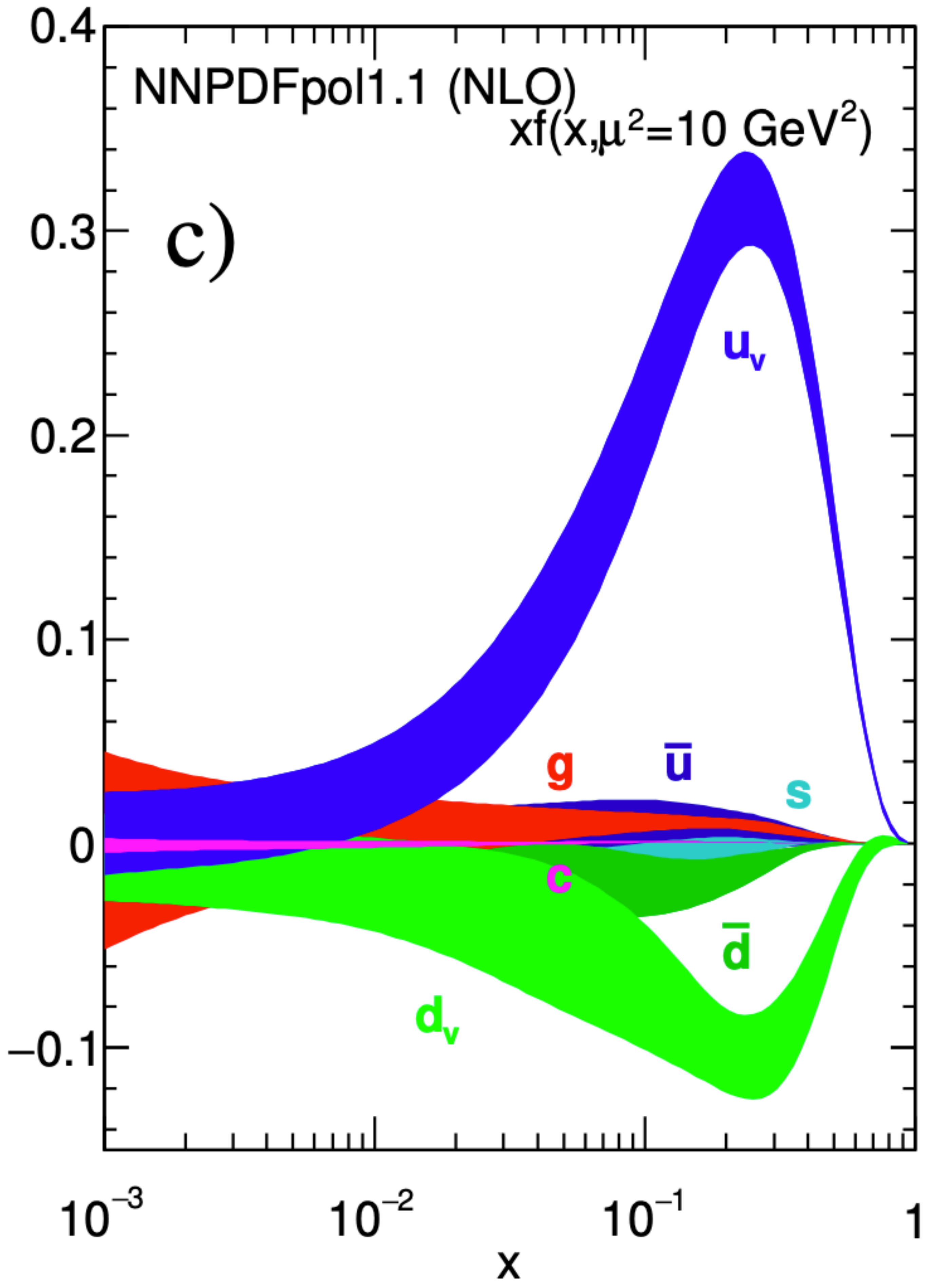
Spin sum-rule and axial charge

→ Proton spin decomposition



$$1/2 = \frac{1}{2}\int_0^1 dx\ \Delta\Sigma(x, Q^2) + \int_0^1 dx\ \Delta g(x, Q^2) + \sum_q L_q + L_g$$

[Jaffee, Manohar, NPB 337 (1990)]



Quasi parton distribution functions

Xiangdong Ji, Phys. Rev. Lett. 110, 262002 (2013)

$$q(x,\mu,P^z) = \int \frac{dz}{4\pi} e^{-ixP^z z} \langle P|\bar{\psi}(0)\gamma^z \exp\left[-ig\int_0^z dz' A^z(z')\right]\psi(z)|P\rangle + \mathcal{O}(\frac{\Lambda^2_{\mathrm{QCD}}}{(P^z)^2}, \frac{M_N^2}{(P^z)^2})$$

$x \in (-\infty, +\infty)$

μ: renormalization scale
P_z: nucleon momentum

Large **M**omentum **E**ffective **T**heory

Spacelike matrix element → can be calculated on the Lattice

No unique definition → **$\Gamma=\gamma^3$** or **$\Gamma=\gamma^0$**

Approaches the PDFs in the limit Pz →∞, or v → 1.

Lattice simulation of QCD: quasi PDFs

Xiangdong Ji, Phys. Rev. Lett. 110, 262002 (2013)

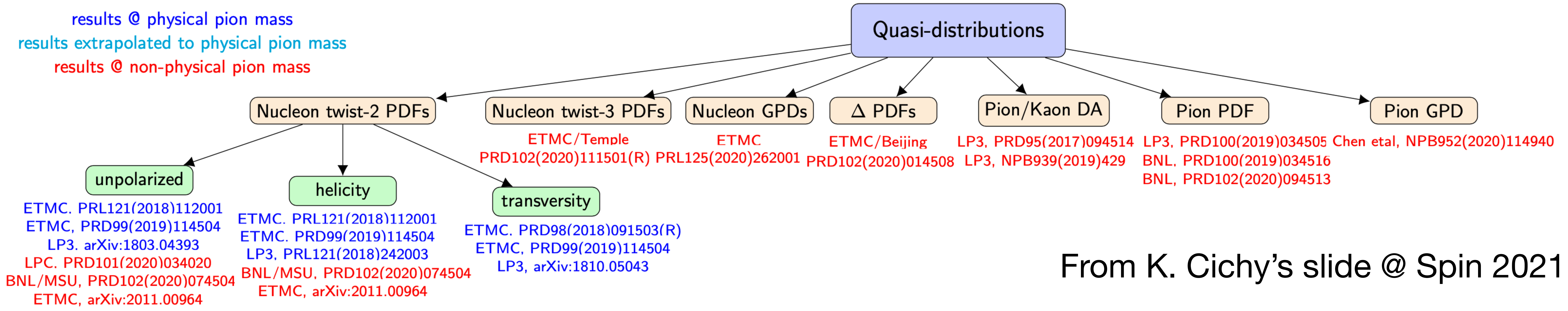
$$q(x,\mu_R,P^z) = \int_{-1}^{1} \frac{dy}{|y|} \underline{C(\frac{x}{y},\frac{\mu_R}{\mu},\frac{\mu}{p^z})} q(y,\mu) + \mathcal{O}(\frac{\Lambda^2_{\rm QCD}}{(P^z)^2},\frac{M_N^2}{(P^z)^2})$$

Perturbative matching coefficients

Extensively studied for the Lattice calculation

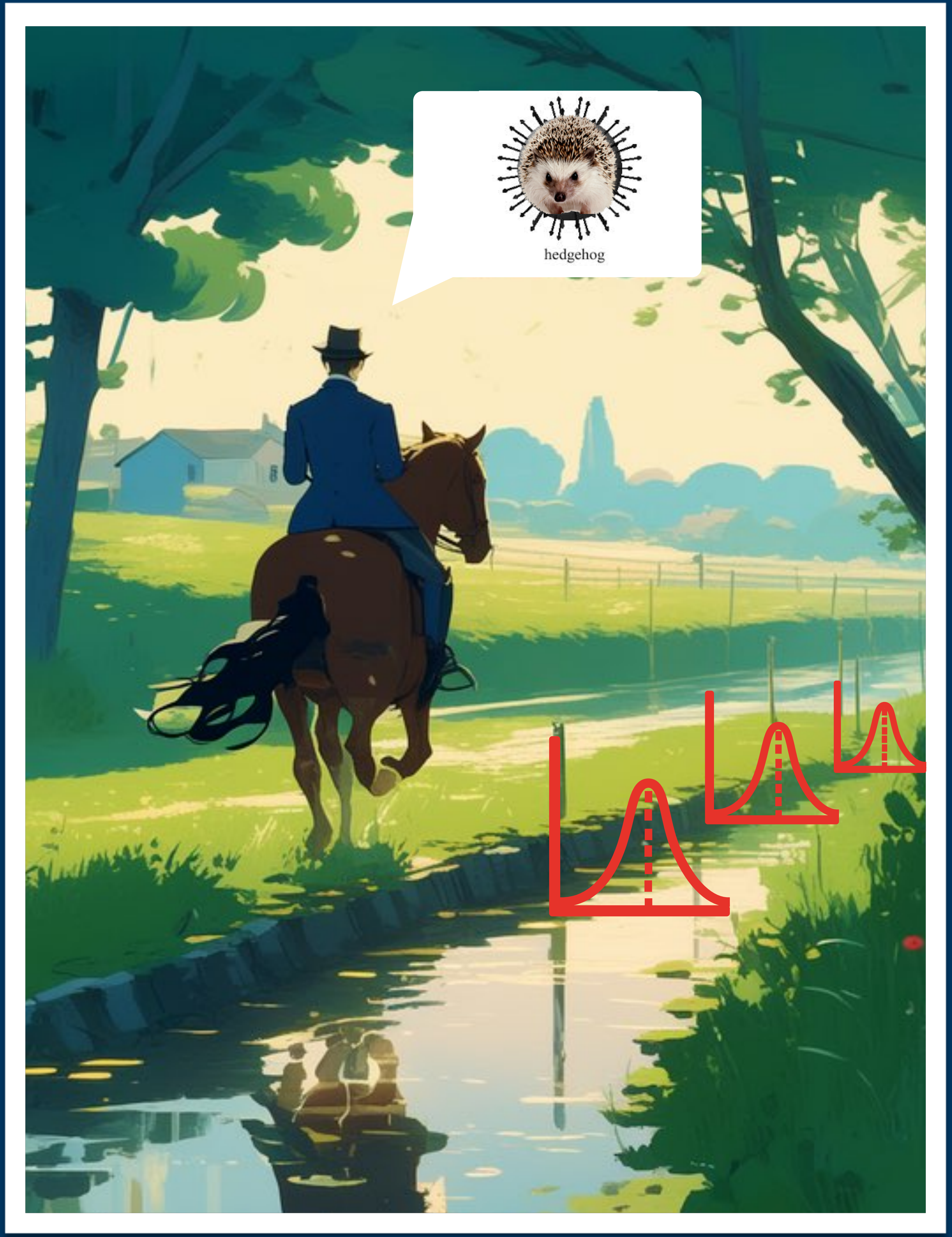
Market results $P_z \sim$ 2-3 GeV

N, π, K / PDFs, DAs, GPDs



From K. Cichy's slide @ Spin 2021

Chiral quark-soliton model



Why do we still focus on effective models?

Model independent approaches

Experiments, Lattice QCD, Effective theories (HQEFT, Large Nc QCD, ChPT)

What we can learn from a model study

Complimentary study for experiment and lattice

Initial state of the partons inside a hadron at low energy scale,

insights via the effective degrees of freedom

A sound effective model should

be firmly planted to the first principle (symmetries)

clear and understandable limitation

not have too much free parameters (self-consistency)

eg. Instanton QCD vacuum,
Chiral quark-soliton model,
…

Nucleon as a chiral soliton in the large Nc limit

In the large Nc limit, nucleon can be seen as a chiral soliton ($m_N \sim \mathcal{O}(N_c)$) , formed by Nc quarks in a self-consistent pion mean-field

[E. Witten, Nucl. Phys. B 160, 57 (1979)]

Skyrme model: Nappi, Adkins, and Witten ‘1983

Baryon number N=1, winding number from Wess-Zumino term

Chiral quark soliton model:

Quarks interact strongly to produce a pion mean-field

Baryon number N=1 given by Nc quarks at the bound level

Quark and antiquark structure can be studied

Large mean-field (gradient expansion) → Skyrme model

Effective quark partition function in the large Nc

[D. Diakonov, V. Petrov, and P. Pobylitsa, Nucl. Phys. B 306, 809 (1988)]

$$Z = \int \mathcal{D}\pi^a d\psi^\dagger d\psi \ \exp \int d^4x \psi^\dagger(x)(i\partial\!\!\!/ + iMU^{\gamma_5})\psi(x)$$

$$U^{\gamma_5}(x) = U(x)\frac{1+\gamma_5}{2} + U^\dagger(x)\frac{1-\gamma_5}{2} \qquad U(x) = \exp\left[\frac{i}{F_\pi}\pi^a(x)\tau^a\right]$$

Low energy effective theory derived from QCD via the **instantons**

Instanton parameters: average size $\bar{\rho} \sim 1/3\ \mathrm{fm}$ & distance $\bar{R} \sim 1\ \mathrm{fm}$ (no more parameters, Λ_{QCD})

Intrinsic renormalisation scale $\Lambda \sim 1/\bar{\rho} \approx 600\ \mathrm{MeV}$

Spontaneous chiral symmetry breaking & dynamically generated quark mass $M = 350\ \mathrm{MeV}$

Fully field theoretic: successfully describes a wide class of baryon properties

Nucleon: chiral soliton in the large Nc, quarks are bound by a self-consistent mean-field [E. Witten, Nucl. Phys. B 160, 57 (1979)]

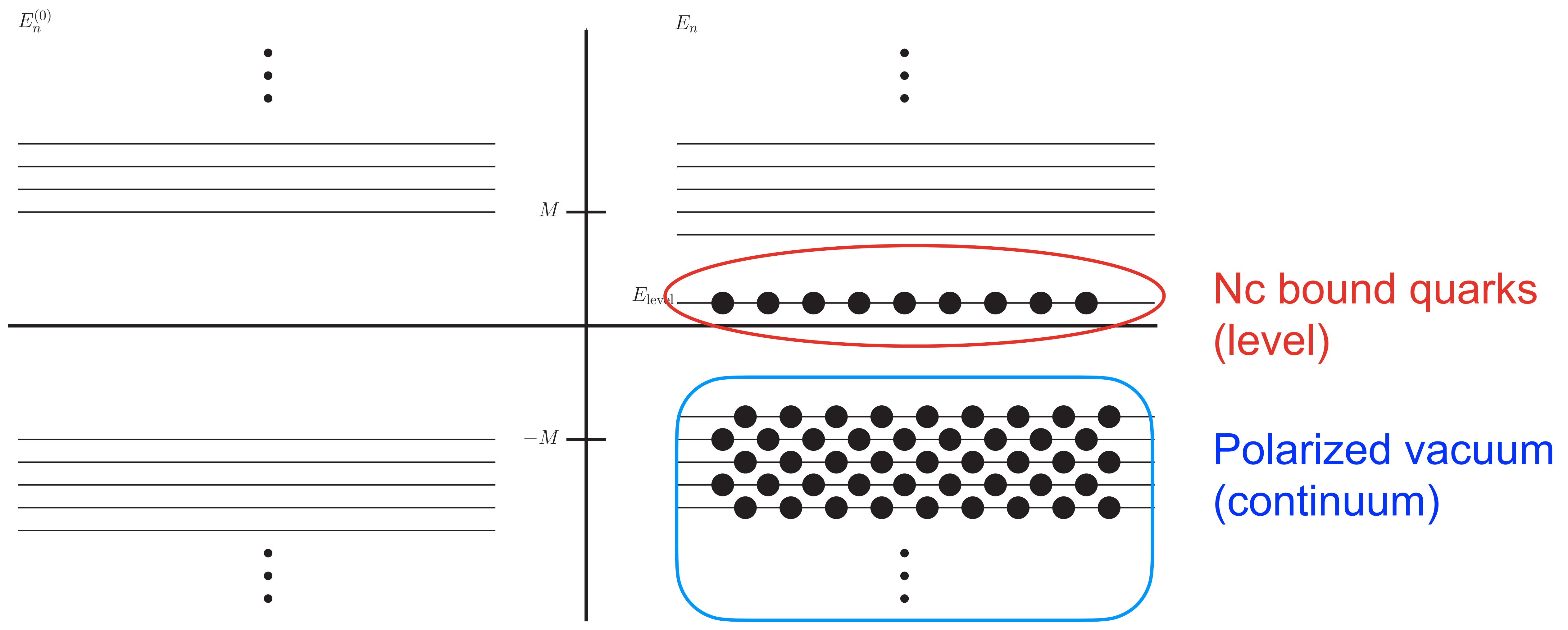
Interplays the naive quark-model and (topological) soliton picture of the baryons

Nucleon as a chiral soliton in the large N_c limit

Nc quarks are bound by

a pion mean-field, self-consistently

generated by their interactions



Hedgehog Ansatz

$$U = \exp[i\gamma_5 \hat{n}^a \tau^a P(r)]$$

Dirac spectra (n): Grandspin K= J + T and Parity P

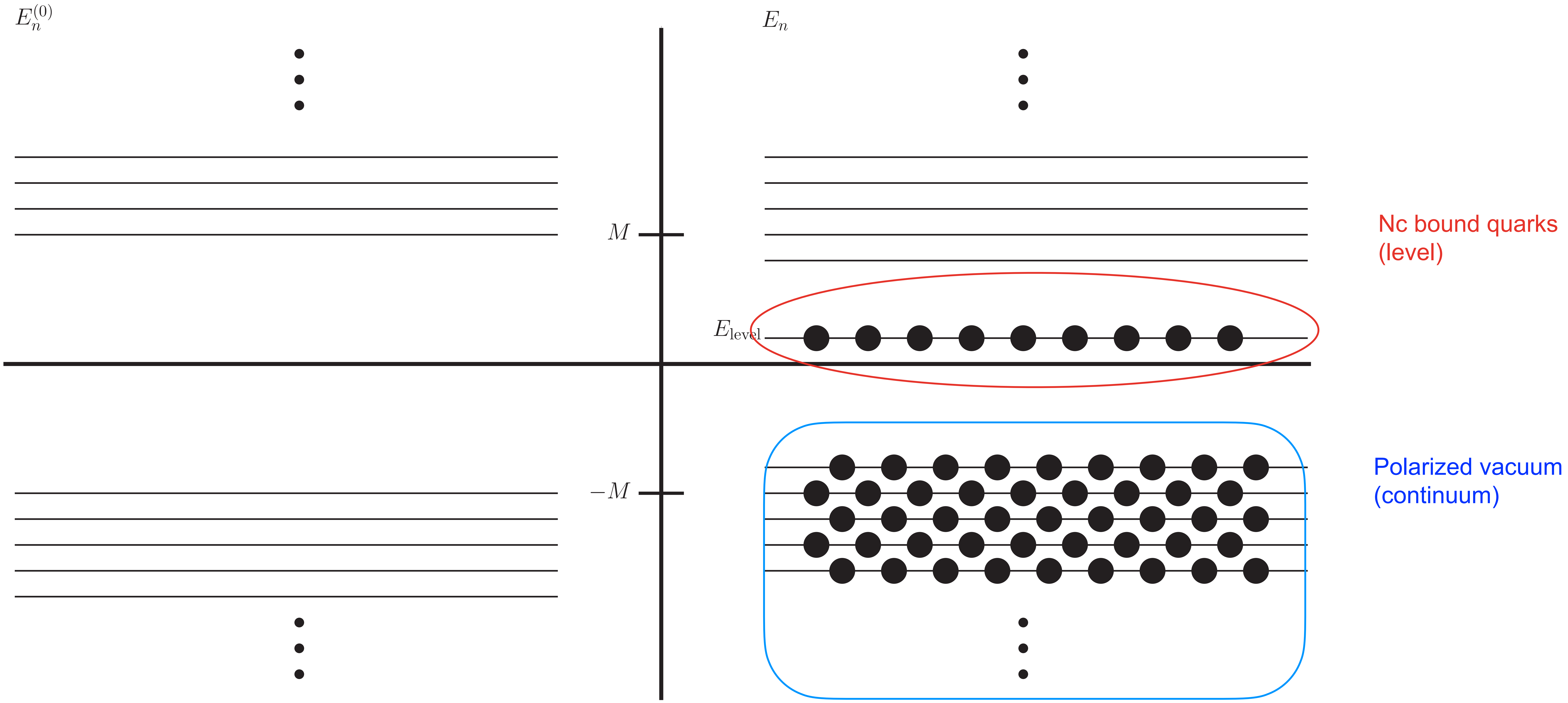
$$H\Phi_n(\vec{x}) = E_n \Phi_n(\vec{x})$$

Classical soliton energy

$$\frac{\delta}{\delta U}(N_c E_{\rm level} + E_{\rm cont.})|_{U=U_c} = 0 \quad \rightarrow \quad M_{sol} = N_c E_{\rm level}(U_c) + E_{\rm cont.}(U_c)$$

Nucleon quantum numbers: quantization around the rotational zero-modes

Nucleon as a chiral soliton in the large N_c limit



Example: isovector axial charge

[Praszalowicz et al. *Phys.Lett.B* 354 (1995) 415-422]

A parametric study shows the behavior of the axial charge, in the soliton size (or the quark-meson coupling M)

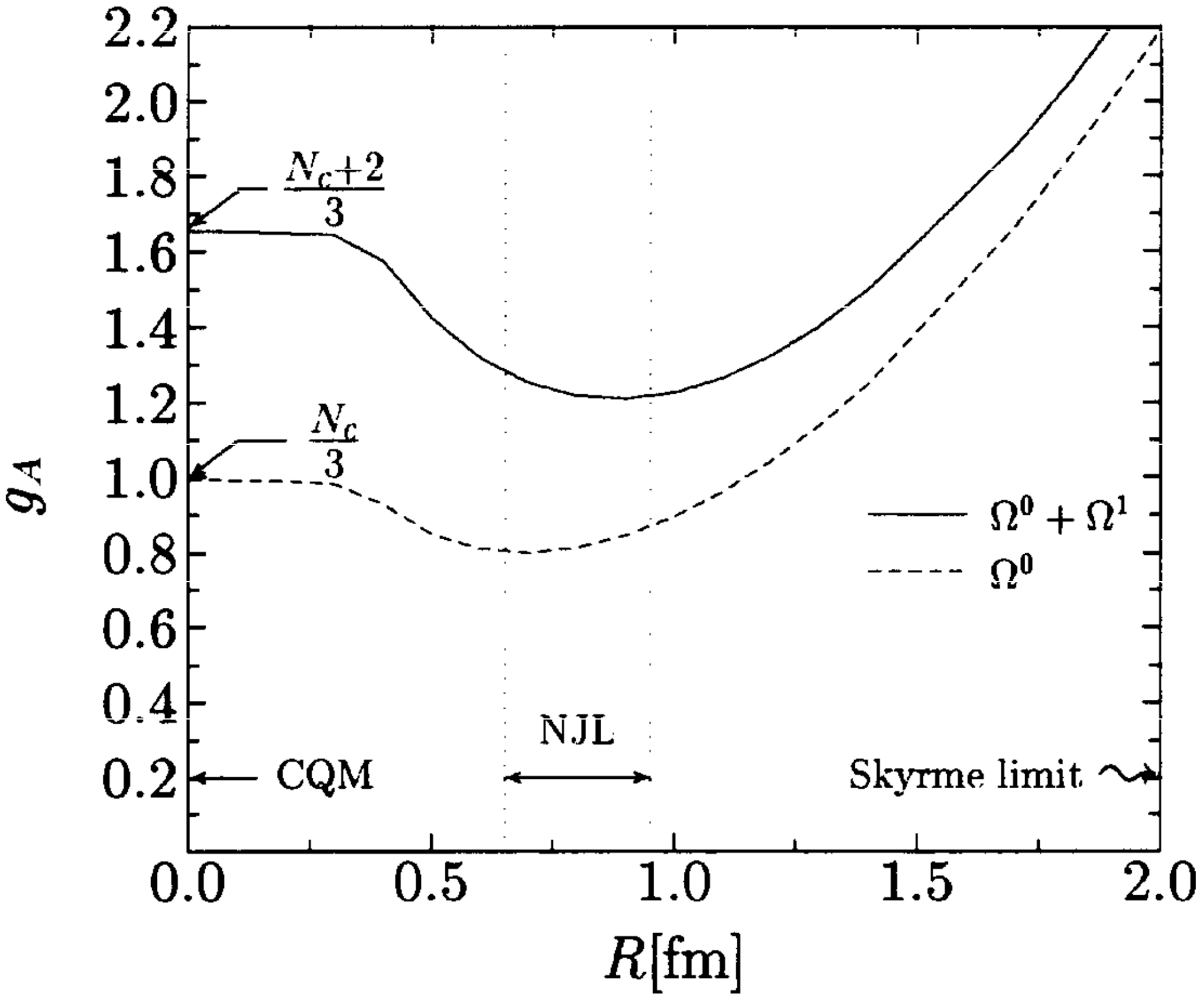
Soliton size R→0, naive quark model results

Using the self-consistent mean-field (M=350MeV),

$$g_A^{(3)} = g_A^{(3)}(\text{level}) + g_A^{(3)}(\text{cont.}) \approx 0.7 + 0.2,$$

in the leading Nc

vs. $g_A^{(3)}(\text{exp.}) = 1.27$



Quark distribution functions in the χQSM

Quark distribution functions in the large Nc

[D. Diakonov, V. Y. Petrov, P. V. Pobylitsa, M. Polyakov, and C. Weiss, 1996, 1997]

In general, in the large Nc limit:

$\rho(N_c x)$: stable in large Nc, $x \sim 1/N_c$

Isosinglet unpolarised

Isovector helicity

Isovector transversity

$$\sim N_c^2 \rho(N_c x)$$

Isovector unpolarised

Isosinglet helicity

Isosinglet transversity

$$\sim N_c \rho(N_c x)$$

Quark distribution functions in the χQSM

Nucleon at rest → Lorentz boost to a inertial frame with velocity v in the z direction

Quasi- quark and antiquark number densities

$$D_f(x,v) = \frac{1}{2E_N}\int \frac{d^3k}{(2\pi)^3}\delta\left(x-\frac{k^3}{P_N}\right)\int d^3x\ e^{-i\mathbf{k}\cdot\mathbf{x}}\langle N_v|\bar{\psi}_f\left(-\frac{\mathbf{x}}{2},t\right)\Gamma\psi_f\left(\frac{\mathbf{x}}{2},t\right)|N_v\rangle$$

$$\bar{D}_f(x,v) = \frac{1}{2E_N}\int \frac{d^3k}{(2\pi)^3}\delta\left(x-\frac{k^3}{P_N}\right)\int d^3x\ e^{-i\mathbf{k}\cdot\mathbf{x}}\langle N_v|\mathrm{Tr}\left[\Gamma\psi_f\left(-\frac{\mathbf{x}}{2},t\right)\bar{\psi}_f\left(\frac{\mathbf{x}}{2},t\right)\right]|N_v\rangle$$

become exact number densities in the limit $v \to 1$

Isoscalar unpolarized quark distribution

$$H\Phi_n(\vec{x}) = E_n\Phi_n(\vec{x})$$

$$\sum_f q_f(x,v) = N_c M_N v\sum_{n,occ}\int\frac{d^3k}{(2\pi)^3}\delta(k^3+vE_n-vM_Nx)\left[\Phi_n^\dagger(\vec{k})(1+v\gamma^0\gamma^3)\gamma_0\Gamma\Phi_n(\vec{k})\right]$$

$\sim N_c^2$

Isovector longitudinally polarized quark distribution (helicity)

$$\Delta u(x,v)-\Delta d(x,v) = -\frac{1}{3}(2T^3)\frac{N_c M_N v}{2\pi}\sum_{n,occ}\int\frac{d^2k_\perp}{(2\pi)^2}\delta(k^3+vE_n-vM_Nx)\left[\Phi_n^\dagger(\vec{k})(1+v\gamma^0\gamma^3)\gamma_0\Gamma\tau^3\gamma^5\Phi_n(\vec{k})\right]$$

$\Gamma=\gamma^0$ and $\Gamma=\gamma^3$ define different quasi-PDFs

Twist-2 quark distribution functions in the large Nc limit

Observations

Positivity for the **antiquark** is guaranteed by the **contribution of the polarized vacuum**

Sea quarks at low renormalization scale $\mu = 1/\rho = 600$ MeV is indispensable

Sum-rules: baryon number & momentum sum-rule, Bjorken and Gottfried sum

Large antiquark flavor asymmetry for the longitudinally polarized quarks

To note

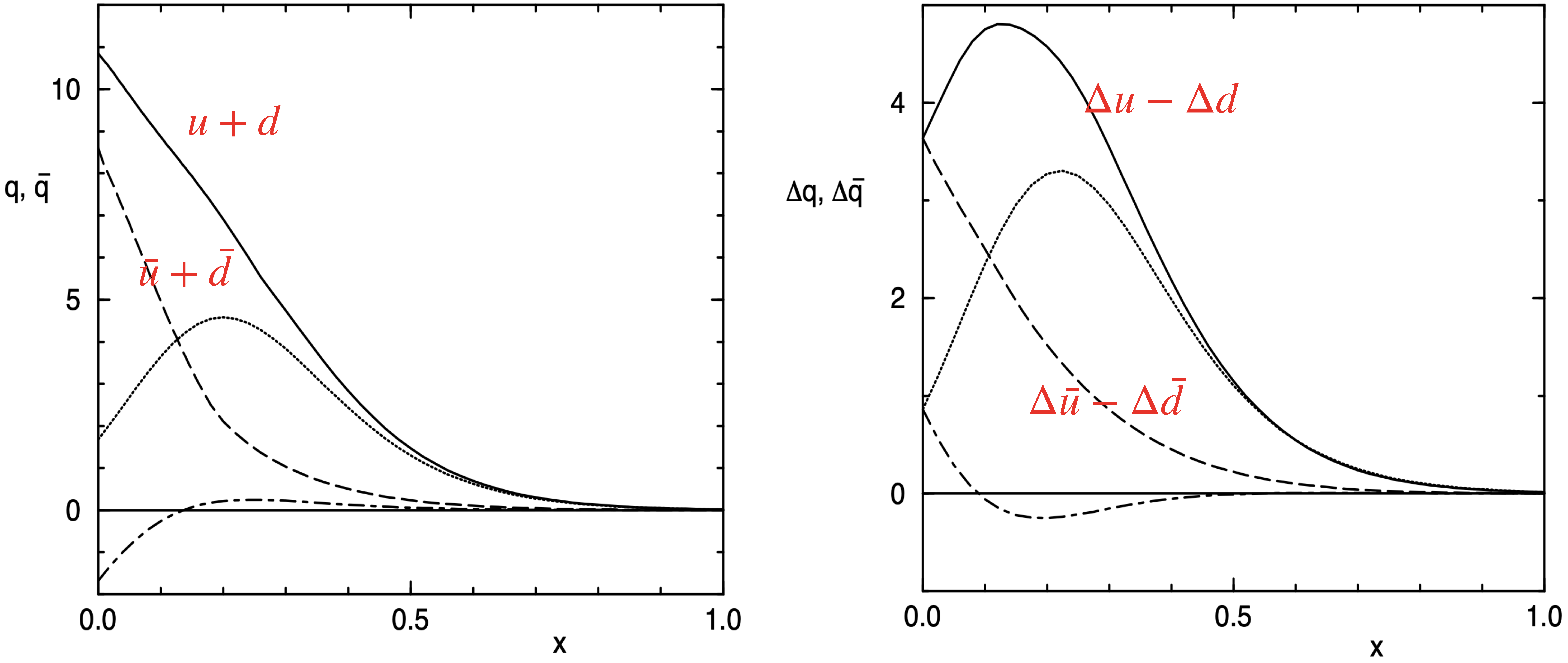
Model has its best resolution at x~1/Nc

Quark distributions at large-x ($x \sim 1$) suffer from the 'soliton tail', e^{-xN_c}.

No gluon degrees of freedom

Quark distribution functions in the χQSM

….., — - —: Level contributions



Antiquark flavor asymmetry

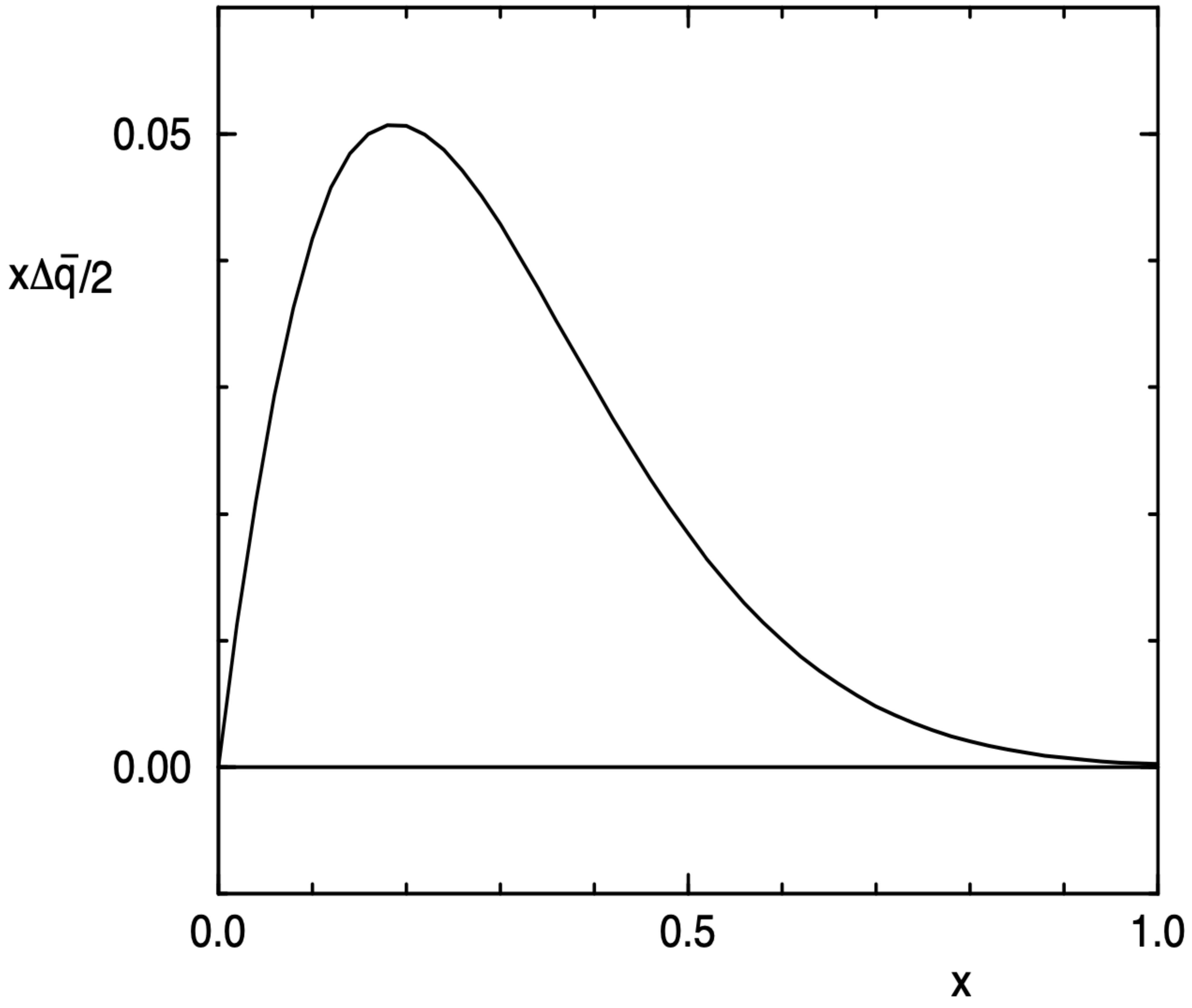


Figure 6: The isovector polarized antiquark distribution, $\frac{1}{2}x[\Delta\bar{u}(x) - \Delta\bar{d}(x)]$. *Solid line:* calculated distribution (total result, *cf.* Fig.2). In the fit of ref.[4] this distribution is assumed to be zero.

[ref.[4] Glück et al., PRD 53 (1996)]

Antiquark asymmetries in the proton

Unpolarized antiquarks: $\bar{d} > \bar{u}$ [Glück, Reya, Vogt, ZPC (1995)]

PDFs from polarized DIS: assumed $\Delta\bar{u} - \Delta\bar{d} = 0$ [Glück, Reya, Volgesang, PLB 359 (1995)]
[Glück et al., PRD 53 (1996)]

χQSM prediction: $\Delta\bar{u} - \Delta\bar{d}$ **is large and positive** [Diakonov et al., NPB (1996) / PRD (1997)]

DIS is insensitive to the antiquark flavor asymmetry, but Drell-Yan is! [Dressler et al, EPJC 14 (2000), EPJC 18 (2001)]
[Kumano and Miyama, PLB 479 (2000)]

Analyses using DIS + SIDIS, Drell-Yan [Glück et al., PRD 63 (2001)]
[De Florian et al, PRD 80 (2009)]
[Nocera et al. (NNPDF), NPB 887 (2014)]

Single spin asymmetry (W-boson) in polarized PP collision is used to study the asymmetry

(STAR collaboration) [L. Adamczyk et al. PRL 113 (2014)]
[A. Adare et al. PRD 98 (2018)]
[J. Adam et al. PRD 99 (2019)]

Global analyses updates:

[De Florian et al. PRD 100 (2019)]
[Cocuzza et al. (JAM) arXiv:2202.03371 (2022)]

Antiquark asymmetries in the proton

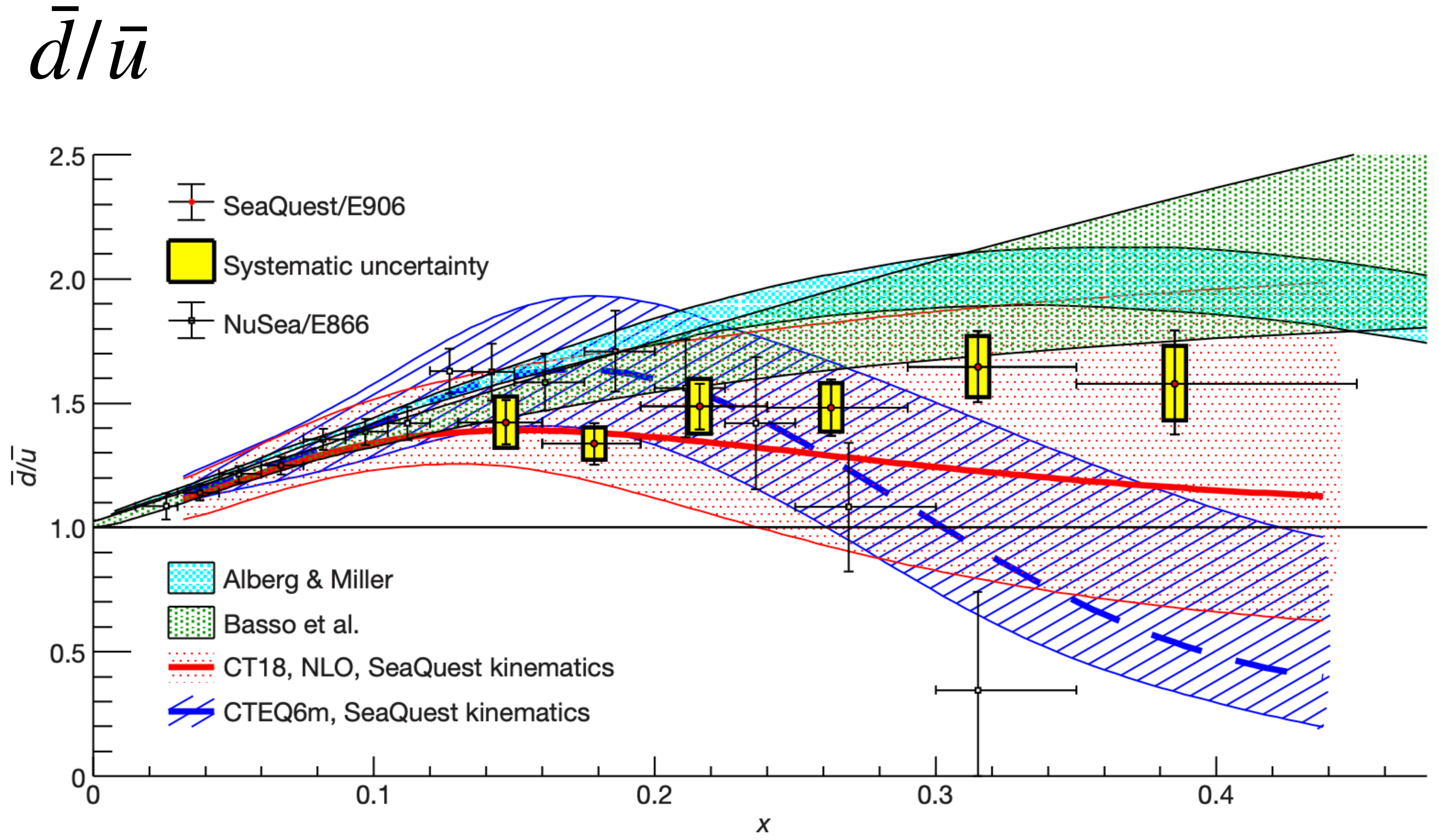


Fig. 2 | Ratios $\bar{d}(x)/\bar{u}(x)$. Ratios $\bar{d}(x)/\bar{u}(x)$ in the proton (red filled circles) with their statistical (vertical bars) and systematic (yellow boxes) uncertainties extracted from the present data based on NLO calculations of the Drell–Yan cross-sections. Also shown are the results obtained by the NuSea experiment (open black squares) with statistical and systematic uncertainties added in quadrature[4]. The cyan band shows the predictions of the meson–baryon model of Alberg & Miller[25] and the green band shows the predictions of the statistical parton distributions of Basso et al.[21]. The red solid (blue dashed) curves show the ratios $\bar{d}(x)/\bar{u}(x)$ calculated with CT18[29] (CTEQ6[35]) parton distributions at the scales of the SeaQuest results. The horizontal bars on the data points indicate the width of the bins.

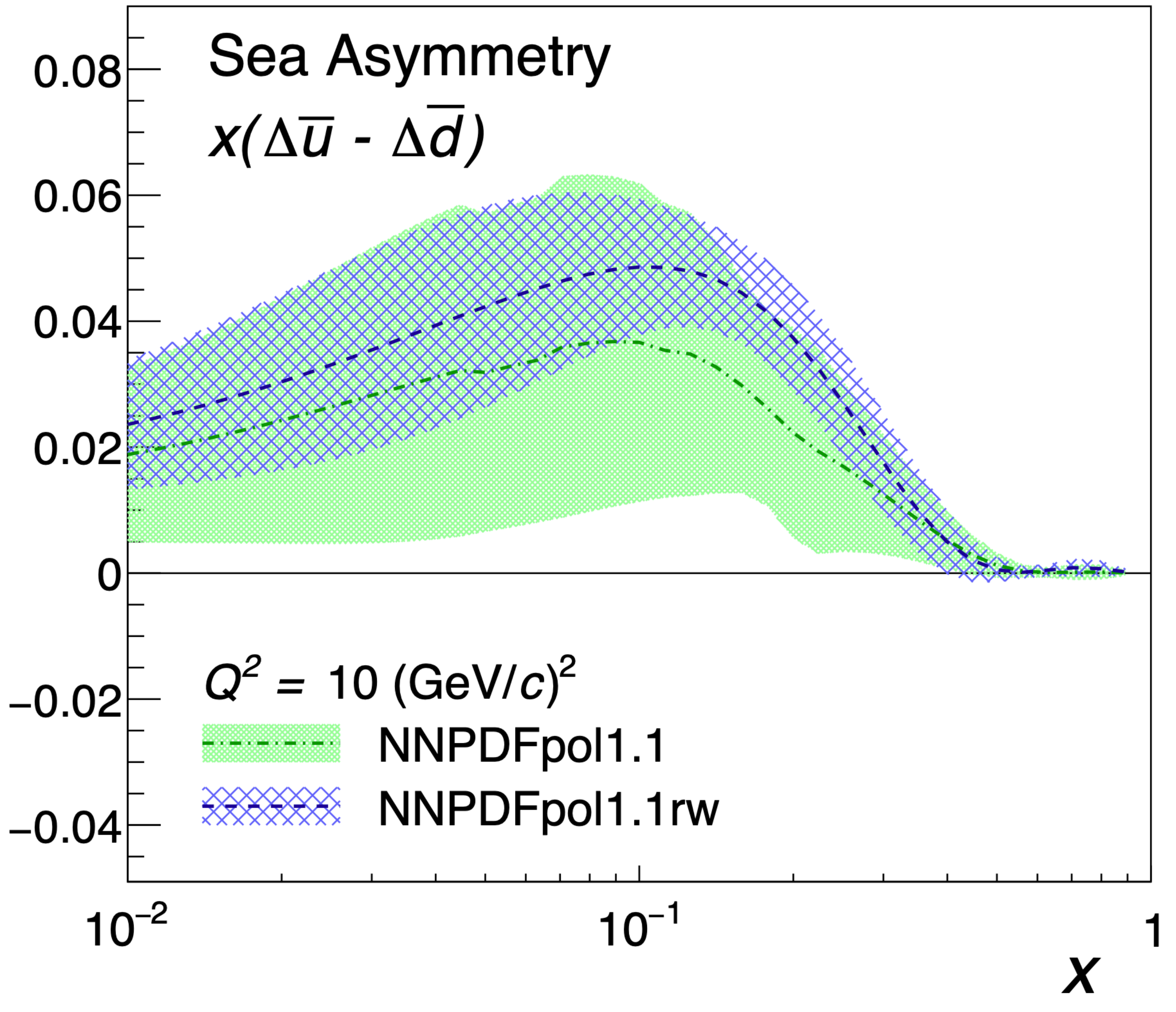
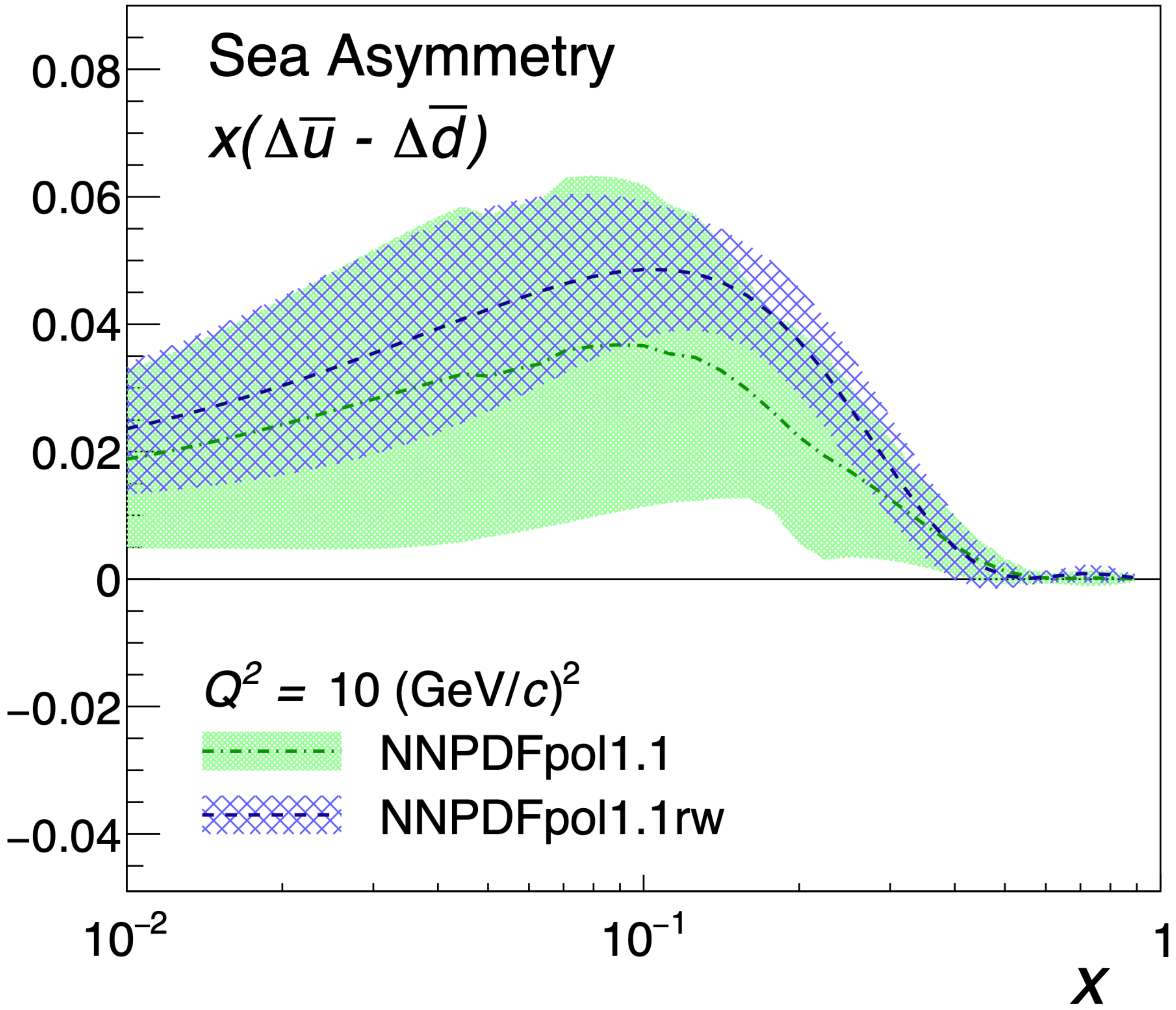


FIG. 6. The difference of the light sea-quark polarizations as a function of x at a scale of $Q^2 = 10\,(\mathrm{GeV}/c)^2$. The green band shows the NNPDFpol1.1 results [1] and the blue hatched band shows the corresponding distribution after the STAR 2013 $W^{\pm}$ data are included by reweighting.

[SeaQuest, Nature 590 (2021) 7847, 561-565]

[STAR collaboration, Phys.Rev.D 99 (2019) 5, 051102]

Polarized antiquark flavor asymmetry: model case



[STAR collaboration, Phys.Rev.D 99 (2019) 5, 051102]

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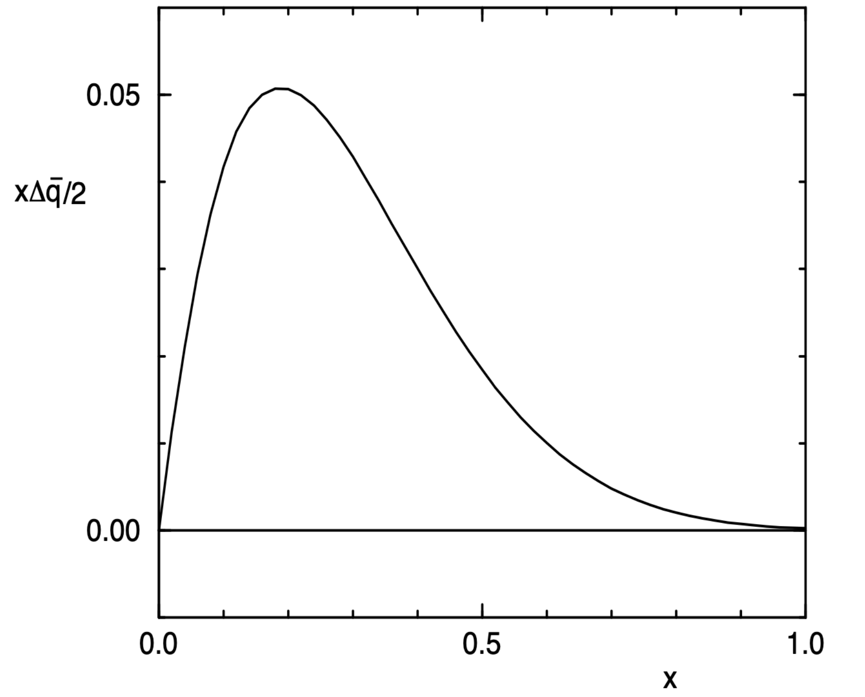


Figure 6: The isovector polarized antiquark distribution, $\frac{1}{2}x[\Delta\bar{u}(x) - \Delta\bar{d}(x)]$. *Solid line:* calculated distribution (total result, *cf.* Fig.2). In the fit of ref.[4] this distribution is assumed to be zero.

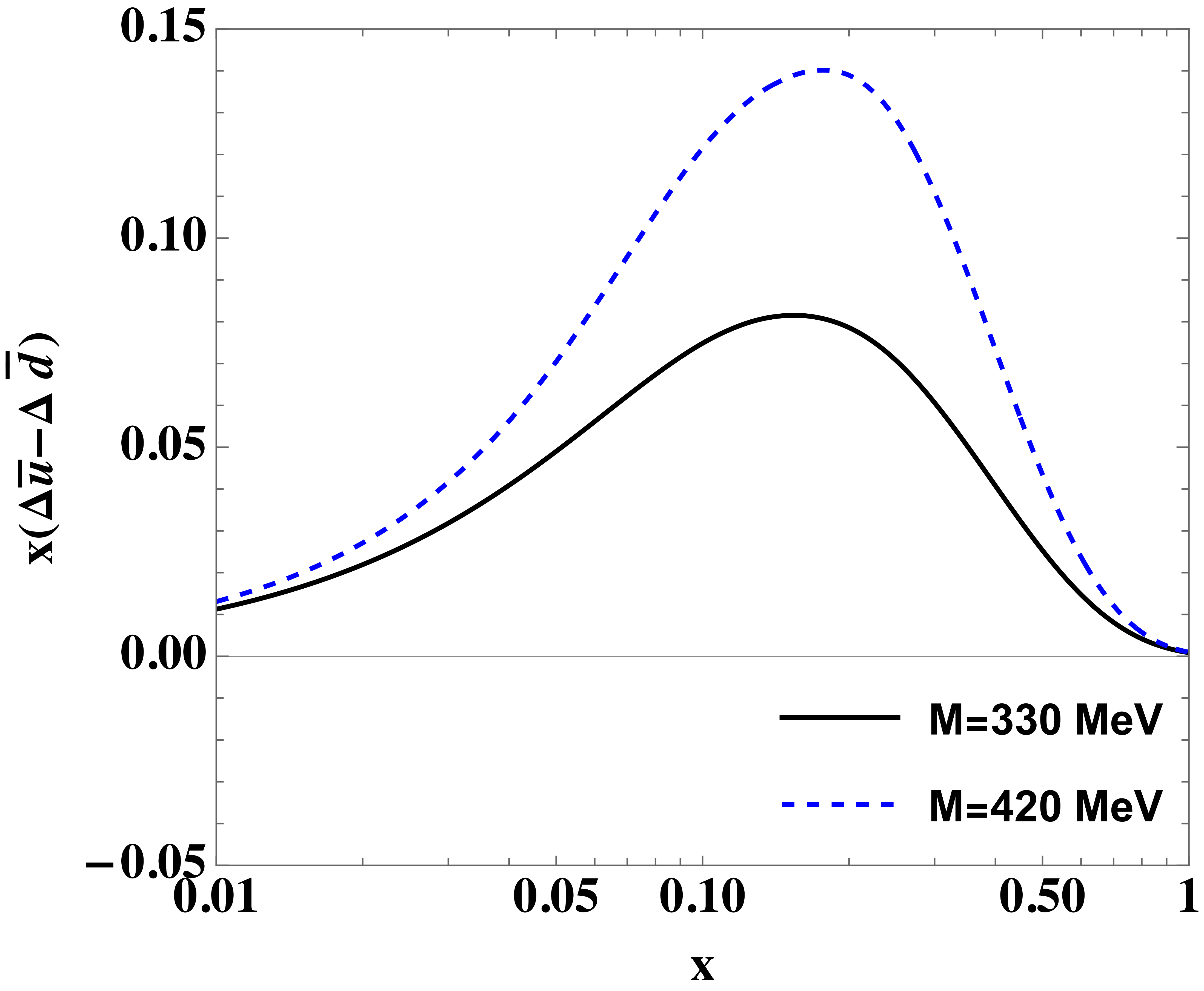
[Diakonov et al., NPB (1996) / PRD (1997)]

M = 350 MeV

Model allows the following parameter window, depending on ρ/R with fixed ρ

M [MeV]	330	420
M_N [MeV]	1161	1077
ρ/R	0.32	0.37
F_π [MeV]	77	90

Polarized antiquark flavor asymmetry: model case



Dependence on M seems to be significant

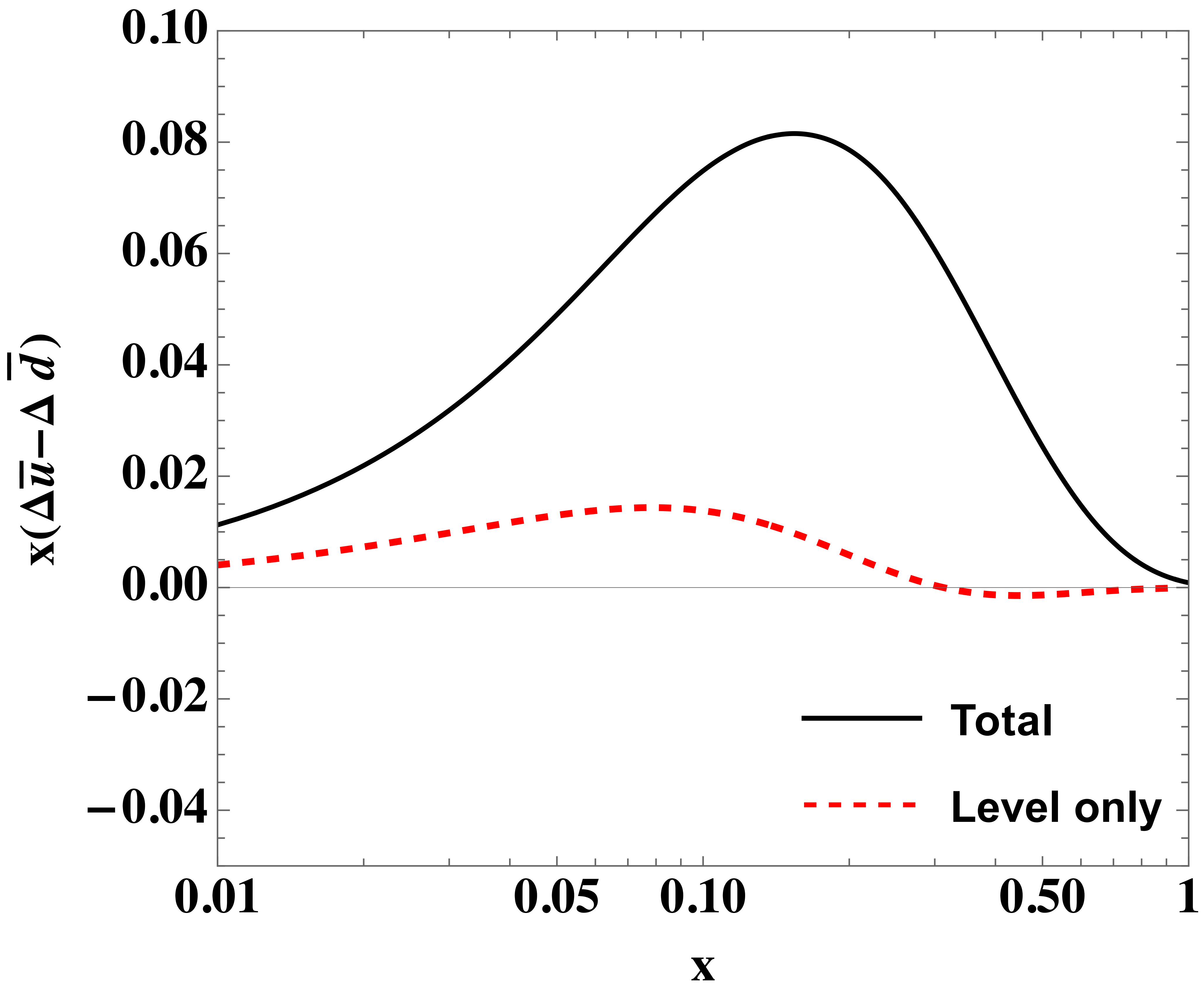
Larger M

→ larger soliton size

→ strongly polarized vacuum

→ larger antiquark flavor asymmetry

Polarized antiquark flavor asymmetry: model case



M=330 MeV

Continuum (polarized vacuum) >> level contribution

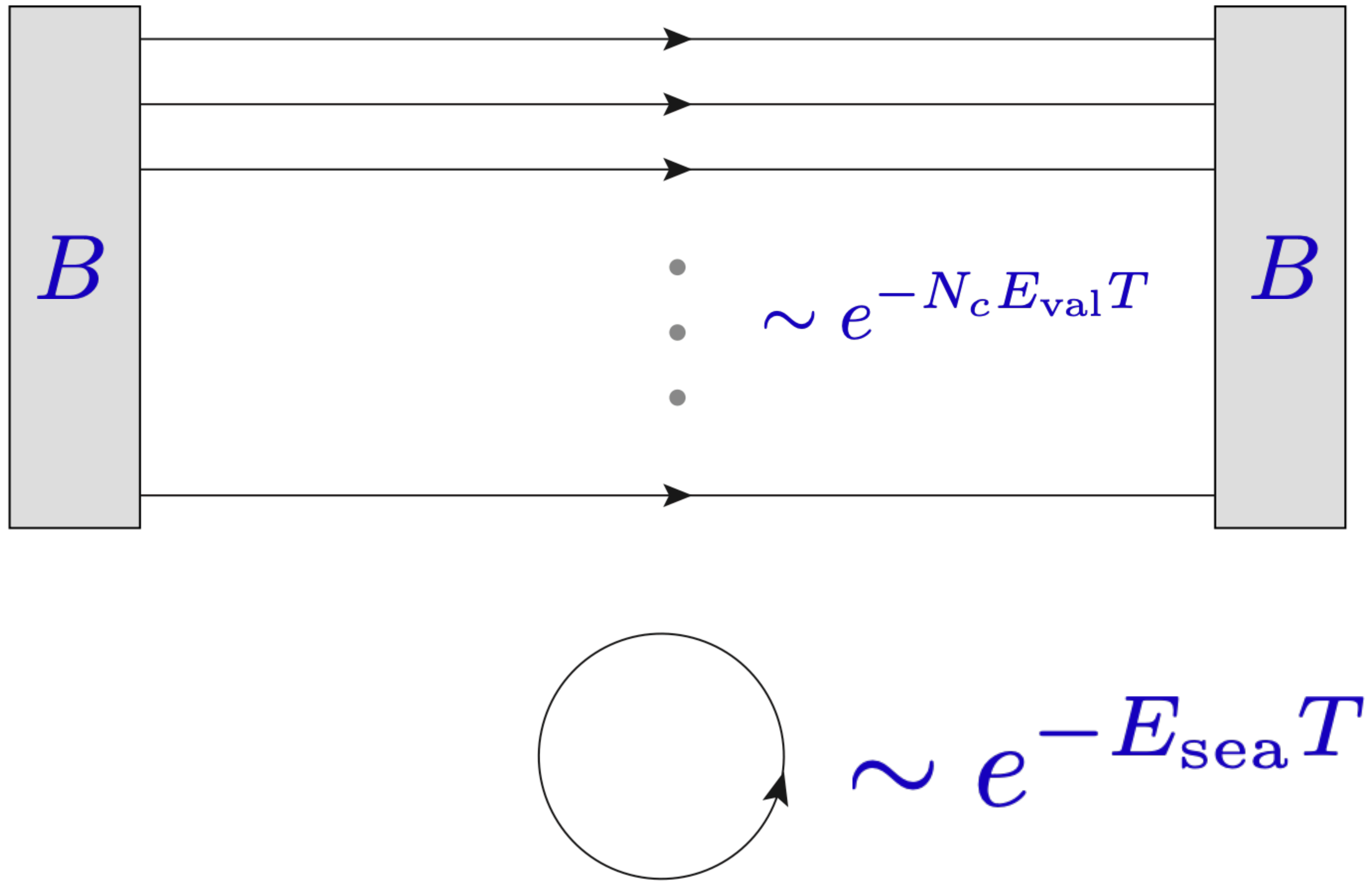
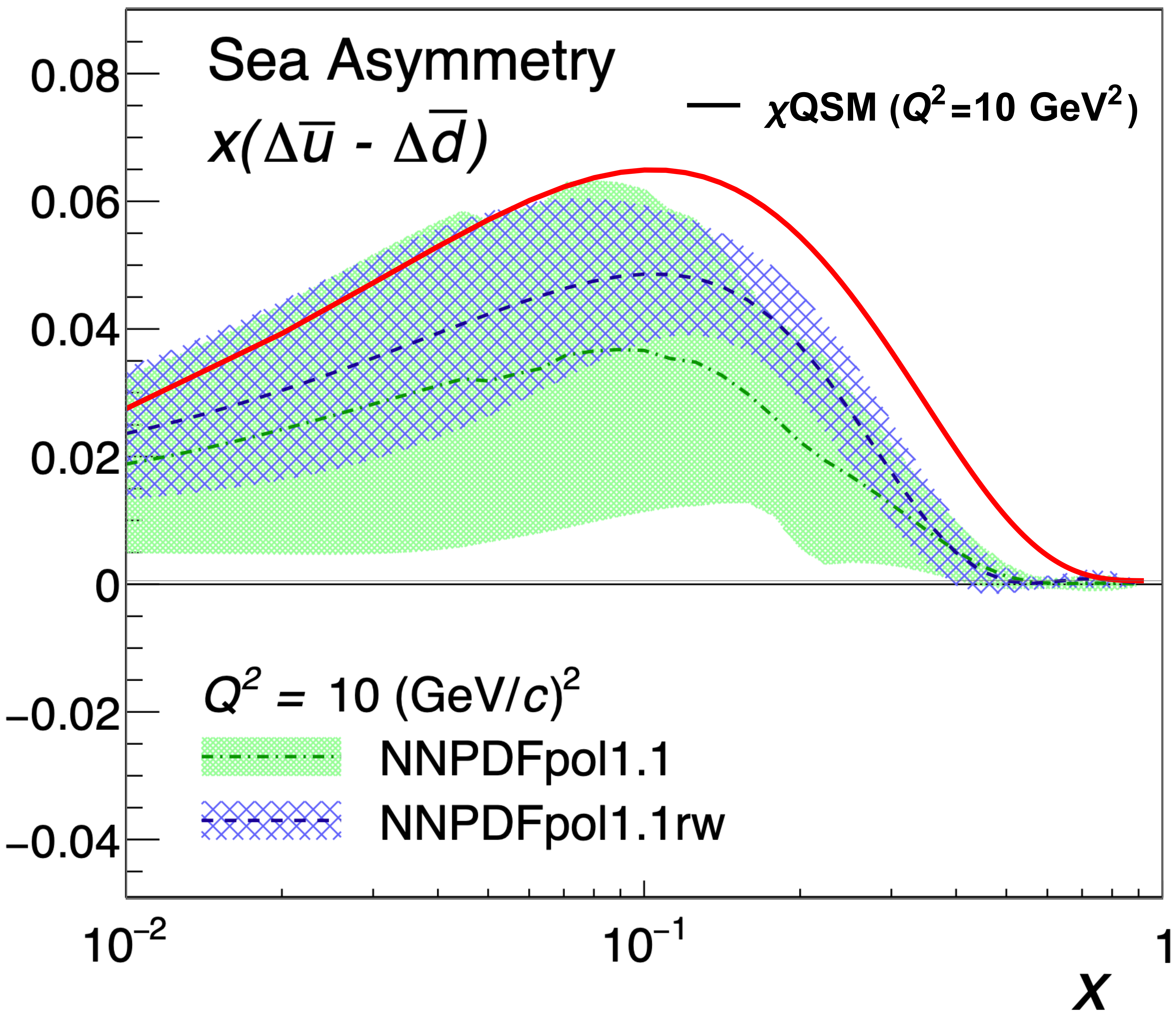


Figure from Hyun-Chul Kim's slide

Antiquark asymmetries in the proton



NLO DGLAP evolution (HOPPET)

decreases the curve ~20% & softens the curves

Notes

- 1/Nc correction ~20% ($g_A^{(3)} = 0.9$)

- Taking into account the quark virtuality from instantons will make the distribution even softer.

[V. Petrov et al, Phys.Rev.D 57 (1998) 4325]

GPD example

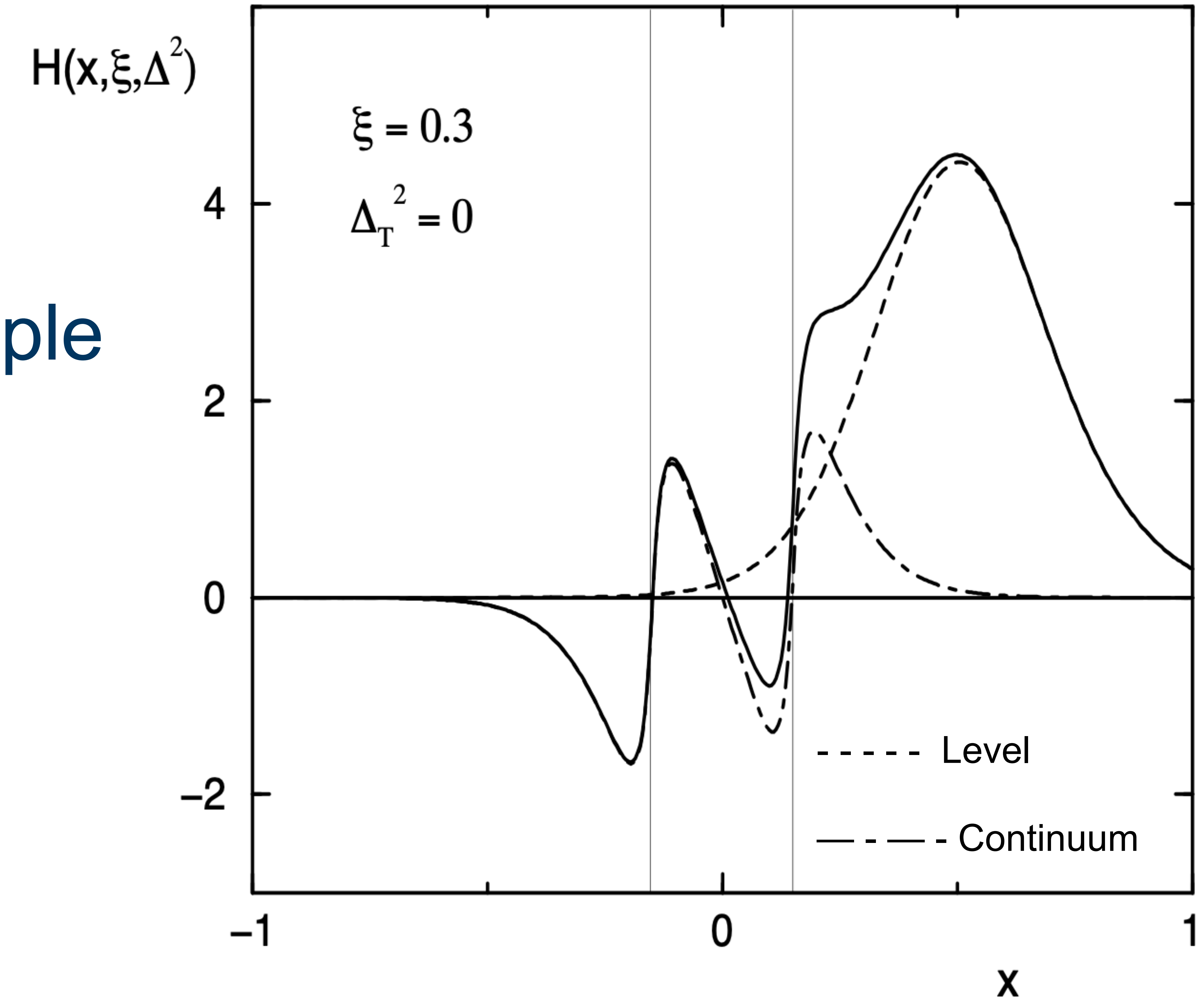


Figure 2: The isosinglet distribution $H(x,\xi,\Delta^2)$ for $\Delta_T^2 = 0$ ($\Delta_T^2 \equiv -\Delta^2 - \xi^2 M_N^2$) and $\xi = 0.3$. *Dashed line*: contribution from the discrete level. *Dashed-dotted line*: contribution from the Dirac continuum according to the interpolation formula, eq. (4.26). *Solid line*: the total distribution (sum of the dashed and dashed-dotted curves). The vertical lines mark the crossover points $x = \pm\xi/2$.

Continuous at $x = \xi$

ERBL region dominated by the continuum

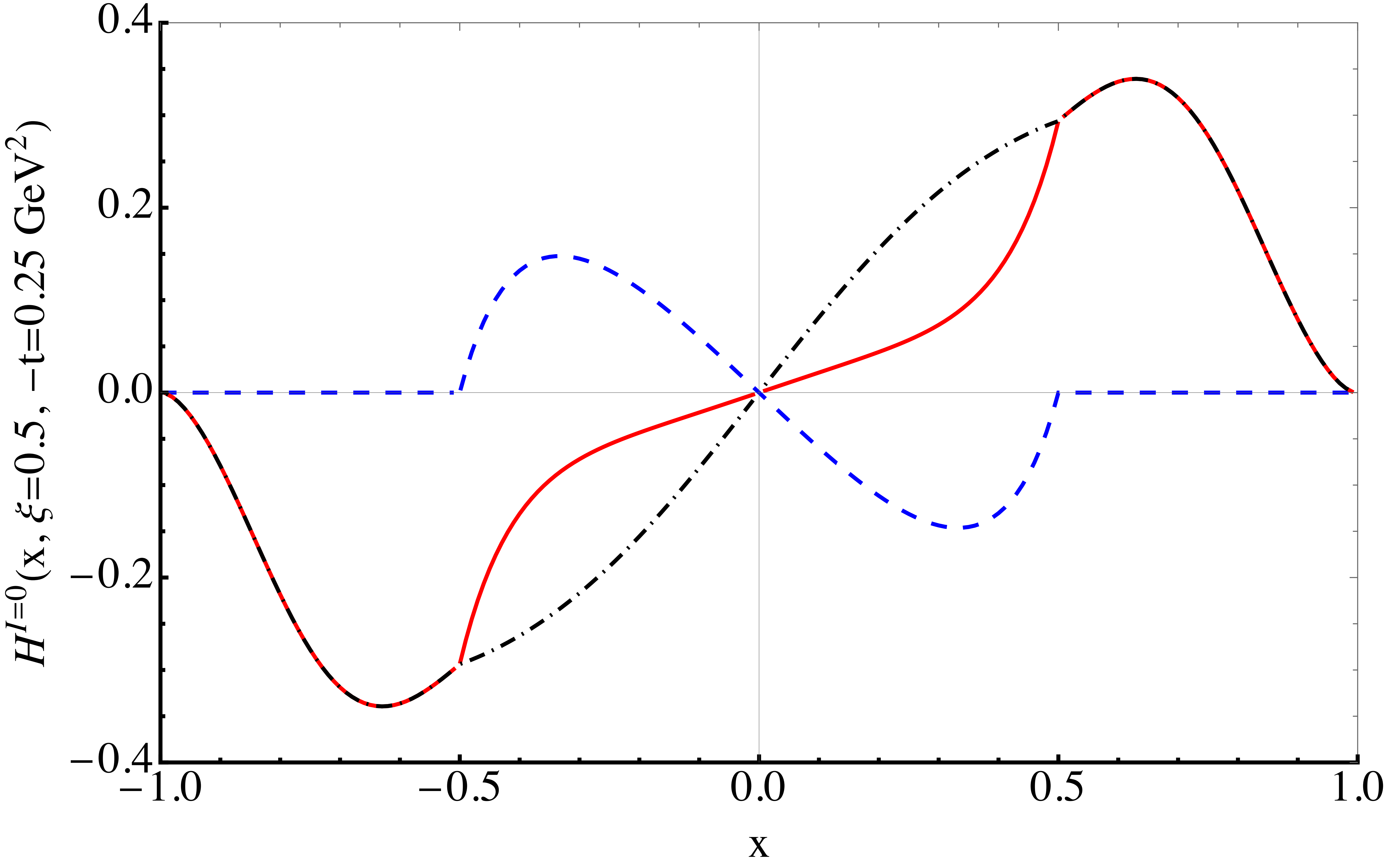
Warning

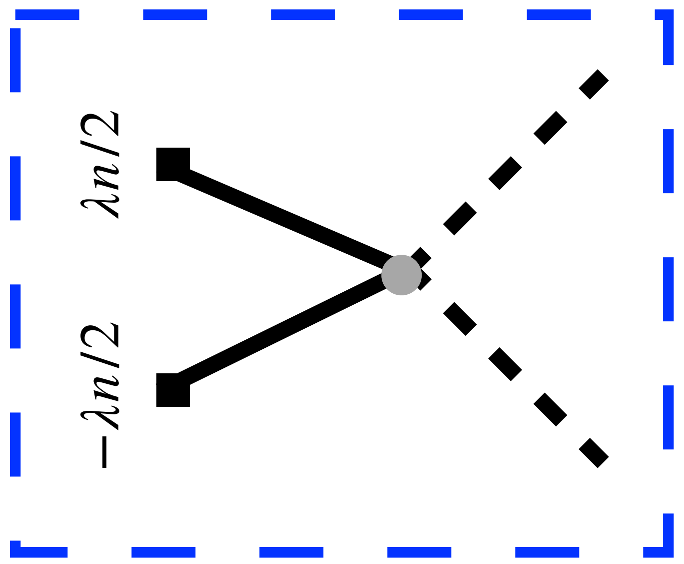
- Original model result shows discontinuity at the cross-over

→ Momentum dependent quark mass introduced by hand (model consistency?), with a smearing width $(M\rho)^2/N_c$

GPD example, Pion case

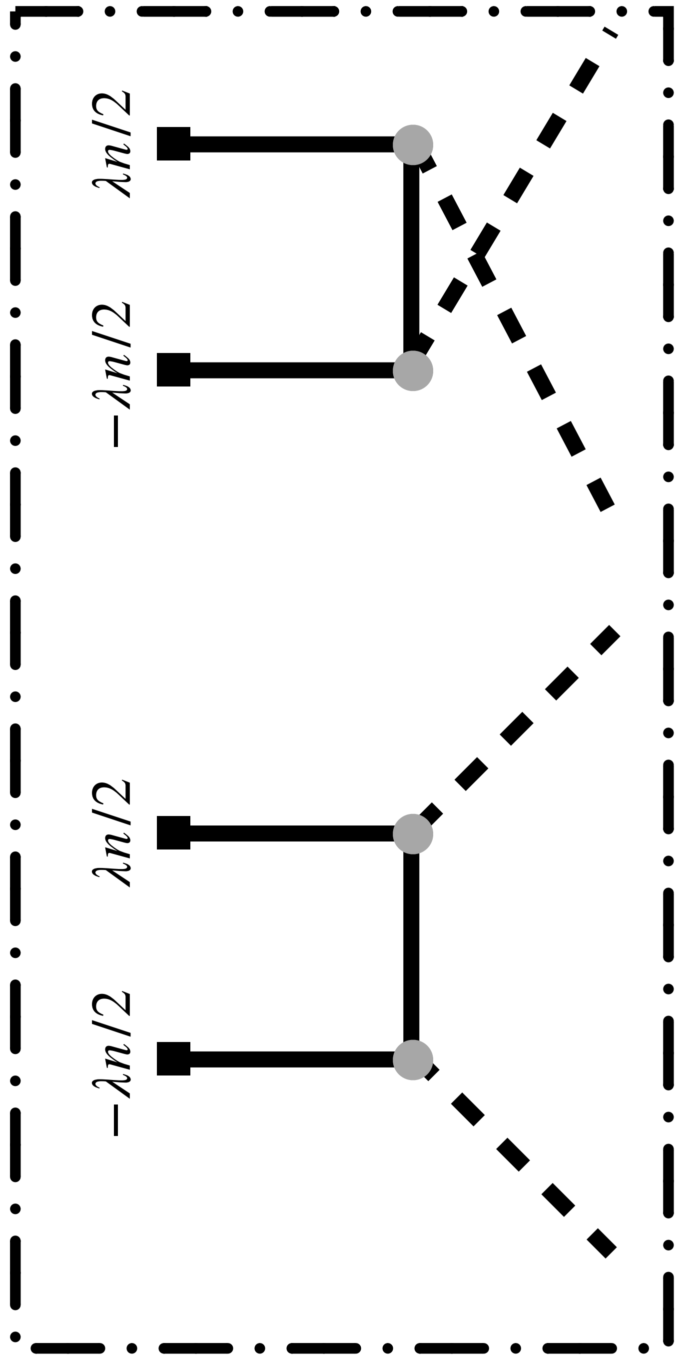
[M. Praszalowicz and A. Rostworowski, Acta Phys. Polon. B 34 (2003) 2699–2730]





Diagrams from two-pion vertex only contribute to the ERBL region

$$-\frac{\delta^{ab}}{2F_\pi^2}\sqrt{M(k_1)}\sqrt{M(k_2)}$$



Usual quark triangle diagrams Contribute to both the ERBL and DGLAP region

Quark quasi-distribution functions in the χQSM

Quasi parton distribution function

Xiangdong Ji, Phys. Rev. Lett. 110, 262002 (2013)

$$q(x,\mu,P^z) = \int \frac{dz}{4\pi} e^{-ixP^z z} \langle P|\bar{\psi}(0)\gamma^z \exp\left[-ig\int_0^z dz' A^z(z')\right]\psi(z)|P\rangle + \mathcal{O}(\frac{\Lambda^2_{\text{QCD}}}{(P^z)^2}, \frac{M_N^2}{(P^z)^2})$$

$x \in (-\infty, +\infty)$

μ: renormalization scale
P_z: nucleon momentum

Large **M**omentum **E**ffective **T**heory

Spacelike matrix element → can be calculated on the Lattice

No unique definition → **$\Gamma=\gamma^3$** or **$\Gamma=\gamma^0$**

Approaches the PDFs in the limit Pz →∞, or v → 1.

Quark quasi-distributions in χQSM

[HDS, A. Tandogan and M. V. Polyakov, Phys. Lett. B 808 (2020) 135665]

[HDS, Phys.Lett.B 838 (2023) 137741]

Effective approach to obtain x-dependence of the light-cone distributions

Benchmark model computation for the convergence in $P_z \to \infty$

Numerical results for $u + d$ and $\Delta u - \Delta d$ (leading in the large Nc)

$\Delta u - \Delta d$ has much better convergence to the light-cone PDF

Sum-rules depend on the defining Dirac matrix, eg. γ^0 or γ^3

Momentum sum-rule with γ^3 involves the non-conserving EMT-ff $\bar{c}(t)$

Sum-rules

[HDS, A. Tandogan and M. V. Polyakov, Phys. Lett. B 808 (2020) 135665]

[HDS, Phys.Lett.B 838 (2023) 137741]

Baryon number

$$\int_{-\infty}^{\infty} dx\ q(x,v) = \begin{cases} N_c B, & \Gamma = \gamma^0 \\ v N_c B, & \Gamma = \gamma^3 \end{cases}$$

Momentum

$$\int_{-\infty}^{\infty} dx\ xq(x,v) = \begin{cases} 1, & \Gamma = \gamma^0 \\ v, & \Gamma = \gamma^3 \end{cases}$$

Bjorken

$$\int_{-\infty}^{\infty} dx\ (\Delta u(x,v) - \Delta d(x,v)) = \begin{cases} v g_A^{(3)}, & \Gamma = \gamma^0 \\ g_A^{(3)}. & \Gamma = \gamma^3 \end{cases}$$

→ **better definition of qPDFs for the convergence to the light-cone PDFs**

→ **Interpretation of the QCD symmetry currents**

Sum-rules

[HDS, A. Tandogan and M. V. Polyakov, Phys. Lett. B 808 (2020) 135665]

[HDS, Phys.Lett.B 838 (2023) 137741]

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Bjorken

$$\int_{-\infty}^{\infty} dx\; (\Delta u(x,v) - \Delta d(x,v)) = \begin{cases} v g_A^{(3)}, & \Gamma = \gamma^0 \\ g_A^{(3)}. & \Gamma = \gamma^3 \end{cases}$$

Momentum sum-rule is satisfied only by quarks

Energy-momentum tensor: momentum flux ($T^{30} \sim \partial_3 \gamma^0$) vs pressure ($T^{33} \sim \partial_3 \gamma^3$)

In general, $M_2^q(\Gamma = \gamma^3) = v\left(A^q(0) - \frac{1-v^2}{v^2}\bar{c}^q(0)\right)$

Smallness of $\bar{c}^q$ in the dilute instanton

[Maxim Polyakov and HDS, JHEP 09 (2018) 156]

Numerical calculation setup

Ansatz for the pion mean field

[D. Diakonov, V. Petrov, and P. Pobylitsa, Nucl. Phys. B 306, 809 (1988)]

$$P(r) = 2\ \mathrm{Arctan}\left(\frac{r_0^2}{r^2}\right) \qquad r_0 \approx 1/M$$

within ~10% from the self-consistent solution

Interpolation formula

$$\frac{pM}{p^2+M^2}(U-1) \ll 1$$

Quasi-PDFs have the same order of divergence as the PDFs (v=1)

with smooth convergence in v →1

Logarithmic divergence: Pauli-Villars regularization

$$q(x,v)^{PV} = q(x,v)^{\mathrm{level}} + q(x,v)_{occ} - \frac{M^2}{M_{PV}^2} q(x,v)_{occ}(M \to M_{PV})$$

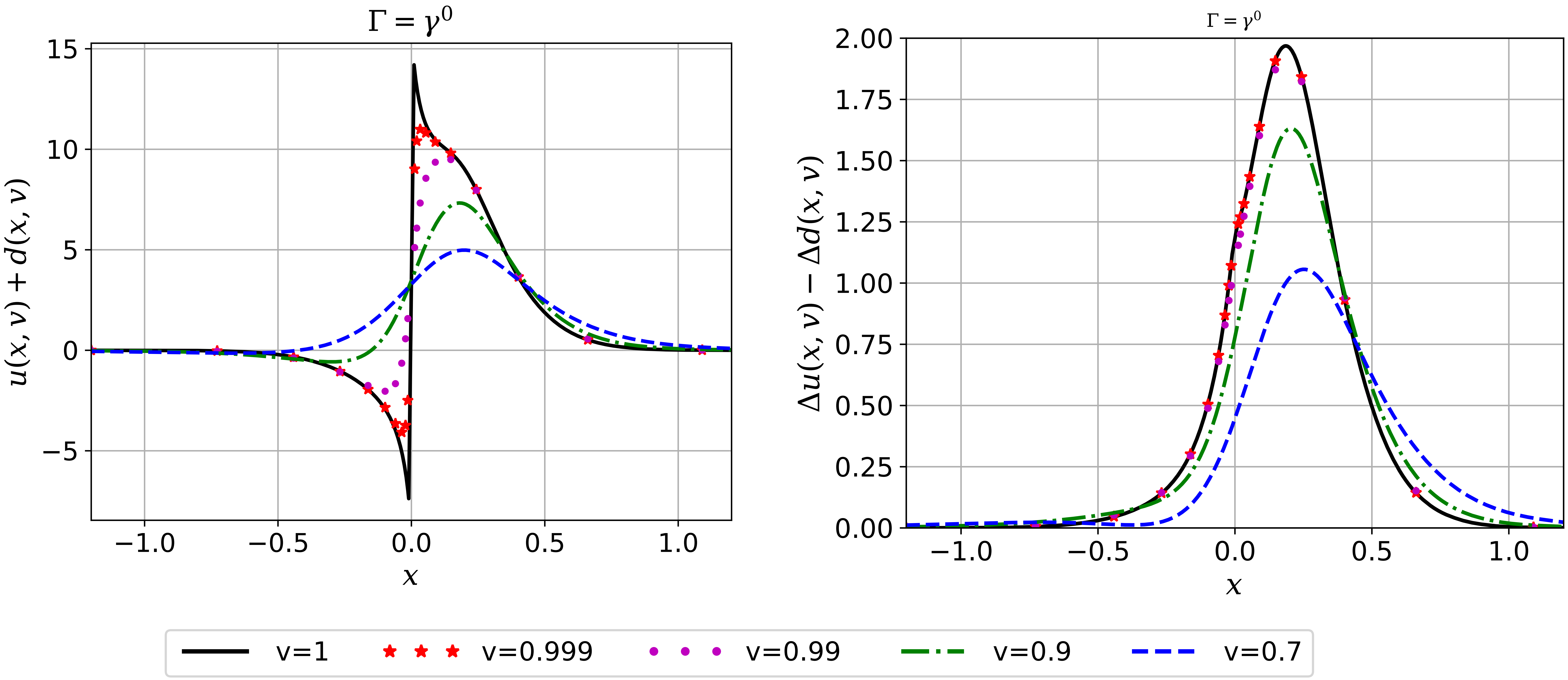
$$F_\pi^2 = \frac{N_c M^2}{4\pi^2}\log(M_{PV}^2/M^2)$$

$$M = 350\ \mathrm{MeV}$$
$$M_{PV} = 557\ \mathrm{MeV}$$

Quark quasi-distributions

[HDS, A. Tandogan and M. V. Polyakov, Phys. Lett. B 808 (2020) 135665]

[HDS, Phys.Lett.B 838 (2023) 137741]



$P_N/M_N=\infty$ 22.3 7.0 2.1 1.0

Light quarks in a heavy baryon

Heavy baryon in the chiral quark-soliton model

Heavy quark symmetry → an approximation scheme for a singly heavy baryon:

Nc-1 chiral quark-soliton in the large Nc

+ Heavy quark in the heavy quark limit

Recent studies on

→ baryon mass spectrum [J.Y-. Kim H.-Ch. Kim, G.-S. Yang, PRD 2018]

→ EM ffs: good agreements with lattice calculations, Axial & Tensor [J.Y-. Kim H.-Ch. Kim, PRD 2018/EPJC 2019,2020]

→ EMT form factors [J.Y-. Kim, H.-Ch. Kim, M. Polyakov, HDS, PRD 2021]

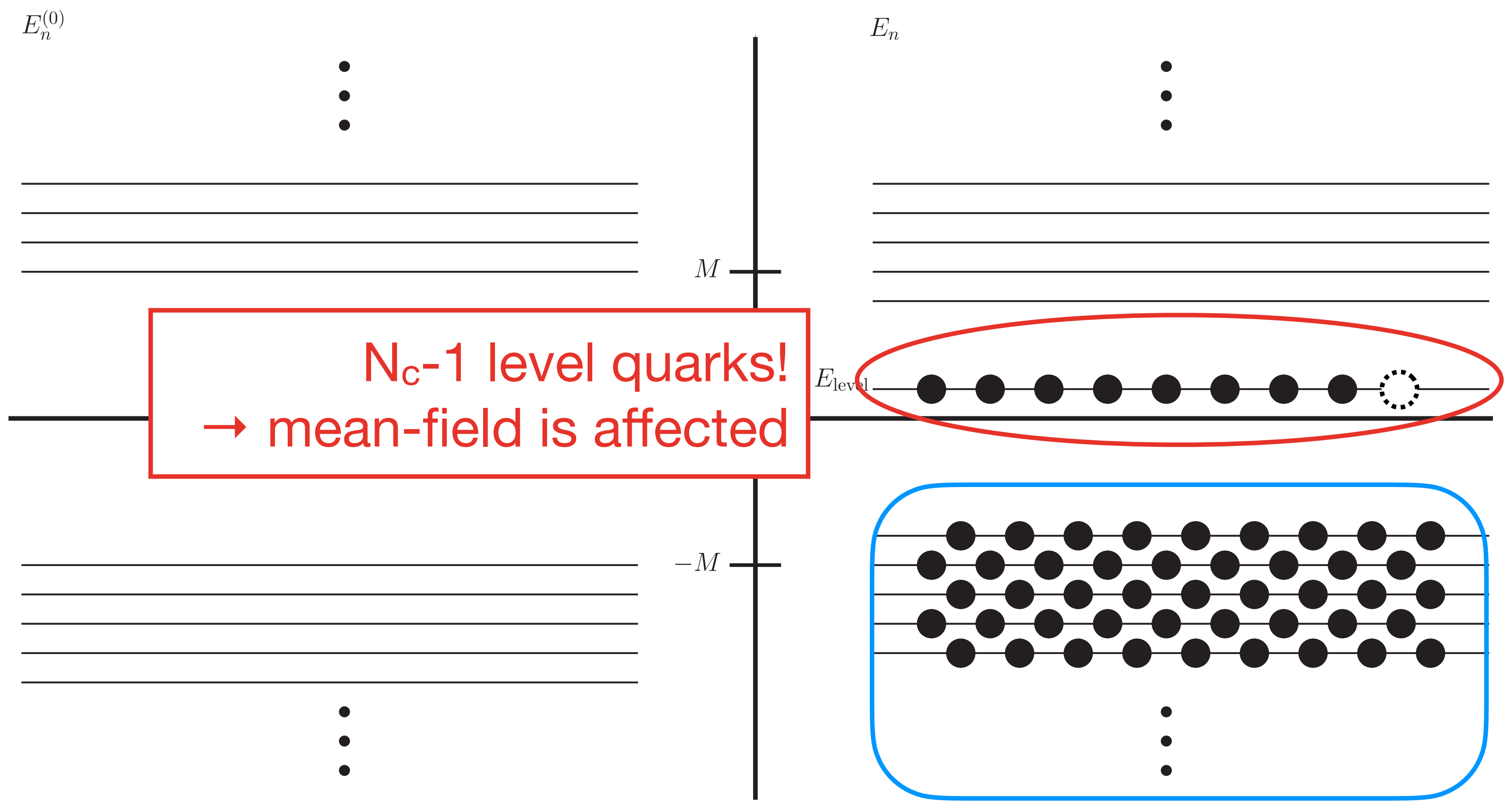
Light quarks in a heavy baryon?

PDF studies for heavy flavor in heavy baryon/meson [Guo, Thomas, Williams, PRD64 (2001)] [J. Lan et al. PRD102 (2020)]

Related to the fragmentation functions by crossing of

the DIS and e^+e^- (Drell-Levi-Yan) [Drell, Levy, Yan, PR 1969, PRD 1970]

Heavy baryon: Nc-1 quark-soliton



+ heavy quark as a color source

$$\frac{\delta}{\delta U}\left[(N_c-1)E_{\rm level}+E_{\rm cont.}\right]\Big|_{U=U_c}=0$$

$$M_{sol}=(N_c-1)E_{\rm level}(U_c)+E_{\rm cont.}(U_c)$$

$$M_h = M_Q + M_{sol}$$

Heavy quark mass $M_Q = (1.3,\, 4.2)$ GeV as parameters

M=420 MeV: strong quark-pion coupling is needed because of Nc-1

Recent studies for the heavy baryons

→ground-state mass spectrum

→EM ffs: good agreements with lattice calculations, Axial & Tensor

→Energy-momentum tensor form factors

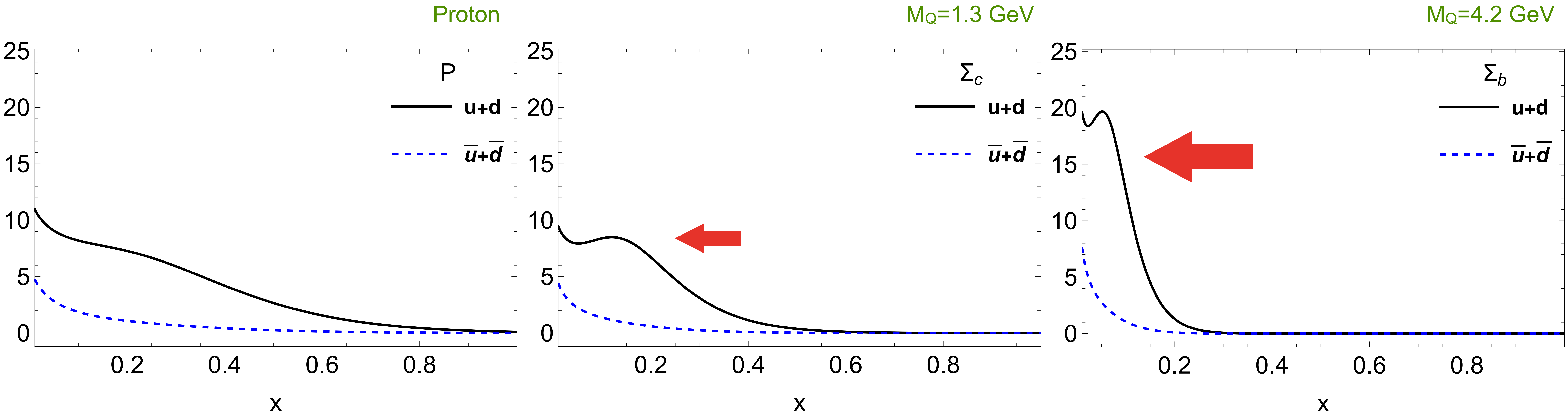
:Nc-1 level quarks produce a self-consistent mean-field

~ key ingredient for the stability

[J.Y-. Kim, H.-Ch. Kim, M. Polyakov, HDS, PRD 2021]

Light quarks in singly heavy baryons

[HDS, H.-Ch. Kim PTEP 2023 (2023) 3, 033D02]



Light quarks inside a heavy baryon are more concentrated at small x region

More probable to find a quark with small momentum fraction

Momentum sum-rule: light quarks are less energetic in a heavy baryon (M_{sol}/M_h)

δ-like heavy quark distribution function $Q(x) = \delta(x - M_Q/M_h)$

$u(x) + d(x)$: naive quark limit

[HDS, H.-Ch. Kim PTEP 2023 (2023) 3, 033D02]

Mean-field size →0, the model exhibit the properties of the naive quark limit

Proton:

$u(x) + d(x) = N_c \delta(x - M/M_N)$, M: constituent quark mass ($M_N = N_c M$)

Momentum sum-rule:

$$\int_0^1 dx\ x\ u(x) + d(x) = N_c M/M_N = 1$$

Heavy baryon:

$u(x) + d(x) = (N_c - 1)\delta(x - M/M_h)$, $M_h = (N_c - 1)M + M_Q$

→ The distribution is squeezed to small x as M_Q grows

Momentum sum-rule:

$$\int_0^1 dx\ x\ u(x) + d(x) = (N_c - 1)M/M_h$$ goes to 0 in the exact limit $M_Q \to \infty$

$u(x) + d(x)$

[HDS, H.-Ch. Kim PTEP 2023 (2023) 3, 033D02]

Momentum sum-rule

$$\int_0^1 dx\,x\,[\underbrace{u(x) + d(x) + \bar{u}(x) + \bar{d}(x)}_{M_{sol}/M_h} + \underbrace{Q(x)}_{M_Q/M_h}] = 1$$

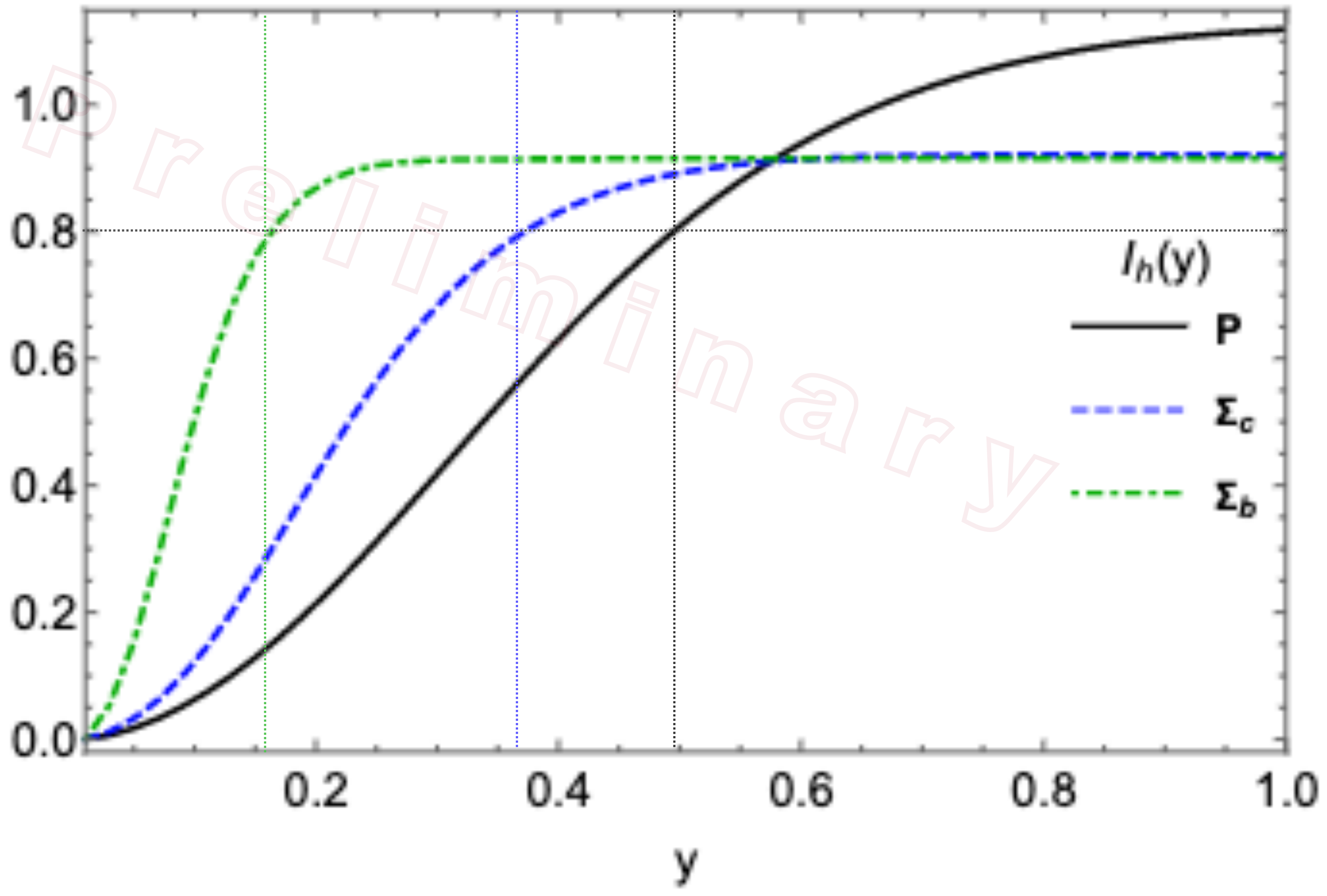
Comparison of the light-quark momenta in (P, Σc, Σb)

Momentum sum-rule: $I_h(y=1) = M_{sol}$

y for I=0.8 GeV: y =(0.5, 0.35, 0.15) for (P, Σc, Σb)

Truncated momentum-sum

$$I_h(y) \equiv M_h \int_0^y dx\,x\,[u(x) + d(x) + \bar{u}(x) + \bar{d}(x)]$$



Closing remarks

Summary

- χQSM provides a reasonable description on the (quasi-)PDFs at low renormalization scale
- Highlights on the flavor asymmetry of the longitudinally polarized antiquarks
- Continuum contribution is large and important
- New studies on the qPDFs and light quarks in a heavy baryon

Future developments

More realistic model: quark virtuality from the instantons

→ momentum dependent quark mass, M(k)

→ necessary to describe the GPDs

→ under development by Yongwoo Choi (Inha)

Flavour SU(3)

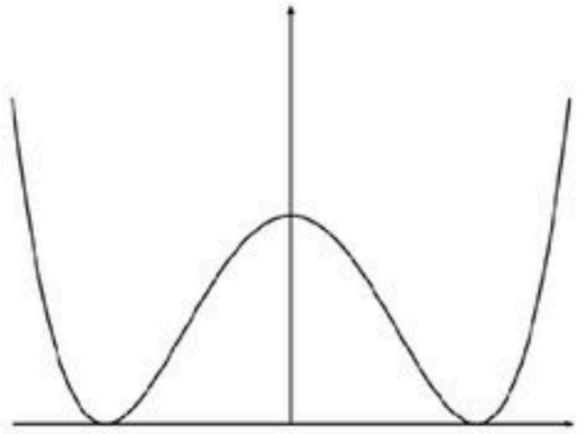
→ strange quark distributions in the nucleon

Gluon operators

→ Gluon structure functions, EMT form factors, higher twist, …

Thank you very much!

Quark interactions from the instanton QCD-vacuum

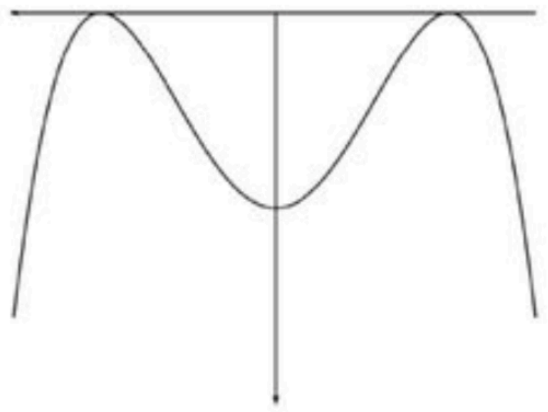


$$V(x) = \frac{1}{4}(x^2 - 1)^2$$

Tunneling amplitude between the minima

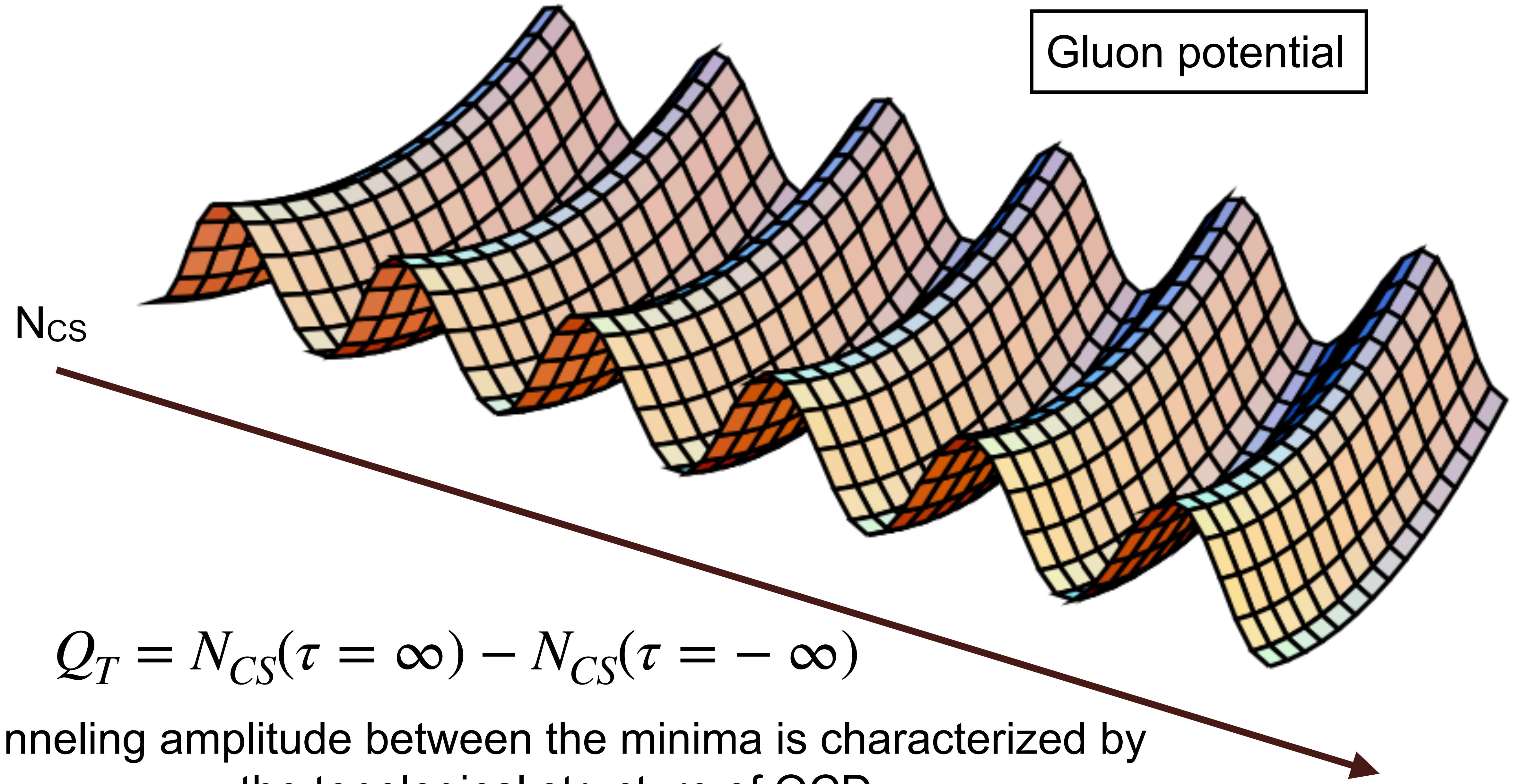
Wick rotation

$it \to \tau$



$$V(x) \to -V(x)$$

Classical path between the apexes



$$Q_T = N_{CS}(\tau = \infty) - N_{CS}(\tau = -\infty)$$

Tunneling amplitude between the minima is characterized by the topological structure of QCD

Classical solution minimizes the Euclidean YM's action

$$F = \tilde{F}$$

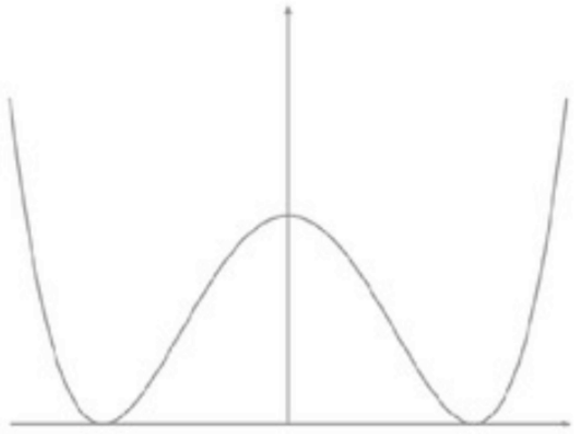
Spatial distribution of the instanton is characterized by

$$\bar{\rho} \approx 0.5/\Lambda_{\overline{MS}}$$

$$\bar{R} \approx 1.35/\Lambda_{\overline{MS}}$$

Diluteness is assumed

Quark interactions from the instanton QCD-vacuum



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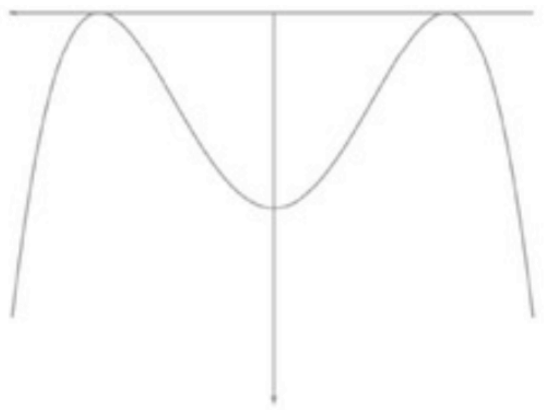

$$V(x) \to -V(x)$$

Tunneling amplitude between the minima

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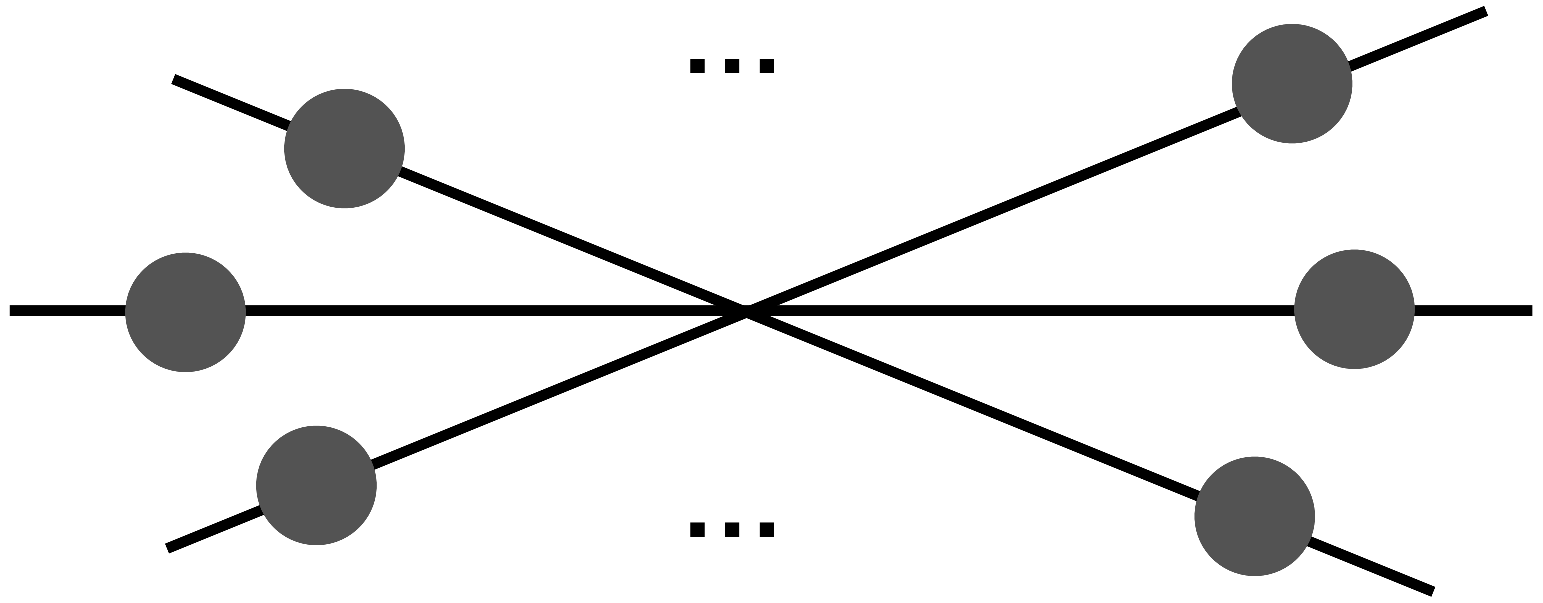
$$\bar{\rho} \approx 0.5/\Lambda_{\overline{MS}}$$

$$\bar{R} \approx 1.35/\Lambda_{\overline{MS}}$$

Diluteness is assumed

't Hooft like 2-N_f quark effective interactions



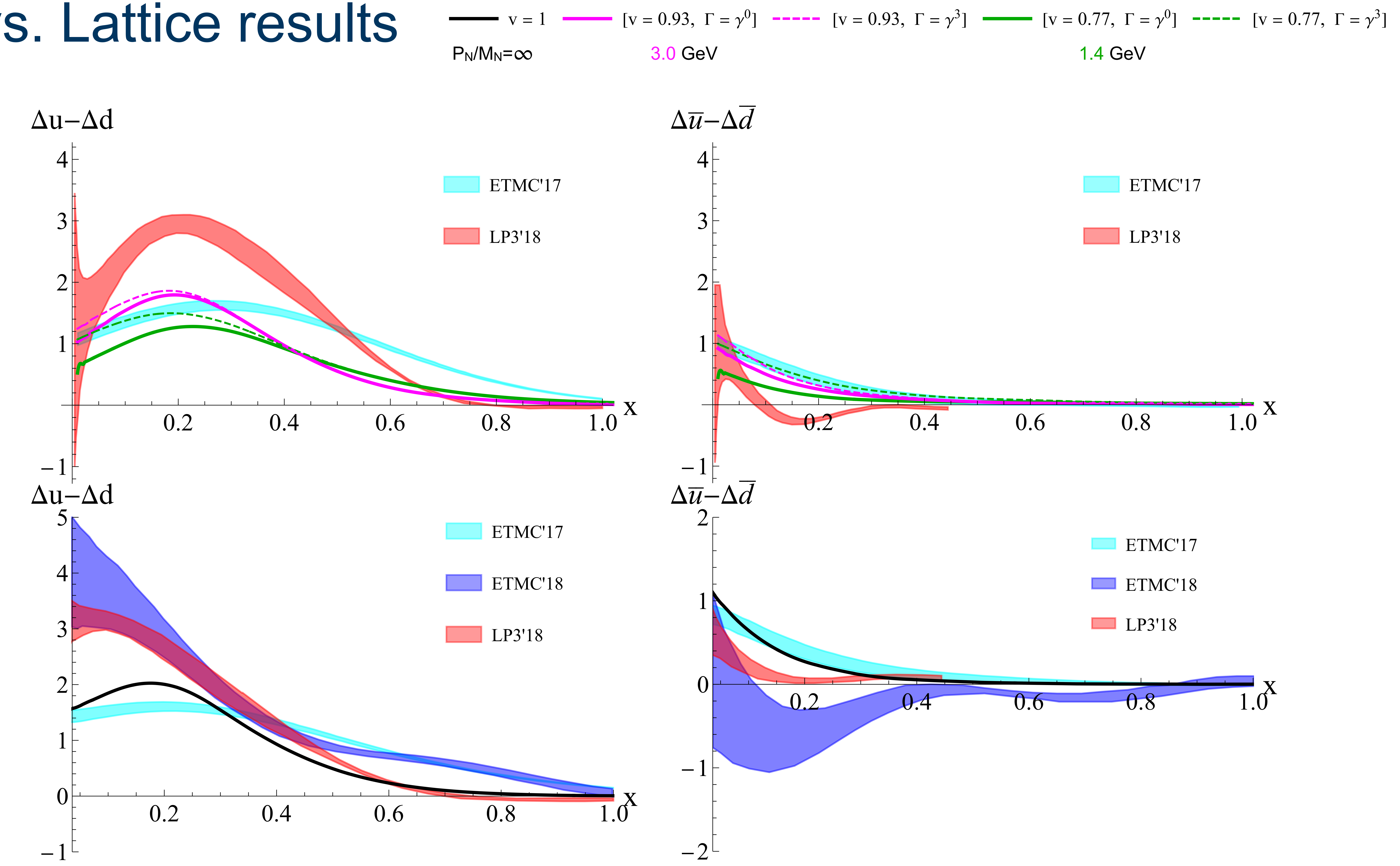
Quark form-factor

$$F(k) = 2t\left[I_0(t)K_1(t) - I_1(t)K_0(t) - \frac{1}{t}I_1(t)K_1(t)\right]\Bigg|_{t=\frac{k\rho}{2}}$$

Dynamical quark mass

$$M \approx 350\ \text{MeV}$$

vs. Lattice results



$(m_\pi, P_z, \mu) =$ (0.37, 1.4, 2.0) **[ETMC'17 Alexandrou et al. Phys. Rev. D, vol. 96, no. 1, p. 014513, 2017]**

(0.13,1.4,2.0) **[ETMC'18 Alexandrou et al. Phys.Rev.Lett. 121 (2018) 11, 112001, 2018]**

(0.135, 3.0, 3.0) **[LP3'18 Lin et al.Phys. Rev. Lett., vol. 121, no. 24, p. 242003, 2018]**

Copied from [STAR collaboration, Phys.Rev.D 99 (2019) 5, 051102]

The leptonic $W^+ \to e^+\nu$ and $W^- \to e^-\bar{\nu}$ decay channels provide sensitivity to the helicity distributions of the quarks, Δu and Δd, and antiquarks, $\Delta\bar{u}$ and $\Delta\bar{d}$, that is free of uncertainties associated with non-perturbative fragmentation. The cross-sections are well described [18]. The primary observable is the longitudinal single-spin asymmetry $A_L \equiv (\sigma_+ - \sigma_-)/(\sigma_+ + \sigma_-)$ where $\sigma_{+(-)}$ is the cross-section when the helicity of the polarized proton beam is positive (negative). At leading order,

$$A_L^{W^+}(y_W) \propto \frac{\Delta\bar{d}(x_1)u(x_2) - \Delta u(x_1)\bar{d}(x_2)}{\bar{d}(x_1)u(x_2) + u(x_1)\bar{d}(x_2)}, \quad (1)$$

$$A_L^{W^-}(y_W) \propto \frac{\Delta\bar{u}(x_1)d(x_2) - \Delta d(x_1)\bar{u}(x_2)}{\bar{u}(x_1)d(x_2) + d(x_1)\bar{u}(x_2)}, \quad (2)$$

where x_1 (x_2) is the momentum fraction carried by the colliding quark or antiquark in the polarized (unpolarized) beam. $A_L^{W^+}$ ($A_L^{W^-}$) approaches $-\Delta u/u$ ($-\Delta d/d$) in the very forward region of W rapidity, $y_W \gg 0$, and $\Delta\bar{d}/\bar{d}$ ($\Delta\bar{u}/\bar{u}$) in the very backward region of W rapidity, $y_W \ll 0$. The observed positron and electron pseudorapidities, η_e, are related to y_W and to the decay angle of the positron and electron in the W rest frame [19]. Higher-order corrections to $A_L(\eta_e)$ are known [20–22] and have been incorporated into the aforementioned global analyses.

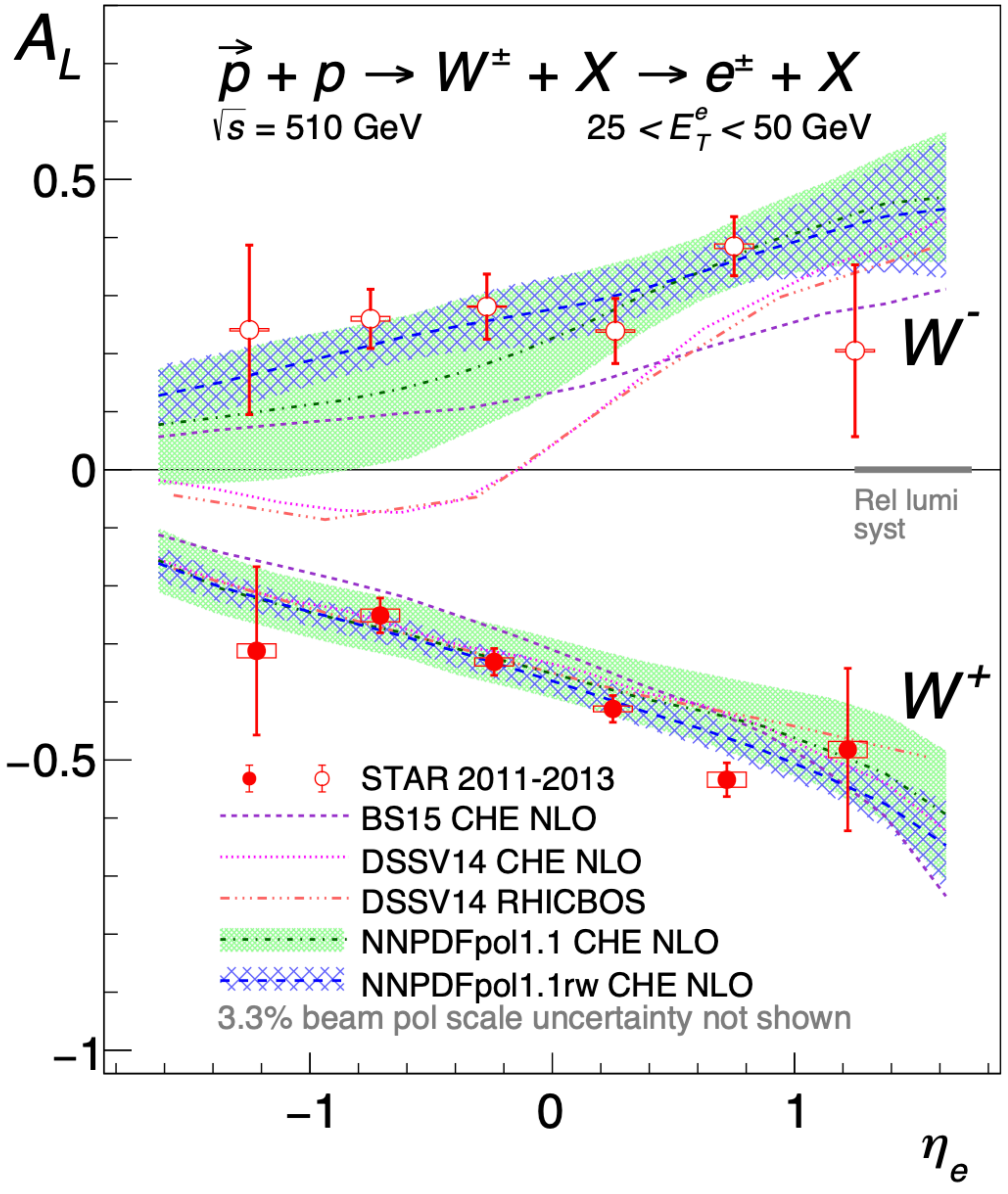


FIG. 5. Longitudinal single-spin asymmetries, A_L, for $W^\pm$ production as a function of the positron or electron pseudorapidity, η_e, for the combined STAR 2011+2012 and 2013 data samples for $25 < E_T^e < 50$ GeV (points) in comparison to theory expectations (curves and bands) described in the text.

List of the works done

Twist-2

Isovector unpolarized — [P. Pobylitsa et al, Phys.Rev.D 59 (1999) 034024]

Transversity — [P. Schweitzer, Phys.Rev.D 64 (2001) 034013]

Isoscalar longitudinally polarized — [M. Penttinen, M. Polyakov, K. Goeke, Phys.Rev.D62 (2000) 014024]

Twist-3

Chiral-odd twist-3 $e^q(x)$

[C. Cebulla et al, *Acta Phys.Polon.B* 39 (2008) 609-640,
P. Schweitzer, Phys. Rev. D 67 (2003) 114010,
M. Wakamatsu and Y. Ohnishi, Phys. Rev. D 67 (2003) 114011,
Y. Ohnishi and M. Wakamatsu, Phys. Rev. D 69 (2004) 114002]

GPDs

Role of the continuum contribution for

$-\xi < x < \xi$ and $-1 < x < -\xi$

Polynomiality

[V. Petrov et al, Phys.Rev.D 57 (1998) 4325
M. Penttinen, M. Polyakov, K. Goeke, Phys.Rev.D62 (2000) 014024]
J. Ossmann et al, Phys.Rev.D 71 (2005) 034011]

Dynamical momentum quark mass is essential for the continuity at $x = \xi$