Interpolation of the ‘t Hooft model between Instant and Light-Front Dynamics

Chueng-Ryong Ji
North Carolina State University

APCTP SEMINAR

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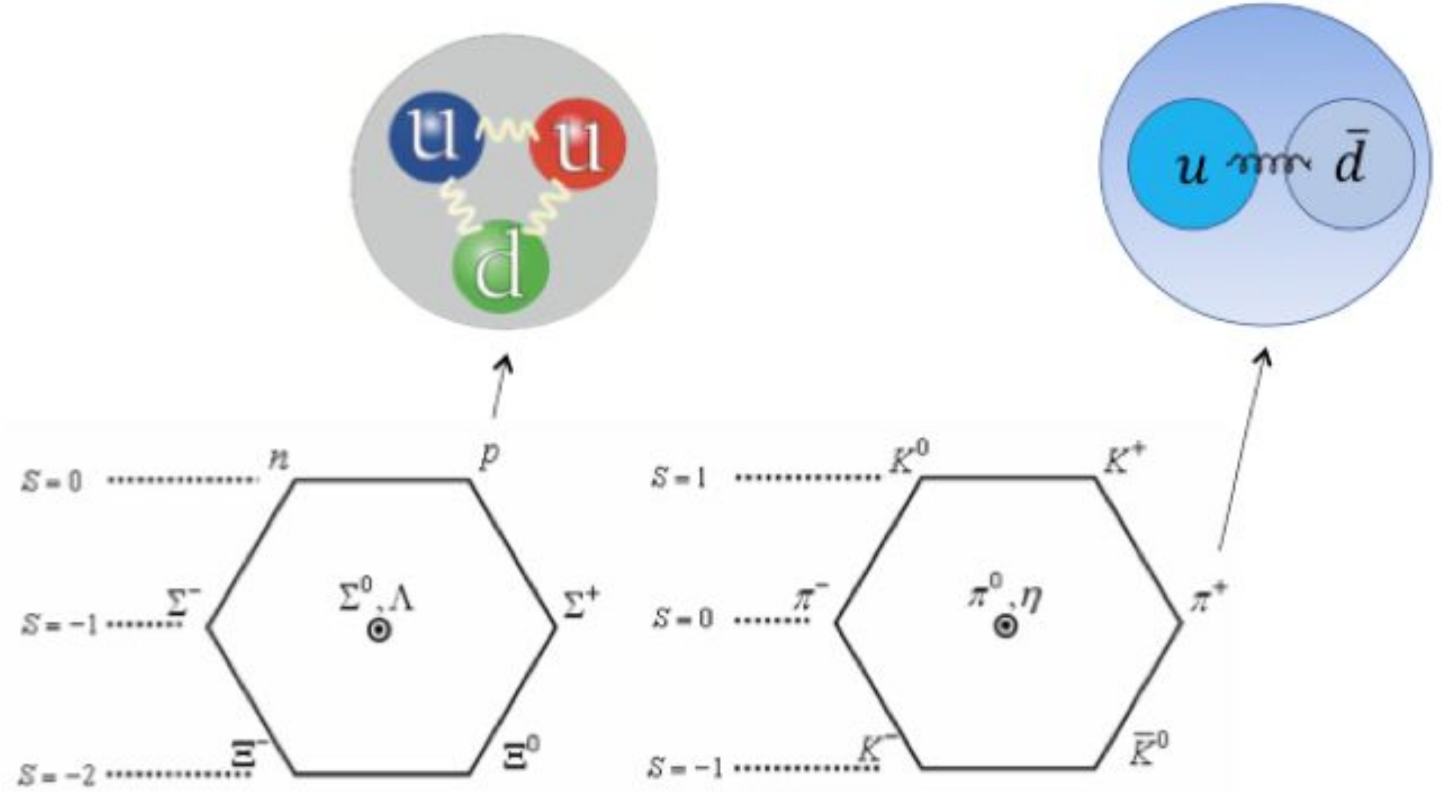
Outline

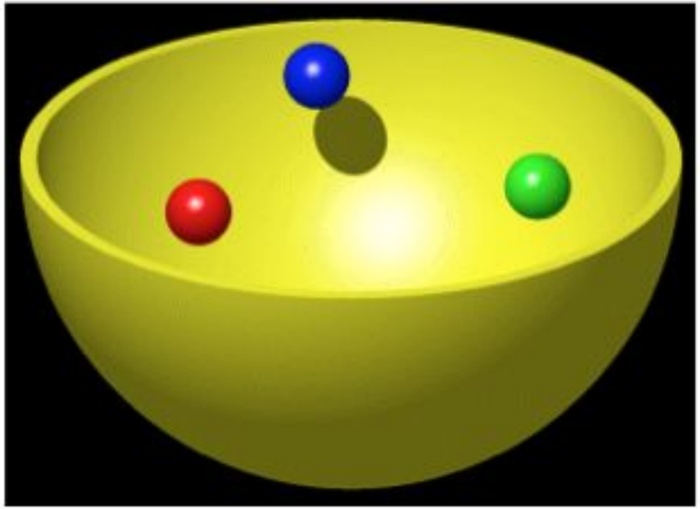
- Motivation
- Why Light-Front?
- Link the Light-Front Dynamics and the Instant Form Dynamics
- ‘t Hooft model as a toy QCD
- Quark mass gap solution
- Quark-Antiquark bound state equation
- Link to the Light-Front Quark Model
- Regge trajectories and the pionic ground state
- Parton Distribution Functions for Hadron Phenomenology
- Conclusions and Outlook

How do we understand the Quark Model in Quantum Chromodynamics?

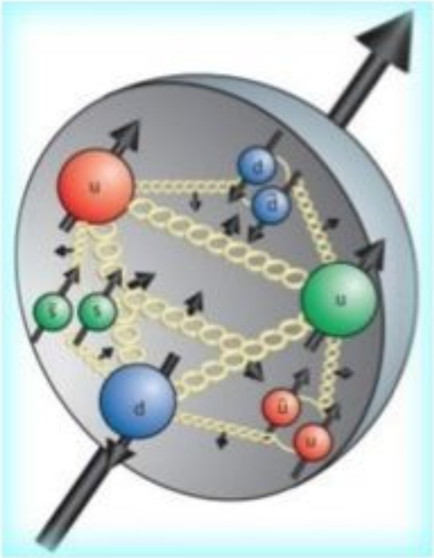


$$M_p = 938.272046 \pm 0.000021\, MeV$$

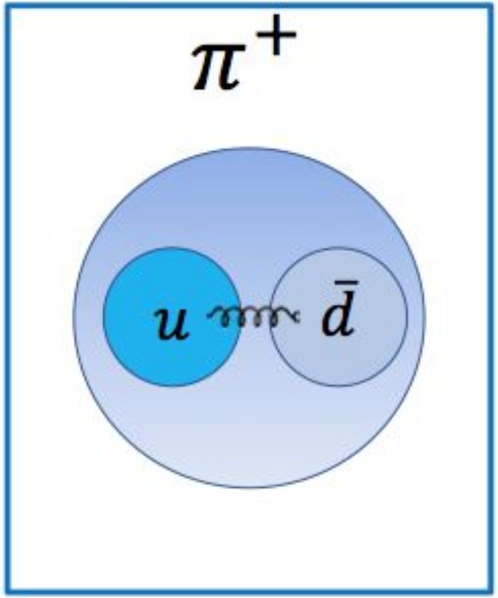
$$M_n = 939.565379 \pm 0.000021\, MeV$$



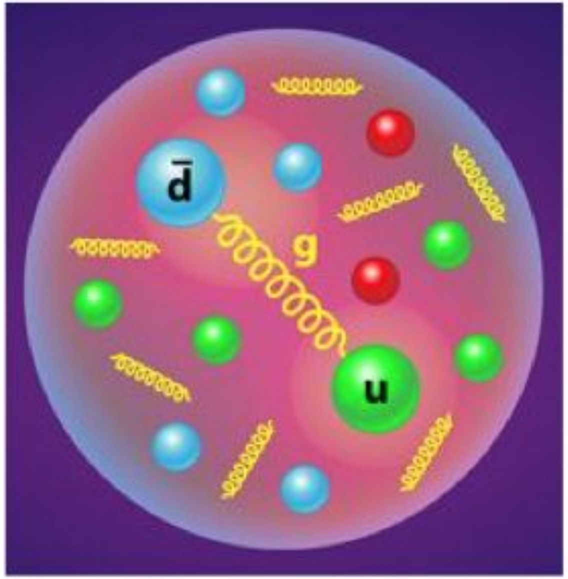
$\Longrightarrow$



$$m_u = 2.3^{+0.7}_{-0.5}\ MeV \quad ; \quad m_d = 4.8^{+0.7}_{-0.3}\ MeV$$



vs.



Constituent Quark Model

$$M = m_1 + m_2 + A\frac{\vec{s}_1 \cdot \vec{s}_2}{m_1 m_2}$$

$$m_u = m_d = 310 MeV/c^2$$

$$A = \left(\frac{2m_u}{\hbar}\right)^2 160 MeV/c^2$$

Quantum Chromodynamics

Isospin symmetry

Chiral symmetry

$$\mathrm{SU}(2)_\mathrm{R} \times \mathrm{SU}(2)_\mathrm{L}$$

Spontaneous symmetry breakdown

Goldstone Bosons

$$F_\pi^2 M_\pi^2 = -(m_u + m_d)\langle 0|\bar{u}u|0\rangle$$

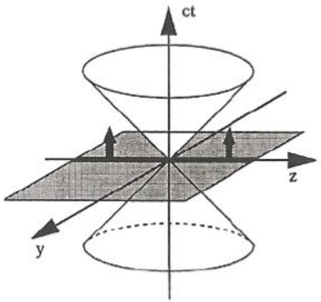
Effective field theory

Why Light-Front?

- Distinguished Vacuum Property
- Maximum Number of Kinematic Operators
- Distinguished Conformal Symmetry
  (Work in progress @ NCSU group meetings)

Dirac's Proposition for Relativistic Dynamics

$k_1^+ = k_2^+ = 0$



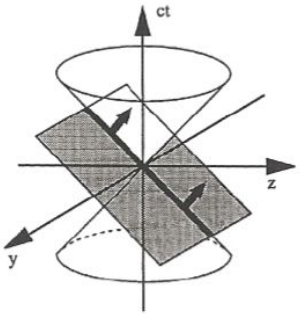
The instant form

Equal t **1949** *Equal τ*

$$p^0 \leftrightarrow p^- = p^0 - p^3$$

$$(p^1, p^2) \leftrightarrow \vec{p}_\perp$$

$$p^3 \leftrightarrow p^+ = p^0 + p^3$$

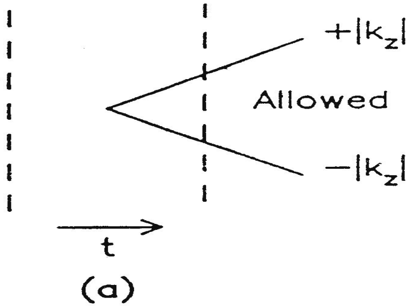


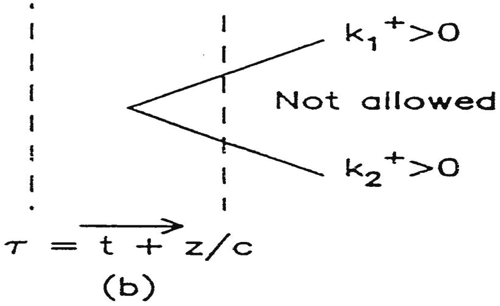
The front form

Energy-Momentum Dispersion Relations

$$p^0 = \sqrt{\vec{p}^{\,2} + m^2}$$

$$p^- = \frac{\vec{p}_\perp^{\,2} + m^2}{p^+}$$





Except zero-modes

$$k_1^+ = k_2^+ = 0$$

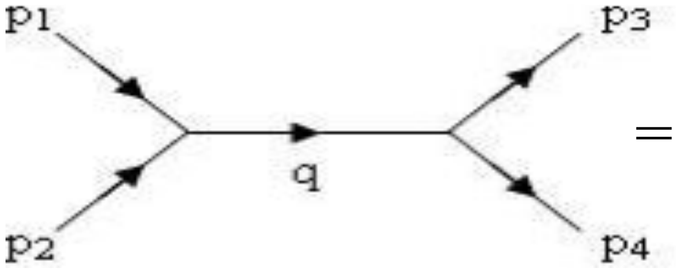
IFD

Instant Form Dynamics

LFD

Light-Front Dynamics

"$e^+e^- \to \mu^+\mu^-$"



$$= \frac{1}{q^2 - m^2} = \frac{1}{s - m^2} \qquad q^2 = (p_1 + p_2)^2 \neq m^2$$

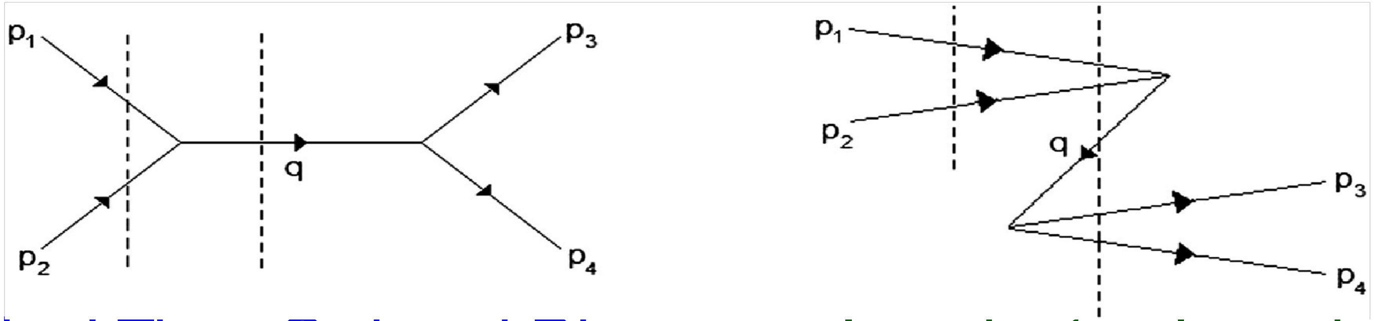
Four-momentum conservation but off-mass-shell

Feynman Diagram: Invariant under all 10 Poincaré generators

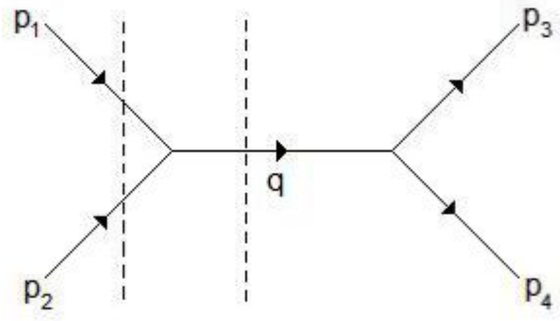
t→ (time evolution; time ordered process in QFT; Energy is not conserved within Δt)

$$(\Delta E)(\Delta t) \sim \hbar$$

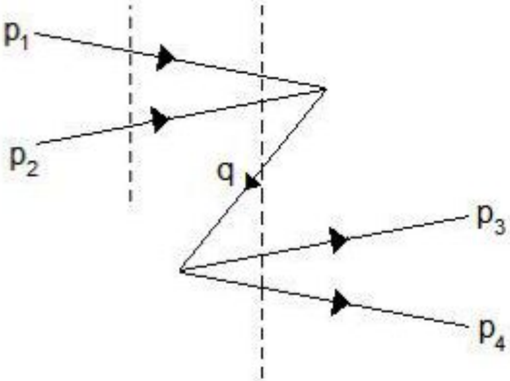
Three-momentum conservation but on-mass-shell



Individual Time-Ordered Diagrams: Invariant only under translation and rotation (6 kinematic generators)



(a)



(b)

$$\Sigma^a_{\mathrm{IFD}} = \frac{1}{2q^0}\left(\frac{1}{p_1^0 + p_2^0 - q^0}\right) \qquad \Sigma^b_{\mathrm{IFD}} = -\frac{1}{2q^0}\left(\frac{1}{p_1^0 + p_2^0 + q^0}\right)$$

$$\Sigma^a_{IFD} + \Sigma^b_{IFD} = \frac{1}{2q^0}\left(\frac{1}{p_1^0 + p_2^0 - q^0} - \frac{1}{p_1^0 + p_2^0 + q^0}\right)$$

$$= \frac{1}{(p_1^0 + p_2^0)^2 - (q^0)^2}$$

$$= \frac{1}{\{(p_1^0 + p_2^0)^2 - (\vec{p}_1 + \vec{p}_2)^2\} - \{(q^0)^2 - \vec{q}^{\,2}\}}$$

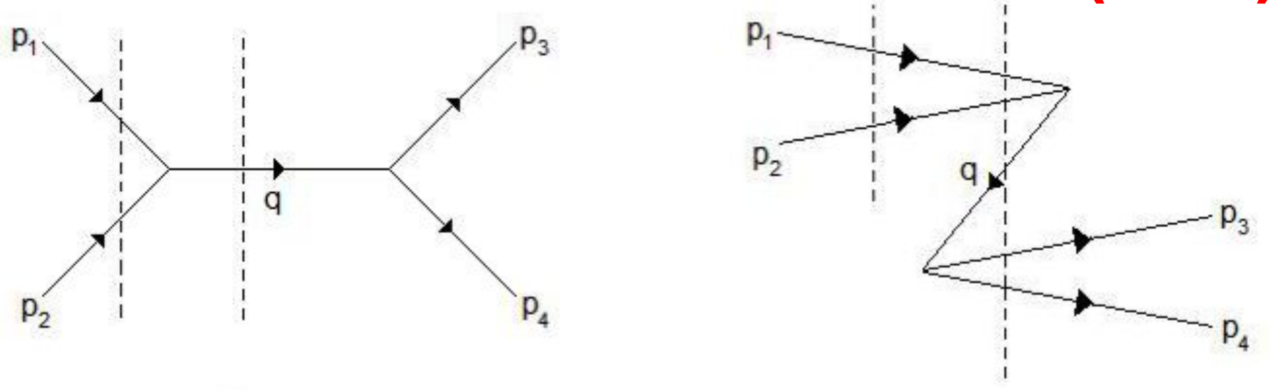
: Three-momentum conservation

$$= \frac{1}{(p_1 + p_2)^2 - q^2}$$

: $q^2 = m^2$; on -mass-shell

$$= \frac{1}{s - m^2}$$

Infinite Momentum Frame (IMF) Approach



(a) (b)

$$\frac{1}{E_1 + E_2 - Eq}$$

$$-\frac{1}{Eq + E_3 + E_4}$$

$$= -\frac{1}{Eq + E_1 + E_2}$$

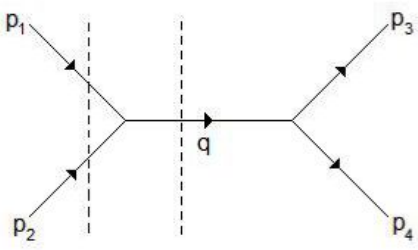
$$\rightarrow \quad 0$$

S.Weinberg, PR158,1638(1967)
"Dynamics at Infinite Momentum"

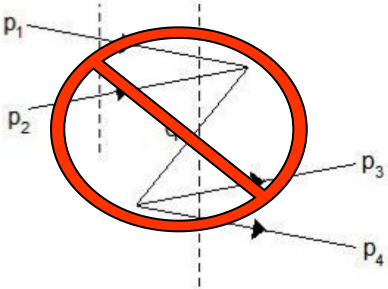
Note that this is still in the instant form (IFD).

However, in LFD, (b) drops for any reference frame (not just for IMF)

τ (= t+z/c) →



(a)



(b)

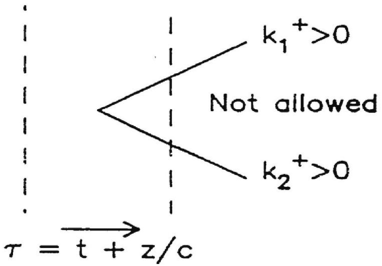
$$\Sigma^a_{LFD} + \Sigma^b_{LFD} = \frac{1}{q^+}\left(\frac{1}{p_1^- + p_2^- - q^-} + 0\right)$$

$$= \frac{1}{q^+\left(\frac{(p_1+p_2)^2 + (\vec{p}_{1\perp} + \vec{p}_{2\perp})^2}{(p_1+p_2)^+} - \frac{m^2 + \vec{q}_\perp^2}{q^+}\right)}$$

$$= \frac{1}{(p_1+p_2)^2 - m^2}$$

$$= \frac{1}{s - m^2}$$

$$p^- = \frac{\vec{p}_\perp^{\,2} + m^2}{p^+}$$



Conformal Symmetry in IFD

1D

	P_0	$\mathfrak{K}_0$	D
P_0	0	$2iD$	iP_0
$\mathfrak{K}_0$	$-2iD$	0	$-i\mathfrak{K}_0$
D	$-iP_0$	$i\mathfrak{K}_0$	0

$$P_0 = i\partial_t$$

$$D = it\partial_t$$

$$\mathfrak{K}_0 = it^2\partial_t$$

2D

	P_0	$\mathfrak{K}_0$	D	$-P_3$	$\mathfrak{K}_3$	K^3
P_0	0	$2iD$	iP_0	0	$2iK^3$	$-iP_3$
$\mathfrak{K}_0$	$-2iD$	0	$-i\mathfrak{K}_0$	$-2iK^3$	0	$-i\mathfrak{K}_3$
D	$-iP_0$	$i\mathfrak{K}_0$	0	iP_3	$i\mathfrak{K}_3$	0
$-P_3$	0	$2iK^3$	$-iP_3$	0	$2iD$	iP_0
$\mathfrak{K}_3$	$-2iK^3$	0	$-i\mathfrak{K}_3$	$-2iD$	0	$-i\mathfrak{K}_0$
K^3	iP_3	$i\mathfrak{K}_3$	0	$-iP_0$	$i\mathfrak{K}_0$	0

Work in progress with Hariprashad Ravikumar et.al.@NCSU group meetings

Conformal Symmetry in LFD

1D

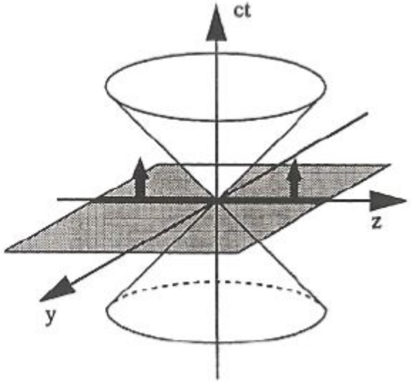
	P_+	$\mathfrak{K}_+$	D_+
P_+	0	$2\sqrt{2}iD_+$	$\sqrt{2}iP_+$
$\mathfrak{K}_+$	$-2\sqrt{2}iD_+$	0	$-\sqrt{2}i\mathfrak{K}_+$
D_+	$-\sqrt{2}iP_+$	$\sqrt{2}i\mathfrak{K}_+$	0

2D

	P_+	$\mathfrak{K}_+$	D_+	P_-	$\mathfrak{K}_-$	D_-
P_+	0	$2\sqrt{2}iD_+$	$\sqrt{2}iP_+$	0	0	0
$\mathfrak{K}_+$	$-2\sqrt{2}iD_+$	0	$-\sqrt{2}i\mathfrak{K}_+$	0	0	0
D_+	$-\sqrt{2}iP_+$	$\sqrt{2}i\mathfrak{K}_+$	0	0	0	0
P_-	0	0	0	0	$2\sqrt{2}iD_-$	$\sqrt{2}iP_-$
$\mathfrak{K}_-$	0	0	0	$-2\sqrt{2}iD_-$	0	$-\sqrt{2}i\mathfrak{K}_-$
D_-	0	0	0	$-\sqrt{2}iP_-$	$\sqrt{2}i\mathfrak{K}_-$	0

$$P_\pm = \frac{P_0 \pm P_3}{\sqrt{2}},\ \mathfrak{K}_\pm = \frac{\mathfrak{K}_0 \mp \mathfrak{K}_3}{\sqrt{2}}\ ,\ \text{and}\ D_\pm = \frac{D \mp K^3}{\sqrt{2}}$$

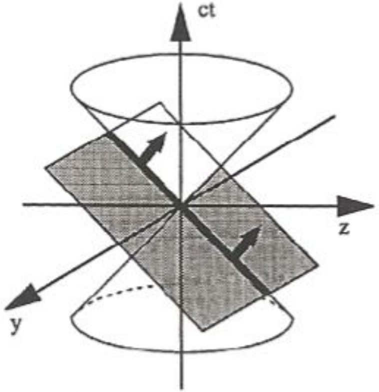
Can IFD and LFD be linked?



The instant form

1949

Yes, they can!



The front form

Traditional approach evolved from NR dynamics

Close contact with Euclidean space

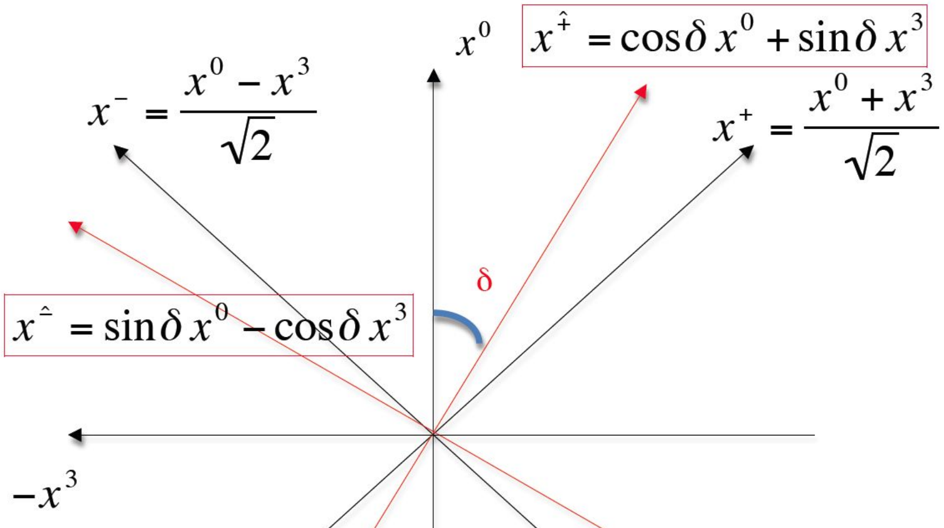
T-dept QFT, LQCD, IMF, etc.

Innovative approach for relativistic dynamics

Strictly in Minkowski space

DIS, PDFs, DVCS, GPDs, etc.

Interpolation between IFD and LFD



$$(IFD) \quad 0 \le \delta \le \frac{\pi}{4} \quad (LFD)$$

$$1 \ge \quad C \equiv \cos(2\delta) \quad \ge 0$$

K. Hornbostel, PRD45, 3781 (1992) – RQFT

C.Ji and S.Rey, PRD53,5815(1996) – Chiral Anomaly

C.Ji and C. Mitchell, PRD64,085013 (2001) – Poincare Algebra

C.Ji and A. Suzuki, PRD87,065015 (2013) – Scattering Amps

C.Ji, Z. Li and A. Suzuki, PRD91, 065020 (2015) – EM Gauges

Z.Li, M. An and C.Ji, PRD92, 105014 (2015) – Spinors

C.Ji, Z.Li, B.Ma and A.Suzuki, PRD98, 036017(2018) – QED

B.Ma and C.Ji, arXiv:2105.09388v1[hep-ph], PRD104, 036004(2021) – QCD_{1+1}

Large N_C QCD in 1+1 dim. ('tHooft Model)

$$\mathcal{L} = -\frac{1}{4} F^a_{\hat{\mu}\hat{\nu}} F^{\hat{\mu}\hat{\nu}a} + \bar{\psi}(i\gamma^{\hat{\mu}} D_{\hat{\mu}} - m)\psi$$

$$D_{\hat{\mu}} = \partial_{\hat{\mu}} - igA^a_{\hat{\mu}} t_a$$

$$F^a_{\hat{\mu}\hat{\nu}} = \partial_{\hat{\mu}} A^a_{\hat{\nu}} - \partial_{\hat{\nu}} A^a_{\hat{\mu}} + g f^{abc} A^b_{\hat{\mu}} A^c_{\hat{\nu}}$$

'tHooft Coupling $\lambda = \frac{g^2 (N_c - 1/N_c)}{4\pi}$ **and** **mass** m

$$g \to 0, N_C \to \infty; \lambda \to finite$$

Short List of LFD vs. IFD References

- G.‘tHooft, NPB75,461(74) - LFD
- Y.Frishman, et al., PRD15(75) - Interpol Gauges IFD&LFD
- I.Bars&M.Green, PRD17,537(78) - IFD(formulation)
- A.Zhitnitsky, PLB165,405(85) - LFD(chiral sym breaking)
- M.Li, et al., JPG13, 915(87) - IFD(rest frame)
- K.Hornbostel, Ph.D. Dissertation(88) - LFD(DLCQ)
- M.Burkardt, PRD53,933(96) - LFD(vacuum condensates)
- Y.Kalashnikov&A.Nefed’ev,Phys.-Usp.45,346(‘02) - IFD(rev)
- Y. Jia, et al., JHEP11, 151(‘17) - IFD(moving frame)
- Y. Jia, et al., PRD98, 054011(‘18) - IFD(quasi-PDFs)
- B.Ma&C.Ji, PRD104,036004(‘21) - Link IFD&LFD

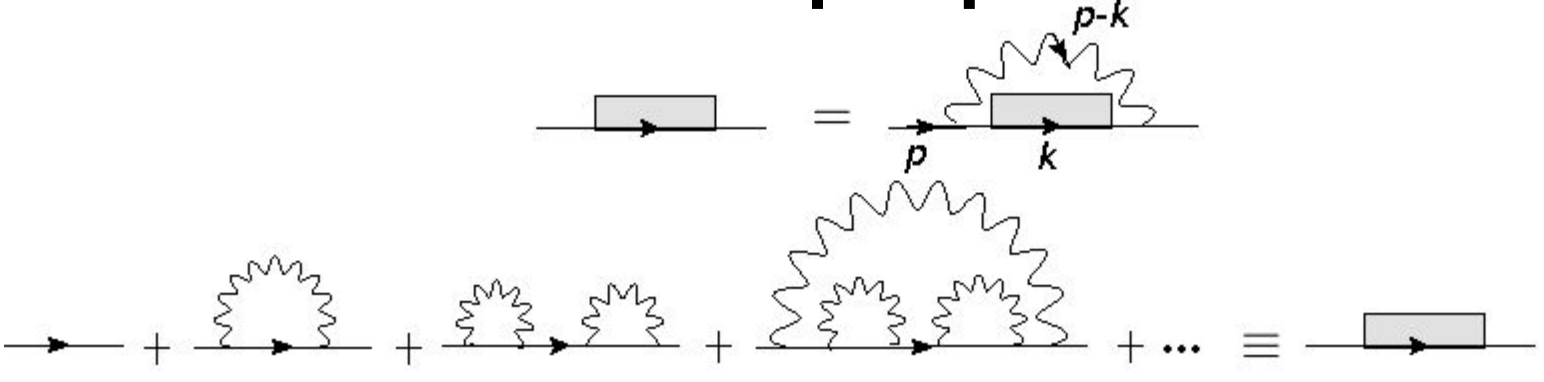
Interpolating Axial Gauge

$$A^a_{\hat{-}} = 0$$

$$A^a_1 = 0 \qquad\qquad A^+_a = 0$$

$$\mathcal{L} = \frac{1}{2}(\partial_{\hat{-}} A^a_{\hat{+}})^2 + \bar{\psi}(i\gamma^{\hat{+}} D_{\hat{+}} + i\gamma^{\hat{-}}\partial_{\hat{-}} - m)\psi$$

Mass Gap Equation



$$\equiv i\,\Sigma(p)$$

$$\Sigma(p_{\hat{-}}) = i\frac{\lambda}{2\pi}\int \frac{dk_{\hat{-}}\,dk_{\hat{+}}}{(p_{\hat{-}} - k_{\hat{-}})^2}\gamma^{\hat{+}}\frac{1}{\not{k} - m - \Sigma(k_{\hat{-}}) + i\epsilon}\gamma^{\hat{+}}$$

Coulomb Gauge vs. Axial Gauge

$$A^0 = 0$$
Coulomb Gauge

Link Using Interpolating Dynamics

$$A^{\hat{+}} = 0$$
Interpolating Coulomb Gauge

Interpolate to LFD

$$A^+ = A_- = 0$$
Light-Front Gauge

Apply Interpolating Axial Gauge

$$A_{\hat{-}} = 0$$
Interpolating Axial Gauge

Interpolate back to IFD

$$A^1 = 0$$
Axial Gauge

Hunter Duggin, C.-R. Ji and Bailing Ma, PoS(SPIN2023)051

Fermion Propagator

Free Propagator

$$S_f(p) = \frac{1}{\not{p} - m + i\varepsilon}$$

Interacting Propagator

$$S(p) = \frac{1}{\not{p} - m - \sum(p) + i\varepsilon}$$

$$= \frac{F(p)}{\not{p} - M(p) + i\varepsilon}$$

$$\Sigma(p) = \Sigma_s(p) + \Sigma_v(p)\not{p}$$

$$F(p) = \left(1 - \Sigma_v(p)\right)^{-1}$$ "Wave function renormalization factor"

$$M(p) = \frac{m + \Sigma_s(p)}{1 - \Sigma_v(p)}$$ "Renormalized fermion mass function"

Energy-Momentum Dispersion Relation

Free particle | **Interacting particle**

$$E=\sqrt{p_z^2+m^2}$$

$$\theta_f=\tan^{-1}(p_z/m)$$

$$\beta=p_z/E$$

$$=\sin\theta_f$$

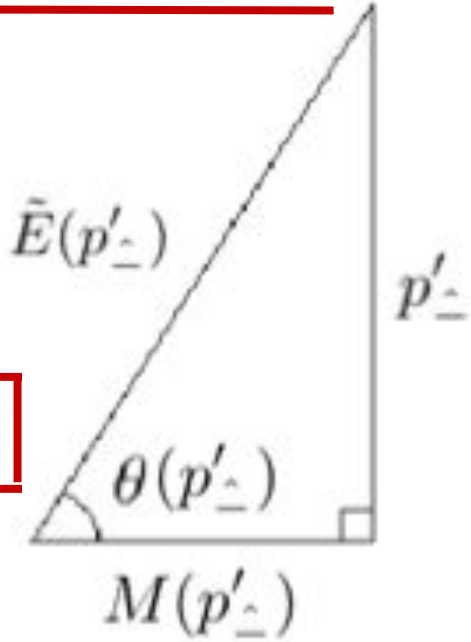
$$=\tanh\eta$$

$$\frac{F(p'_-)E(p'_-)}{\sqrt{C}}=\sqrt{p'^2_- + M(p'_-)^2}\equiv\tilde{E}(p'_-)$$

$$\theta(p'_-)=\theta_f(p'_-)+2\zeta(p'_-)$$

$$\begin{pmatrix} b^i(p'_-) \\ d^{+i}(p'_-) \end{pmatrix}=\begin{pmatrix} \cos\zeta(p'_-) & -\sin\zeta(p'_-) \\ \sin\zeta(p'_-) & \cos\zeta(p'_-) \end{pmatrix}\begin{pmatrix} b^i_f(p'_-) \\ d^{+i}_f(p'_-) \end{pmatrix}$$

$$b^i_f|0>=0,\ d^i_f|0>=0 \quad \text{vs.} \quad b^i|\Omega>=0,\ d^i|\Omega>=0$$



Interpolation

$$(E,p_z)\Rightarrow(p^+/\sqrt{C},\, p_-/\sqrt{C}\equiv p'_-)$$

Mass Gap Equation in Scaled Variables

$$\bar{p}'_\perp = \frac{\bar{p}_\perp}{\sqrt{\mathbb{C}}}, \; \bar{E}' = \frac{\bar{E}}{\sqrt{\mathbb{C}}}, \bar{p}_\perp = \frac{p_\perp}{\sqrt{2\lambda}}, \; \bar{E} = \frac{E}{\sqrt{2\lambda}}, \bar{m} = \frac{m}{\sqrt{2\lambda}}$$

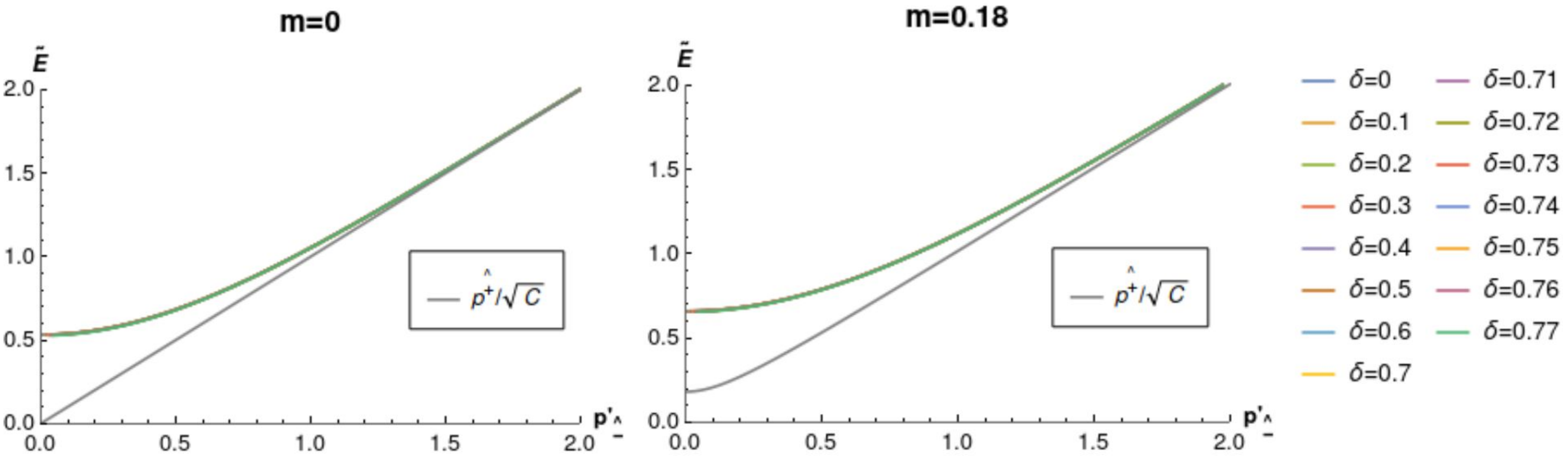
$$\bar{p}'_\perp \cos\theta(\bar{p}'_\perp) - \bar{m}\sin\theta(\bar{p}'_\perp) = \frac{1}{4}\int \frac{d\bar{k}'_\perp}{(\bar{p}'_\perp - \bar{k}'_\perp)^2} \sin\Big(\theta(\bar{p}'_\perp) - \theta(\bar{k}'_\perp)\Big)$$

$$\bar{E}'(\bar{p}'_\perp) = \bar{p}'_\perp \sin\theta(\bar{p}'_\perp) + \bar{m}\cos\theta(\bar{p}'_\perp) + \frac{1}{4}\int \frac{d\bar{k}'_\perp}{(\bar{p}'_\perp - \bar{k}'_\perp)^2} \cos\Big(\theta(\bar{p}'_\perp) - \theta(\bar{k}'_\perp)\Big)$$

$$\frac{p_\perp}{\mathbb{C}}\cos\theta(p_\perp) - \frac{m}{\sqrt{\mathbb{C}}}\sin\theta(p_\perp) = \frac{\lambda}{2}\int \frac{dk_\perp}{(p_\perp - k_\perp)^2}\sin\left(\theta(p_\perp) - \theta(k_\perp)\right)$$

$$E(p_\perp) = p_\perp \sin\theta(p_\perp) + \sqrt{\mathbb{C}}m\cos\theta(p_\perp) + \frac{\mathbb{C}\lambda}{2}\int \frac{dk_\perp}{(p_\perp - k_\perp)^2}\cos\left(\theta(p_\perp) - \theta(k_\perp)\right)$$

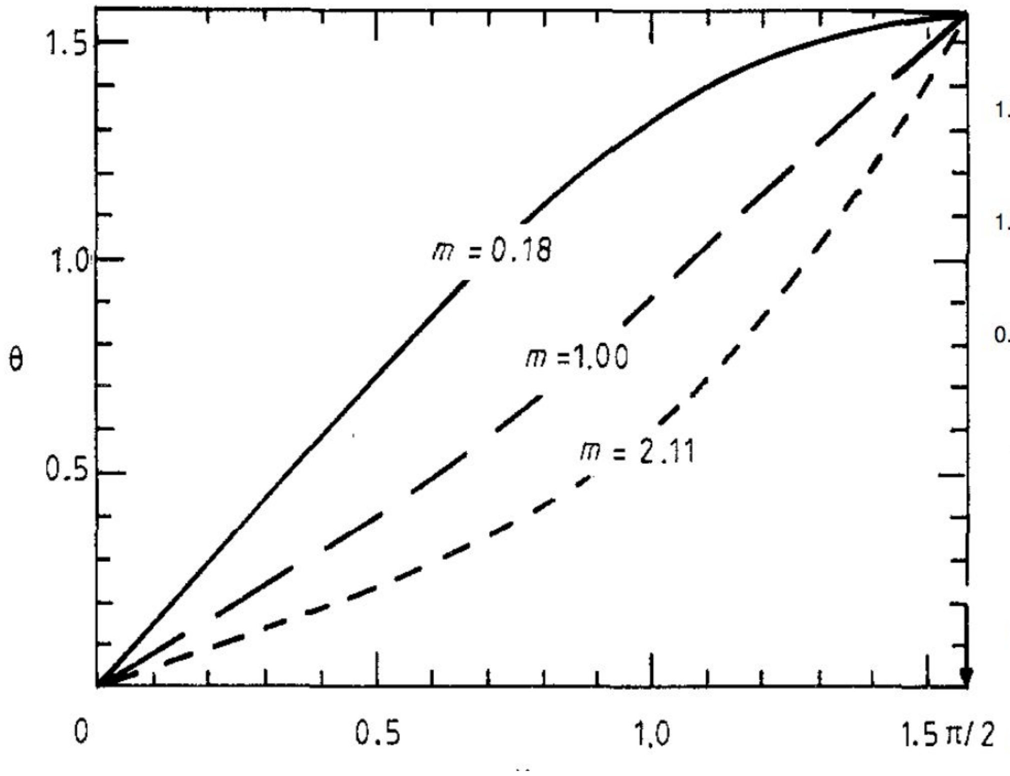
Mass Gap Solutions

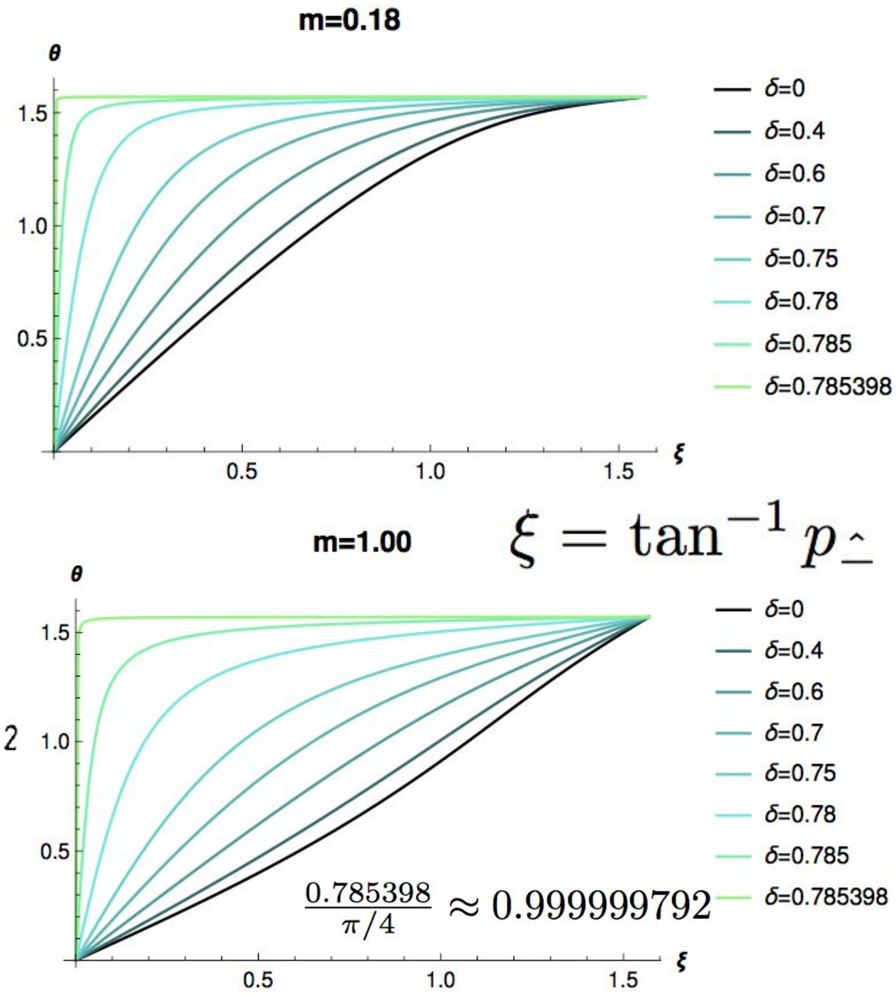


$$\tilde{E}(0) = \frac{F(0)E(0)}{\sqrt{\mathbb{C}}} = M(0)$$

m	0	0.045	0.18	0.749	1.00	2.11	4.23
$M(0)$	0.532778	0.563644	0.659112	1.10105	1.31167	2.30969	4.34358
$F(0)$	−0.495173	−0.584175	−0.987673	4.11079	2.17976	1.22134	1.05526

$$m \lesssim 0.56$$

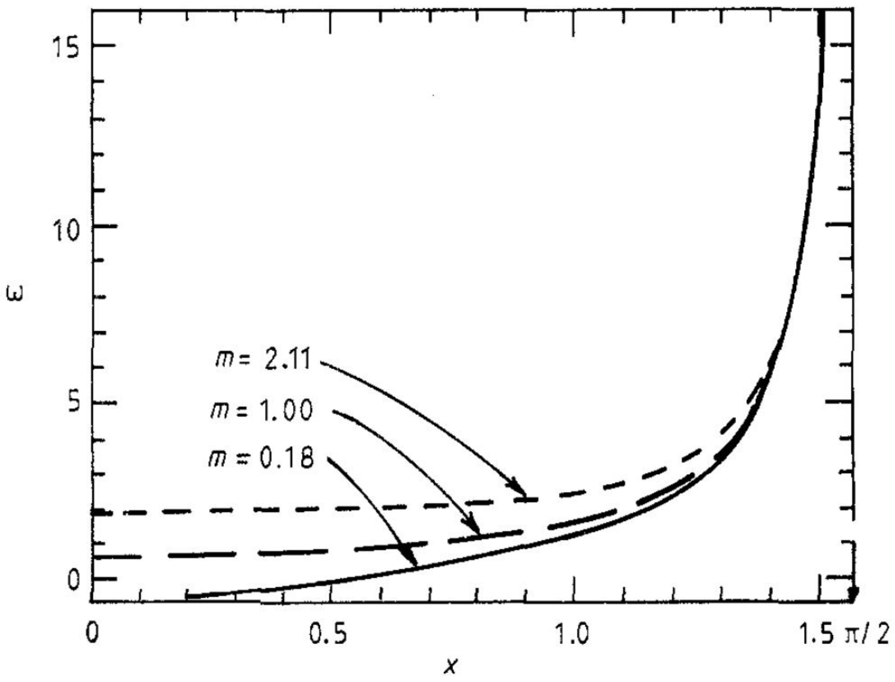




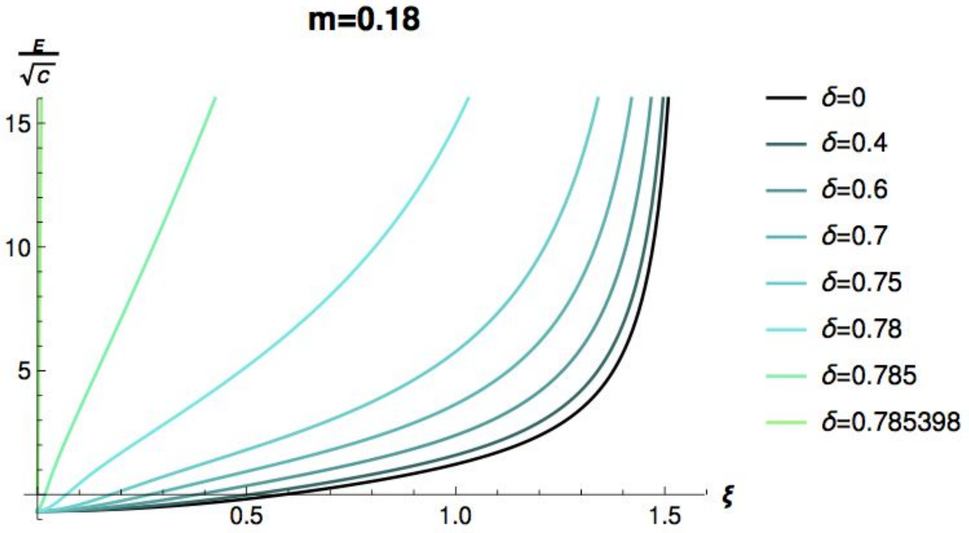
$$\xi = \tan^{-1} p_{\perp}$$

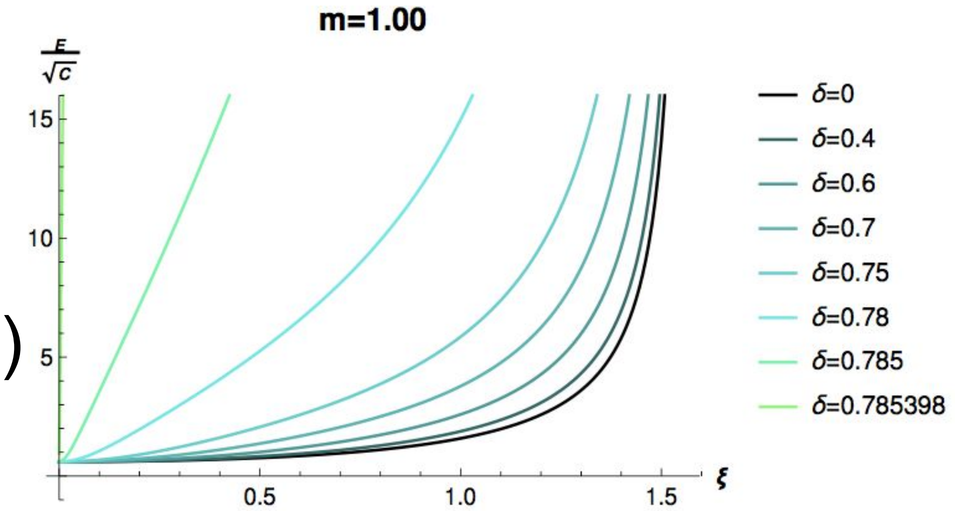
$$\frac{0.785398}{\pi/4} \approx 0.999999792$$

- M.Li, et al., JPG13, 915(87)
  - IFD(rest frame)

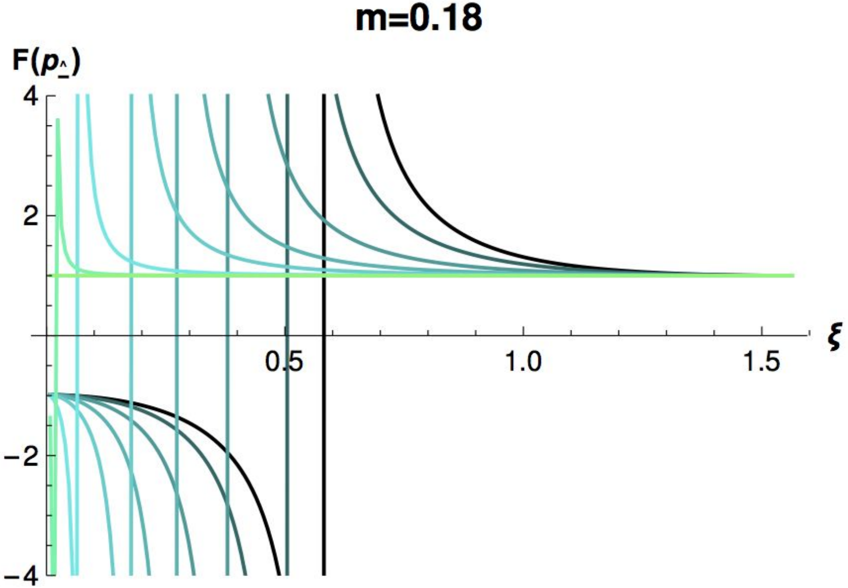


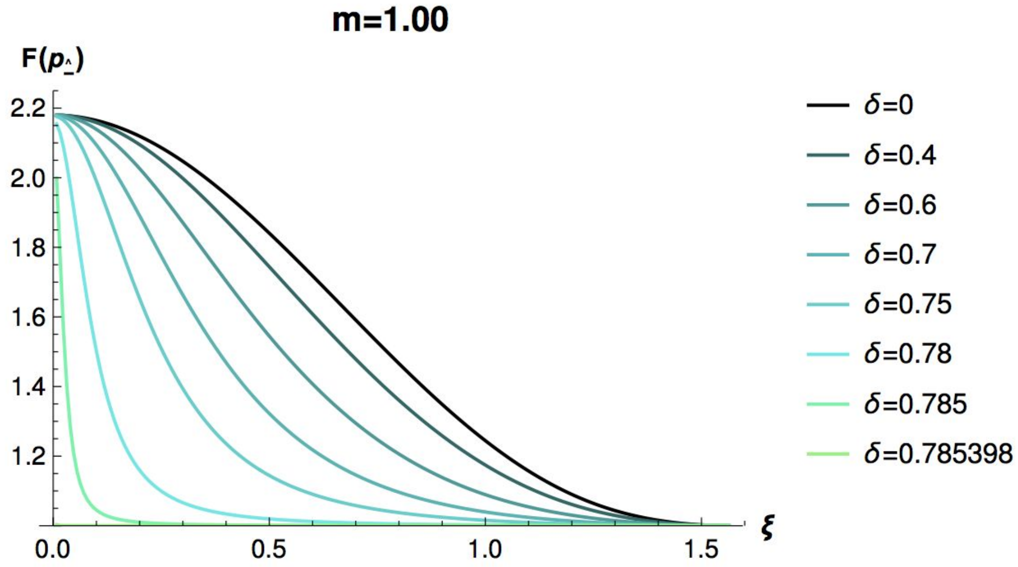
- M.Li, et al., JPG13, 915(87)
  - IFD(rest frame)

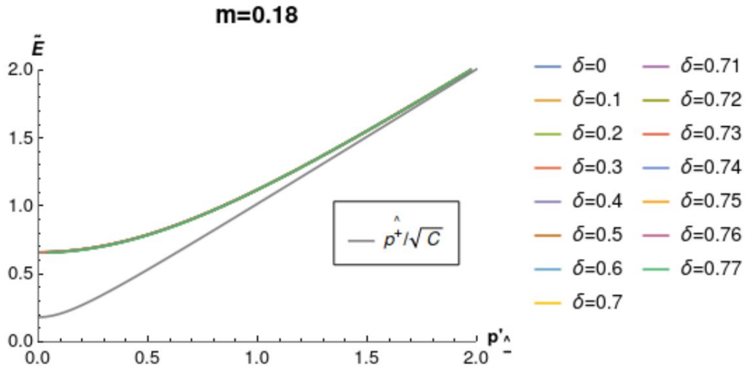




Wave Function Renormalization Factors



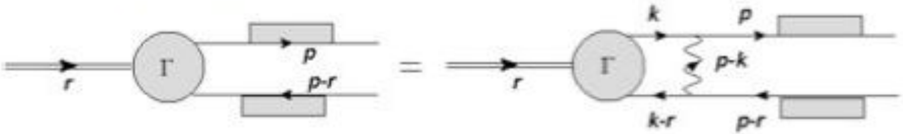




$$\frac{F(p'_{\hat{-}})E(p'_{\hat{-}})}{\sqrt{C}} = \sqrt{p'^{2}_{\hat{-}} + M(p'_{\hat{-}})^{2}} \equiv \tilde{E}(p'_{\hat{-}})$$

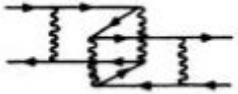
BOUND-STATE EQUATION

$$\Gamma(r,p) = \frac{i\lambda}{2\pi} \fint \frac{dk_{\hat{-}} dk_{\hat{+}}}{(p_{\hat{-}} - k_{\hat{-}})^2} S(p)\gamma^{\hat{+}}\Gamma(r,k)\gamma^{\hat{+}}S(p-r)$$



$$\left[-r_{\hat{+}} + \frac{-\mathbb{S}p_{\hat{-}} + E(p_{\hat{-}})}{\mathbb{C}} + \frac{\mathbb{S}(p_{\hat{-}} - r_{\hat{-}}) + E(p_{\hat{-}} - r_{\hat{-}})}{\mathbb{C}}\right] \hat{\phi}_+(r_{\hat{-}}, p_{\hat{-}})$$
$$= \lambda \fint \frac{dk_{\hat{-}}}{(p_{\hat{-}} - k_{\hat{-}})^2} \left[C(p_{\hat{-}}, k_{\hat{-}}, r_{\hat{-}})\hat{\phi}_+(r_{\hat{-}}, k_{\hat{-}}) - S(p_{\hat{-}}, k_{\hat{-}}, r_{\hat{-}})\hat{\phi}_-(r_{\hat{-}}, k_{\hat{-}})\right],$$

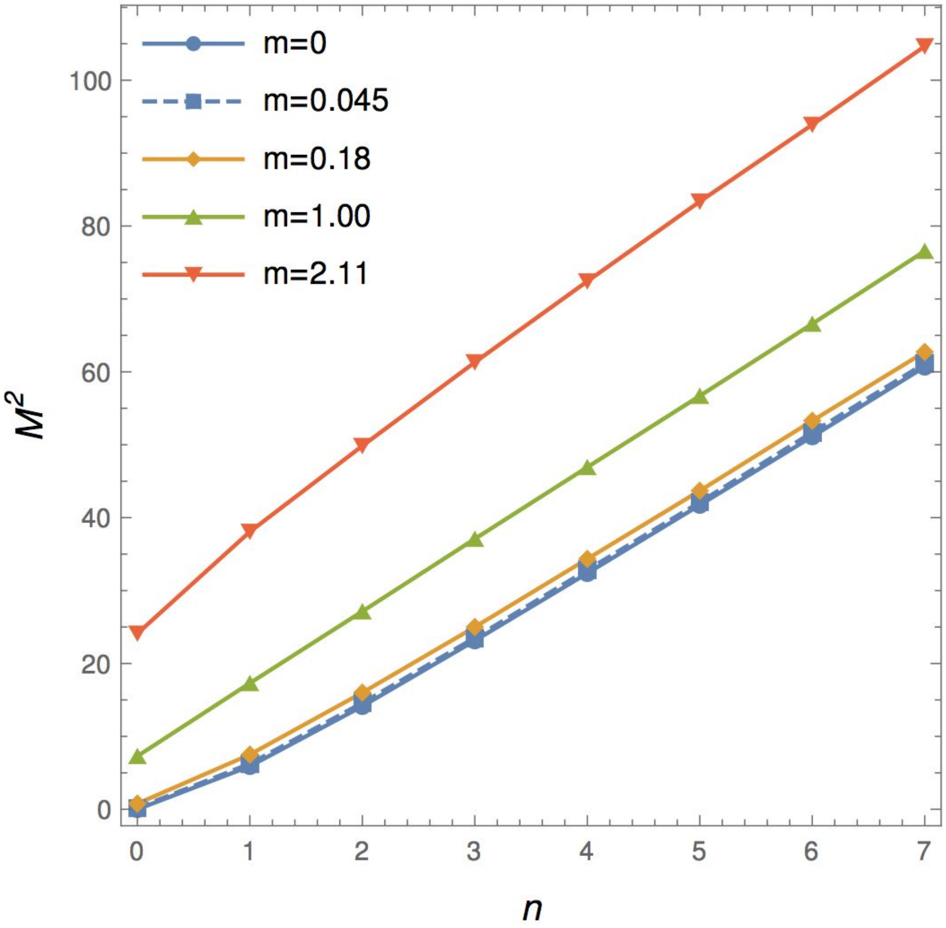
$$\left[r_{\hat{+}} + \frac{-\mathbb{S}(p_{\hat{-}} - r_{\hat{-}}) + E(p_{\hat{-}} - r_{\hat{-}})}{\mathbb{C}} + \frac{\mathbb{S}p_{\hat{-}} + E(p_{\hat{-}})}{\mathbb{C}}\right] \hat{\phi}_-(r_{\hat{-}}, p_{\hat{-}})$$
$$= \lambda \fint \frac{dk_{\hat{-}}}{(p_{\hat{-}} - k_{\hat{-}})^2} \left[C(p_{\hat{-}}, k_{\hat{-}}, r_{\hat{-}})\hat{\phi}_-(r_{\hat{-}}, k_{\hat{-}}) - S(p_{\hat{-}}, k_{\hat{-}}, r_{\hat{-}})\hat{\phi}_+(r_{\hat{-}}, k_{\hat{-}})\right].$$



LFD $\left[\mathcal{M}^2 - \frac{m^2 - 2\lambda}{x} - \frac{m^2 - 2\lambda}{1-x}\right] \phi(x) = -2\lambda \fint_0^1 \frac{dy}{(x-y)^2} \phi(y)$

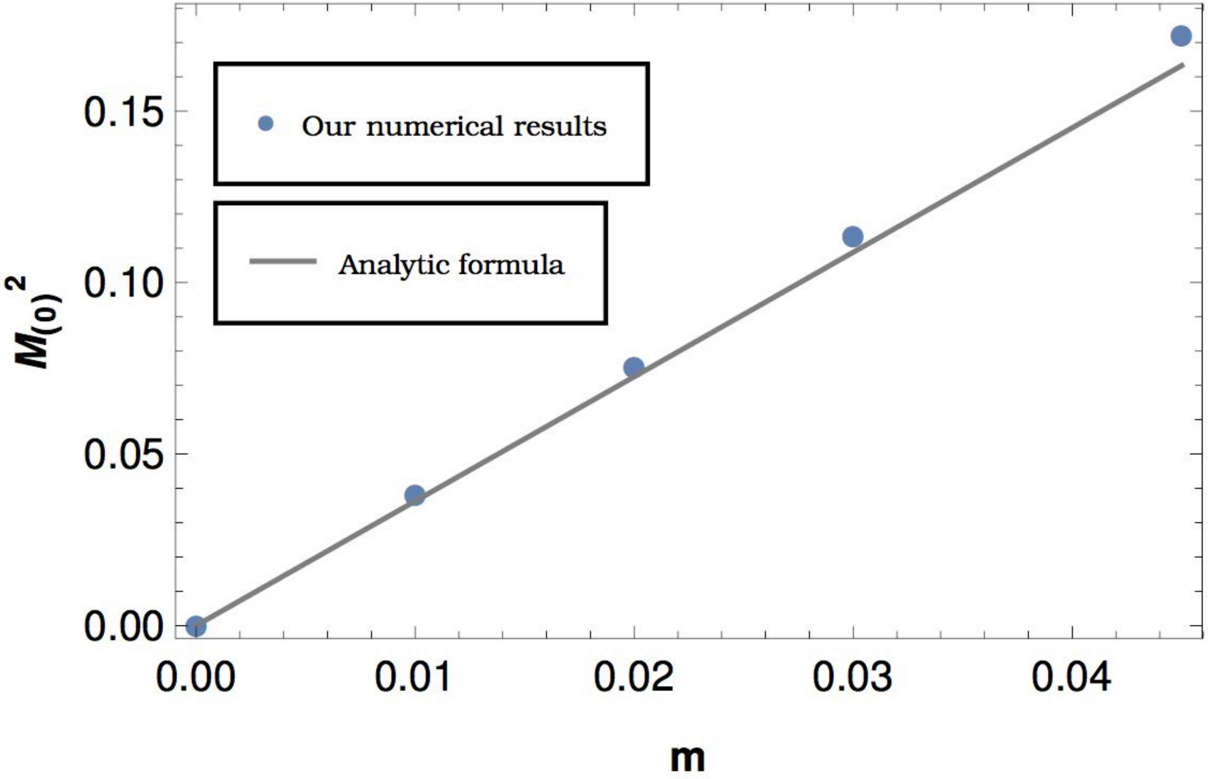
Meson Spectroscopy

- G.'tHooft, NPB75, 461(74) - LFD



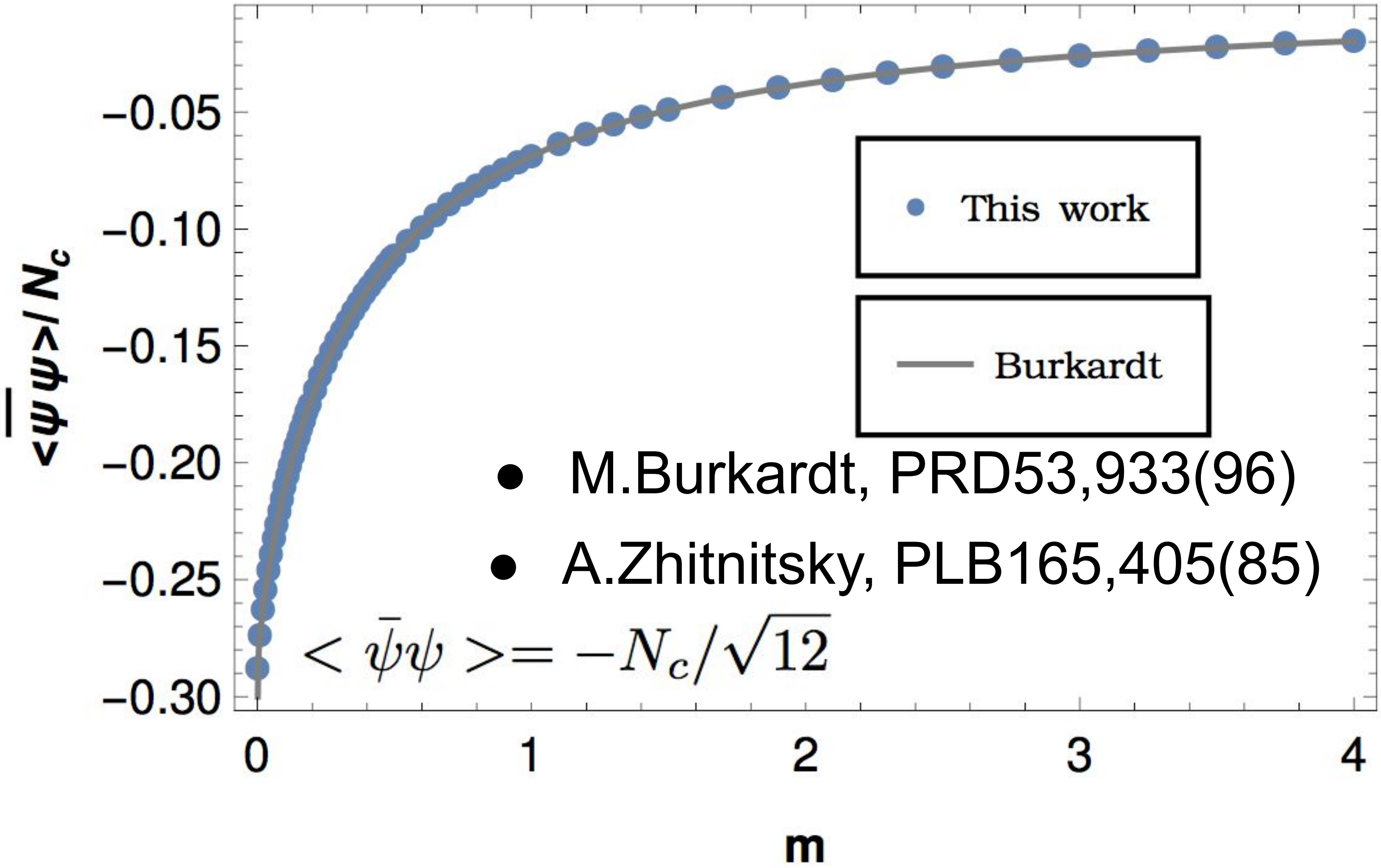
- M.Li, et al., JPG13, 915(87) - IFD (rest frame)
- Y. Jia, et al., JHEP11, 151('17) - IFD (moving frame)

Gell-Mann - Oaks - Renner Relation



$$\mathcal{M}_\pi^2 = -\frac{4m < \bar{\psi}\psi >}{f_\pi^2} = \sqrt{\frac{8\pi^2 m^2 \lambda}{3}}$$

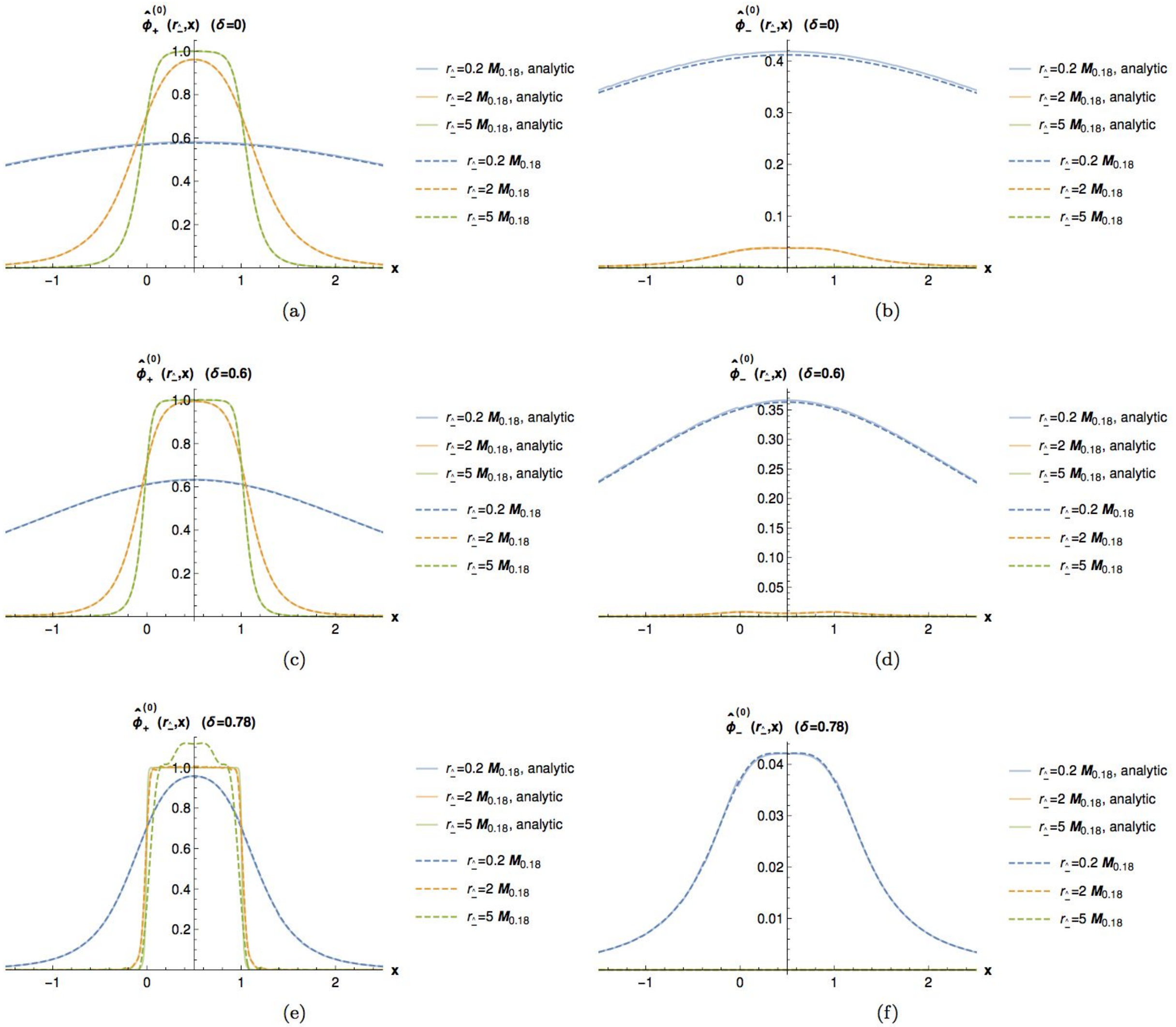
$$f_\pi = \sqrt{N_c/\pi}$$



$<\bar{\psi}\psi>/N_c$
−0.05
−0.10
−0.15
−0.20
−0.25
−0.30
0
1
2
3
4
m
This work
Burkardt
• M.Burkardt, PRD53,933(96)
• A.Zhitnitsky, PLB165,405(85)
$<\bar{\psi}\psi>=-N_c/\sqrt{12}$

Meson Ground-state Wave-function for m=0 case

$$\hat{\phi}_{\pm}^{(0)}(r_{\hat{-}}, p_{\hat{-}}) = \frac{1}{2}\left(\cos\frac{\theta(r_{\hat{-}} - p_{\hat{-}}) - \theta(p_{\hat{-}})}{2} \pm \sin\frac{\theta(r_{\hat{-}} - p_{\hat{-}}) + \theta(p_{\hat{-}})}{2}\right)$$



Parton Distribution Functions (PDFs)

$$q_n(x) = \int_{-\infty}^{+\infty} \frac{d\xi^-}{4\pi} e^{-ixP^+\xi^-}$$

$$\times \langle P_n^-, P^+ | \bar{\psi}(\xi^-)\gamma^+ \mathcal{W}[\xi^-, 0]\psi(0) | P_n^-, P^+ \rangle_C,$$

$$\mathcal{W}[\xi^-, 0] = \mathcal{P}\left[\exp\left(-ig_s \int_0^{\xi^-} d\eta^- A^+(\eta^-)\right)\right]$$

A^+=0 Gauge in LFD

Quasi-PDFs

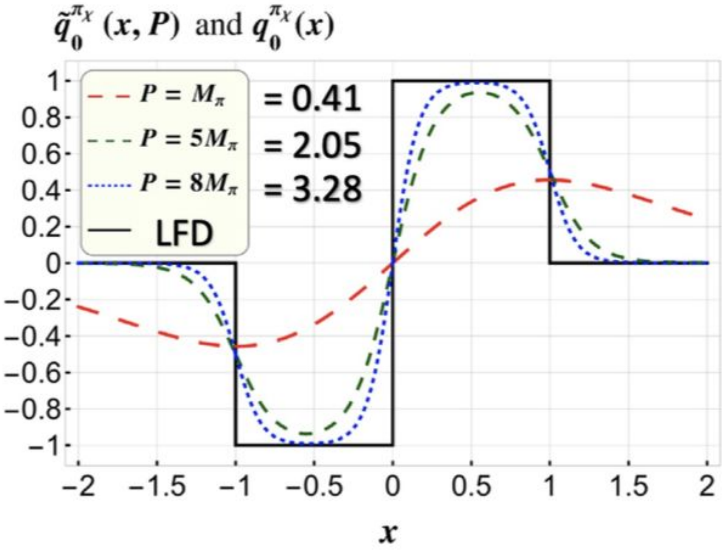
$$\tilde{q}_{(n)}(r_{\hat{-}}, x) = \int_{-\infty}^{+\infty} \frac{dx^{\hat{-}}}{4\pi} \, \mathrm{e}^{ix^{\hat{-}} r_{\hat{-}}}$$

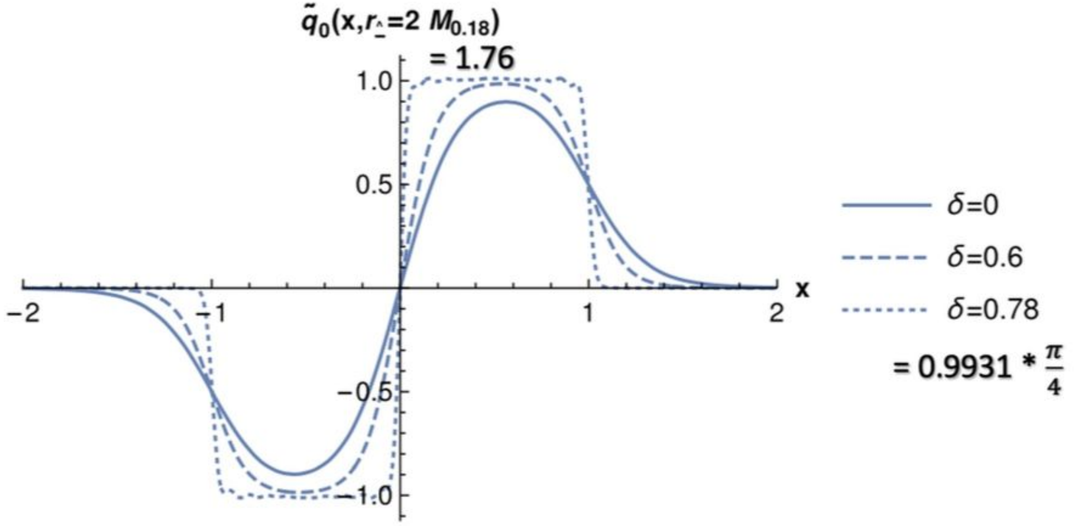
$$\times < r^{\hat{+}}_{(n)}, r_{\hat{-}} \mid \bar{\psi}(x^{\hat{-}}) \, \gamma_{\hat{-}} \, \mathcal{W}[x^{\hat{-}}, 0] \, \psi(0) \mid r^{\hat{+}}_{(n)}, r_{\hat{-}} >_C,$$

$$\mathcal{W}[x^{\hat{-}}, 0] = \mathcal{P}\left[\exp\left(-ig \int_0^{x^{\hat{-}}} dx'^{\hat{-}} A_{\hat{-}}(x'^{\hat{-}})\right)\right]$$

Interpolating dynamics

Y. Jia, et al., PRD98, 054011('18)
- IFD (quasi-PDFs)





B.Ma&C.Ji,PRD104,036004('21)
- Interpolating Dynamics

Extended Wick Rotation

$$p^0 \rightarrow \tilde{P}^0 = ip^0 \qquad (\delta = 0)$$

$For \quad 0 < \delta < \pi / 4,$

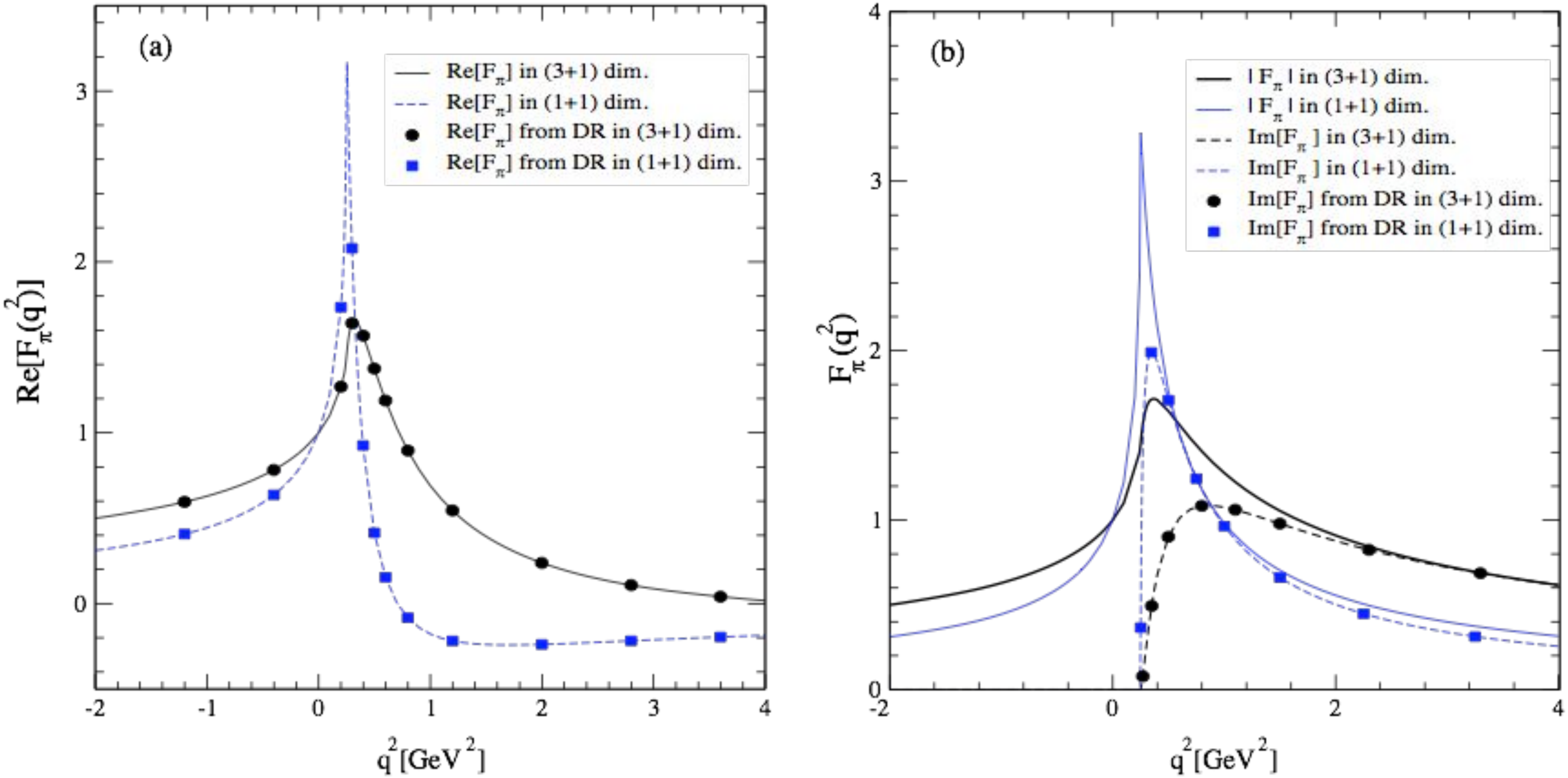
$$p^{\hat{+}} / \sqrt{C} \rightarrow \tilde{P}^{\hat{+}} / \sqrt{C} = ip^{\hat{+}} / \sqrt{C} \, .$$

Correspondence to Euclidean Space

$$p'^{2}_{\underline{\wedge}} = p^{2}_{\underline{\wedge}} / C \leftrightarrow -\tilde{P}^2$$

Conclusions and Outlook

- Interpolating ‘t Hooft model between IFD and LFD hints a plausible link between QCD and LFQM.
- Mass gap in 1+1D LFD is entirely provided by LF ZMs.
- Interpolation between IFD and LFD reveals the nature of LF ZMs and clarifies the prevailing notion of equivalence between the IMF approach and the LFD.
- Chiral condensate, GOR and Regge trajectories identical between IFD and LFD indicate the persistence of nontrivial vacuum even in LFD.
- LF vacuum is nontrivial due to LF ZMs.
- Interpolating quasi-PDFs offers a versatile tool to remedy the issue of LaMET approach in lattice QCD.
- Interpolating gauges (Coulomb vs. Axial), conformal algebra, 3+1D extension and mass gap timelike region deserve further investigation.



Yongwoo Choi, H.-M.Choi, C.-R. Ji and Y. Oh, PRD103,076002(2021)