Ultralight dark matter & Mysteries of galaxies

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Outline

1. Brief review on DM models

2. Fuzzy DM and its applications

3. Self-interacting ULDM

Galaxy DM & Flat Rotation curves

NASA

Invisible DM Halo: size $1\sim10^3\,kpc$
mass $10^7\sim10^{13}\ M_\odot$

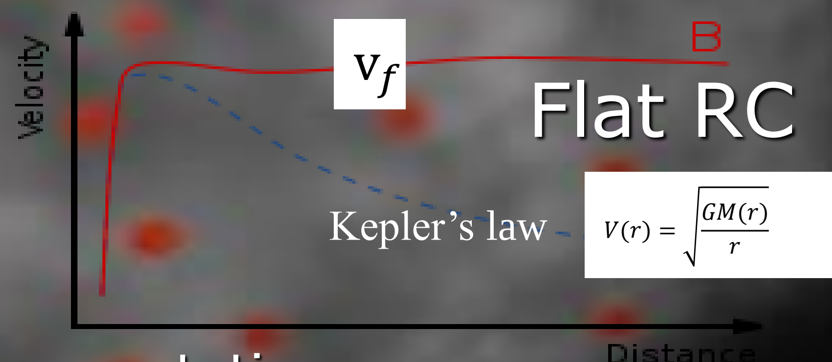
Globular Cluster

SMBH

Bulge

Disk

Vera Rubin 70s



We still can't explain RCs well!

3 ways to solve the missing mass problem

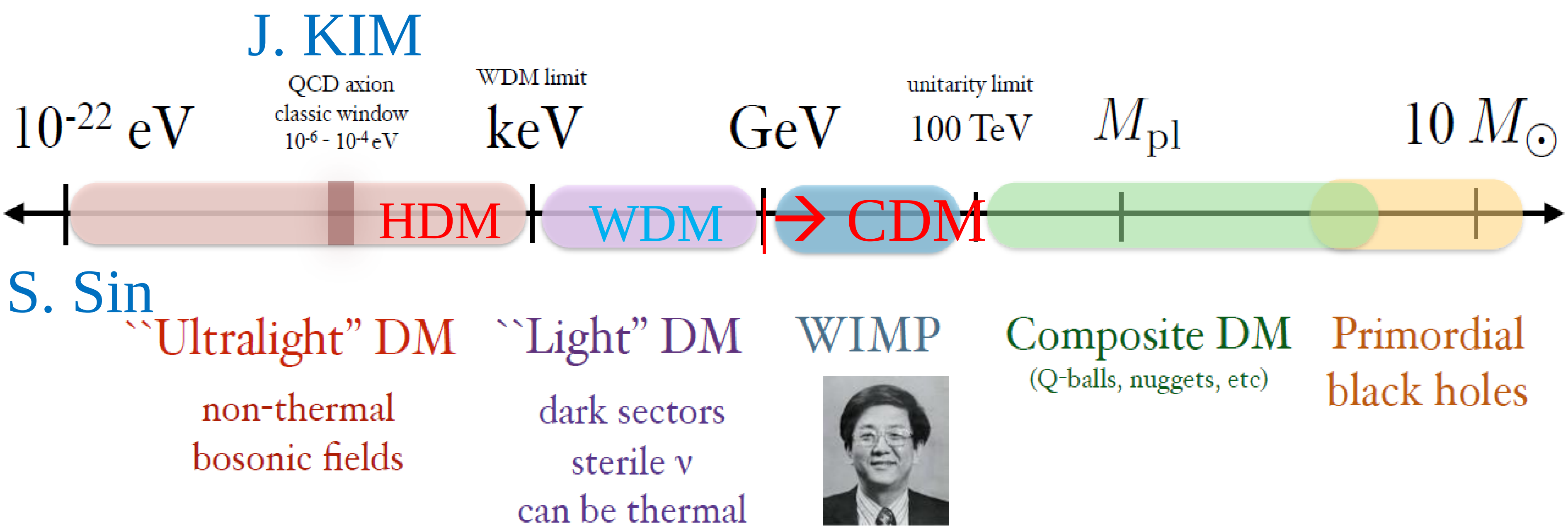
$$\frac{GM(r)}{r^2} = \frac{\mathrm{v}^2}{r} \quad \Rightarrow \quad \mathrm{v}(r) = \sqrt{\frac{GM(r)}{r}} \quad \text{rotation curve}$$

for large r to make v ~ const.

1. Change M(r) ~ r → DM (missing mass)
2. Change LHS to $\frac{GM(r)}{r^1}$ → modified gravity
   (No DM, but galaxy cluster gravity is not so different from that of the solar system)
3. Change RHS to $\left(\frac{\mathrm{v}^2}{r}\right)^2$ → MOND (Modified Newtonian)
   (for $a < a_0 = 1.2 \times 10^{-10} meter/s^2$
   No DM, hard to explain galaxy clusters or CMBR)

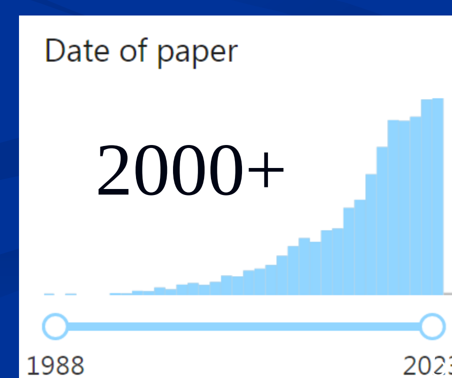
Mass scale of dark matter

(not to scale) TASI lectures by Lin arXiv:1904.07915

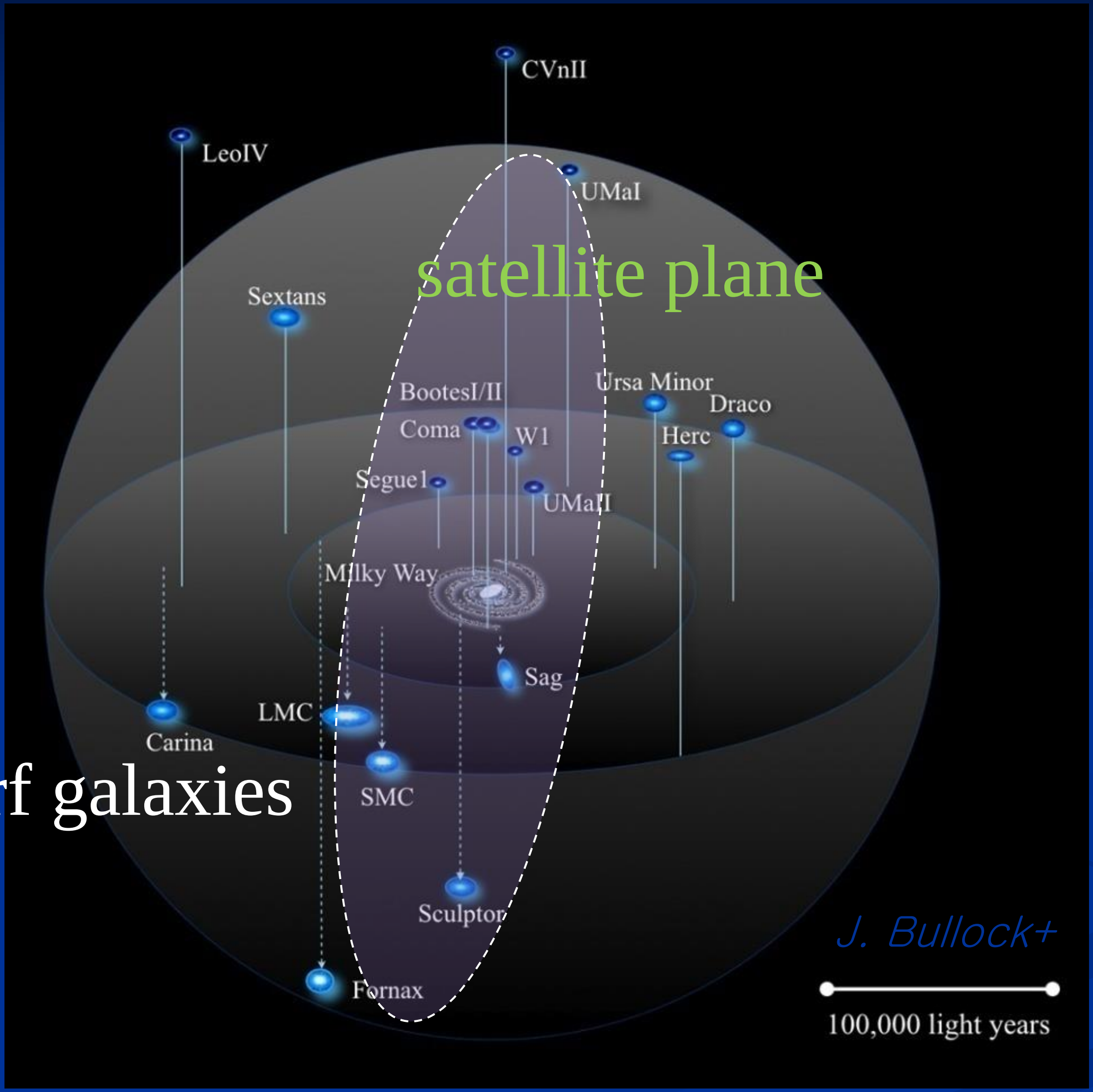


Reviews on ULDM = Fuzzy DM, ULA, BEC, SF, Wave, ψ,…

- JWL 1704.05057 (history)
- HOTW 1610.08297 (Witten et al)
- Marsh 1510.07633 (ULA)
- Ferreira et al. 2005.03254



Galaxies observed

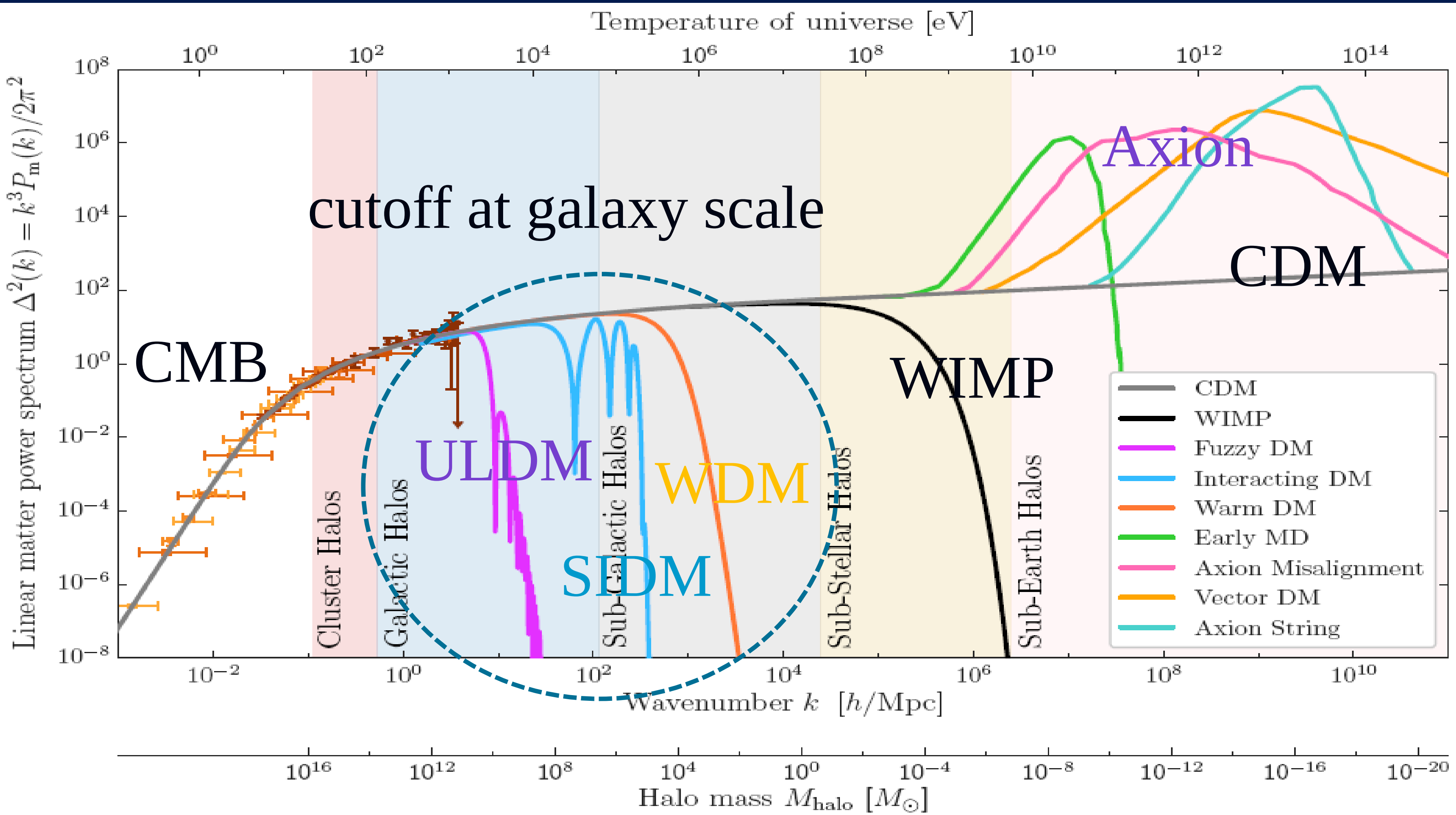


dwarf galaxies

Minimum mass
$\sim 10^6$ Ms

Any good DM model should explain observed galaxies!

Linear Power spectrum



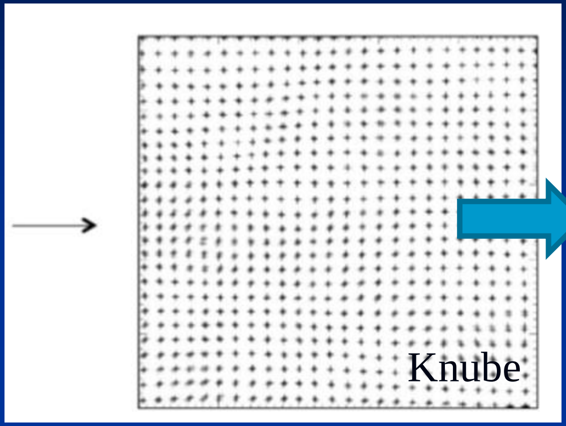
2022 Snowmass Summer Study

Dwarf galaxies are the smallest DM-dominated objects

Galaxy mass M_G from DM particle mass m

DM Model	Galaxy mass
SIDM	$M_J = \frac{\pi\bar{\rho}}{6}\left(\frac{\pi c_S^2}{G\bar{\rho}}\right)^{3/2}$ ← λ_J (pressure)
CDM, WDM (no pressure)	$M_{fs} = 10^{10}\left(\frac{m}{1keV}\right)^{-3.33} Ms$
Baryon (H)	$M_J(z\sim1000) \approx 10^6(\Omega_B h^2)^{-1/2} M_s$
ULDM (quantum pressure)	$M_{QJ} \approx \left(\frac{\hbar}{G^{1/2}m}\right)^{3/2} \bar{\rho}^{1/4}$ ← λ_{QJ}
self-interacting ULDM (pressure +quantum pressure)	$M = \sqrt{\Lambda}\frac{m_P^2}{m} = \frac{m_P^3\sqrt{\lambda}}{2m^2\sqrt{\pi}}$?

Non-linear evolution (usually N-body)



time

N-body
ex) Gadget..

similar to
observations

ITC @ University of Zurich

Challenges for ΛCDM

•Numerical ΛCDM is very successful at large scale, BUT
•ΛCDM encounters concerns like

1. Hubble tension:
 mismatch of H between CMB estimation and SN→ not Λ?

2. S_8 tension:
mismatch of Mpc scale denstity perturbation
between CMB estimation and SDSS→ not CDM?

3. Small scale crisis (at galaxy scale) → not CDM?
 predicts too many small structures not observed

4. Galaxy cluster mysteries (speed and offset)→ DM self-interacting?

5. Li problem …etc

ΛCDM Tensions with Dwarf Galaxies

No tension	Uncertain	Weak tension	Strong tension
Missing satellites	$M_\star$-$M_{\rm halo}$ relation	Too big to fail	Diversity of rotation curves
	Core-cusp	Diversity of dwarf sizes	Satellite planes
		Quiescent fractions	

Sales+ 2206.05295

Can baryon physics + precise numerical simulation + more observations save CDM?

Mysteries of galaxies in CDM model

- Min./Max. size and mass of galaxies
- Small scale crisis: Missing Satellites, Cusp, TBTF…
- Scaling laws: RAR, BTFR, fundamental plane, M-Sigma…
- Galaxies without DM or Visible M
- Angular momentum catastrophe, fraction of disk galaxies
- Impossibly early galaxies (James Webb)
- Impossibly early SMBH (James Webb)
- Final pc problem of SMBH
- Ripples, Rings and more …

Conjecture (or, hopes?)

FDM with $m \sim 10^{-22} eV$ (+ baryons) can explain these mysteries of galaxies

Solutions to Small scale problems

- CDM : m ~ G eV

Baryon physics (SN, BH jets,…) → Not enough baryon

- WDM : m ~ k eV

→ Catch 22 problem (cusp)

Suppression of the power spectrum prohibiting the formation of the dwarf galaxy

- SIDM: $\sigma/m \sim 0.5\text{-}1\ cm^2/g$

→ velocity dependent? (missing satellite)

- ULDM: $m \sim 10^{-22}$ eV

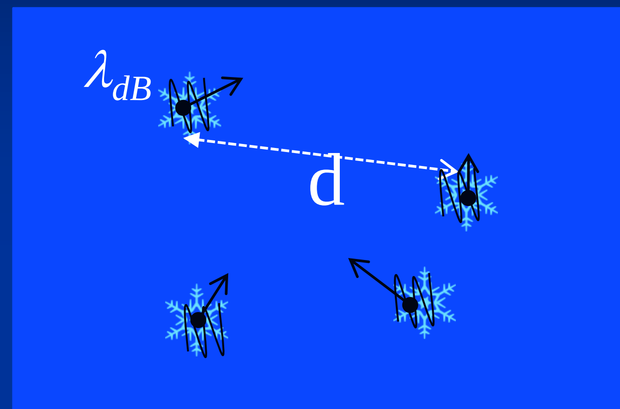
→ Lyman alpha favors $m > 10^{-21}$ eV ?

ULDM: DM is ultra-light & in Bose-Einstein Condensate!

For a history review JWL, 1704.05057

CDM (WIMP)

- *Heavy, $m > GeV$*
- *Particle-like*
- $d \gg \lambda_{dB} > 1/m$
- *Newton's eq*
- *Random motion*
- *No scale for DM halo*



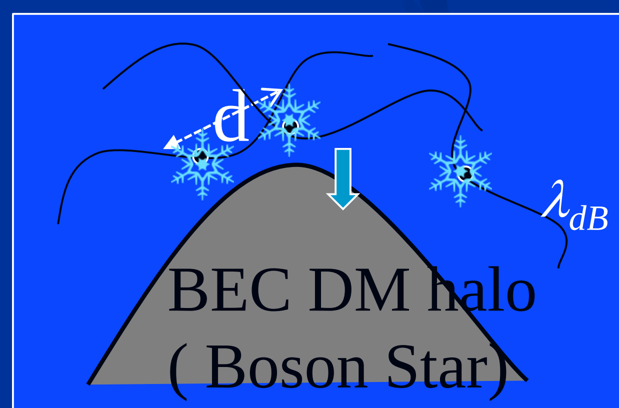
d: inter-particle distance
m: DM particle mass

ULDM

- *Ultra-light, $m \sim 10^{-22}$ eV*
- *high # density $n \sim 10^{25}$ /cm^3*

→ $(d \ll \lambda_{dB})$

- *wave-like at galactic scale*
- *SPE*
- *coherent motion*
- *Min. scale for DM halo*



$m \approx 10^{-22} eV$

$BEC \quad T_c = \left(\frac{3n}{m}\right)^{2/3}$

$\sim 5 \times 10^7 GeV,$

But ULDM acts as CDM at super-galactic scale!

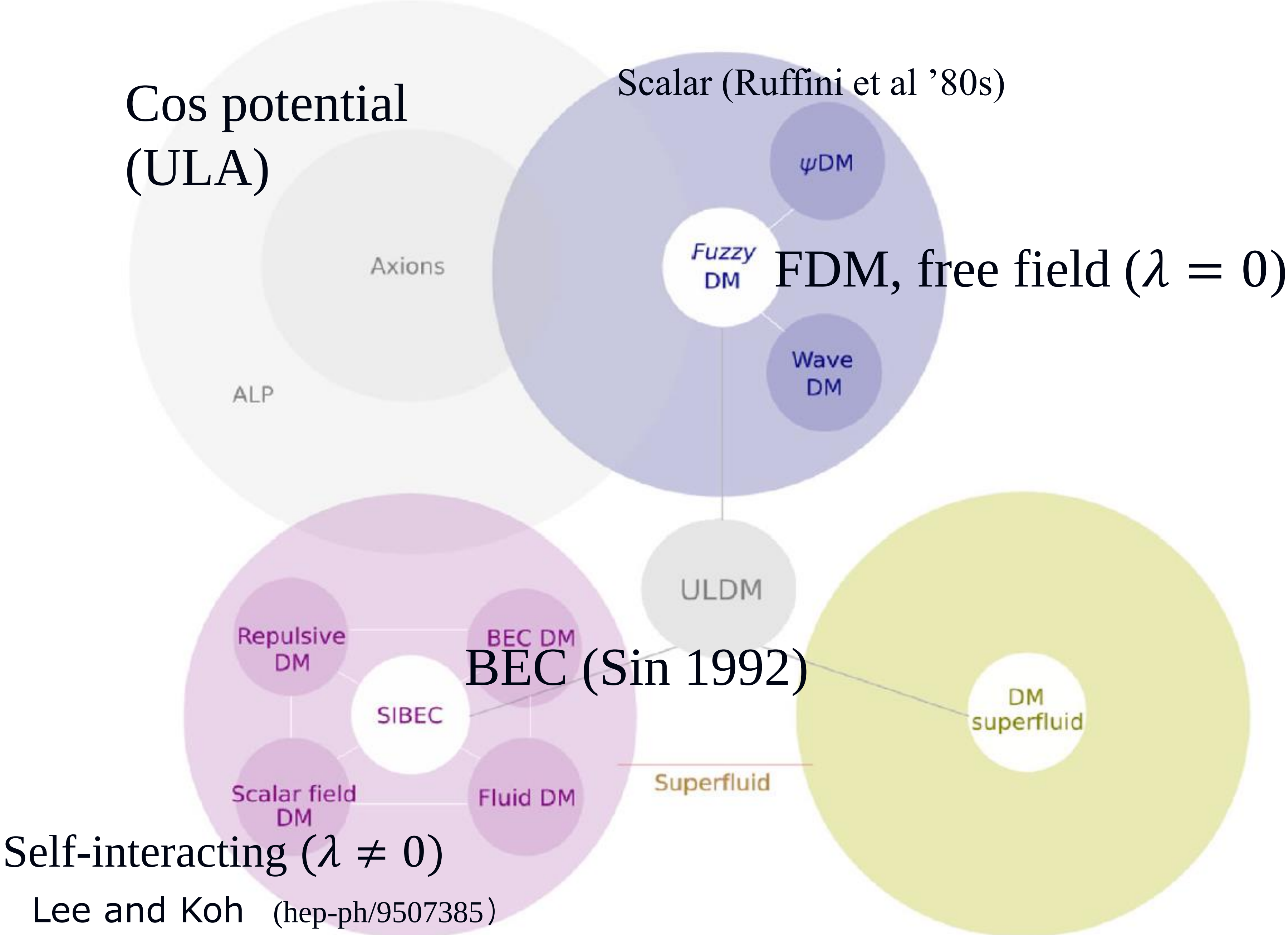


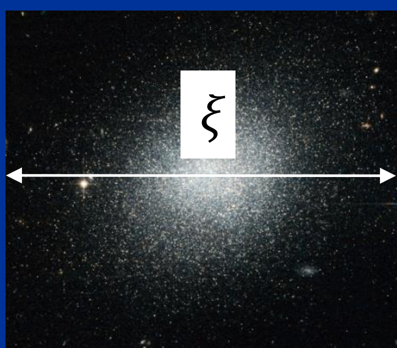
Fig. 9 Map of the ULDM classes of models

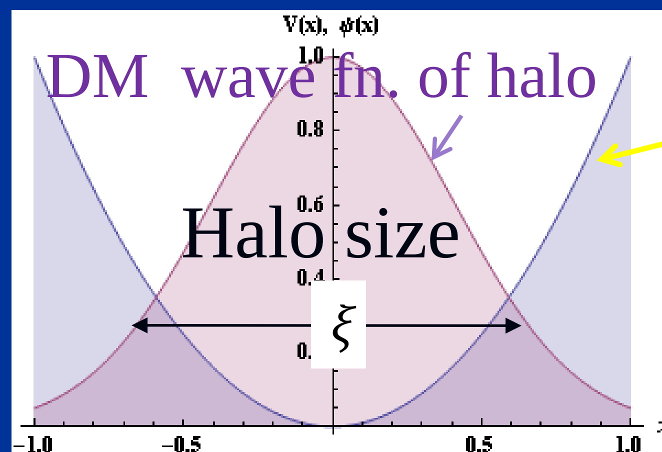
Ferreira et al.

ULDM

- Core of galactic halo is a single self-gravitating soliton (boson star) made of BEC bosons having a single macroscopic wave fn.
- Quantum pressure (from uncertainty principle) prevents collapse
- Minimum length scale ξ ~ de Broglie wave length

→ $m \sim 10^{-22}$ eV





Self-gravitating potential well V

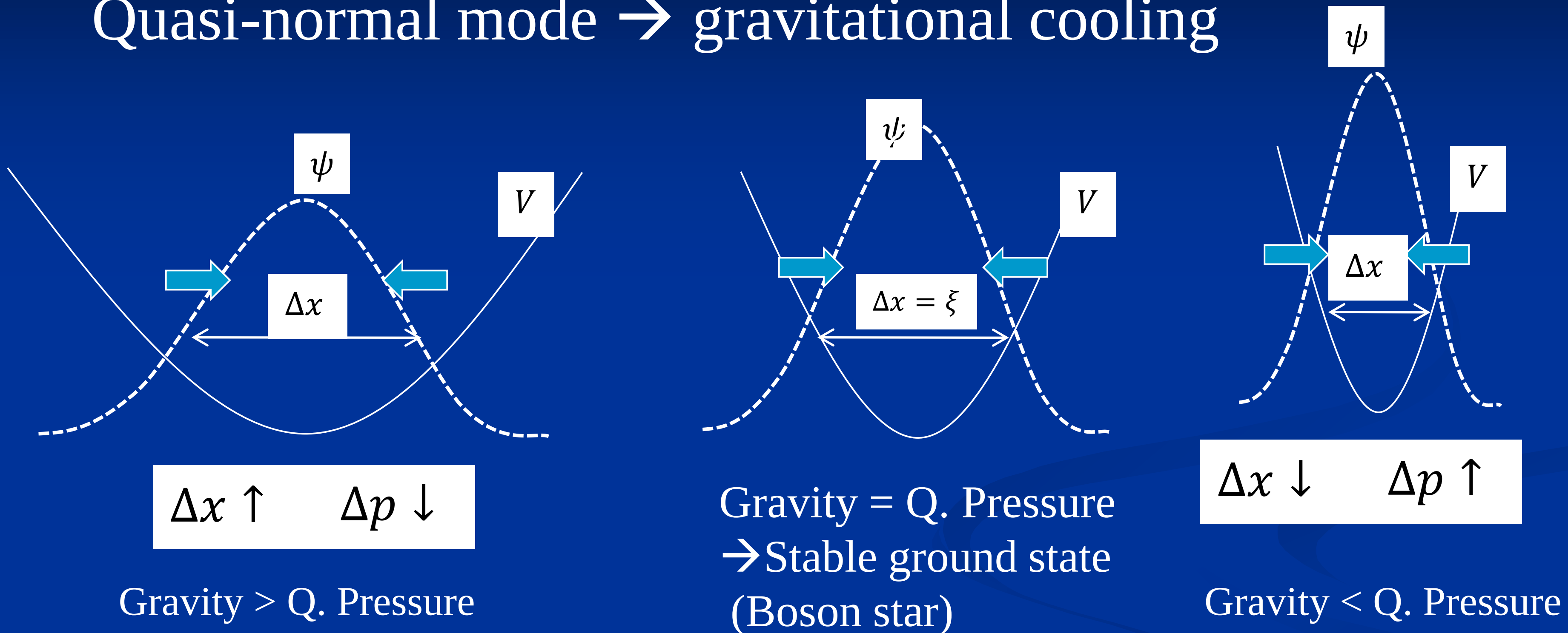
***Schunck**, CQG*

Schrodinger -Poisson (SP)

$$\begin{cases} i\hbar\partial_t\psi = -\dfrac{\hbar^2}{2m}\nabla^2\psi + mV\psi \\ \nabla^2 V = 4\pi G(\rho_d + \rho_v) \quad , \quad \rho_d = m|\psi|^2 \end{cases}$$

Quantum pressure

Quasi-normal mode → gravitational cooling



$$\lambda_{dB} = \hbar/(mv) = \hbar/(m\sqrt{GM/\lambda_{dB}}),$$

Typical length scale
= Gravitational Bohr radius
~ Q. Jeans length

$$\lambda_{dB} = \frac{h}{mv} \approx R_{Gal} = \frac{GM_{Gal}}{v^2}$$

$$\to v \sim \frac{GM_{Gal}m}{h} \to R_{Gal} \sim \frac{h^2}{GM_{Gal}m^2}$$

For $R_{Gal} \approx kpc, \quad M_{Gal} \approx 10^8 M_{\odot}$

$\to m \approx 5 \times 10^{-22} eV$

arXiv > astro-ph > arXiv:2312.00254

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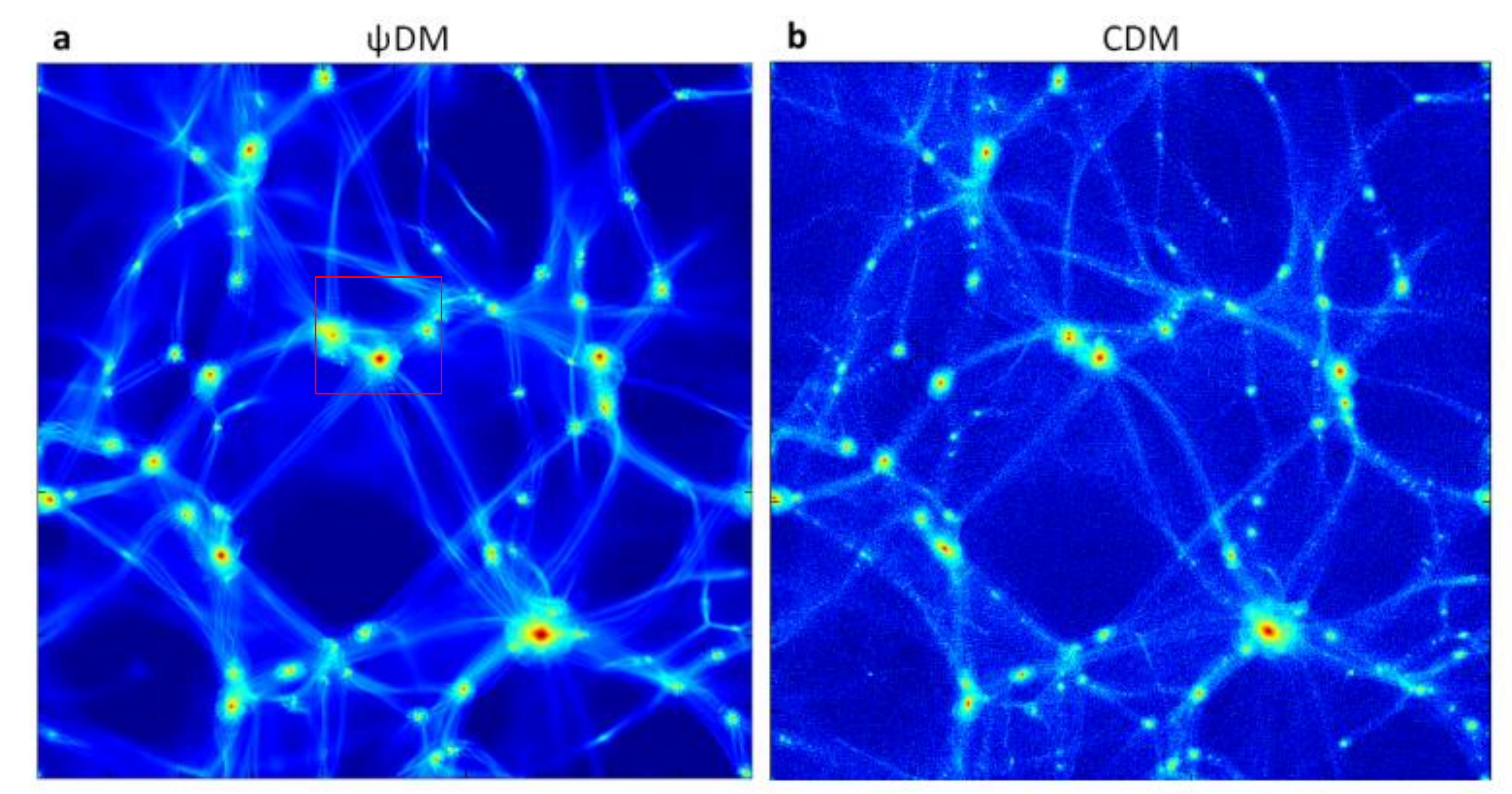
[Submitted on 30 Nov 2023]

Short Review of the main achievements of the Scalar Field, Fuzzy, Ultralight, Wave, BEC Dark Matter model

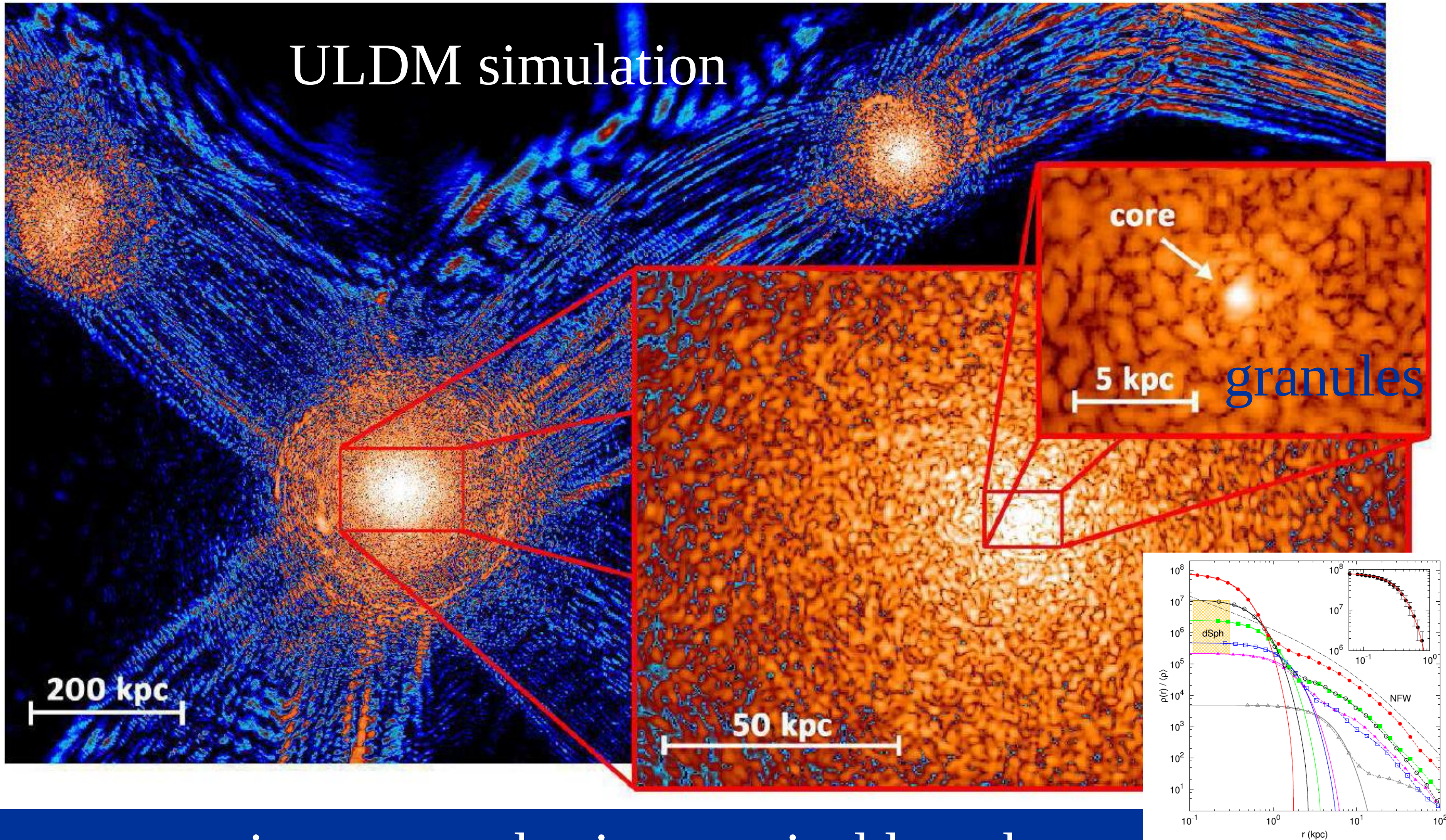
Tonatiuh Matos, Luis A. Ureña-López, Jae-Weon Lee

The Scalar Field Dark Matter model has been known in various ways throughout its history; Fuzzy, BEC, Wave, Ultralight, Axion-like Dark Matter, etc. All of them consist in proposing that the dark matter of the universe is a spinless field Φ that follows the Klein-Gordon (KG) equation of motion $\Box\Phi - dV/d\Phi = 0$, for a given scalar field potential V. The difference between different models is sometimes the choice of the scalar field potential V. In the literature we find that people usually work in the nonrelativistic, weak-field limit of the KG equation where it transforms into the Schrödinger equation and the Einstein equations into the Poisson equation, reducing the KG-Einstein system, to the Schrödinger-Poisson system. In this paper, we review some of the most interesting achievements of this model from the historical point of view and its comparison with observations.

ULDM vs CDM



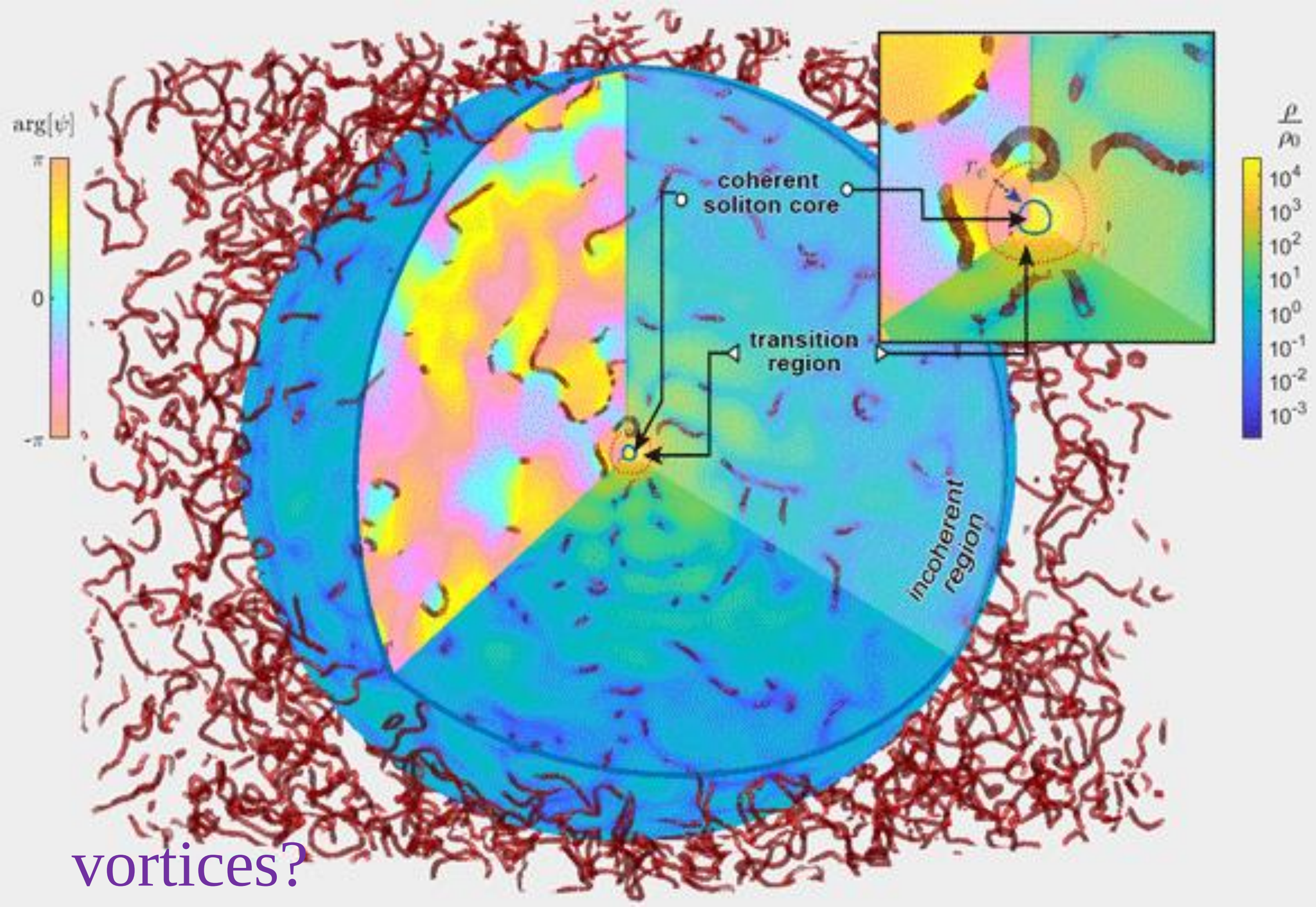
Schive etal , Nature physics 2014



core size ~ granule size~ typical length

AMR (GAMER) with GPU cluster

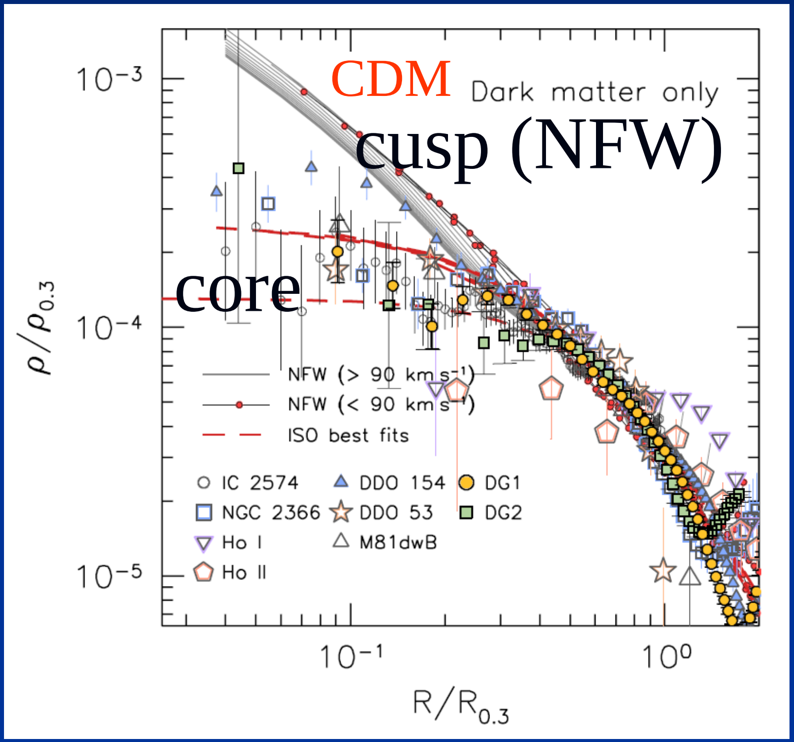
Schive etal , Nature physics 2014



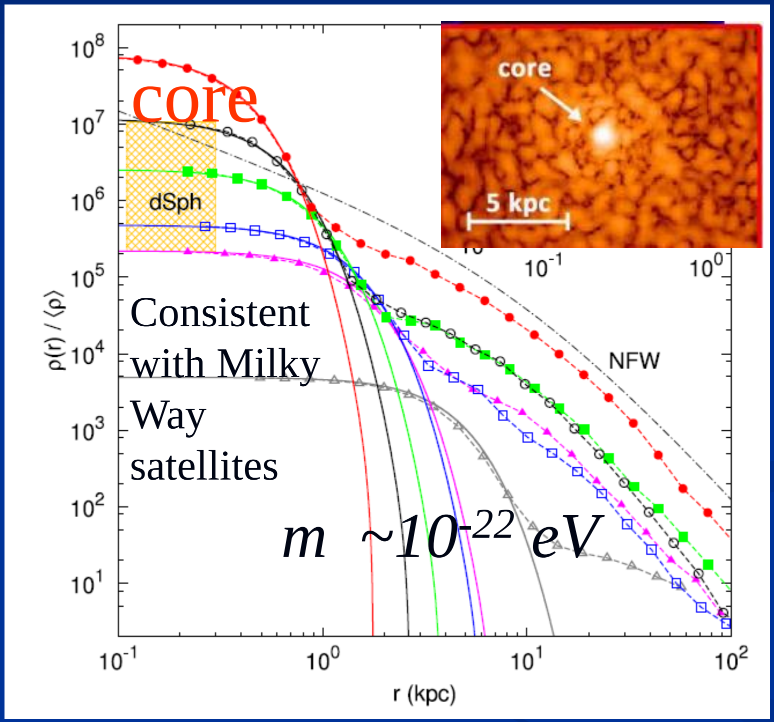
I-Kang Liu+ MNRAS

Core/Cusp problem of CDM

Observation



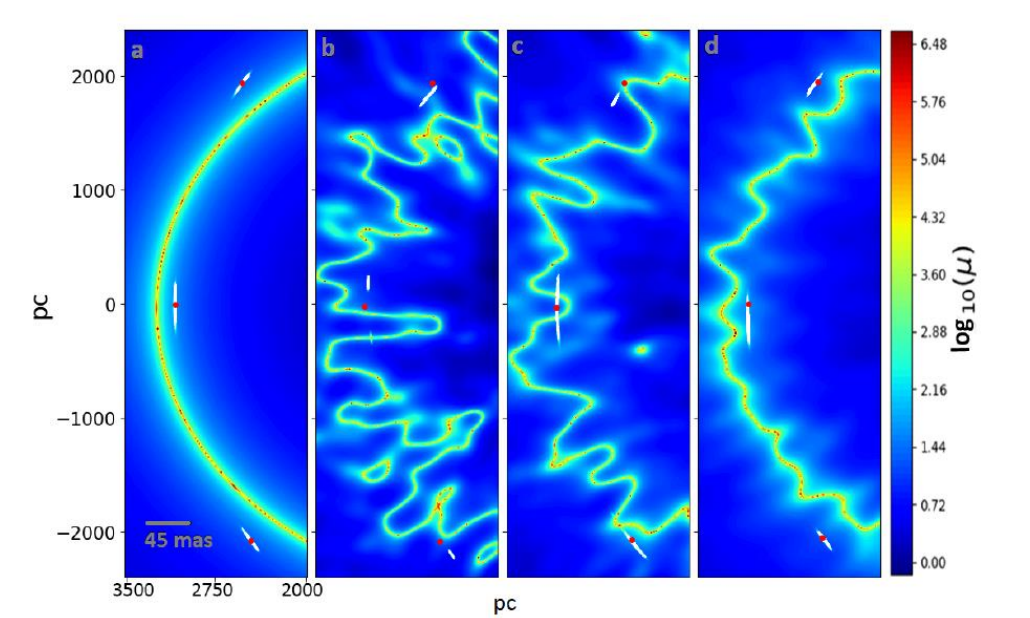
ULDM simulation→
Core ~ de Broigle wave len.



Schive etal , Nature physics 2014

Density profile from rotation curves of small galaxies strongly disfavors CDM

→ ULDM well explains the core profile!

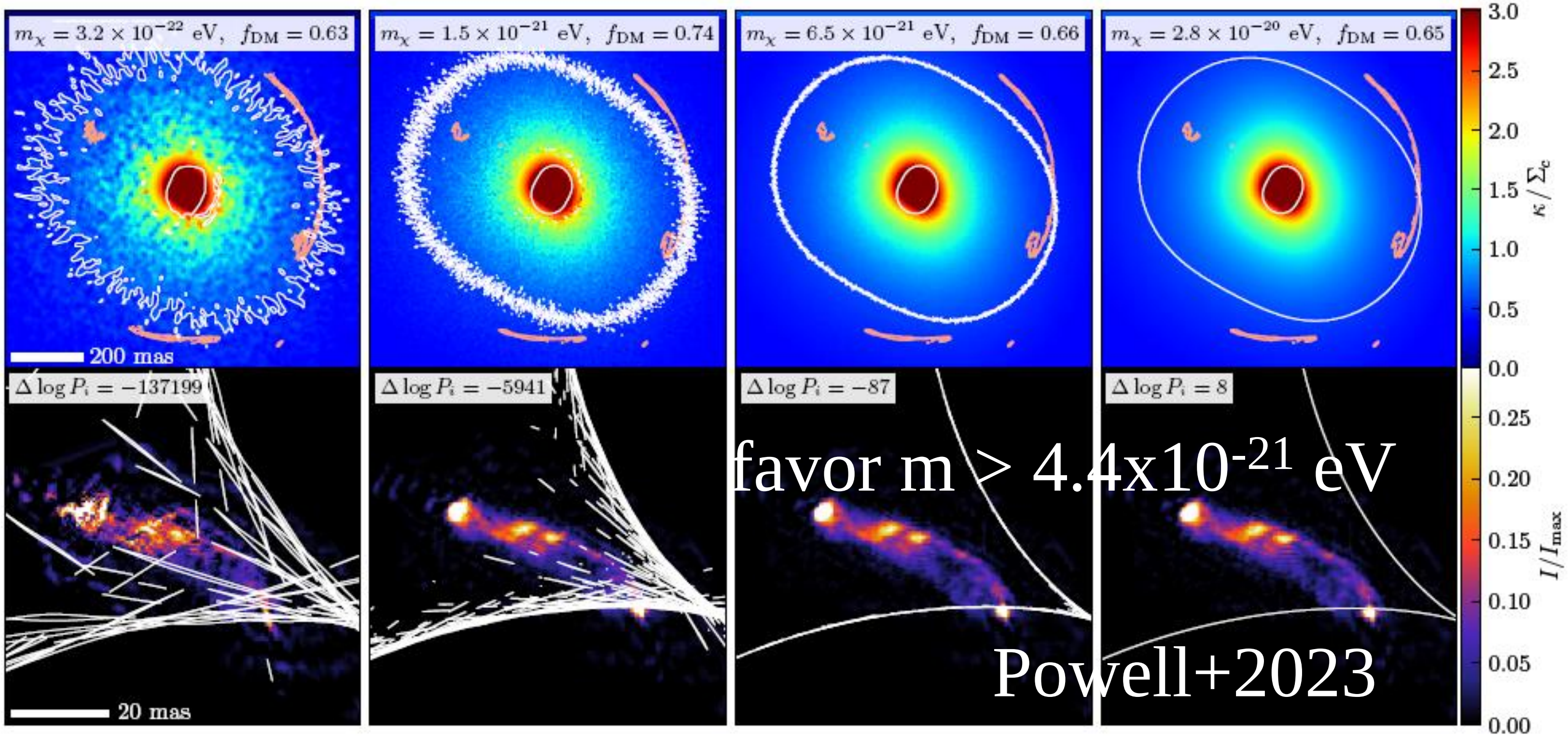


ULDM well reproduce lens of radio objects

Armurth+2023

Small m → more fuzzy

Smoking gun?



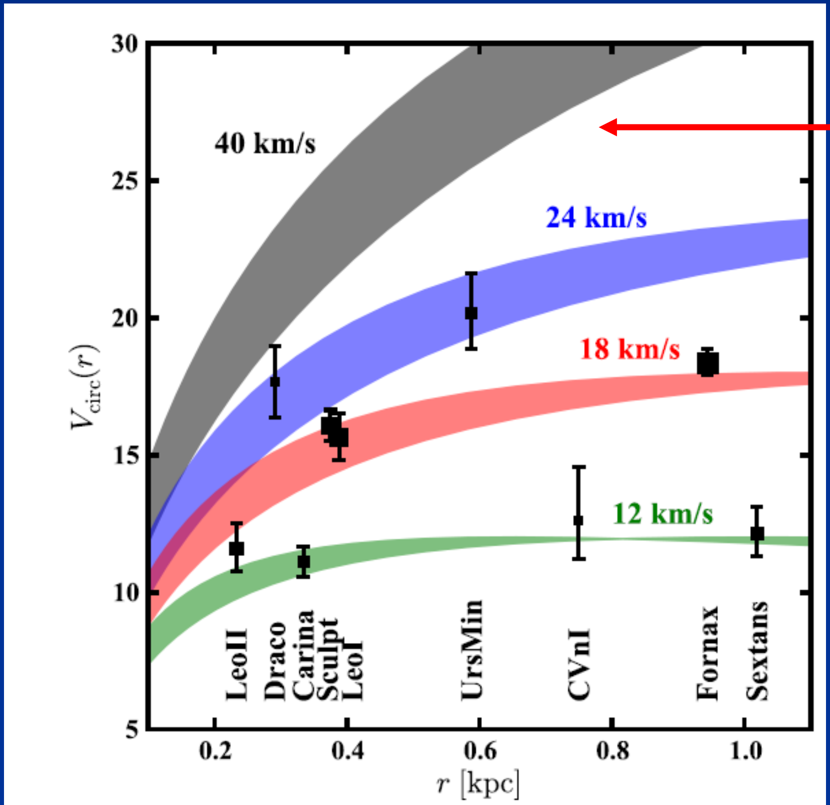
favor m > 4.4×10^{-21} eV

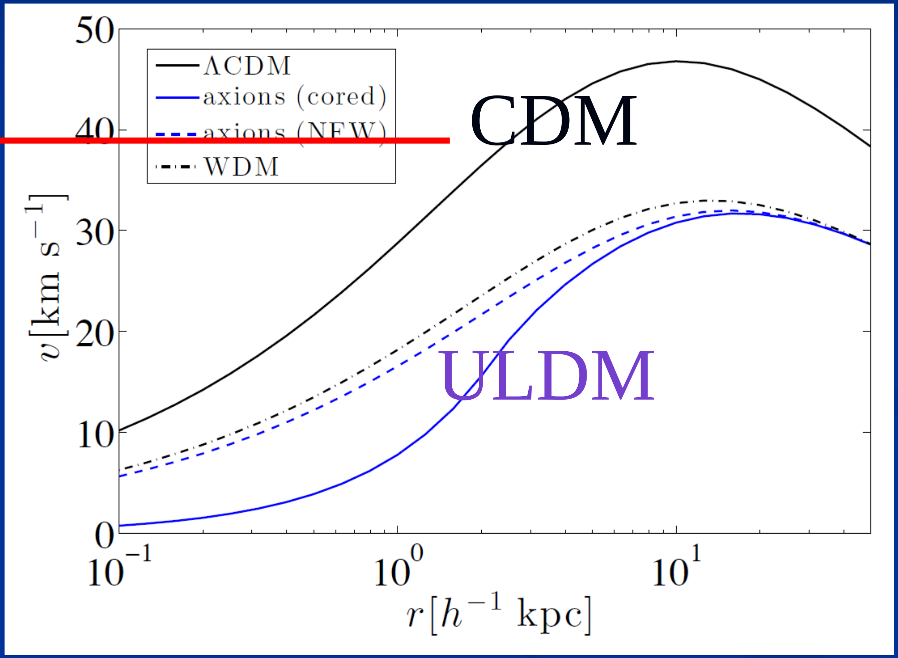
Powell+2023

Too big to fail= no dense satellite

Absence of bright satellites

Where are these bright satellites?





Marsh & Silk 2013

Boylan etal 2012

Density pert. Of ULDM

ULDM has only 2 parameters m and density ρ_0

a=scale factor

Madelung representation

$$i\hbar\left(\frac{\partial\psi}{\partial t}+\frac{3}{2}H\psi\right)=-\frac{\hbar^2}{2ma^2}\Delta\psi+mV\psi+\text{self. int.}$$

$$perturbation \quad \text{with } \psi=\sqrt{\rho}e^{iS}, \qquad v\equiv\frac{\hbar}{ma}\nabla S\Rightarrow$$

$$\begin{cases}\partial_t\rho+3H\rho+\frac{1}{a}\nabla\cdot(\rho v)=0\\ \partial_t v+\frac{1}{a}v\cdot\nabla v+Hv+\frac{1}{\rho a}\nabla p+\frac{1}{a}\nabla V+\frac{\hbar^2}{2m^2a^3}\nabla\left(\frac{\Delta\sqrt{\rho}}{\sqrt{\rho}}\right)=0\end{cases}$$

Quantum Pressure

Density contrast (k space)

$$\text{perturbation } \delta=\delta_k=\delta\rho/\rho_0$$

$$\Rightarrow\partial_t^2\delta+2H\partial_t\delta+\left(\left(\frac{\hbar^2k^2}{4m^2a^2}+c_s^2\right)\frac{k^2}{a^2}-4\pi G\rho_0\right)\delta=0$$

Quantum Jeans length $\lambda_J=\frac{2\pi}{k_J}a=\pi^{3/4}\hbar^{1/2}(G\rho_0 m^2)^{-1/4}\propto 1/\sqrt{mH}$

- CDM-like on super-galactic scale (for a small $k < k_J$)
- Suppress sub-galactic structure (for a large $k > k_J$)

Typical scales of ULDM 2310.01442

is a function of $\frac{\hbar}{m} = 0.019 \times \left(\frac{10^{-22} eV}{m}\right) pc^2/year$

1) time

$t_c \simeq (G\bar{\rho})^{-1/2}$: Hubble time

2) length $\lambda_{dB} = O(\lambda_{QJ})$

$$x_c = \lambda_{dB} = \left(\frac{\hbar}{m}\right)^2 \frac{1}{GM} = 854.8\, pc \left(\frac{10^{-22} eV}{m}\right)^2 \frac{10^8 M_\odot}{M} = \sqrt{\frac{\hbar}{m}} (G\bar{\rho})^{-1/4}$$

3) velocity

$$v_c \equiv x_c/t_c = GM\, m/\hbar = 22.4\, km/s \left(\frac{M}{10^8 M_\odot}\right)\left(\frac{m}{10^{-22} eV}\right) \simeq \sqrt{\frac{\hbar}{m}} (G\bar{\rho})^{1/4},$$

4) Angular momentum

$$L_c = M x_c v_c = \hbar \frac{M}{m} = N\hbar$$

L eigenstates?

$$= 1.1 \times 10^{96} \hbar \left(\frac{M}{10^8 M_\odot}\right)\left(\frac{10^{-22} eV}{m}\right) \simeq \frac{\left(\frac{\hbar}{m}\right)^{5/2} \bar{\rho}^{1/4}}{G^{3/4}}$$

5) acceleration

$$a_c = x_c/t_c^2 = G^3 m^4 M^3/\hbar^4$$

$$= 1.9 \times 10^{-11} meter/s^2 \left(\frac{m}{10^{-22} eV}\right)^4 \left(\frac{M}{10^8 M_\odot}\right)^3 \simeq \sqrt{\frac{\hbar}{m}} (G\bar{\rho})^{3/4}$$

cf) MOND scale

$$a_0 = 1.2 \times 10^{-10} meter/s^2$$

6) potential

$$V_c = \frac{m^2}{\hbar^2}(4\pi G M)^2 = 8.8 \times 10^{-7} c^2 \sim \left(\frac{m}{10^{-22} eV}\right)^2 \left(\frac{M}{10^8 M_\odot}\right)^2$$

$$\psi_c = \frac{m^3}{\hbar^3}(GM)^{\frac{3}{2}} = 4 \times 10^{-5} pc^{-3/2} \left(\frac{m}{10^{-22} eV}\right)^3 \left(\frac{M}{10^8 M_\odot}\right)^{3/2}$$

$$\simeq \left(\frac{\hbar}{m}\right)^{-3/4} (G\bar{\rho})^{3/8}$$

$$\int |\psi|^2 \, d^3x = 1$$

density

$$\rho_c = \frac{G^3 m^6 M^4}{\hbar^6} = 0.16\, M_\odot / p\, c^3 \left(\frac{m}{10^{-22} eV}\right)^6 \left(\frac{M}{10^8 M_\odot}\right)^4$$

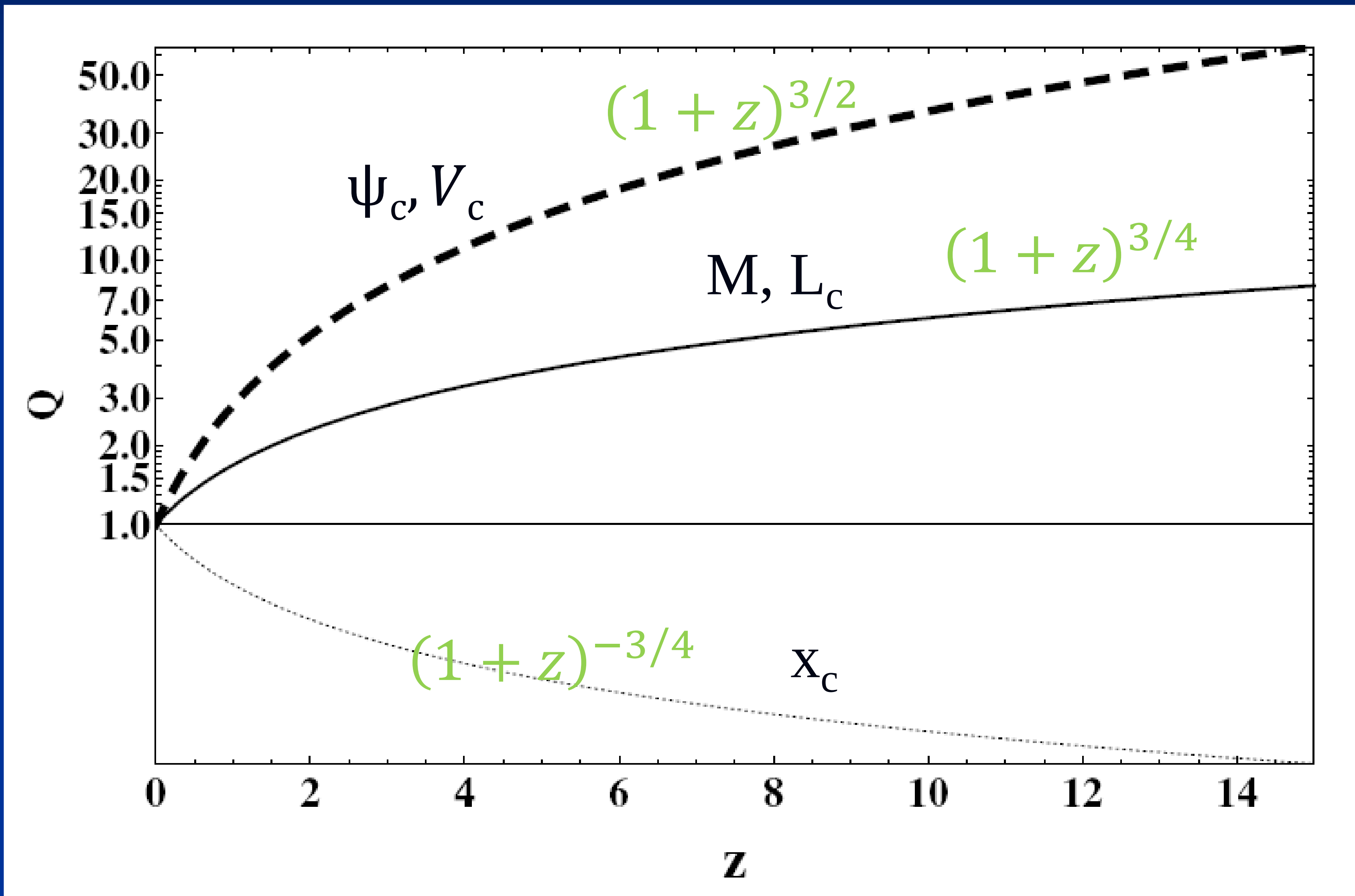
flux

$$J_c = \frac{\hbar}{m} Im(\psi_c \nabla \psi_c^*) \simeq \frac{\hbar}{m} \frac{\psi_c^2}{x_c} = \frac{G^4 m^7 M^4}{\hbar^7} \simeq \frac{Gm\bar{\rho}}{\hbar}$$

mass flux

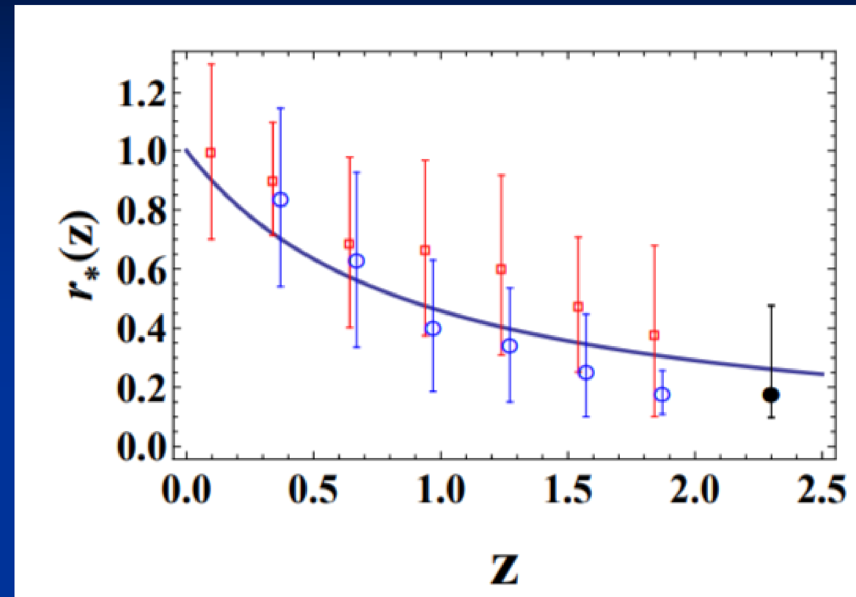
$$MJ_c = 3.66 \times 10^{-6} M_\odot / pc^2 / year \left(\frac{m}{10^{-22} eV}\right)^7 \left(\frac{M}{10^8 M_\odot}\right)^5$$

time evolution of scales

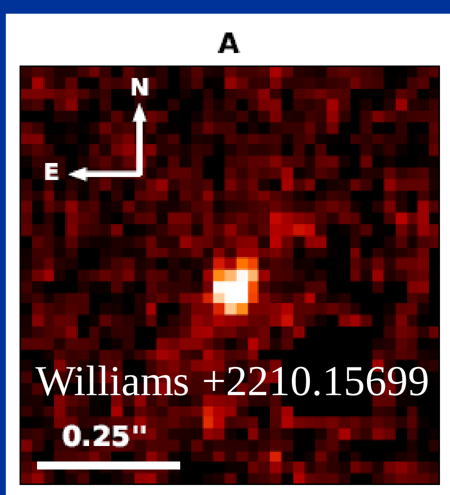


Size evolution

JWL PLB 2009



James Webb found massive compact early galaxies at z= 7~13



$$\text{length}\,\xi\,(z) \approx \frac{\hbar^2}{GM_J m^2} \approx \frac{\hbar^{\frac{1}{2}}}{(Gm\,^2\rho_d(z))^{\frac{1}{4}}}$$

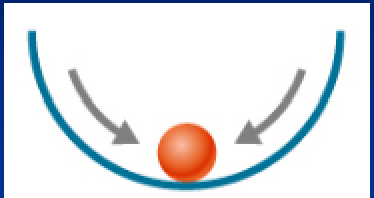
visible size→ $(1+z)^{-1.125}$

z = 9.51 ± 0.01 (510 million years)

radius =16.2−7.2+4.6 pc

$$\text{mass}\ \log(M_*/\mathrm{M}_\odot) = 7.63^{+0.22}_{-0.24}$$

Maximum scales (independent of time)



$$t_c = \frac{\hbar}{mc^2} = 0.208\, year \left(\frac{10^{-22} eV}{m}\right)$$

→ *can be detected by PTA*

$$\rho_c = \frac{c^4 m^2}{G\hbar^2} = 5.1 \times 10^{15}\, M_\odot / pc^3 \left(\frac{m}{10^{-22} eV}\right)^2$$

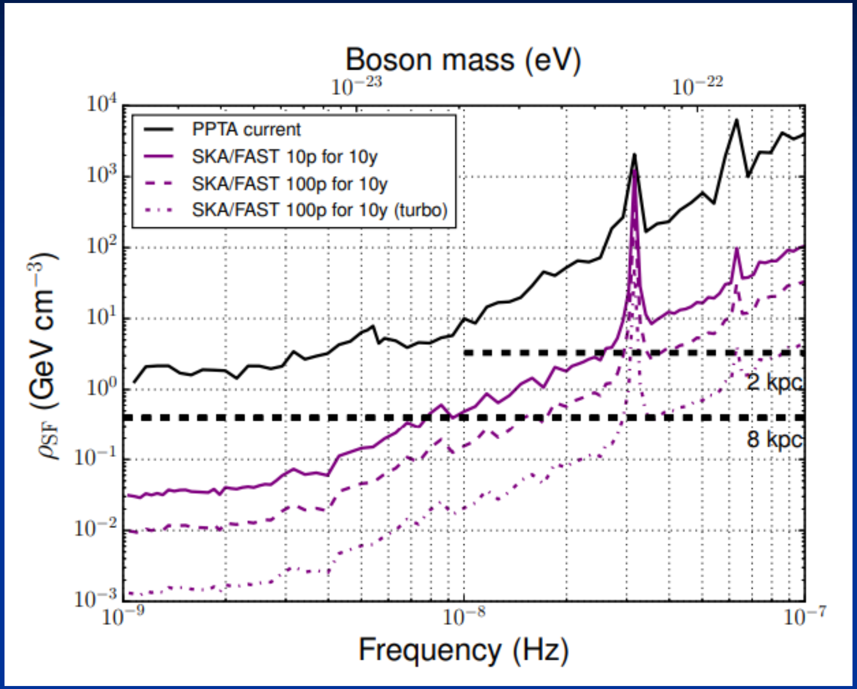
$$J_c = \frac{c^5 m^2}{4\pi G \hbar^2}$$

$$= 1.24 \times 10^{14}\, M_\odot / pc^2 / year \left(\frac{m}{10^{-22} eV}\right)^3,$$

$$a_c = c^3 m/\hbar = 45.5 meter/s^2 \left(\frac{m}{10^{-22} eV}\right)$$

max. acceleration ULDM structures can have!

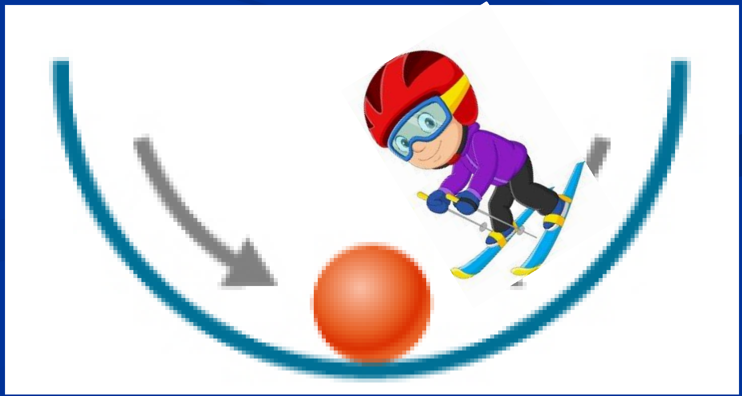
GW background detected by pulsar timing array



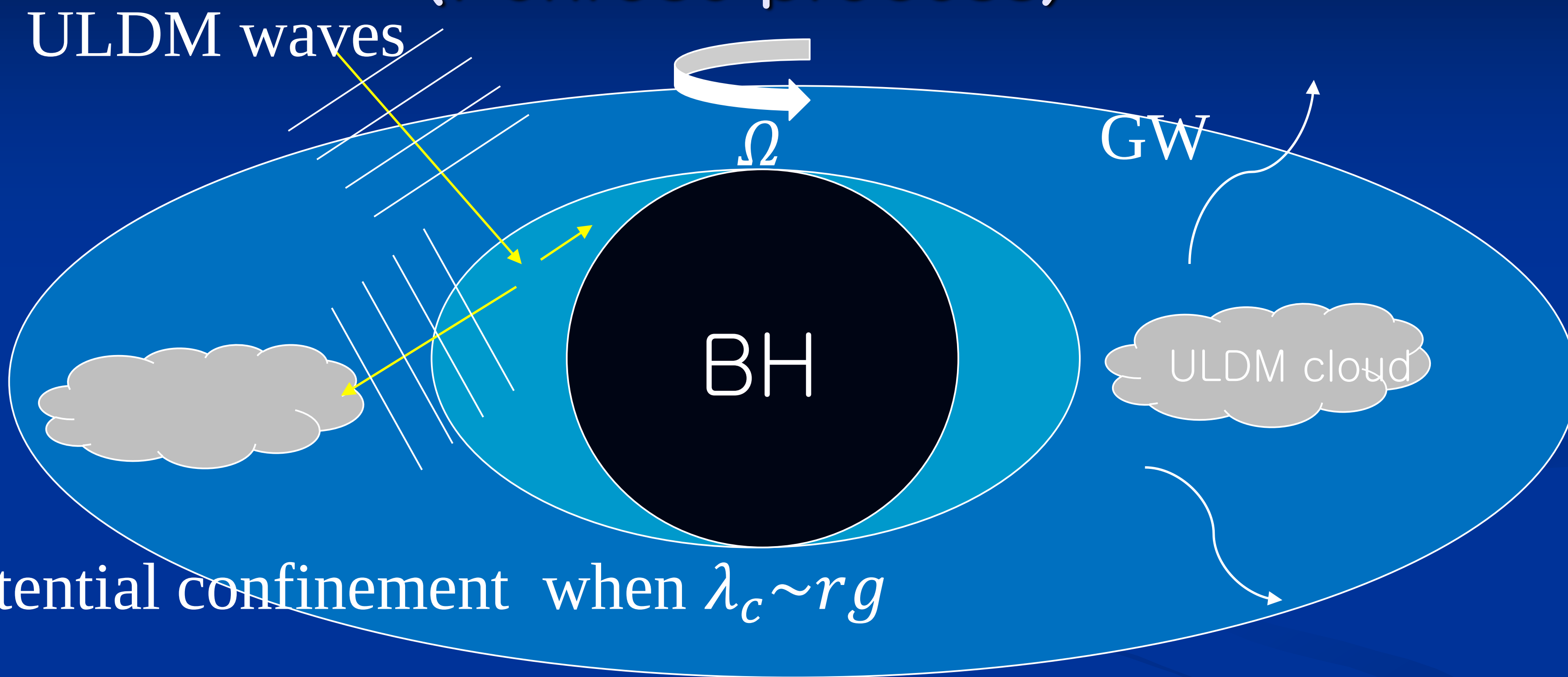
1810.03227

ULDM has
intrinsic osc time scale
1/m ~ yrs

$$\omega = \frac{1}{2.5 months}\frac{m}{10^{-22} eV}$$



Superradiance
(Penrose process)

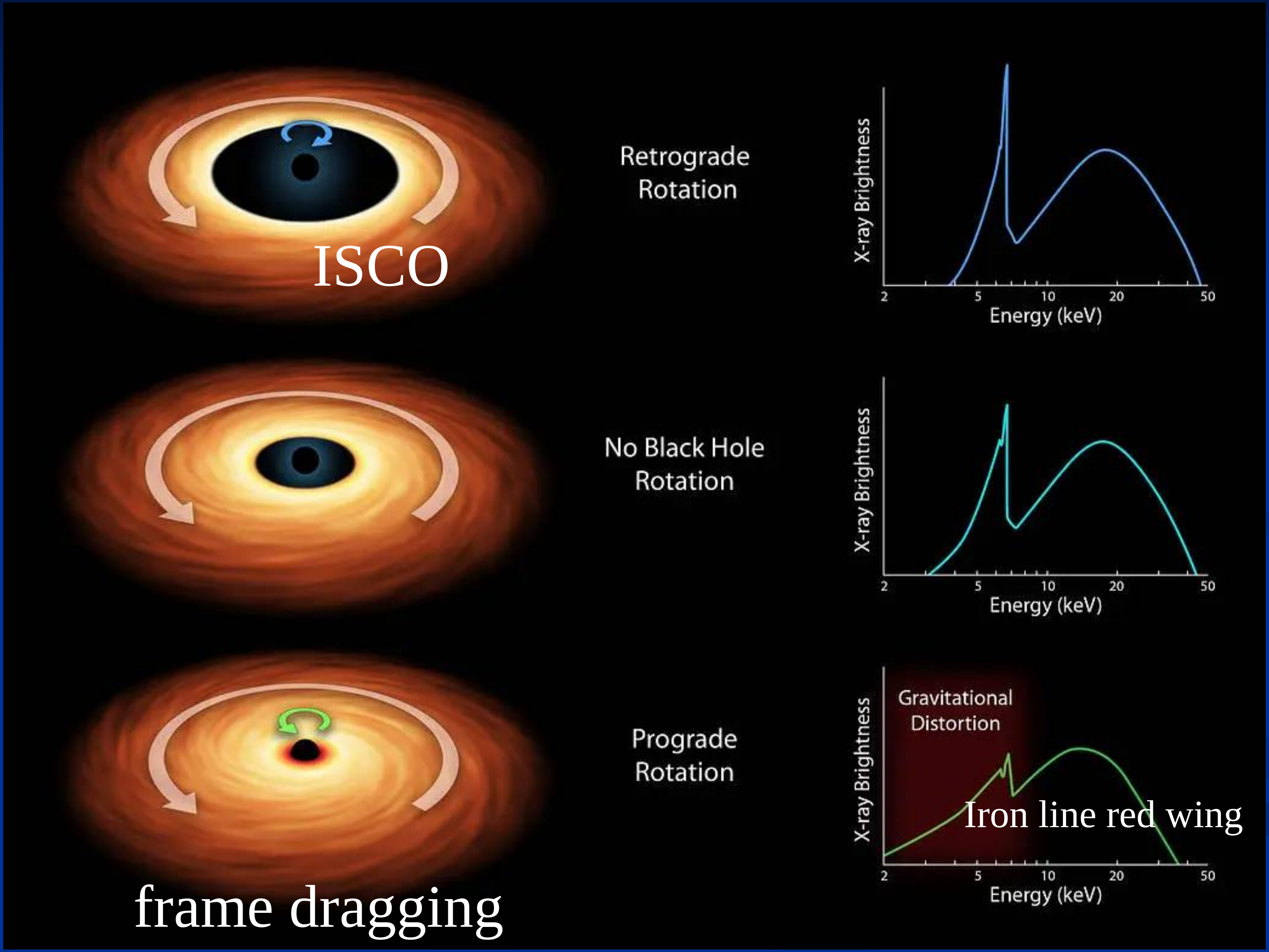


potential confinement when $\lambda_c \sim rg$

- growing mode if $\omega_{nlm} < m\Omega$ → BH spin decreases
  magnetic q. number
- can change GW patterns from a BH and BH binary

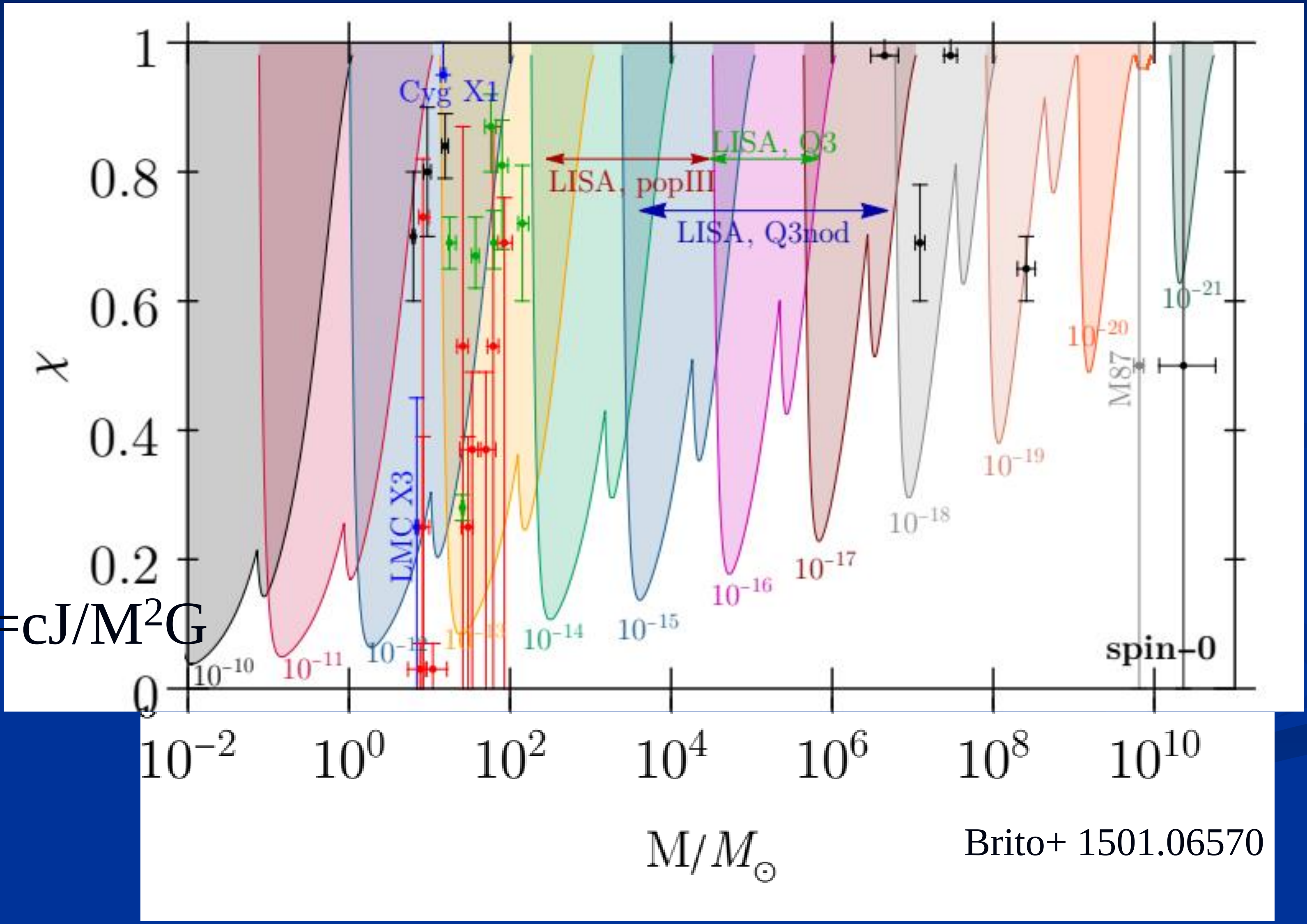
See Zhang & Yang 2018

How to measure spin of BHs



NASA

Bounds from BH-spin measurements (Regge plane)



$=cJ/M^2G$

Brito+ 1501.06570

Brief History of SFDM

JWL, Arxiv: 1704.05057

- 1983: Ruffini et al (m=10^{-24}eV), 1989: Membrado et al (ground state)
- 1992: Sin's BEC DM for halo (excited state, RC fitting m=3×10^{-23}eV)
- 1992, 1995: Lee & Koh Boson star (with self-interaction λ) model
- 2000: Fuzzy (Hu et al, λ=0)
  (suppress small scale→ m=10^{-22}eV), Core (Rioto),
  Satellite prob. (Matos), SFDM(Guzman)
- 2007: Bohmer & Harko, BEC DM in details
- 2008: Lee & Lim, min mass and size of galaxies → m=10^{-22}eV
- 2009: ULA: Mielke & Perez, Hwang & H. Noh, Sikivie & Yang
- 2010: Spiral arms (H. Bray), Tully-Fisher,
- 2010: Superfluid universe, Inflation, DE, DM, Kerson Huang et al
- 2013: Cosmological constraints, Bohua Li et al, and others
- 2014 : high precision structure formation simulation, Schive et al
- 2016: JEKim (pt. model) , Hui, Ostriker, Tremaine & E. Witten (review & pt. model)

Typical scales and dimensionless SPE

Quasi normal mode period

$$v_c = x_c/t_c = \sqrt{G\,M/x_c}$$
$$= \frac{\hbar}{m x_c}$$

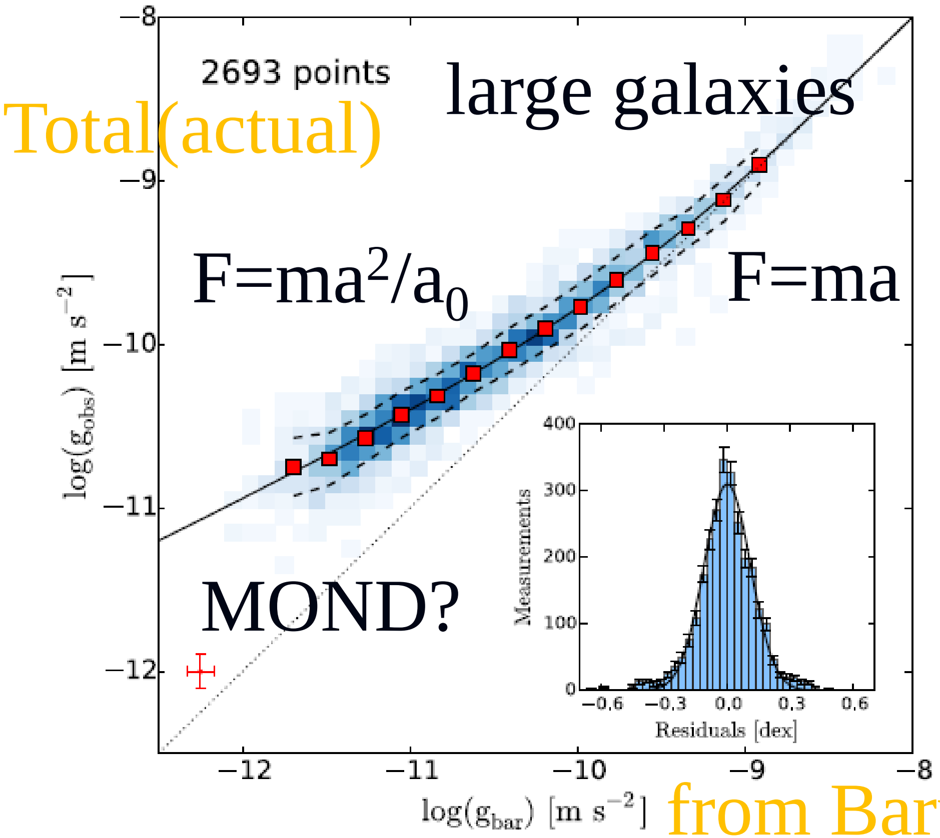
These scales are typical scales of galaxies

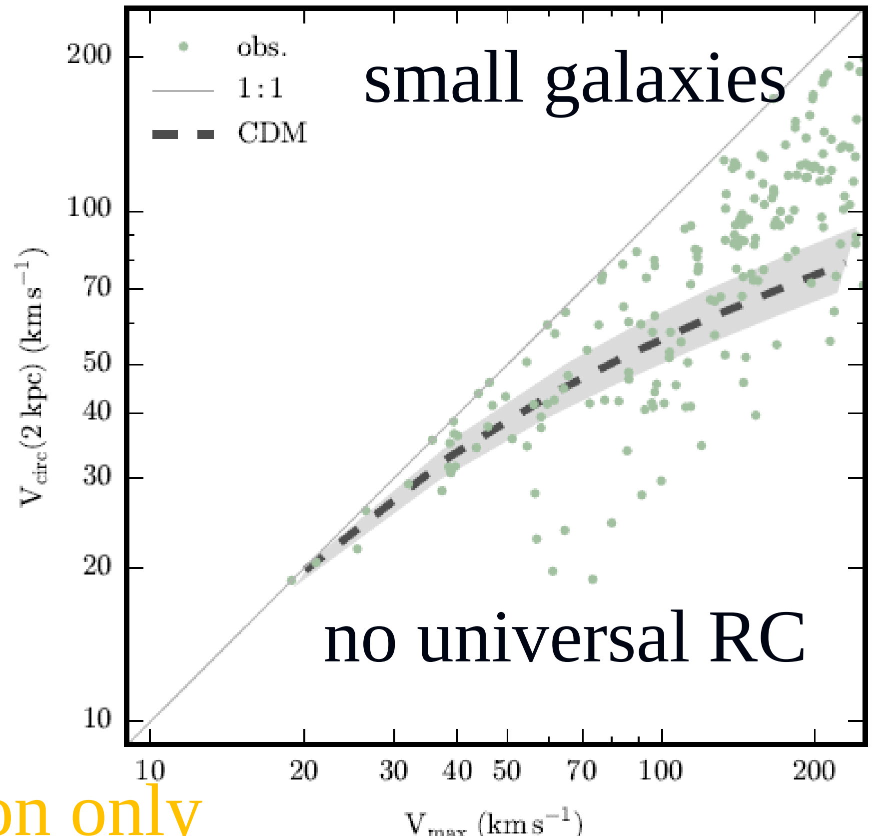
$$t \equiv t_c\,\hat{t} = \frac{\hbar^3}{m^3}\frac{1}{(GM)^2}\hat{t},$$
$$\mathrm{x} \equiv \mathrm{x}_c\,\hat{\mathrm{x}} = \frac{\hbar^2}{m^2}\frac{1}{GM}\,\hat{\mathrm{x}},$$
$$\psi \equiv \psi_c\,\hat{\psi} = \frac{m^3}{\hbar^3}(GM)^{\frac{3}{2}}\,\hat{\psi},$$
$$V \equiv V_c\,\hat{V} = \frac{m^2}{\hbar^2}(4\pi GM)^2\,\hat{V},$$

dim. less SPE

$$i\,\partial_t\hat{\psi}(\hat{\mathrm{x}},\hat{t}) = -\frac{1}{2}\nabla^2\hat{\psi}(\hat{\mathrm{x}},\hat{t}) + \hat{V}(\hat{\mathrm{x}},\hat{t})\hat{\psi}(\hat{\mathrm{x}},\hat{t}),$$
$$\nabla^2\hat{V}(\hat{\mathrm{x}},\hat{t}) = 4\pi\,|\hat{\psi}|^2(\hat{\mathrm{x}},\hat{t}).$$

Regularity vs. Diversity.





$$a_0 = 1.2 \times 10^{-10}\, m/s^2$$

from ULDM scale? LKL PLB, 1901.00305

Numerical Methods for SP

1. Schrödinger

1) Pseudo-Spectral Solver using FFT

Fast & Easy

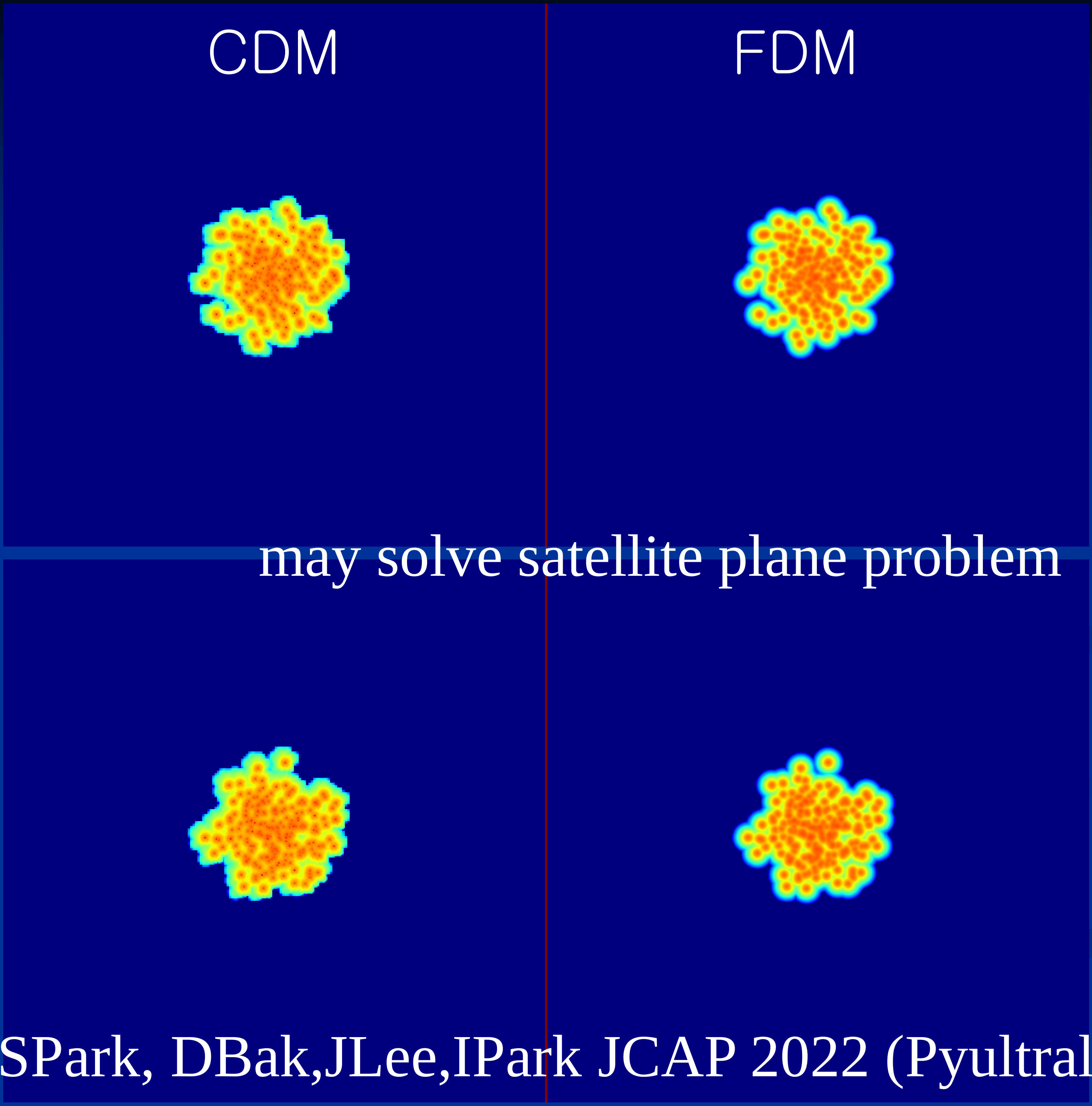
Periodic artifact & can't use AMR

$$\psi_n = \exp[iHdt]\psi_{n-1}$$
$$= (IFT)\exp[0.5i(|\vec{k}|^2/2m)dt](FT)\exp[-iV(x)dt](IFT)\exp[0.5i(|\vec{k}|^2/2m)dt](FT)\psi_{n-1}$$

2) Finite difference (RK4…)

2. Poisson $\nabla^2 \Phi = -4\pi\rho \rightarrow k^2 \Phi_k = -4\pi\rho_k$

using FFT

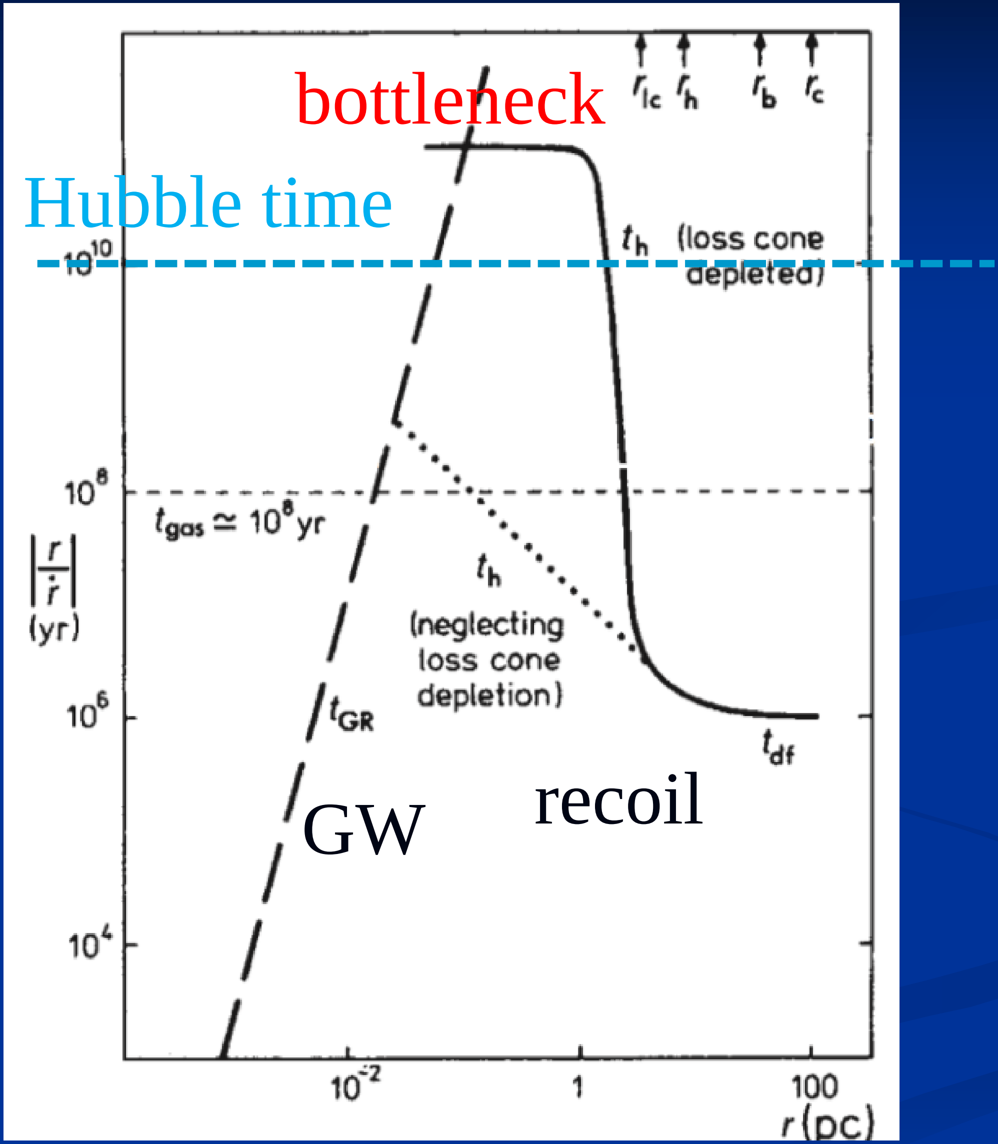


may solve satellite plane problem

시립대

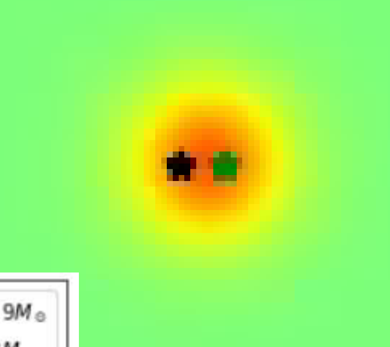
SPark, DBak,JLee,IPark JCAP 2022 (Pyultralight)

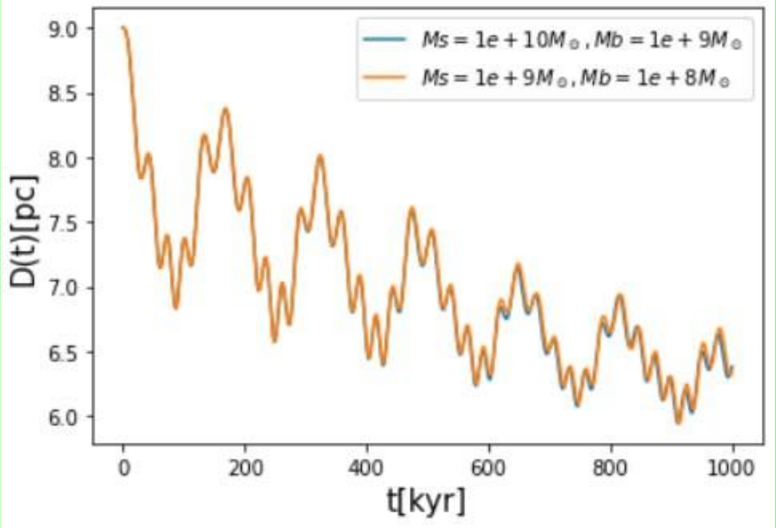
Final pc problem



Begelman+ (1980)

BH binary (Pyultralight)





may solve final pc problem
(Koo+ 2311.03412)

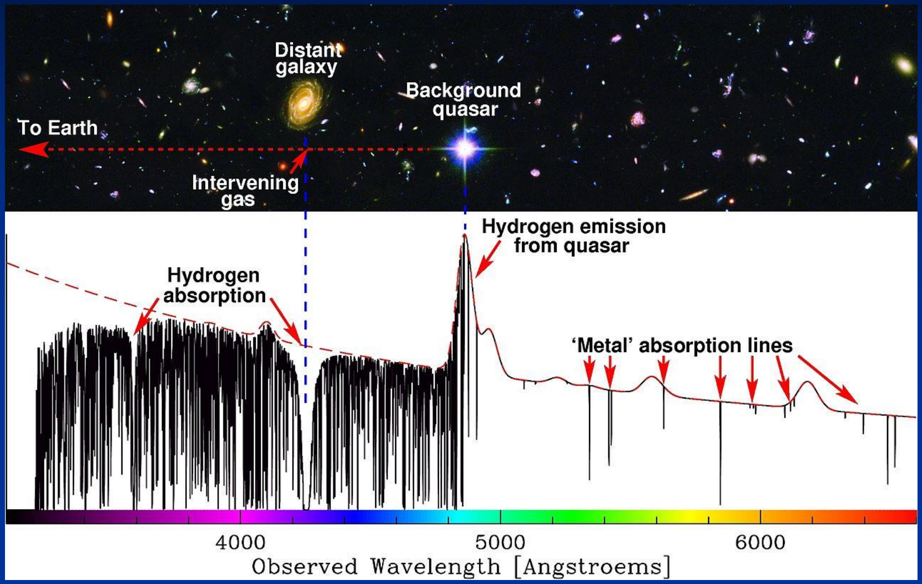
Other cosmological Constraints

- BEC phase transition before nucleosynthesis: $m < 10^{2}$ eV
- field oscillation before equality $m > 10^{-28}$eV
- Maximum mass of galaxies from BS theory

  spiral $1.04 \times 10^{12} M_s < O(1)\, M_p^2/m \rightarrow m < O(1)\ 1.28 \times 10^{-22}$eV

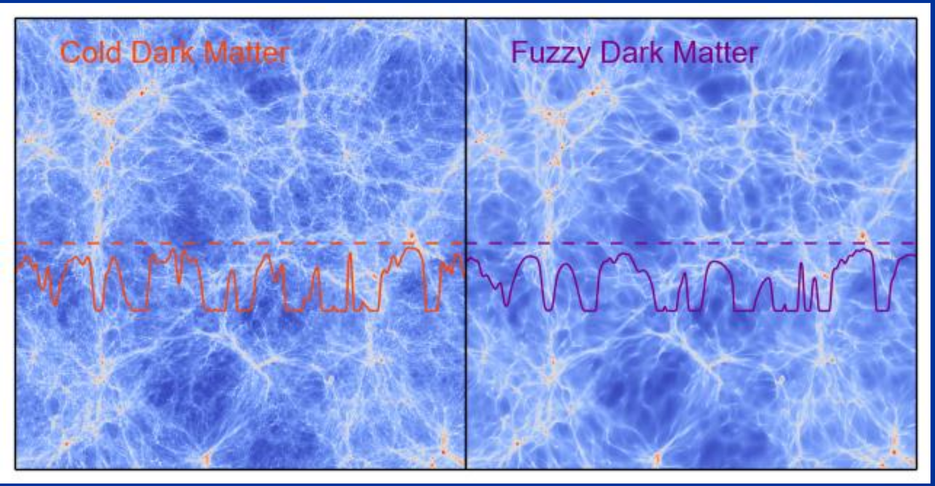
  elliptical $1 \times 10^{13} M_s < O(1)\, M_p^2/m \rightarrow m < O(1)\ 1.28 \times 10^{-23}$eV
- Lyα forest $m > 10^{-21}$eV
- high-redshift galaxy luminosity $\rightarrow m > 1.2 \times 10^{-22}$eV
- Stella subpopulations in Fornax $\rightarrow m < 1.1 \times 10^{-22}$eV
- Ultra-faint dSphs $\rightarrow m \sim 3.7\text{-}5.6 \times 10^{-22}$eV

Lyman alpha tension?



$m > 10^{-21}eV$

PRL 2017 (Irisic et al)

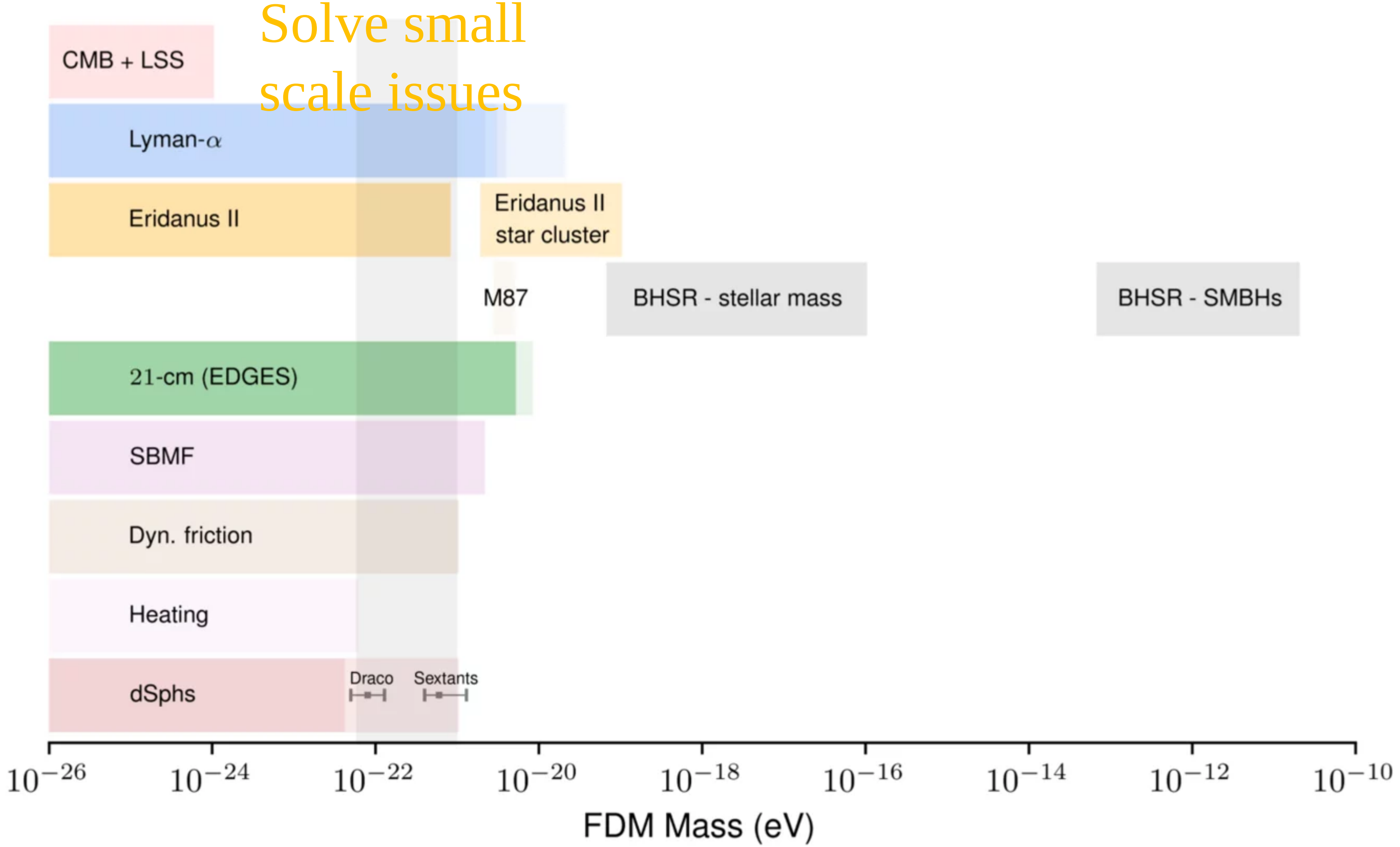


Hydrosimulation uncertainty is large

WDM (1, 3.3) keV ~ FDM (1, 20) x 10^{-22}eV

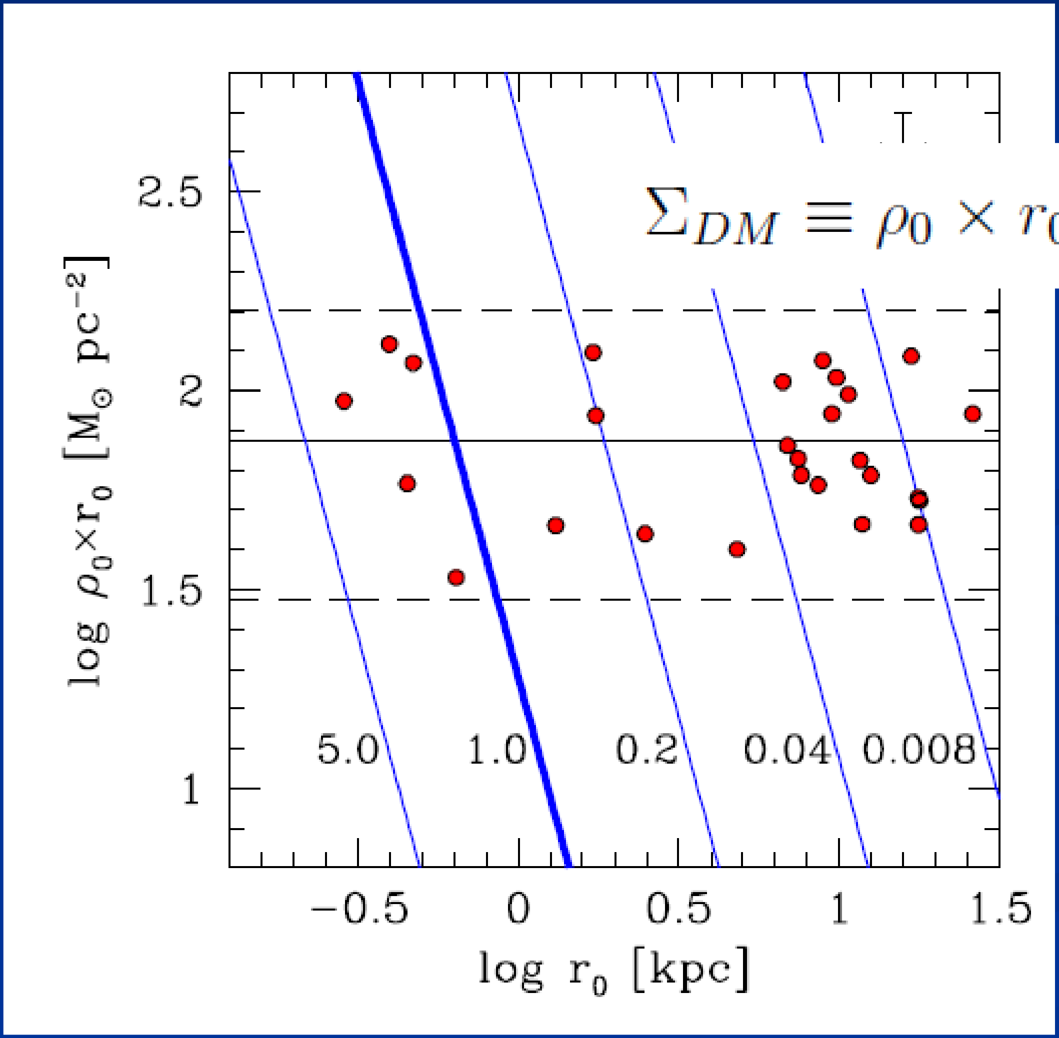
Constraints on FDM mass

Solve small scale issues



favors $m > 10^{-21}$ eV?

universal surface density



$$\Sigma_{DM} \equiv \rho_0 \times r_0 = 75^{+55}_{-45} M_\odot pc^{-2}$$

Burkert

Merits of studying self-interacting ULDM

We can

- allow wider mass range

→avoid some tensions of FDM

- study direct, or indirect detection of ULDM
- calculate abundance
- understand particle model

Interacting ULDM

Lee and Koh (PRD 53, 2236,1996, hep-ph/9507385)

Galactic DM halo is a big boson star
→ galactic DM is described by coherent scalar field

Action

$$S=\int\sqrt{-g}d^4x\left[\frac{-R}{16\pi G}-\frac{g^{\mu\nu}}{2}\phi^*_{;\mu}\phi_{;\nu}-\frac{m^2}{2}|\phi|^2-\frac{\lambda}{4}|\phi|^4\right]$$

typical phi4 theory with gravity

Metric

$$ds^2=-B(r)dt^2+A(r)dr^2+r^2d\Omega$$

Spherical.

Field

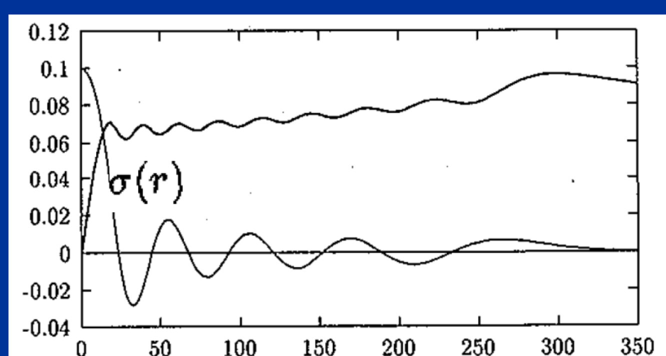
$$\phi(r,t)=(4\pi G)^{-\frac{1}{2}}\sigma(r)e^{-i\omega t}$$

Stationary spherical

Einstein-KGE

$$\begin{cases}\frac{A'}{A^2x}+\frac{1}{x^2}\left[1-\frac{1}{A}\right]=\left[\frac{\Omega^2}{B}+1\right]\sigma^2+\frac{\Lambda}{2}\sigma^4+\frac{\sigma'^2}{A},\\ \frac{B'}{ABx}-\frac{1}{x^2}\left[1-\frac{1}{A}\right]=\left[\frac{\Omega^2}{B}-1\right]\sigma^2-\frac{\Lambda}{2}\sigma^4+\frac{\sigma'^2}{A},\\ \sigma''+\left[\frac{2}{x}+\frac{B'}{2B}-\frac{A'}{2A}\right]\sigma'+A\left[\left(\frac{\Omega^2}{B}-1\right)\sigma-\Lambda\sigma^3\right]=0,\end{cases}$$

Even tiny self-interaction changes the scales drastically!



Gives similar rotation curves as Sin's BEC model for weak gravity

$$v_{rot}=\sqrt{\frac{xB'(x)}{2B(x)}}$$

Density pert. Of ULDM

ULDM has only 2 parameters m and density ρ_0

a=scale factor

$$i\hbar\left(\frac{\partial\psi}{\partial t}+\frac{3}{2}H\psi\right)=-\frac{\hbar^2}{2ma^2}\Delta\psi+mV\psi+\text{self. int.}$$

Madelung representation

$$perturbation \quad \text{with } \psi=\sqrt{\rho}e^{iS}, \qquad v\equiv\frac{\hbar}{ma}\nabla S\Rightarrow$$

$$\begin{cases}\partial_t\rho+3H\rho+\frac{1}{a}\nabla\cdot(\rho v)=0\\ \partial_t v+\frac{1}{a}v\cdot\nabla v+Hv+\frac{1}{\rho a}\nabla p+\frac{1}{a}\nabla V+\frac{\hbar^2}{2m^2a^3}\nabla\left(\frac{\Delta\sqrt{\rho}}{\sqrt{\rho}}\right)=0\end{cases}$$

Quantum Pressure

perturbation $\delta=\delta_k=\delta\rho/\rho_0$

Density contrast (k space)

$$\Rightarrow \partial_t^2\delta+2H\partial_t\delta+\left(\left(\frac{\hbar^2k^2}{4m^2a^2}+c_s^2\right)\frac{k^2}{a^2}-4\pi G\rho_0\right)\delta=0$$

Thomas-Fermi limit

TF limit

Lee and Koh (PRD 53, 2236,1996, hep-ph/9507385

$$For \quad \lambda \neq 0, \quad \Lambda >> 1 \quad \text{(TF limit)}$$

$$\nabla^2\sigma = \gamma\sigma,$$

$$\nabla^2\gamma = 2\sigma^2,$$

$$\gamma \equiv 1 - \frac{\Omega^2}{B}$$

Analytic sol. for ground field

$$\gamma = -\gamma_0 \frac{Sin(\sqrt{2}x_*)}{\sqrt{2}x_*},$$

$$\sigma_* = \sqrt{\frac{\gamma_0 Sin(\sqrt{2}x_*)}{\sqrt{2}x_*}},$$

$$x_* = x\Lambda^{-1/2}$$

$$\Lambda \equiv \frac{\lambda m_P^2}{4\pi m^2}$$

$$R \approx \sqrt{\Lambda}/m$$

$$M_{\max} = \sqrt{\Lambda}\frac{m_P^2}{m}$$

$$observed\ M < M_{\max}$$

$$\rightarrow \lambda^{1/2}\left(\frac{m_P}{m}\right)^2 \geq 10^{50}$$

Typical scales for self-int ULDM

$$\Lambda \equiv \frac{\lambda m_P^2}{4\pi m^2} >> 1 \quad \text{even for tiny m}$$

$$x \approx \sqrt{\Lambda}/m = \frac{\mathrm{m_p}\sqrt{\lambda}}{2m^2\sqrt{\pi}} = t$$

$$M \quad = \sqrt{\Lambda}\frac{m_P^2}{m} \quad = \frac{m_P^3\sqrt{\lambda}}{2m^2\sqrt{\pi}}$$

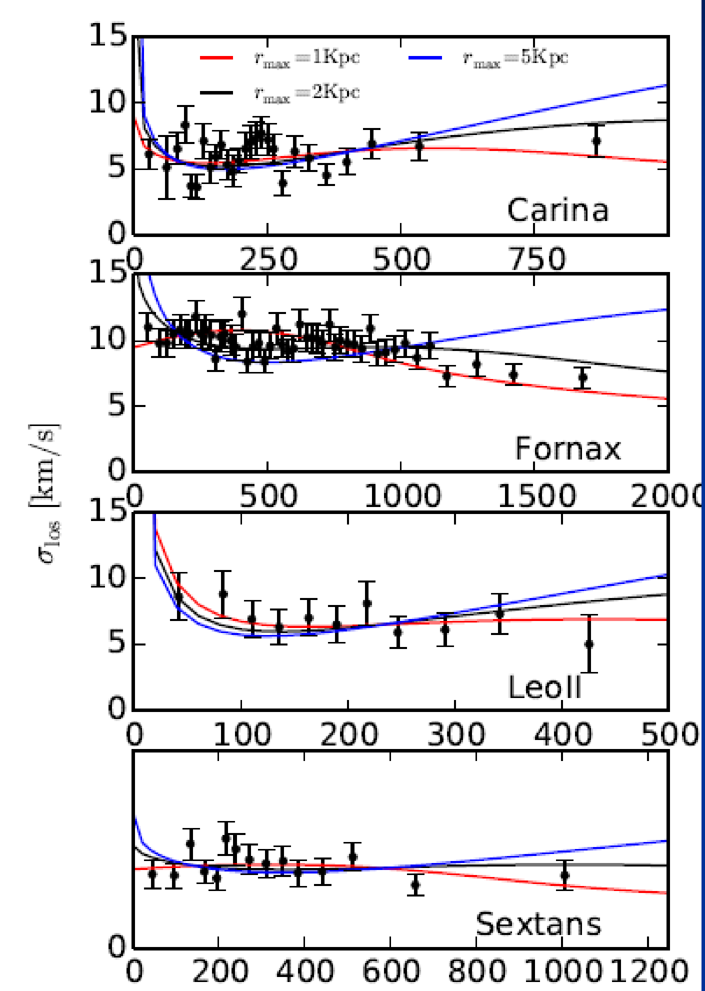
$$\rho_c = (m^2 Mp^2)/\Lambda = (4m^4\pi)/\lambda$$

$$\mathrm{a_c} = \frac{\mathrm{m}}{\sqrt{\Lambda}} = (2\mathrm{m}^2\sqrt{\pi})/\left(Mp\sqrt{\Lambda}\right)$$

$$L_c = \frac{Mp^2\Lambda}{m^2} = \frac{Mp^4\lambda}{4m^4\pi}$$

$$\psi_c = \mathrm{m}^3/\Lambda^{(3/2)} = \frac{8m^6\pi^{3/2}}{Mp^3\lambda^{3/2}}$$

Alberto Diez-Tejedor etal PRD 2014



$$\lambda^{1/2}\left(\frac{m_P}{m}\right)^2 \geq 10^{50} \rightarrow \left(\frac{m_P}{10^{25}}\right) \approx 1221 eV \geq \frac{m}{\lambda^{1/4}}$$

$$\rho = \sigma^2 = \rho_0 \frac{\sin(\pi r / r_{\max})}{\pi r / r_{\max}}$$

$$r_{\max} \equiv \sqrt{\frac{\pi^2 \Lambda}{2}} \simeq 49\left(\frac{\lambda^{1/4}}{m / eV}\right)^2 kpc$$

dSphs $\quad r_{\max} \sim 1 kpc \rightarrow m / \lambda^{1/4} \sim 7 eV$

some other constraints

1) perturbative $\lambda < 1$ & $\lambda^{1/2}\left(\frac{m_P}{m}\right)^2 \geq 10^{50}$

$\rightarrow m \leq 10^3 eV$

$\rightarrow m^4 < \lambda \times 2.2 \times 10^{12} eV^4 \rightarrow 10^{-100} < \lambda$

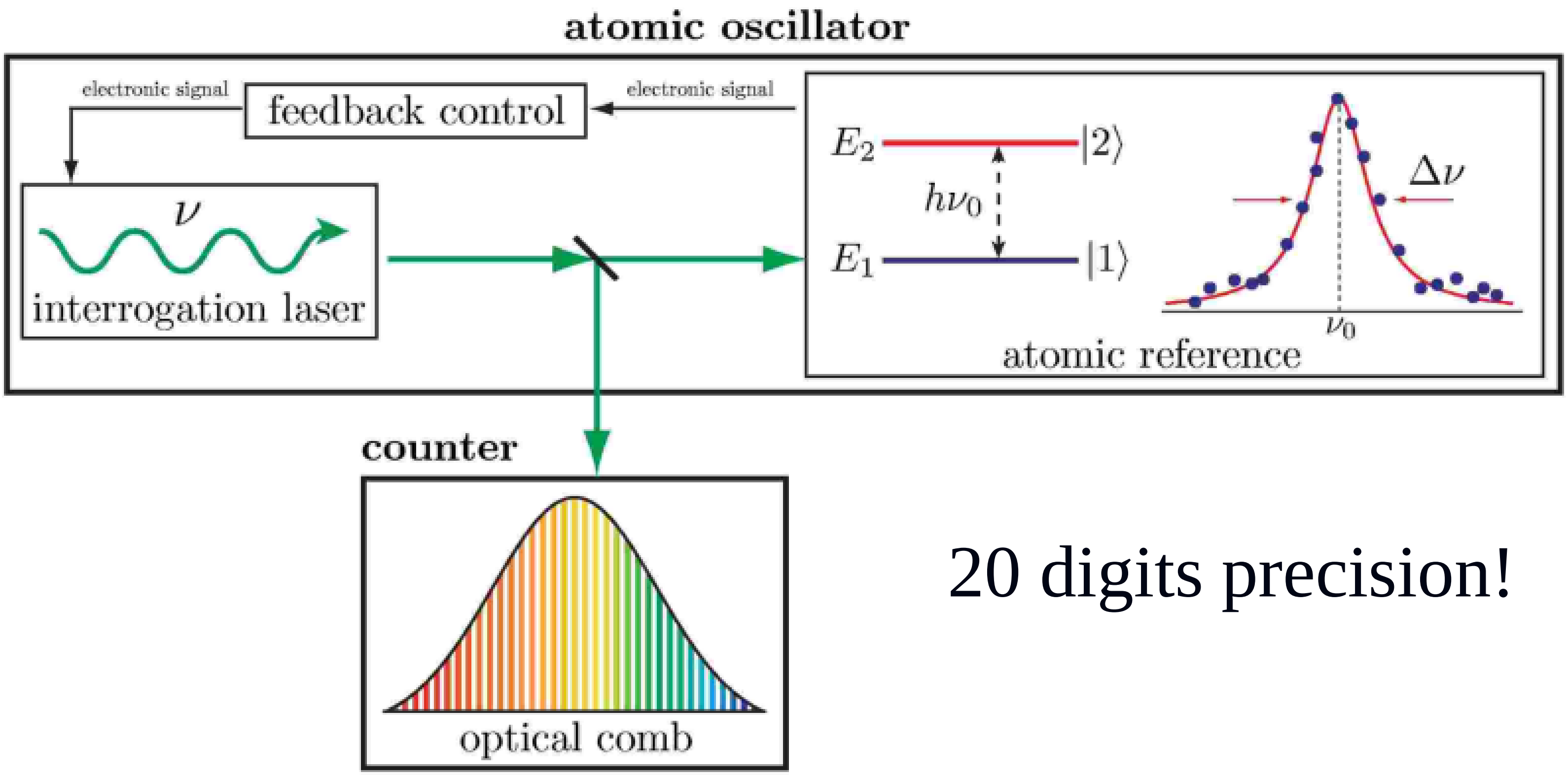
2) interparticle distance $<$ Compton wavelength

$\rightarrow m \leq 10^{-2} eV$

3) UMi & Fornax

$m^4/\lambda < 0.55 \times 10^3 eV^4$

detection by atomic clock



20 digits precision!

Fig. 2. – Schematic view of an optical atomic clock: the local oscillator (laser) is resonant with the atomic transition. A correction signal is derived from atomic spectroscopy that is fed back to the laser. An optical frequency synthesizer (optical frequency comb) is used to divide the optical frequency down to countable microwave or radio frequency signals.

arXiv:1401.2378v2

ULDM detection

ULDM makes coherent waves → if coupled to EM field
→ change in effective coupling constants (fine structure)
→ oscillation in frequency

sinusoidal modulation

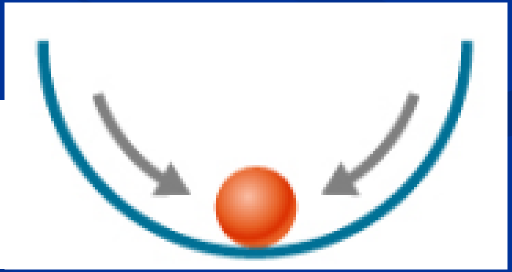
$$\frac{1}{4g^2}F_{\mu\nu}F^{\mu\nu}+\frac{\varphi d_e}{4g^2}F_{\mu\nu}F^{\mu\nu}$$

$$\alpha(t) \approx \alpha\ [1 + d_e\varphi_0 \cos(\omega t + \delta)]$$

$$\rho_\varphi = \frac{1}{2}m^2\varphi^2 = 0.3\mathrm{GeV/cm^3}$$

similar to 5th force

$$\omega = \frac{1}{2.5months}\frac{m}{10^{-22}eV}$$



Arvanitaki et al., PRD 91

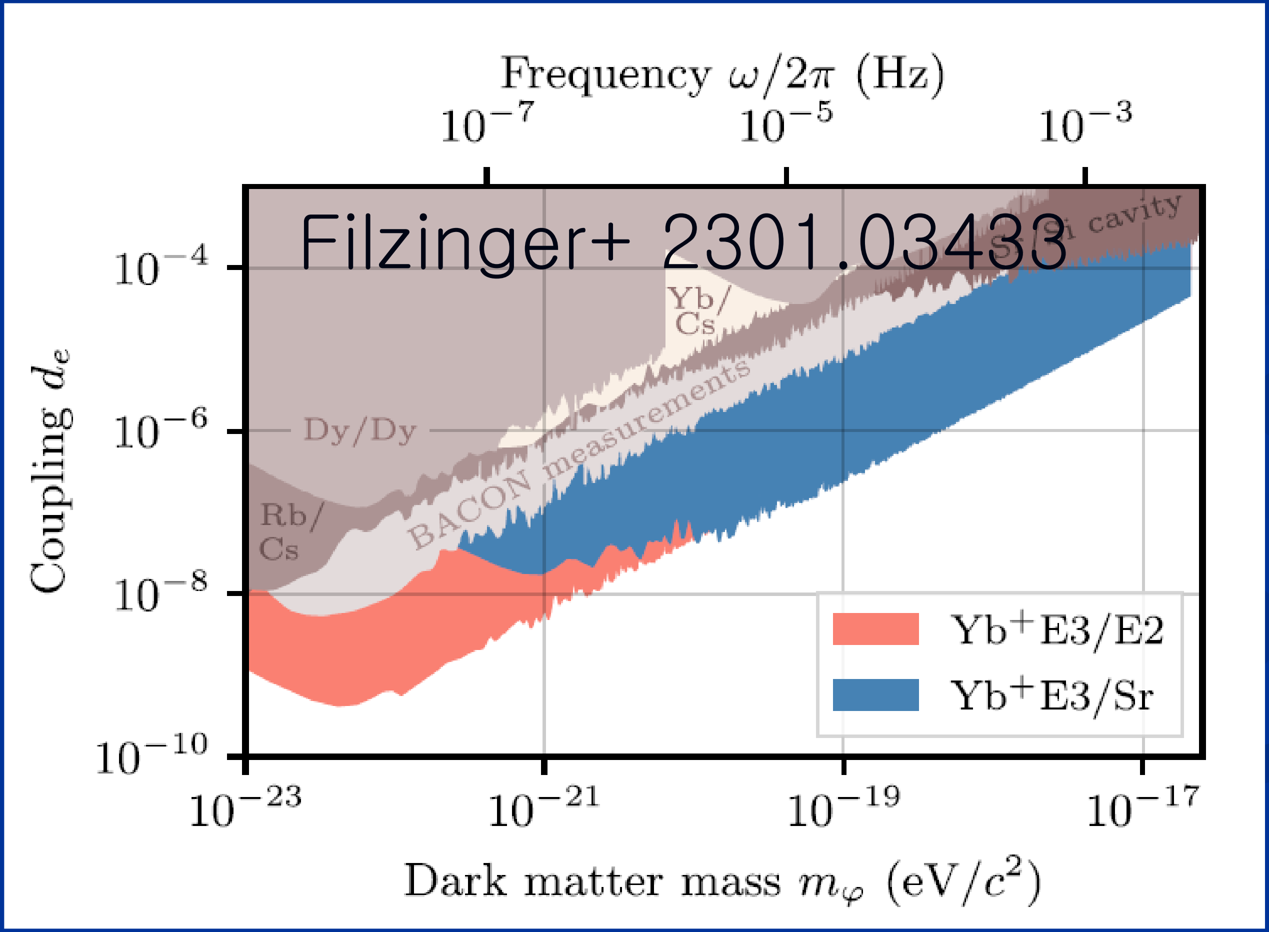
Questions

1. effective coupling (radiative correction)
2. oscillation frequency 1/m?
3. Oscillation really happens?

detection

Oscillation of fine structure constant

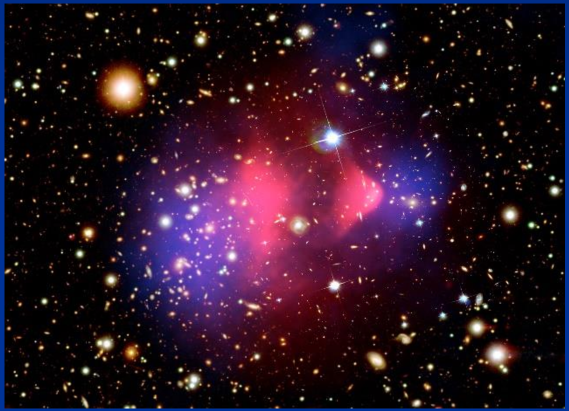
$$\alpha(t) \approx \alpha\left[1 + d_e \varphi_0 \cos(\omega t + \delta)\right]$$



Cross section

$$\sigma(\phi\phi \to \phi\phi) = \lambda^2/128\pi m^2$$

$$\frac{\sigma}{m} = \frac{\lambda^2}{128\pi m^3} \approx 1\, cm^2 g^{-1}$$ for galactic cluster



a cosmological constraint

$$if\ m \sim 10^{-22} eV \to \lambda \sim 10^{-92}$$

$$\frac{m}{\lambda^{1/4}} \sim 0.1\mathrm{eV}$$

One loop correction

if gauge interaction, $g A_\mu \phi$

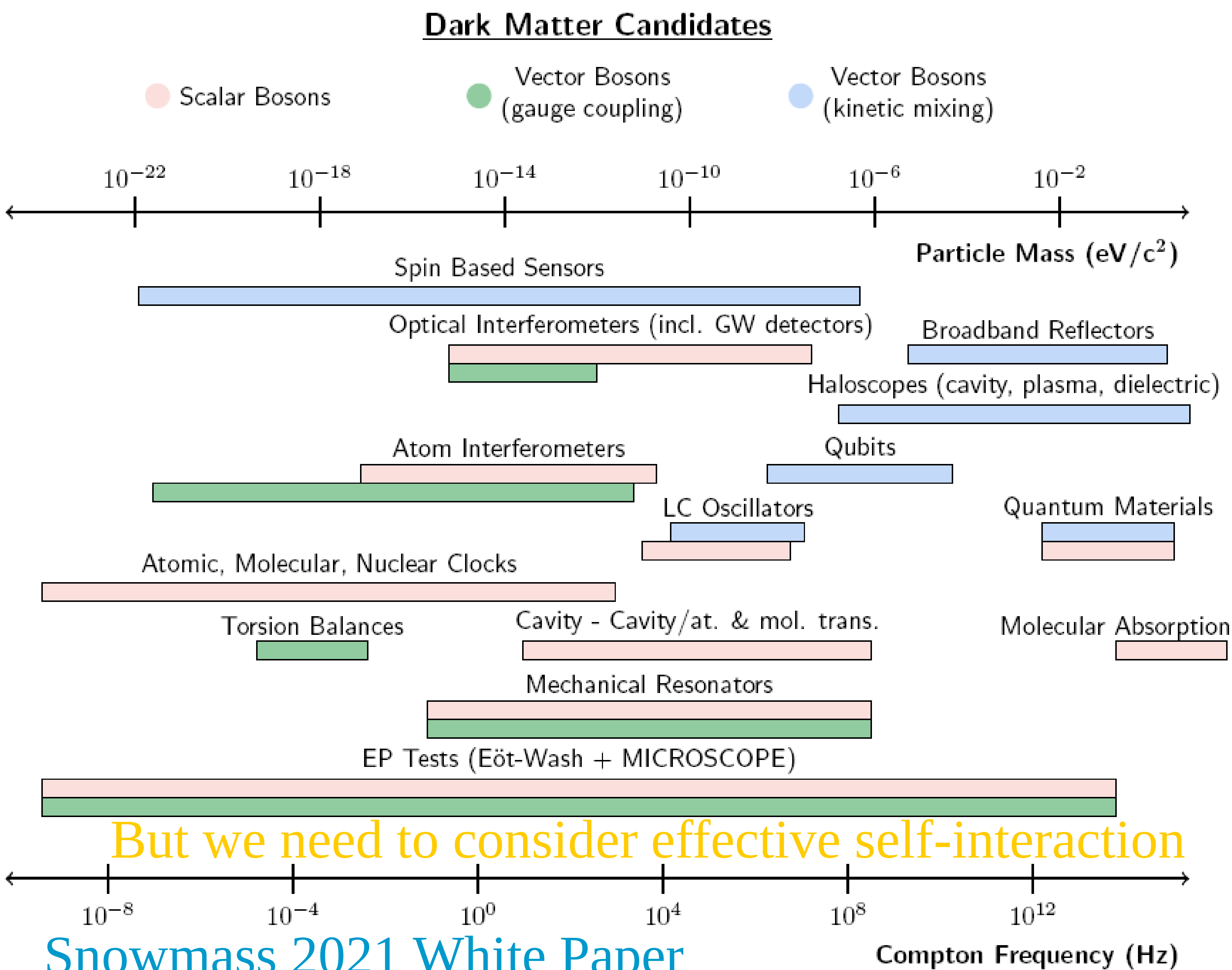
effective self–int. coupling $\lambda' \sim O(\frac{3g^4}{64\pi^2})$

$$\lambda' < \lambda_{obs} \Rightarrow g \leq 10^{-22.5} \Rightarrow d_e \sim g^2 \leq 10^{-45}$$

→ gauge coupling should be extremely small

→ hard to direct detect, if not impossible

in preparation



But we need to consider effective self-interaction

Snowmass 2021 White Paper

Conclusions

ULDM with m~10^{-22} eV or

self-interacting ULDM with $\frac{m}{\lambda^{1/4}} \sim 1 eV$

seems to be a viable alternative to CDM

→ direct detection is questionable

universal surface density

observation

$$\Sigma_{DM} \equiv \rho_0 \times r_0 = 75^{+55}_{-45} \mathrm{M_\odot pc^{-2}}$$

*$FDM\ Scaling\ law$

$(t, x, V, \psi, \rho) \rightarrow (k^{-2}t, k^{-1}x, k^2 V, k^2 \psi, k^4 \rho)$

$(M, E, v, L) \rightarrow (kM, k^3 E, kv, kL)$

*Surface density

$\Sigma \approx \mathrm{M}/\, x^2 \rightarrow k^3 \Sigma$ not scale invariant! → not universal

*ULDM in TF limit

$$\Sigma \approx \mathrm{M}/\, R^2 \approx \sqrt{\Lambda}\frac{m_P^2}{m}/(\sqrt{\Lambda}/m)^2 = \frac{m_P^2 m}{\sqrt{\Lambda}} \approx \frac{m_P\, m^2}{\sqrt{\lambda}} = \text{const.}$$

→ $\frac{m}{\lambda^{1/4}} \approx (75\ \mathrm{M_\odot\ pc^{-2}}/\ m_P\ )^{1/2} \approx 0.0005\mathrm{eV}$

JLee in preparation

Mass scale

1) $$M \simeq \bar{\rho} x_c^3 \simeq \bar{\rho} \left(\frac{\hbar}{m}\right)^6 \left(\frac{1}{GM}\right)^3$$

$$\simeq G^{-\frac{3}{4}} \left(\frac{\hbar}{\mathrm{m}}\right)^{3/2} \bar{\rho}^{1/4}$$

($\bar{\rho}$= avg. DM density)

2) from quantum Jeans length, Jeans mass

$$M_J(z) = \frac{4}{3} \pi^{\frac{13}{4}} G^{-\frac{3}{4}} \left(\frac{\hbar}{\mathrm{m}}\right)^{3/2} \bar{\rho}^{1/4}$$

We can use parameters either (m, M) or (m, $\bar{\rho}$)

Time evolution

SPE has a scaling symmetery

$$\{t, x, \psi, \rho, V\} \rightarrow \{\lambda^{-2}t, \lambda^{-1}x, \lambda^{2}\psi, \lambda^{4}\rho, \lambda^{2}V\},$$

$$\{M, E, L\} \rightarrow \{\lambda M, \lambda^{3}E, \lambda L\},$$

During the matter dominated era $\bar{\rho}$ *scales as* $(1+z)^3$, *thus by setting* $\lambda = (1+z)^{3/4}$ *we can easily estimate the time evolution of the galactic halos*

$$\{t, x, \psi, \rho, V\} \rightarrow \{(1+z)^{-3/2}t, (1+z)^{-3/4}x, (1+z)^{3/2}\psi, (1+z)^{3}\rho, (1+z)^{3/2}V\},$$

$$\{M, E, L\} \rightarrow \{(1+z)^{3/4}M, (1+z)^{9/4}E, (1+z)^{3/4}L\}$$

Particle model (ULA)

$$I = \int d^4x\sqrt{g}\left[\frac{1}{2}F^2 g^{\mu\nu}\partial_\mu a\partial_\nu a - \mu^4(1-\cos a)\right]$$

$$m = \frac{\mu^2}{F}$$

$$\ddot{a} + 3H\dot{a} + m^2 \sin a = 0$$

oscillation starts at $\frac{T_0^2}{M_P} = m$

MDE starts at $T_1 \sim 1eV \rightarrow \frac{\mu^4(DM)}{T_0^4(rad)} \rightarrow \frac{\mu^4 T_0}{T_0^4 T_1} \sim 1$

$$F = \frac{\mu^2}{m} \sim \frac{M_P^{3/4} T_1^{1/2}}{m^{1/4}} \sim 0.5 \times 10^{17} GeV$$

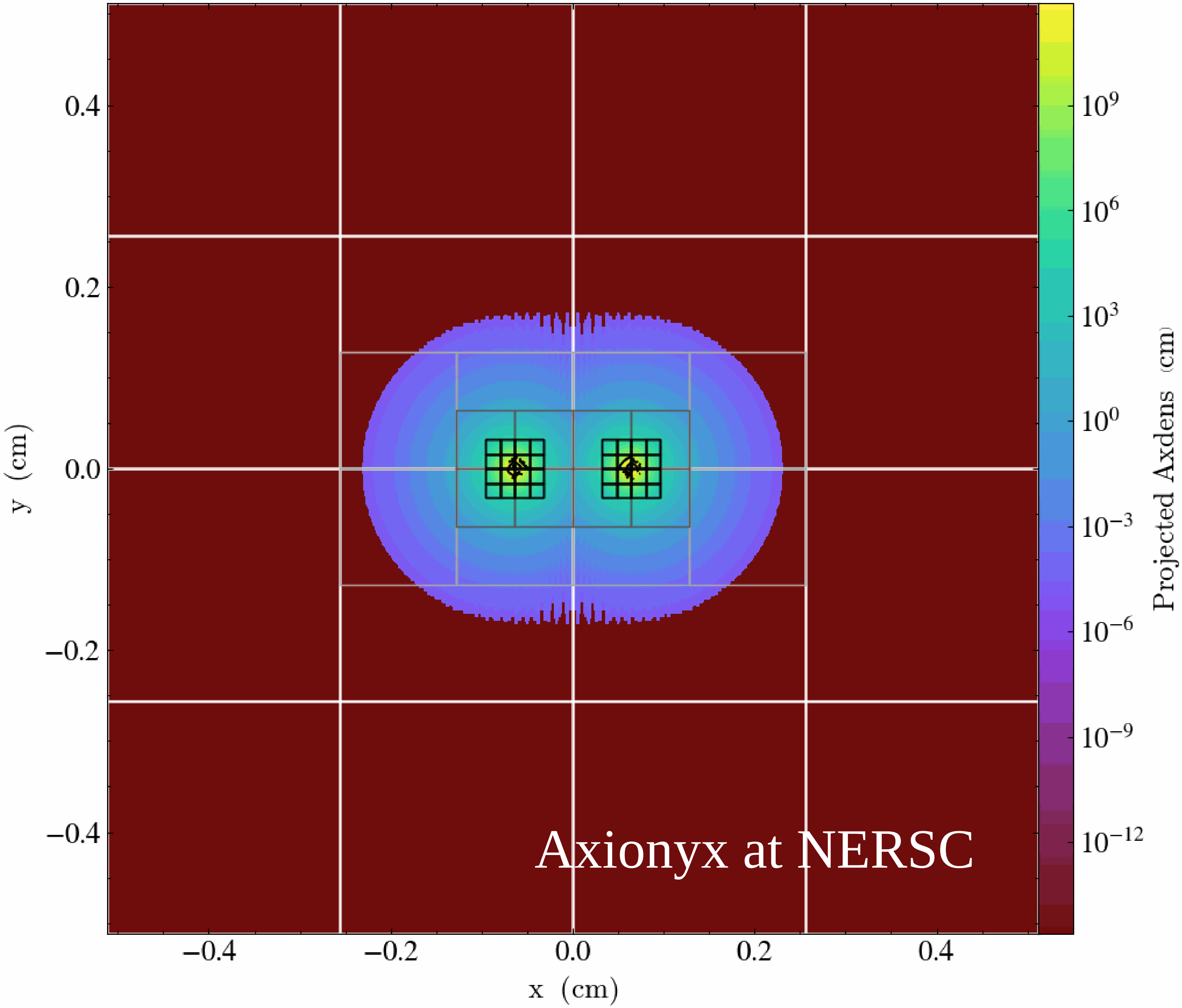
$\Omega_a \sim 0.1 \left(\frac{F}{10^{17}GeV}\right)^2 \left(\frac{m}{10^{-22}eV}\right)^{1/2}$ ULA miracle?

Hui etal 2017

2 axion model, Kim JE, Marsh 2016, axiverse

Some open codes with ULDM

Name	Hydrodynamics	MPI	AMR	Mixed DM	Relativistic	Self interaction	
Pyultralight	X	X	X	O	X	O	Python
Axionyx	O	O	O	O	X	X	Nyx
Enzo	O	O	O	O	X	X	Wave, fluid community
GRChombo	X	O	O	O	O	X	community



0.4
0.2
0.0
−0.2
−0.4
y (cm)
−0.4
−0.2
0.0
0.2
0.4
x (cm)
10^9
10^6
10^3
10^0
10^{-3}
10^{-6}
10^{-9}
10^{-12}
Projected Axdens (cm)
Axionyx at NERSC

Relativistic Scaling

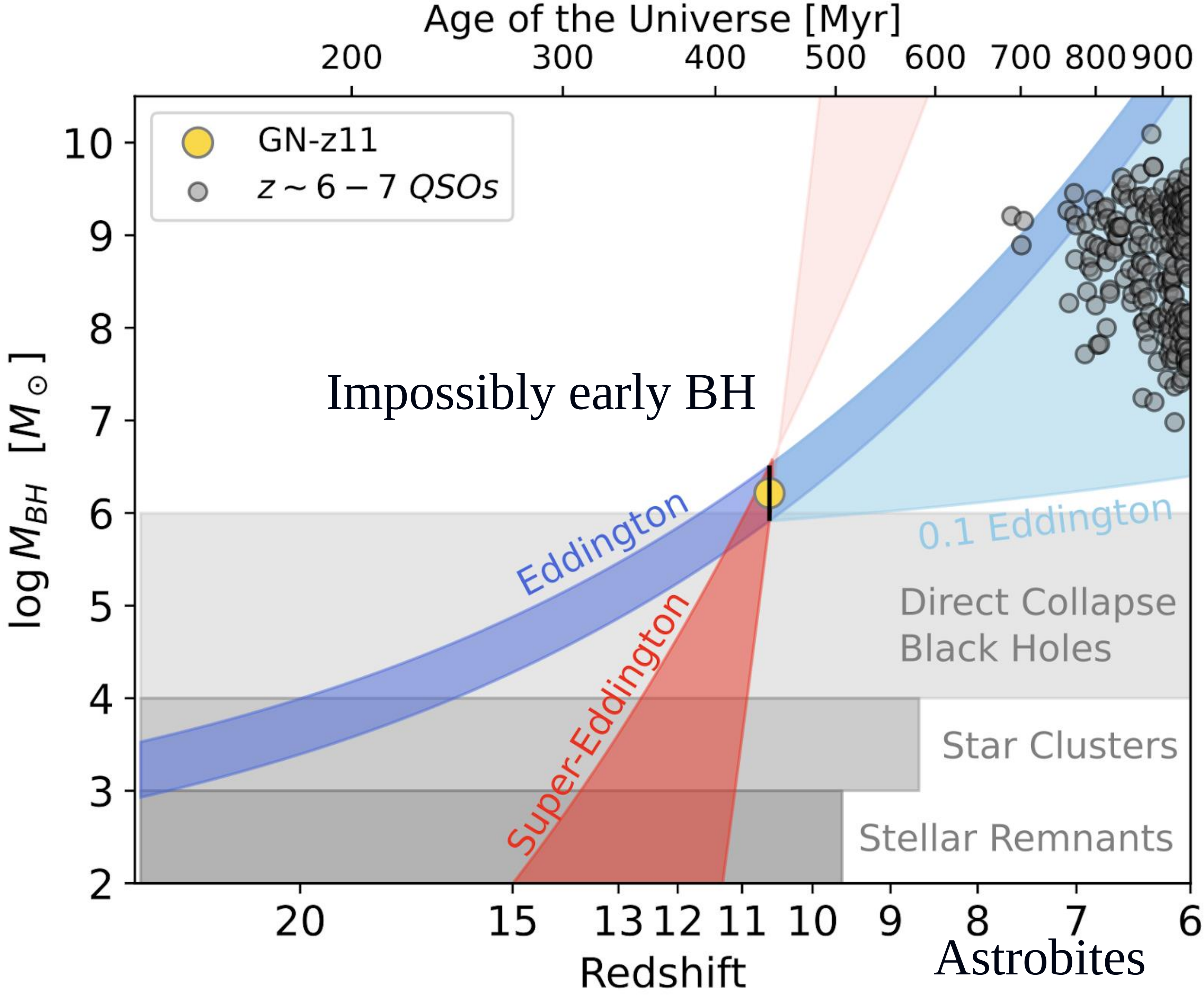
$$t \equiv t_c \hat{t} = \frac{\hbar}{mc^2} \hat{t},$$

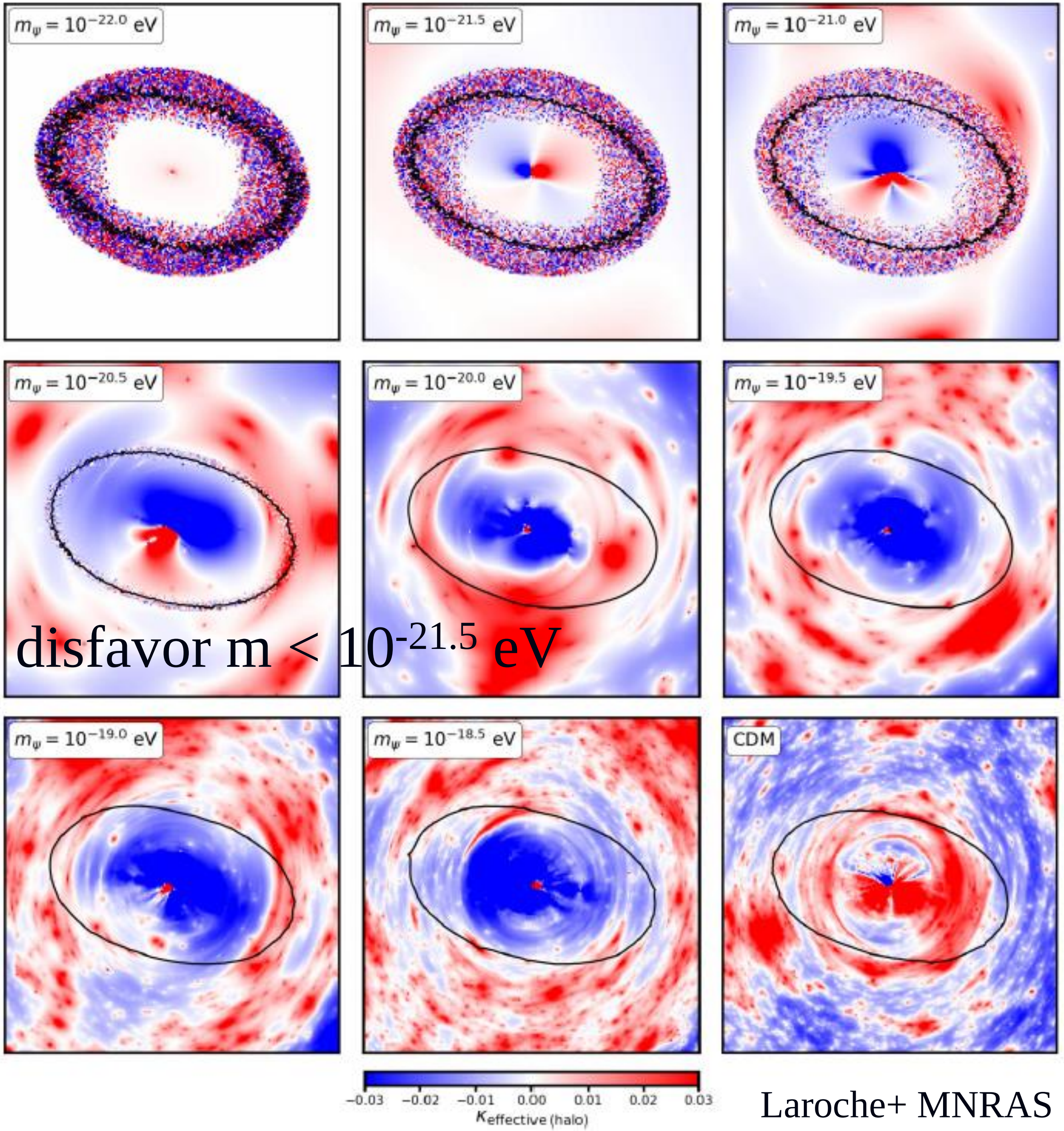
$$\mathbf{x} \equiv \mathbf{x}_c \hat{\mathbf{x}} = \frac{\hbar}{mc} \hat{\mathbf{x}},$$

$$\psi \equiv \psi_c \hat{\psi} = \frac{mc^2}{\hbar\sqrt{4\pi G}} \hat{\psi},$$

$$V \equiv V_c \hat{V} = c^2 \hat{V},$$

$$4\pi \int |\psi|^2 \, d^3x = M$$


Age of the Universe [Myr]
200
300
400
500
600
700
800
900
GN-z11
z ~ 6 − 7 QSOs
Impossibly early BH
Eddington
0.1 Eddington
Super-Eddington
Direct Collapse
Black Holes
Star Clusters
Stellar Remnants
log M_BH [M_⊙]
10
9
8
7
6
5
4
3
2
20
15
13
12
11
10
9
8
7
6
Redshift
Astrobites



disfavor m < $10^{-21.5}$ eV

Laroche+ MNRAS

BTFR from fuzzy DM

LKL PLB, 1901.00305

$g(r^\dagger) \simeq g^\dagger \rightarrow r^\dagger \simeq \sqrt{GM_b/g^\dagger} \Leftarrow g^\dagger = \dfrac{\hbar^2}{2m^2\xi^3}$

Using $r^\dagger$ one can estimate the constant roation velocity

($M_b(r^\dagger) = \; M_b/2 = M_d(r^\dagger) \rightarrow M(r^\dagger) = \; M_b$)

$v_f \equiv \sqrt{r^\dagger g^\dagger} = \sqrt{GM_b/r^\dagger} \simeq \left(Gg^\dagger M_b\right)^{1/4}$

$\Rightarrow M_b = Av_f^4$ (BTFR) with

$A = \left(Gg^\dagger\right)^{-1} = 34.16\left(\dfrac{m}{10^{-22}eV}\right)^2\left(\dfrac{\xi}{300pc}\right)^2 M_\odot/(km/s)^4$

$= 47M_\odot/(km/s)^4$ for $m = 1.17 \times 10^{-22}eV$ and $\xi = 300pc$

$g_b(r \gg r^\dagger) \approx \dfrac{GM_b}{r^2} = \dfrac{g_{obs}^2}{g^\dagger} \rightarrow \quad g_{obs} = \sqrt{g_b g^\dagger} \qquad (MOND!)$

- We theoretically derived the coefficient A for the first time.
- Faber-Jackson Relation for elliptical and universal surface density can be derived similarly
- Is ξ universal?

Ultra-light axion (ULA)

potential $\mathrm{V(a)} = \mu^4\left(1-\cos(\frac{a}{f_a})\right)$

$\simeq \mu^4\left(1-(1-\frac{1}{2}(\frac{a}{f_a})^2+\frac{1}{4!}(\frac{a}{f_a})^4+\cdots\right)$

$\simeq \frac{1}{2}(\frac{\mu^4}{f_a^2})a^2-\frac{1}{4!}(\frac{\mu}{f_a})^4 a^4+\cdots=\frac{1}{2}m^2a^2-\lambda a^4+\cdots$

$\frac{\text{interaction term}}{\text{mass term}} \sim (\frac{a}{f_a})^2 < 1 \quad \because a{\sim}10^{-2}f_a$

Brief History of BEC/SFDM

- 1983: Ruffini et al (m=10^{-24}eV), 1989 Membrado et al (ground state)
  1990 Press, et al (Soft Boson), 1993 Widrow (simulation methods)
- 1992: Sin's BEC DM for halo (QM approach, excited state, RC fitting m=3x10^{-23}eV)
- 1992, 1995: Lee & Koh Boson star (with self-interaction λ) model
- 1998: Shunck , massless scalar (m=0)
- 2000: Fuzzy (Hu et al, λ=0) (m=10^{-22}eV),
  Cusp prob. (Rioto),
  Satellite prob. (Matos), SFDM(Guzman)
- 2001: Dark Fluid (DM & DE) (Arbey)
- 2003: CDM-like (Matos)
- 2007: Bohmer & Harko, BEC DM in details
- 2009: ULA: Mielke & Perez, Hwang & H. Noh, Sikivie & Yang
  String Axiverse (Arvanitaki et al)
- 2010: Spiral arms (H. Bray), Tully-Fisher,
- 2010: Superfluid universe, Inflation,DE, DM, Kerson Huang et al
- 2013: Cosmological constraints, Bohua Li et al, and others
- 2014 :Structure formation simulation, Schive et al
- 2015: Constraints on ULA, Marsh, Guth…
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