Quantum Orientation Entanglement

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06/14/2024

Quantum Correlation originated from the orientation entanglement of spins

- The angle should be rotated to get the same initial configuration.

  - Spin- 0 → Any angle
  - Spin-1/2 → 720^0
  - Spin-1 → 360^0

Rotation by 180^0

Spin-0 particles

$$|0,0> \longrightarrow |0,0>$$

Spin-1/2 particles

$$|1/2,1/2> \longrightarrow |1/2,-1/2>$$
$$|1/2,-1/2> \longrightarrow -|1/2,1/2>$$

Spin-1 particles

$$|1,1> \longrightarrow |1,-1>$$
$$|1,0> \longrightarrow -|1,0>$$
$$|1,-1> \longrightarrow |1,1>$$

Quantum–orientation entangled states of spins

$$d^{(j)}_{m',m}(\beta) = < j,m'|exp(\frac{-ij_y\beta}{\hbar})|j,m>$$

(Wigner-d function)

How can we see quantum orientation entanglement of spins of a moving particle ?

- We consider the helicity states of spin (1/2, 1) spinors and spin -1 polarization vectors.

  - Spin orientation
  - Momentum of the particle

Can we rotate the helicity states and see the exact quantum orientation entanglement of spins in helicity states?

- IFD

- Rotations IFD (J^1 ,J^2, J^3) are kinematic operators, therefore the helicity in IFD is invariant under rotation.
- Helicity is defined with respect to the moving direction (not with respect to the spin direction)
  - ➢ We can't obtain quantum orientation entanglement in helicity states, by rotating helicities in IFD.

- LFD

- Transverse rotation becomes (F^1 ,F^2) a dynamical problem in the light-front quantization. Because the quantization surface $x^+ = 0$ is not invariant under the transverse rotation whose direction is perpendicular to the direction of the quantization axis z at equal x^+.
- Light-front transverse angular momentum operator involves the interaction that changes the particle number.
  - ➢ We can't obtain quantum orientation entanglement in helicity states using light- front transverse rotations as they change the particle number.
- Longitudinal rotation LFD (J^3) is a kinematic operator, and helicity is invariant under light-front longitudinal rotation.

As helicity definitions can vary with the different forms of dynamics, how we can understand helicities in general?

- Using interpolating helicity transformation matrix ($T = e^{i\beta_1 \mathcal{K}^{\hat{1}} + i\beta_2 \mathcal{K}^{\hat{2}}} e^{-i\beta_3 K^3}$) we can calculate the interpolating helicity spin (1/2, 1) spinors and spin -1 polarization vectors between Instant form and light-front form dynamics.

$$\mathcal{K}^{\hat{1}} = -K^1 \sin\delta - J^2 \cos\delta, \qquad (\delta \to 0),\ \mathcal{K}^{\hat{1}} \to -J^2,\ \mathcal{K}^{\hat{2}} \to J^1$$

$$\mathcal{K}^{\hat{2}} = J^1 \cos\delta - K^2 \sin\delta, \qquad (\delta \to \pi/4),\ \mathcal{K}^{\hat{1}} \to -E_1,\ \mathcal{K}^{\hat{2}} \to -E_2$$

❑ Interpolating helicity spin-1 polarization vectors

$$\epsilon_{\hat{\mu}}(P,+) = -\frac{1}{\sqrt{2}\mathbb{P}}\left(\mathbb{S}|\mathbf{P}_\perp|, \frac{P_1 P_{\hat{-}} - iP_2\mathbb{P}}{|\mathbf{P}_\perp|}, \frac{P_2 P_{\hat{-}} + iP_1\mathbb{P}}{|\mathbf{P}_\perp|}, -\mathbb{C}|\mathbf{P}_\perp|\right),$$

$$\epsilon_{\hat{\mu}}(P,-) = \frac{1}{\sqrt{2}\mathbb{P}}\left(\mathbb{S}|\mathbf{P}_\perp|, \frac{P_1 P_{\hat{-}} + iP_2\mathbb{P}}{|\mathbf{P}_\perp|}, \frac{P_2 P_{\hat{-}} - iP_1\mathbb{P}}{|\mathbf{P}_\perp|}, -\mathbb{C}|\mathbf{P}_\perp|\right),$$

$$\epsilon_{\hat{\mu}}(P,0) = \frac{P^{\hat{+}}}{M\mathbb{P}}\left(P_{\hat{+}} - \frac{M^2}{P^{\hat{+}}}, P_1, P_2, P_{\hat{-}}\right),$$

- The interpolating transverse polarization vectors respect the guage condition $A^{\hat{+}} = 0$ and $\partial_{\hat{-}} + \partial_\perp \cdot A_\perp \mathbb{C} = 0$ which links the Coulomb gauge $\nabla \cdot A = 0$ in IFD and light-front guage $A^+ = 0$ in the LFD.
- The real photon's helicity λ takes only $+$ or $-$,but not 0 as $M \to 0$ limit, $\epsilon_{\hat{\mu}}(P,0)$ doesn't exist

How we confirm the helicity state of the given interpolating helicity state?

Generalized helicity operator

$$\mathcal{J}_i = T J_i T^{-1}$$

$$\mathcal{J}_3 |p; j, m\rangle_\delta = T J_3 T^{-1} T |0; j, m\rangle = m |p; j, m\rangle_\delta,$$

$$\mathcal{J}_3 = \frac{1}{\mathbb{P}} \left(P_{\hat{-}} J_3 + P^1 \mathcal{K}^{\hat{2}} - P^2 \mathcal{K}^{\hat{1}} \right)$$

$\delta \to 0$

$\delta \to \frac{\pi}{4}$

$$\mathcal{J}_3 = \frac{\mathbf{P}.\mathbf{J}}{|\mathbf{P}|}$$

Helicity defined in IFD

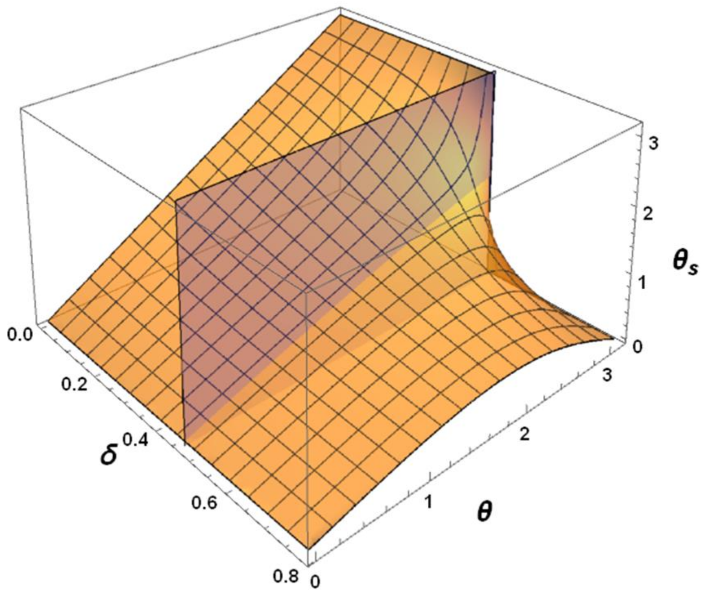
$$\mathcal{J}_3 = J_3 + \frac{(P^2 E_1 - P^1 E_2)}{P+}$$

Helicity defined in LFD

$$\mathcal{J}_3 \epsilon^{\hat{\mu}}(P, \lambda) = \lambda \epsilon^{\hat{\mu}}(P, \lambda)$$

How can we point out the differences between helicity defined in IFD and LFD using the spin orientation?

- By comparing helicity transformation matrix ($T = e^{i\beta_1 \mathcal{K}^{\hat{1}} + i\beta_2 \mathcal{K}^{\hat{2}}} e^{-i\beta_3 K^3}$) with the $B(\eta)D(\hat{\boldsymbol{m}}.\theta_s) = e^{-i\eta \boldsymbol{K}} e^{-i\widehat{\boldsymbol{m}}.\boldsymbol{J}\theta_s}$ transformation, we found a relationship between spin orientation , particle' momentum direction and the interpolation angle. (Relations of spin orientation in the interpolating helicity)
- We confirm that this unique relationship holds for spin (1/2, 1) spinors and spin -1 polarization vectors.



- Spin orientation changes with the particle's momentum direction and the interpolation angle in the rest frame (Positive helicity spinor/ polarization vector)

Boosts to momentum **P**
$\hat{n} = (\, sin\theta cos\varphi, sin\theta sin\varphi, cos\theta)$

Rotates the spin around the axis by a unit vector $\widehat{m} = (\,-sin\varphi_s, cos\varphi_s, 0)$ by angle θ_s .

$$\cos\theta_s = \frac{\cos\alpha(1+\cosh\beta_3) + \cosh\beta_3 - \cosh\eta}{1+\cosh\eta}$$

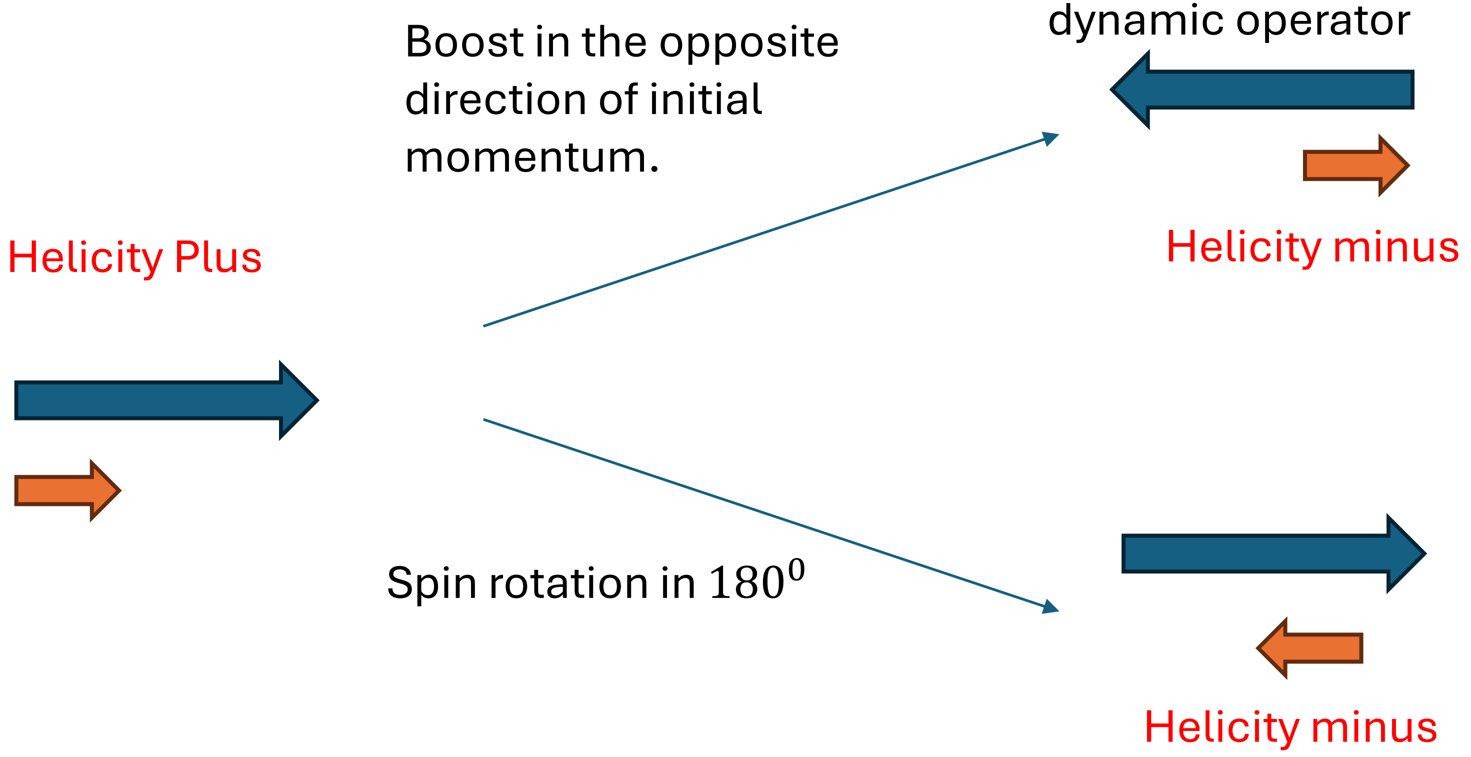
$$\cos\phi_s = \frac{\beta_1^2}{\sqrt{\beta_1^2+\beta_2^2}} = \cos\phi \qquad \sin\phi_s = \frac{\beta_2^2}{\sqrt{\beta_1^2+\beta_2^2}} = \sin\phi$$

Correlation between spin-orientation and longitudinal boost

Since we can't use rotation operator on helicity and show the quantum orientation entanglement of spins in the helicity states , what is the way of obtaining it / What is the counterpart of rotating only the spins of a helicity state that effectively leads to change the helicity.

❑ Instant Form Dynamics

- Longitudinal boost is a dynamic operator



- Transverse rotation is a kinematic operator

Momentum direction

Spin direction

❑ Light-front Form Dynamics

- Light-front longitudinal boost is a kinematic operator
- Light-front transverse rotation is a dynamic operator

Quantum Correlation in interpolating helicity spinors and vectors

When we fix the particle's initial momentum direction as +z ,

$$\theta_s = 0, \ \ \cos\alpha = P_{\hat{-}}/|P_{\hat{-}}| \rightarrow 1 \qquad P_{\hat{-}} > 0$$

$$\theta_s = \pi, \ \ \cos\alpha = P_{\hat{-}}/|P_{\hat{-}}| \rightarrow -1 \qquad P_{\hat{-}} < 0$$

- Quantum–orientation entangled states of interpolating spinors and polarization vectors

• Spin-1/2 spinors	• Spin-1 spinors	• Spin-1 polarization vectors
$U^{+1/2}(P_{\hat{-}} > 0) \Rrightarrow U^{-1/2}(P_{\hat{-}} > 0)$ $U^{-1/2}(P_{\hat{-}} > 0) \Rrightarrow -U^{+1/2}(P_{\hat{-}} > 0)$	$U^{+1}(P_{\hat{-}} > 0) \Rrightarrow U^{-1}(P_{\hat{-}} > 0)$ $U^{0}(P_{\hat{-}} > 0) \Rrightarrow -U^{0}(P_{\hat{-}} > 0)$ $U^{-1}(P_{\hat{-}} > 0) \Rrightarrow U^{+1}(P_{\hat{-}} > 0)$	$\epsilon^{+1}(P_{\hat{-}} > 0) \Rrightarrow \epsilon^{-1}(P_{\hat{-}} > 0)$ $\epsilon^{0}(P_{\hat{-}} > 0) \Rrightarrow -\epsilon^{0}(P_{\hat{-}} > 0)$ $\epsilon^{-1}(P_{\hat{-}} > 0) \Rrightarrow \epsilon^{+1}(P_{\hat{-}} > 0)$

$P_{\hat{-}} = 0 \longrightarrow$

- Initial direction of particles' momentum
- Interpolation angle
- Boost of the frame

Interpolating Lorentz Transformation

$$\begin{pmatrix} P'_{\hat{+}} \\ P'_1 \\ P'_2 \\ P'_{\hat{-}} \end{pmatrix} = \begin{pmatrix} \gamma(1-\beta\mathbb{S}) & 0 & 0 & \gamma\beta\mathbb{C} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \gamma\beta\mathbb{C} & 0 & 0 & \gamma(1+\beta\mathbb{S}) \end{pmatrix} \begin{pmatrix} P_{\hat{+}} \\ P_1 \\ P_2 \\ P_{\hat{-}} \end{pmatrix}$$

$P'_{\hat{-}} = 0 \qquad \beta = -(P_{\hat{-}}/P^{\hat{+}})$

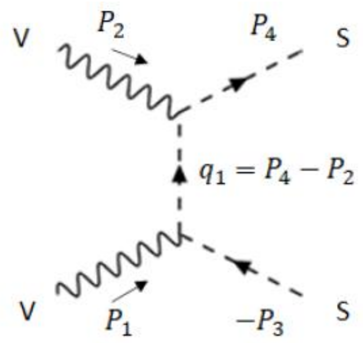
At light-front , $\beta \to -1 \Rightarrow P'_{\hat{-}} \to P^{+} = 0$ (Light-front zero-mode)

- we can show the quantum orientation entanglement of spins in the helicity states ,using the Lorentz transformation for all interpolation angle except light front, Since light-front boost is kinematic, and helicity defined in LF is boost invariance

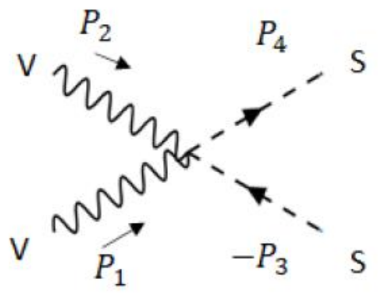
❑ Key Points

- Orientation entanglement of interpolating spin 1 spinors and polarization vectors distinguishes the helicity "0" state of vector particle from the helicity "0" state of scalar particle.
- Helicity zero state of vector particle internally has parity odd behavior.
- Helicity zero state of scalar particle internally has parity even behavior.

Scalar particle and its anti-particle production by two massive spin-1 polarization vectors (VV→ SS)



(a)



(b)

Fig. (a) t-channel Feynman diagram, the cross channel (u-channel) can be drawn by crossing the two final states' particles. Fig. (b) is drawn for the seagull channel.

V -> Vector particle (Spin-1)
S -> Scalar particle (Spin-0)

❑ Interpolating helicity amplitudes

$$M_t^{\lambda_1\lambda_2} = (-p_3 + q_1)^{\hat{\mu}} \varepsilon_{\hat{\mu}}(p_1,\lambda_1) \frac{1}{q_1^2 - m_s{}^2} (p_4 + q_1)^{\hat{\nu}} \varepsilon_{\hat{\nu}}(p_2,\lambda_2)$$

$$M_u^{\lambda_1\lambda_2} = (-p_3 + q_2)^{\hat{\nu}} \varepsilon_{\hat{\nu}}(p_2,\lambda_2) \frac{1}{q_2^2 - m_s{}^2} (-p_4 + q_2)^{\hat{\mu}} \varepsilon_{\hat{\mu}}(p_1,\lambda_1)$$

$$M_{se}^{\lambda_1\lambda_2} = -2 g_{\hat{\mu}\hat{\nu}} \varepsilon^{\hat{\mu}} (p_1,\lambda_1)\, \varepsilon^{\hat{\nu}} (p_2,\lambda_2)$$

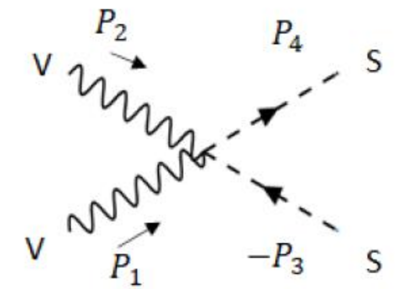
Where $q_2 = p_3 - p_2$

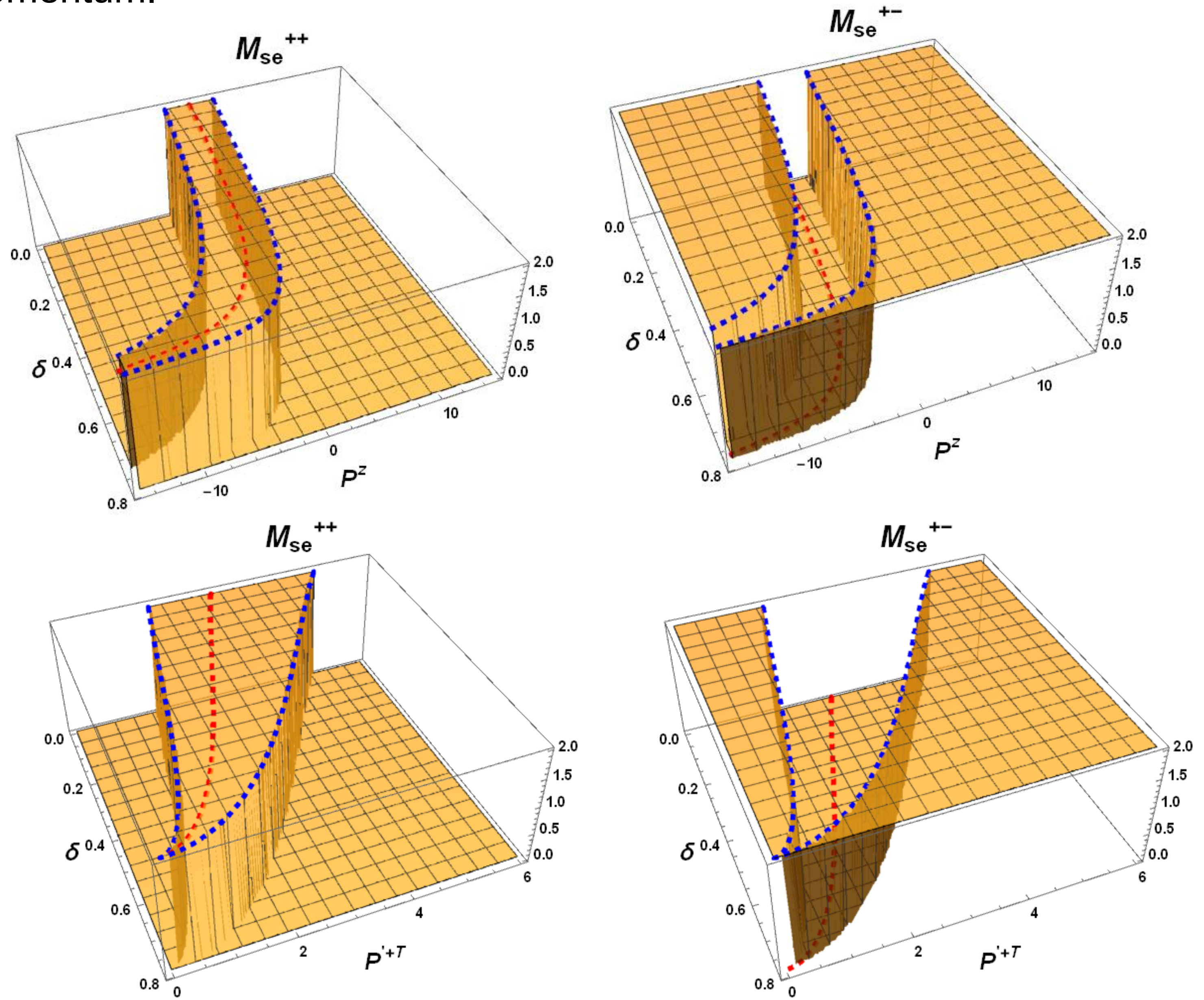
❑ T and U channels have time-ordered interpolating helicity amplitudes.

- Scalar particle and its anti-particle production by real photons ($\gamma\gamma \rightarrow SS$)

Seagull channels with transverse polarization vectors

- Contact interaction - Angular momentum is conserved without involving orbital angular momentum.



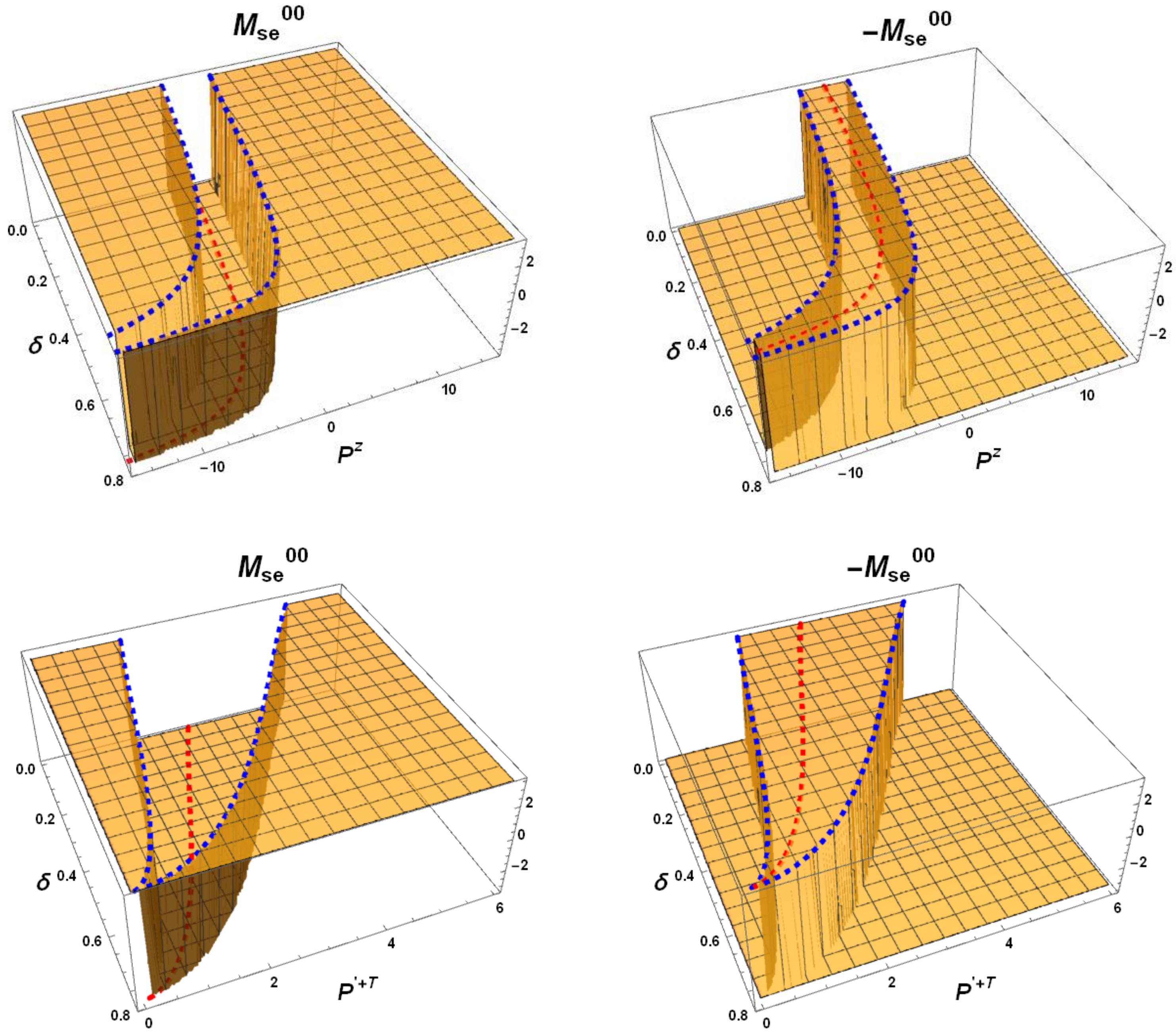


- Quantum correlation /helicity changes

$$\epsilon^{+1}(P_{\hat{-}} > 0) \Rrightarrow \epsilon^{-1}(P_{\hat{-}} > 0)$$
$$\epsilon^{-1}(P_{\hat{-}} > 0) \Rrightarrow \epsilon^{+1}(P_{\hat{-}} > 0)$$

- Boundaries approaching to the LF zero-mode at the vicinity of the LF end.

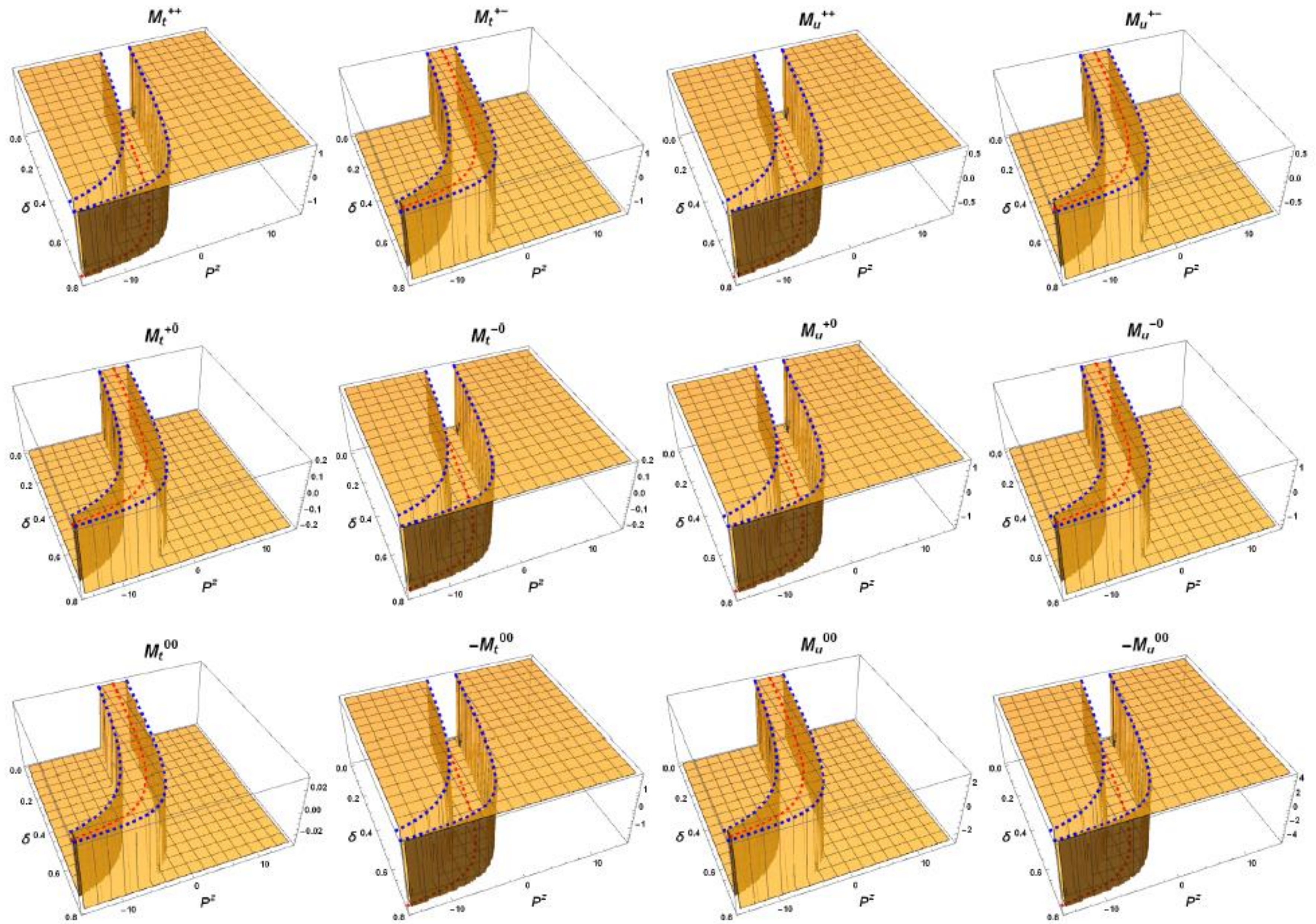
Seagull channel with longitudinal polarization vectors



- Phase changes due to QC

$$\epsilon^0(P_{\hat{-}} > 0) \Rrightarrow -\epsilon^0(P_{\hat{-}} > 0)$$

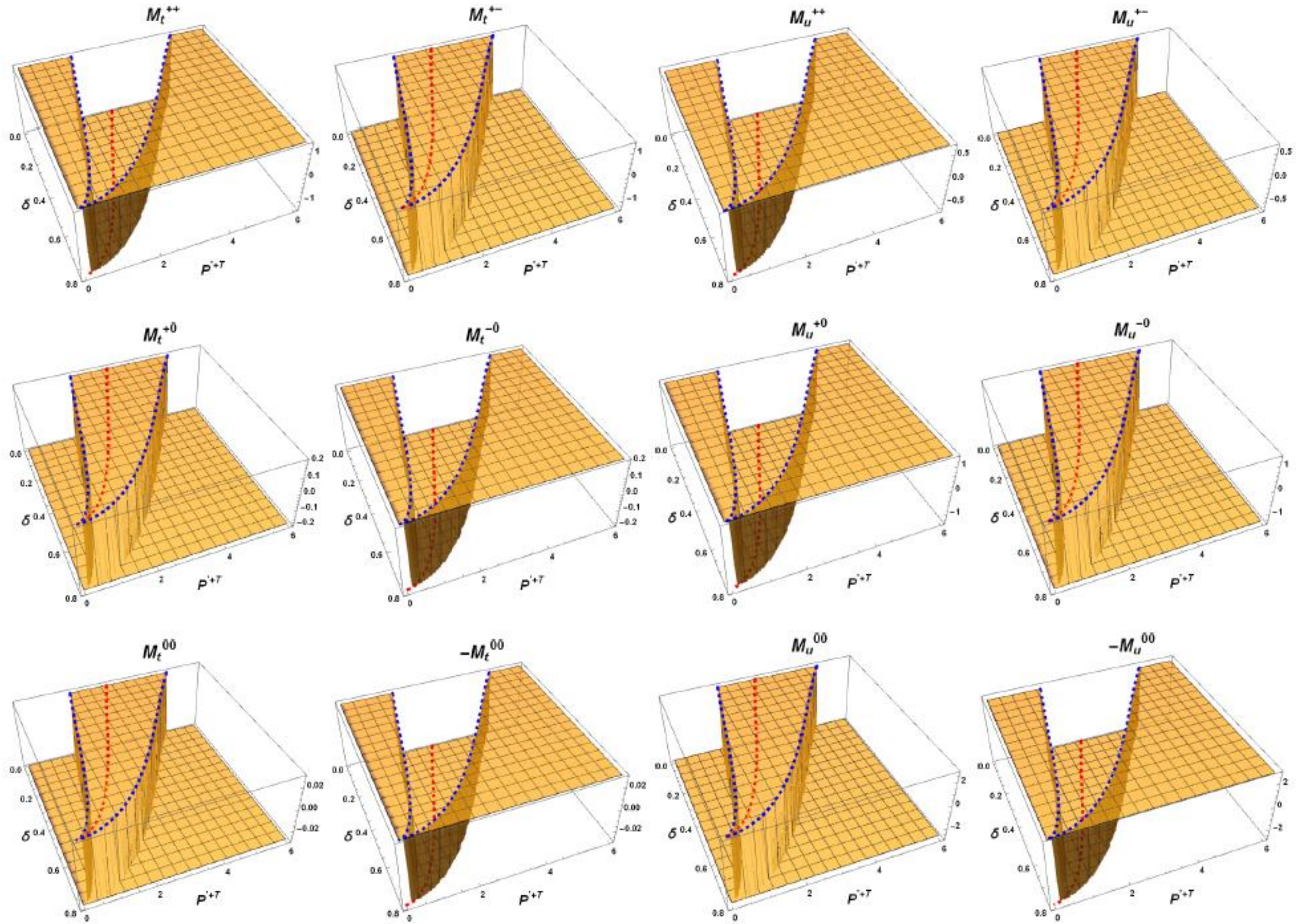
Orientation Entanglement in T and U channels



Processes Take place within some impact parameter

Involve orbital angular momentum, $\theta = \frac{\pi}{3}$

Orientation Entanglement in T and U channels



Symmetries between covariant helicity amplitudes

$M_t^{++} = M_t^{--}$ $M_u^{++} = M_u^{--}$ $M_{se}^{++} = M_{se}^{--}$	$M_t^{+-} = M_t^{-+}$ $M_u^{+-} = M_u^{-+}$ $M_{se}^{+-} = M_{se}^{-+}$
$M_t^{0+} = -(M_t^{0-})$ $M_u^{0+} = -(M_u^{0-})$ $M_{se}^{0+} = -(M_{se}^{0-})$	$M_t^{+0} = -(M_t^{-0})$ $M_u^{+0} = -(M_u^{-0})$ $M_{se}^{+0} = -(M_{se}^{-0})$

- <u>t – u Symmetry</u>

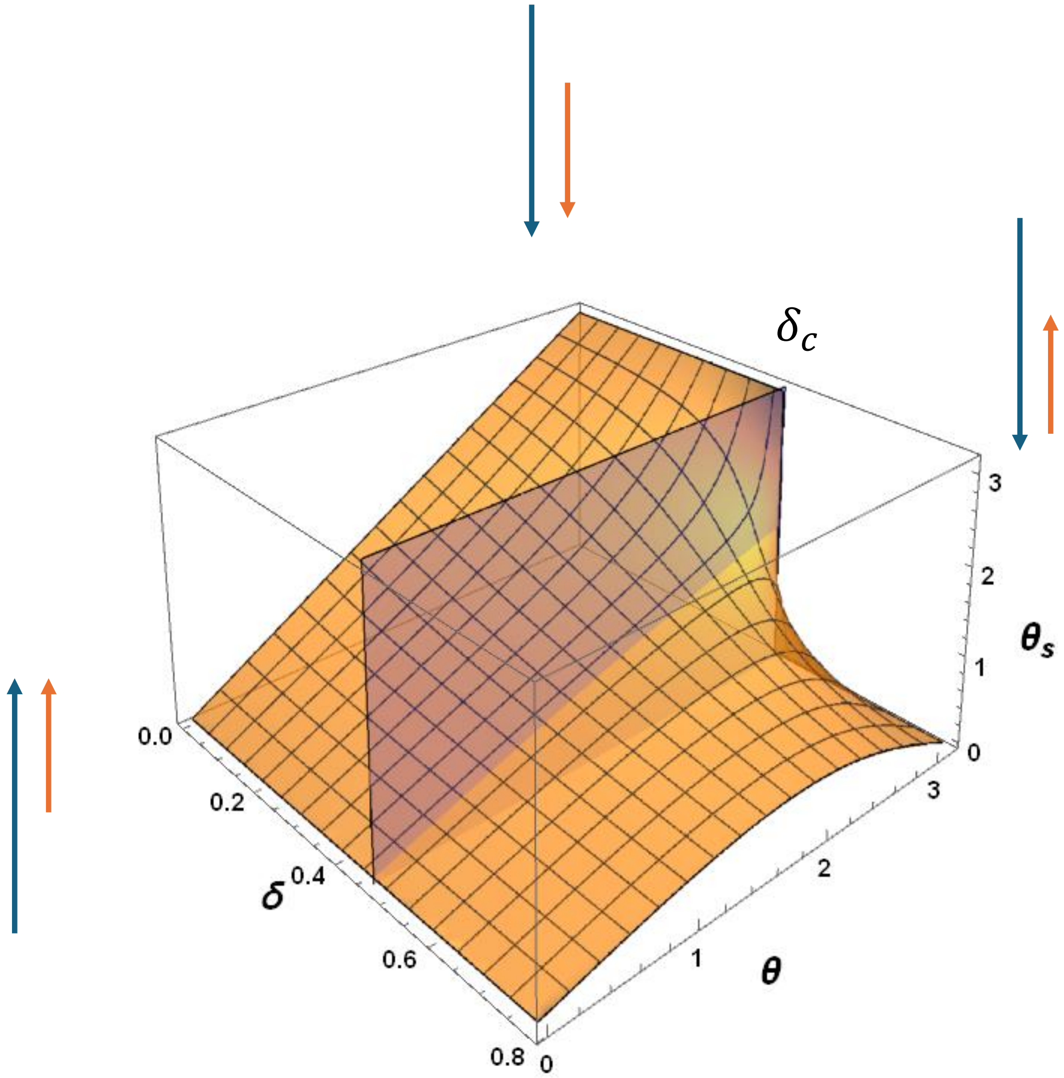
$$\boxed{\begin{aligned} M_t^{++}(\theta) &= M_u^{++}(\pi-\theta) \\ M_t^{+-}(\theta) &= M_u^{-+}(\pi-\theta) \\ M_t^{0+}(\theta) &= M_u^{+0}(\pi-\theta) \\ M_t^{+0}(\theta) &= M_u^{0+}(\pi-\theta) \\ M_t^{00}(\theta) &= M_u^{00}(\pi-\theta) \end{aligned}}$$

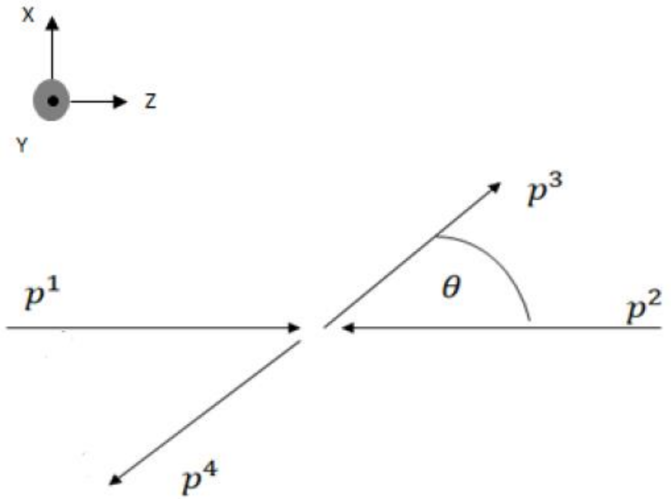
- Helicity amplitudes satisfy symmetry based on parity conservation.

$$M(-\lambda', -\lambda) = (-1)^{\lambda'-\lambda} M(\lambda', \lambda)$$

  - All time-ordered diagrams also satisfy the same symmetries mentioned above.

How can we see orientation entanglement in the LF?





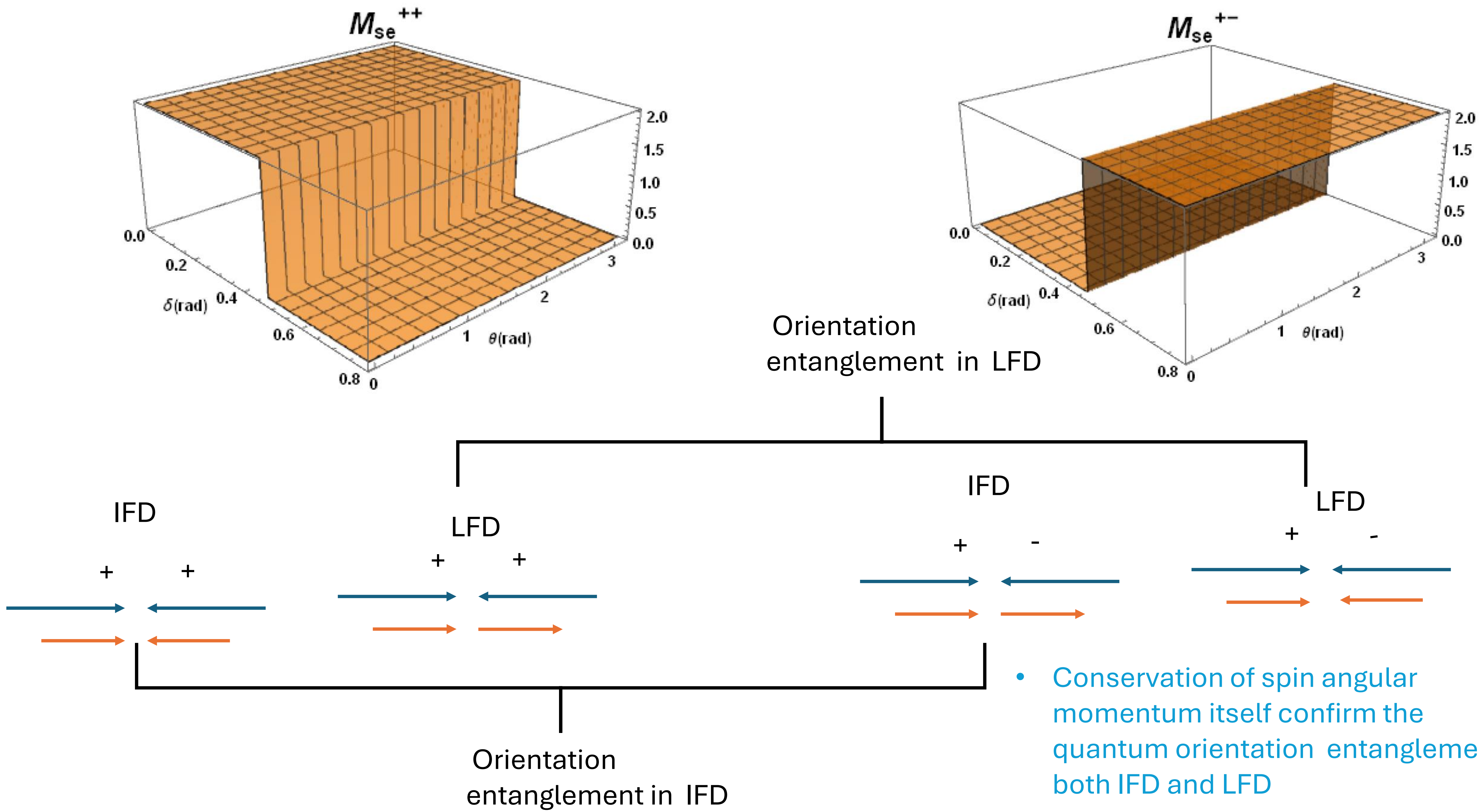
$$p^1 = (E_0, 0, 0, P_v)$$
$$p^2 = (E_0, 0, 0, -P_v)$$
$$p^3 = (E_0, P_s \sin\theta, 0, P_s \cos\theta)$$
$$p^4 = (E_0, -P_s \sin\theta, 0, -P_s \cos\theta)$$

$$\delta_c = ArcTan\left(\frac{p_v}{E_0}\right)$$

Particle move in negative z-direction rotates it's IFD spin 180^o in the LFD end.

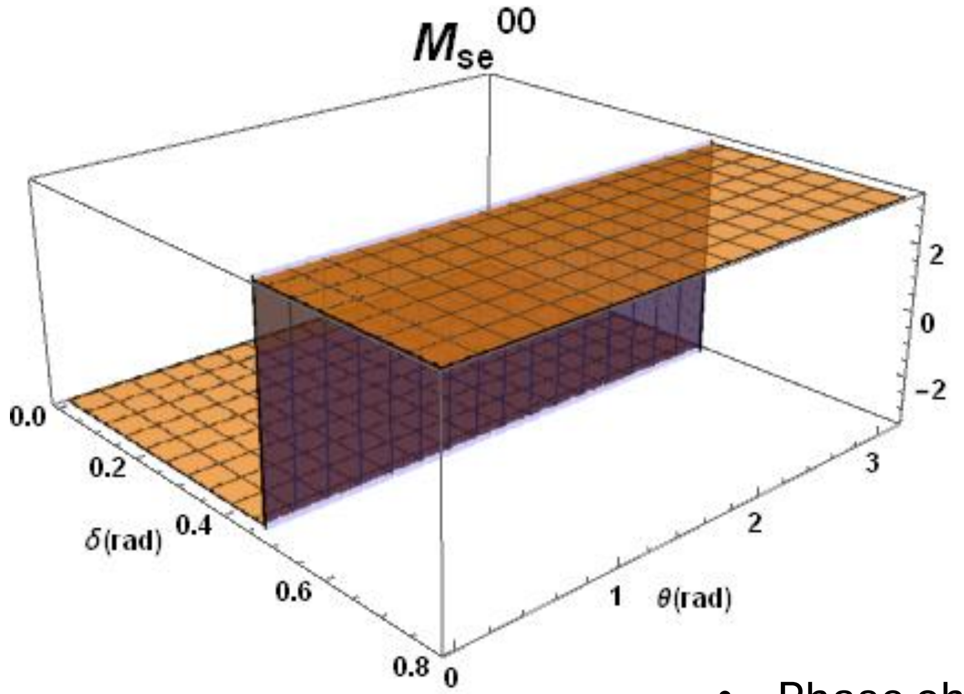
- Spin orientation changes with the particle's momentum direction and the interpolation angle in the rest frame (Positive helicity spinor/ polarization vector)

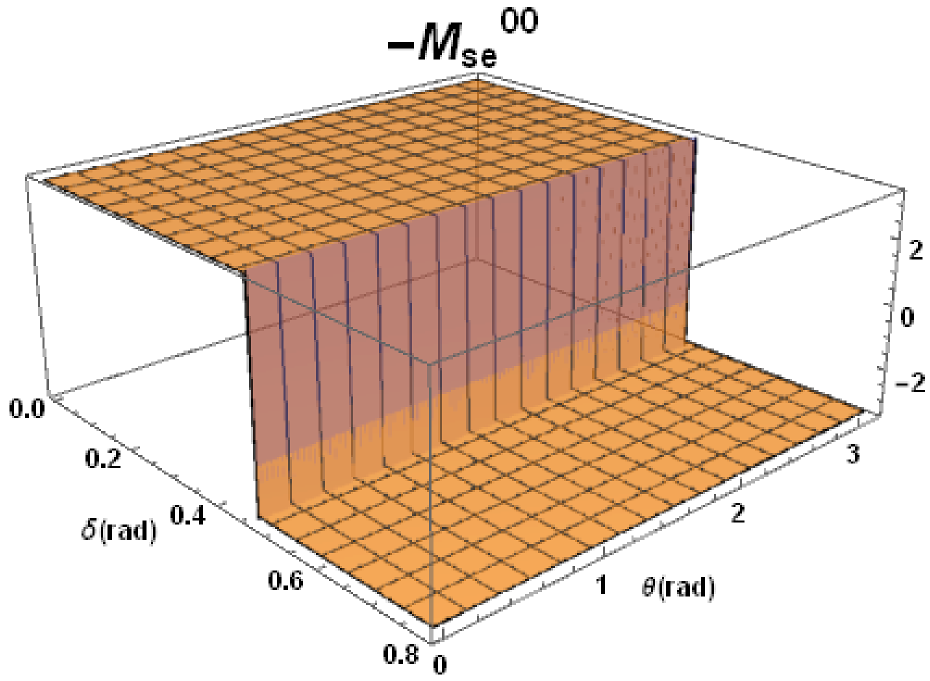
Orientation entanglement in the interpolating angular distribution



- Conservation of spin angular momentum itself confirm the quantum orientation entanglement in both IFD and LFD

Orientation entanglement in the interpolating angular distribution





- Phase changes due to QC

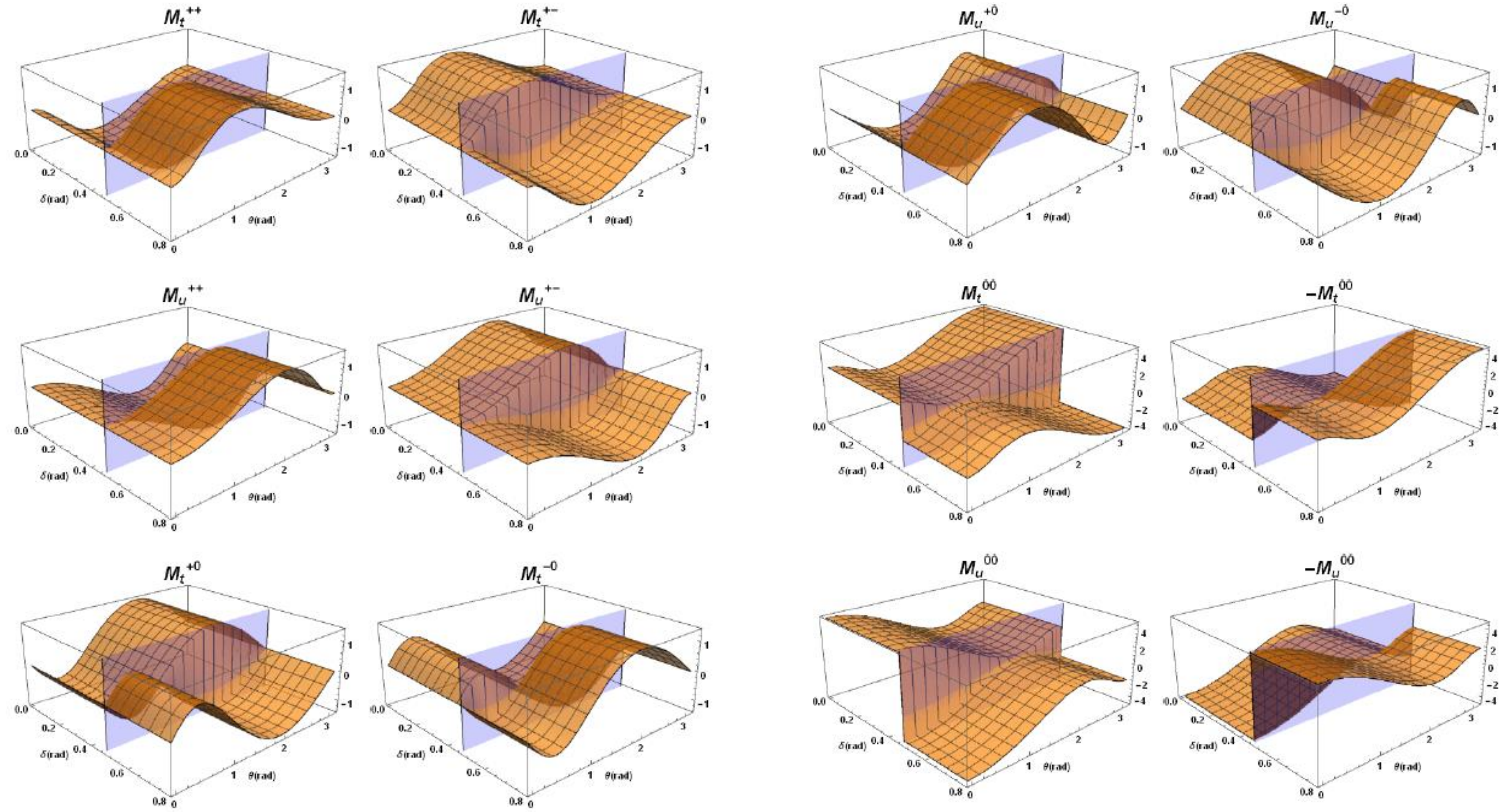
$$\epsilon^0(P_{\hat{-}} > 0) \Rrightarrow -\epsilon^0(P_{\hat{-}} > 0)$$

- $Mse^{+0} = Mse^{-0} = Mse^{0+} = Mse^{0-} = 0$ Do not satisfy the conservation of total angular momentum in any interpolation angle

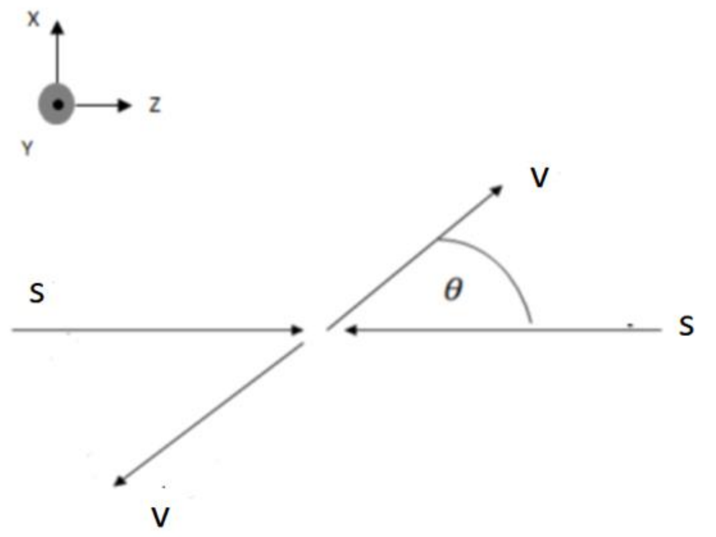
Reminder

Scalar particle and its anti-particle production by two massive spin-1 polarization vectors (VV→ SS) process particles with helicities move in +z and –z directions

Orientation entanglement in the interpolating angular distribution (T and U channels)



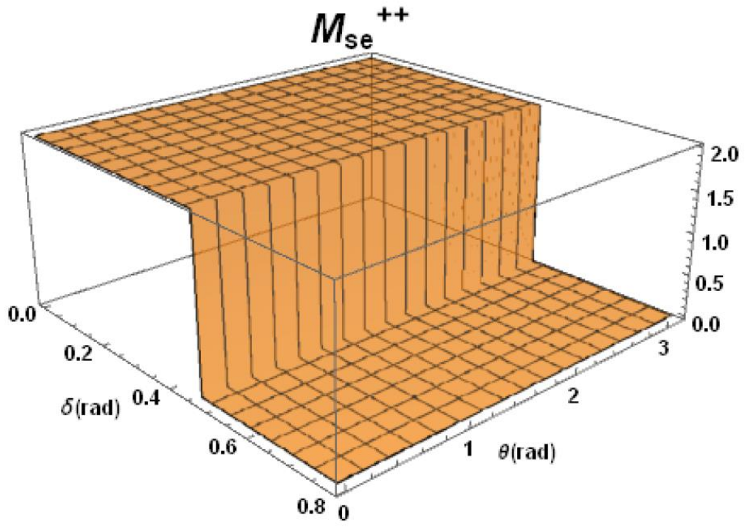
Orientation entanglement in the other directions



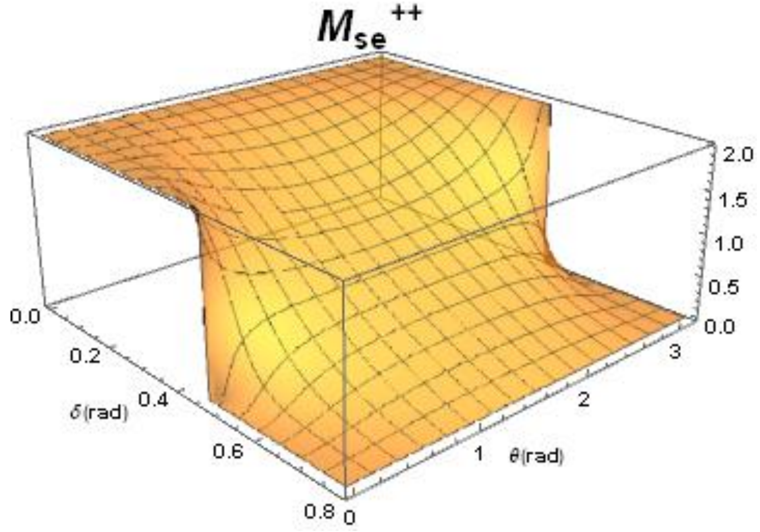
Two massive spin-1 polarization vectors production by scalar particle and its anti-particle process. $(SS \to VV)$

Colinear processes explicitly show the time reversal

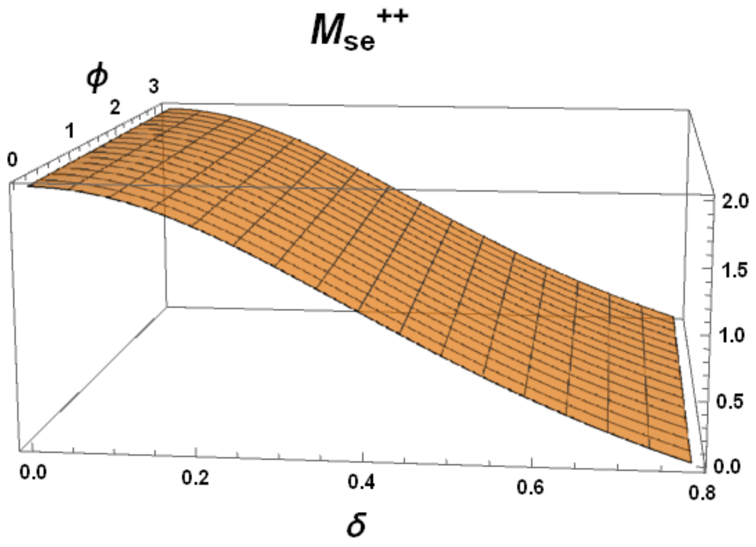
$vv(z) \to ss$



$ss(z) \to vv$

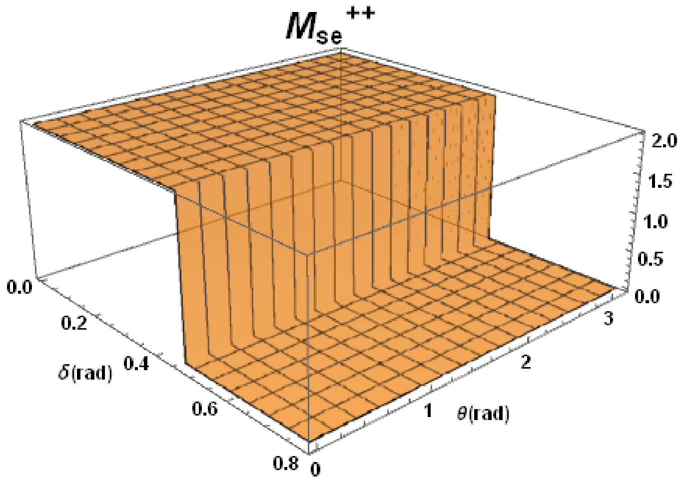


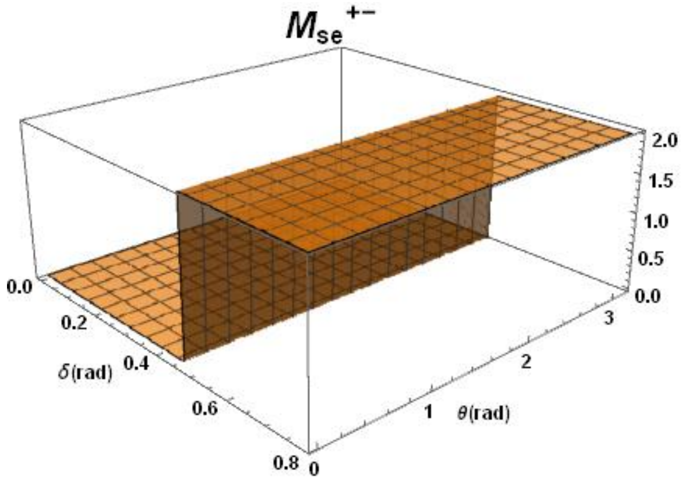
$vv(x) \to ss$



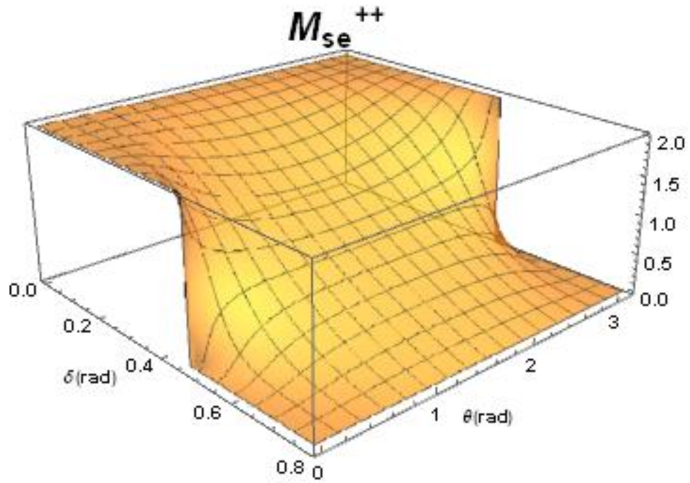
Orientation entanglement in the other directions

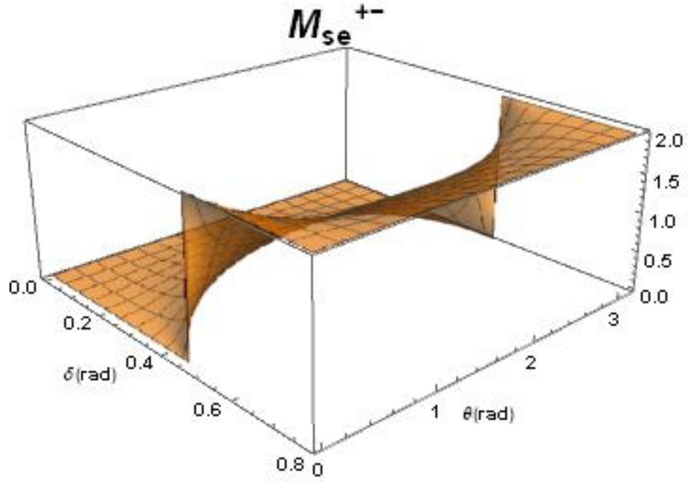
$vv(z) \rightarrow ss$



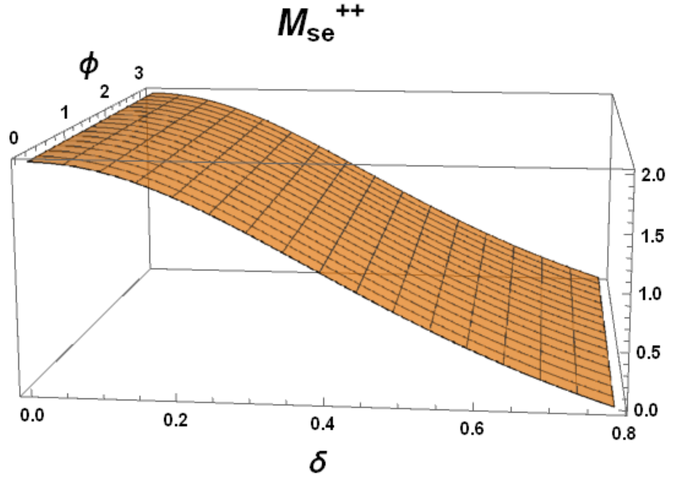


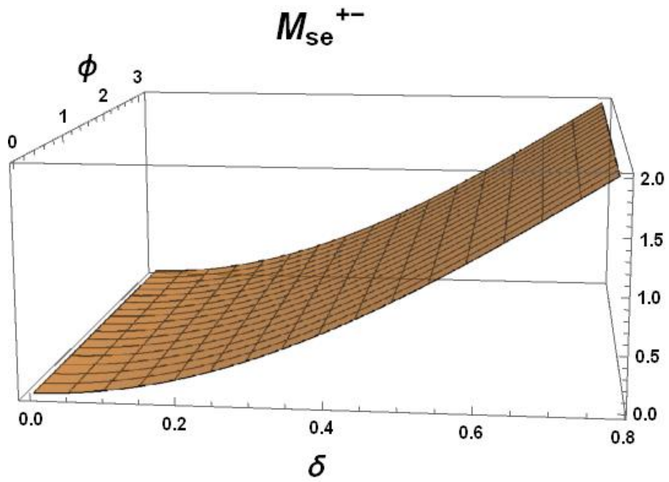
$ss(z) \rightarrow vv$





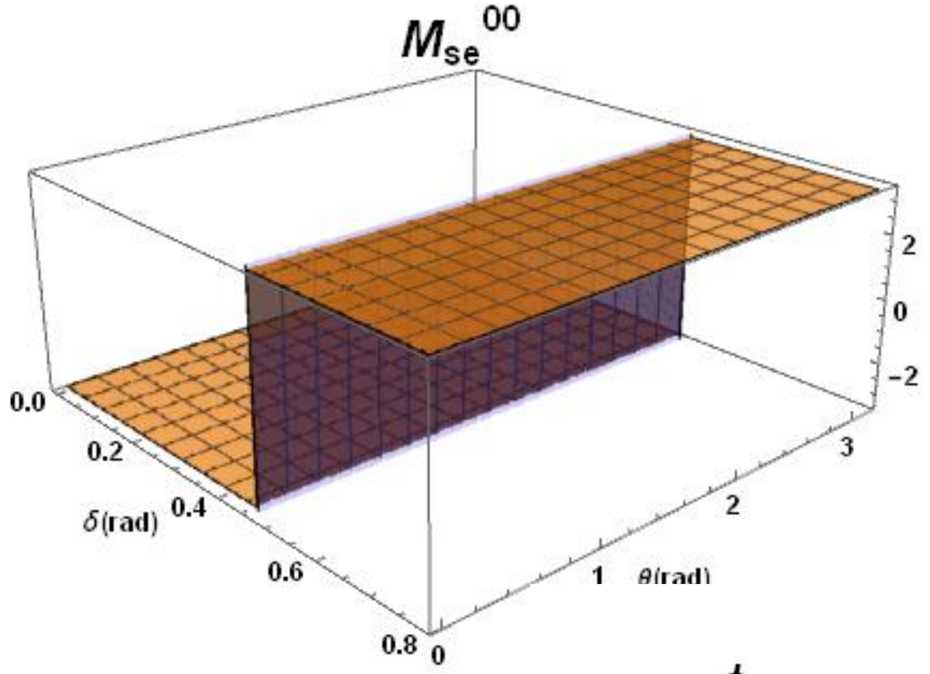
$vv(x) \rightarrow ss$



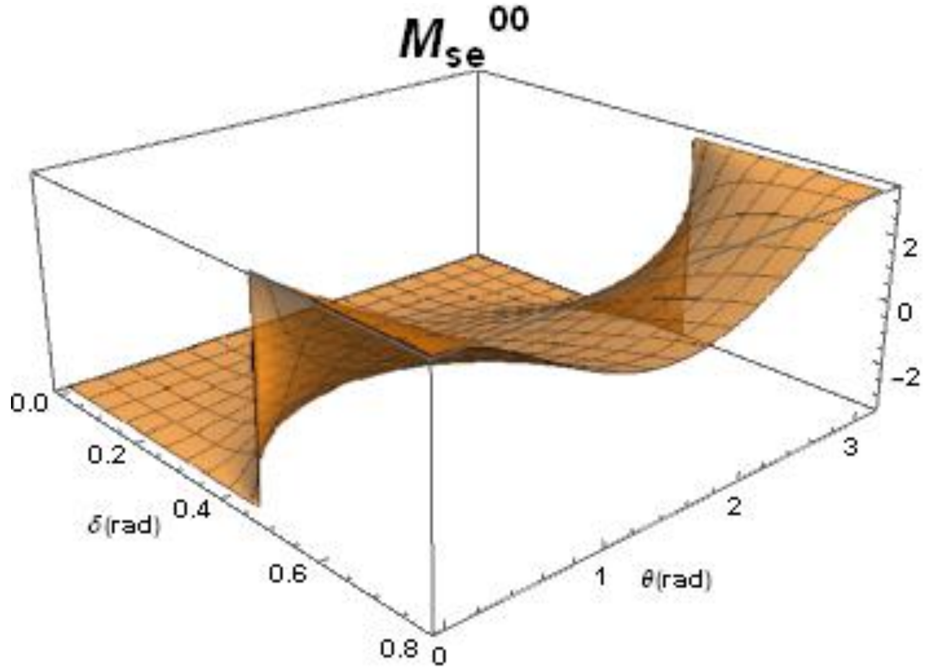


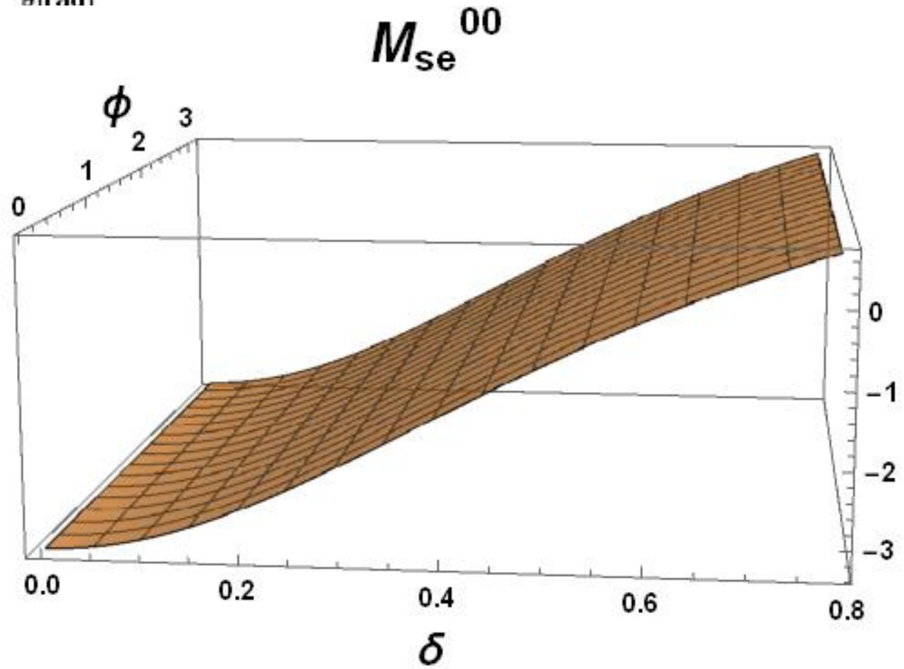
Orientation entanglement in the other directions

$vv(z) \rightarrow ss$



$ss(z) \rightarrow vv$

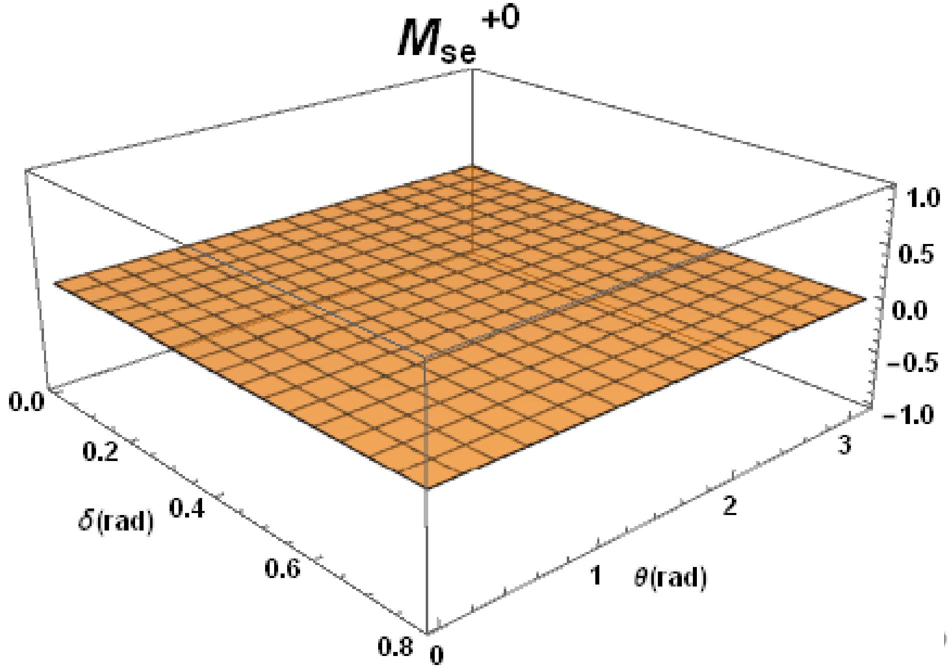




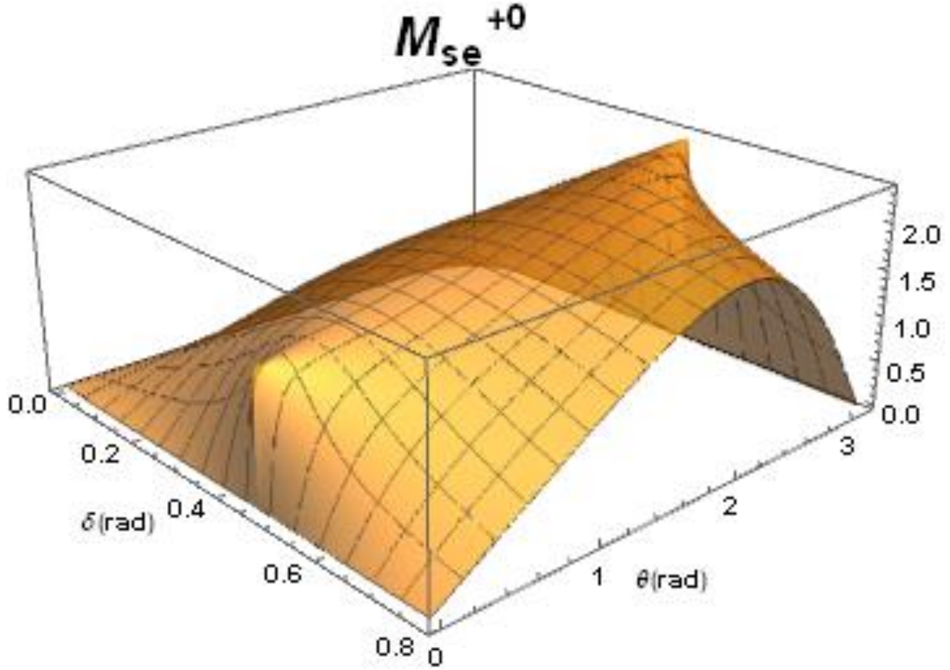
$vv(x) \rightarrow ss$

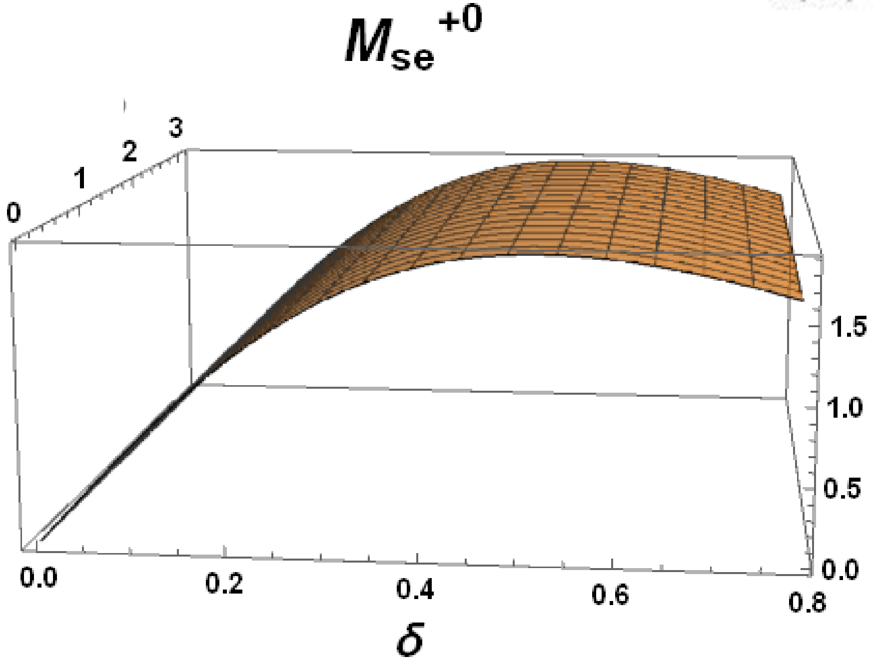
Orientation entanglement in the other directions

$vv(z) \rightarrow ss$



$ss(z) \rightarrow vv$





$vv(x) \rightarrow ss$

Other observations

- What will happen if we have processes involving real photons?
  - The helicity of the real photon is a relativistic invariant that matches its chirality.
  - No boundaries corresponding to the quantum correlation in this process.
  - We demonstrate it by considering $\gamma\gamma \rightarrow SS$ and $SS \rightarrow \gamma\gamma$ processes .

- What are the other important observations we made by considering these processes.
  - Decomposition of scalar propagator at the LF end due to the rational relationship of the light-front energy dispersion relationship.
  - Analysis of time-ordered helicity amplitudes proves that the decomposition of scalar propagator in LF limit results in some time-ordered helicity amplitudes vanishing exactly at the LF end regardless the boost of the frame (P^z) . It clarifies the general misnomer of equivalence between the light-front dynamics and infinity momentum approach (IMF) in the instant form dynamics as $P^z \rightarrow \infty$ and $P^z \rightarrow -\infty$ yield very different results from each other.

Conclusion

- In this research work, we are able to manifest the quantum correlation originated from orientation entanglement of spins in the interpolating helicity amplitudes between Instant Form Dynamic (IFD) and Light-Front Dynamic (LFD) for scattering/annihilation processes involve scalar and vector particles.

- According to our calculations, the orientation entangled states of interpolating spin-1 spinors and polarization vectors show helicity changes in “+" and “-” helicity states but and phase changes in “0" helicity state which clearly distinguish “0" helicity state of scalar particle.

- We confirm that one can interpret the orientation entanglement of spinors in IFD as the Jacob and Wick helicity defined in IFD due to the variance of helicity under the boost, and such interpretation can't be used in the LFD as the helicity defined in the LFD is boost invariant and the light-front transverse angular operator involves changing the particle number. But our interpolation formalism enable us to show the orientation entanglement of spinors at the vicinity of the LF, that arm the significance of using interpolating formalism in electrodynamic calculations.

- Conservation of spin angular momentum itself confirm the quantum orientation entanglement in both IFD and LFD, which can be clearly seen in the angular distribution of interpolating helicity amplitudes,