Light-Front Scaled Interpolating Basis and Operators

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Outline

- Scaled interpolating basis and transformation.
- New light-front Scaled interpolating basis and transformation.
- Scaled interpolating operators.
- Possible applications of scaled interpolating dynamics .
- Light-front scaled interpolating operators.
- Possible application of light-front scaled interpolating dynamics.
- Summary and Conclusion.

Scaled Interpolating Transformation

$$x^N = H.x$$

$$\begin{pmatrix} \frac{x^{\hat{+}}}{\sqrt{\mathbb{C}}} \\ x^{\hat{1}} \\ x^{\hat{2}} \\ \frac{x_{\hat{-}}}{\sqrt{\mathbb{C}}} \end{pmatrix} = \begin{pmatrix} \frac{\cos\delta}{\sqrt{\mathbb{C}}} & 0 & 0 & \frac{\sin\delta}{\sqrt{\mathbb{C}}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{\sin\delta}{\sqrt{\mathbb{C}}} & 0 & 0 & \frac{\cos\delta}{\sqrt{\mathbb{C}}} \end{pmatrix} \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}$$

In the IFD ($\delta \to 0$), $\frac{x^{\hat{+}}}{\sqrt{\mathbb{C}}} \to x^0, x^{\hat{1}} \to x^1, x^{\hat{2}} \to x^2, \frac{x_{\hat{-}}}{\sqrt{\mathbb{C}}} \to x^3$, In the LFD ($\delta \to \frac{\pi}{4}$) , $x^{\hat{+}} \to x^+$, $x_{\hat{-}} \to x^+$ and $\sqrt{\mathbb{C}} \to 0$. Therefore $\frac{x^{\hat{+}}}{\sqrt{\mathbb{C}}}$ and $\frac{x_{\hat{-}}}{\sqrt{\mathbb{C}}}$ become indeterminate unless we consider $x^+ = 0$.

$$\left(\frac{x^{\hat{+}}}{\sqrt{\mathbb{C}}}\right)_{\delta \to \frac{\pi}{4} - \epsilon} = \frac{x^+}{\sqrt{2\epsilon}} + \frac{x^-\epsilon}{\sqrt{2}} - \frac{x^+\epsilon^{3/2}}{6\sqrt{2}} \ldots\ldots$$

$$\left(\frac{x_{\hat{-}}}{\sqrt{\mathbb{C}}}\right)_{\delta \to \frac{\pi}{4} - \epsilon} = \frac{x^+}{\sqrt{2\epsilon}} - \frac{x^-\epsilon}{\sqrt{2}} - \frac{x^+\epsilon^{3/2}}{6\sqrt{2}} \ldots\ldots$$

- Reduction of degrees of freedom in the LF end.
- How this reduction can be seen in the light-front perspective.

New Light-front scaled interpolating basis

The new basis $x^{\bar{l}} = L.x^{l}$. We defined $x^{\bar{l}}$ as the new interpolating coordinates in light front.

$$x^{\bar{+}} = \frac{\cos\delta + \sin\delta}{\sqrt{\mathbb{C}}} x^{+} = \frac{1}{\sqrt{2}}\left(\frac{x^{\hat{+}}}{\sqrt{\mathbb{C}}} + \frac{x^{\hat{-}}}{\sqrt{\mathbb{C}}}\right)$$

$$x^{\bar{-}} = \frac{\cos\delta - \sin\delta}{\sqrt{\mathbb{C}}} x^{-} = \frac{1}{\sqrt{2}}\left(\frac{x^{\hat{+}}}{\sqrt{\mathbb{C}}} - \frac{x^{\hat{-}}}{\sqrt{\mathbb{C}}}\right)$$

- In the light perspective, scaled interpolating basis can be seen as an orthogonal basis

$$\begin{pmatrix} x^{\bar{+}} \\ x^{\bar{1}} \\ x^{\bar{2}} \\ x^{\bar{-}} \end{pmatrix} = \begin{pmatrix} \frac{\cos\delta+\sin\delta}{\sqrt{\mathbb{C}}} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{\cos\delta-\sin\delta}{\sqrt{\mathbb{C}}} \end{pmatrix} \begin{pmatrix} x^{+} \\ x^{1} \\ x^{2} \\ x^{-} \end{pmatrix}$$

$(\delta \to 0), x^{\bar{+}} \to x^{+}, x^{\bar{1}} \to x^{1}, x^{\bar{2}} \to x^{2}, x^{\bar{-}} \to x^{-},$

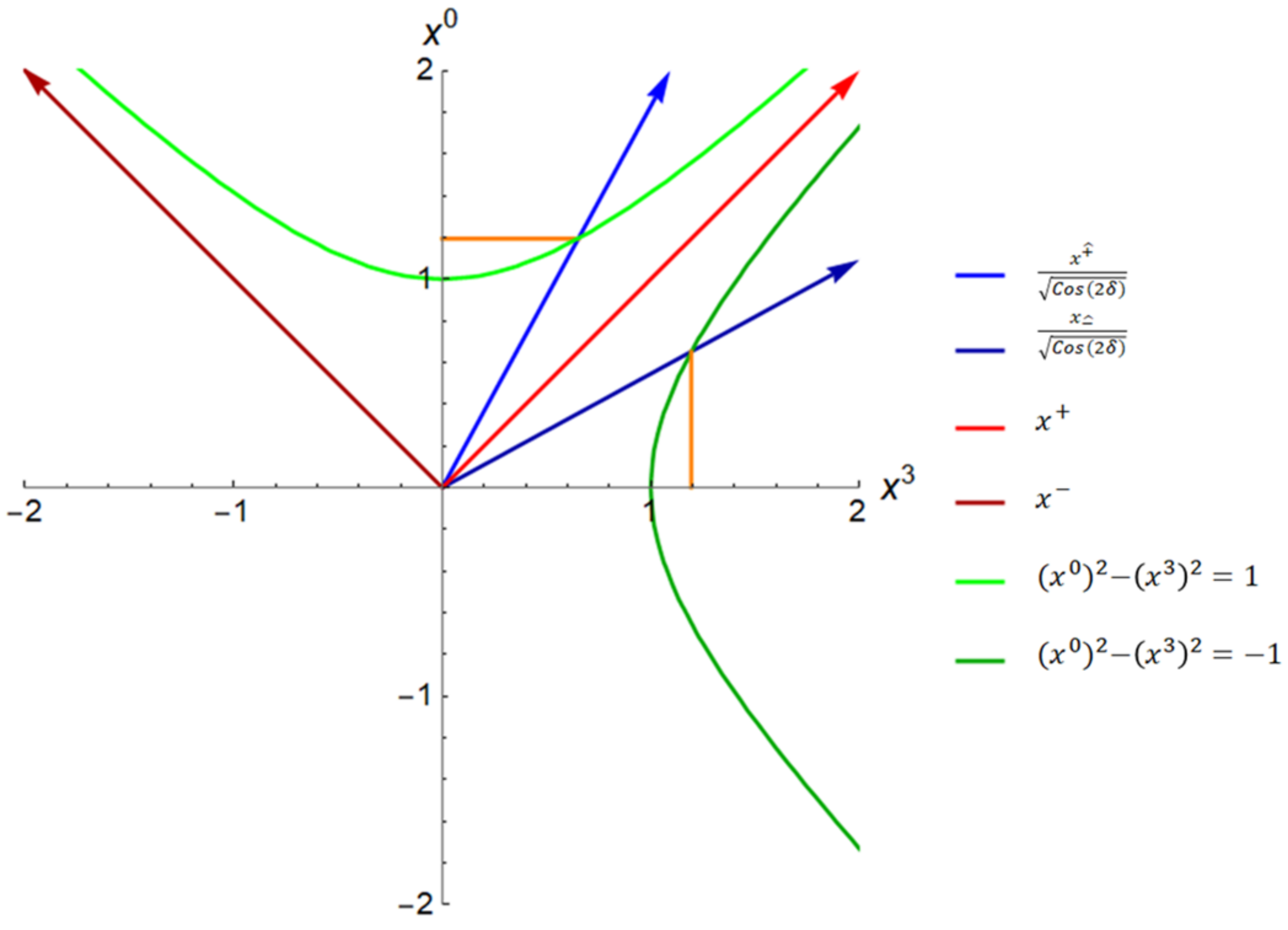
$(\delta \to \frac{\pi}{4})$, $x^{\bar{+}} \to \infty$, $x^{\bar{-}} \to 0$

- Recover the reduction of degrees of freedom in the LF end.

$$(x^{\bar{-}})_{\delta \to \frac{\pi}{4} - \epsilon} = \frac{x^{-}\sqrt{\epsilon}}{\sqrt{2}} + \frac{x^{-}\epsilon^{5/2}}{6\sqrt{2}} \ldots\ldots$$

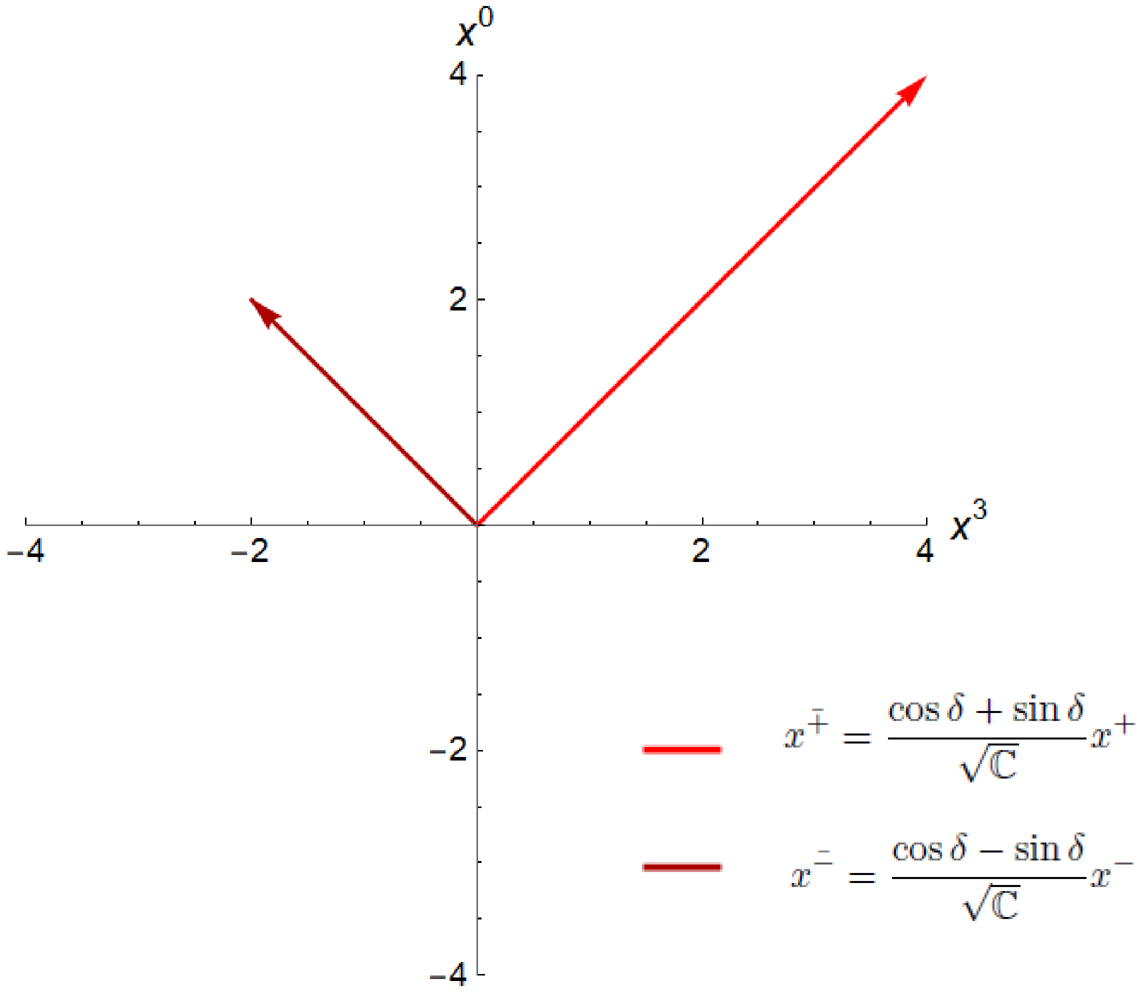
$$(x^{\bar{+}})_{\delta \to \frac{\pi}{4} - \epsilon} = \frac{x^{+}}{\sqrt{2\epsilon}} - \frac{x^{+}\epsilon^{3/2}}{6\sqrt{2}} \ldots\ldots$$

Scaled Interpolating Basis



$$(x^0)^2 - (x^3)^2 = \left(\frac{x^{\hat{+}}}{\sqrt{\mathbb{C}}}\right)^2 - \left(\frac{x_{\hat{-}}}{\sqrt{\mathbb{C}}}\right)^2$$

Light-Front Scaled Interpolating Basis



$$2x^+x^- = 2x^{\bar{+}}x^{\bar{-}}$$

Complete Space-time invariant

$$s^2 = x^{\mu} x_{\mu} = x^{\hat{\mu}} x_{\hat{\mu}} = \left(\frac{x^{\hat{+}}}{\sqrt{\mathbb{C}}}\right)^{\hat{2}} - (x^1)^2 - (x^2)^2 - \left(\frac{x_{\hat{-}}}{\sqrt{\mathbb{C}}}\right)^2 = 2x^+ x^- - (x^1)^2 - (x^2)^2 = 2x^{\bar{+}} x^{\bar{-}} - (x^1)^2 - (x^2)^2$$

- Space-time matrix tensor in scaled interpolating dynamic

$$g^N = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- Even though g^N is equal to the Makowski space-time matrix , this space-time matrix valid for any δ

- Space-time matrix tensor in new scaled light-front interpolating dynamic

$$g^{\bar{\mu}\bar{\nu}} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

Scaled Interpolating Poincare Operators

$$M^N = \begin{pmatrix} 0 & \frac{E^{\hat{1}}}{\sqrt{\mathbb{C}}} & \frac{E^{\hat{2}}}{\sqrt{\mathbb{C}}} & K^3 \\ -\frac{E^{\hat{1}}}{\sqrt{\mathbb{C}}} & 0 & J^3 & \frac{\mathcal{K}^{\hat{1}}}{\sqrt{\mathbb{C}}} \\ -\frac{E^{\hat{2}}}{\sqrt{\mathbb{C}}} & -J^3 & 0 & \frac{\mathcal{K}^{\hat{2}}}{\sqrt{\mathbb{C}}} \\ -K^3 & -\frac{\mathcal{K}^{\hat{1}}}{\sqrt{\mathbb{C}}} & -\frac{\mathcal{K}^{\hat{2}}}{\sqrt{\mathbb{C}}} & 0 \end{pmatrix}$$

- In terms of only the contravariant interpolating operators.

$$M^N = \begin{pmatrix} 0 & \frac{E^{\hat{1}}}{\sqrt{\mathbb{C}}} & \frac{E^{\hat{2}}}{\sqrt{\mathbb{C}}} & K^3 \\ -\frac{E^{\hat{1}}}{\sqrt{\mathbb{C}}} & 0 & J^3 & \frac{\mathbb{C}F^{\hat{1}}-\mathbb{S}E^{\hat{1}}}{\sqrt{\mathbb{C}}} \\ -\frac{E^{\hat{2}}}{\sqrt{\mathbb{C}}} & -J^3 & 0 & \frac{\mathbb{C}F^{\hat{2}}-\mathbb{S}E^{\hat{2}}}{\sqrt{\mathbb{C}}} \\ -K^3 & -\frac{\mathbb{C}F^{\hat{1}}-\mathbb{S}E^{\hat{1}}}{\sqrt{\mathbb{C}}} & -\frac{\mathbb{C}F^{\hat{2}}-\mathbb{S}E^{\hat{2}}}{\sqrt{\mathbb{C}}} & 0 \end{pmatrix}$$

Scaled Interpolating Light-Front Poincare Operators

$$M^{\bar{\mu}\bar{\nu}} = \begin{pmatrix} 0 & \frac{E^1(\cos\delta+\sin\delta)}{\sqrt{\mathbb{C}}} & \frac{E^2(\cos\delta+\sin\delta)}{\sqrt{\mathbb{C}}} & -K^3 \\ -\frac{E^1(\cos\delta+\sin\delta)}{\sqrt{\mathbb{C}}} & 0 & J^3 & -\frac{F^1(\cos\delta-\sin\delta)}{\sqrt{\mathbb{C}}} \\ -\frac{E^2(\cos\delta+\sin\delta)}{\sqrt{\mathbb{C}}} & -J^3 & 0 & -\frac{F^2(\cos\delta-\sin\delta)}{\sqrt{\mathbb{C}}} \\ K^3 & \frac{F^1(\cos\delta-\sin\delta)}{\sqrt{\mathbb{C}}} & \frac{F^2(\cos\delta-\sin\delta)}{\sqrt{\mathbb{C}}} & 0 \end{pmatrix}$$

- Reduction in number of light-front operators in the limit of $\delta \to \pi/4$.
- This is due to the reduction in degrees of freedom.

$$\begin{aligned} E^{\hat{1}} &= J^2\sin\delta + K^1\cos\delta, & \mathcal{K}^{\hat{1}} &= -K^1\sin\delta - J^2\cos\delta, \\ E^{\hat{2}} &= K^2\cos\delta - J^1\sin\delta, & \mathcal{K}^{\hat{2}} &= J^1\cos\delta - K^2\sin\delta, \\ F^{\hat{1}} &= K^1\sin\delta - J^2\cos\delta, & \mathcal{D}^{\hat{1}} &= -K^1\cos\delta + J^2\sin\delta, \\ F^{\hat{2}} &= K^2\sin\delta + J^1\cos\delta, & \mathcal{D}^{\hat{2}} &= -J^1\sin\delta - K^2\cos\delta. \end{aligned}$$

- Infinitesimal transformation matrices of scaled interpolating operators in the scaled interpolating basis coincide with the Infinitesimal transformation operators of rotation and boost operators in the standard basis

$$H.\frac{E^{\hat{1}}}{\sqrt{\mathbb{C}}}.H^{-1} = K^1 \qquad H.\frac{\mathcal{K}^{\hat{1}}}{\sqrt{\mathbb{C}}}.H^{-1} = -J^2$$

$$H.\frac{E^{\hat{2}}}{\sqrt{\mathbb{C}}}.H^{-1} = K^2 \qquad H.\frac{\mathcal{K}^{\hat{2}}}{\sqrt{\mathbb{C}}}.H^{-1} = J^1$$

$$H.K^3.H^{-1} = K^3 \qquad H.J^3.H^{-1} = J^3$$

$$\frac{E^{\hat{1}}}{\sqrt{\mathbb{C}}} = \begin{pmatrix} 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \frac{E^{\hat{2}}}{\sqrt{\mathbb{C}}} = \begin{pmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad K_3 = \begin{pmatrix} 0 & 0 & 0 & i \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}$$

$$\frac{\mathcal{K}^{\hat{1}}}{\sqrt{\mathbb{C}}} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix}, \quad \frac{\mathcal{K}^{\hat{2}}}{\sqrt{\mathbb{C}}} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix}, \quad J_3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

  - Even though scaled interpolating operators seems to depend on the interpolation angle , infinitesimal transformation matrices do not depend on the interpolation angle

Poincare Matrix

- Satisfy same Lie algebra

$$[M^{\mu\nu}, M^{\rho\lambda}] = i(g^{\nu\rho}M^{\mu\lambda} - g^{\nu\lambda}M^{\mu\rho} - g^{\mu\rho}M^{\nu\lambda} + g^{\mu\lambda}M^{\nu\rho})$$

$$M^{\mu\nu} = \begin{pmatrix} 0 & K^1 & K^2 & K^3 \\ -K^1 & 0 & J^3 & -J^2 \\ -K^2 & -J^3 & 0 & J^1 \\ -K^3 & J^2 & -J^1 & 0 \end{pmatrix}$$

Time- evolution parameter of the scaled interpolating basis $\frac{x^{\hat{+}}}{\sqrt{\mathbb{C}}}$

Dynamic operators in the scaled interpolating dynamic

$$e^{-i\beta_1 \frac{E^{\hat{1}}}{\sqrt{\mathbb{C}}}}$$

$$\begin{pmatrix} \frac{x'^{\hat{+}}}{\sqrt{\mathbb{C}}} \\ x'^{\hat{1}} \\ x'^{\hat{2}} \\ \frac{x'_{\hat{-}}}{\sqrt{\mathbb{C}}} \end{pmatrix} = \begin{pmatrix} \cosh\beta_1 & \sinh\beta_1 & 0 & 0 \\ \sinh\beta_1 & \cosh\beta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{x^{\hat{+}}}{\sqrt{\mathbb{C}}} \\ x^{\hat{1}} \\ x^{\hat{2}} \\ \frac{x_{\hat{-}}}{\sqrt{\mathbb{C}}} \end{pmatrix}$$

$$e^{-i\beta_2 \frac{E^{\hat{2}}}{\sqrt{\mathbb{C}}}}$$

$$\begin{pmatrix} \frac{x'^{\hat{+}}}{\sqrt{\mathbb{C}}} \\ x'^{\hat{1}} \\ x'^{\hat{2}} \\ \frac{x'_{\hat{-}}}{\sqrt{\mathbb{C}}} \end{pmatrix} = \begin{pmatrix} \cosh\beta_2 & 0 & \sinh\beta_2 & 0 \\ 0 & 1 & 0 & 0 \\ \sinh\beta_2 & 0 & \cosh\beta_2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{x^{\hat{+}}}{\sqrt{\mathbb{C}}} \\ x^{\hat{1}} \\ x^{\hat{2}} \\ \frac{x_{\hat{-}}}{\sqrt{\mathbb{C}}} \end{pmatrix}$$

$$e^{-i\beta_3 K^3}$$

$$\begin{pmatrix} \frac{x'^{\hat{+}}}{\sqrt{\mathbb{C}}} \\ x'^{\hat{1}} \\ x'^{\hat{2}} \\ \frac{x'_{\hat{-}}}{\sqrt{\mathbb{C}}} \end{pmatrix} = \begin{pmatrix} \cosh\beta_3 & 0 & 0 & \sinh\beta_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \sinh\beta_3 & 0 & 0 & \cosh\beta_3 \end{pmatrix} \begin{pmatrix} \frac{x^{\hat{+}}}{\sqrt{\mathbb{C}}} \\ x^{\hat{1}} \\ x^{\hat{2}} \\ \frac{x_{\hat{-}}}{\sqrt{\mathbb{C}}} \end{pmatrix}$$

Kinematic operators in the scaled interpolating dynamic

$$e^{-i\theta_1 \frac{\mathcal{K}^{\hat{2}}}{\sqrt{\mathbb{C}}}}$$

$$\begin{pmatrix} \frac{x'^{\hat{+}}}{\sqrt{\mathbb{C}}} \\ x'^{\hat{1}} \\ x'^{\hat{2}} \\ \frac{x'_{\hat{-}}}{\sqrt{\mathbb{C}}} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\theta_1 & -\sin\theta_1 \\ 0 & 0 & \sin\theta_1 & \cos\theta_1 \end{pmatrix} \begin{pmatrix} \frac{x^{\hat{+}}}{\sqrt{\mathbb{C}}} \\ x^{\hat{1}} \\ x^{\hat{2}} \\ \frac{x_{\hat{-}}}{\sqrt{\mathbb{C}}} \end{pmatrix}$$

$$e^{-i\theta_2 \frac{\mathcal{K}^{\hat{1}}}{\sqrt{\mathbb{C}}}}$$

$$\begin{pmatrix} \frac{x'^{\hat{+}}}{\sqrt{\mathbb{C}}} \\ x'^{\hat{1}} \\ x'^{\hat{2}} \\ \frac{x'_{\hat{-}}}{\sqrt{\mathbb{C}}} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta_2 & 0 & -\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & \sin\theta_2 & 0 & \cos\theta_2 \end{pmatrix} \begin{pmatrix} \frac{x^{\hat{+}}}{\sqrt{\mathbb{C}}} \\ x^{\hat{1}} \\ x^{\hat{2}} \\ \frac{x_{\hat{-}}}{\sqrt{\mathbb{C}}} \end{pmatrix}$$

$$e^{-i\theta_3 J^3}$$

$$\begin{pmatrix} \frac{x'^{\hat{+}}}{\sqrt{\mathbb{C}}} \\ x'^{\hat{1}} \\ x'^{\hat{2}} \\ \frac{x'_{\hat{-}}}{\sqrt{\mathbb{C}}} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta_3 & -\sin\theta_3 & 0 \\ 0 & \sin\theta_3 & \cos\theta_3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{x^{\hat{+}}}{\sqrt{\mathbb{C}}} \\ x^{\hat{1}} \\ x^{\hat{2}} \\ \frac{x_{\hat{-}}}{\sqrt{\mathbb{C}}} \end{pmatrix}$$

Translation operators

$$\begin{pmatrix} \frac{x'^{\hat{+}}}{\sqrt{\mathbb{C}}} \\ x'^{\hat{1}} \\ x'^{\hat{2}} \\ \frac{x'_{\hat{-}}}{\sqrt{\mathbb{C}}} \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & \frac{a^{\hat{+}}}{\sqrt{\mathbb{C}}} \\ 0 & 1 & 0 & 0 & a^{\hat{1}} \\ 0 & 0 & 1 & 0 & a^{\hat{1}} \\ 0 & 0 & 0 & 1 & \frac{a_{\hat{-}}}{\sqrt{\mathbb{C}}} \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{x^{\hat{+}}}{\sqrt{\mathbb{C}}} \\ x^{\hat{1}} \\ x^{\hat{2}} \\ \frac{x_{\hat{-}}}{\sqrt{\mathbb{C}}} \\ 1 \end{pmatrix}$$

Infinitesimal translation operators in scaled interpolating basis

$$H.\frac{P^{\hat{+}}}{\sqrt{\mathbb{C}}}.H^{-1} = P^0 \qquad H.P^2.H^{-1} = P^2$$

$$H.P^1.H^{-1} = P^1 \qquad H.\frac{P_{\hat{-}}}{\sqrt{\mathbb{C}}}.H^{-1} = P^3$$

Table 01: Kinematic and dynamic generators in scaled interpolating basis

	Kinematic	Dynamic
$\delta = 0$	$\frac{\mathcal{K}^{\hat{1}}}{\sqrt{\mathbb{C}}} = -J^2, \frac{\mathcal{K}^{\hat{2}}}{\sqrt{\mathbb{C}}} = J^1, J^3, P^1, P^2, \frac{P_{\hat{-}}}{\sqrt{\mathbb{C}}} = P^3$	$\frac{E^{\hat{1}}}{\sqrt{\mathbb{C}}} = K^1, \frac{E^{\hat{2}}}{\sqrt{\mathbb{C}}} = K^2, K^3, \frac{P^{\hat{+}}}{\sqrt{\mathbb{C}}} = P^0$
$0 \leq \delta < \pi/4$	$\frac{\mathcal{K}^{\hat{1}}}{\sqrt{\mathbb{C}}}, \frac{\mathcal{K}^{\hat{2}}}{\sqrt{\mathbb{C}}}, J^3, P^1, P^2, \frac{P_{\hat{-}}}{\sqrt{\mathbb{C}}}$	$\frac{E^{\hat{1}}}{\sqrt{\mathbb{C}}}, \frac{E^{\hat{2}}}{\sqrt{\mathbb{C}}}, K^3, \frac{P^{\hat{+}}}{\sqrt{\mathbb{C}}}$

❑ In the interpolating parameter space between $0 \leq \delta < \frac{\pi}{4}$ (Space-like region)

- Scaled interpolating dynamic operators satisfy exact lie algebra of standard Poincare operators
    - Their infinitesimal transformation matrices are equal
    - Scaled interpolating space-time matrix is equal to the Minkowski space-time matrix tensor

- Number of kinematic and dynamic operators are equal, and they are foam invariant
- We can correspond scaled interpolating space to the Euclidean space.
- To make the correspondence which depend on the interpolation angle we need the extend the wick rotation

General Wick Rotation

Minkowski space-time → Euclidean Space-time $(\delta \to 0)$

$$x^0 \to \tilde{x}^0 = ix^0$$

$0 \leq \delta < \pi/4$ (Space- like region) $\quad \frac{x^{\hat{+}}}{\sqrt{\mathbb{C}}} \to \frac{\tilde{x}^{\hat{+}}}{\sqrt{\mathbb{C}}} = i\frac{x^{\hat{+}}}{\sqrt{\mathbb{C}}}$

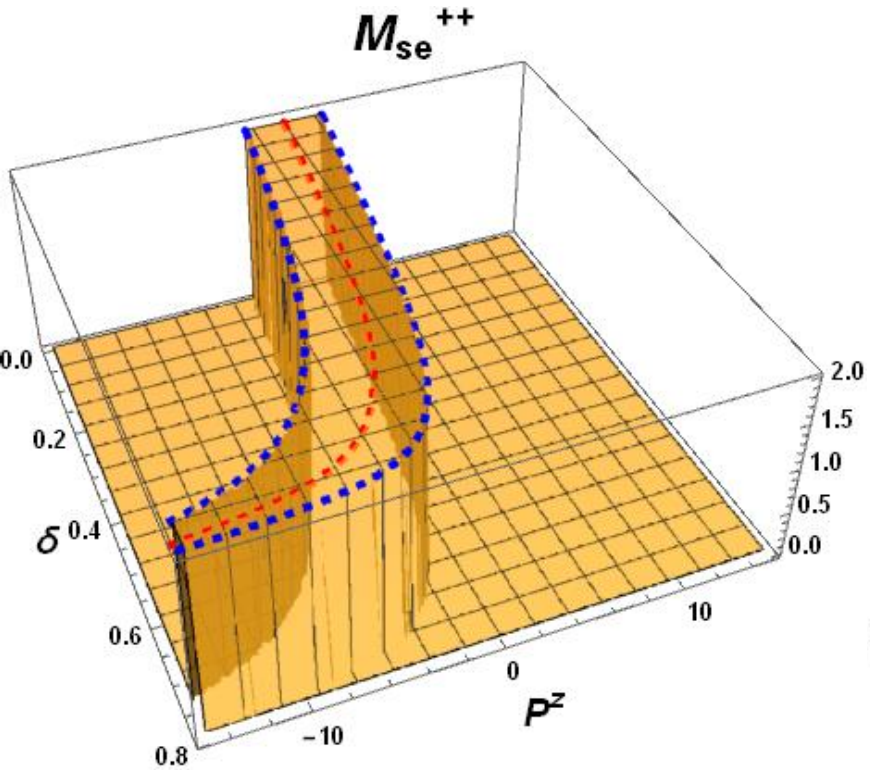
Space-Time interval in the scaled interpolating Euclidean Space

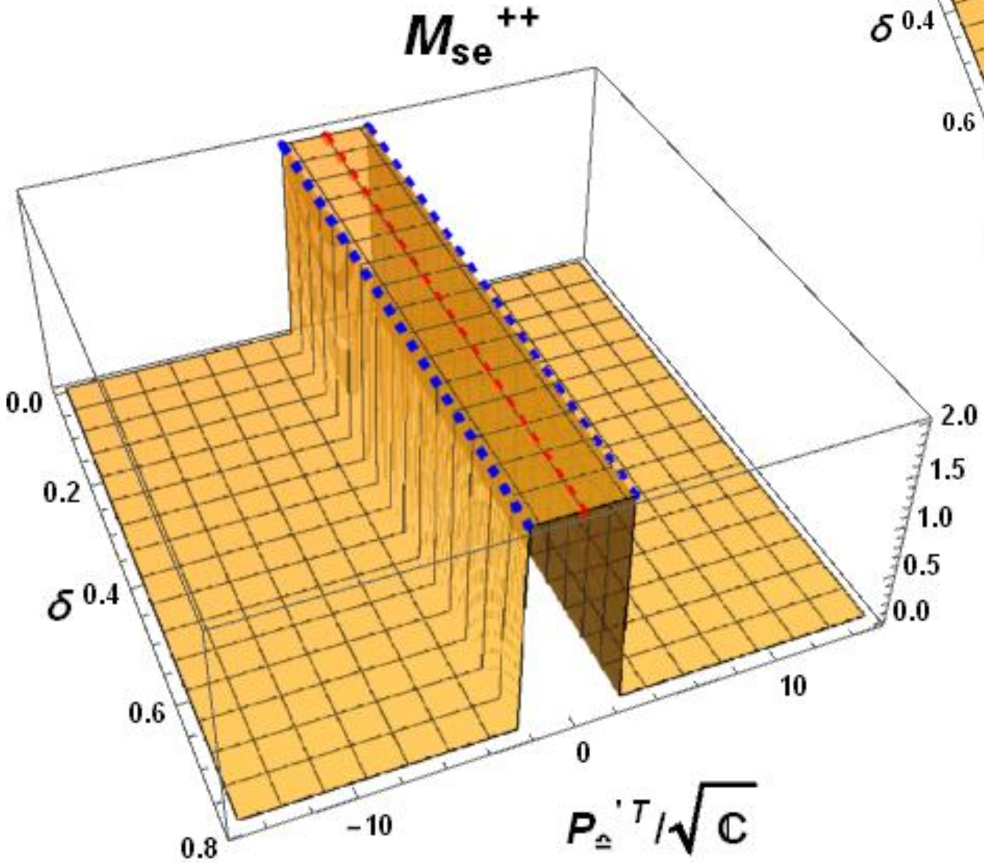
$$s_E^2 = (\tilde{x}^0) + (x^1)^2 + (x^2)^2 + (x^3)^2 = (\frac{\tilde{x}^{+}}{\sqrt{\mathbb{C}}})^2 + (x^1)^2 + (x^2)^2 + (\frac{x^{\hat{-}}}{\sqrt{\mathbb{C}}})^2$$

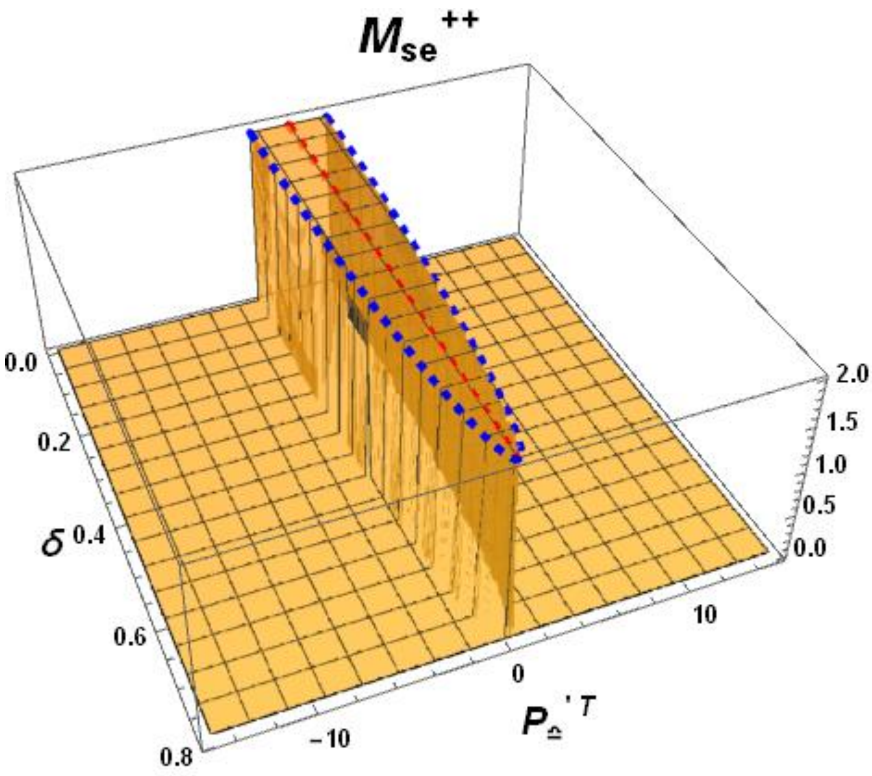
Application

- In lattice gauge theory, the spacetime is wick rotated into Euclidean space and discretized into a lattice with sites separated by distance "a"
- Large-Momentum Effective Theory (or LaMET) advocates a direct approach to simulate Parton Physics in Euclidean lattice QCD theory.
- We propose simulating Parton Physics in Euclidean lattice QCD theory using low momentum corporates with interpolation angle.

Seagull channel helicity amplitude of a $VV \to SS$ process







Scaled interpolating variable

- Infinitesimal transformation matrix of scaled light-front interpolating operators in the scaled light-front interpolating basis

$$E^{\bar{1}} = \frac{E^1(\cos\delta + \sin\delta)}{\sqrt{\mathbb{C}}} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix}, \quad E^{\bar{2}} = \frac{E^2(\cos\delta + \sin\delta)}{\sqrt{\mathbb{C}}} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & i & 0 \end{pmatrix}, \quad K^{\bar{3}} = K^3 = \begin{pmatrix} i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \end{pmatrix}$$

$$F^{\bar{1}} = \frac{F^1(\cos\delta - \sin\delta)}{\sqrt{\mathbb{C}}} = \begin{pmatrix} 0 & i & 0 & 0 \\ 0 & 0 & 0 & i \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad F^{\bar{2}} = \frac{F^2(\cos\delta - \sin\delta)}{\sqrt{\mathbb{C}}} = \begin{pmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & i \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad J^{\bar{3}} = J^3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

- Light –front scaled interpolating operators seems to depend on the interpolation angle, but infinitesimal transformation matrices do not depend on the interpolation angle

- Satisfy same light-front Lie algebra

$$M^{\bar{\mu}\bar{\nu}} = \begin{pmatrix} 0 & E^{\bar{1}} & E^{\bar{2}} & -K^3 \\ -E^{\bar{1}} & 0 & J^3 & -F^{\bar{1}} \\ -E^{\bar{2}} & -J^3 & 0 & -F^{\bar{1}} \\ K^3 & F^{\bar{1}} & F^{\bar{2}} & 0 \end{pmatrix}$$

Time- evolution parameter of the light-front scaled interpolating basis $x^{\bar{+}}$

Kinematic operators in the scaled interpolating dynamic

$$e^{-i\beta_1 E^{\bar{1}}}$$

$$\begin{pmatrix} x'^{\bar{+}} \\ x'^{\bar{1}} \\ x'^{\bar{2}} \\ x'^{\bar{-}} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ \beta_1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{\beta_1^2}{2} & \beta_1 & 0 & 1 \end{pmatrix} \begin{pmatrix} x^{\bar{+}} \\ x^{\bar{1}} \\ x^{\bar{2}} \\ x^{\bar{-}} \end{pmatrix}$$

$$e^{-i\beta_2 E^{\bar{2}}}$$

$$\begin{pmatrix} x'^{\bar{+}} \\ x'^{\bar{1}} \\ x'^{\bar{2}} \\ x'^{\bar{-}} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \beta_2 & 0 & 1 & 0 \\ \frac{\beta_2^2}{2} & 0 & \beta_2 & 1 \end{pmatrix} \begin{pmatrix} x^{\bar{+}} \\ x^{\bar{1}} \\ x^{\bar{2}} \\ x^{\bar{-}} \end{pmatrix}$$

$$e^{-i\beta_3 K^3}$$

$$\begin{pmatrix} x'^{+} \\ x'^{1} \\ x'^{2} \\ x'^{-} \end{pmatrix} = \begin{pmatrix} e^{\beta_3} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{-\beta_3} \end{pmatrix} \begin{pmatrix} x^{+} \\ x^{1} \\ x^{2} \\ x^{-} \end{pmatrix}$$

$$e^{-i\eta_3 J^3}$$

$$\begin{pmatrix} x'^{\bar{+}} \\ x'^{\bar{1}} \\ x'^{\bar{2}} \\ x'^{\bar{-}} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\eta_3 & -\sin\eta_3 & 0 \\ 0 & \sin\eta_3 & \cos\eta_3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x^{\bar{+}} \\ x^{\bar{1}} \\ x^{\bar{2}} \\ x^{\bar{-}} \end{pmatrix}$$

Dynamic operators in the scaled interpolating dynamic

$$e^{-i\eta_1 F^{\bar{1}}}$$

$$\begin{pmatrix} x'^{\bar{+}} \\ x'^{\bar{1}} \\ x'^{\bar{2}} \\ x'^{\bar{-}} \end{pmatrix} = \begin{pmatrix} 1 & \eta_1 & 0 & \frac{\eta_1^2}{2} \\ 0 & 1 & 0 & \eta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x^{\bar{+}} \\ x^{\bar{1}} \\ x^{\bar{2}} \\ x^{\bar{-}} \end{pmatrix}$$

$$e^{-i\eta_2 F^{\bar{2}}}$$

$$\begin{pmatrix} x'^{\bar{+}} \\ x'^{\bar{1}} \\ x'^{\bar{2}} \\ x'^{\bar{-}} \end{pmatrix} = \begin{pmatrix} 1 & 0 & \eta_2 & \frac{\eta_2^2}{2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \eta_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x^{\bar{+}} \\ x^{\bar{1}} \\ x^{\bar{2}} \\ x^{\bar{-}} \end{pmatrix}$$

Translation operators

$$\begin{pmatrix} x'^{\bar{+}} \\ x'^{\bar{1}} \\ x'^{\bar{2}} \\ x'^{\bar{-}} \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & a^{\bar{+}} \\ 0 & 1 & 0 & 0 & a^{\bar{1}} \\ 0 & 0 & 1 & 0 & a^{\bar{2}} \\ 0 & 0 & 0 & 0 & a^{\bar{-}} \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x^{\bar{+}} \\ x^{\bar{1}} \\ x^{\bar{2}} \\ x^{\bar{-}} \\ 1 \end{pmatrix}$$

Table 01: Kinematic and dynamic generators in scaled interpolating Light font basis

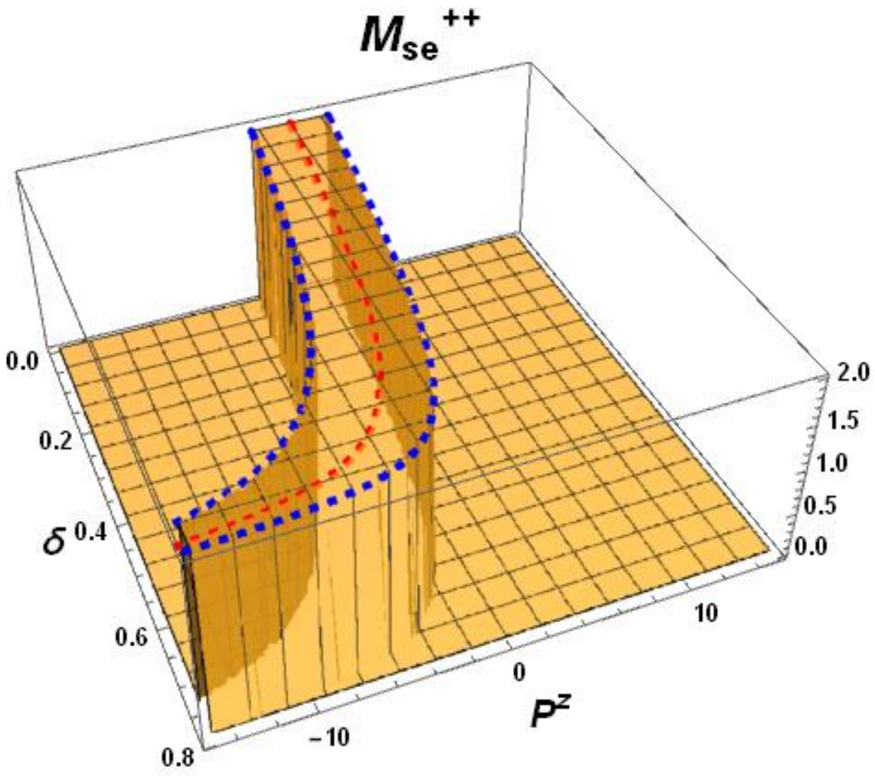
	Kinematic	Dynamic
$\delta = 0$	$E^{\bar{1}} = E^1, E^{\bar{2}} = E^2, K^3, J^3, P^1, P^2, P^{\bar{+}} = P^+$	$F^{\bar{1}} = F^1, F^{\bar{2}} = F^2, P^{\bar{-}} = P^-$
$0 \leq \delta < \pi/4$	$E^{\bar{1}}, E^{\bar{2}}, K^3, J^3, P^1, P^2, P^{\bar{+}}$	$F^{\bar{1}}, F^{\bar{2}}, P^{\bar{-}}$

- In the interpolating parameter space between $0 \leq \delta < \frac{\pi}{4}$
  - Scaling light-front won't change the number of kinematic generators and dynamic generators in the light-front.
  - Even though basis and operators depend on the interpolation angle matrices are independent on interpolation angle, and they are form-invariant with the light-front form

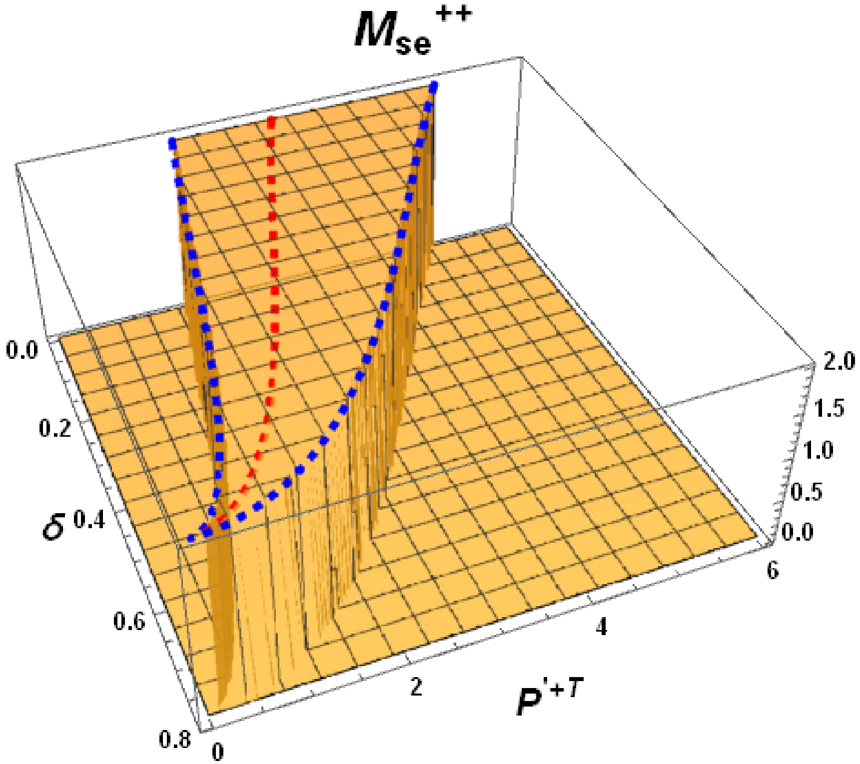
Remark

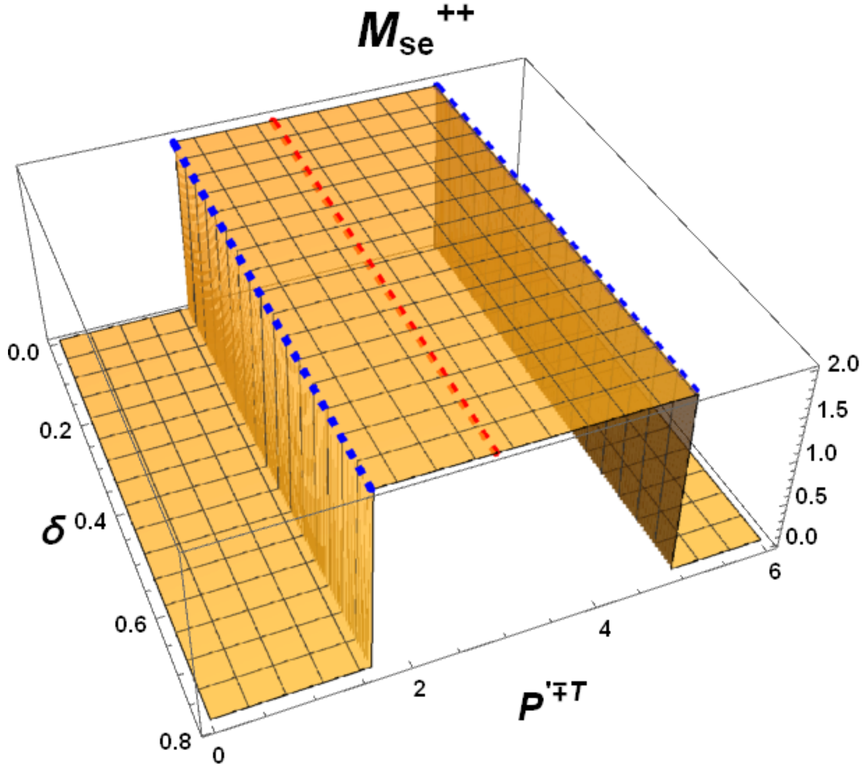
In original interpolation formalism, $\delta = 0$ symbolizes the IFD and $0 \leq \delta < \frac{\pi}{4}$ is the space-like region. Here we look at the same region from the perspective of light-front . When we do so ,the light-front coordinate appears to be scaled and δ becomes scaling parameter.

Seagull channel helicity amplitude of a $VV \to SS$ process



Light-front scaled coordinates





Can’t recover the light-front results

5-dimensional Anti-de Sitter space-time S0(3,2) AdS_5

Scaled interpolating operators

$$R^N = \begin{pmatrix} 0 & \frac{E^{\hat{1}}}{\sqrt{\mathbb{C}}} & \frac{E^{\hat{2}}}{\sqrt{\mathbb{C}}} & K^3 & -\frac{\Pi^{\hat{+}}}{\sqrt{\mathbb{C}}} \\ -\frac{E^{\hat{1}}}{\sqrt{\mathbb{C}}} & 0 & J^3 & \frac{\mathcal{K}^{\hat{1}}}{\sqrt{\mathbb{C}}} & -\Pi^{\hat{1}} \\ -\frac{E^{\hat{2}}}{\sqrt{\mathbb{C}}} & -J^3 & 0 & \frac{\mathcal{K}^{\hat{2}}}{\sqrt{\mathbb{C}}} & -\Pi^{\hat{2}} \\ -K^3 & -\frac{\mathcal{K}^{\hat{1}}}{\sqrt{\mathbb{C}}} & -\frac{\mathcal{K}^{\hat{2}}}{\sqrt{\mathbb{C}}} & 0 & -\frac{\Pi^{\hat{-}}}{\sqrt{\mathbb{C}}} \\ \frac{\Pi^{\hat{+}}}{\sqrt{\mathbb{C}}} & \Pi^{\hat{1}} & \Pi^{\hat{2}} & \frac{\Pi^{\hat{-}}}{\sqrt{\mathbb{C}}} & 0 \end{pmatrix}$$

Satisfy Anti-de sitter standard lie algebra

Light-front scaled interpolating operators

$$J^{\bar{\alpha}\bar{\beta}} = \begin{pmatrix} 0 & E^{\bar{1}} & E^{\bar{2}} & -K^3 & -\Pi^{\bar{+}} \\ -E^{\bar{1}} & 0 & J^3 & -F^{\bar{1}} & -\Pi^1 \\ -E^{\bar{2}} & -J^3 & 0 & -F^{\bar{2}} & -\Pi^2 \\ K^3 & F^{\bar{1}} & F^{\bar{2}} & 0 & -\Pi^{\bar{-}} \\ \Pi^{\bar{+}} & \Pi^1 & \Pi^2 & \Pi^{\bar{-}} & 0 \end{pmatrix}$$

Satisfy Anti-de sitter light-front lie algebra

$$ds_5{}^2 = Cosh^2\chi dT^2 - l^2[d\chi^2 + Sinh^2\chi(d\theta^2 + Sin^2\theta d\varphi^2)]$$

$$ds_4{}^2 = dT^2 - [dr^2 + r^2(d\theta^2 + Sin^2\theta d\varphi^2)]$$

- Ten homogenous operators $^5C_2$
- When vacuum energy density becomes zero
    - Reduction in umber of degrees of freedom.
    - Operators → 6 homogenous operators + 4 inhomogeneous operators ISO (3,1)

$^4C_2$ $^4C_1$

What's happening exactly at $\delta \to \pi/4$ in the scaled light-front coordinates

- Reduction of degrees of freedom in the LF end

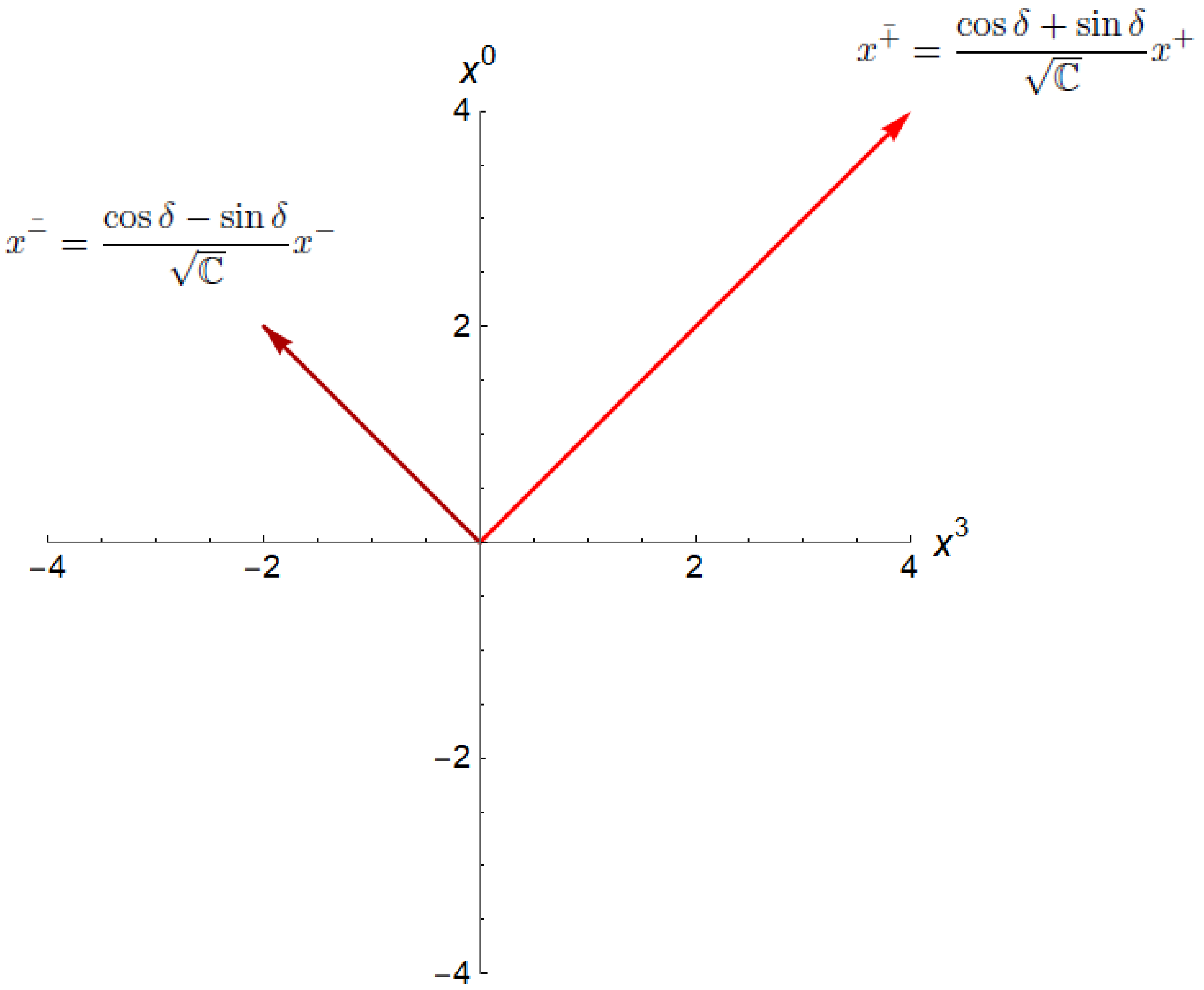
It appears as we end up having , 3D space-time

$^3C_2 = 3$ Number of homogenous operators

$^3C_1 = 3$ Number of inhomogeneous operators

$$2x^+x^- = 2x^{\bar{+}}x^{\bar{-}}$$ $\Rightarrow$ Finite

Four-dimensional space-time invariant



$$x^{\mp} = \frac{\cos\delta + \sin\delta}{\sqrt{\mathbb{C}}} x^{+} \qquad x^{\bar{-}} = \frac{\cos\delta - \sin\delta}{\sqrt{\mathbb{C}}} x^{-}$$

$$\delta \to \frac{\pi}{4} \qquad x^{\mp} \dashrightarrow \infty \qquad x^{\bar{-}} \dashrightarrow 0$$

Similarity with

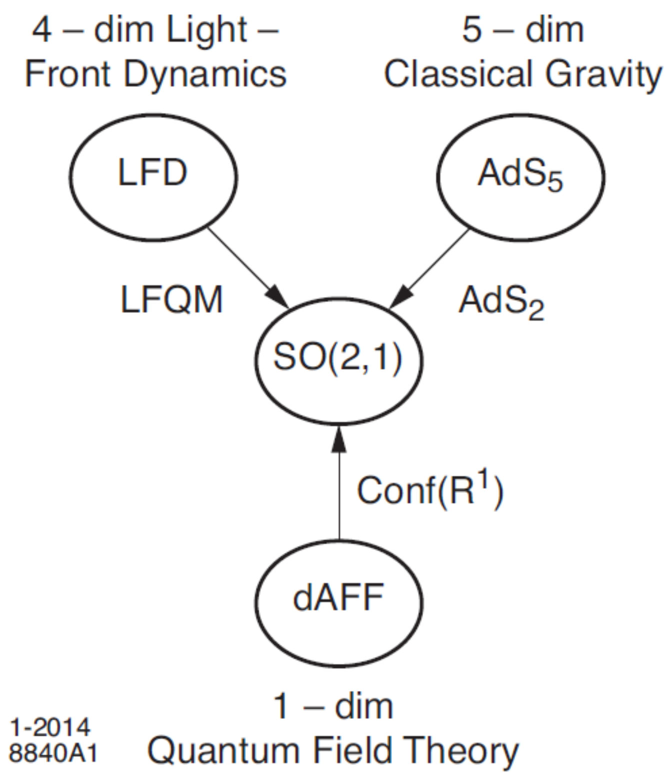
Infinite time dilation
(Time freeze)

Infinite length contraction

It seems we effectively end up having only a 2D space,
operators -> x and y translation and rotation of x-y plane.

=> $\delta \to \frac{\pi}{4}$ limit still should be investigated more in the scaled interpolating methods

Possible application



Light-front Holographic QCD and Emerging Confinement

Stanley J. Brodsky , Joshua Erlich , Hans Gunter Dosch, Guy F.de Teramond

"In this report we explore the remarkable connections between light-front dynamics, its holography mapping to gravity in a higher dimensional anti-de Sitter space and conformal quantum mechanics"

$$[H, D] = i\,H, \quad [H, K] = 2\,i\,D, \quad [K, D] = -i\,K.$$

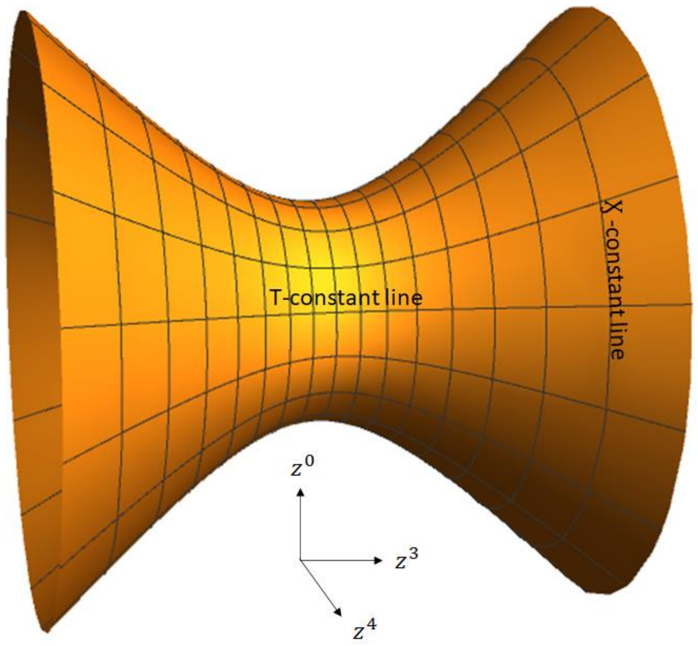
Conf(R^1) – 1-dimentional conformal group

⟶ Finite time- dimension

dAFF → de Alfaro, Fubini and Furlan

Anti-de Sitter space-time

- can be visualized as the hyperboloid in flat five-dimensional space.
    - Two dimensions are suppressed in the figures.



AdS_5

Space-time invariant

$$z_0^2 - z_1^2 - z_2^2 - z_3^2 + z_4^2 = l^2$$

$$l_2 = \sqrt{\frac{-3}{\Lambda_2}}$$

l_2="Anti de Sitter radius"

- $z^0 = l_2 Cosh(Ȣ) Sin\left(\frac{T}{l_2}\right)$
- $z^1 = l_2 Sinh(Ȣ)\ Sin(\theta) Cos(\varphi)$
- $z^2 = l_2 Sinh(Ȣ) Sin(\theta) Sin(\varphi)$
- $z^3 = l_2 Sinh(Ȣ)\ Cos(\theta)$
- $z^4 = l_2 Cosh(Ȣ) Cos\left(\frac{T}{l_2}\right)$

$$ds_2{}^2 = Cosh^2 Ȣ dT^2 - l_2{}^2[dȢ^2 + Sinh^2 Ȣ (d\theta^2 + Sin^2\theta d\varphi^2)]$$

AdS_2

$$X_0^2 - X_1^2 + X_{-1}^2 = R^2$$

$$\begin{aligned} X_{-1} &= R\frac{\sin\tau}{\cos\rho}, \\ X_0 &= R\frac{\cos\tau}{\cos\rho}, \\ X_i &= R\,\Omega_i \tan\rho. \end{aligned}$$

1+1 conformal algebra in light front

Commutation relations among all light-front conformal generators in two dimensional

	P_+	$\mathfrak{K}_-$	D_-	P_-	$\mathfrak{K}_+$	D_+
P_+	0	$2\sqrt{2}iD_-$	$\sqrt{2}iP_+$	0	0	0
$\mathfrak{K}_-$	$-2\sqrt{2}iD_-$	0	$-\sqrt{2}i\mathfrak{K}_-$	0	0	0
D_-	$-\sqrt{2}iP_+$	$\sqrt{2}i\mathfrak{K}_-$	0	0	0	0
P_-	0	0	0	0	$2\sqrt{2}iD_+$	$\sqrt{2}iP_-$
$\mathfrak{K}_+$	0	0	0	$-2\sqrt{2}iD_+$	0	$-\sqrt{2}i\mathfrak{K}_+$
D_+	0	0	0	$-\sqrt{2}iP_-$	$\sqrt{2}i\mathfrak{K}_+$	0

where, $D_\pm = \frac{D \pm K^3}{\sqrt{2}}$, $\mathfrak{K}_\pm = \frac{\mathfrak{K}_0 \pm \mathfrak{K}_3}{\sqrt{2}}$, and $P_\pm = \frac{P_0 \pm P_3}{\sqrt{2}}$.

Hari’s presentation slide

1+1 conformal algebra in scaled interpolating light front

	$P_{\mp}$	$\mathfrak{K}_{=}$	$D_{=}$	$P_{=}$	$\mathfrak{K}_{\mp}$	$D_{\mp}$
$P_{\mp}$	0	$2\sqrt{2}iD_{=}A$	$\sqrt{2}iP_{\mp}\,\mathrm{Ai}$	0	0	0
$\mathfrak{K}_{=}$	$-2\sqrt{2}iD_{=}A$	0	$-\sqrt{2}i\mathfrak{K}_{=}\mathrm{Ai}$	0	0	0
$D_{=}$	$-\sqrt{2}iP_{\mp}\,\mathrm{Ai}$	$\sqrt{2}i\mathfrak{K}_{=}\,\mathrm{Ai}$	0	0	0	0
$P_{=}$	0	0	0	0	$2\sqrt{2}iD_{\mp}\mathrm{Ai}$	$\sqrt{2}iP_{=}\,A$
$\mathfrak{K}_{\mp}$	0	0	0	$-2\sqrt{2}iD_{\mp}\mathrm{Ai}$	0	$-\sqrt{2}i\mathfrak{K}_{\mp}A$
$D_{\mp}$	0	0	0	$-\sqrt{2}iP_{=}A$	$\sqrt{2}i\mathfrak{K}_{\mp}A$	0

$$A = \left(\frac{\sqrt{\mathrm{Cos}[2\delta]}}{(\mathrm{Cos}[\delta] + \mathrm{Sin}[\delta])}\right) \qquad \mathrm{Ai} = \left(\frac{\sqrt{\mathrm{Cos}[2\delta]}}{(\mathrm{Cos}[\delta] - \mathrm{Sin}[\delta])}\right)$$

$\delta \to \frac{\pi}{4}$ Non-Vanishing operators $\mathfrak{K}_{=}$ $D_{=}$ $P_{=}$

	$P_{\mp}$	$\mathfrak{K}_{=}$	$D_{=}$	$P_{=}$	$\mathfrak{K}_{\mp}$	$D_{\mp}$
$P_{\mp}$	0	$2\sqrt{2}iD_-$	$\sqrt{2}iP_+$	0	0	0
$\mathfrak{K}_{=}$	$-2\sqrt{2}iD_-$	0	$-\sqrt{2}i\mathfrak{K}_{-\mathrm{Ai}^2}$	0	0	0
$D_{=}$	$-\sqrt{2}iP_+$	$\sqrt{2}i\mathfrak{K}_{-\mathrm{Ai}^2}$	0	0	0	0
$P_{=}$	0	0	0	0	$2\sqrt{2}iD_+$	$\sqrt{2}iP_-$
$\mathfrak{K}_{\mp}$	0	0	0	$-2\sqrt{2}iD_+$	0	$-\sqrt{2}i\mathfrak{K}_{+\mathrm{A}^2}$
$D_{\mp}$	0	0	0	$-\sqrt{2}iP_-$	$\sqrt{2}i\mathfrak{K}_{+\mathrm{A}^2}$	0

- Need to investigate effect of scaled interpolating on the dilatation and special conformal transformation to make a conclusion.

Summary and Conclusion

- In the interpolating parameter space between $0 \leq \delta < \frac{\pi}{4}$
  - Scaled interpolating dynamic space is form-invariant with the standard instant form dynamics, but scaled interpolating coordinates are not orthogonal to each other.
  - Light-front scaled interpolating dynamic space is form-invariant with the light-front form dynamics, while light-front scaled interpolating coordinates are orthogonal to each other.

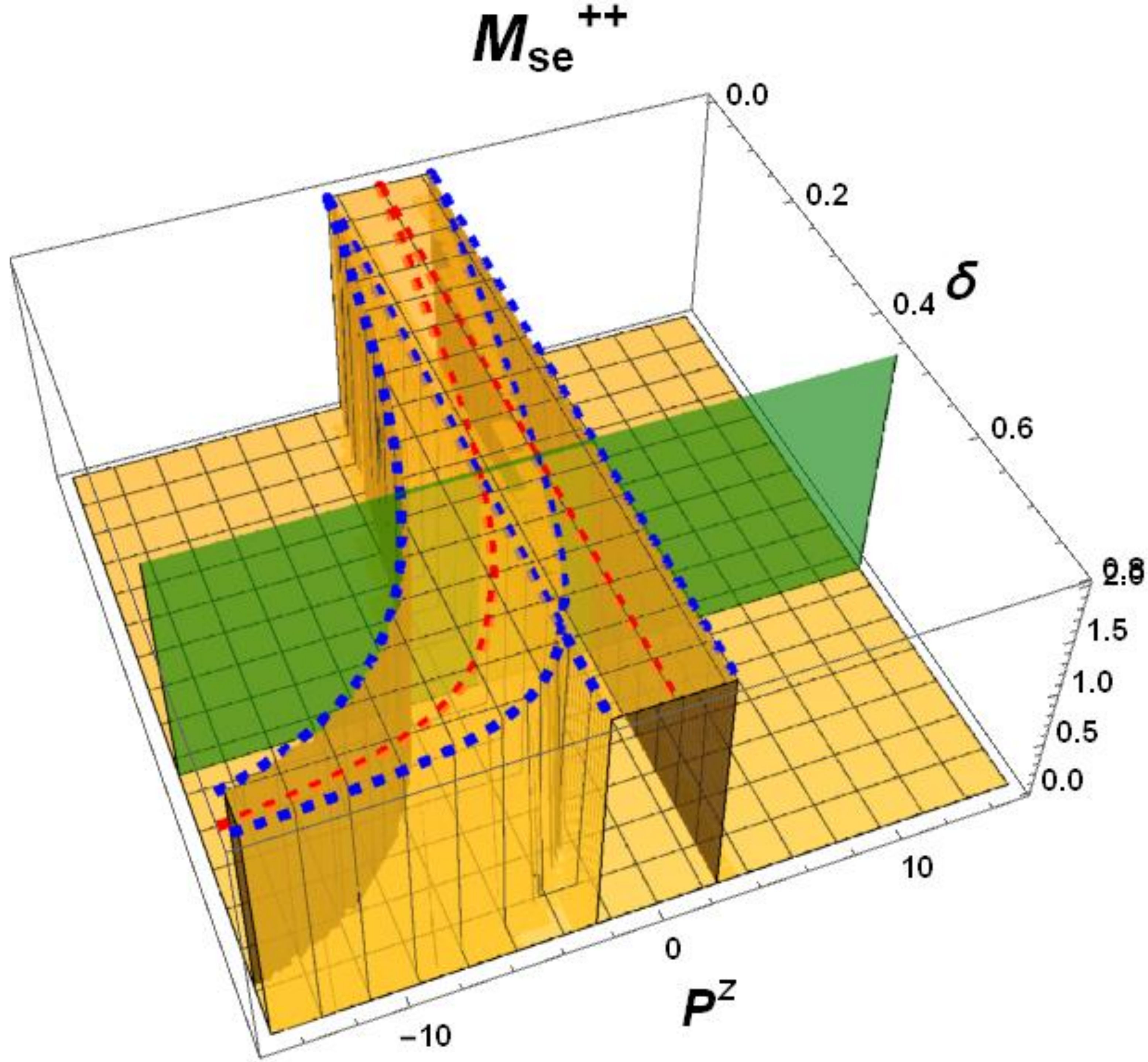
- In the limit $\delta \rightarrow \frac{\pi}{4}$, Both scaled interpolating formalism show the reduction of degrees of freedom and reduction in the number of operators

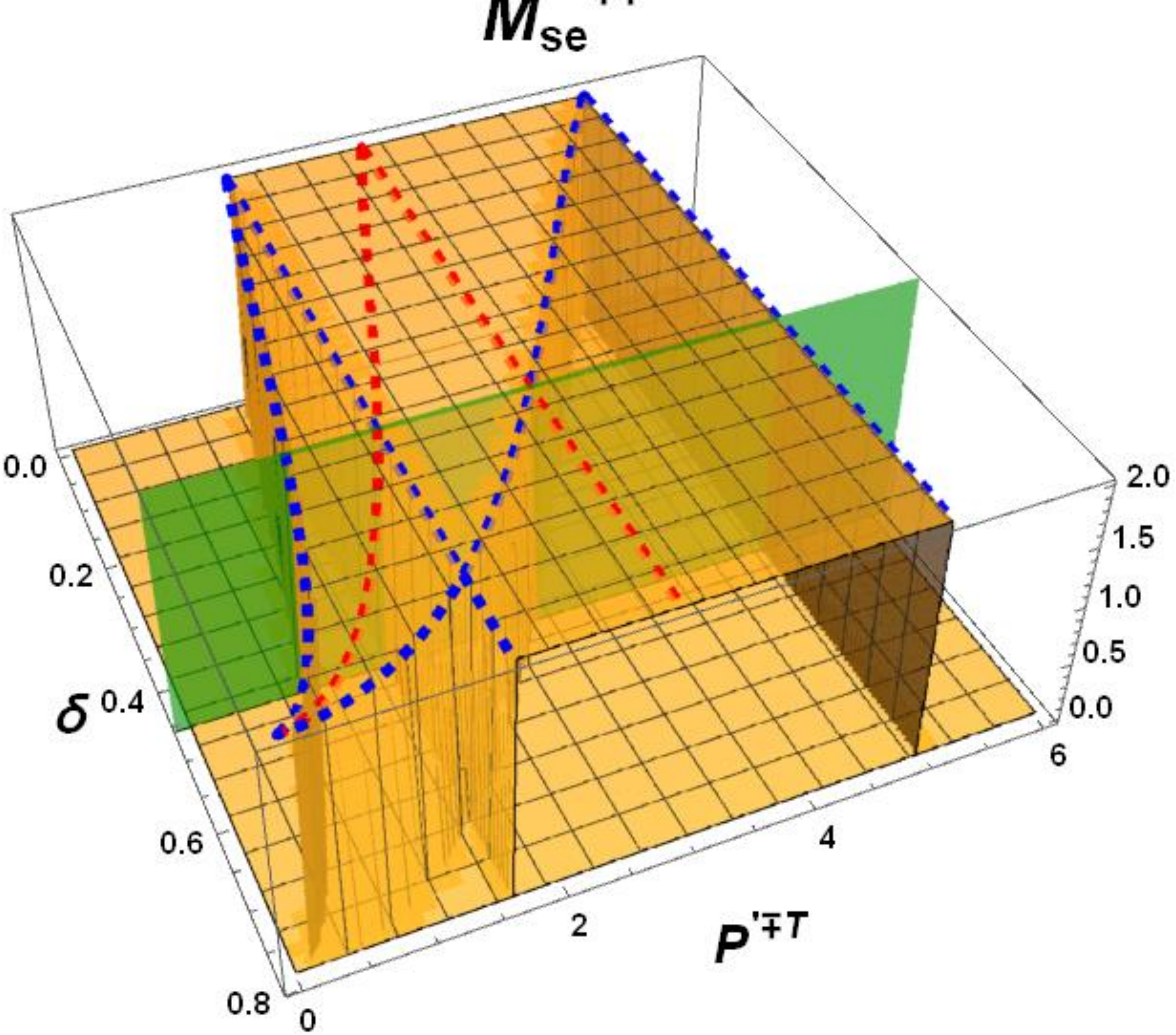
Future work

- $\delta \rightarrow \frac{\pi}{4}$ limit still should be investigated more in the scaled interpolating methods
- Study AdS_2 curve space-time. (Contraction of $\mathrm{AdS}_5 \rightarrow \mathrm{AdS}_2$).
- Need to investigate effect of scaled interpolating on the dilatation and special conformal transformation

Possible future application

- Scaled interpolating basis in the lattice QCD
- Light-front scaled basis in the holographic QCD


M_{se}^{++}
0.0
0.2
0.4
δ
0.6
0.8
2.0
1.5
1.0
0.5
0.0
10
0
P^z
−10



M_{se}
0.0
0.2
δ 0.4
0.6
0.8
0
2
4
6
$P'^{\mp T}$
2.0
1.5
1.0
0.5
0.0