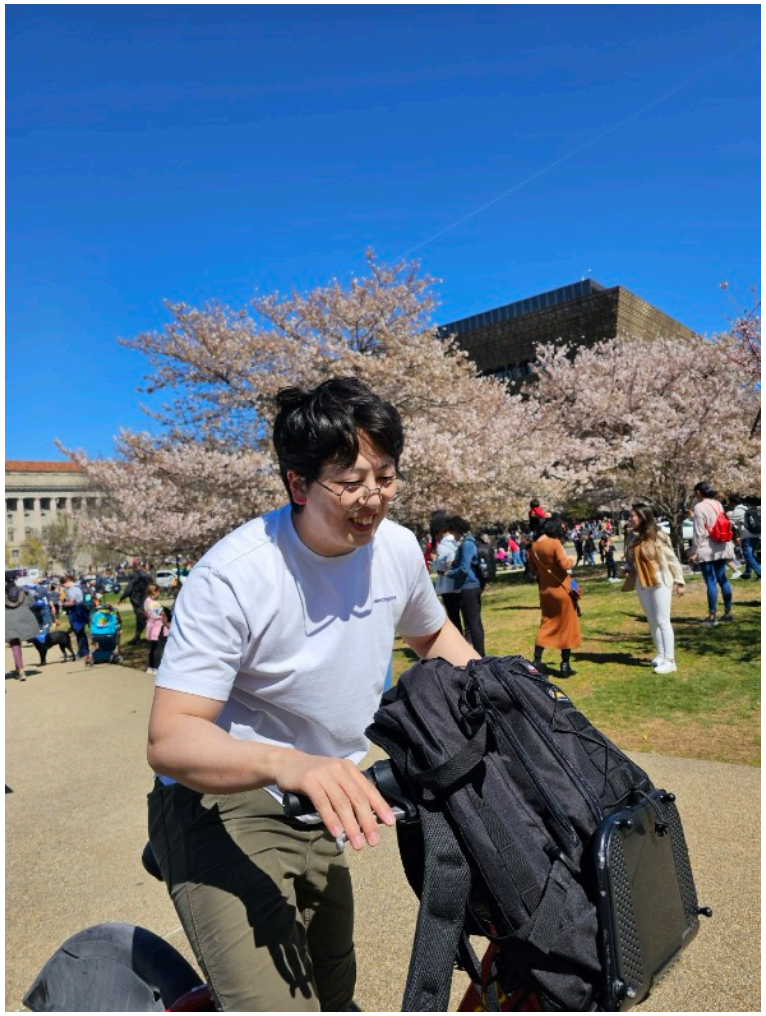
Angular momentum decomposition in the effective theory

Group meeting at NCSU

Jun-Young Kim

March 29, 2024

Introduction



Cherry blossom festival at
Washington D. C.

Inha University (2014-2019) / Military service (2014-2016)

near the Incheon airport, South Korea / supervisor : H.-Ch. Kim

Ruhr-University Bochum (2019-2022)

Nordrhein-Westfalen, Germany / supervisor : M. V. Polyakov, E. Epelbaum

Jefferson Lab (2022-)

Virginia, USA / working with C. Weiss and J. L. Goity

Research Interest

[EIC Yellow Report]

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PHYSICAL REVIEW LETTERS 131, 021901 (2023)

First Measurement of Hard Exclusive $\pi^-\Delta^{++}$ Electroproduction Beam-Spin Asymmetries off the Proton

S. Diehl,[34,6] N. Trotta,[6] K. Joo,[6] P. Achenbach,[39] Z. Akbar,[46,12] W. R. Armstrong,[1] H. Atac,[38] H. Avakian,[39] L. Baashen,[11]

JLAB-PHY-23-3840

Strong Interaction Physics at the Luminosity Frontier with 22 GeV Electrons at Jefferson Lab

Energy-momentum tensor (EMT)

Upcoming EIC projects : Mass generation, spin decomposition, and 3D tomography

Energy-momentum tensor, Generalized parton distributions

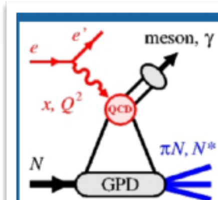
Mechanical interpretation

Transition generalized parton distributions (GPDs) and transition EMT

CLAS collaboration ($N \to \Delta$ DVCS, $\pi^-\Delta^{++}$ electroproduction)

The 22 GeV upgrade of JLab → ideal conditions for the study of the 3D structure of nucleon resonances via transition GPDs

Workshop “Exploring resonance structure with transition GPDs”, ECT* Trento, 21-25 Aug 2023



ECT*-APCTP Joint Workshop: Exploring resonance structure with transition GPDs

Matrix element of the QCD operator

QCD operator

Non-local operator

$$\Gamma = \bar{\psi}(z/2)\gamma^{\mu}[z/2; -z/2]\psi(-z/2)$$

Factorization theorem allows us to separate the non-perturbative object from the perturbative one in the hard exclusive process

It involves the gauge fields and scale-dependent

Hadronic matrix element of the QCD operator

$$\langle N' | \mathcal{O}_{\rm QCD} | N \rangle, \quad \langle \Delta | \mathcal{O}_{\rm QCD} | N \rangle \quad \cdots$$

Local operators

Gluon structure / higher-twist operator

$$\Gamma = F^{\mu\eta}F_{\eta\nu}, \bar{\psi}\gamma^{\mu}A^{\nu}\psi, \ldots$$

Twist-2 GPDs

$$\bar{\psi}\gamma^{\{\mu}\overleftrightarrow{\nabla}^{\{\nu_1}\overleftrightarrow{\nabla}^{\nu_2}\ldots\overleftrightarrow{\nabla}^{\nu_n\}}\psi - \text{traces}$$

$$F^{\eta\{\mu}\overleftrightarrow{\nabla}^{\{\nu_1}\overleftrightarrow{\nabla}^{\nu_2}\ldots\overleftrightarrow{\nabla}^{\nu_n}F_{\eta}^{\nu\}} - \text{traces}$$

Expansion of the non-local operator → Infinite tower of the quark-gluon local operators

Mellin moment of the GPDs

- Vector current / Axial-vector current
- Energy-momentum tensor / Parity-odd Energy-momentum tensor

Provides the mechanical interpretation

Energy-momentum tensor

$$T_q^{\mu\nu} = \bar{\psi}\gamma^\mu i \overleftrightarrow{\nabla}^\nu \psi, \quad \sum_{q,g} \partial_\mu T_a^{\mu\nu} = 0$$

$$T_{q,\text{spin}-2}^{\mu\nu} = \bar{\psi}\gamma^{\{\mu} i \overleftrightarrow{\nabla}^{\nu\}} \psi - \text{traces}$$

$$T_{q,\text{spin}-1}^{\mu\nu} = \bar{\psi}\gamma^{[\mu} i \overleftrightarrow{\nabla}^{\nu]} \psi = -\frac{1}{4}\epsilon^{\mu\nu\alpha\beta}\partial_\alpha[\bar{\psi}\gamma_\beta\gamma_5\psi]$$

QCD equation of motion

$$T_{q,\text{spin}-0} = \bar{\psi}\gamma^\mu i \overleftrightarrow{\nabla}_\mu \psi = O(m)$$

Energy-momentum tensor current

Conservation of the EMT current / trace anomaly

Nucleon matrix element is composed of the five independent form factors

Nucleon matrix element of the EMT current $T^{\mu\nu}$

[twist-2] EMT form factors

$$A(t), J(t), \text{ and } D(t)$$

Connected to the leading twist vector GPDs

[twist-3] Angular momentum decomposition

$$J = \sum_q S^q + \sum_q L^q + J^g$$

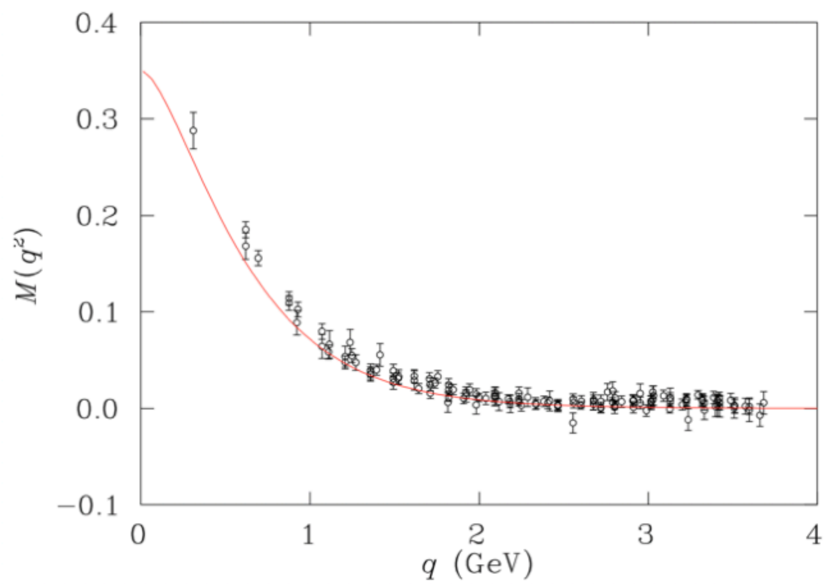
[twist-4] Interplay between quark and gluon subsystem

$$\bar{c}^q(t) + \bar{c}^g(t) = 0$$

QCD equation of motion provides strong constraint on the higher-twist operators

Effective theory: Chiral symmetry

Chiral symmetry



ChiSB is very important for understanding the matrix element of the QCD operator at the low normalization point

Chiral symmetry breaking (ChiSB) governs long-range hadron structure: Chiral perturbation theory, massless pion

ChiSB is related to the nucleon mass generation: Constituent quark picture, phenomenology, and etc.

Can we use this knowledge to compute/estimate matrix elements of QCD operators?

We need to construct an effective theory

- Chiral symmetry breaking mechanism from the QCD
- Degrees of freedom : pion, dynamical quark
- Fully relativistic and field theoretical approach

Effective theory: Effective operator

$$S_{\rm QCD} \to S_{\rm eff}$$

- The effective action should be derived from the QCD (QCD action).

$$O_{\rm QCD} \to O_{\rm eff}$$

- “QCD operator” should be translated into the “effective operator”
- This connection should be trackable (very important)!

$$V^{\mu}_{\rm QCD}, T^{\mu\nu}_{\rm QCD} \to V^{\mu}_{\rm eff}, T^{\mu\nu}_{\rm eff}$$

- For example, the conserved current (vector, EMT current) can be understood as an effective operator from the Noether theorem in a given theory.

$$\bar{\psi}\gamma^{\mu}\nabla^{\nu}\tau^{3}\psi \to \, ?$$

- However, a non-conserved operator cannot be obtained (e.g. isovector component of the EMT current, twist-3 operators)
- So, the effective operator should be derived from the QCD, consistently with effective dynamics.

Program

Construct description of QCD vacuum based on “Instanton vacuum” [Diakonov, Petrov (1984)/ Shuryak (1982)]

Derive effective theory (dynamics), also known as “chiral quark-soliton model”, from chiral symmetry breaking → baryon structure [Diakonov, Petrov, Pobylitsa (1986)]

Derive effective operators resulting from QCD operators using the following scheme [Diakonov et al, (1996) /J. Balla, M.V. Polyakov, C. Weiss (1999)]

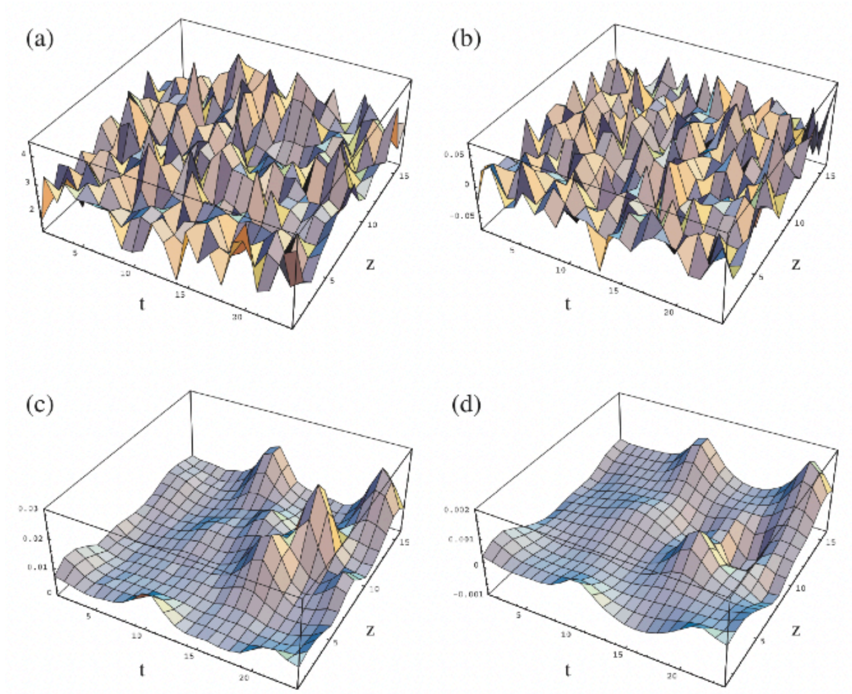
- Use systematic parametric approximation: Packing fraction, 1/Nc
- Obtain effective operators consistent with effective dynamics
- Preserve operator relations from QCD equation of motion [J.-Y. Kim, C. Weiss, PLB (2024)]

Estimate the nucleon matrix element of the derived effective operator within the effective theory using the same scheme (1/Nc and packing fraction):

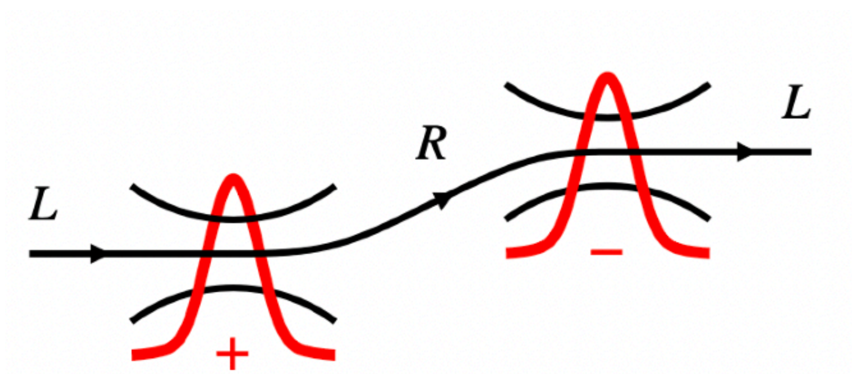
- Energy-momentum tensor form factors (twist-2)
  [K. Goeke, et al PRD (2007) / H.-Y. Won, H.-Ch Kim, JYKim PRD (2023) …]
- Angular momentum decomposition (twist-3) → Main dish
  [J.-Y. Kim, H-Y. Won, J. L. Goity , C. Weiss in progress] [J.-Y. Kim, H-Y. Won, H.-Ch. Kim , C. Weiss, ArXiv]
- cbar form factor (twist-4) → Under investigation
  [J.-Y. Kim, C. Weiss in progress]

Effective theory

Instanton vacuum



[Chu et al, PRL 70 (1993) 225; PRD 49 (1994) 6039]



[C. Weiss, presentation in JLab seminar (2023)]

Instanton

Chiral symmetry breaking is caused by topological fluctuations of the gauge fields in the QCD vacuum (instantons)

Instantons: Classical solution of the Yang-Mill equation (self-duality / localized) in Euclidean time

Typical size $\bar{\rho} \sim 0.3\ \mathrm{fm}$, separation $\bar{R} \sim 1\ \mathrm{fm}$

Fermionic zero modes

Dirac equation (in the presence of the instanton field background) → $(i\nabla^{\mu}\gamma_{\mu} + im)\Psi = \lambda\Psi$

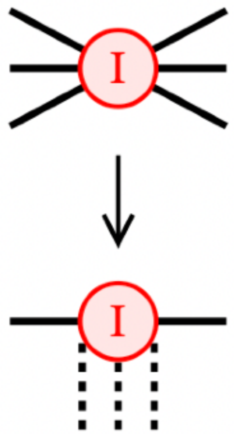
Zero mode $\lambda = 0$ (dominant in the low energy regime)

Chiral symmetry breaking

Quarks experience chirality flip → order parameter $\langle\bar{\psi}\psi\rangle \neq 0$;

Main outcome: massless bosons $\langle\bar{\psi}^a_L\psi^b_R\rangle \sim U^{ab}$; generation of the dynamical quark mass

Instanton vacuum: effective theory



[J.Y. Kim, C. Weiss, PLB (2024)]

Effective action

Zero mode induces the multi-fermion interaction (Hoof't vertex) / Bosonization $\bar{\psi}^a \psi^b \sim U^{ab}$

Semi-bosonized Effective action

Systematic expansions: Diluteness of the instanton ($M\bar{\rho}$), $1/N_c$ expansion

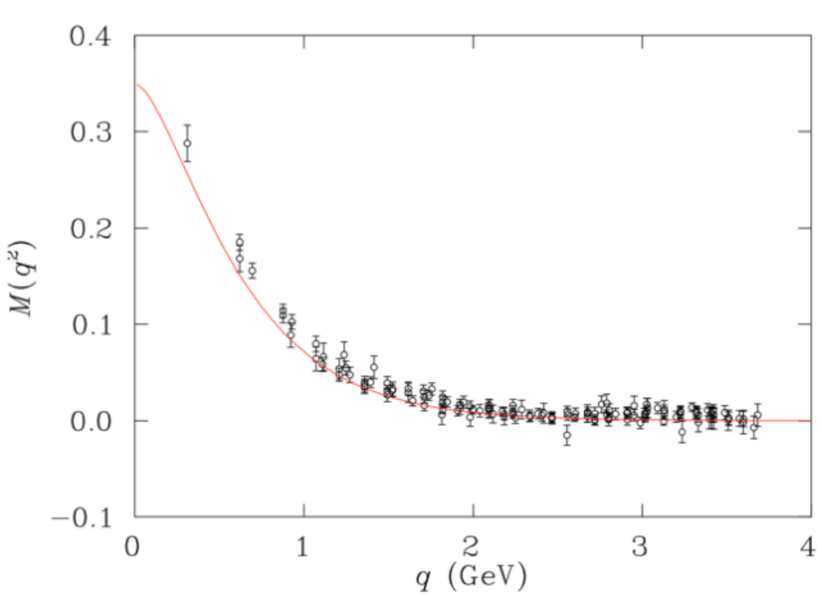
$$Z = \int [DA]_{\text{low}} \int [DA]_{\text{high}} \exp[-S_{\text{YM}}] \ \ [\times \text{ fermions}] \quad \longrightarrow \quad S = -\int d^4x \bar{\psi}[i\partial^\mu \gamma_\mu + MU^{\gamma_5}]\psi$$

QCD action

Effective chiral action

$$U^{\gamma_5}(x) = \frac{1+\gamma_5}{2}U(x) + \frac{1-\gamma_5}{2}U^\dagger(x)$$

Effective chiral theory



Degrees of freedom→Goldstone boson + Dynamical quark mass $(p < \rho^{-1} \sim 600 \text{ MeV})$

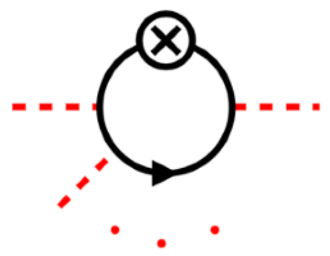
UV cutoff by zero mode form factor $M(p)$

Typical size of the dynamical quark mass $M(0) \sim 0.3 - 0.4 \text{ GeV}$

Quark-pion coupling $g_{\pi qq} \sim 4$

Instanton vacuum: Fully bosonized chiral theory

Fully bosonized effective chiral action



[C. Weiss, presentation in JLab seminar (2023)]

Chiral perturbation theory in large N_c limit of QCD

Integrating out the quark fields

Chiral expansion [Expanding action in the power pion momentum (∂U^n)]

Infinite towers of the pion momentum appear

F_π is determined by quark-loop integral

$$S = -\int d^4x \bar{\psi}[i\partial^\mu \gamma_\mu + MU^{\gamma_5}]\psi \quad \longrightarrow \quad S = \int d^4x \left\{ \frac{F_\pi^2}{4} \mathrm{tr}[\partial_\alpha U^\dagger \partial_\alpha U] + \mathcal{O}(\partial U^4) \right\}$$

Effective chiral action | Fully bosonized effective chiral action

Outcome

Effective action in the chiral perturbation theory with dynamically determined low-energy constants

Wess-Zumino term, topological baryon charge, and axial charge …

Truncated version of the fully bosonized theory is the Skyrme model

Effective operator

Instanton vacuum: Evaluating QCD operators

Vector and energy-momentum tensor currents can be obtained from Noether's theorem in a given effective theory.

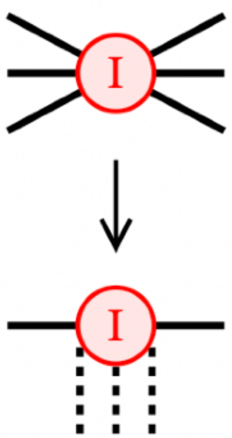
However, QCD operators containing gauge fields cannot measured from a given effective theory.

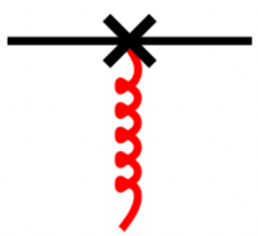
Thus, we need to trace down how the effective operators are dynamically generated from the QCD operators

QCD operators normalized at scale $\bar{\rho}^{-1} \sim 0.6$ GeV

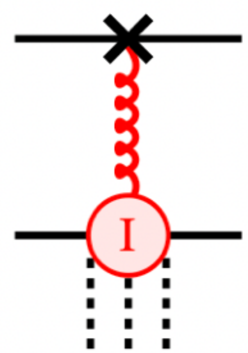
Use approximations Diluteness, saddle-point approximation $1/N_c$

Effective operators satisfy QCD equation of motion





[J.Y. Kim, C. Weiss, PLB (2024)]



$$\mathcal{F}[A] = \sum_{I+\bar{I}} \mathcal{F}[A_I] + O(\rho^4/R^4)$$

$$\langle \ldots \mathcal{F}[A] \ldots \rangle_{\text{inst}} \to \langle \ldots \mathcal{F}[\bar{\psi}, \psi] \ldots \rangle_{\text{eff}}$$

[Diakonov et al, (1996) /J. Balla, M.V. Polyakov, C. Weiss (1999)]

Gluon operator in the instanton vacuum

Coupling through the quark zero modes ('tHooft vertex)

Local QCD gluon operator $\mathcal{F}[A]$

Single-instanton approximation (packing fraction)

Gauge field is converted into the effective fermion operator in the effective theory

QCD operators: Twist-2

$$O_q^{\mu\nu} = \bar{\psi}\gamma^{\{\mu} i\nabla^{\nu\}}\tau\psi - \text{traces}$$

$$O_g^{\mu\nu} = F^{\beta\{\mu}F^{\nu\}}_{\beta} - \text{traces}$$

QCD operator

Twist-2 spin-2 local operator; $\nabla := \partial - iA$ contain gauge potential

Spin-projected EMT current

Nucleon matrix element = moments of PDF/GPD (EMT form factor)

Light-front momentum $A(t)$, total angular momentum $J(t)$, and D-term form factor $D(t)$

↓

$$O_q^{\mu\nu} = \bar{\psi}\gamma^{\{\mu} i\partial^{\nu\}}\tau\psi - \text{traces}$$

$$O_g^{\mu\nu} = 0 + O(\rho^4/R^4)$$

Effective operator [Diakonov et al, (1996) /J. Balla, M.V. Polyakov, C. Weiss (1999)]

Use of the pure derivative operator can be justified in the estimation of the EMT form factor

The EMT operator obtained from Noether's theorem coincides with that of the effective operator formalism

Gluon operators are suppressed in packing fraction

Under the scale evolution, the gluon contribution starts to increase. In turn, we would have a finite value of the gluon contribution to the energy-momentum tensor at $\mu \sim 2.0$ GeV.

QCD operators: Twist-3

$$O_q^{\mu\nu} = \bar{\psi}\gamma^{[\mu} i \nabla^{\nu]} \tau \psi$$

$$= -\frac{1}{4}\epsilon^{\mu\nu\alpha\beta}\partial_\alpha[\bar{\psi}\gamma_\beta\gamma_5\tau\psi]$$

$$\downarrow$$

$$O_q^{\mu\nu} = \bar{\psi}\left(\gamma^{[\mu} i\partial^{\nu]}\tau + \frac{i}{4}\sigma^{\alpha\beta}[MU^{\gamma_5}, \tau]\right)\psi$$

$$= -\frac{1}{4}\epsilon^{\mu\nu\alpha\beta}\partial_\alpha[\bar{\psi}\gamma_\beta\gamma_5\tau\psi]$$

QCD operator

Twist-3 spin-1 local operator / antisymmetrized EMT current

QCD equation of motion → total derivative of the axial-vector current

Nucleon matrix element = moments of twist-3 PDF/GPD

Ji's relation / orbital angular momentum

$$J = S^q + L^q + J^g$$

Effective operator **[JYKim ,C. Weiss PLB (2024)]**

Gauge-dependent part $\nabla := \partial - iA$ → the additional spin-flavor-dependent term ("potential" term)

QCD relation is also satisfied in the effective theory.

A number of literatures encounter the inconsistency in the values of the orbital angular momentum (due to the lack of knowledge of the effective operator) **[E. Leader, C. Lorce Phys.Rept (2014)]**

OAM from Ji's relation vs. direct measurement of OAM

Similar effects in QCD operator with $\gamma^\mu\gamma_5$ describing the spin-orbit correlation

QCD operators: Twist-4

$$O_q = \bar{\psi}\gamma^\mu i\nabla_\mu \tau\psi = O(m)$$

$$O_g = \frac{\beta}{2g}F^2 + O(m)$$

$$\downarrow$$

$$O_q = \bar{\psi}\gamma^\mu i\nabla_\mu \tau\psi = 0$$

$$O_g = \frac{\beta}{2g}F^2$$

QCD operator

Twist-4 spin-0 local operator

QCD equation of motion → trace of the quark part = 0 (chiral limit)

Trace anomaly; force between quark and gluon subsystems $\bar{c}(t)$;

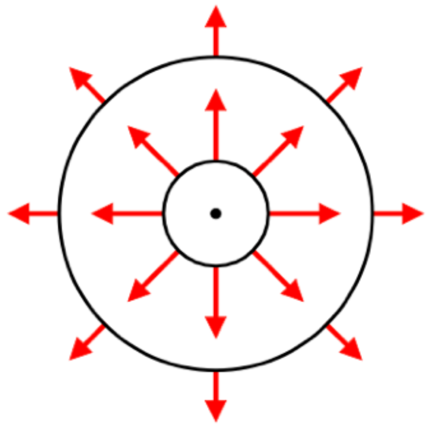
Effective operator **[Diakonov et al, (1996) /J. Balla, M.V. Polyakov, C. Weiss (1999)]**

QCD relation is also satisfied in the effective theory.

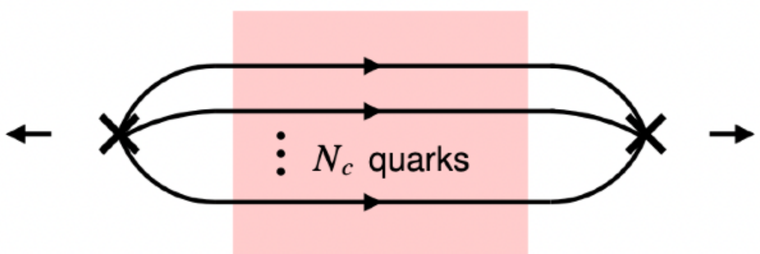
Instanton vacuum realizes the low-energy theorem from trace

Nucleon matrix element

Instanton vacuum: Correlation function



$U_{\rm cl}(\boldsymbol{x})$ isospin



$U_{\rm cl}$ classical chiral field

Chiral theory in the large N_c limit of QCD

$$S = -\int d^4x \bar{\psi}[i\partial^\mu \gamma_\mu + MU^{\gamma_5}]\psi$$

Effective action

$$U = e^{iP(r)\mathbf{r}\cdot\tau}$$

Hedgehog symmetry → key role in the spin-flavor symmetry

Nucleon correlation function

$$\Pi_N(T) = \int dUF[U]e^{-S_{\rm eff}[U]}$$

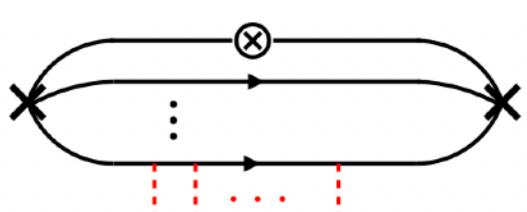
Saddle point approximation in $1/N_c$ expansion

$$\lim_{T\to\infty} \Pi_N(T) \sim e^{-M_N T} \qquad \left.\frac{\delta S[U]}{\delta U}\right|_{U=U_{\rm cl}} = 0$$

Saddle point equation

Nucleon mass [conn+disc]

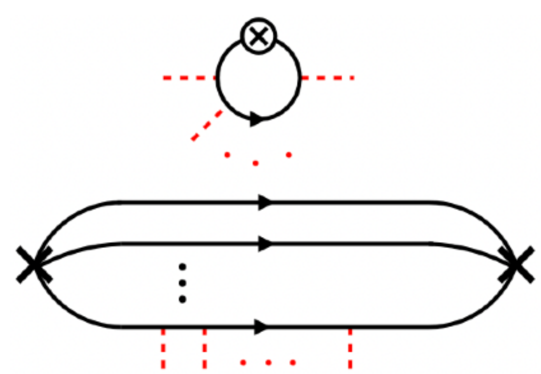
$M_{\rm cl}$ =



[Valence quark]

$$\sim \Pi_{i=1}^{N_c} \langle 0, T/2 | \frac{i}{i\partial_\tau + H} | 0, -T/2 \rangle_i$$

+



[Sea quark]

$$\sim \exp[-N_c \mathrm{Tr}\ln(i\partial_\tau - H)]$$

Nucleon mass: first-quantized representation

Spectral representation

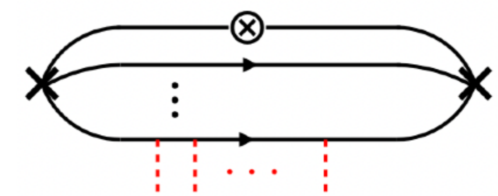
$$H\Phi_n = (-i\gamma_0\gamma\cdot\nabla + \gamma_0 M U^{\gamma_5})\Phi_n = E_n\Phi_n$$

$$\langle x'|\frac{i}{i\partial_\tau - H}|x\rangle = \Theta(t-t')\sum_{E_n>0} e^{-iE_n(t'-t)}\Phi_n(\mathbf{x}')\Phi_n^\dagger(\mathbf{x})$$
$$+\Theta(t-t')\sum_{E_n<0} e^{-iE_n(t-t')}\Phi_n(\mathbf{x}')\Phi_n^\dagger(\mathbf{x})$$

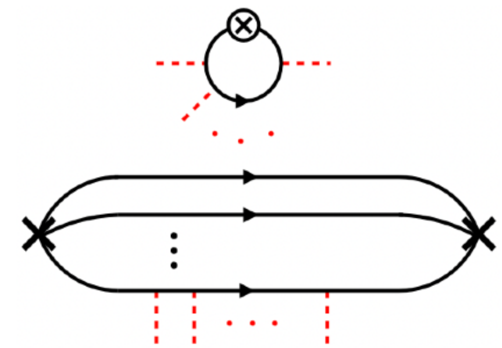
Dirac Hamiltonian / Finite box method

[S. Kahana, G. Ripka]

Quark propagator



[Valence quark]



[Sea quark]

Nucleon mass

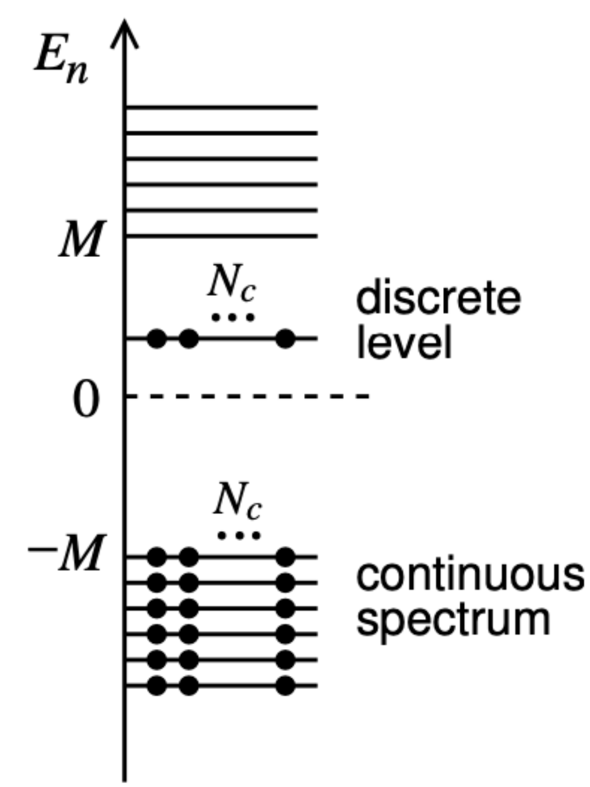
$$P(r) = 2\arctan R^2/r^2$$

$$M_N = \int d^3x[N_c E_{\rm lev}\Phi^\dagger_{\rm lev}(\mathbf{x})\Phi_{\rm lev}(\mathbf{x})$$
$$+N_c\sum_{\rm n\ neg\ cont} E_n\Phi^\dagger_{\rm n}(\mathbf{x})\Phi_{\rm n}(\mathbf{x})] \sim 1.207\ {\rm GeV}$$

Self-consistent profile function / variational principle

[Diakonov, Petrov (1988)]

Nucleon mass: First quantized representation



[JYKim, H.-Y. Won, H-Ch.Kim,C. Weiss (ArXiv)]

Nucleon mass: two limits

Non-relativistic quark limit [$MR \sim 0$]

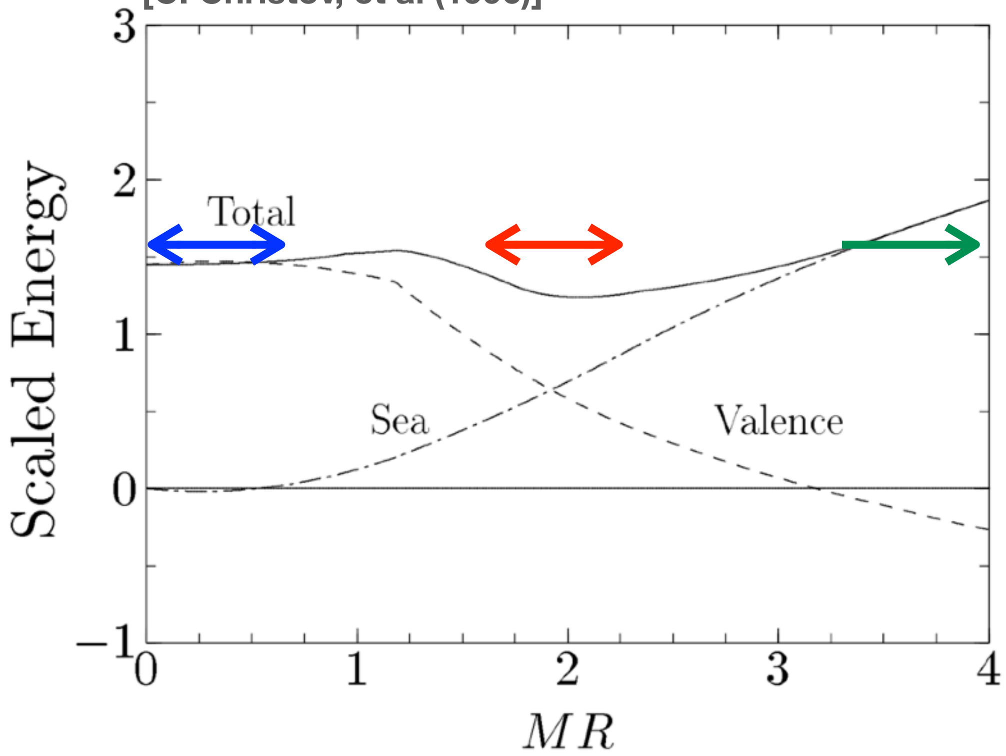
$R \to 0$, all the pion cloud effects are suppressed and the valence quark contribution becomes dominant (NQM).

Relativistic motion of the quark vanishes

Plateau → quarks are not bound

Total energy= valence (100%) + sea (0%)

[C. Christov, et al (1996)]



Realistic picture [$MR \sim 1-2$]

Use self-consistent profile function

Local minimum of the nucleon energy appears

Non-topological soliton

Total energy= valence (~50%) + sea (~50%)

Lattice results

$$\langle x \rangle_p \sim 0.5\,|_{\rm conn} + 0.3\,|_{\rm disc} + 0.3\,|_{\rm gluon}$$

Skyrme picture [$MR \gg 2$]

Chiral expansion — slowly varying chiral field ($R \to \infty$) allows us to expand the quark propagator in the pion momentum ($\partial_\mu U$).

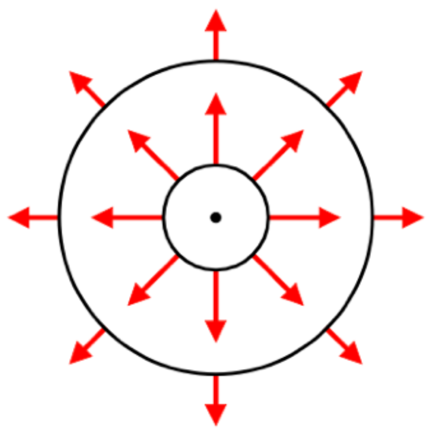
Taking leading order of pion momentum/ truncated version in the chiral expansion (Skyrme model)

Valence quark dive into the sea level; topological soliton (Wess-Zumino term)

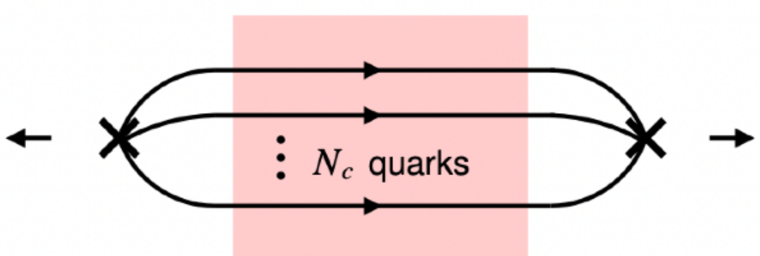
Total energy= valence (0%) + sea (100%)

Effective chiral theory naturally interpolates the NQM and Skyrme model

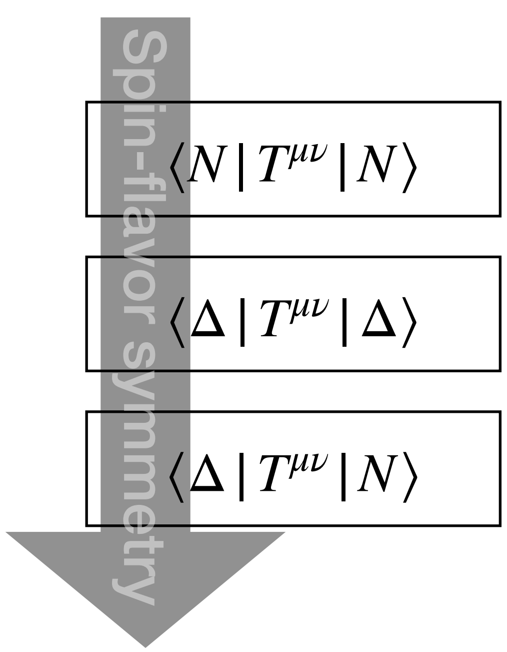
Instanton vacuum: zero mode quantization



$U_{\rm cl}(\boldsymbol{x})$ isospin



$U_{\rm cl}$ classical chiral field



Zero mode quantization [C. Christov, et al (1996)]

$$\int DU\,\mathscr{F}[U] \to \int DTDA\; TA\mathscr{F}[U]A^\dagger T^\dagger$$

Correlation function

$$T = \exp\left[-i\mathbf{P}\cdot\mathbf{X}\right], \quad A = \exp\left[\frac{i}{2}\boldsymbol{\Omega}\cdot\tau\right]$$

Translational rotational zero modes

Collective Hamiltonian

$$H_{\rm coll} = M_{\rm cl} + \frac{\mathbf{S}^2}{2I} + \frac{\mathbf{P}^2}{2M_{cl}}$$

Collective Hamiltonian

$$I = O(N_c) \text{ and } M_{\rm cl} = O(N_c)$$

Inertia parameters

$$M_\Delta - M_N = \frac{3}{2I} \sim 270 \text{ MeV}$$

$N - \Delta$ mass splitting

Extension

Thanks to the spin-flavor symmetry, the matrix element of the $N, \Delta,$ and $N \to \Delta$ can be studied in a one framework.

It can be extended to the flavor SU(3) sector (octet and decuplet baryons) by using the trivial embedding

Results

Energy-momentum tensor: effective operator

Effective EMT operator

[twist-2]

$$T^{\mu\nu}_{q,\text{spin}-2} = i\bar{\psi}\overleftrightarrow{\partial}^{\{\mu}\gamma^{\nu\}}\tau\psi - \text{traces}$$

$$T^{\mu\nu}_{g,\text{spin}-2} = 0$$

[J Balla, M. V. Polyakov, C. Weiss NPB 1999]
$A(t), J(t), D(t)$ form factors

[twist-3]

$$T^{\mu\nu}_{q,\text{spin}-1} = i\bar{\psi}\overleftrightarrow{\partial}^{[\mu}\gamma^{\nu]}\tau\psi - \frac{M}{4}\bar{\psi}\left[\gamma^{[\mu}\gamma^{\nu]}\tau U^{\gamma_5} + \gamma^{[\nu}\gamma^{\mu]}U^{\gamma_5}\tau\right]\psi$$

$$T^{\mu\nu}_{g,\text{spin}-1} = 0$$

[JYKim, C. Weiss PLB 2024]
$L(t), S(t)$ form factors

[twist-4]

$$T^{\mu\nu}_{q,\text{spin}-0} = 0$$

$$T^{\mu\nu}_{g,\text{spin}-0} = g^{\mu\nu}\frac{\beta}{8g}F^2 .$$

[D. Diakonov, et al NPB 1996]
$\bar{c}(t)$ form factors

Implication of the effective EMT operator in the nucleon form factors

$$J = L_q + S_q + J_g$$

All QCD relations are satisfied in the effective theory / Ji's relation / All sum rules are satisfied [JYKim, C. Weiss PLB (2024)]

$$\bar{c}_q + \bar{c}_g = 0 \qquad A_q + A_g = 1$$

Gluon contributions to the leading twist form factor are suppressed

$$A_g = 0 \qquad J_g = 0$$

Large values of both the $\bar{c}_{q,g}$ form factors [JYKim, C. Weiss in preparation]

$$\bar{c}_q = -\frac{1}{4} \qquad \bar{c}_g = \frac{1}{4}$$

Energy-momentum tensor: Fully bosinized theory

Effective EMT operator $R_\mu = U\partial^\mu U^\dagger, \quad L_\mu = U^\dagger \partial^\mu U$ $\qquad \bar\psi O^{\mu\nu}\psi = -iN_c\left[O^{\mu\nu}\langle x|\frac{1}{i\gamma^\mu\partial_\mu - MU^{\gamma_5}}|x\rangle\right]$

Fully bosonized effective EMT current in the presence of the background pion mean field.

We perform the chiral expansion in the leading order.

Axial vector current is defined by $A^\mu = -\frac{F_\pi^2}{2i}\mathrm{tr}[\tau^3(R^\mu - L^\mu)]$.

QCD equation of motion is satisfied.

$$\langle T^{\mu\nu}_{q,\mathrm{spin}-2}\rangle_U = -\frac{F_\pi^2}{2}\mathrm{tr}[L^{\{\mu}L^{\nu\}} - \frac{1}{2}g^{\mu\nu}L^\beta L_\beta] - \text{traces}$$

$$\langle T^{\mu\nu}_{q,\mathrm{spin}-1}\rangle_U = 0$$

[Isoscalar]

$$\langle T^{\mu\nu}_{q,\mathrm{spin}-2}\rangle_U = 0 \qquad \sim J^{u\pm d}$$

$$\langle T^{\mu\nu}_{q,\mathrm{spin}-1}\rangle_U = -\frac{1}{4}\epsilon^{\mu\nu\alpha\beta}\partial_\alpha A_\beta \qquad \sim S^{u\pm d}$$

[Isovector]

AM decomposition [JYKim, HYWon, J.L. Goity, C. Weiss in preparation]

$$J^{u+d} = L^{u+d}, \quad S^{u+d} = 0.$$

[isoscalar] The total nucleon spin (isoscalar) comes from the OAM “OAM(100%) vs. spin(0%)”

$$J^{u-d} = 0, \quad S^{u-d} = -L^{u-d}.$$

[isovector] The total nucleon spin (isovector) is zero “OAM(50%) vs. spin(50%)”

Angular momentum: N_c scaling

Matrix element of the effective EMT current

$$\langle N'|T^{\mu\nu}_{\mathrm{spin}-n}(0)|N\rangle = \lim_{T\to\infty}\frac{1}{Z}\mathcal{N}e^{ip_4\frac{T}{2}-ip'_4\frac{T}{2}}\int d^3x\int d^3y e^{-i\mathbf{p}'\cdot\mathbf{y}+i\mathbf{p}'\cdot\mathbf{x}}$$
$$\times\int D\psi D\psi^\dagger DU J_{N'}(\mathbf{y},T/2)T^{\mu\nu}_{\mathrm{spin}-n}(0)J_N(\mathbf{x},-T/2)\exp[-S]$$

Correlation function

$\mathbf{N_c}$ scaling [K. Goeke, et al (2007) /H.-Y. Won, H-Ch.Kim,JYKim PLB (2024)]

$$A^{u+d}\sim O(N_c^0)\quad\gg\quad A^{u-d}\sim O(N_c^{-1})\qquad D^{u+d}\sim O(N_c^2)\quad\gg\quad D^{u-d}\sim O(N_c^1)$$

$$J^{u+d}\sim O(N_c^0)\quad\ll\quad J^{u-d}\sim O(N_c^1)\qquad \bar{c}^{u+d}\sim O(N_c^0)\quad\gg\quad \bar{c}^{u-d}\sim O(N_c^{-1})$$

AM decomposition: $\mathbf{N_c}$ scaling

Selected results (AM decomposition in the leading order in the $1/N_c$ expansion) will be presented.

Isovector AM is parametrically larger than the isoscalar component.

N_c scalings of the OAM and spin are equivalent to that of the total AM.

Once we decompose the total AM into OAM and spin, the effective operator is subject to large instanton effects.

AM decomposition is a dynamical question in the $1/N_c$ expansion

$$L^{u-d}\sim O(N_c^1)\quad\approx\quad S^{u-d}\sim O(N_c^1)$$

Angular momentum: First-quantized representation

AM decomposition: First-quantized representation

[JYKim, H.-Y. Won, J. L. Goity, C. Weiss in preparation]

$$J^{u-d} = -\frac{N_c}{12}\int d^3r \sum_{n=\mathrm{occ}} \Phi_n^\dagger(\mathbf{x})\tau^3[2L^3 + 2E_{\mathrm{lev}}(\mathbf{r}\times\mathbf{\Sigma})^3\gamma_5]\Phi_n(\mathbf{x})$$

Total angular momentum

$$S^{u-d} = -\frac{N_c}{3}\int d^3x \sum_{n=\mathrm{occ}} \Phi_n^\dagger(\mathbf{x})\left[\tau^3\frac{\Sigma^3}{2}\right]\Phi_n(\mathbf{x})$$

Axial vector charge

$$L^{u-d} = -\frac{N_c}{3}\int d^3r \sum_{n=\mathrm{occ}} \Phi_n^\dagger(\mathbf{x})\left[\tau^3 L^3\right]\Phi_n(\mathbf{x})$$

Naive OAM

$$+\frac{MN_c}{18}\int d^3x \sum_{n=\mathrm{occ}} \Phi_n^\dagger(\mathbf{x})\left[r\sin P(r)\gamma^0[(\hat{\mathbf{r}}\cdot\tau)(\hat{\mathbf{r}}\cdot\mathbf{\Sigma}) - (\mathbf{\Sigma}\cdot\tau)]\right]\Phi_n(\mathbf{x})$$

Potential term (instanton effects)

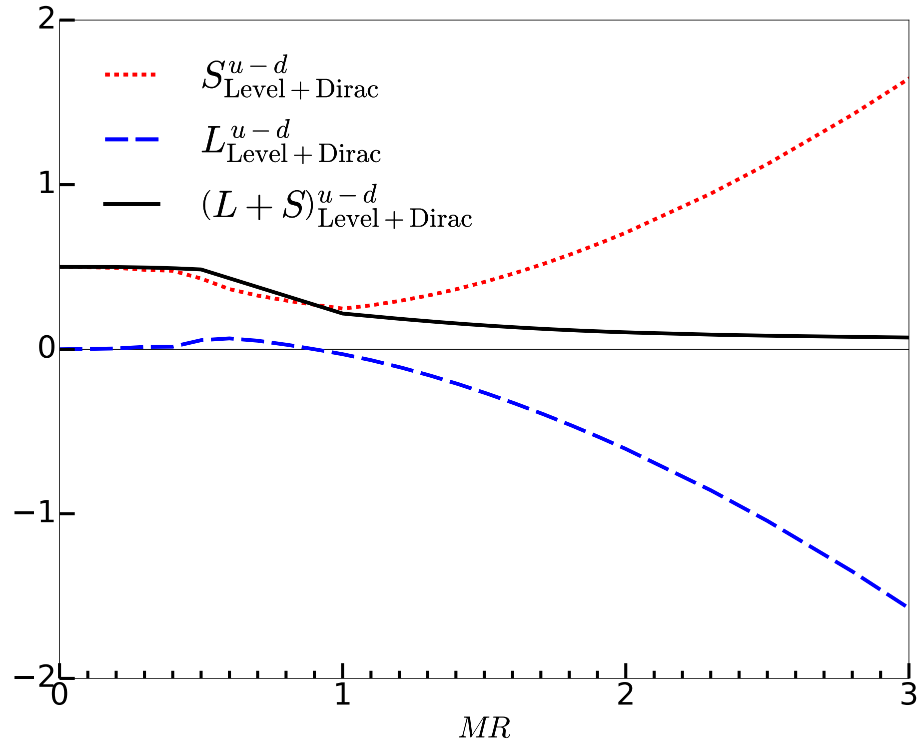
OAM composed of the kinetic and potential terms/ fully relativistic contributions

Chiral interaction is essential in the effective theory to comply with the QCD equation of motion.

$$S^{u-d}[0.52] \quad + \quad \overbrace{L^{u-d}[-0.32]} \quad = \quad J^{u-d}[0.20]$$

$$L^{u-d}[\mathrm{kin}] = -0.11 \quad \ll \quad L^{u-d}[\mathrm{pot}] = -0.21$$

Angular momentum: two limits



[JYKim, H.-Y. Won, J. L. Goity, C. Weiss in preparation]

Non-relativistic limit

AM J^{u-d} comes from the quark spin alone.

Relativistic effects are suppressed.

Sub-leading order in $1/N_c$ provides correct axial charge in the quark model.

$$J^{u-d} = S^{u-d} = \frac{1}{2} g_A = \frac{N_c}{6} + \frac{1}{3}\ [\mathrm{sub-leading\ order}]$$

$$L^{u-d} = 0$$

Realistic picture

Spin dominates total spin of the nucleon

Large relativistic effects

$$S^{u-d}[0.52] \quad + \quad L^{u-d}[-0.32] \quad = \quad J^{u-d}[0.20]$$

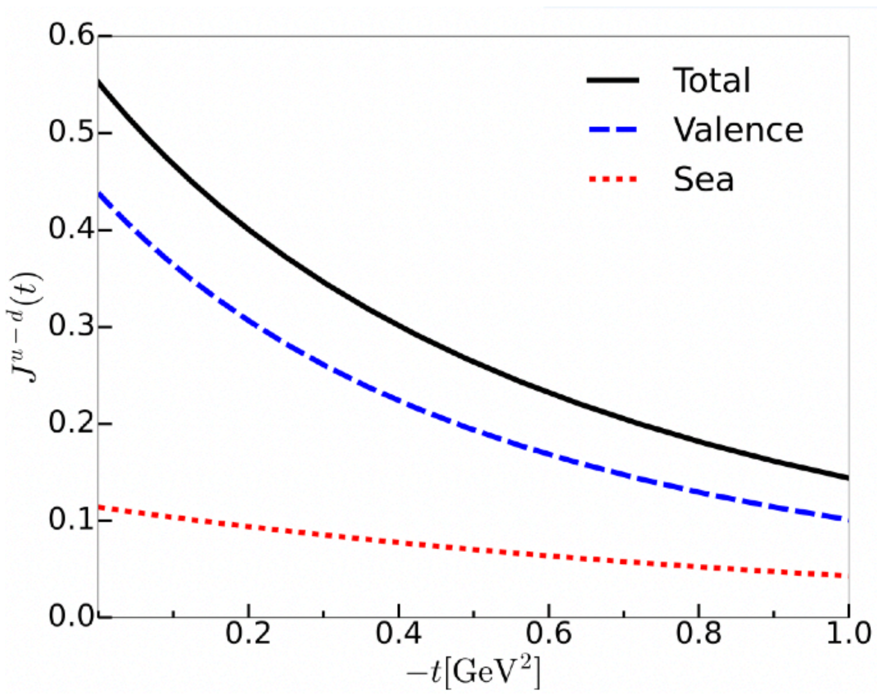
Skyrme limit

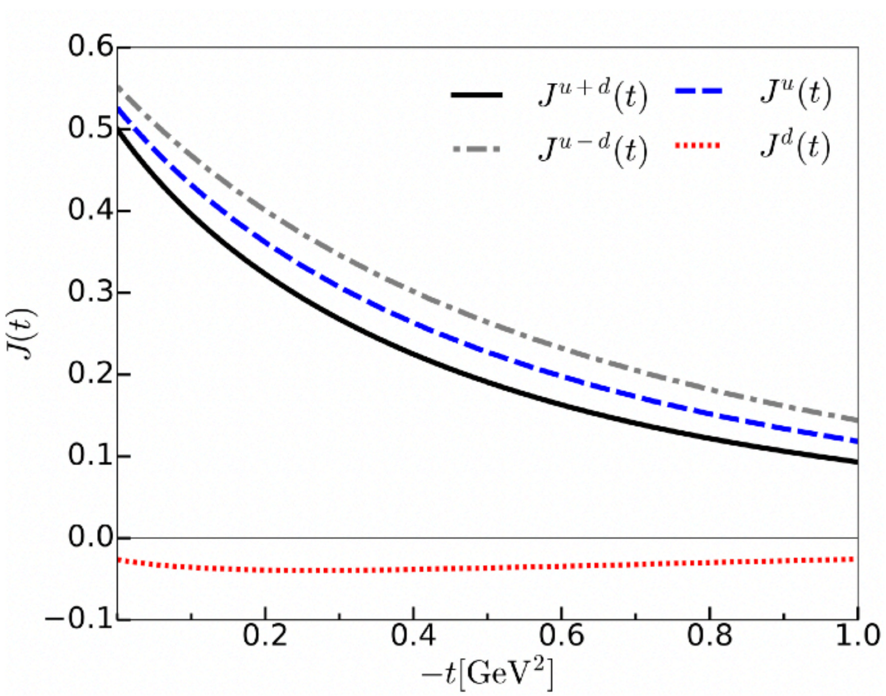
AM J^{u-d} is zero

The magnitude of the OAM becomes the same with spin, but opposite sign.

$$J^{u-d} = 0 \qquad S^{u-d} = \frac{1}{2} g_A \qquad L^{u-d} = -\frac{1}{2} g_A$$

Angular momentum: form factors





[H.-Y. Won, H-Ch.Kim,JYKim PLB (2024)]

Angular momentum form factor

We include sub-leading order corrections in the $1/N_c$ expansion.

This correction is essential to have a correct non-relativistic quark limit.

Valence quark contributions are dominant in the isovector AM form factor.

Flavor decomposition

Isovector AM is numerically larger than the isoscalar AM $\rightarrow$ consistent with the N_c scalings

$$J^{u+d} = 0.5, \quad J^{u-d} = 0.56$$

While u-quark contribution to AM is very large, d-quark contribution is very small $\rightarrow$ in good agreement with results from the lattice QCD

$$J^{u} = 0.53, \quad J^{d} = -0.03$$

J^u	J^d
0.214(27)	−0.001(27)

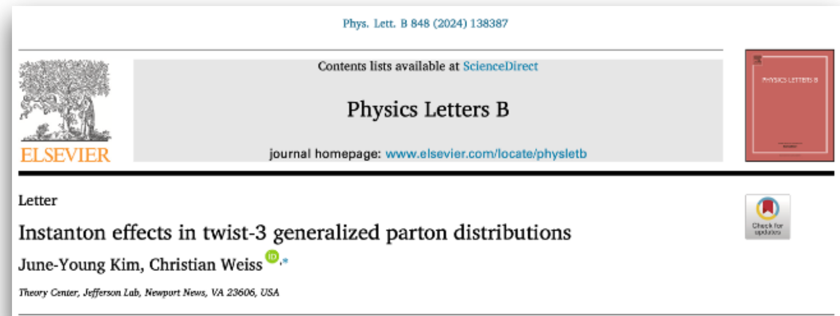
[LHPC collaboration, PRD (2008)]

Extension & Summary

Extension: spin-orbit correlation

$$\frac{1}{2}\int d^3x\bar{\psi}(x)\gamma^+(\mathbf{x}_\perp \times i\overleftrightarrow{\mathbf{D}})^z\psi(x) = L^z_{q_R} + L^z_{q_L}$$

$$\frac{1}{2}\int d^3x\bar{\psi}(x)\gamma^+(\mathbf{x}_\perp \times i\overleftrightarrow{\mathbf{D}})^z\gamma_5\psi(x) = L^z_{q_R} - L^z_{q_L}$$



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Letter

Instanton effects in twist-3 generalized parton distributions

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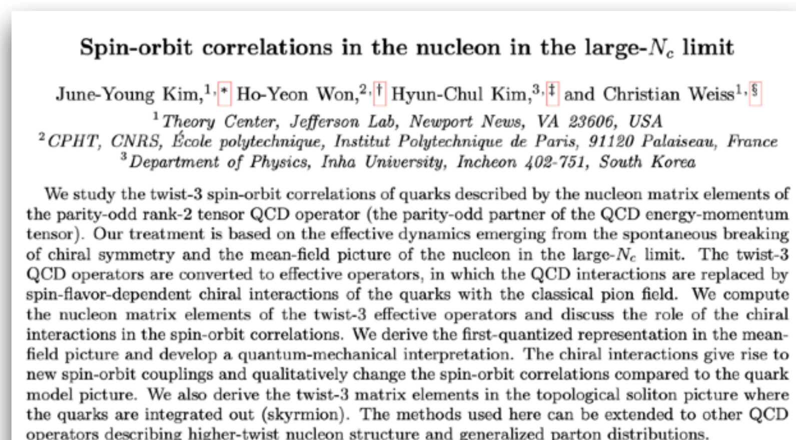
[JYKim ,C. Weiss PLB (2024)]



Spin-orbit correlations in the nucleon in the large-N_c limit

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We study the twist-3 spin-orbit correlations of quarks described by the nucleon matrix elements of the parity-odd rank-2 tensor QCD operator (the parity-odd partner of the QCD energy-momentum tensor). Our treatment is based on the effective dynamics emerging from the spontaneous breaking of chiral symmetry and the mean-field picture of the nucleon in the large-N_c limit. The twist-3 QCD operators are converted to effective operators, in which the QCD interactions are replaced by spin-flavor-dependent chiral interactions of the quarks with the classical pion field. We compute the nucleon matrix elements of the twist-3 effective operators and discuss the role of the chiral interactions in the spin-orbit correlations. We derive the first-quantized representation in the mean-field picture and develop a quantum-mechanical interpretation. The chiral interactions give rise to new spin-orbit couplings and qualitatively change the spin-orbit correlations compared to the quark model picture. We also derive the twist-3 matrix elements in the topological soliton picture where the quarks are integrated out (skyrmion). The methods used here can be extended to other QCD operators describing higher-twist nucleon structure and generalized parton distributions.

[JYKim, H.-Y. Won, H-Ch.Kim,C. Weiss (ArXiv)]

Quark spin (chiral) decomposition [C. Lorce , PLB (2014)]

By introducing the parity-odd counterpart, we can decompose the OAM into the quark chirality

This operator is known as the spin-orbit correlation

Chiral decomposition can be performed for any EMT form factors by introducing the parity-odd EMT current

Spin-orbit correlation (twist-3)

Very recently, we have studied the spin-orbit correlation in the nucleon in the large N_c limit of QCD

Derive the effective operator / estimate nucleon matrix element / N_c scalings

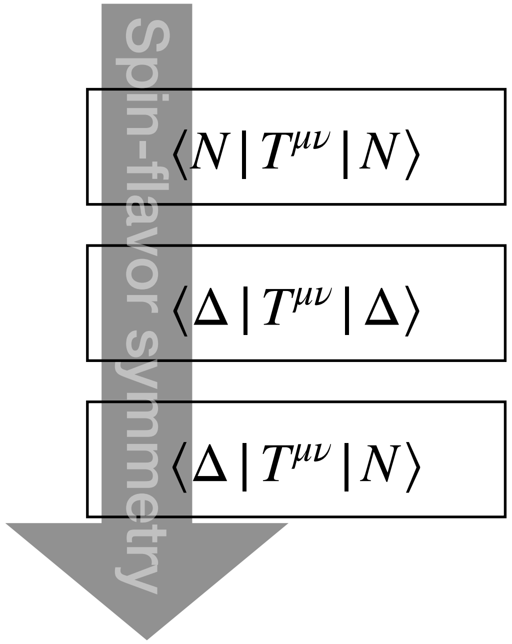
All QCD relation is also satisfied in the effective theory

Instanton effects are found to be very important!

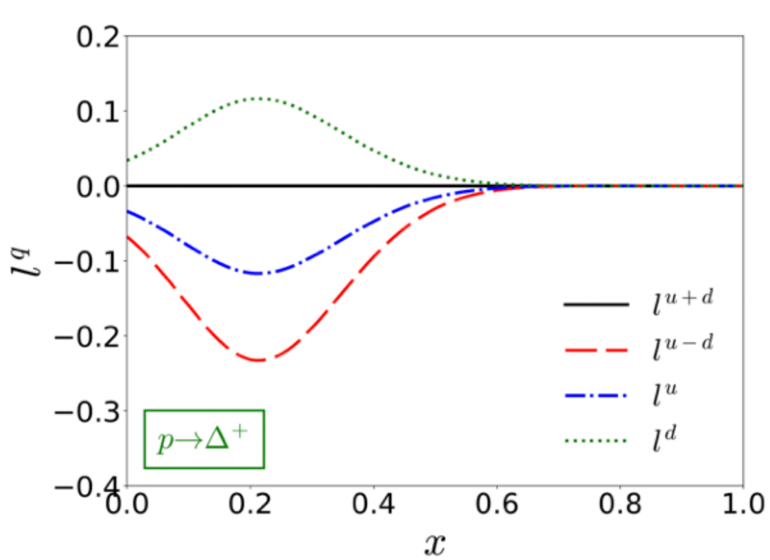
Study of the 3D/2D distributions are under investigation

[H.-Y. Won, JYKim, H-Ch.Kim, C. Lorce ,C. Weiss in progress]

Extension: $N \to \Delta$ spin structure



Lattice QCD	$J^S_{p\to p}$	$J^S_{\Delta^+\to\Delta^+}$	$J^V_{p\to p}$	$J^V_{p\to\Delta^+}$	$J^V_{\Delta^+\to\Delta^+}$
[9] $\mu^2 = 4\,\mathrm{GeV}^2$	0.33^*	0.33	0.41^*	0.58	0.08
[10] $\mu^2 = 4\,\mathrm{GeV}^2$	0.21^*	0.21	0.22^*	0.30	0.04
[11] $\mu^2 = 4\,\mathrm{GeV}^2$	0.24^*	0.24	0.23^*	0.33	0.05
[12] $\mu^2 = 1\,\mathrm{GeV}^2$	–	–	0.23^*	0.33	0.05
[13] $\mu^2 = 4\,\mathrm{GeV}^2$	–	–	0.17^*	0.24	0.03



Spin-flavor symmetry in the large N_c limit of QCD

Spin-flavor symmetry in the large N_c limit of QCD is a powerful tool to investigate the $N \to \Delta$ matrix element.

Realization of the dynamical symmetry → Chiral soliton approach

Transition EMT

Using spin-flavor symmetry and dynamical input from lattice QCD, we estimate the $N \to \Delta$ transition AM

applicable to the spin-orbit correlation

$$J^{u-d}_{p\to p} = 5 J^{u-d}_{\Delta^+\to\Delta^+} = \frac{1}{\sqrt{2}} J^{u-d}_{p\to\Delta^+}$$

[JYKim, H.-Y. Won, J.L.Goity, C.Weiss PLB (2023)]

Transition GPDs and PDFs

x-dependence of the GPDs / Transition GPDs

[JYKim PRD (2023) / JYKim, H.-Y. Won, J.L.Goity, C.Weiss in preparation]

Extension: gluon structure

Chiral-odd twist-3 ($e^q(x)$) [JYKim, C.Weiss in progress]

Huge instanton effects in the chiral-odd twist-3 operator

$$\bar{\psi}\overleftrightarrow{\nabla}^{\mu}\psi \to \bar{\psi}(x)\partial^{\mu}\psi(x) + O(1)_{\text{instanton effect}}$$

Need to revisit the $e^q(x)$ calculation with the correct effective operator

Wandzura-Wilczek approximation

Wandzura-Wilczek approximation can be verified via instanton vacuum systematically [$1/N_c$, diluteness of instantons]

Twist-4 form factor $\bar{c}_{q,g}(t)$ [JYKim, C.Weiss in progress]

Reconstruct the GPD and PDFs / higher Mellin moments [JYKim, C.Weiss in progress]

$$\bar{\psi}\gamma^{[\mu}\overleftrightarrow{\nabla}^{\{\nu_1]}\overleftrightarrow{\nabla}^{\nu_2}\ldots\overleftrightarrow{\nabla}^{\nu_n\}}\psi$$

x-dependence of the Orbital angular momentum

$$\bar{\psi}\gamma^{[\mu}\overleftrightarrow{\nabla}^{\{\nu_1]}\overleftrightarrow{\nabla}^{\nu_2}\ldots\overleftrightarrow{\nabla}^{\nu_n\}}\gamma_5\psi$$

x-dependence of the spin-orbit correlation

Summary

Main strategy: instanton vacuum, effective theory, $1/N_c$ expansion, spin-flavor symmetry

Construct description of QCD vacuum based on “Instanton vacuum”

Derive effective theory (dynamics), also known as “chiral quark-soliton model”, from chiral symmetry breaking → baryon structure

Derive effective operators resulting from QCD operators in same scheme

- Use systematic parametric approximation: Packing fraction, 1/Nc
- Obtain effective operators consistent with effective dynamics
- Preserve operator relations from QCD equation of motion

Estimate the nucleon matrix element of the derived effective operator within the effective theory using the same scheme (1/Nc and packing fraction):

- Angular momentum decomposition (twist-3)

Conclusion: Gluon contribution (instanton effect) is very important in the AM decomposition

Thank you very much!