Application of gaussian-expansion method in the light-front quark model

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NCSU physics seminar, 26 april 2024

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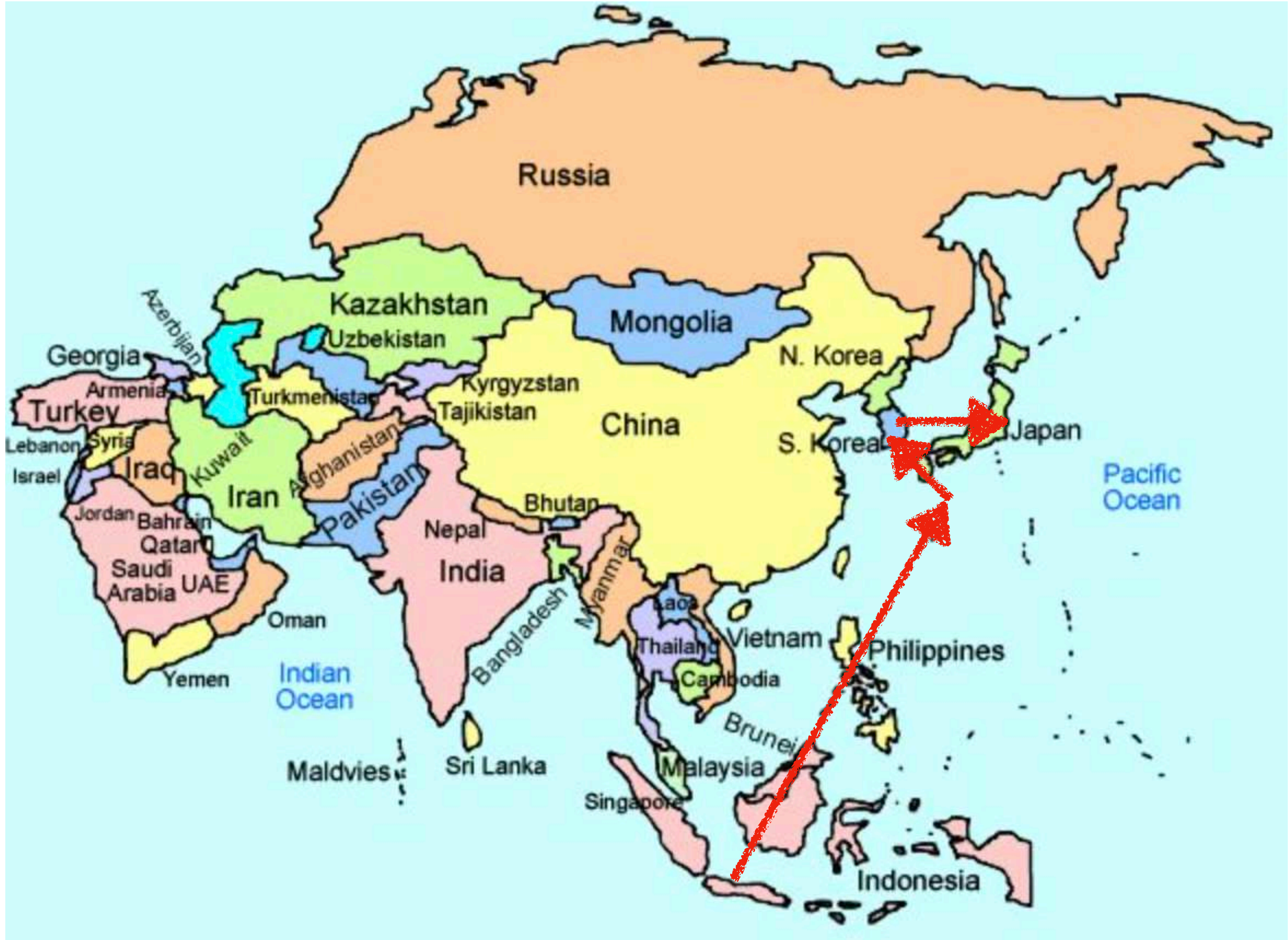

https://ajarifi.github.io

- **Bachelor, University of Indonesia (2011 - 2015)**
  —> Prof. Mart [Kaon photo-production]
- **Internship, JAEA (2019 - 2020)**
  —> Prof. Tanida [Analysis of Belle data]
- **Ph.D. & Postdoc, Osaka University (2015 - 2021)**
  —> Prof. Hosaka [Quark model & Dalitz plot]
- **Postdoc, APCTP, South Korea (2021-2023)**
  —> Profs. Ji and Choi [Light-Front Quark Model]
  —> Prof. Tsushima [MIT bag model]
- **Postdoc, RIKEN (2023 - Now)**
  —> Profs. Hiyama and Oka [Gaussian Expansion Method]

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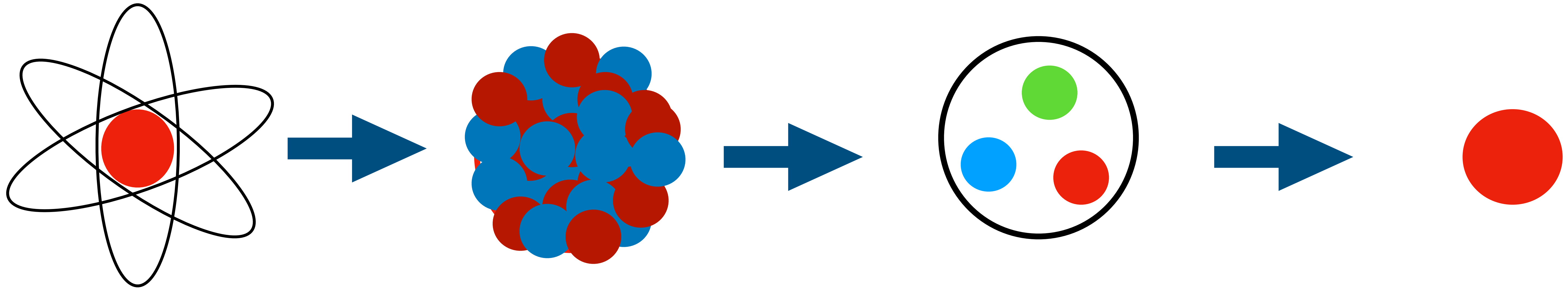
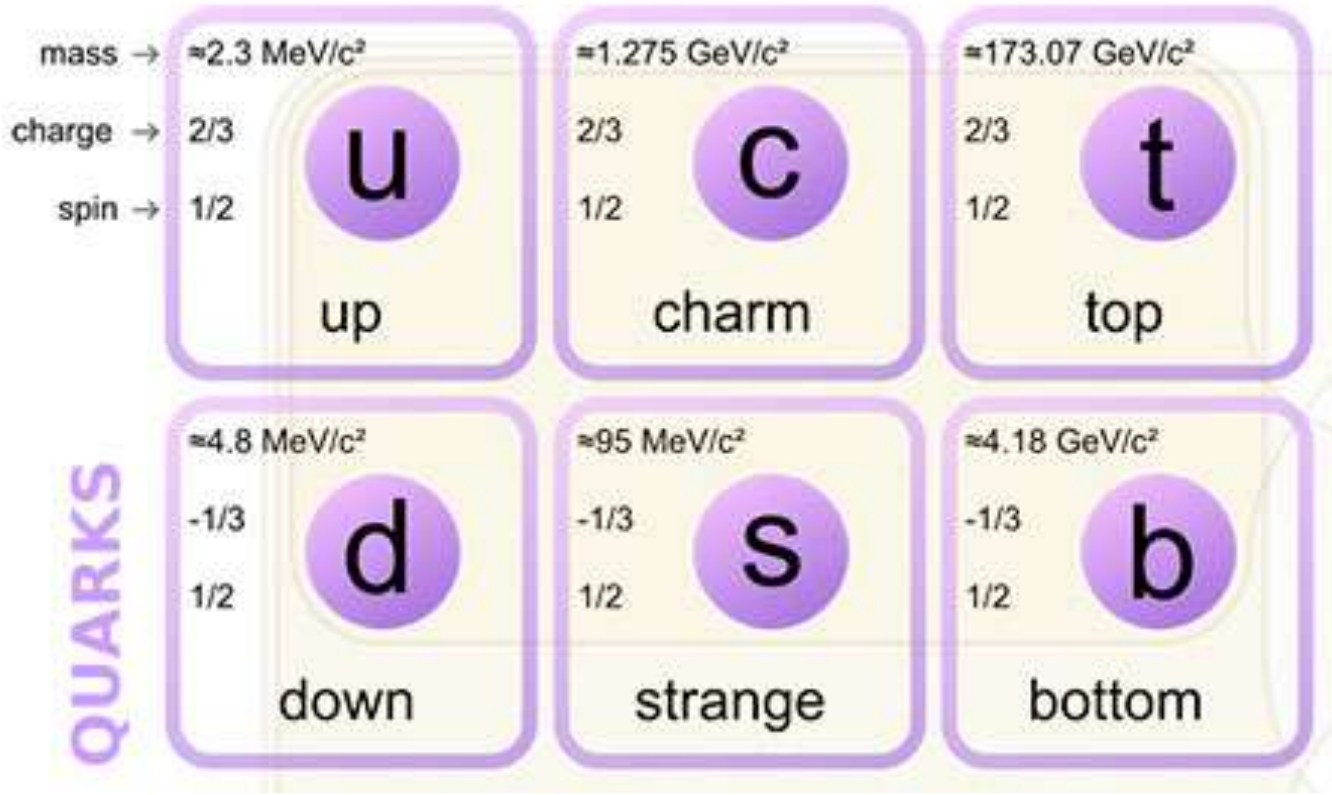
Finding hadrons

Hierarchy of matter



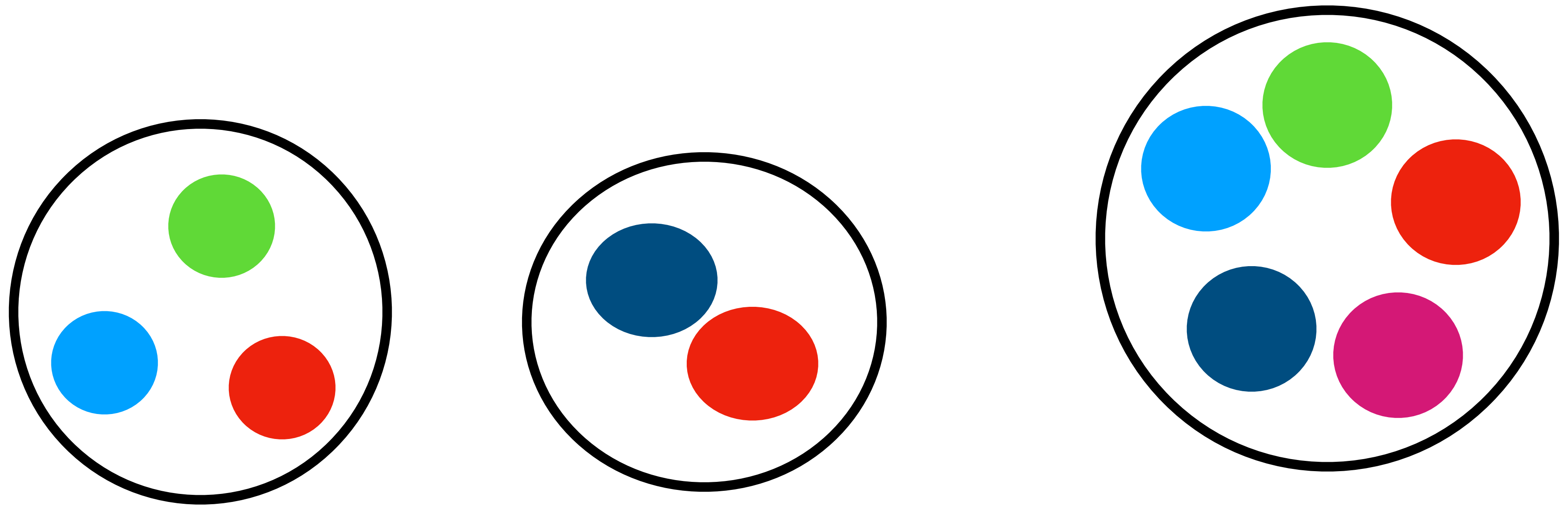
Atom Nucleus Nucleon Quark

Quark flavor



Types of hadrons

pdg.lbl.gov



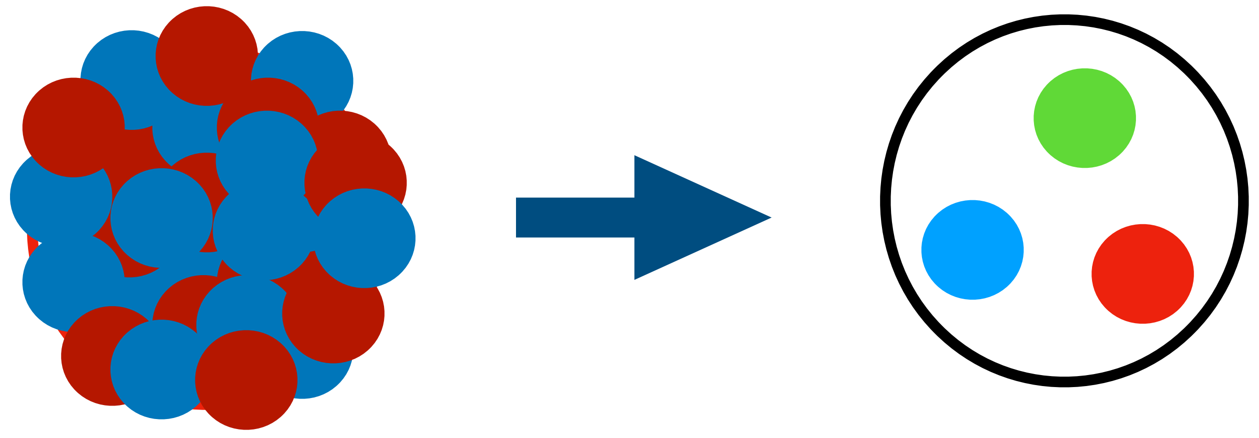
Baryon Meson Multiquark/ Exotic

Main goal

- *To understand the structure and spectroscopy of hadrons*

Where are hadrons?

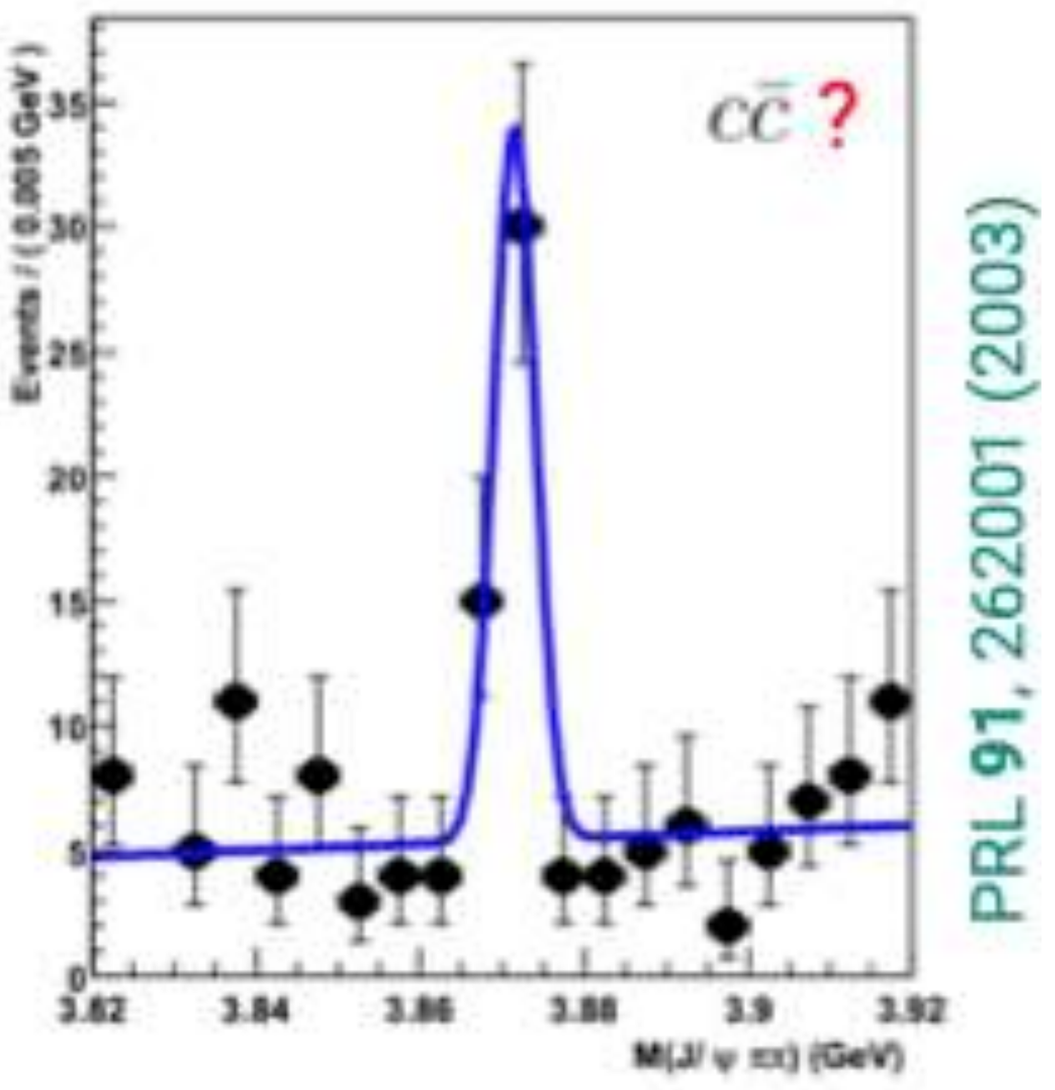
Inside an atom



Proton (uud)

Neutron (udd)

Nucleus



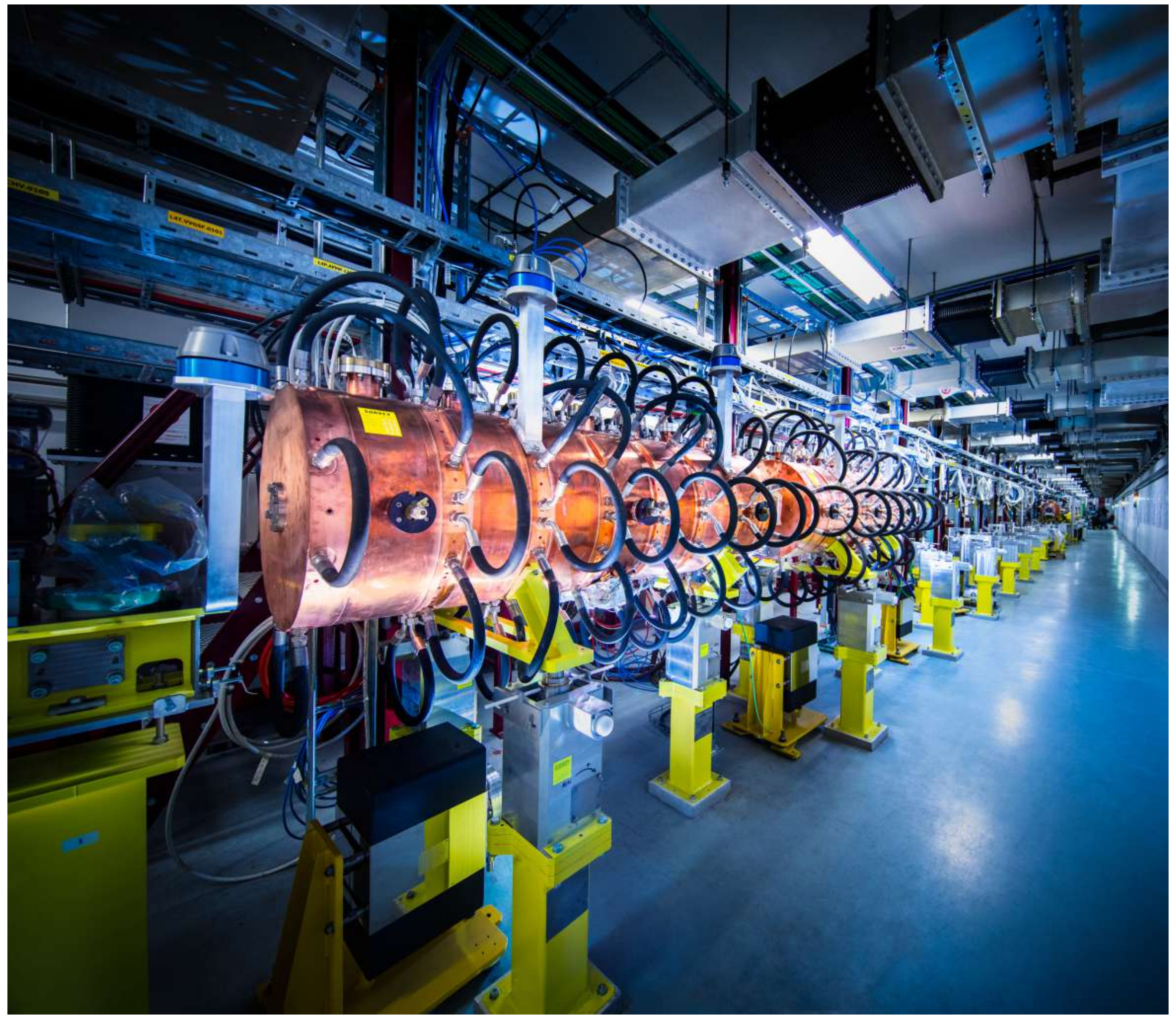
A bump!!

- Is it a hadron?
- Which hadron?

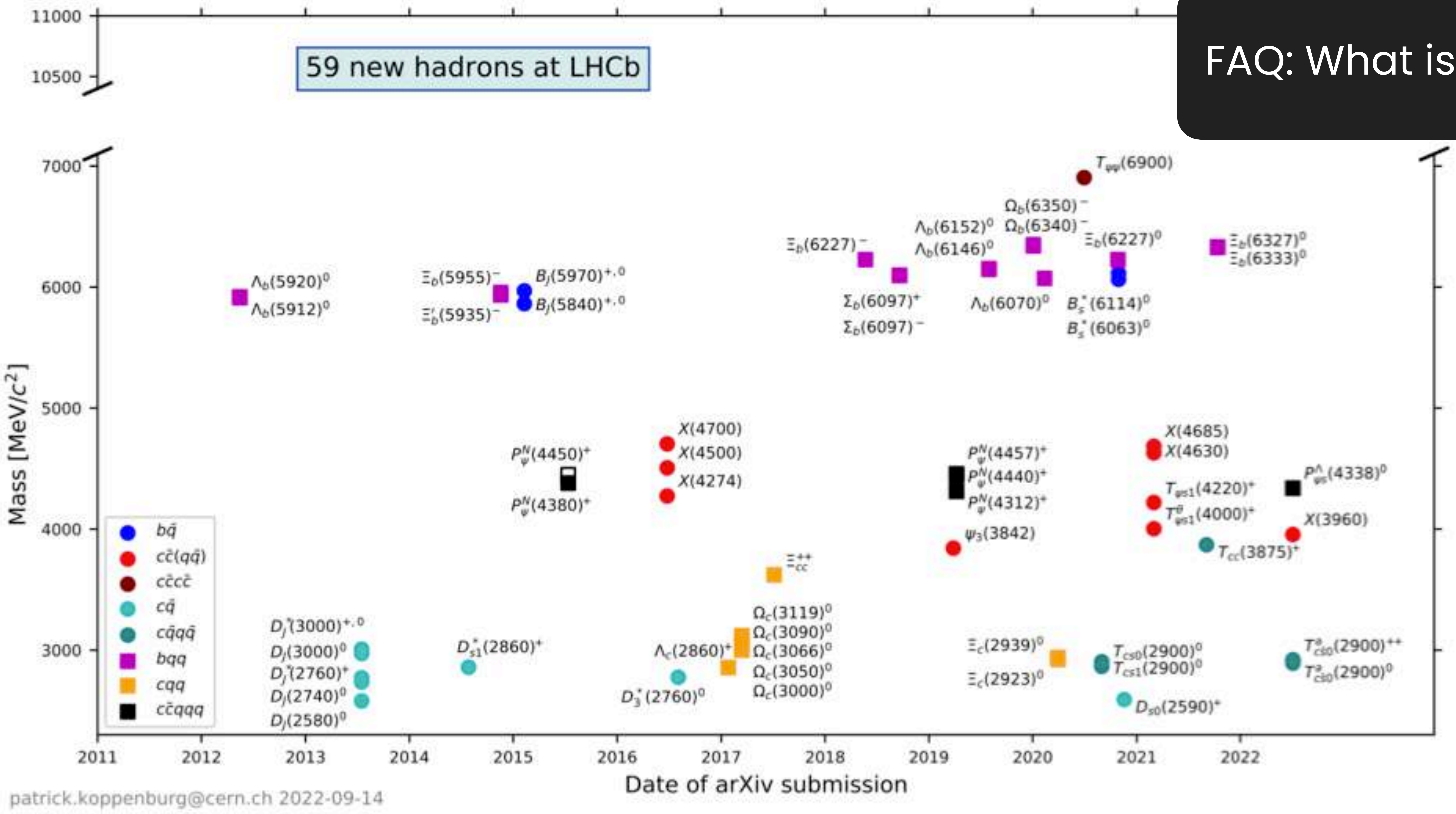
$X(3872)$

Where are the others?

- Cosmic ray, etc
- Most of them are produced in experiments: Scattering or decays



Hadron spectroscopy: many new states!

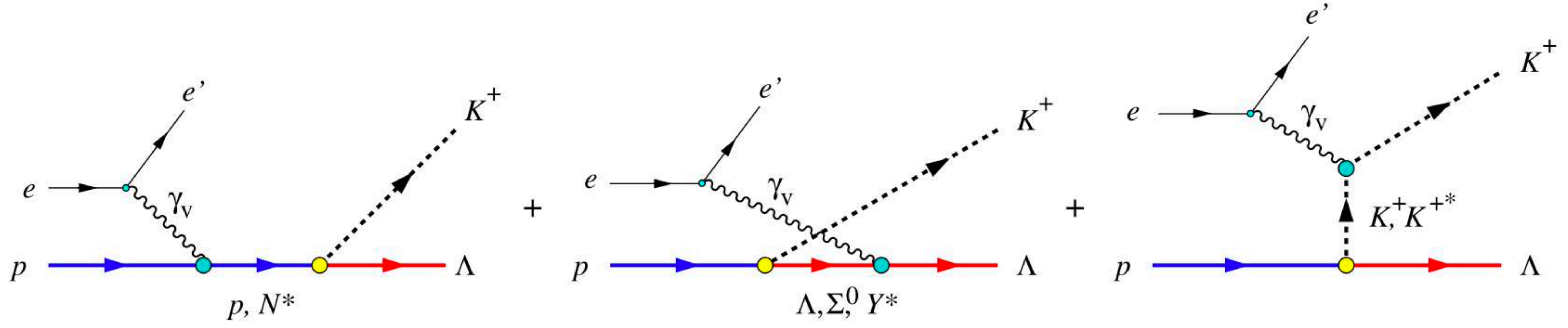


FAQ: What is its internal structure?

- Mass, Width?
- Pole position?
- Branching ratio?
- Coupling?
- Radius?
- Spin-parity?
- Compact?
- Molecule?
- …

Hadron experiments: LHC, KEK, J-PARC, BESIII, J-Lab, EIC, etc…

Hadron resonances via scattering



- $\gamma p \to K\Lambda$ scattering,
- Many other scattering data.

Observables

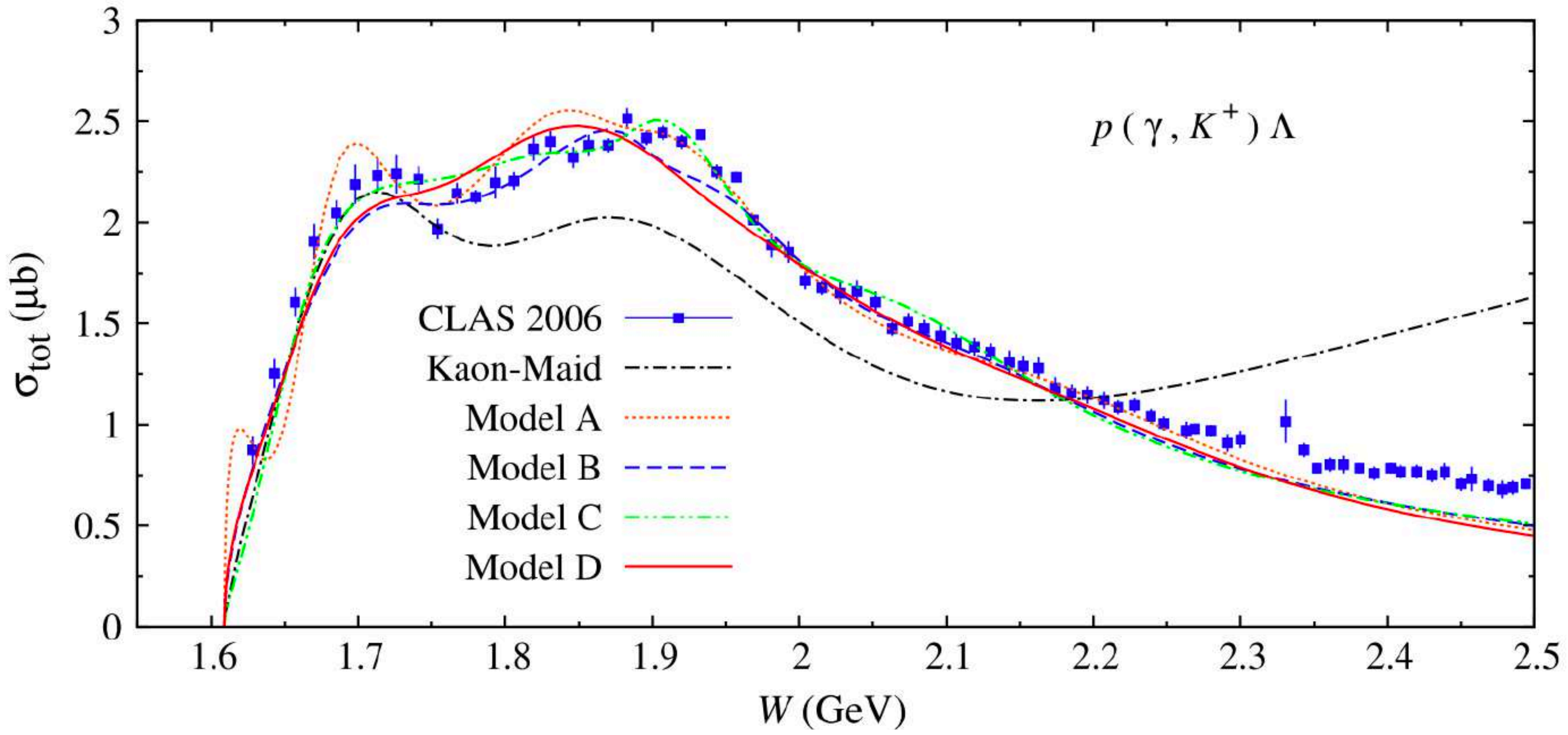
- Total and differential cross section,
- Polarization observables

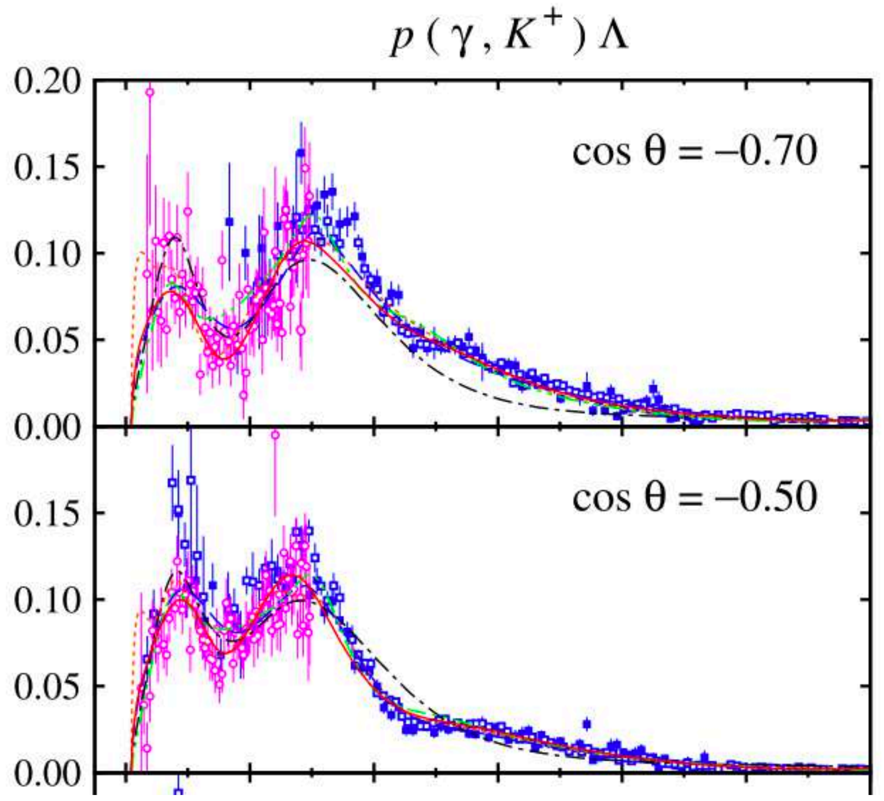
Methods

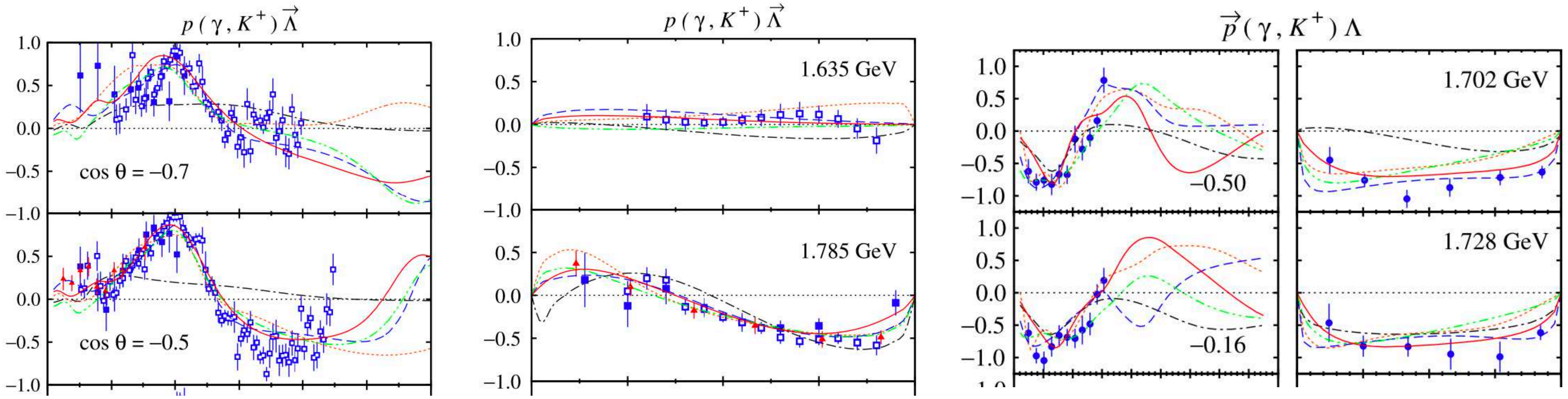
- Effective Lagrangian approach
- Coupled-channel approach
- χ^2 fit to the data, (N>7000 data)

Analysis

- Extracted nucleon resonances







Nucleon resonances in PDG

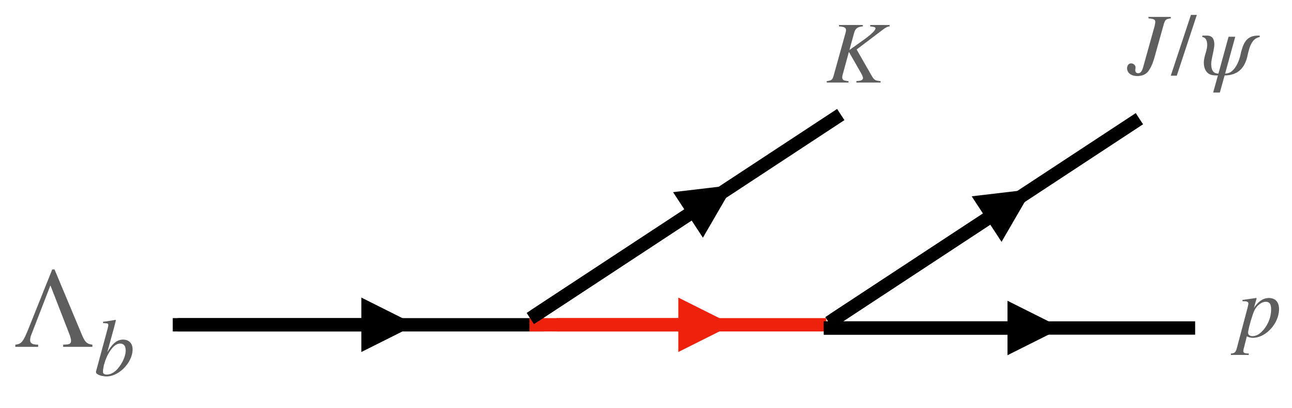
- There are many nucleon resonances discovered.

p	$1/2^+$	****
n	$1/2^+$	****
$N(1440)$	$1/2^+$	****
$N(1520)$	$3/2^-$	****
$N(1535)$	$1/2^-$	****
$N(1650)$	$1/2^-$	****
$N(1675)$	$5/2^-$	****
$N(1680)$	$5/2^+$	****
$N(1700)$	$3/2^-$	***
$N(1710)$	$1/2^+$	****
$N(1720)$	$3/2^+$	****
$N(1860)$	$5/2^+$	**
$N(1875)$ was $N(2080)$	$3/2^-$	***

How to understand this spectrum?

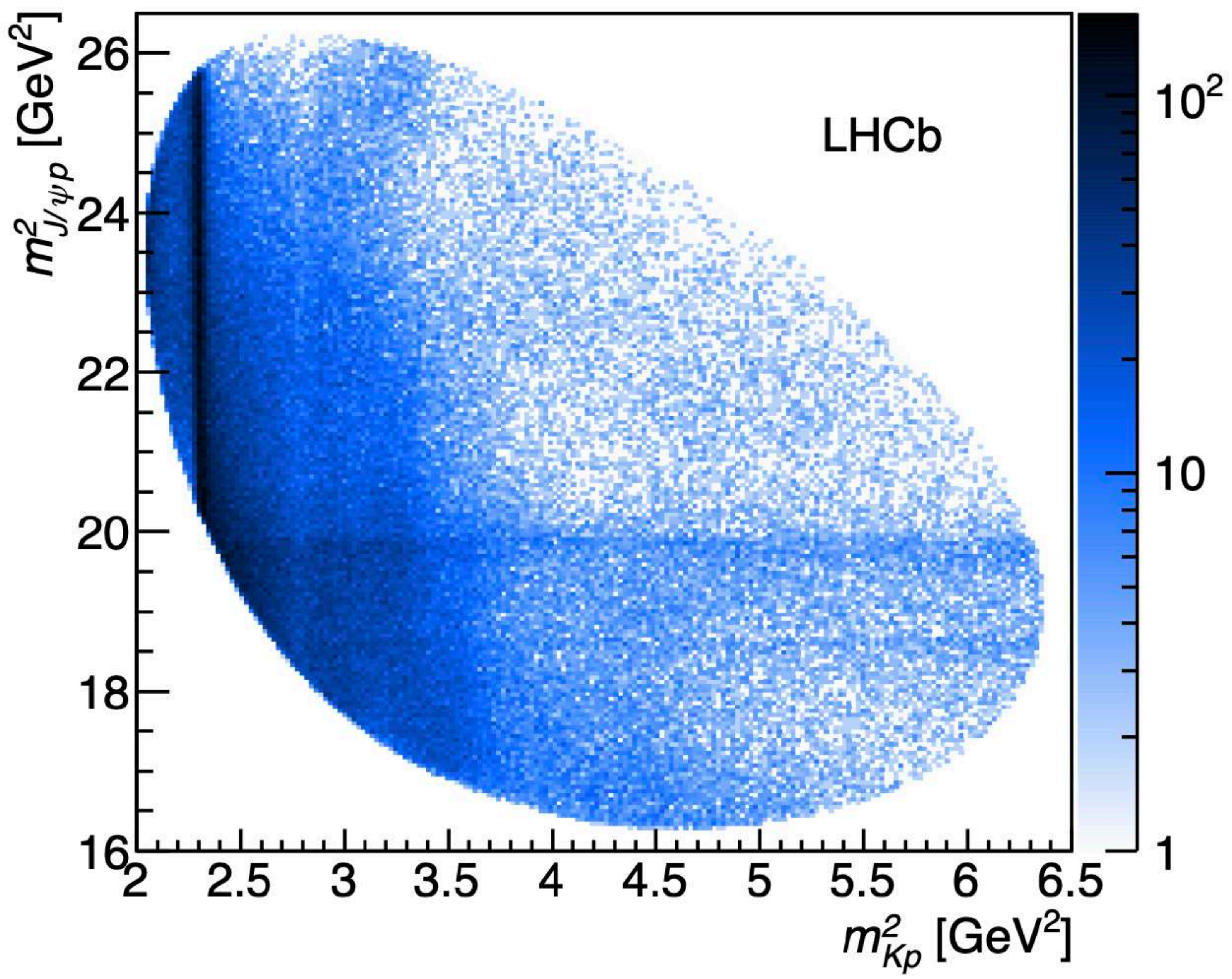
- **Hadron excitations**
  - Evidence that it is a composite particle.
  - Problem: Nonperturbative!!
- **Quark model predictions.**
  - Too many states?
  - Missing resonance problem?
  - Coupled-channel effect?
- **Existence of exotic states?**

Hadron resonances via multi-body decay

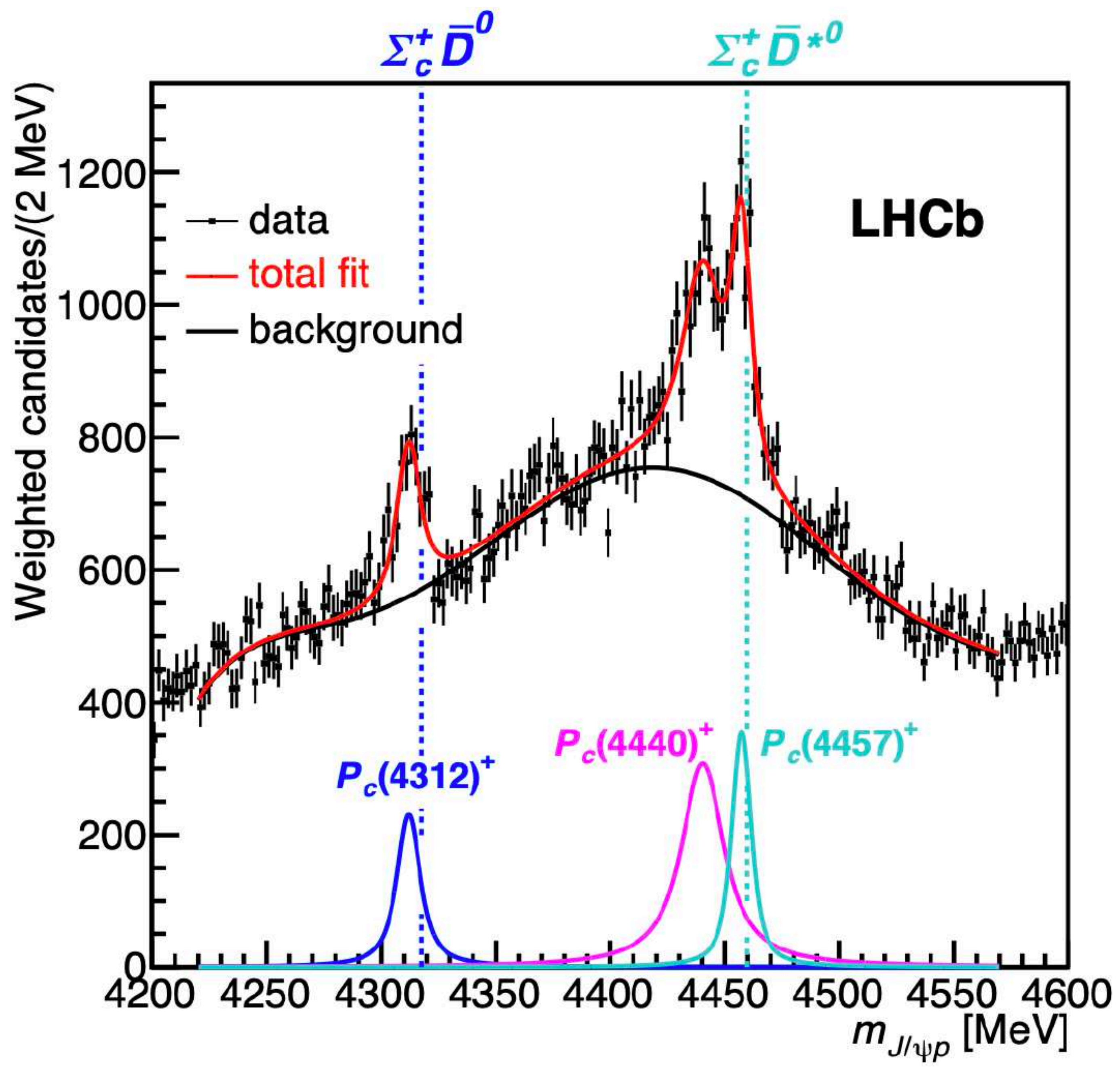


Three-body decay

- Another way to search for hadron resonances.
- proton-proton, electron-positron collisions:
  - Produce heavy particles and decays into smaller ones.



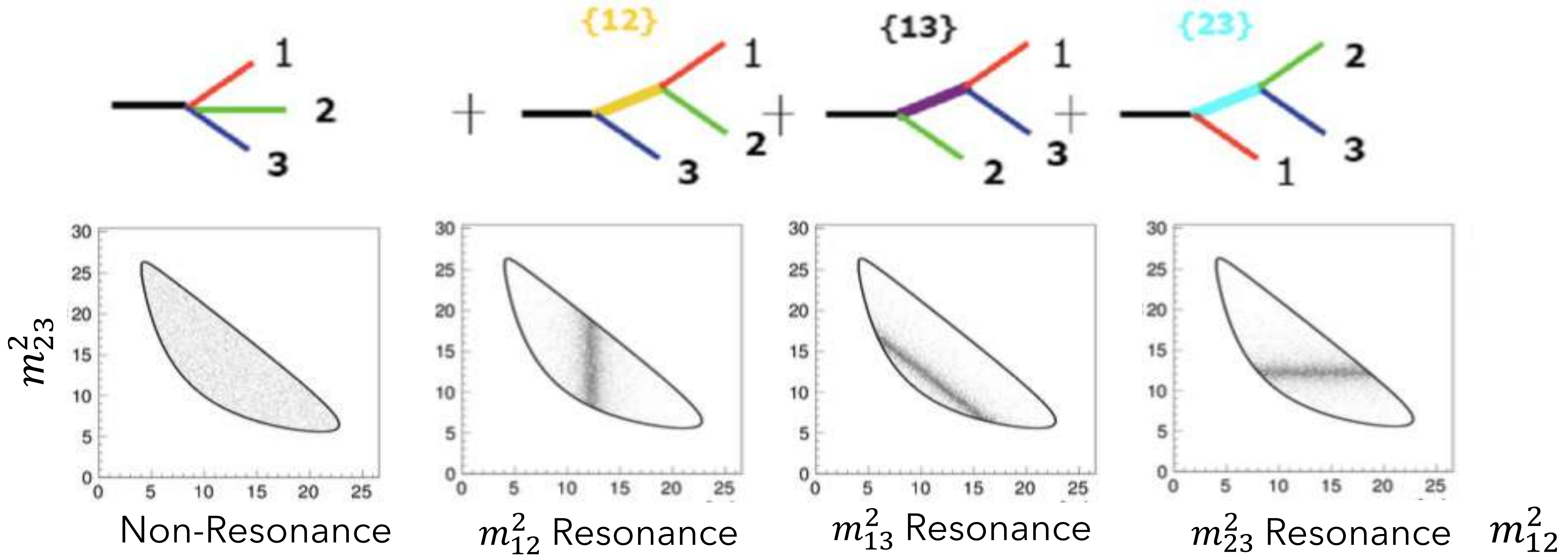
Dalitz plot

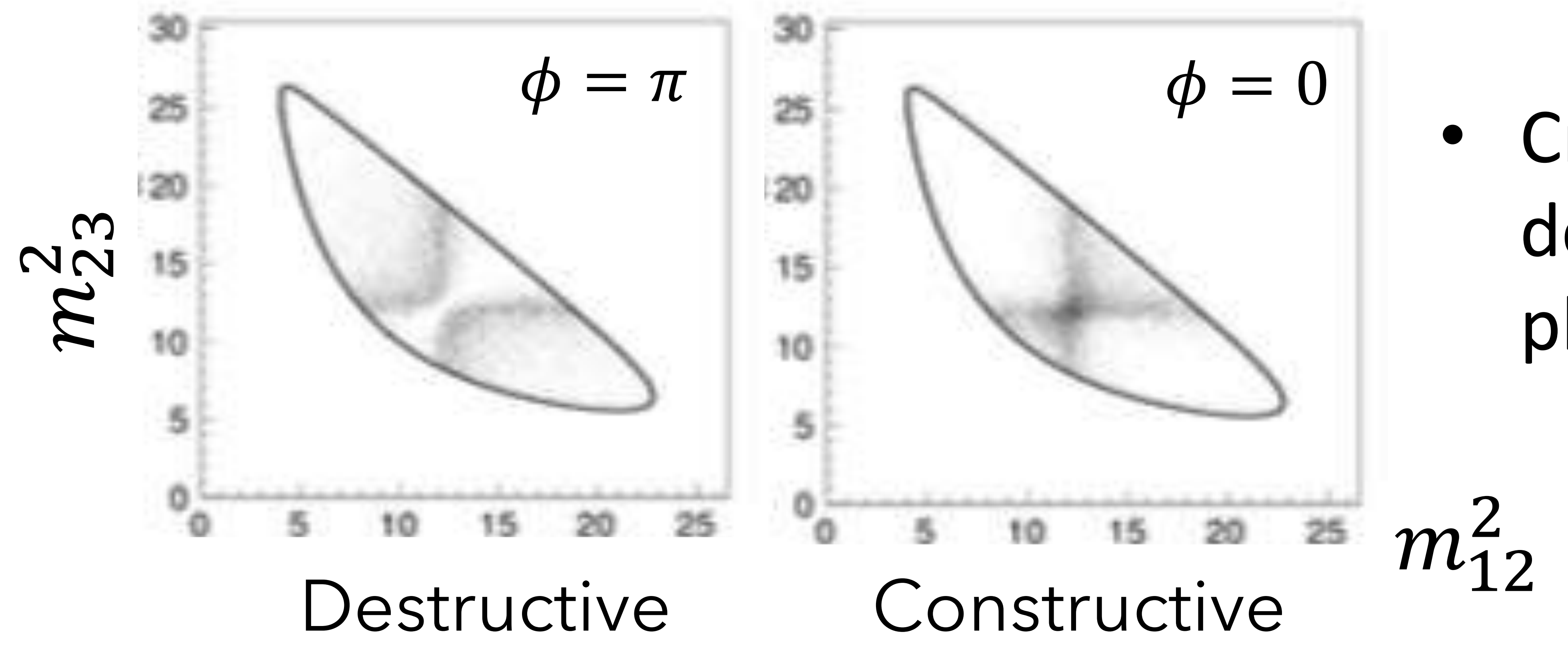


Invariant mass plot (a projection)

- Pc pentaquarks were discovered in this way.

Dalitz plots and resonance bands





- Cross channel interference depending on the relative phase.

Observables

- Dalitz plots
- Invariant mass distribution
- Angular correlation

Methods

- (Helicity) Amplitude analysis
- χ^2 fit to the data

Analysis

- Extract resonance information
- Determine quantum number

Analysis of $\Lambda_b(6072)$

LHCb Observation

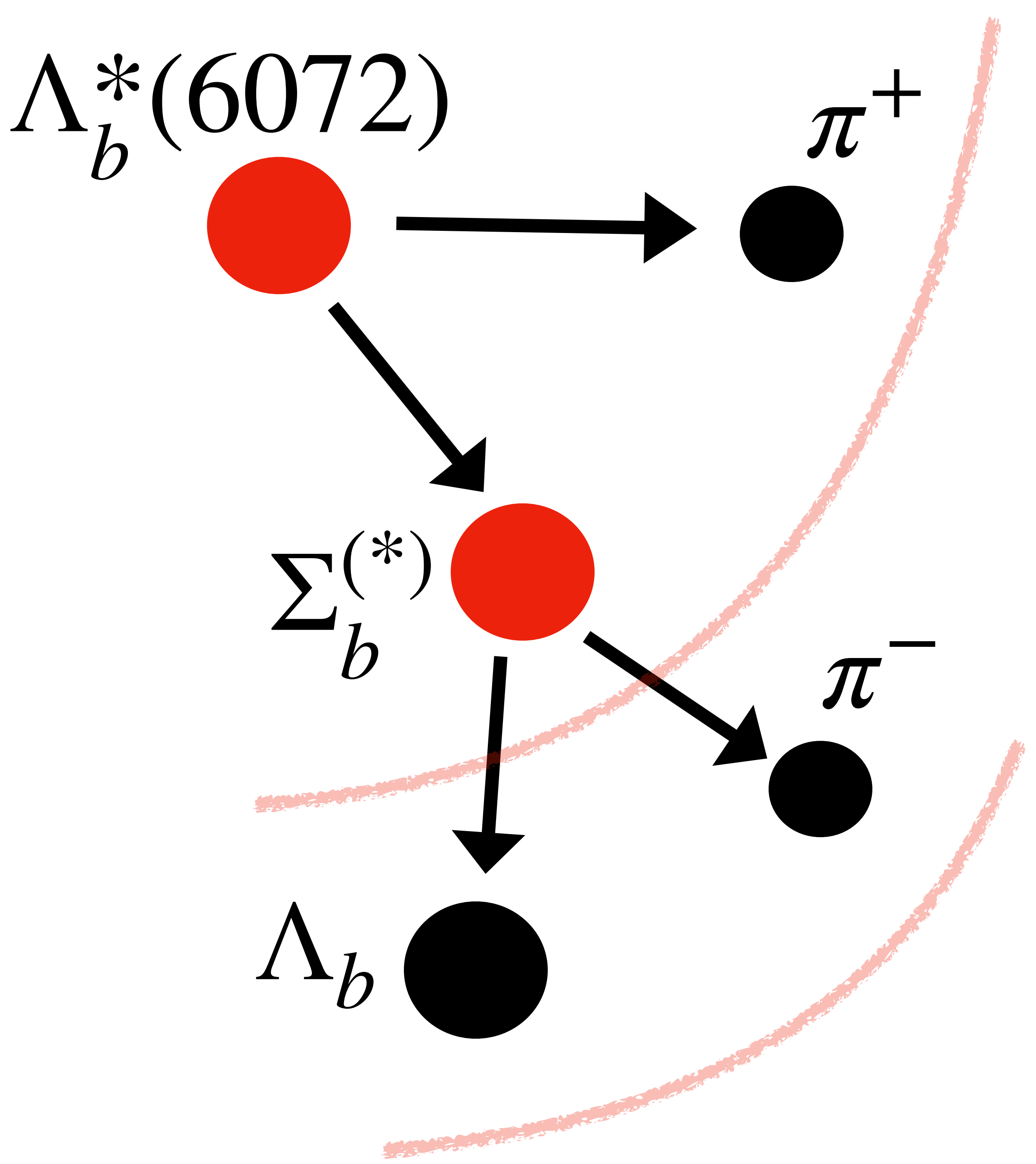
- Observed resonance [qqb]
  - >> $M = 6072$ MeV
  - >> $\Gamma = 72$ MeV
  - >> $\Lambda_b^* \to \Lambda_b \pi^+ \pi^-$

LHCb, JHEP 06, 136 (2020).

- What is its spin-parity?
  - -> By analyzing its decay, we can determine them

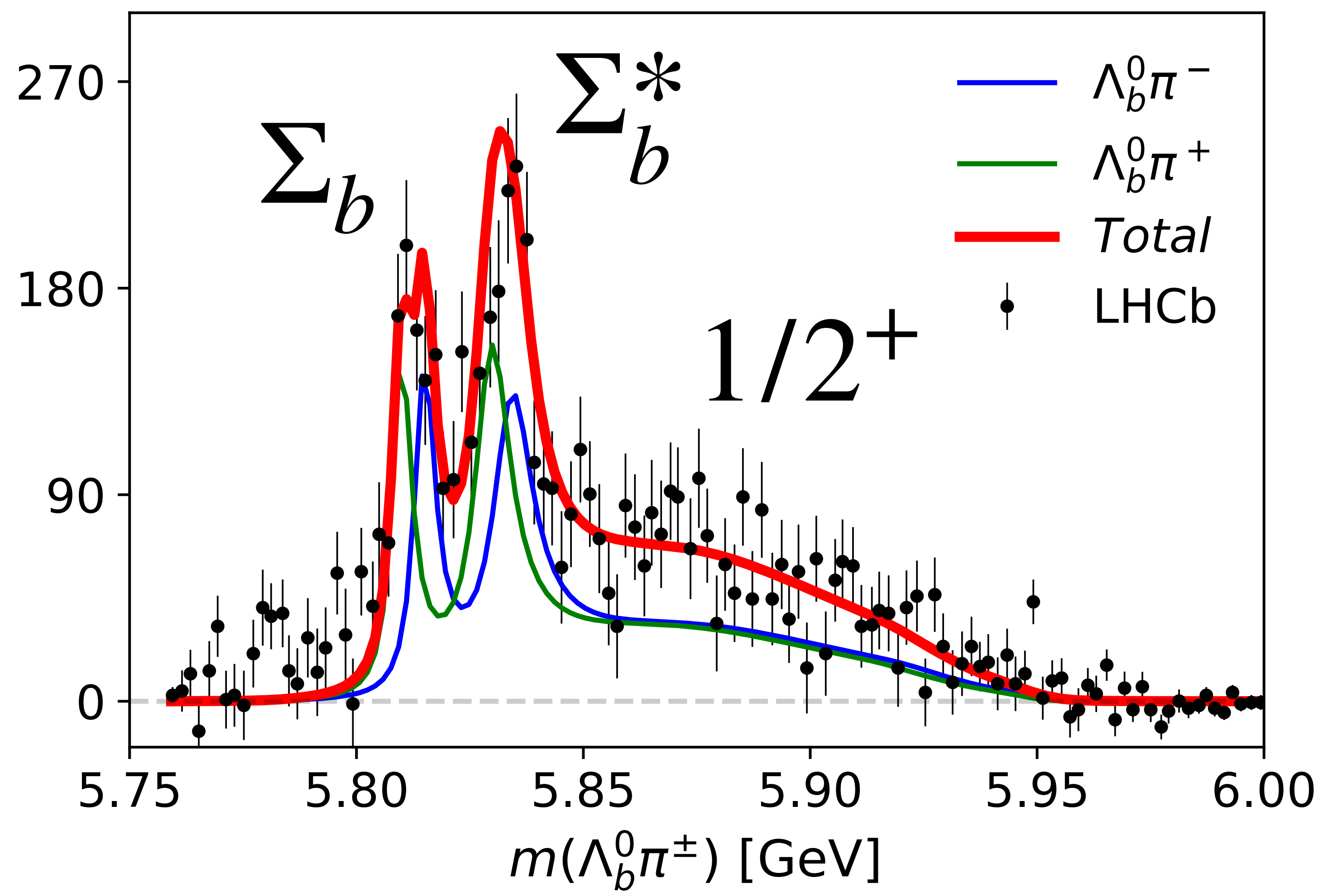
PRD101, 111502 (R) (2020)

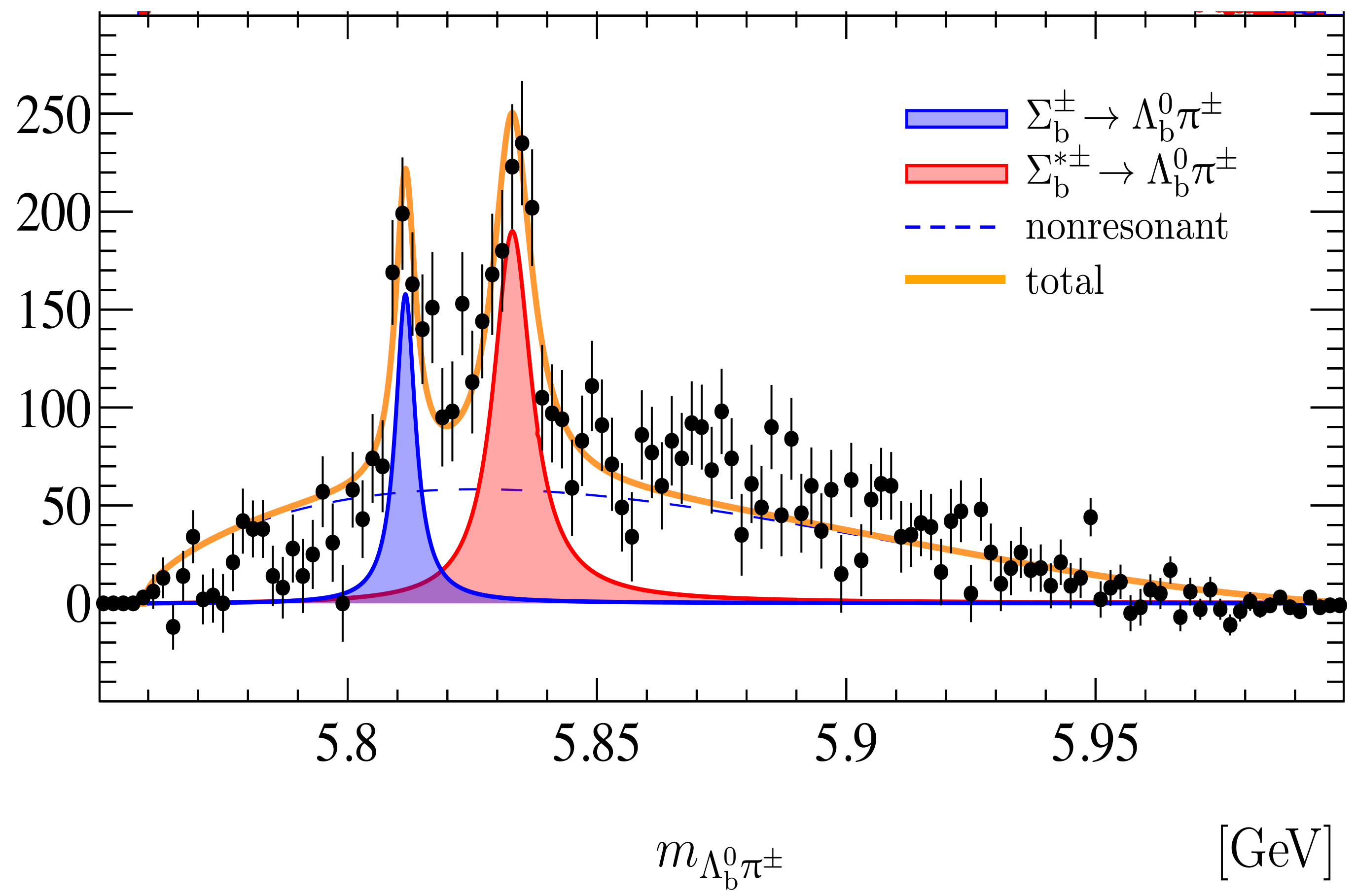
Sequential decay



- We need a reaction or decay model.

Invariant mass distribution

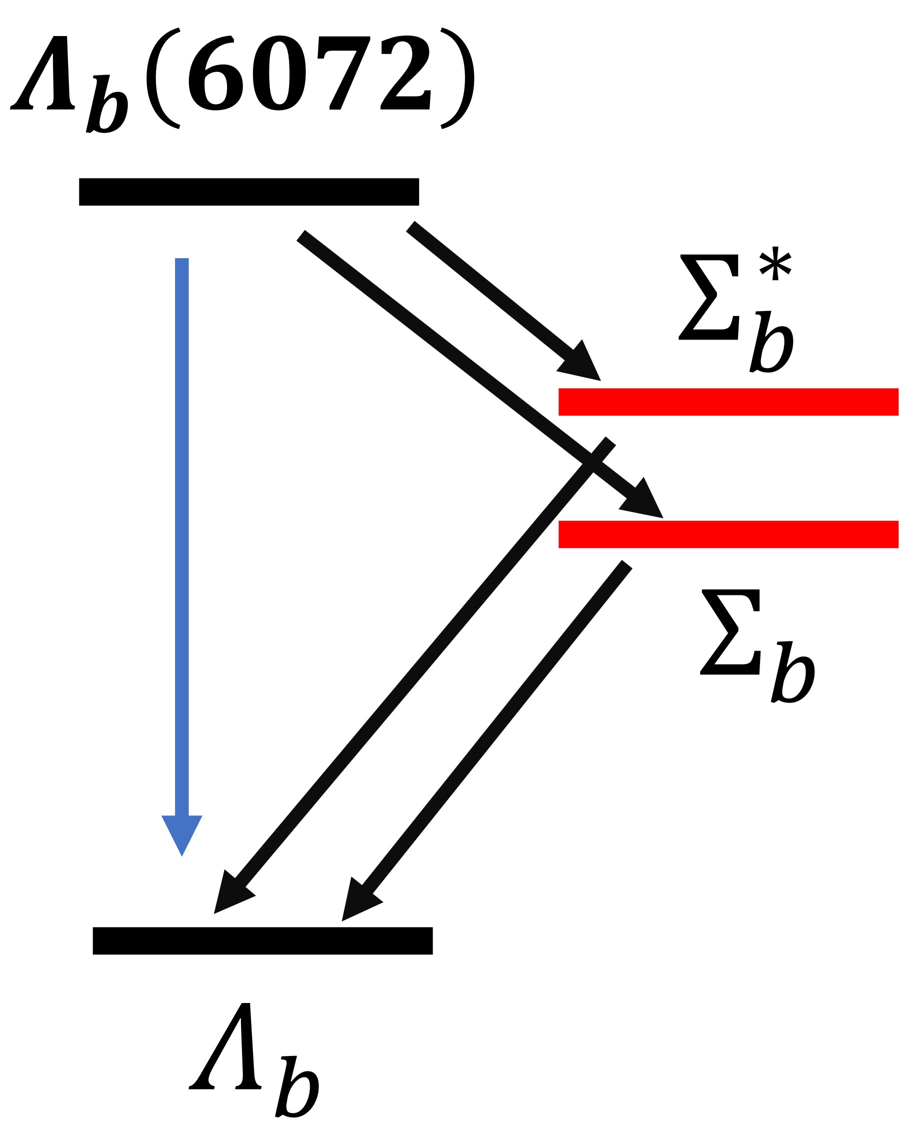


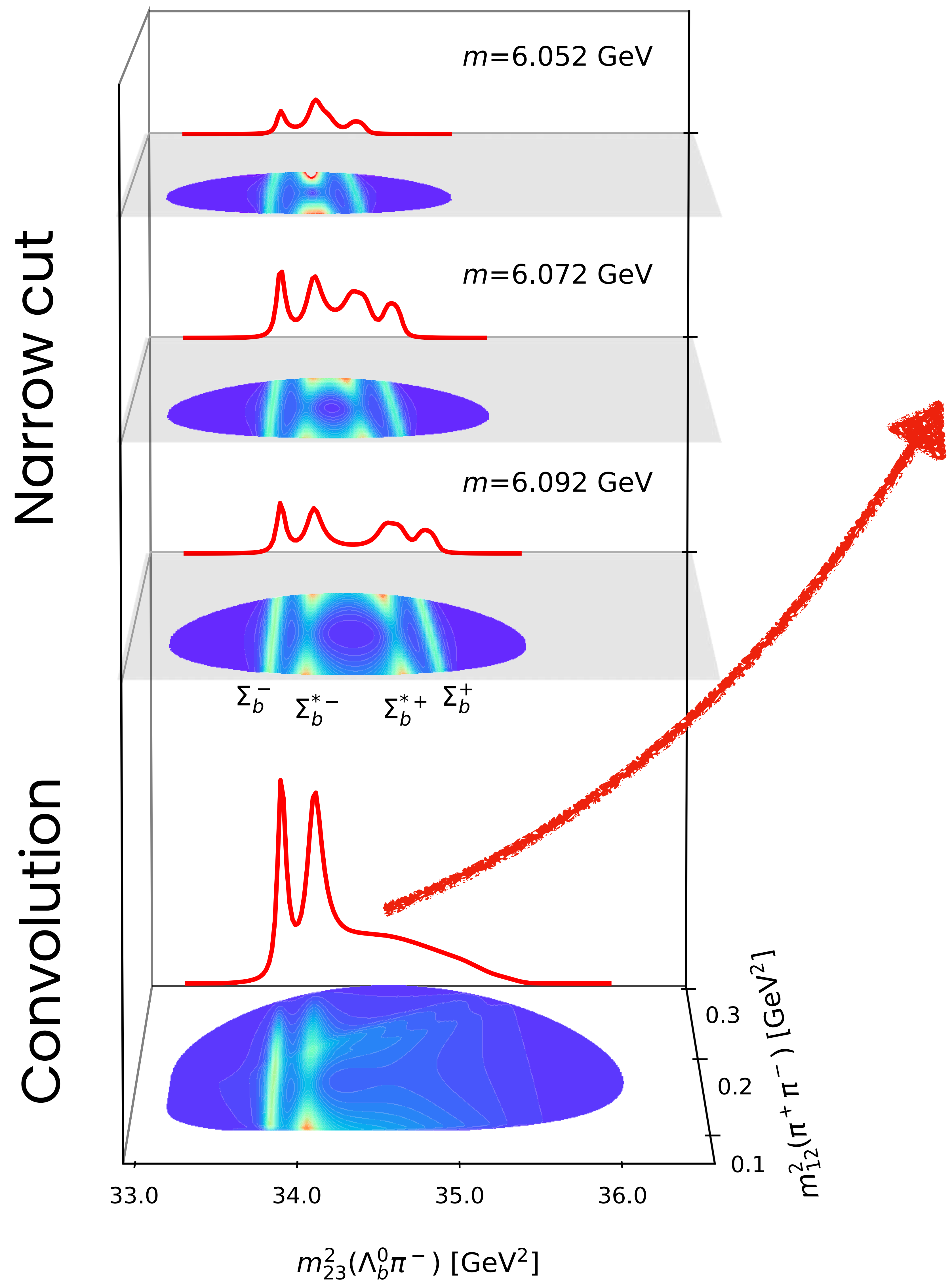


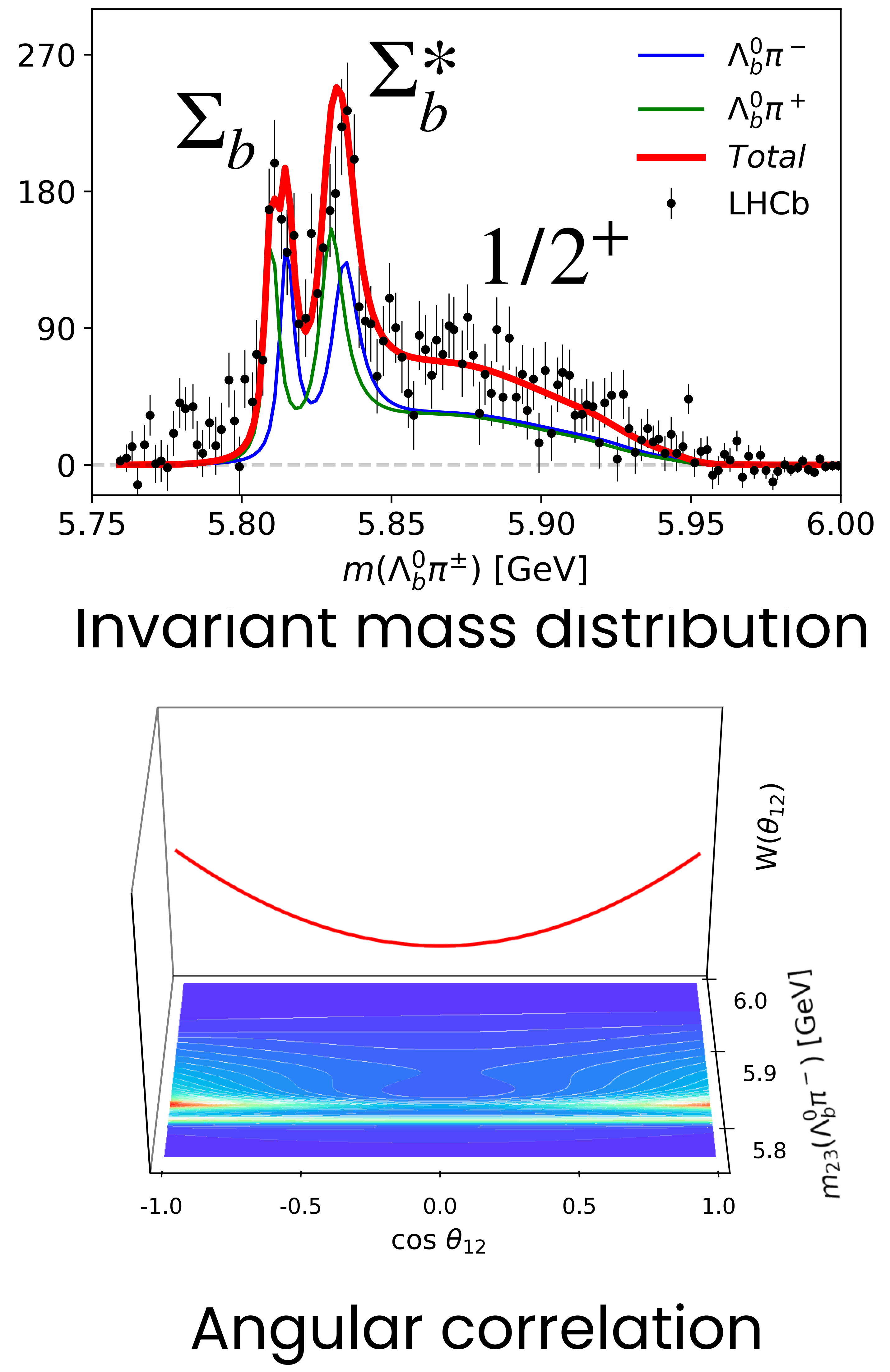
Dalitz plot of $\Lambda_b(6072)$

- Resonance [qqb]
  - >> $M = 6072$ MeV
  - >> $\Gamma = 72$ MeV
  - >> $\Lambda_b^* \to \Lambda_b \pi^+ \pi^-$

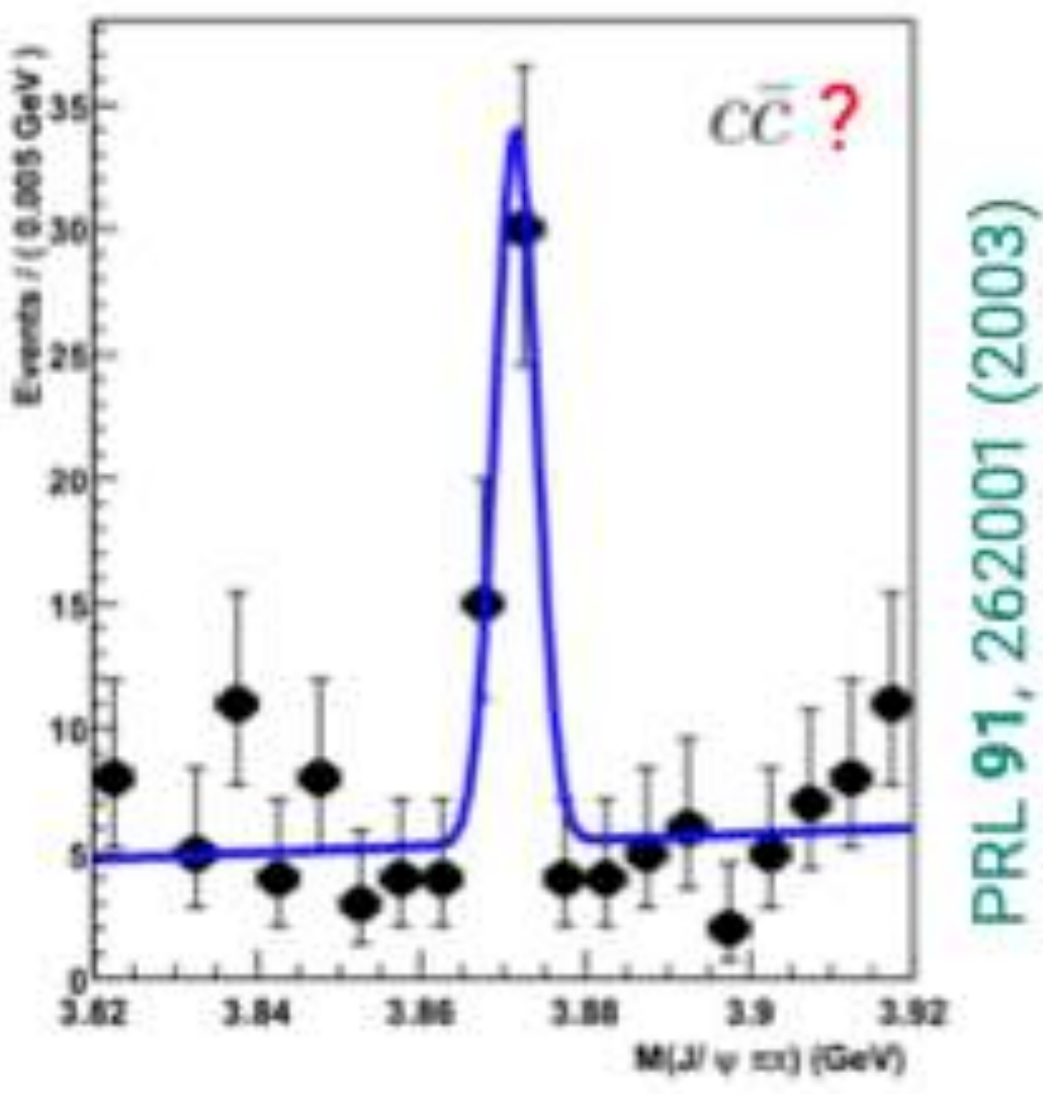
LHCb, JHEP 06, 136 (2020)







Quark (gluon) structure of hadrons



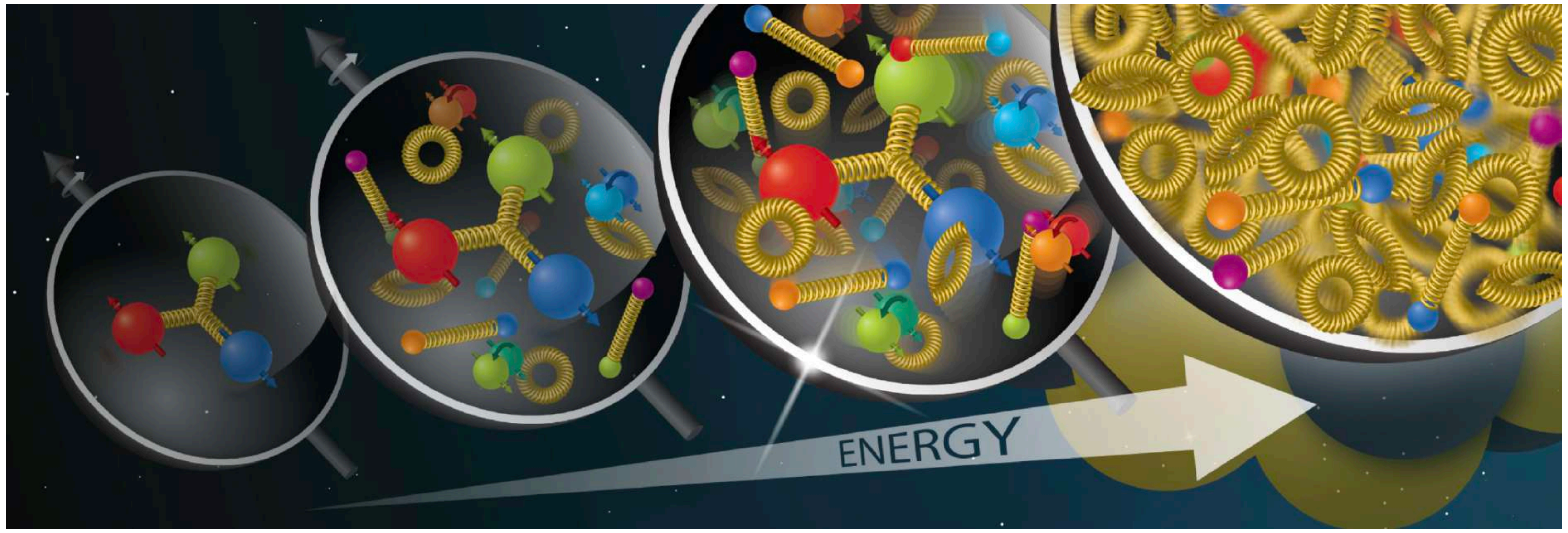
What we see!

- Hadron,
- not a quark

What is inside hadron?

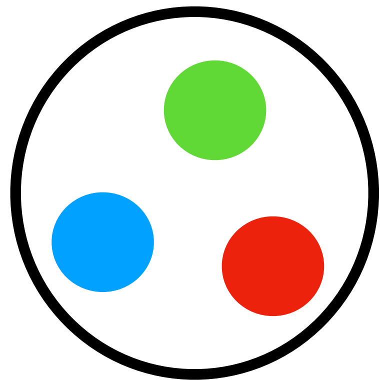
- Not known exactly
- Need a model

Scale dependence



Where is the quark?

- Cannot be directly observed
- Color confinement.



Only color neutral is observed.

Quantum Chromodynamics (QCD)

- Part of Standard Model
- Nonperturbative nature
- “Least precise theory.”

- Electron Ion Collider [EIC]
- Lattice QCD

Constraining the structure

In-medium modifications

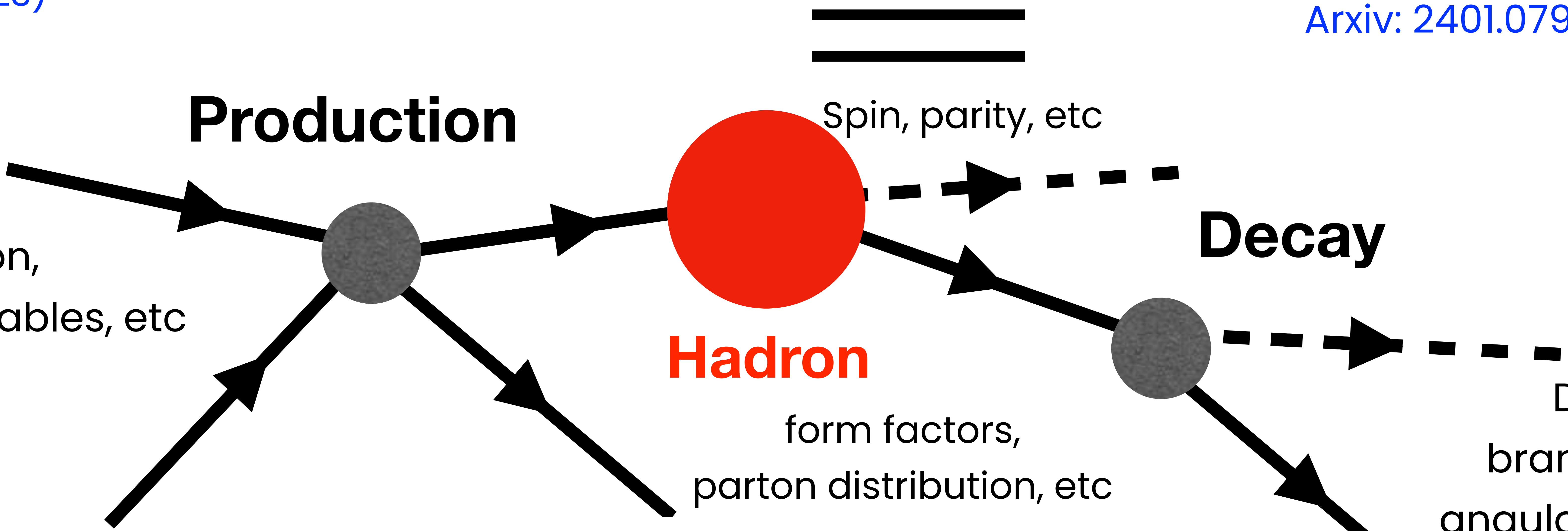
Phys. Rev. D **107**, 114010 (2023)

Mass, mass splitting, etc

mass spectrum

Gaussian expansion method

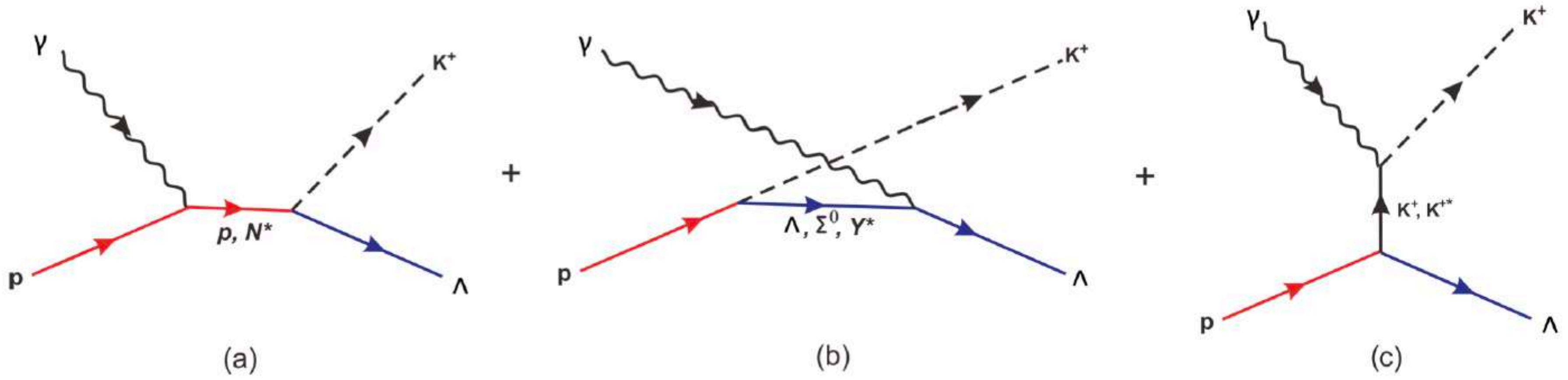
Arxiv: 2401.07933



Cross section,
polarization observables, etc

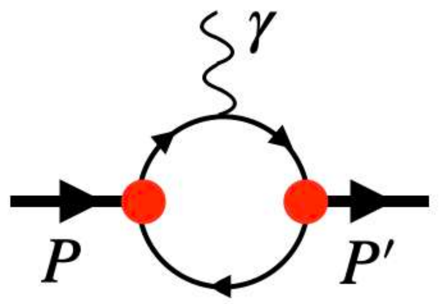
form factors,
parton distribution, etc

Decay rates,
branching fraction,
angular distribution, etc



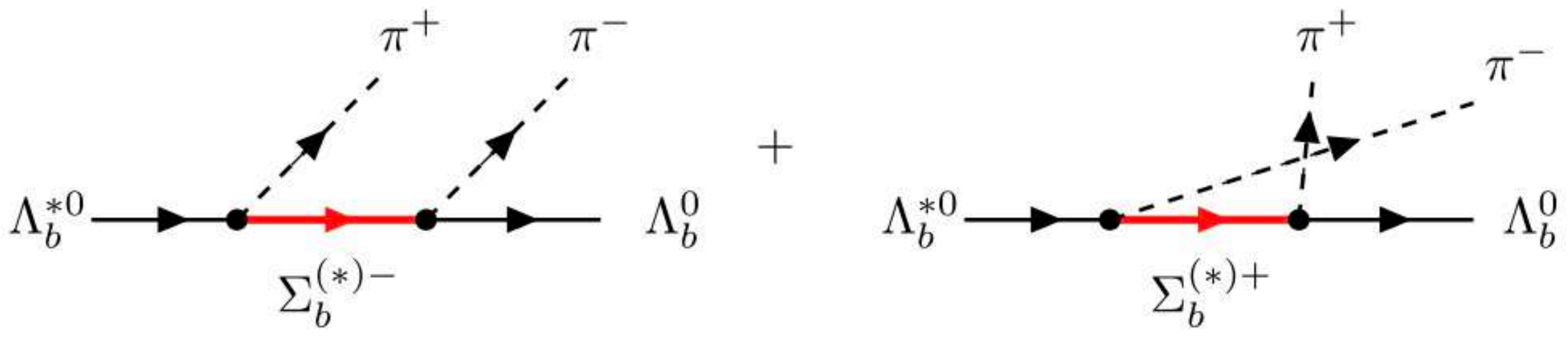
Kaon photo-production

PRD **92**, 094019 (2015)



Heavy meson structure

PRD **106**, 011009 (2022)



Heavy baryon decays

PRD **101**, 111502 (R) (2020)

Gaussian expansion method

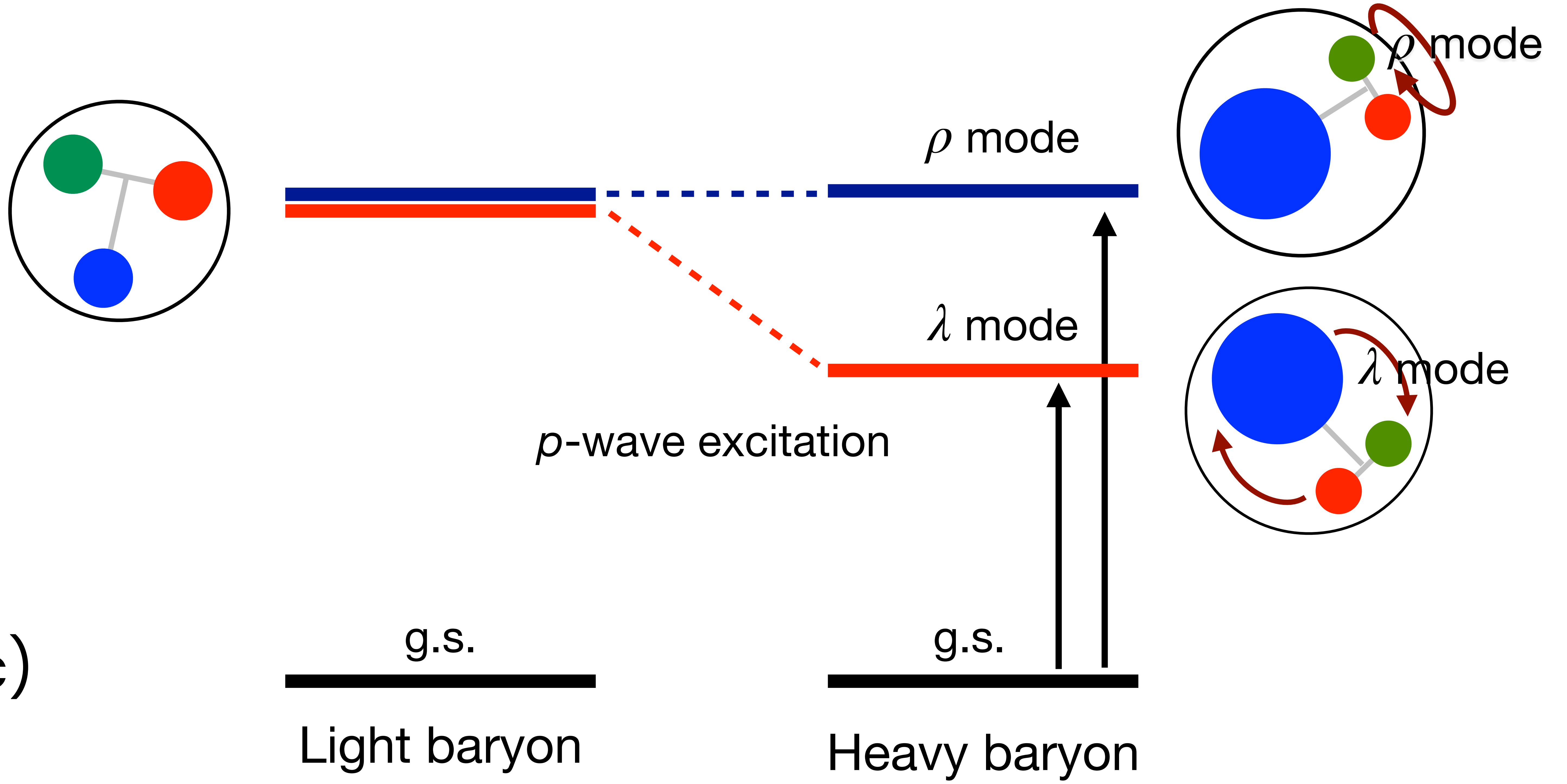
At low resolution: constituent quark

Quark model wave function

$$|\,\text{hadron} > \; = \; |\,\text{spatial} > \otimes \; |\,\text{spin} > \otimes \; |\,\text{flavor} > \otimes \; |\,\text{color} >$$

light | u d s | **c b** | heavy

quark



Spatial part

- Harmonic oscillator WF (analytic)
- From Schrödinger equation

$$H\,\big|\psi\rangle = E\,\big|\psi\rangle$$

Chiral symmetry **Heavy-quark symmetry**

Variational methods in quantum mechanics

Variational principle

Approximation methods

Given a system with a time-independent Hamiltonian. If ψ is a well-behaved trial wave function of the system that satisfied the boundary conditions of the problem, then

$$\frac{\langle\psi|\hat{H}|\psi\rangle}{\langle\psi|\psi\rangle} \geq E_0.$$

Example:

1D Harmonic oscillator

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2 = \frac{-\hbar^2}{2m}\frac{\mathrm{d}^2}{\mathrm{d}x^2} + \frac{1}{2}m\omega^2x^2.$$

Trial wave function

$$\psi(x) = e^{-\alpha x^2},$$

Minimization of variational parameter

$$\frac{\mathrm{d}f}{\mathrm{d}\alpha} = 0 \qquad \alpha = \pm\frac{m\omega}{2\hbar}.$$

Excited states can be computed from orthogonality.

A numerical method to solve Schrödinger equation.

PPNP51, 223 (2003)

- Solving Schrödinger equation

$$H\left|\psi\right\rangle = E\left|\psi\right\rangle$$

- Gaussian basis functions

$$\psi = \sum_{n=1}^{\max} c_n \phi_n^G$$

$$\phi_n^G(r) = \frac{(2\nu_n)^{3/4}}{\pi^{3/4}} \mathrm{e}^{-\nu_n r^2}$$

Why Gaussian basis?

- Generalized Eigenvalue equation

$$\boldsymbol{H}_h \boldsymbol{c} = M_h \boldsymbol{S}\boldsymbol{c} \qquad \boldsymbol{H} = \left\langle \phi_n^G \middle| \hat{H} \middle| \phi_m^G \right\rangle \qquad \boldsymbol{S} = \left\langle \phi_n^G \mid \phi_m^G \right\rangle$$

- Geometric progression $[r_1, r_{\max}]$

$$\nu_n = \frac{1}{r_n^2} \qquad r_n = r_1 a^{n-1} \qquad a = \left(\frac{r_{\max}}{r_1}\right)^{\frac{1}{n_{\max}-1}}$$

- Normalization

$$\left\langle \psi \mid \psi \right\rangle = \sum_{m,n} c_n^* S_{nm} c_m = 1$$

Matrix elements in GEM

Matrix elements

$$\langle A\rangle = \langle\psi_{lm}|\hat{A}|\psi_{lm}\rangle$$
$$= \sum_{i=1}^{n_{\max}}\sum_{j=1}^{n_{\max}} c_i c_j \langle\phi^{\mathrm{G}}_{ilm}|\hat{A}|\phi^{\mathrm{G}}_{jlm}\rangle$$

$$\psi_{lm}(\boldsymbol{r}) = \sum_{n=1}^{n_{\max}} c_{nl}\phi^{\mathrm{G}}_{nlm}(\boldsymbol{r})$$

$$\iint Y^*_{lm}(\theta,\varphi)Y_{l'm'}(\theta,\varphi)\sin\theta\,\mathrm{d}\theta\mathrm{d}\varphi = \delta_{ll'}\delta_{mm'}$$

Matrix elements in a Gaussian basis

$$T_{ij} = \langle\phi^{\mathrm{G}}_{ilm}| -\frac{1}{2\mu}\nabla^2|\phi^{\mathrm{G}}_{jlm}\rangle = \frac{1}{\mu}\frac{(2l+3)\nu_i\nu_j}{\nu_i+\nu_j}\left(\frac{2\sqrt{\nu_i\nu_j}}{\nu_i+\nu_j}\right)^{l+\frac{3}{2}}$$

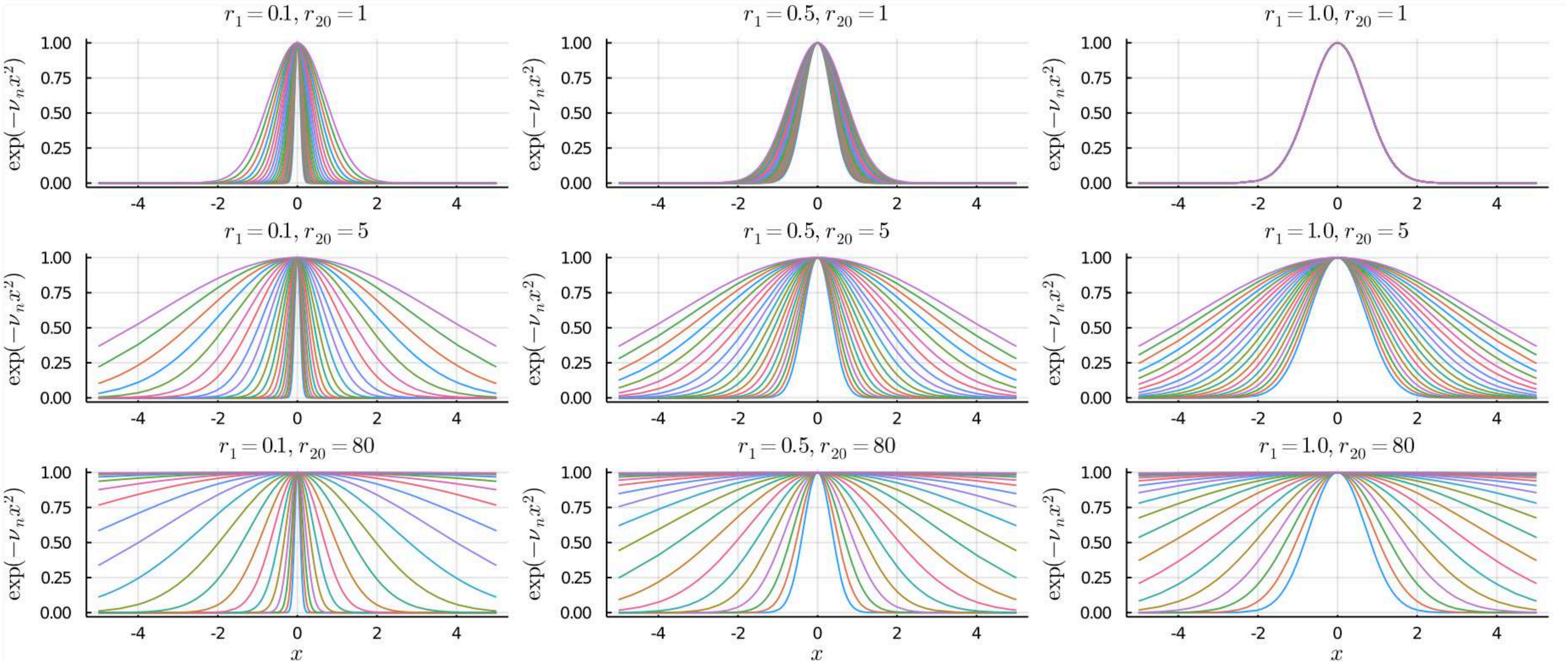
$$V_{ij} = \langle\phi^{\mathrm{G}}_{ilm}| -\frac{1}{r}|\phi^{\mathrm{G}}_{jlm}\rangle = -\frac{2}{\sqrt{\pi}}\frac{2^l l!}{(2l+1)\,!!}\sqrt{\nu_i+\nu_j}\left(\frac{2\sqrt{\nu_i\nu_j}}{\nu_i+\nu_j}\right)^{l+\frac{3}{2}}$$

$$\langle\phi^{\mathrm{G}}_{ilm}|r|\phi^{\mathrm{G}}_{jlm}\rangle = \frac{2}{\sqrt{\pi}}\frac{2^l(l+1)!}{(2l+1)\,!!}\frac{1}{\sqrt{\nu_i+\nu_j}}\left(\frac{2\sqrt{\nu_i\nu_j}}{\nu_i+\nu_j}\right)^{l+\frac{3}{2}}$$

$$\langle\phi^{\mathrm{G}}_{ilm}|r^2|\phi^{\mathrm{G}}_{jlm}\rangle = \frac{l+\frac{3}{2}}{\nu_i+\nu_j}\left(\frac{2\sqrt{\nu_i\nu_j}}{\nu_i+\nu_j}\right)^{l+\frac{3}{2}}$$

$$\langle\phi^{\mathrm{G}}_{ilm}|\hat{\mu}|\phi^{\mathrm{G}}_{jlm}\rangle = z_1\langle\phi^{\mathrm{G}}_{ilm}|r_1|\phi^{\mathrm{G}}_{jlm}\rangle$$

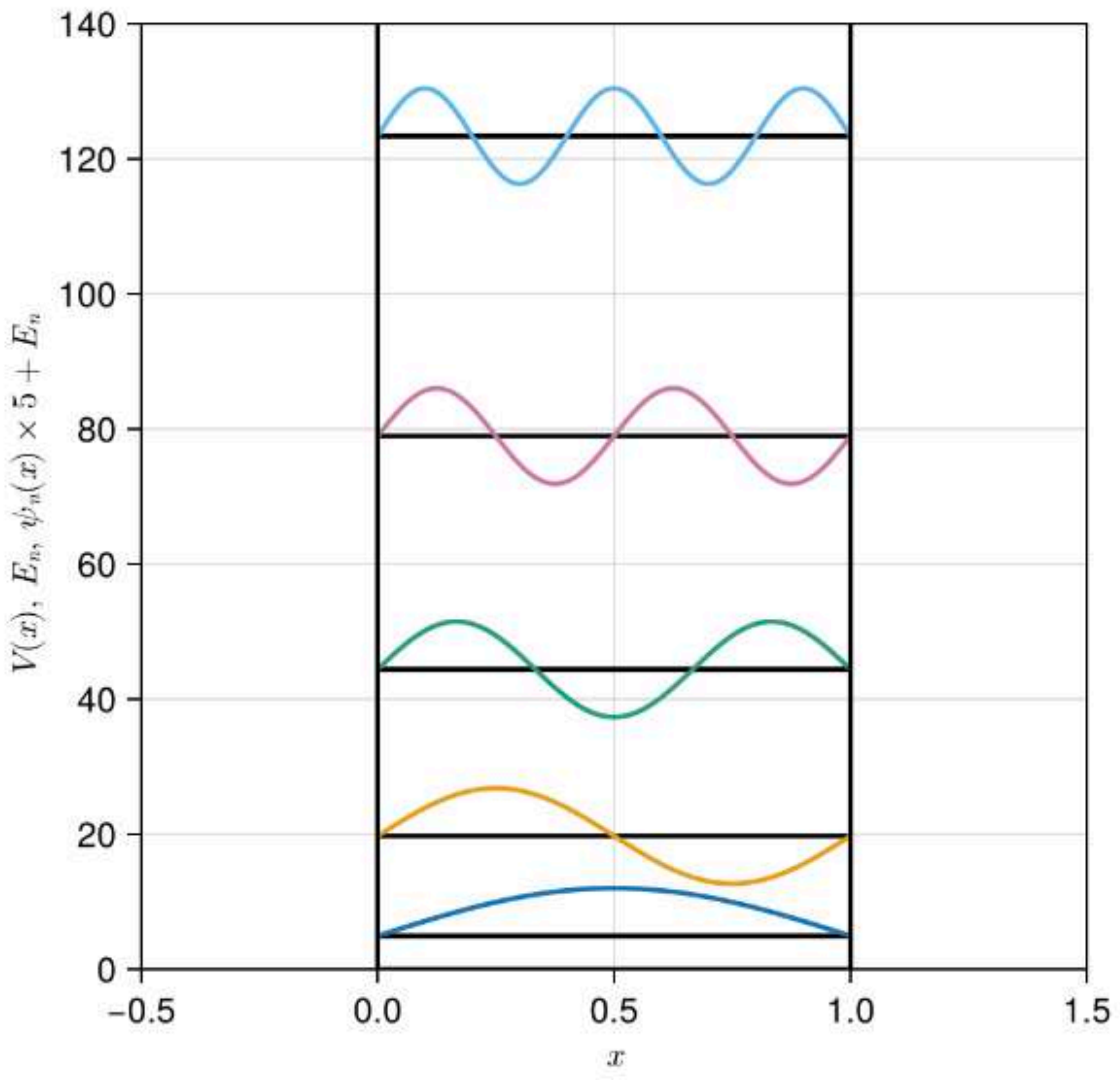
Range parameters in Gaussian basis functions



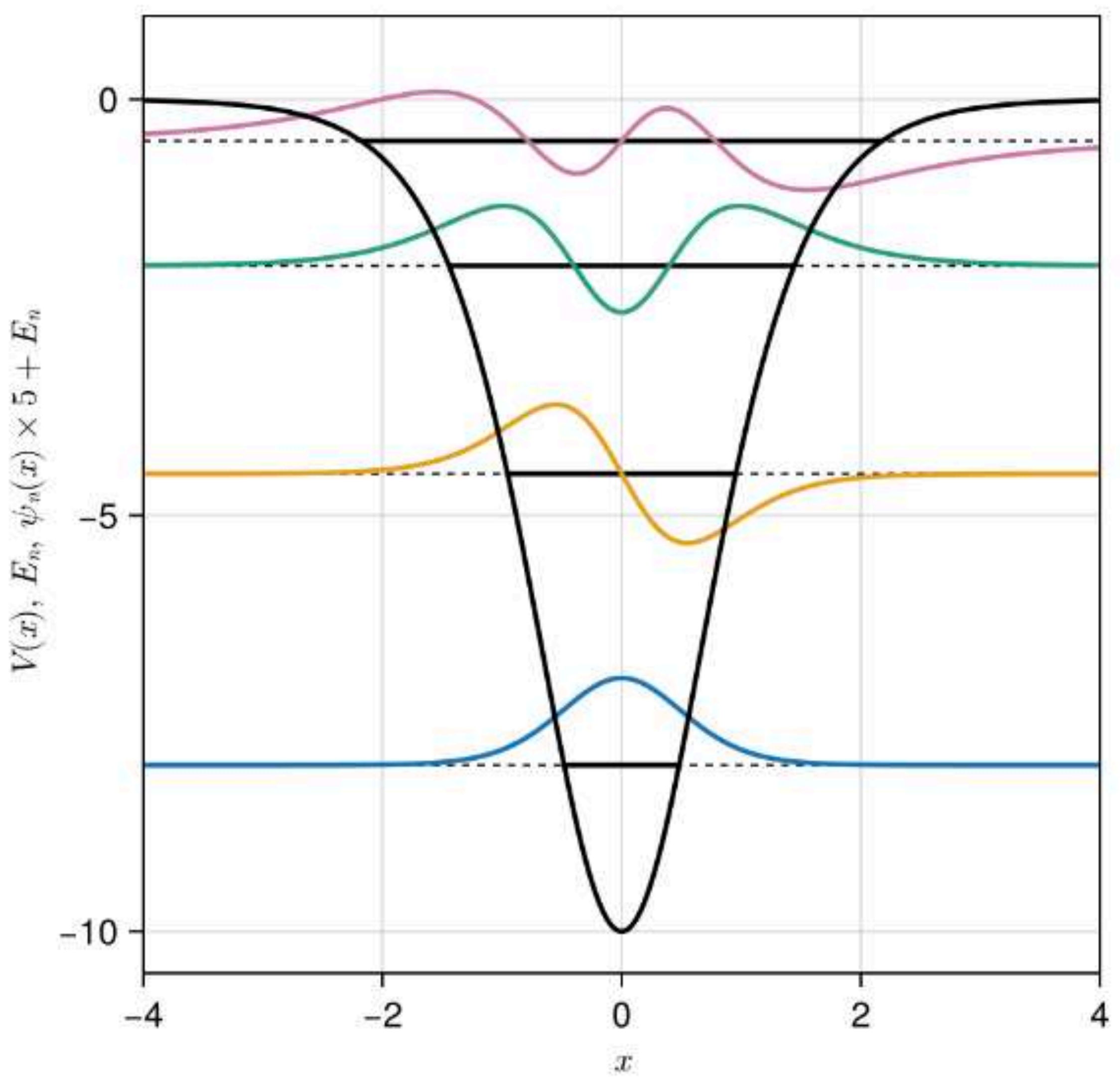
antique.jl package

Analytical Solutions of Quantum Mechanical Equations

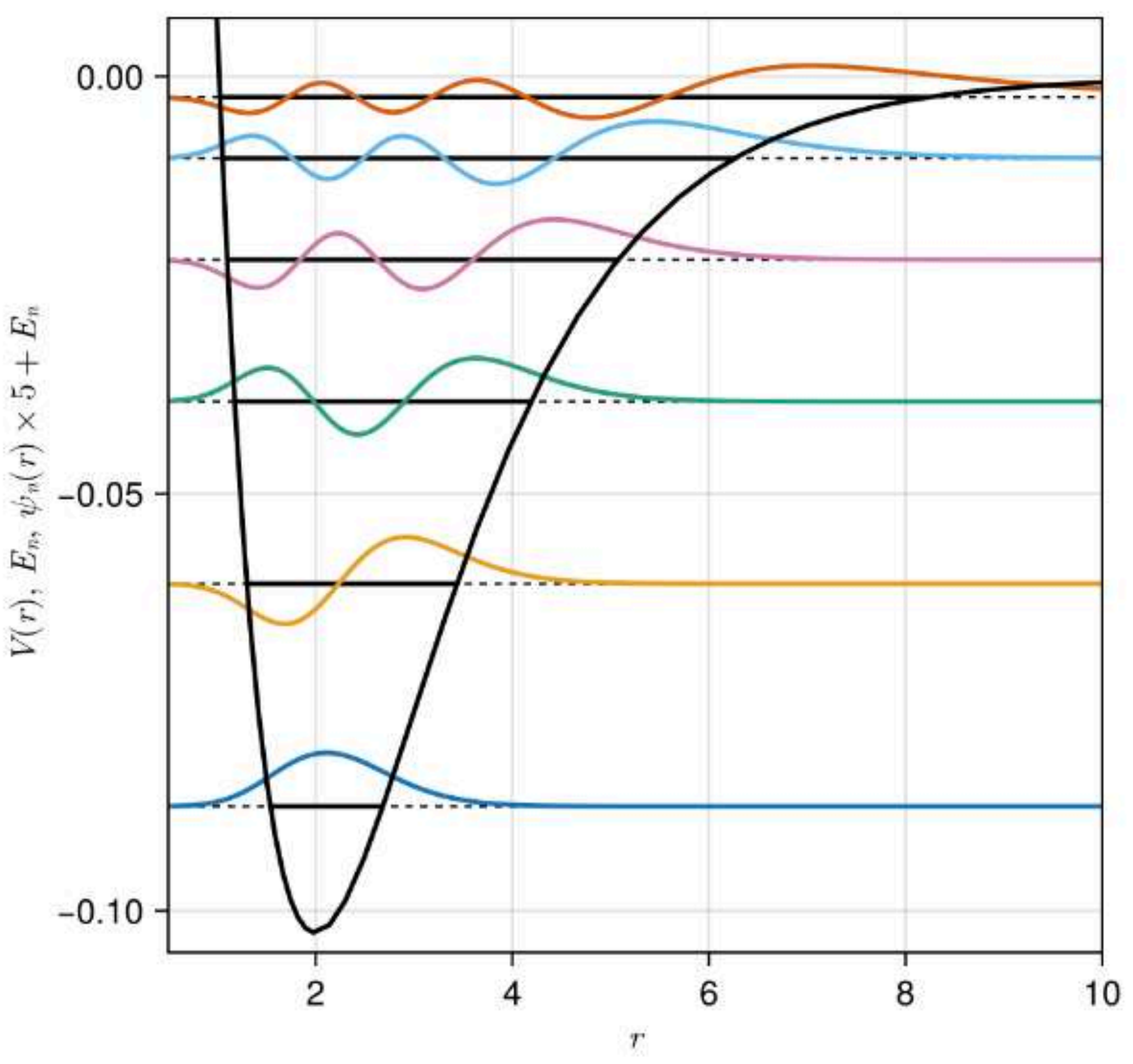
- For a benchmark of numerical method of solving Schrödinger equation
- For a basis function of trial wave functions.



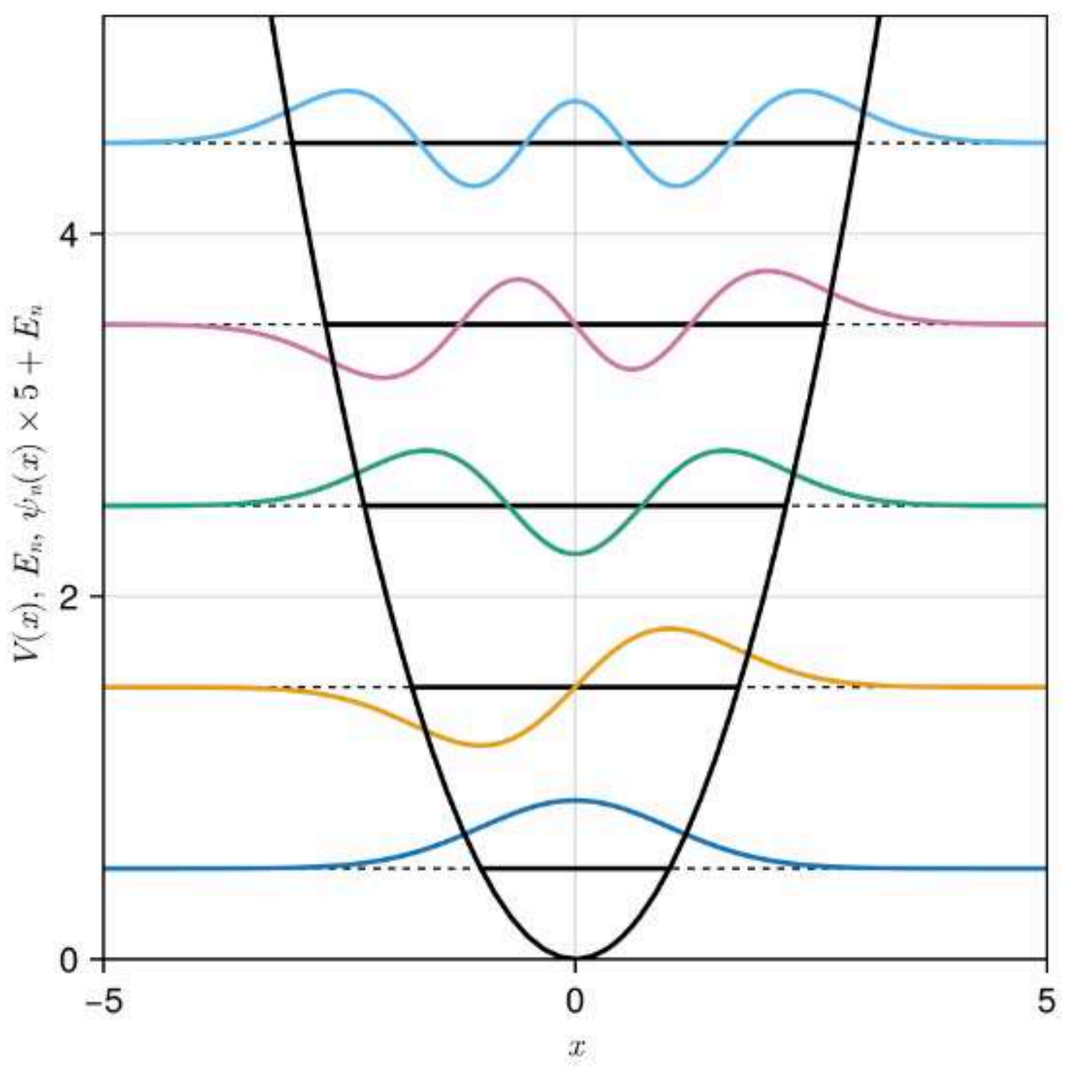
`InfinitePotentialWell`



`PoschlTeller`



`MorsePotential`

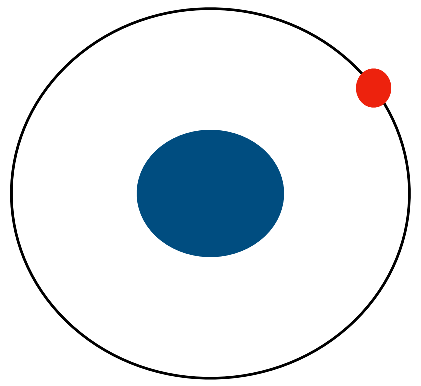


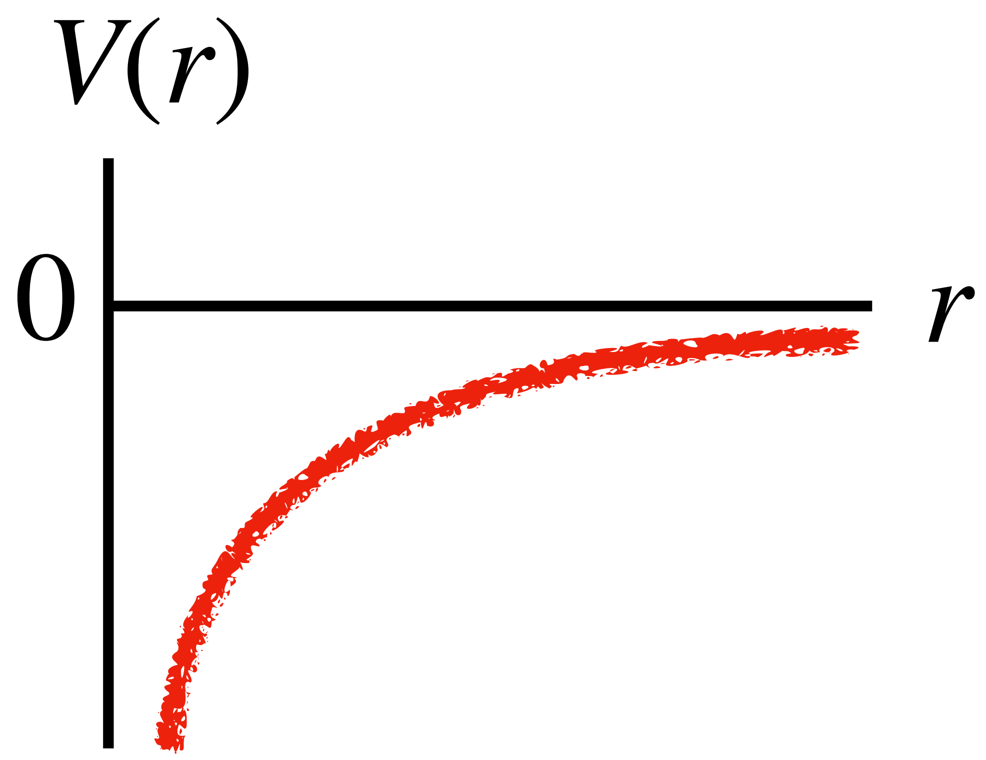
`HarmonicOscillator`

Give us a star on Github!

A benchmark test of GEM: Hydrogen atom

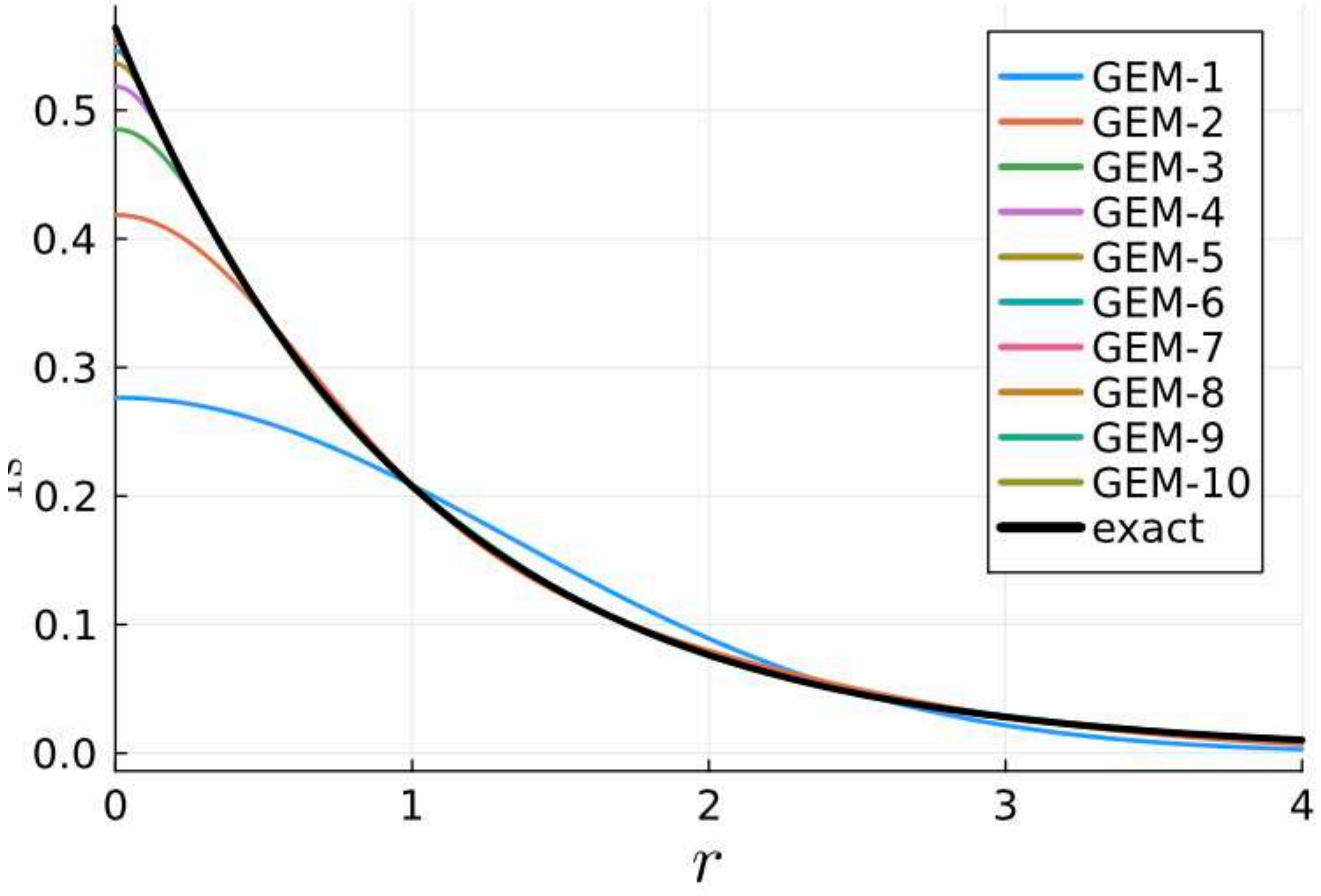
Hydrogen atom

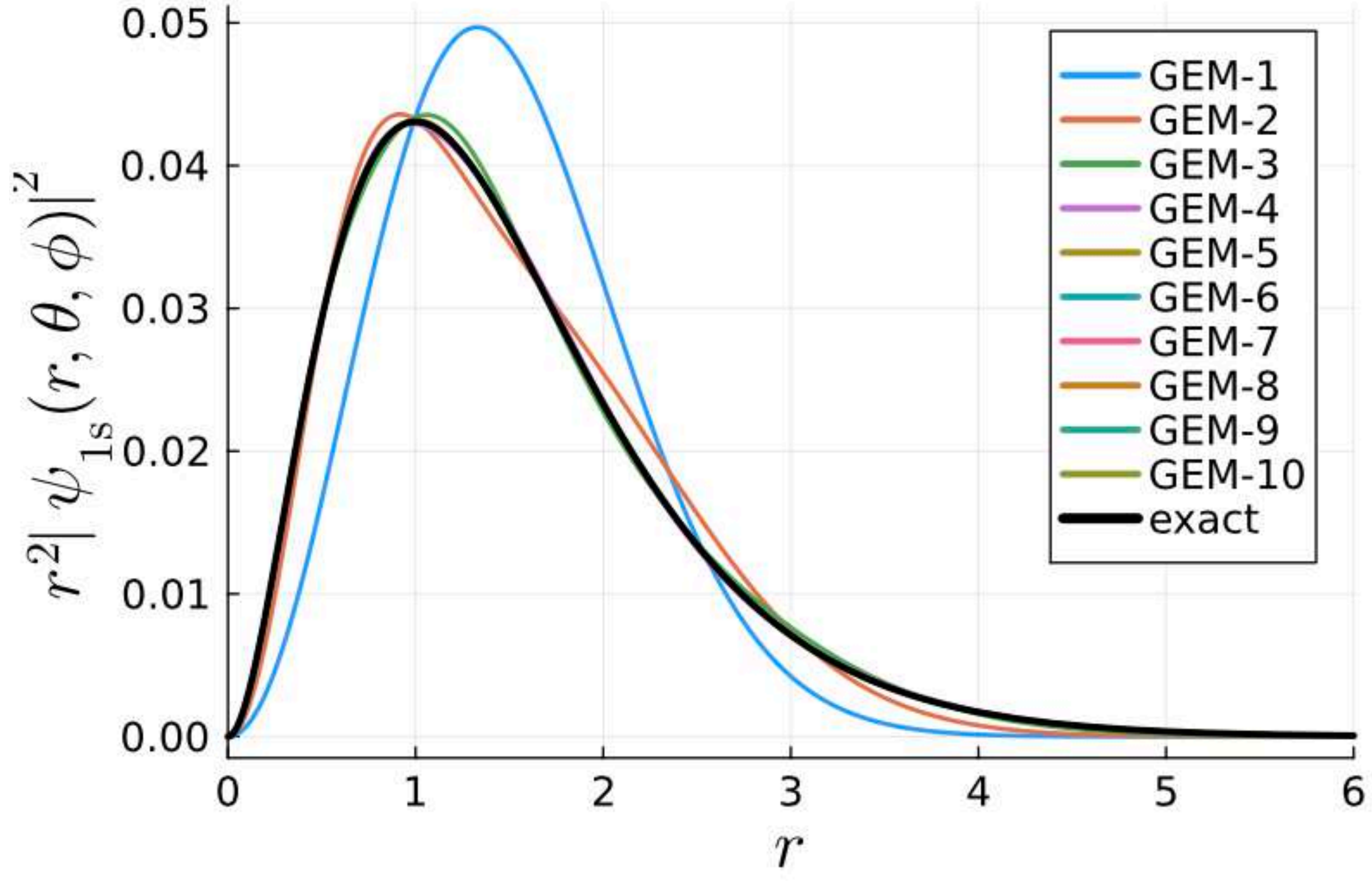




Coulomb potential

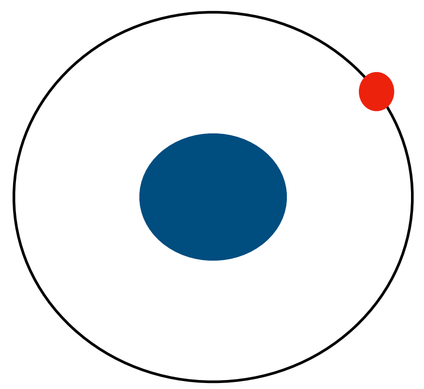
$$V(r) \propto -\frac{\alpha}{r}$$



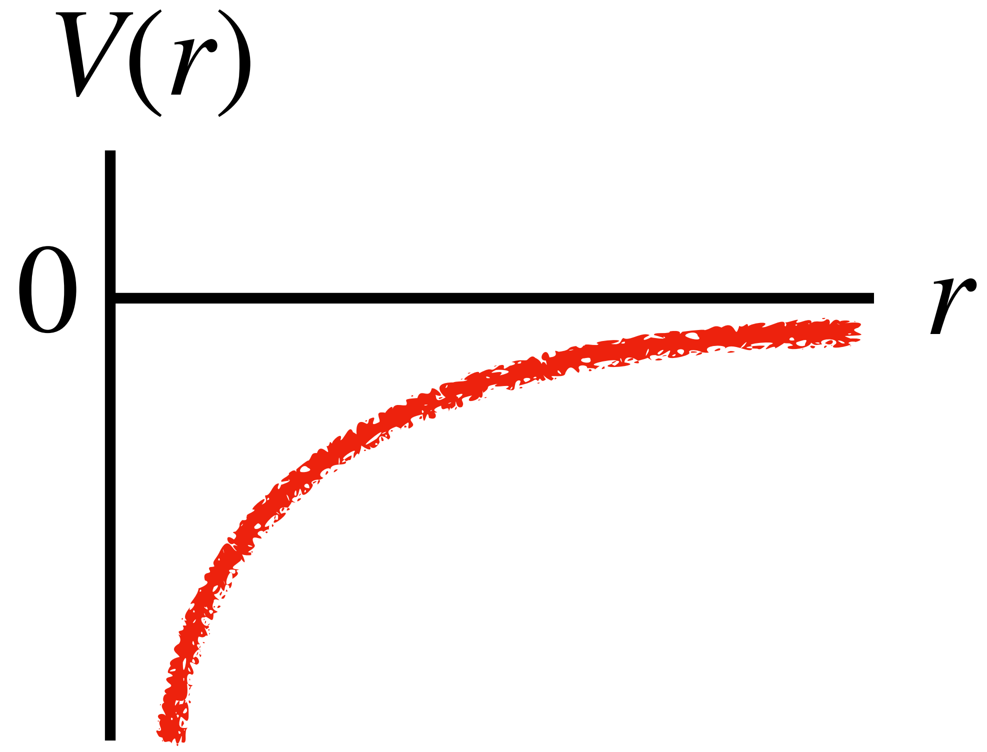


Hydrogen vs Hadron spectrum

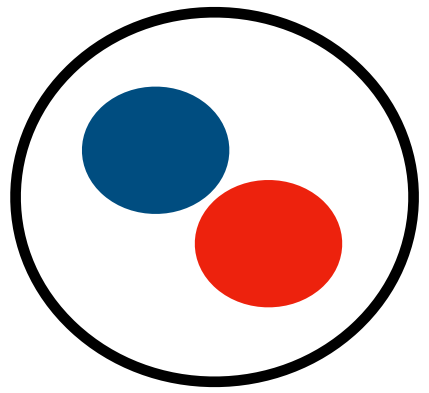
Hydrogen atom



$$V(r) \propto -\frac{\alpha}{r}$$

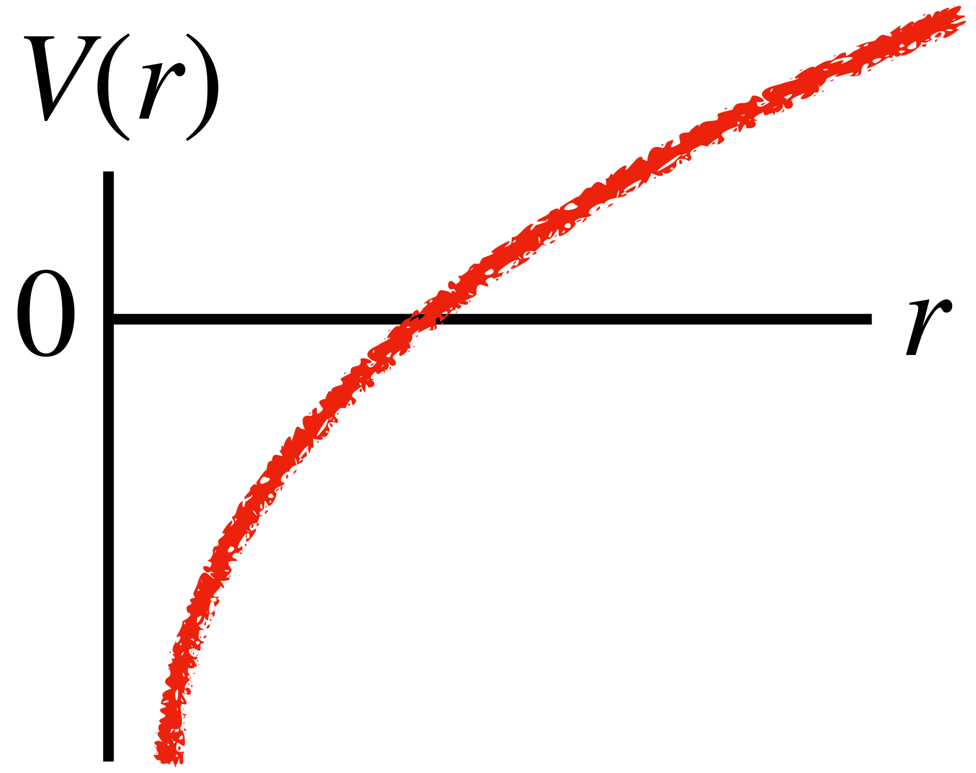


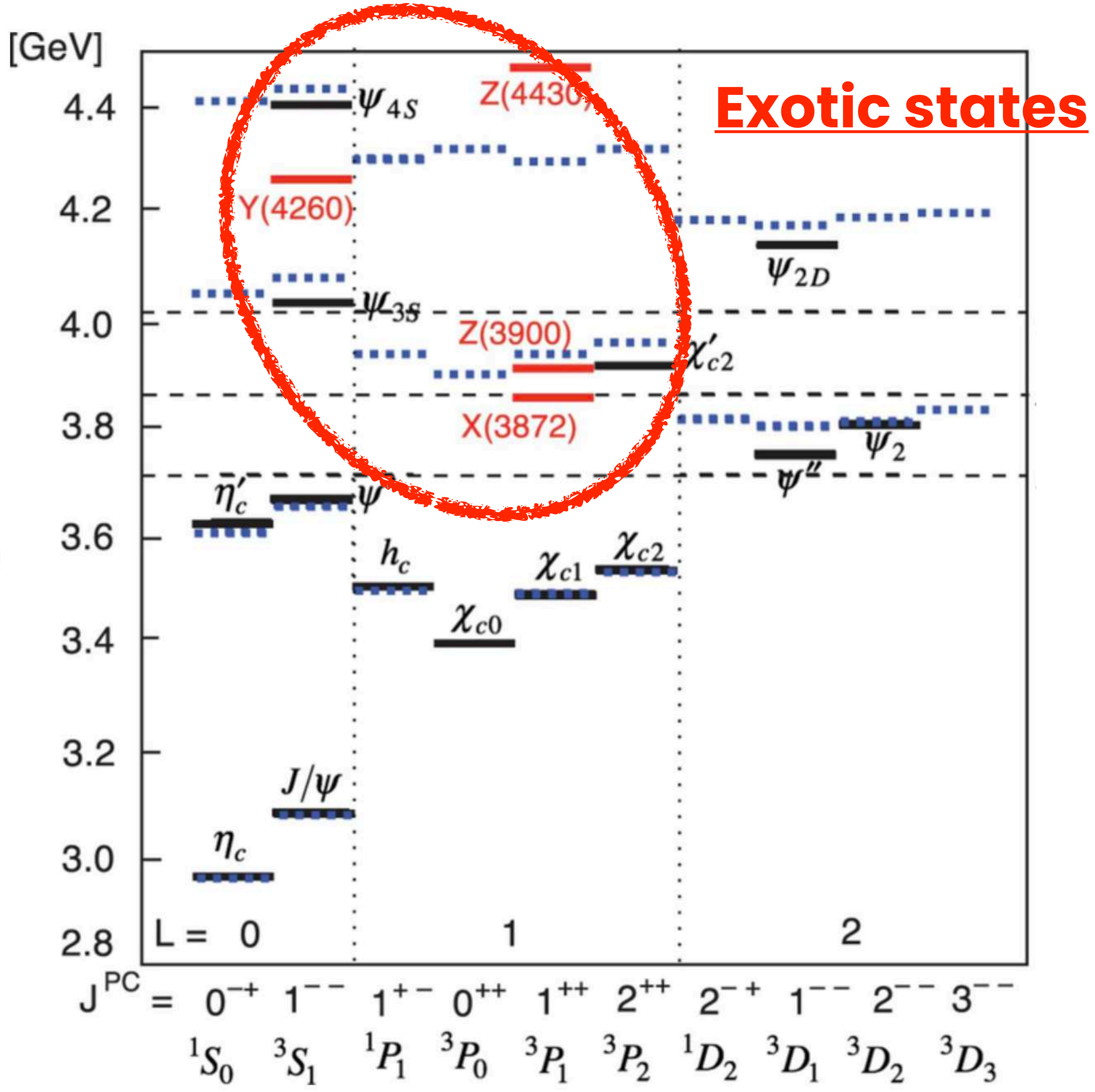
Charmonium



$(c\bar{c})$

$$V(r) \propto -\frac{\alpha_s}{r} + br$$





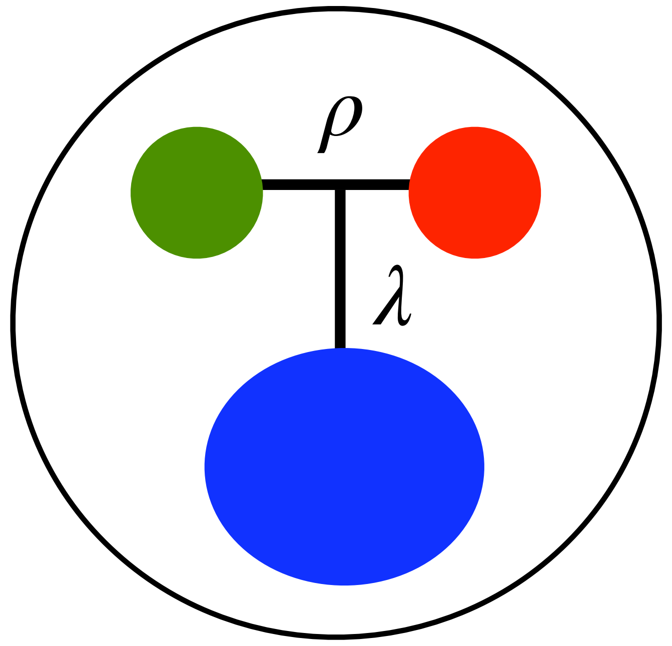
Heavy baryon spectrum

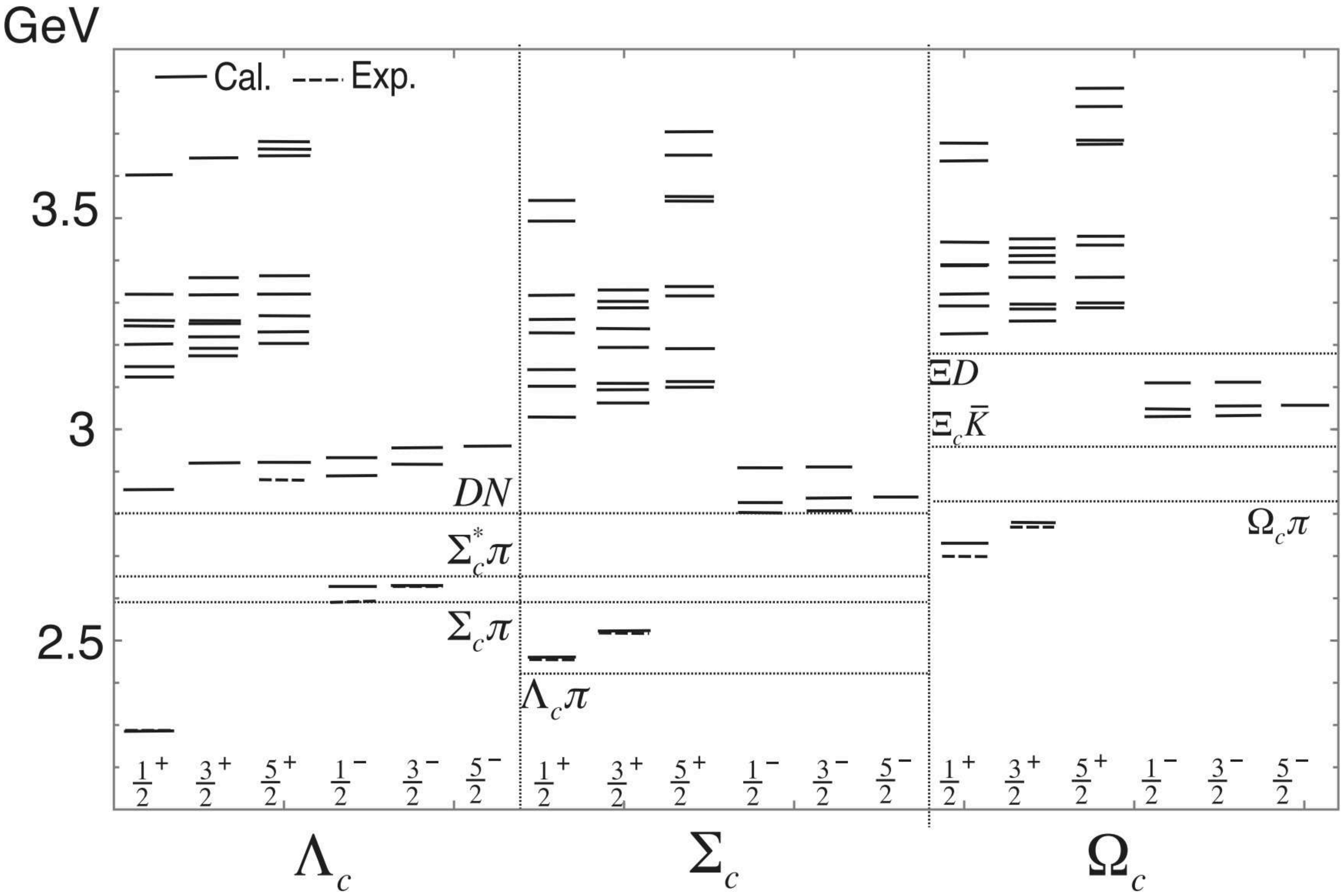
- Interaction:

$$H = K + V_{\mathrm{conf}} + V_{\mathrm{short}}$$

$$K = \sum_i \left(m_i + \frac{p_i^2}{2m_i} \right) - K_G$$

PRD92, 114029 (2015)

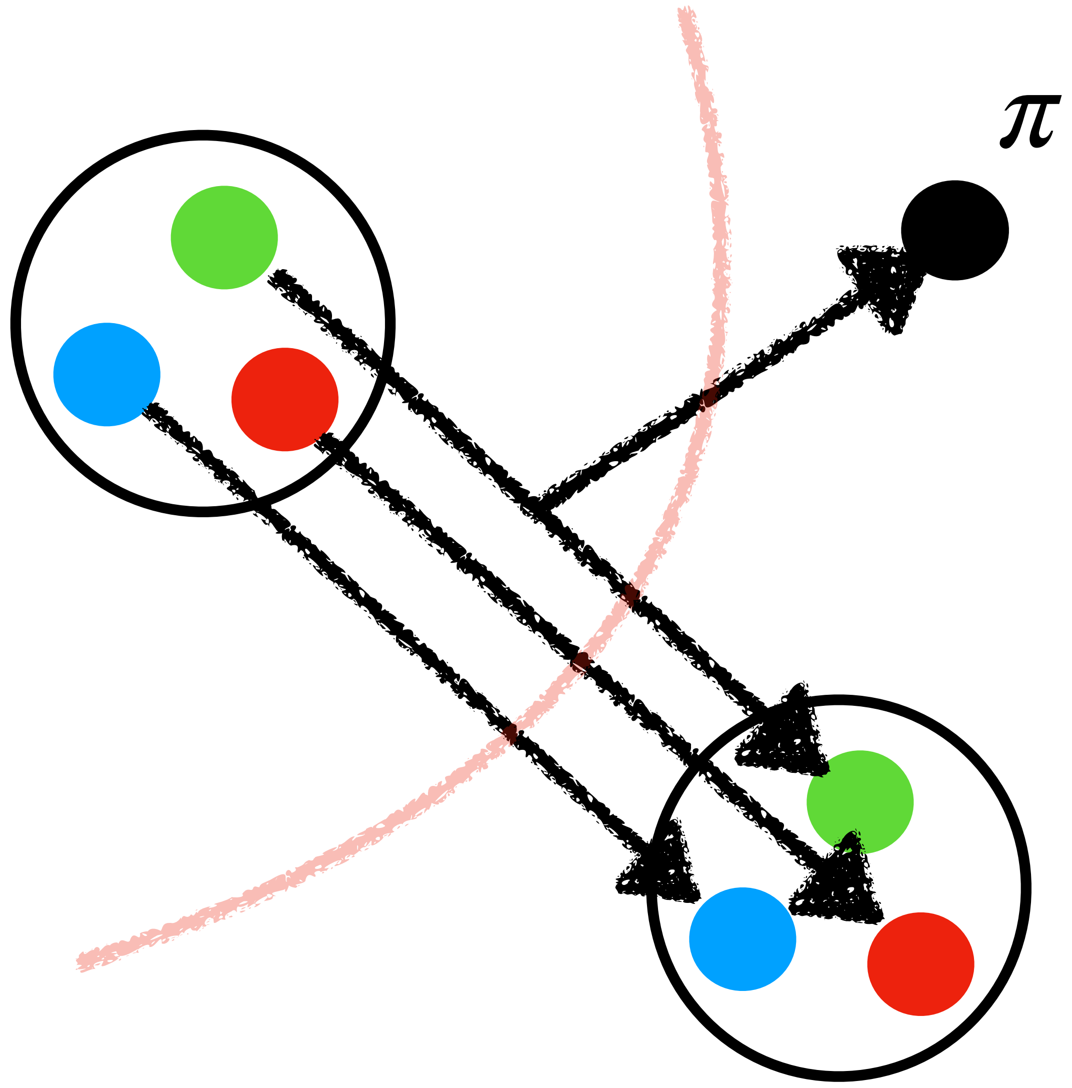




Heavy baryon decay

Nonrelativistic QM

$$B_i \to B_f + \pi$$



- Hadron wave function
- Chiral interaction

PRD 103, 094003 (2021)

Quark-meson interaction

$$\mathcal{T} = \langle \pi \, \Lambda_c | \mathcal{L}_{\pi qq} | \Lambda_c^* \rangle$$

$$\mathcal{L}_{\pi qq} = \frac{g_A^q}{2 f_\pi} \bar{q} \gamma^\mu \gamma_5 \vec{\tau} q \cdot \partial_\mu \vec{\pi}$$

Non-relativistic expansion

$$H_{NR} = g \left[\boldsymbol{\sigma} \cdot \boldsymbol{q} - \frac{\omega_\pi}{2m} \boldsymbol{\sigma} \cdot (\boldsymbol{p}_i + \boldsymbol{p}_f) \right]$$

$$H_{RC} = \frac{g}{8m^2} \left[m_\pi^2 \boldsymbol{\sigma} \cdot \boldsymbol{q} - 2\boldsymbol{\sigma} \cdot (\boldsymbol{p}_i + \boldsymbol{p}_f) \times (\boldsymbol{q} \times \boldsymbol{p}_i) \right]$$

State	Multiplet	Channel	$\Gamma_{\rm NR}$	$\Gamma_{\rm NR+RC}$	$\Gamma_{\rm Exp.}$
$\Sigma_c(2455)^{++}$	$\Sigma_c(1S, 1/2(1)^+)$	$\Lambda_c\pi$	4.27 - 4.34	0.36 - 1.95	1.84 ± 0.04
$\Sigma_c(2520)^{++}$	$\Sigma_c(1S, 3/2(1)^+)$	$\Lambda_c\pi$	29.8 - 31.4	2.70 - 14.1	14.77 ± 0.25
$\Lambda_c(2595)^+$	$\Lambda_c(1P_\lambda, 1/2(1)^-)$	$\Sigma_c(2455)\pi$	1.35 - 3.16	1.36 - 3.20	2.6 ± 0.6
$\Lambda_c(2625)^+$	$\Lambda_c(1P_\lambda, 3/2(1)^-)$	$\Sigma_c(2455)\pi$	0.08 - 0.15	0.01 - 0.06	
		$\Sigma_c(2520)\pi$	0.07 - 0.18	0.08 - 0.20	
		Sum	0.15 - 0.33	0.09 - 0.26	< 0.97
$\Lambda_c(2765)^+$	$\Lambda_c(2S_{\lambda\lambda}, 1/2(0)^+)$	$\Sigma_c(2455)\pi$	0.71 - 2.66	5.56 - 26.1	
		$\Sigma_c(2520)\pi$	0.67 - 2.04	5.26 - 22.9	
		Sum	1.38 - 4.70	10.8 - 49.0	73 ± 5

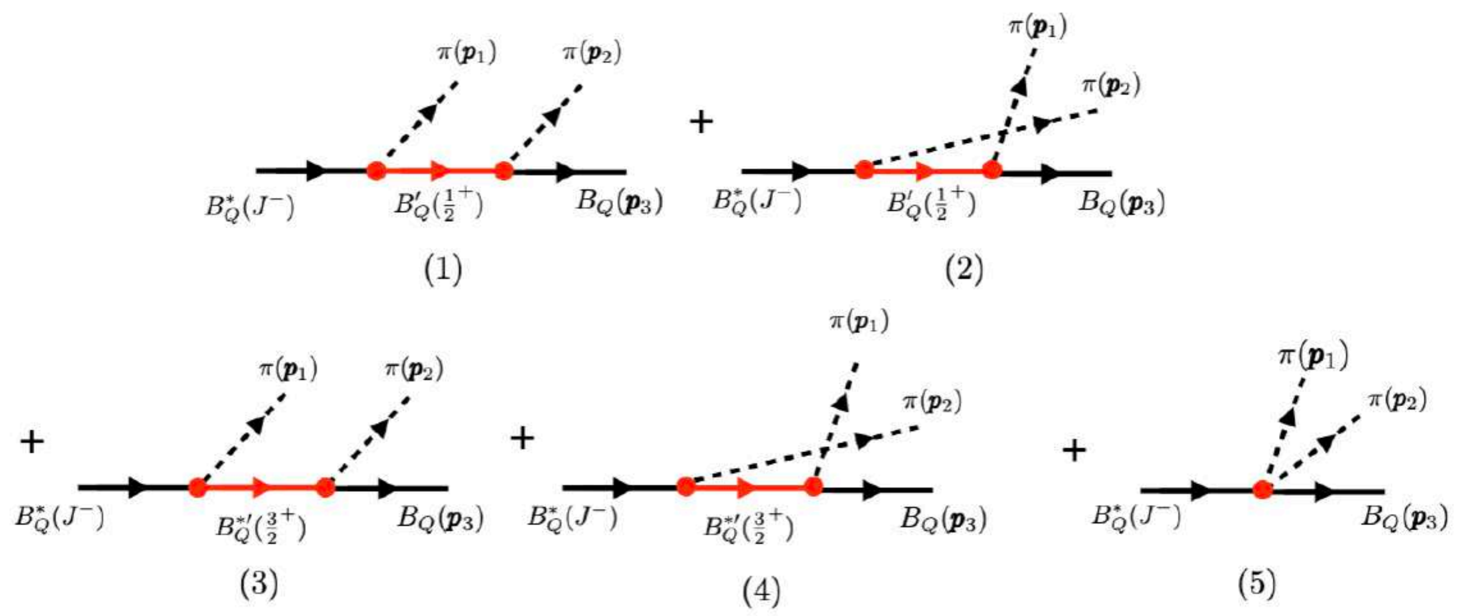
Analysis of $\Lambda_c(2625) \to \Lambda_c^+ \pi^+ \pi^-$

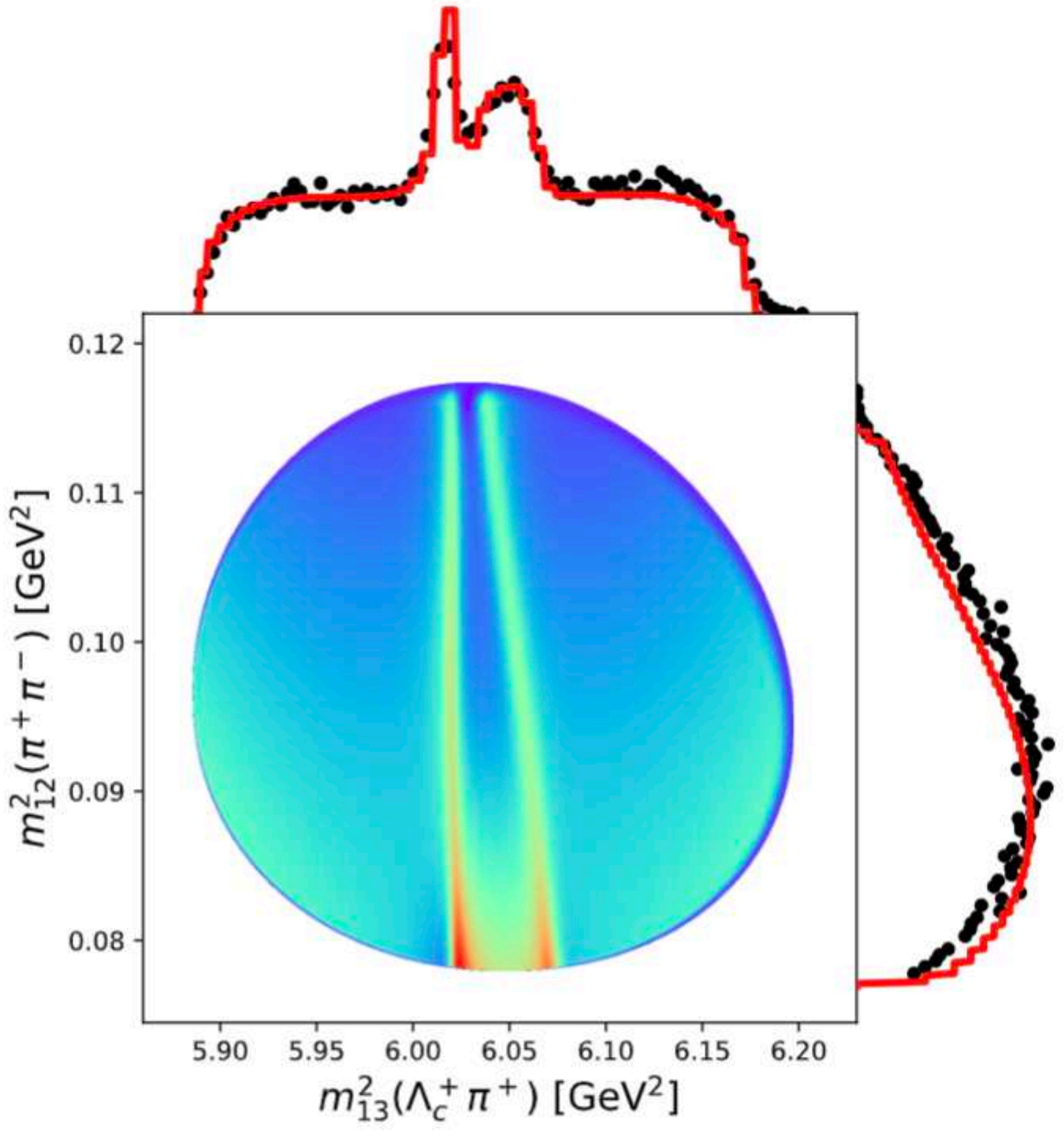
Belle data

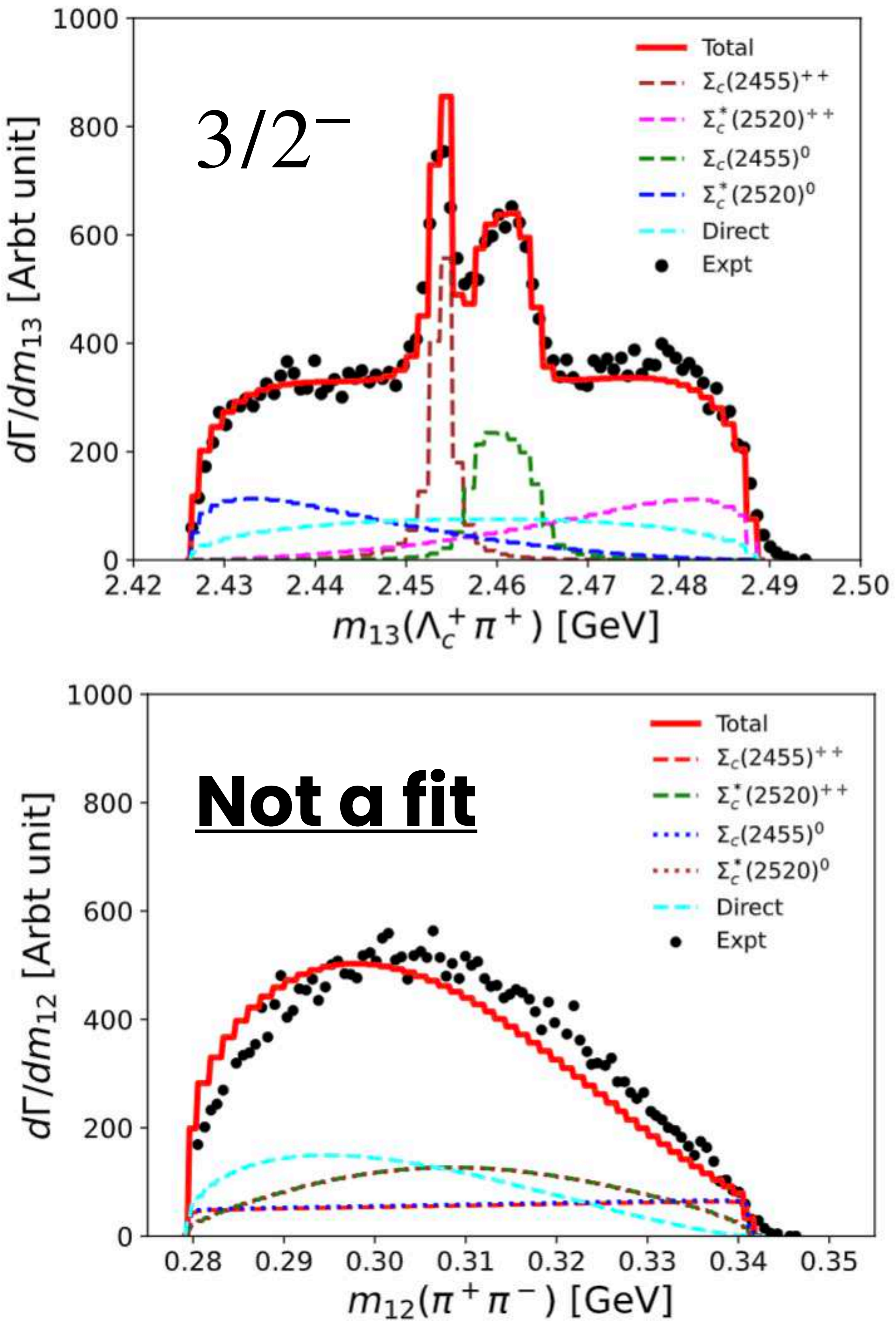
- Observed resonance [qqc]
  - >> $M = 2625$ MeV
  - >> $\Gamma < 0.52$ MeV
  - >> $\Lambda_c^* \to \Lambda_c \pi^+ \pi^-$

Belle, PRD 107, 032008 (2023).

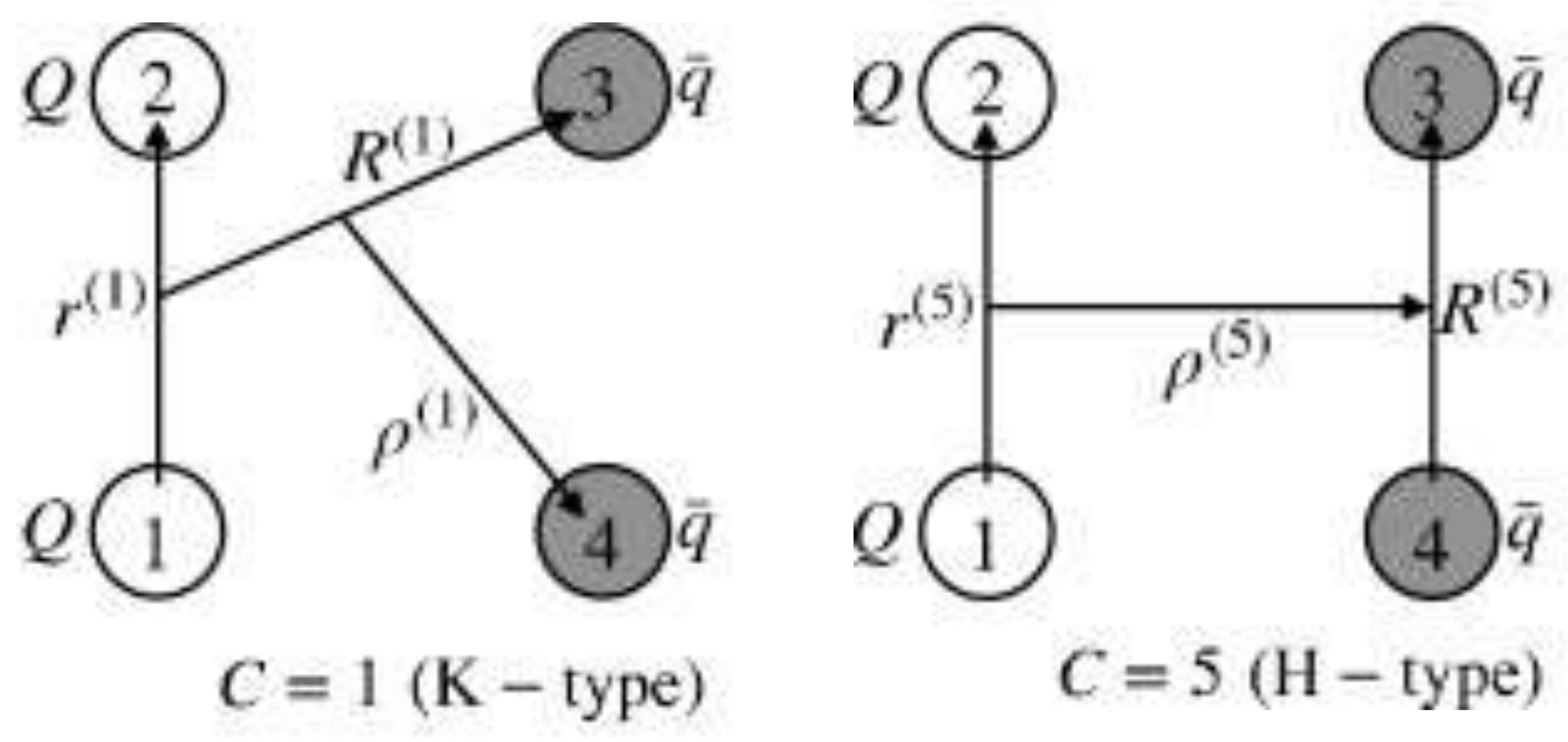
- Comparison with quark model?
  - -> assign to a QM state.
  - -> lambda-mode, $3/2^-$.
  - -> direct coupling.

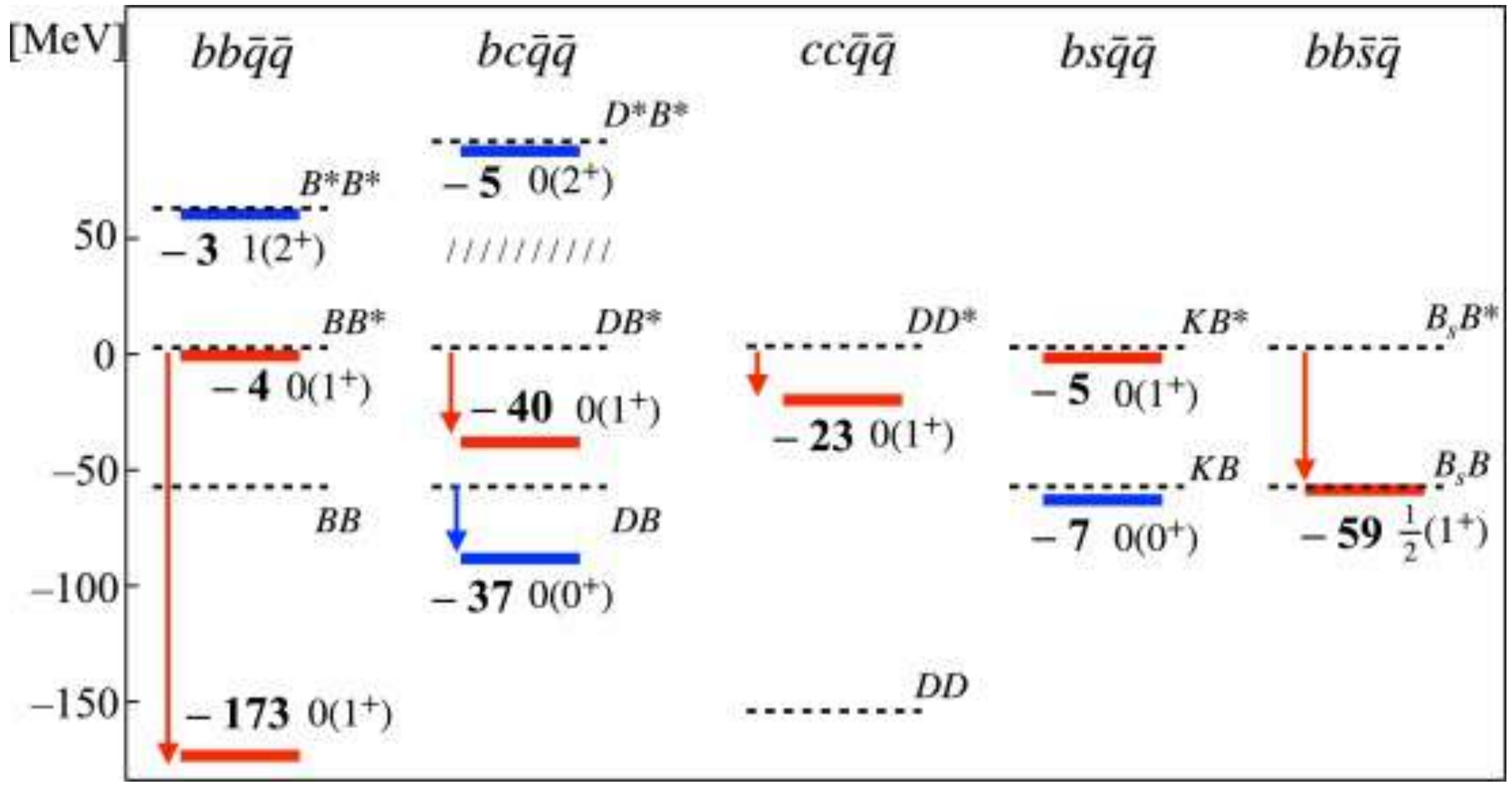






Doubly heavy Tetraquarks





- Looks simple, but numerically challenging.

PLB 814, 136095 (2021)

Many more!!

Hadrons under a magnetic field

Potential in the quark model

$$H_{\rm rel} = \frac{\boldsymbol{K}^2}{2M} - \frac{\nabla^2}{2\mu} + \frac{q^2B^2}{8\mu}\rho^2 + \frac{qB}{4\mu}K_x y - \frac{qB}{4\mu}K_y x + V(r) + \sum_{i=1}^{2}[-\boldsymbol{\mu}_i \cdot \boldsymbol{B} + m_i].$$

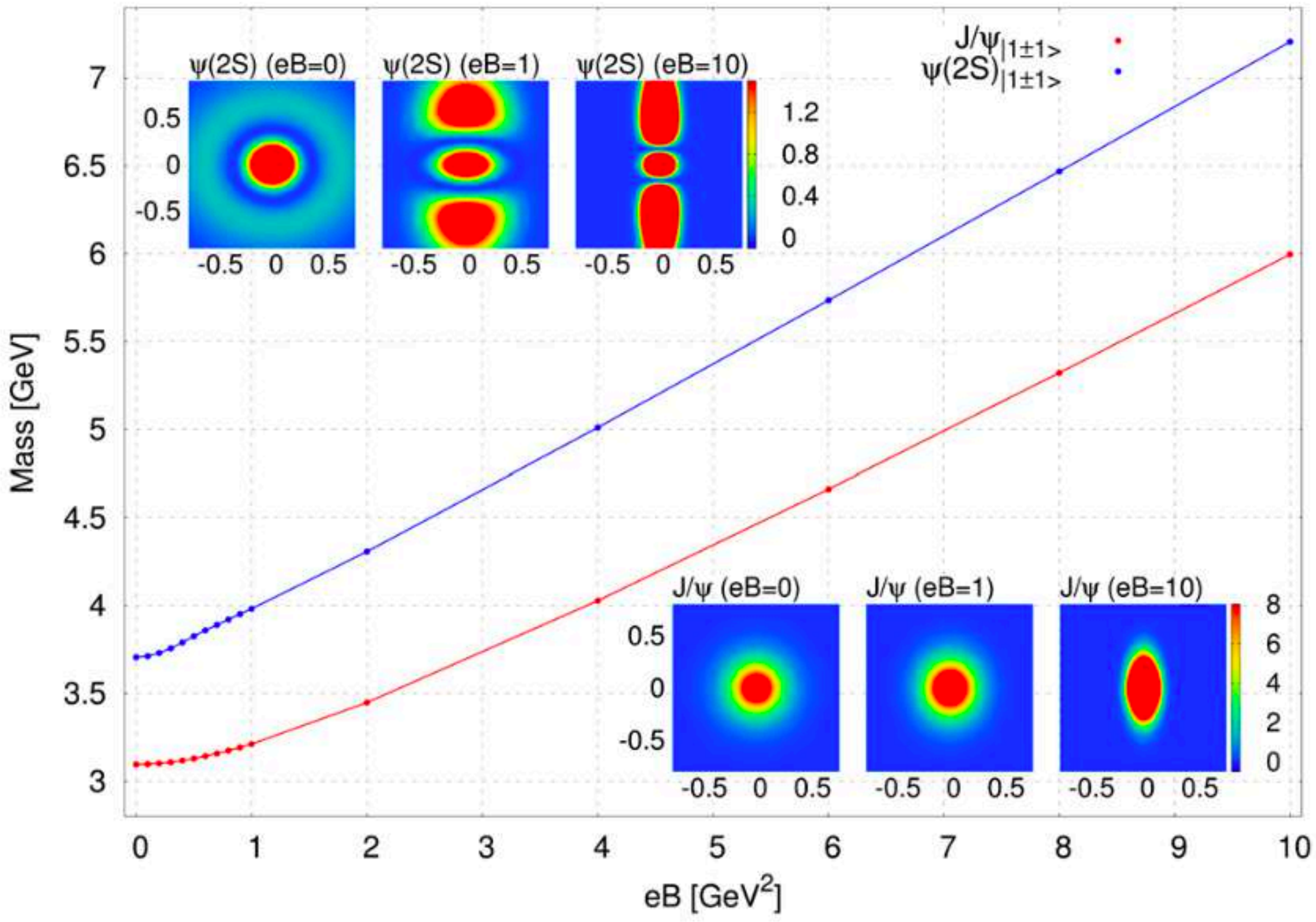
$$V(r) = \sigma r - \frac{A}{r} + \alpha(\boldsymbol{S}_1 \cdot \boldsymbol{S}_2)e^{-\Lambda r^2} + C = \sigma\sqrt{\rho^2+z^2} - \frac{A}{\sqrt{\rho^2+z^2}} + \alpha(\boldsymbol{S}_1 \cdot \boldsymbol{S}_2)e^{-\Lambda(\rho^2+z^2)} + C,$$

Cylindrical GEM

$$\Psi(\rho, z, \phi) = \sum_{n=1}^{N} C_n \Phi_n(\rho, z, \phi),$$

$$\Phi_n(\rho, z, \phi) = N_n e^{-\beta_n \rho^2} e^{-\gamma_n z^2},$$

PRD93, 051502(R) (2016)

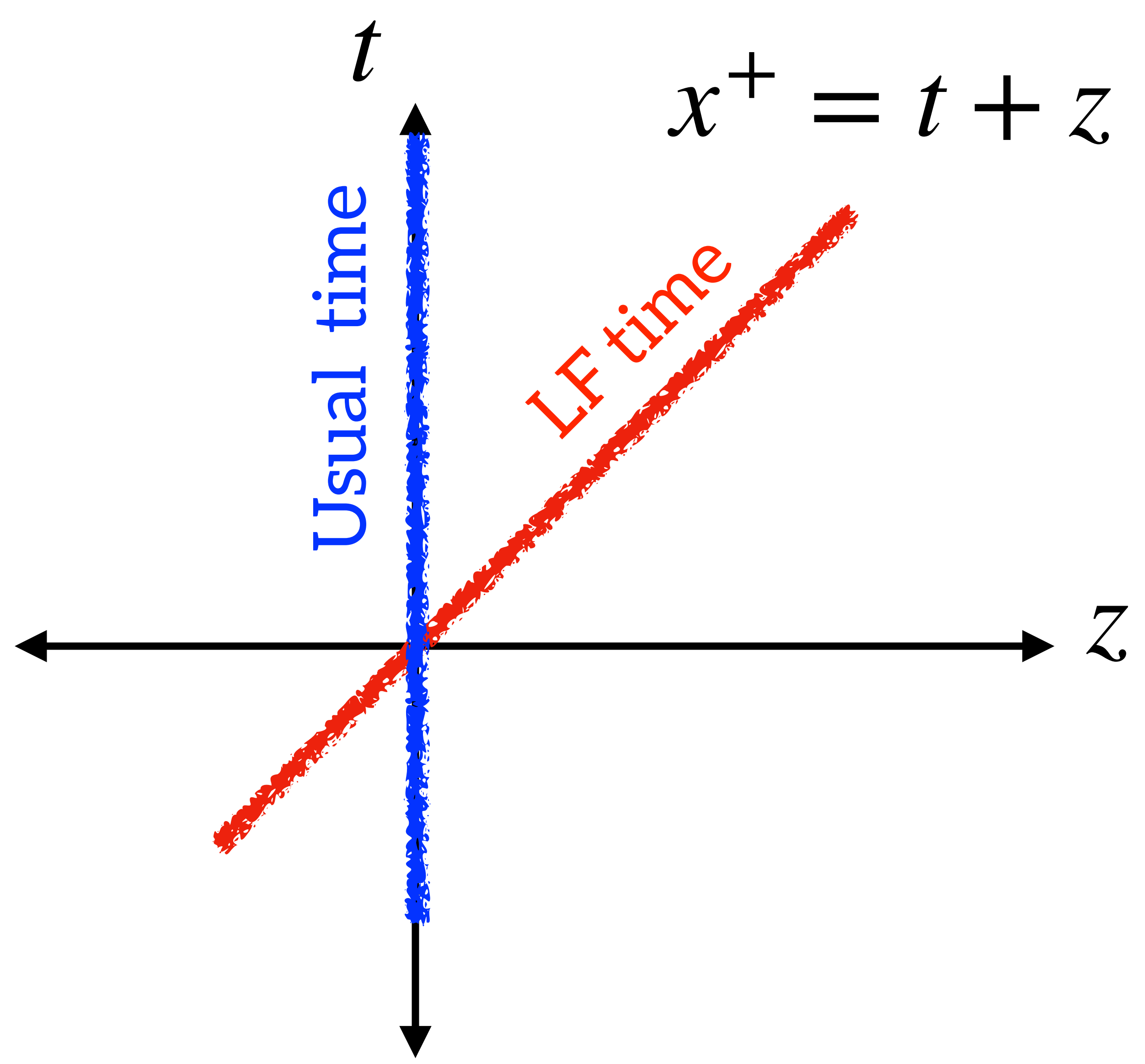


- Landau level (transverse confinement)
- Zeeman effect

Light-front quark model

Light-front dynamics

	Instant form	Light-Front form
Time	x^0	$x^+ = x^0 + x^3$
Space	x^1, x^2, x^3	$x^- = x^0 - x^3, \; \mathbf{x}_\perp = (x^1, x^2)$
Hamiltonian	p^0	$p^- = p^0 - p^3$
Momentum	p^1, p^2, p^3	$p^+ = p^0 + p^3, \; \mathbf{p}_\perp = (p^1, p^2)$
Product	$x \cdot p = x^0 p^0 - \mathbf{x} \cdot \mathbf{p}$	$x \cdot p = (x^+ p^- + x^- p^+)/2 - \mathbf{x}_\perp \cdot \mathbf{p}_\perp$
Vacuum	very complex	can only contain zero-mode excitations



Formalism

- Proposed by Dirac (1949)

Why LFD?

- Handle relativistic effect properly
- Maximal Poincare kinematic operator
- Relevant for high-energy process
- Vacuum becomes simpler

Light-front wave functions (LFWFs)

Many approaches

- Diagonalizing light-front Hamiltonian
- Bethe-Salpeter approach
- Continuum approach
- Ansatz

Instantaneous Hamiltonian

- Connection to the previous discussion
- Diagonalizing usual Hamiltonian
- Transform the wave function to LFWFs
- With relativized kinematics

Solving this Hamiltonian

$$H_{q\bar{q}} |\Psi_{q\bar{q}}\rangle = M_{q\bar{q}} |\Psi_{q\bar{q}}\rangle ,$$

$$H_{q\bar{q}} = H_0 + V_{q\bar{q}}$$

$$H_0 = \sqrt{m_q^2 + \boldsymbol{k}^2} + \sqrt{m_{\bar{q}}^2 + \boldsymbol{k}^2},$$

$$V_{q\bar{q}} = a + br - \frac{4\alpha_s}{3r} + \frac{32\pi\alpha_s \tilde{\delta}^3(r)}{9 m_q m_{\bar{q}}} (\boldsymbol{S}_q \cdot \boldsymbol{S}_{\bar{q}}),$$

Relativized quark model

- Hadron is a relativistic object.
- Relativized Schrödinger equation:

$$H\left|\psi\right\rangle = E\left|\psi\right\rangle$$

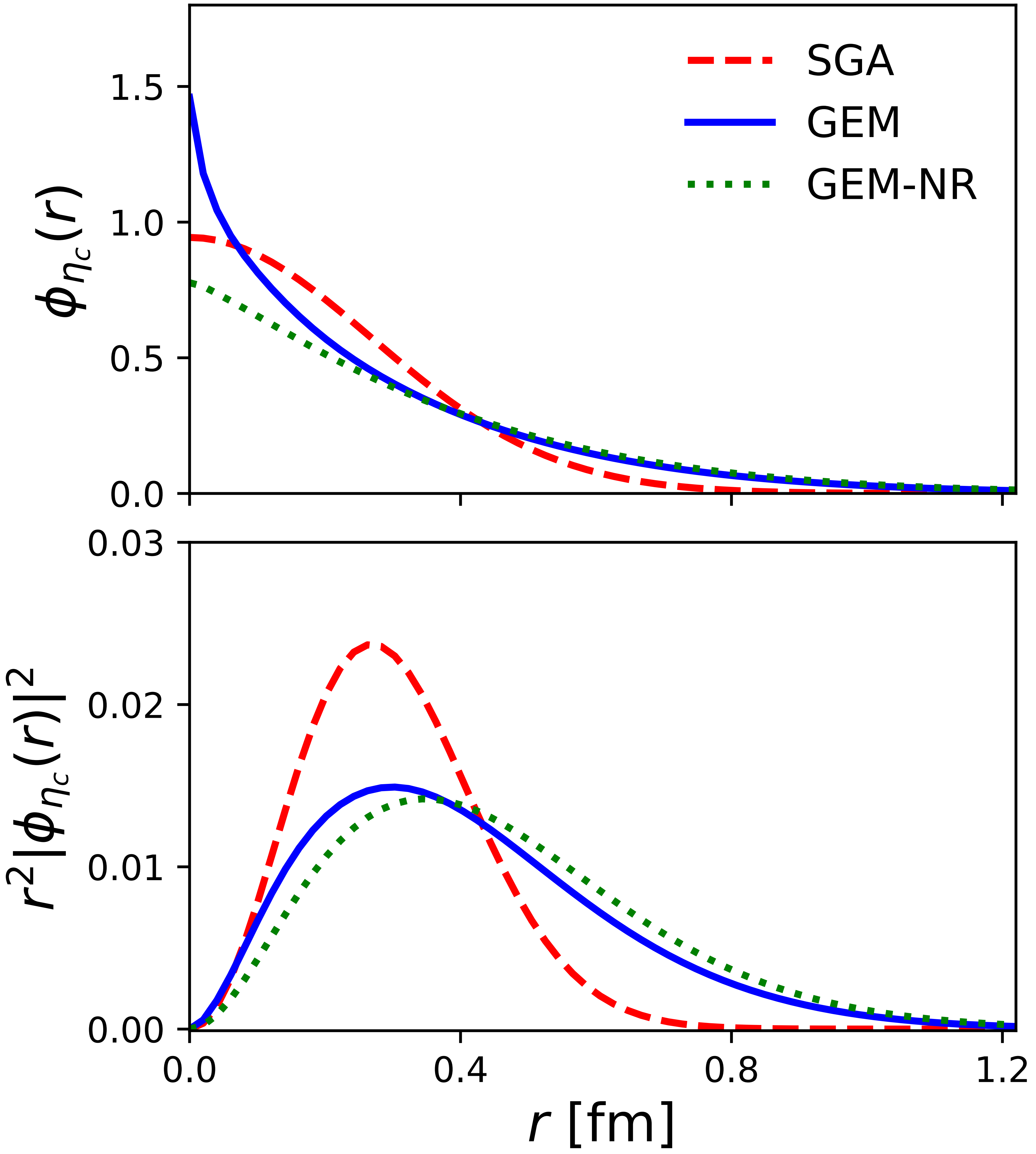
- Relativistic kinematics

$$H_0 = \frac{p^2}{2m} \to \sqrt{m^2 + p^2}$$

- Gaussian Expansion Method (GEM) to solve the equation.

Hiyama, PPNP51, 223 (2003)

- Wave function is divergent at the origin



Light-front quark model

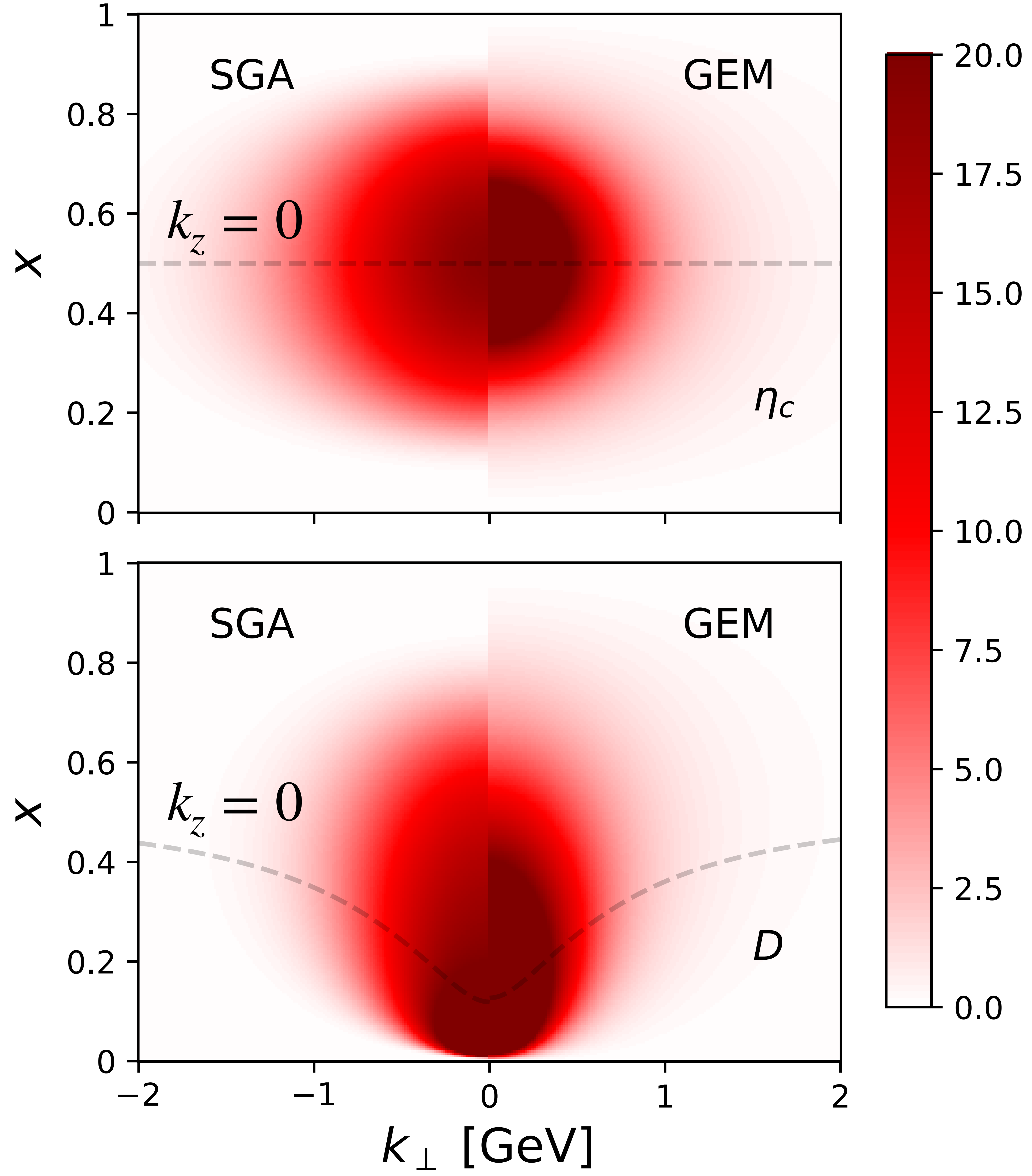
- Transform the wave function to the light front

$$\left|\psi_{IF}(k_z, k_\perp)\right\rangle \to \left|\psi_{LF}(x, k_\perp)\right\rangle$$

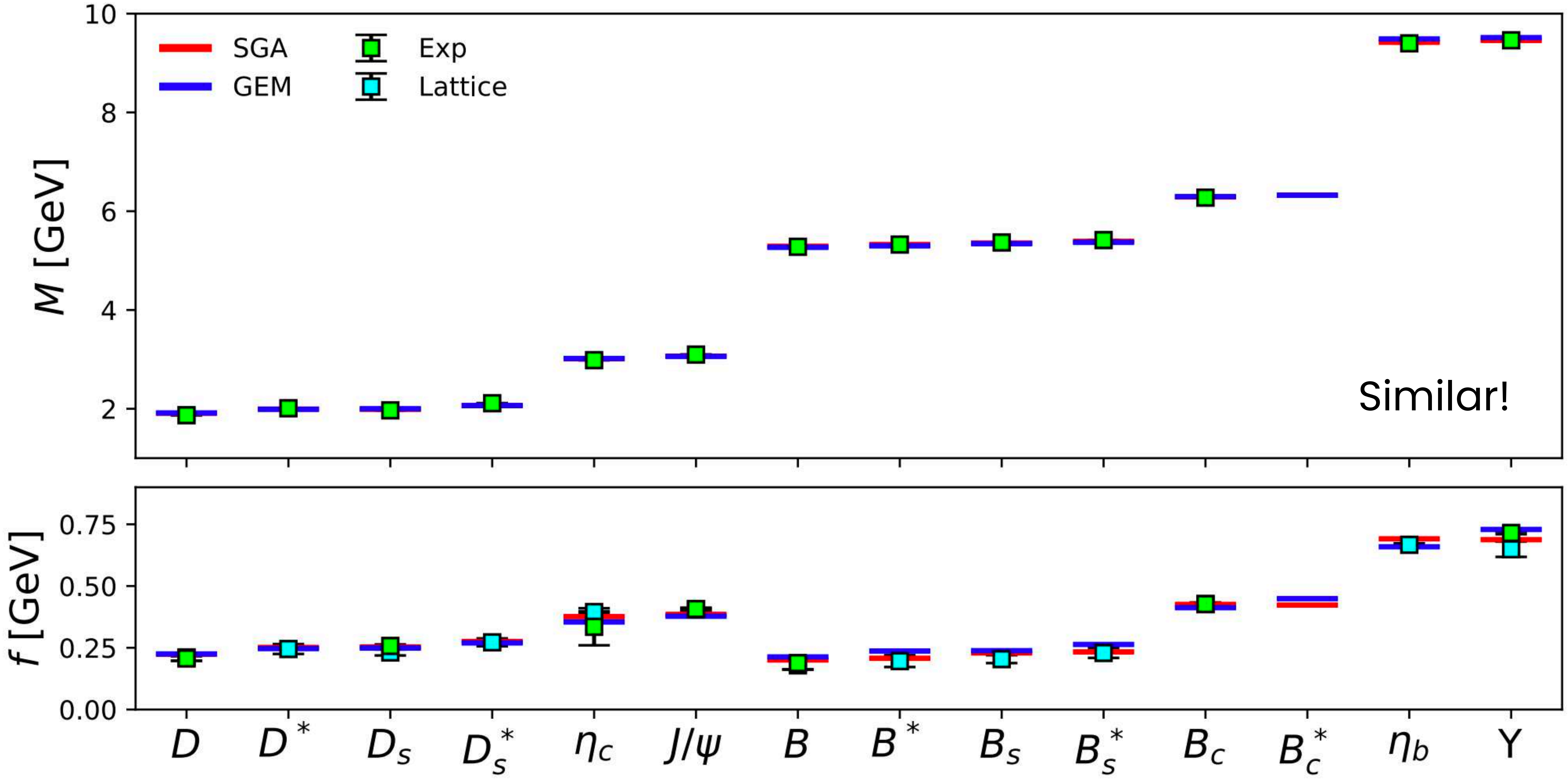
$$k_z \to x = \frac{p_1^+}{P^+}$$

- LFWF contains:
  - -> Radial part
  - -> Spin-obrit part (Melosh transformation)

$$\psi^{JJ_z}_{\lambda_1\lambda_2}(x, k_\perp) = \phi(x, k_\perp)\mathscr{R}^{JJ_z}_{\lambda_1\lambda_2}(x, k_\perp)$$

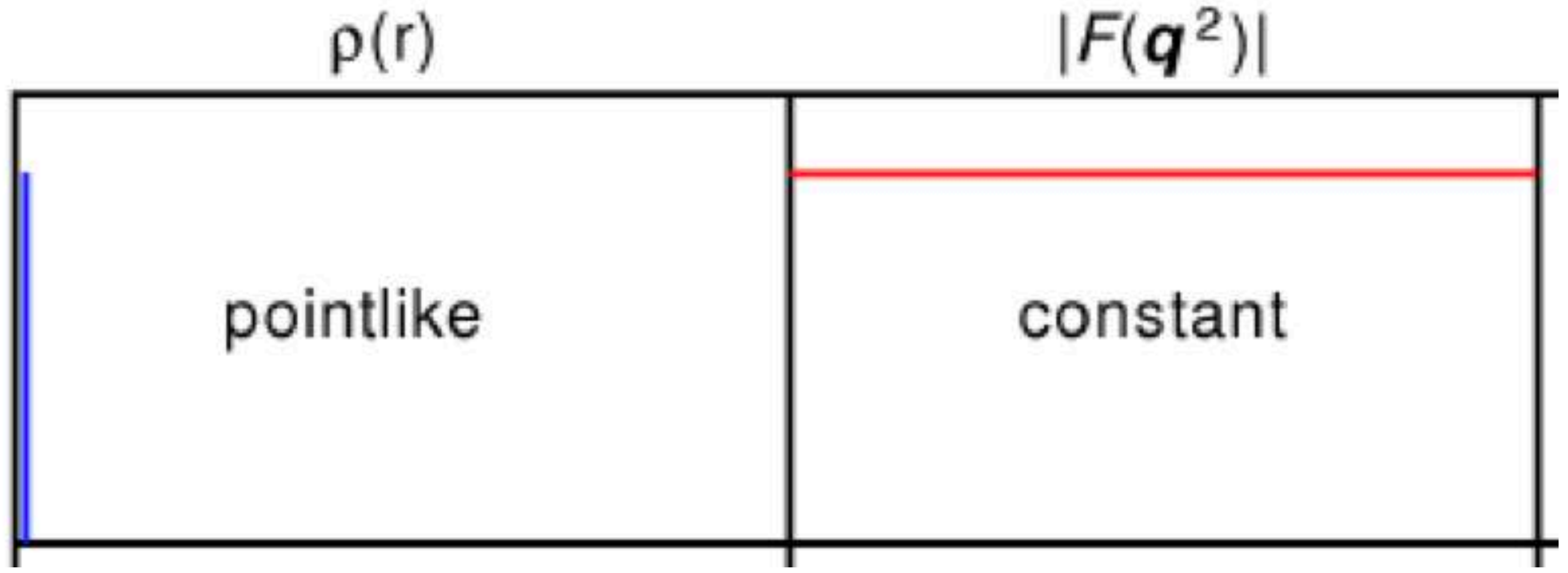


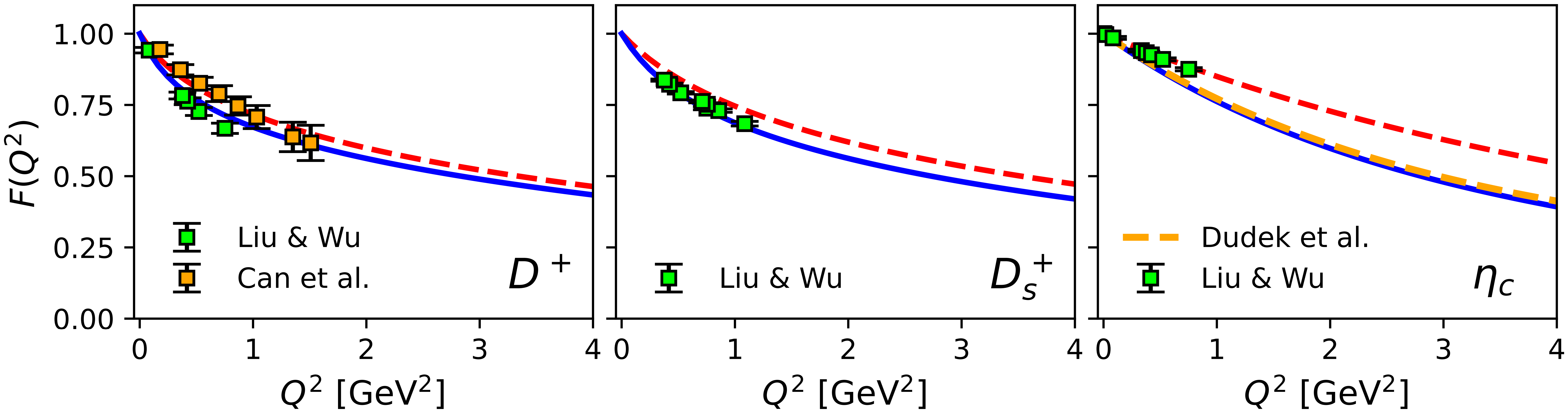
Mass and decay constant



Electromagnetic form factor

- EM form factor
  => hadron is not a point-like object.
  => related to charge distribution and radius.

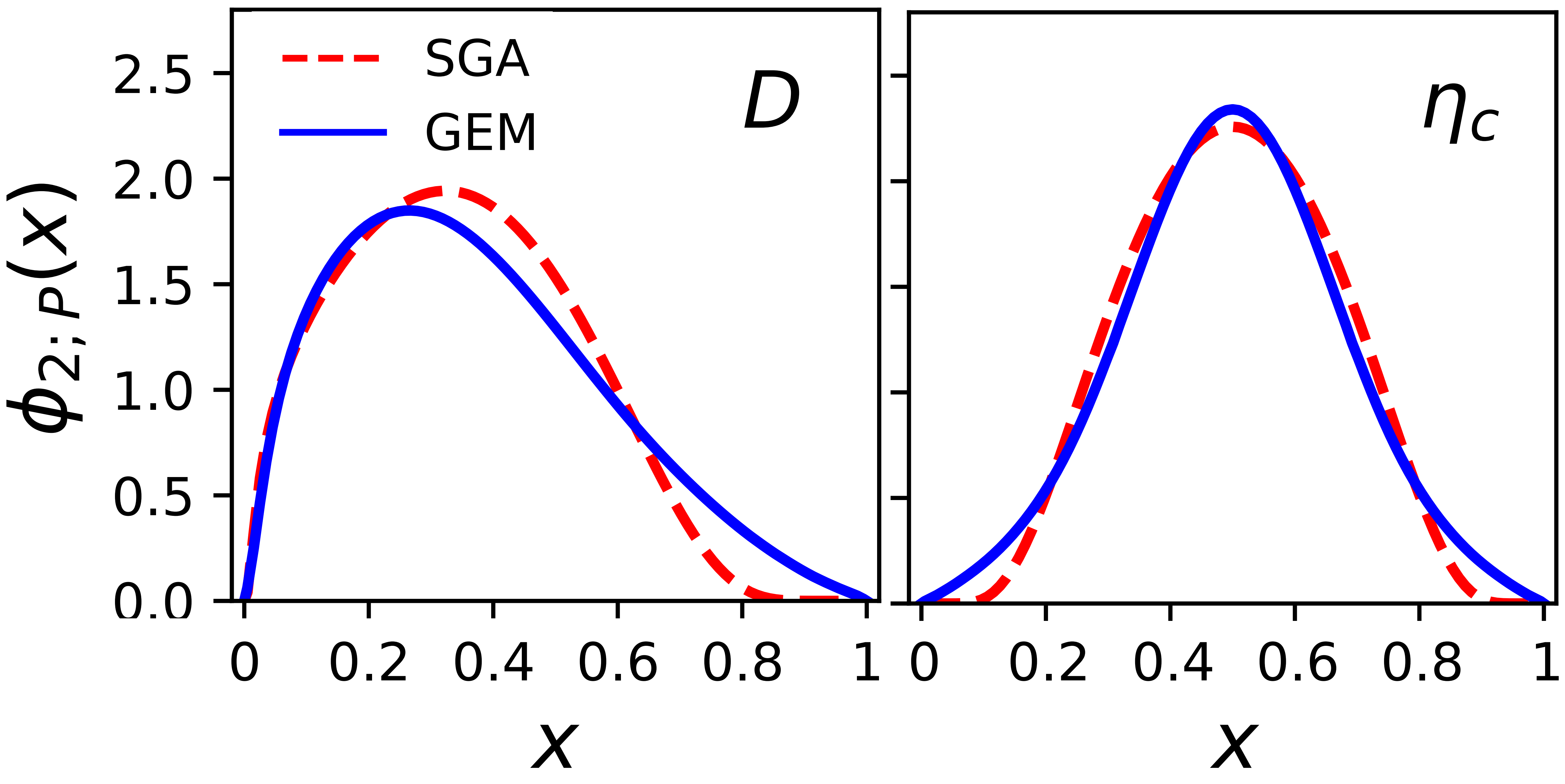




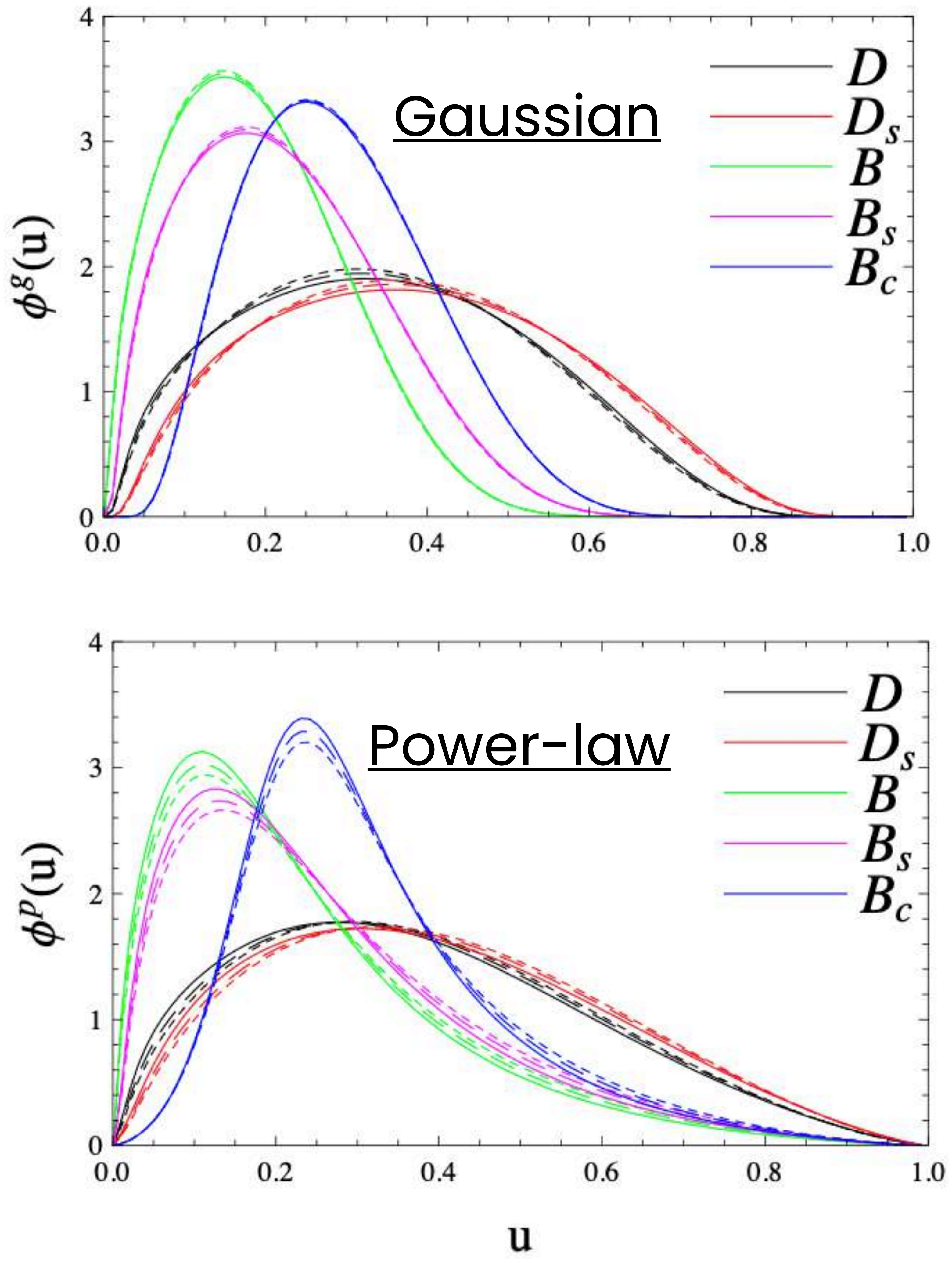
- LFQM can reproduce lattice data
- LFQM is good for low and high energy regions.

Distribution amplitude: GEM vs SGA

- Probability to find quark at given momentum fraction x.



Arxiv: 2401.07933



PRD81, 114024 (2010)

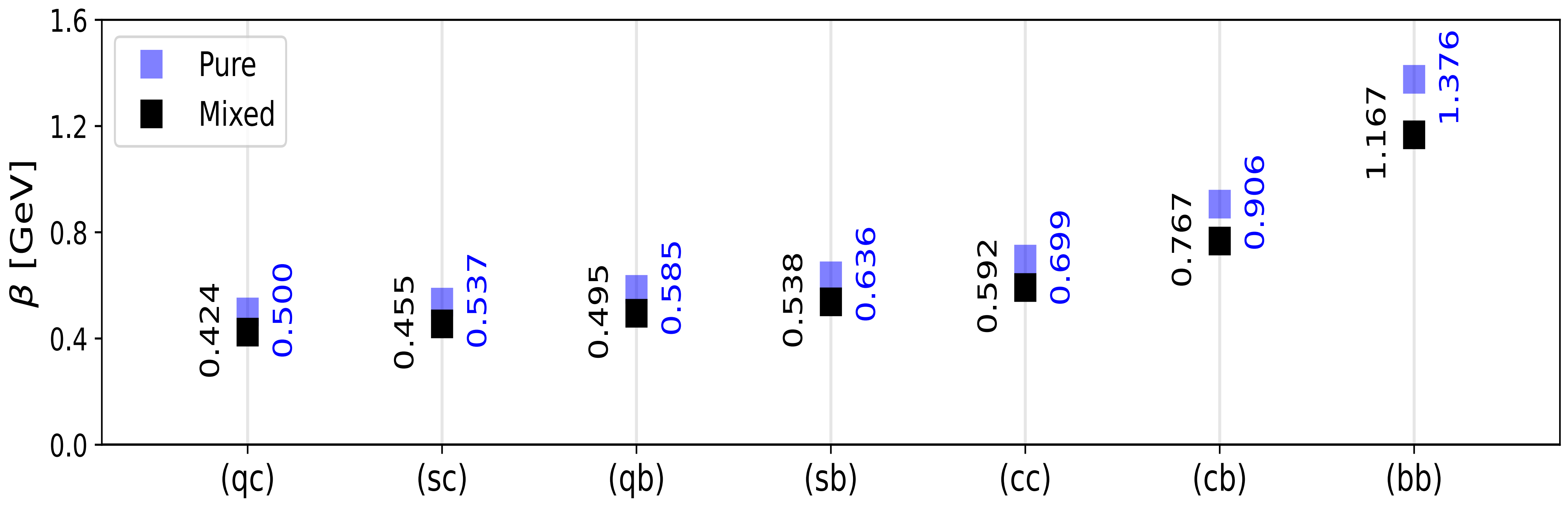
- This LFQM-GEM can provide a realistic approach.
- Further application to hadron tomography.

HO wave function for excited states

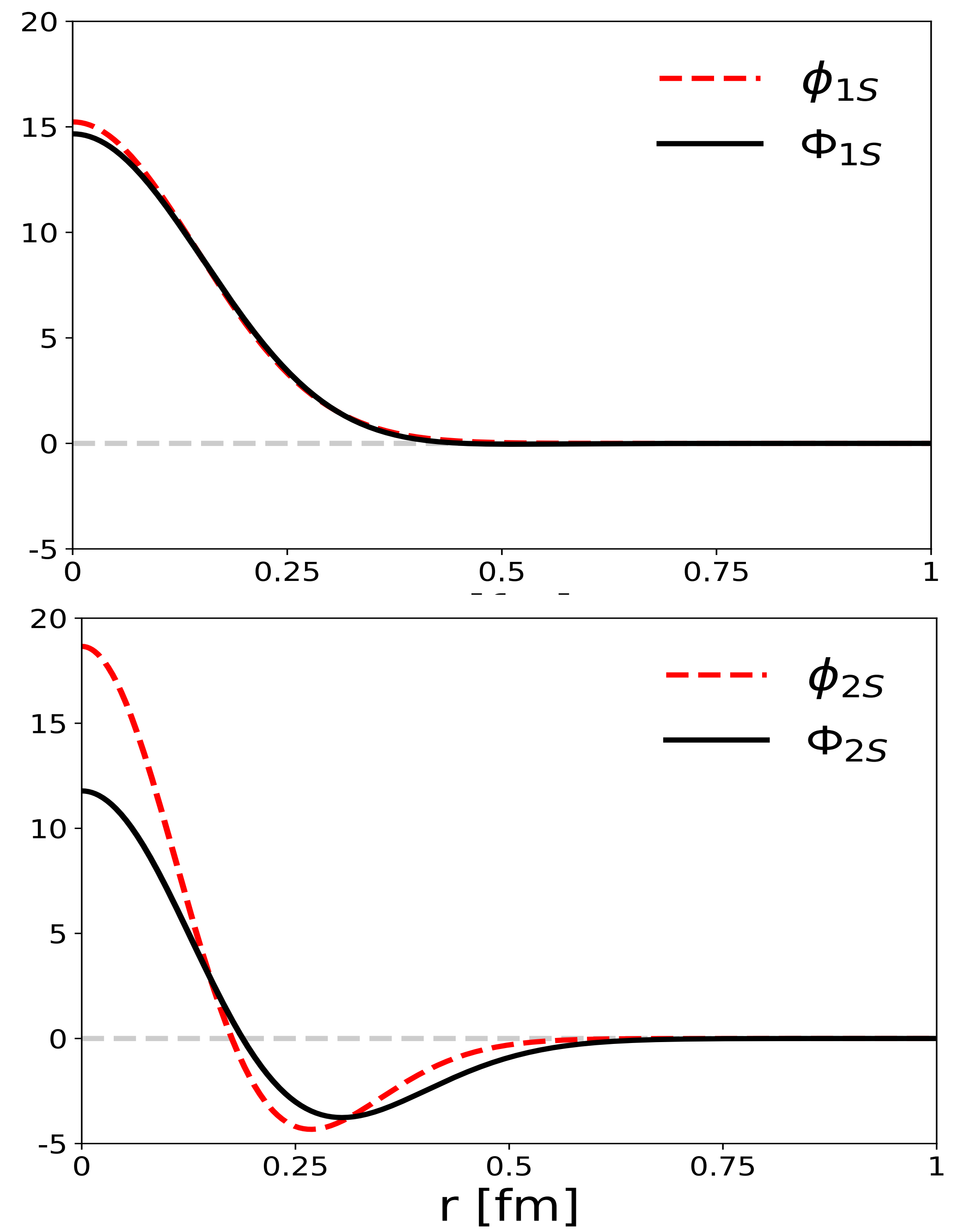
- HO basis expansion

$$\Phi_{1S} = \phi_{1S}^{H0} \qquad \Phi_{1S} = \cos\theta\ \phi_{1S}^{H0} + \sin\theta\ \phi_{2S}^{H0}$$

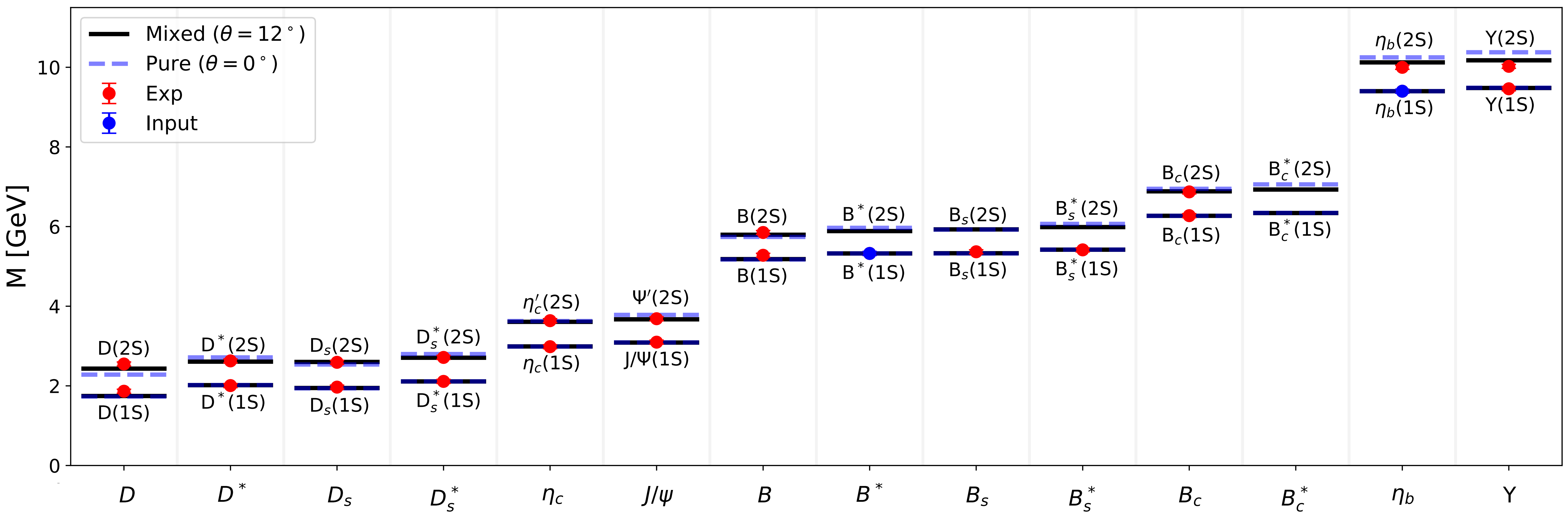
$$\Phi_{2S} = \phi_{2S}^{H0} \qquad \Phi_{2S} = -\sin\theta\ \phi_{1S}^{H0} + \cos\theta\ \phi_{2S}^{H0}$$



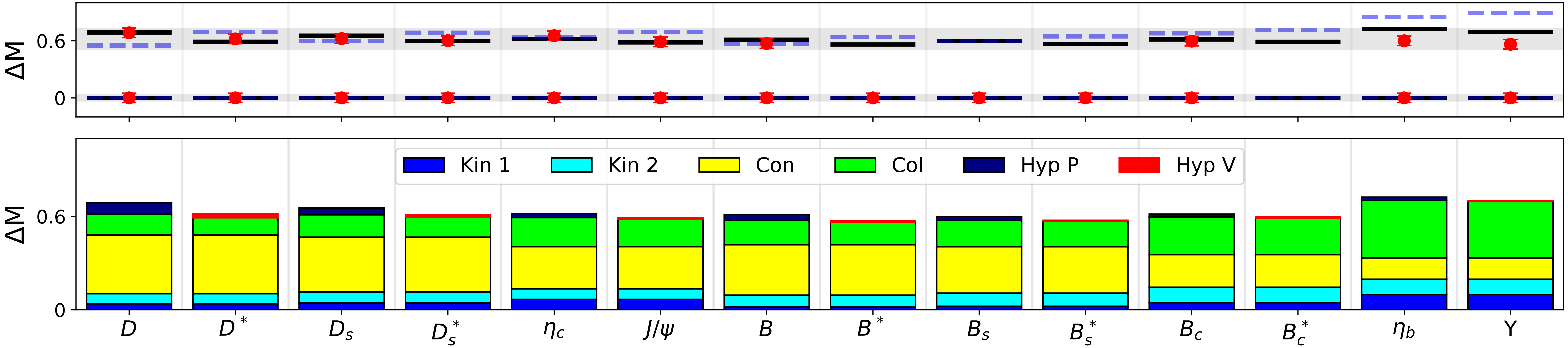
- The universal θ parameter —> good approximation



- Mass spectra $V_{q\bar{q}} = a + br - \frac{4\alpha_s}{3r} + \frac{2}{3}\frac{\mathbf{S}_q \cdot \mathbf{S}_{\bar{q}}}{m_q m_{\bar{q}}} \nabla^2 V_{\text{Coul}}$



Mass splitting $[M_{2S} - M_{1S}]$



<> Competing contribution:

—> Confinement int

$$\Delta M_{conf} \propto \frac{1}{\beta}$$

—> Coulomb-like int

$$\Delta M_{colmb} \propto \beta$$

<> Hyperfine int

—> Small, but very important

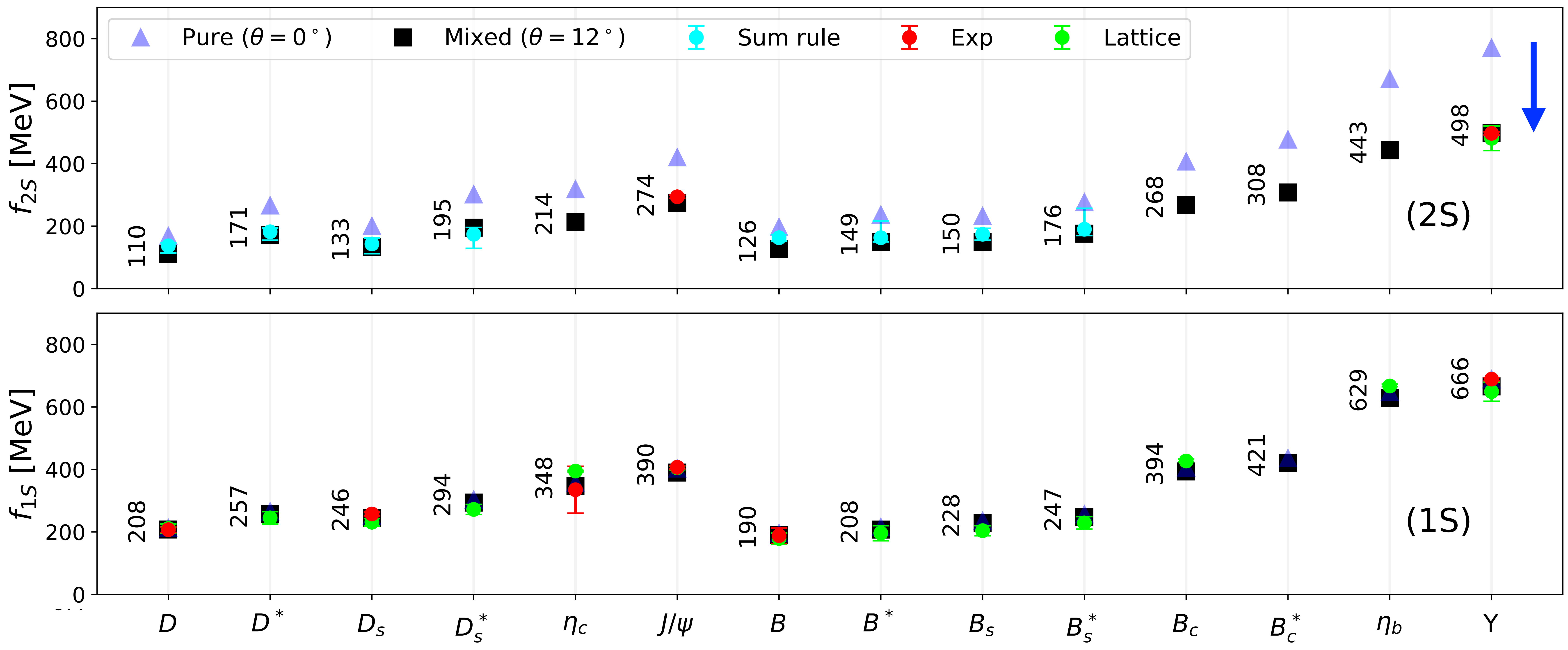
—> Mixing is needed

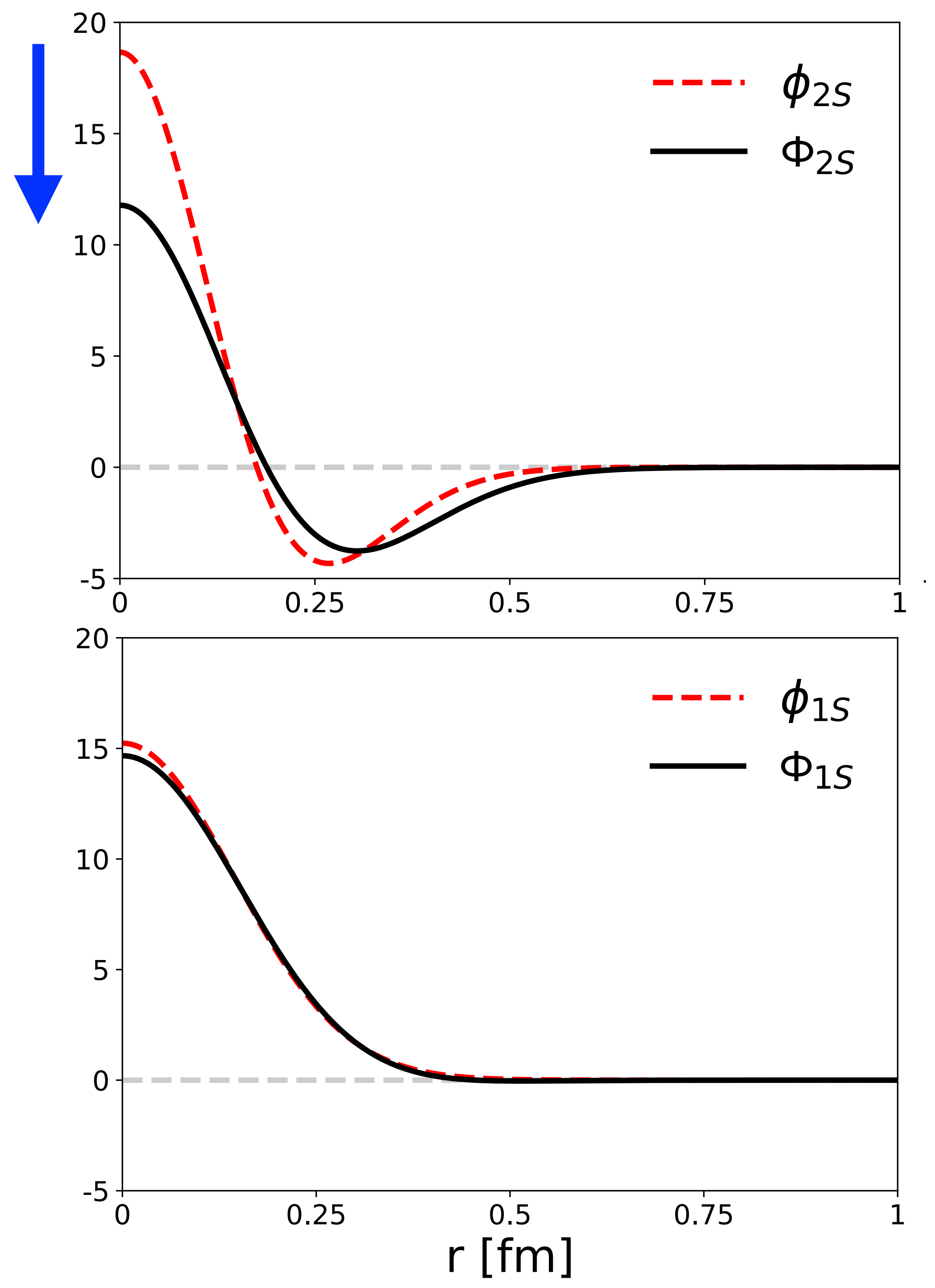
—> $\Delta M_P > \Delta M_V$

$$\Delta M_{hyp} \propto (S_q \cdot S_{\bar{q}})(\cos 2\theta - 2\sqrt{6} \sin 2\theta)$$

—> $\theta_c \approx 6^\circ$

Decay constants for 2S states





$$\langle 0|\,\bar{q}\gamma^{\mu}\gamma_5 q\,|\mathrm{P}(P)\rangle = i f_{\mathrm{P}} P^{\mu},$$
$$\langle 0|\,\bar{q}\gamma^{\mu} q\,|\mathrm{V}(P,J_z)\rangle = f_{\mathrm{V}} M \epsilon^{\mu}(J_z),$$

- Exp. data (2S State)
  => $V \to e^+e^-$
- Lattice data?
  –> only for $\Upsilon(2S)$

A global analysis (fit): light and heavy mesons

$\sigma^b_{model} = 0.7\,\%$

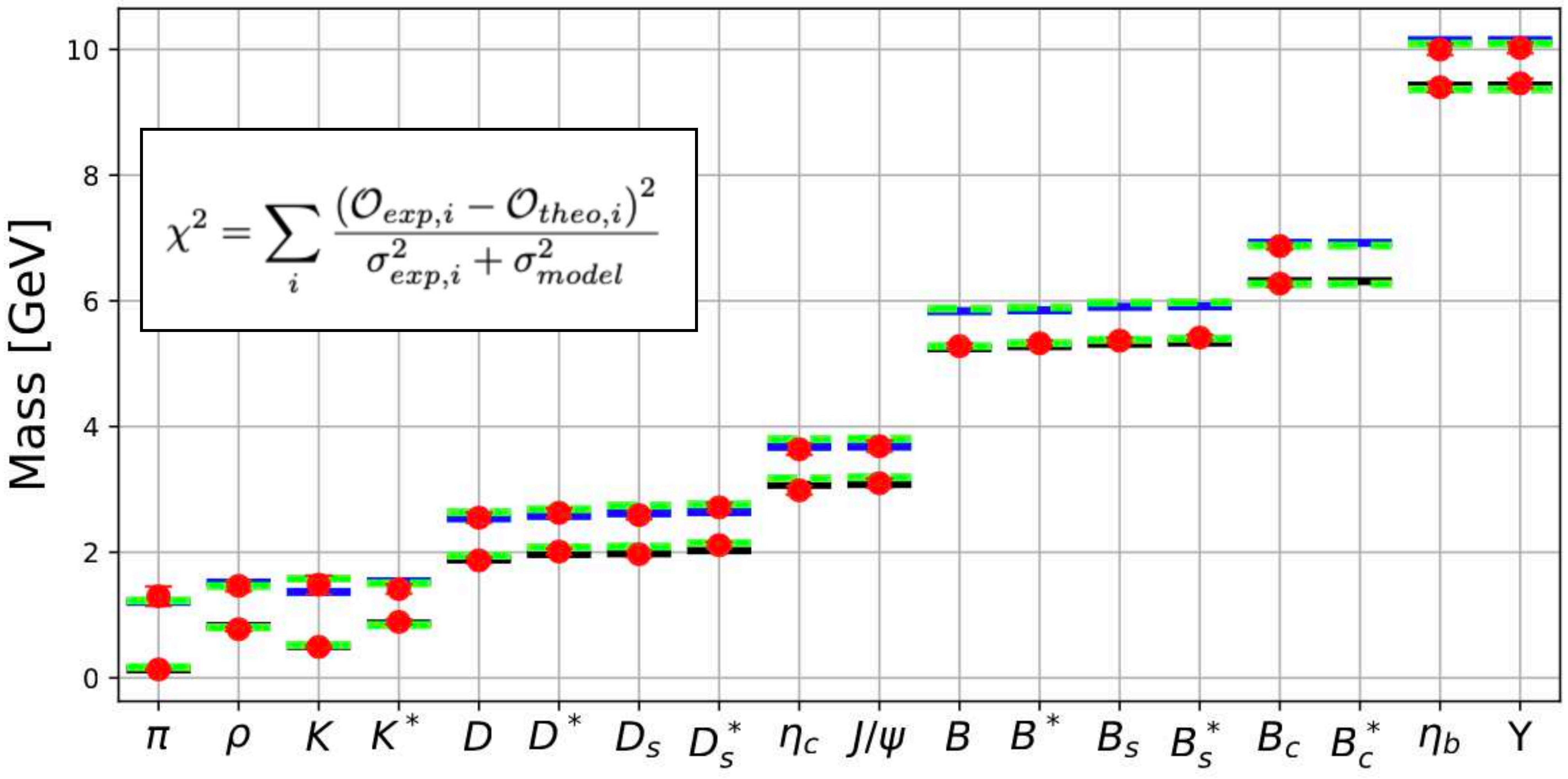
$\theta = 12.9^{\circ}(5^{\circ})$

$\sigma^c_{model} = 2.0\,\%$

$\theta = 10^{\circ}(1^{\circ})$

$\sigma^q_{model} = 3.6\,\%$

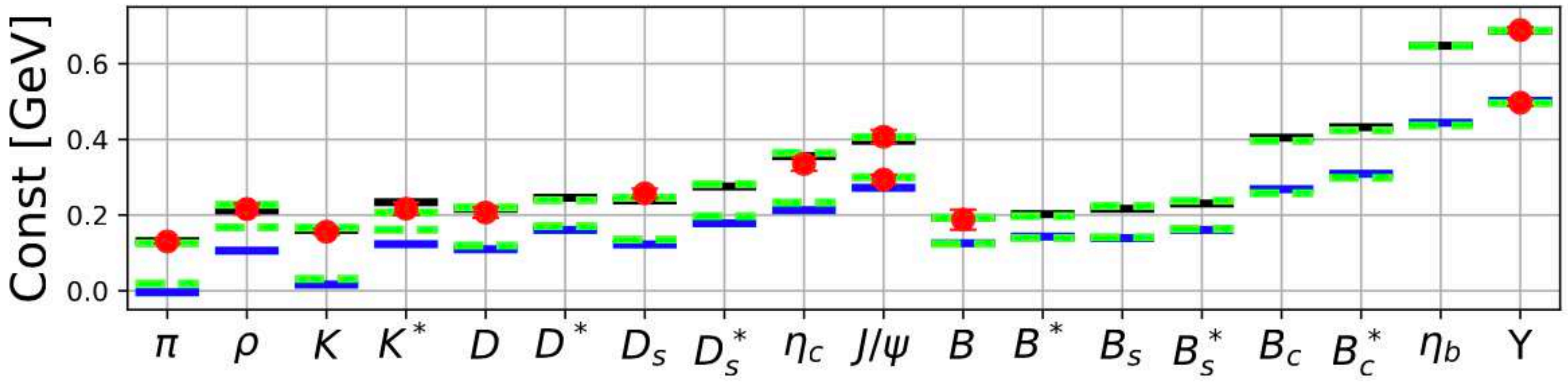
$\theta = 2^{\circ}(8^{\circ})$



$\sigma_{model} = \lambda \sigma^i_{model}$

$\lambda = 1.2$

$\theta = 12.6^{\circ}(5^{\circ})$



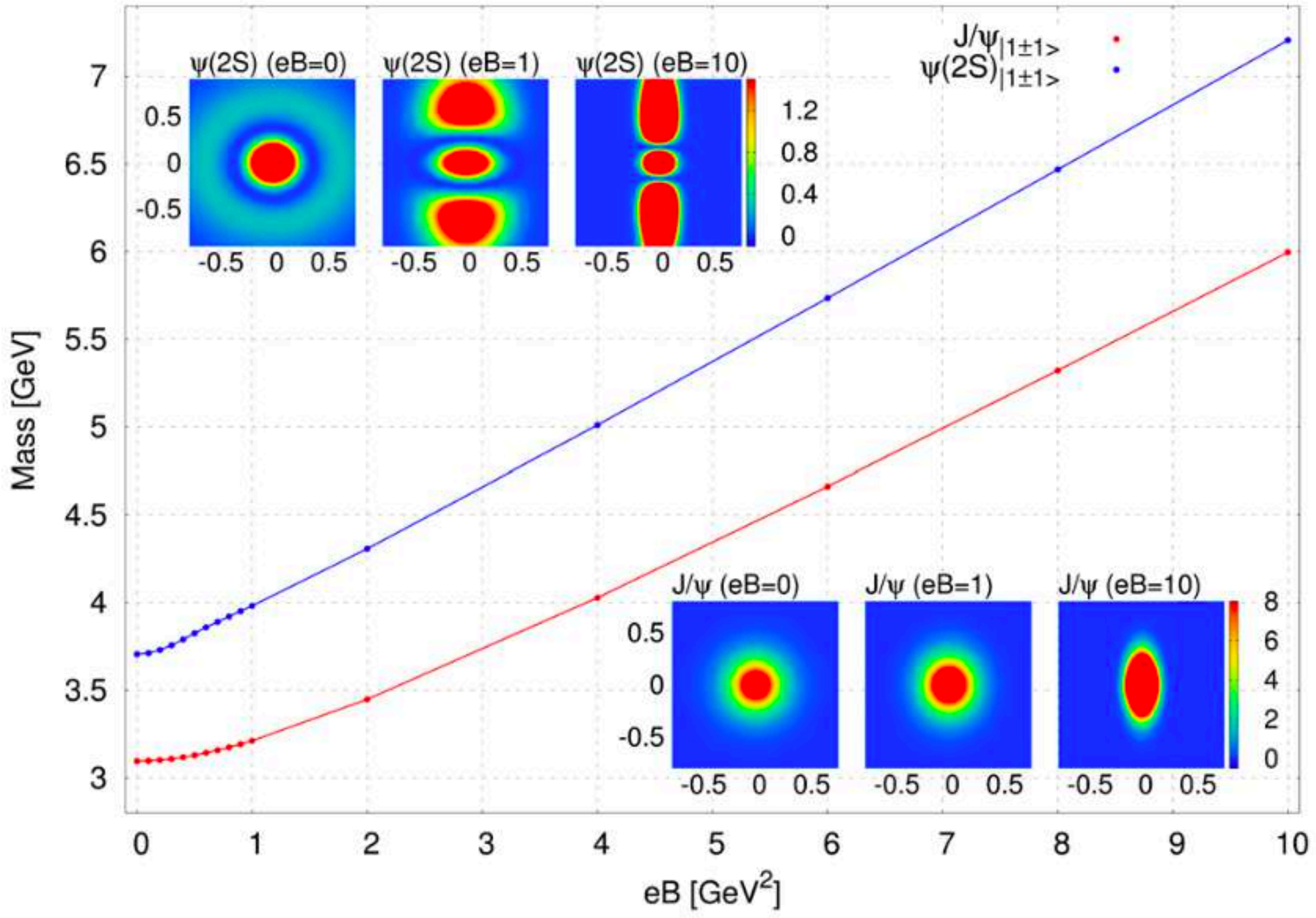
- Global fit
  —> iMinuit (python)
- Trial WF
  —> $\theta = 12.6^{\circ}(5^{\circ})$
- Model error
  —> less than 5%
- LFWF
  —> other properties

Deformed wave functions and LFWFs

Cylindrical GEM

$$\Psi(\rho, z, \phi) = \sum_{n=1}^{N} C_n \Phi_n(\rho, z, \phi),$$

$$\Phi_n(\rho, z, \phi) = N_n e^{-\beta_n \rho^2} e^{-\gamma_n z^2},$$



Light-front Hamiltonian (BLFQ)

$$H_{\text{eff}} = \frac{\vec{k}_\perp^2 + m_q^2}{x} + \frac{\vec{k}_\perp^2 + m_{\bar{q}}^2}{1-x} + \kappa^4 \vec{\zeta}_\perp^2 - \frac{\kappa^4}{(m_q + m_{\bar{q}})^2} \partial_x (x(1-x)\partial_x) - \frac{C_F 4\pi\alpha_s(Q^2)}{Q^2} \bar{u}_{s'}(k')\gamma_\mu u_s(k) \bar{v}_{\bar{s}}(\bar{k})\gamma^\mu v_{\bar{s}'}(\bar{k}').$$

Basis functions (HO & power-law)

$$\phi_{nm}(\vec{q}_\perp; b) = b^{-1}\sqrt{\frac{4\pi n!}{(n+|m|)!}}\left(\frac{q_\perp}{b}\right)^{|m|} \exp(-q_\perp^2/(2b^2)) L_n^{|m|}(q_\perp^2/b^2) \exp(im\theta_q),$$

$$\chi_l(x; \alpha, \beta) = \sqrt{4\pi(2l+\alpha+\beta+1)} \sqrt{\frac{\Gamma(l+1)\Gamma(l+\alpha+\beta+1)}{\Gamma(l+\alpha+1)\Gamma(l+\beta+1)}} x^{\frac{\beta}{2}}(1-x)^{\frac{\alpha}{2}} P_l^{(\alpha,\beta)}(2x-1).$$

Problem of restoring the covariance

<> The basic idea: the observables are computed with good current. J^+

—> Practically, it is easy

—> Containing kinematic operators.

<> By definition, the results obtained from good or bad currents are the same.

—> Practically, it is not easy

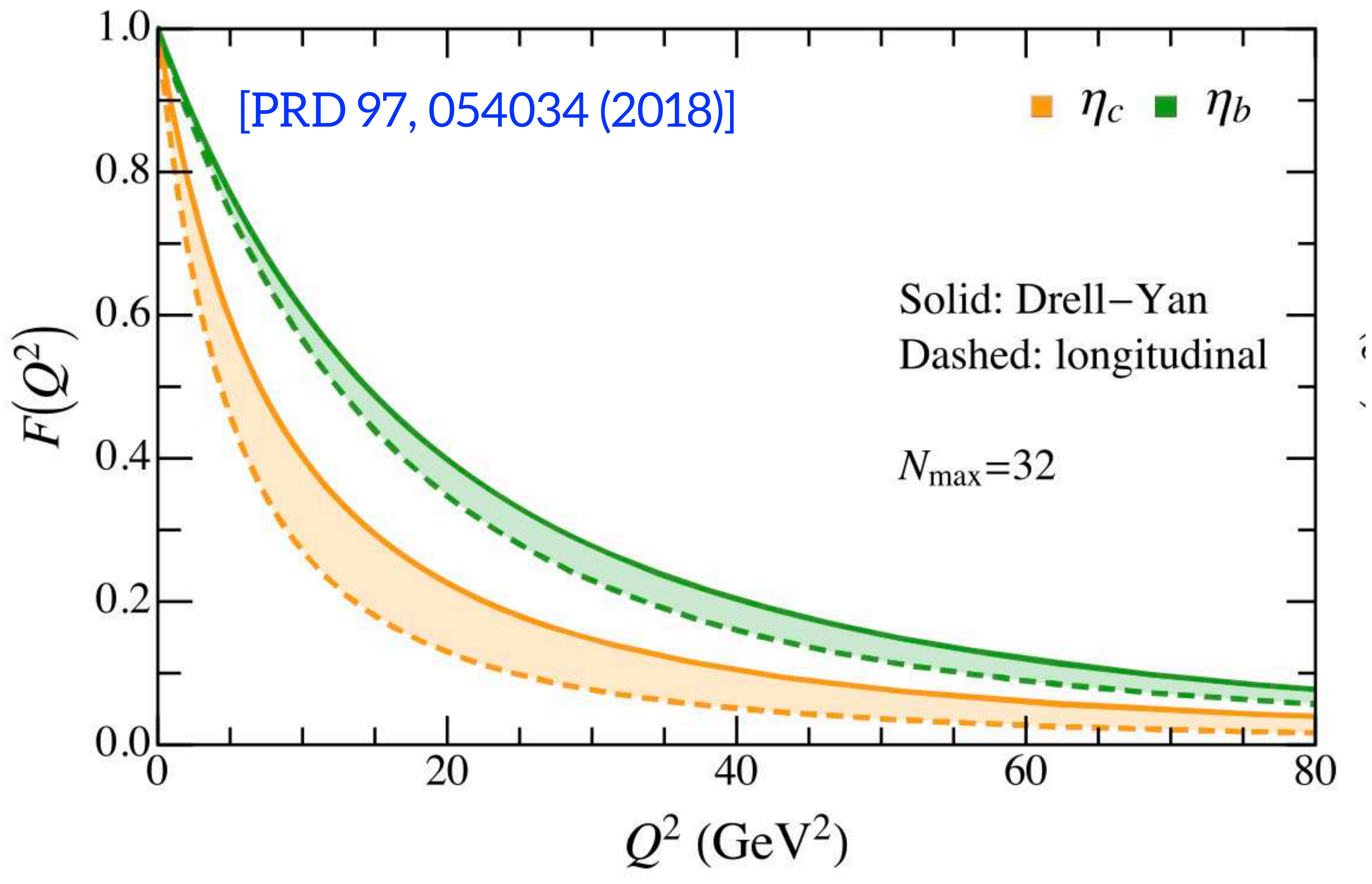
—> Minus current J^-

>> nontrivial dynamics

<> People claim the obtained results are different.

—> Fock space truncation

—> Zero-mode contribution



LFQM: non-interacting quark basis

- In general, the quark bound inside a hadron is off-shell.
- In light-front BS model:

$$S = S_{on} + S_{inst} + S_{z.m.}$$

- But, there is another (easier) way.

Bakamjian-Thomas construction

- How to add the interaction to the noninteracting basis

[Adv. Nucl. Phys. 20, 225 (1991)]

Noninteracting quark and antiquark

$$|P_1, P_2, S_1, S_2\rangle = |P_1, S_1\rangle \otimes |P_2, S_2\rangle \qquad P^\mu = P_1^\mu + P_2^\mu$$

Interaction is added to mass Casimir operator

$$M = M_0 + V$$

Meson wave function

$$|M, P, s, s_z\rangle = \sum_s \int d^3p \ \ |M_0, P_0, s_0, s_{0z}\rangle \, \delta^3(P - P_0)\delta_{ss_0}\Psi_M(M_0, s_z)$$

Eigenvalue equation

$$(\lambda - M_0)\Psi_M(M_0, s_z) = \sum_s \int d^3p_0' \left\langle M_0, s_z | V^s | M_0', s_z' \right\rangle \Psi_M(M_0', s_z')$$

Decay constants: various currents and polarizations

$$\begin{aligned}\langle 0|\,\bar{q}\gamma^\mu\gamma_5 q\,|\mathrm{P}(P)\rangle &= i f_\mathrm{P} P^\mu,\\ \langle 0|\,\bar{q}\gamma^\mu q\,|\mathrm{V}(P,J_z)\rangle &= f_\mathrm{V} M \epsilon^\mu(J_z),\\ \langle 0|\,\bar{q}\sigma^{\mu\nu} q\,|\mathrm{V}(P,J_z)\rangle &= i f_\mathrm{V}^T\left[\epsilon^\mu(J_z)P^\nu - \epsilon^\nu(J_z)P^\mu\right],\end{aligned}$$

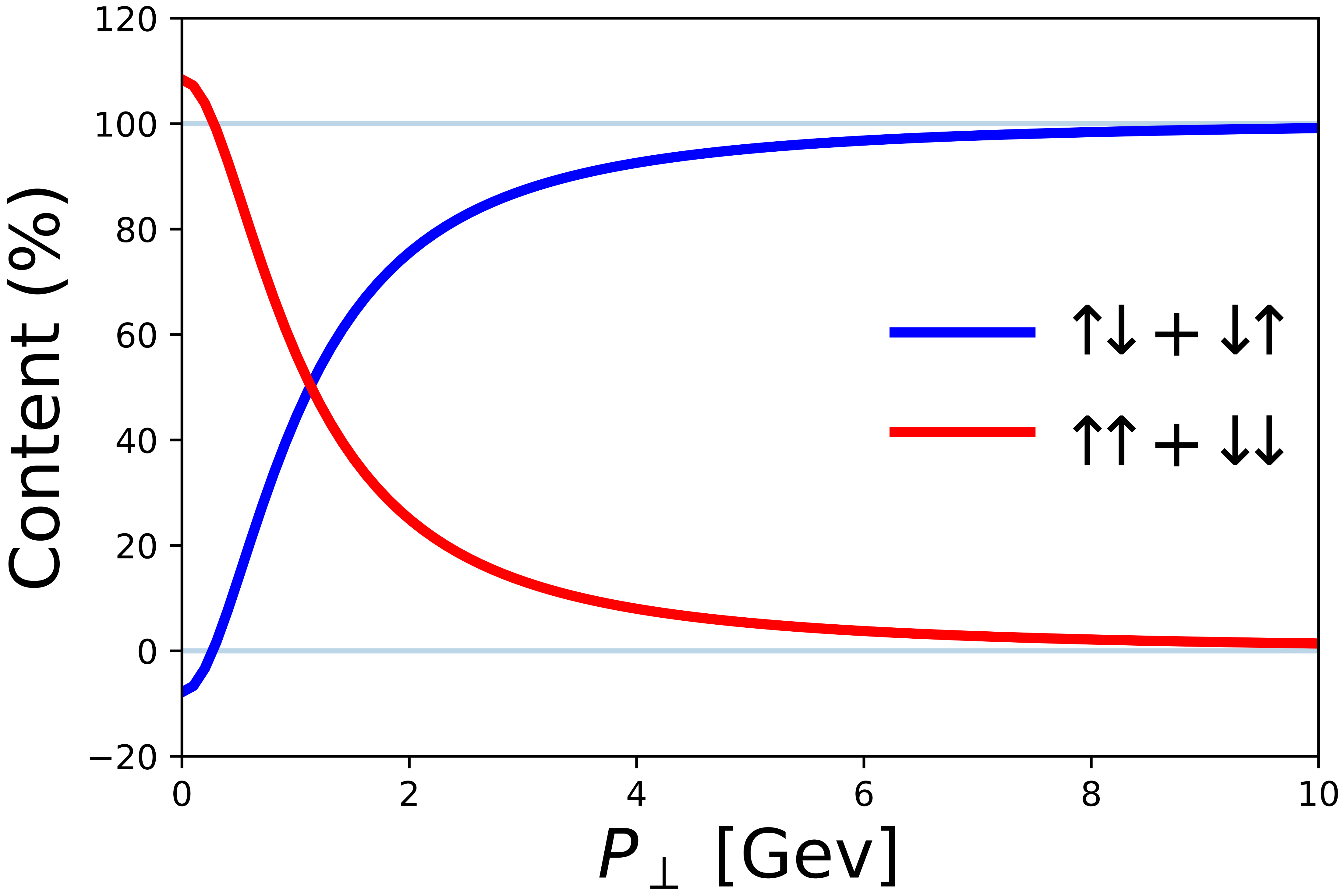
$$\mathcal{F} = \sqrt{6}\int_0^1 \mathrm{d}x \int \frac{\mathrm{d}^2\mathbf{k}_\perp}{16\pi^3}\,\frac{\phi(x,\mathbf{k}_\perp)}{\sqrt{m^2+\mathbf{k}_\perp^2}}\,\mathcal{O}(x,\mathbf{k}_\perp),$$

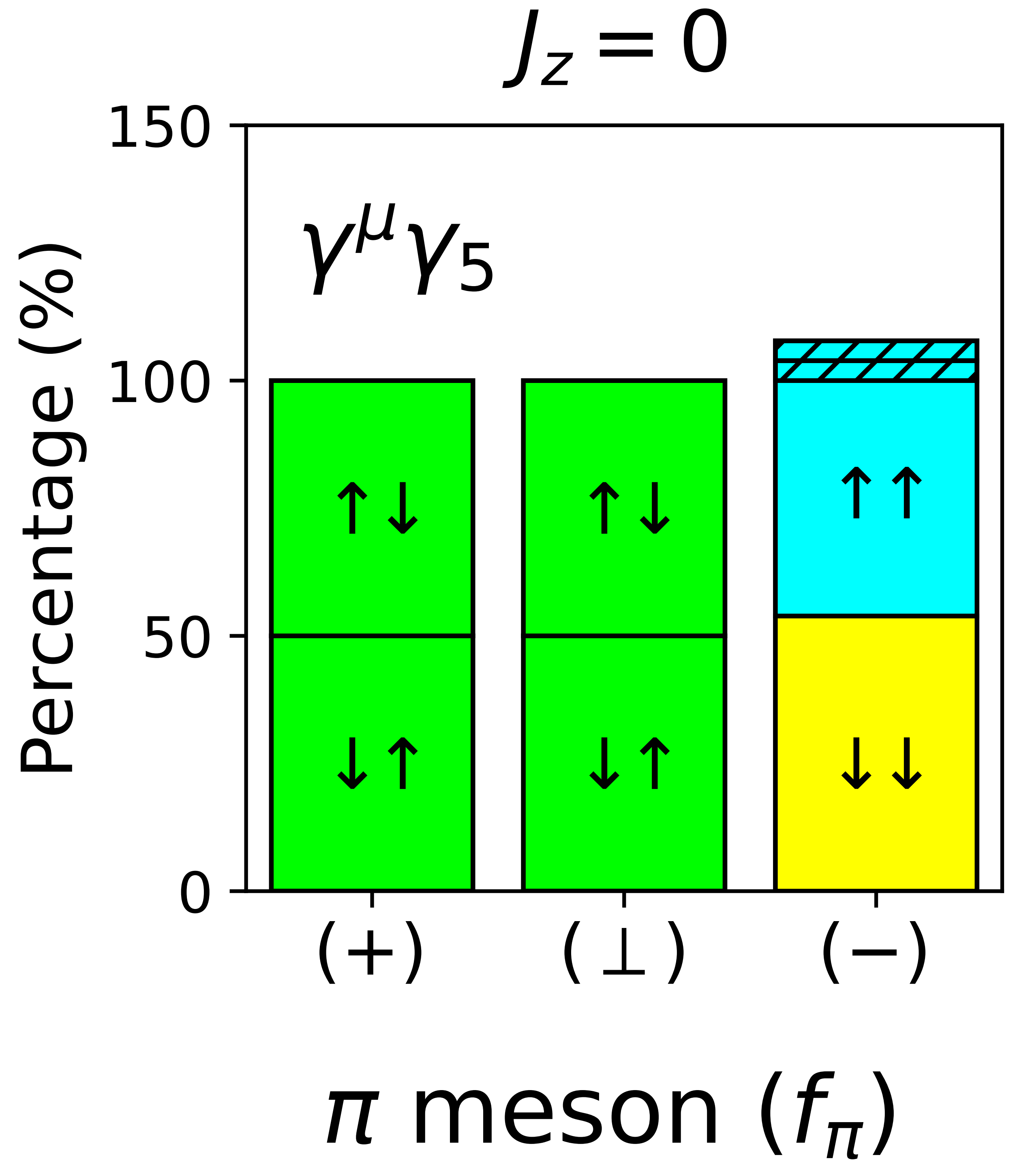
$\mathcal{F}$	$\mathcal{G}$	$\epsilon(J_z)$	$H_{\uparrow\uparrow}$	$H_{\uparrow\downarrow}$	$H_{\downarrow\uparrow}$	$H_{\downarrow\downarrow}$	$\mathcal{O}$
f_P	$\gamma^{(+,\perp)}\gamma_5$		0	m	m	0	$2m$
	$\gamma^-\gamma_5$		$\frac{2m\mathbf{k}_\perp^2}{x_1x_2(M_0^2+\mathbf{P}_\perp^2)}$	$m-\frac{2m\mathbf{k}_\perp^2}{x_1x_2(M_0^2+\mathbf{P}_\perp^2)}$	$m-\frac{2m\mathbf{k}_\perp^2}{x_1x_2(M_0^2+\mathbf{P}_\perp^2)}$	$\frac{2m\mathbf{k}_\perp^2}{x_1x_2(M_0^2+\mathbf{P}_\perp^2)}$	$2m$
f_V	$\gamma^{(+,\perp)}$	$\epsilon(0)$	0	$m+\frac{2\mathbf{k}_\perp^2}{\mathcal{D}_0}$	$m+\frac{2\mathbf{k}_\perp^2}{\mathcal{D}_0}$	0	$2m+\frac{4\mathbf{k}_\perp^2}{\mathcal{D}_0}$
	γ^-	$\epsilon(0)$	0	$m+\frac{2\mathbf{k}_\perp^2}{\mathcal{D}_0}$	$m+\frac{2\mathbf{k}_\perp^2}{\mathcal{D}_0}$	0	$2m+\frac{4\mathbf{k}_\perp^2}{\mathcal{D}_0}$
	$\gamma^{(\perp,-)}$	$\epsilon(+1)$	$M_0-\frac{(M_0+m)\mathbf{k}_\perp^2}{x_1x_2M_0\mathcal{D}_0}$	$\frac{x_1(x_1M_0+m)\mathbf{k}_\perp^2}{x_1x_2M_0\mathcal{D}_0}$	$\frac{x_2(x_2M_0+m)\mathbf{k}_\perp^2}{x_1x_2M_0\mathcal{D}_0}$	0	$M_0-\frac{2\mathbf{k}_\perp^2}{\mathcal{D}_0}$
f_V^T	$\sigma^{\perp+}$	$\epsilon(+1)$	$2m+\frac{2\mathbf{k}_\perp^2}{\mathcal{D}_0}$	0	0	0	$2m+\frac{2\mathbf{k}_\perp^2}{\mathcal{D}_0}$
	$\sigma^{\perp-}$	$\epsilon(+1)$	$2m-\frac{2m(m+M_0)\mathbf{k}_\perp^2}{x_1x_2M_0^2\mathcal{D}_0}$	$\frac{2m(m+x_1M_0)\mathbf{k}_\perp^2}{x_1x_2M_0^2\mathcal{D}_0}$	$\frac{2m(m+x_2M_0)\mathbf{k}_\perp^2}{x_1x_2M_0^2\mathcal{D}_0}$	$\frac{2\mathbf{k}_\perp^4}{x_1x_2M_0^2\mathcal{D}_0}$	$2m+\frac{2\mathbf{k}_\perp^2}{\mathcal{D}_0}$
	σ^{+-}	$\epsilon(0)$	$\frac{\mathbf{k}_\perp^2}{2x_1x_2\mathcal{D}_0}-\frac{2\mathbf{k}_\perp^2}{\mathcal{D}_0}$	$\frac{M_0}{2}-\frac{\mathbf{k}_\perp^2}{2x_1x_2\mathcal{D}_0}$	$\frac{M_0}{2}-\frac{\mathbf{k}_\perp^2}{2x_1x_2\mathcal{D}_0}$	$\frac{\mathbf{k}_\perp^2}{2x_1x_2\mathcal{D}_0}-\frac{2\mathbf{k}_\perp^2}{\mathcal{D}_0}$	$M_0-\frac{4\mathbf{k}_\perp^2}{\mathcal{D}_0}$

Transverse momentum dependence

$\mathcal{F}$	$\mathcal{G}$	$\epsilon(J_z)$	$H_{\uparrow\uparrow}$	$H_{\uparrow\downarrow}$	$H_{\downarrow\uparrow}$	$H_{\downarrow\downarrow}$	$\mathcal{O}$
f_P	$\gamma^{(+,\perp)}\gamma_5$		0	m	m	0	$2m$
	$\gamma^-\gamma_5$		$\frac{2m\mathbf{k}_\perp^2}{x_1x_2(M_0^2+\mathbf{P}_\perp^2)}$	$m-\frac{2m\mathbf{k}_\perp^2}{x_1x_2(M_0^2+\mathbf{P}_\perp^2)}$	$m-\frac{2m\mathbf{k}_\perp^2}{x_1x_2(M_0^2+\mathbf{P}_\perp^2)}$	$\frac{2m\mathbf{k}_\perp^2}{x_1x_2(M_0^2+\mathbf{P}_\perp^2)}$	$2m$

$$P^- = \frac{M^2+\mathbf{P}_\perp^2}{P^+}$$





Bad and good currents

$\mathcal{F}$	$\mathcal{G}$	$\epsilon(J_z)$	$H_{\uparrow\uparrow}$	$H_{\uparrow\downarrow}$	$H_{\downarrow\uparrow}$	$H_{\downarrow\downarrow}$	$\mathcal{O}$
f_{P}	$\gamma^{(+,\perp)}\gamma_5$		0	m	m	0	$2m$
	$\gamma^-\gamma_5$		$\frac{2m\mathbf{k}_\perp^2}{x_1x_2(M_0^2+\mathbf{P}_\perp^2)}$	$m-\frac{2m\mathbf{k}_\perp^2}{x_1x_2(M_0^2+\mathbf{P}_\perp^2)}$	$m-\frac{2m\mathbf{k}_\perp^2}{x_1x_2(M_0^2+\mathbf{P}_\perp^2)}$	$\frac{2m\mathbf{k}_\perp^2}{x_1x_2(M_0^2+\mathbf{P}_\perp^2)}$	$2m$

$(\mu = +)$ $$\mathcal{O}_P^+ = \mathcal{A}$$

$(\mu = -)$ $$\mathcal{O}_P^- = \frac{\mathcal{A}}{(M_0^2+\mathbf{P}_\perp^2)}\left[\frac{\mathbf{k}_\perp^2(\mathcal{A}'/\mathcal{A})+m_1m_2}{x(1-x)}+\mathbf{P}_\perp^2\right]$$

The difference

$$\mathcal{O}_P^- - \mathcal{O}_P^+ = \frac{(m_2-m_1)M_0}{(\mathbf{P}_\perp^2+M_0^2)}(-2k_z)$$

$$P^- - P^+ = -2P_z$$

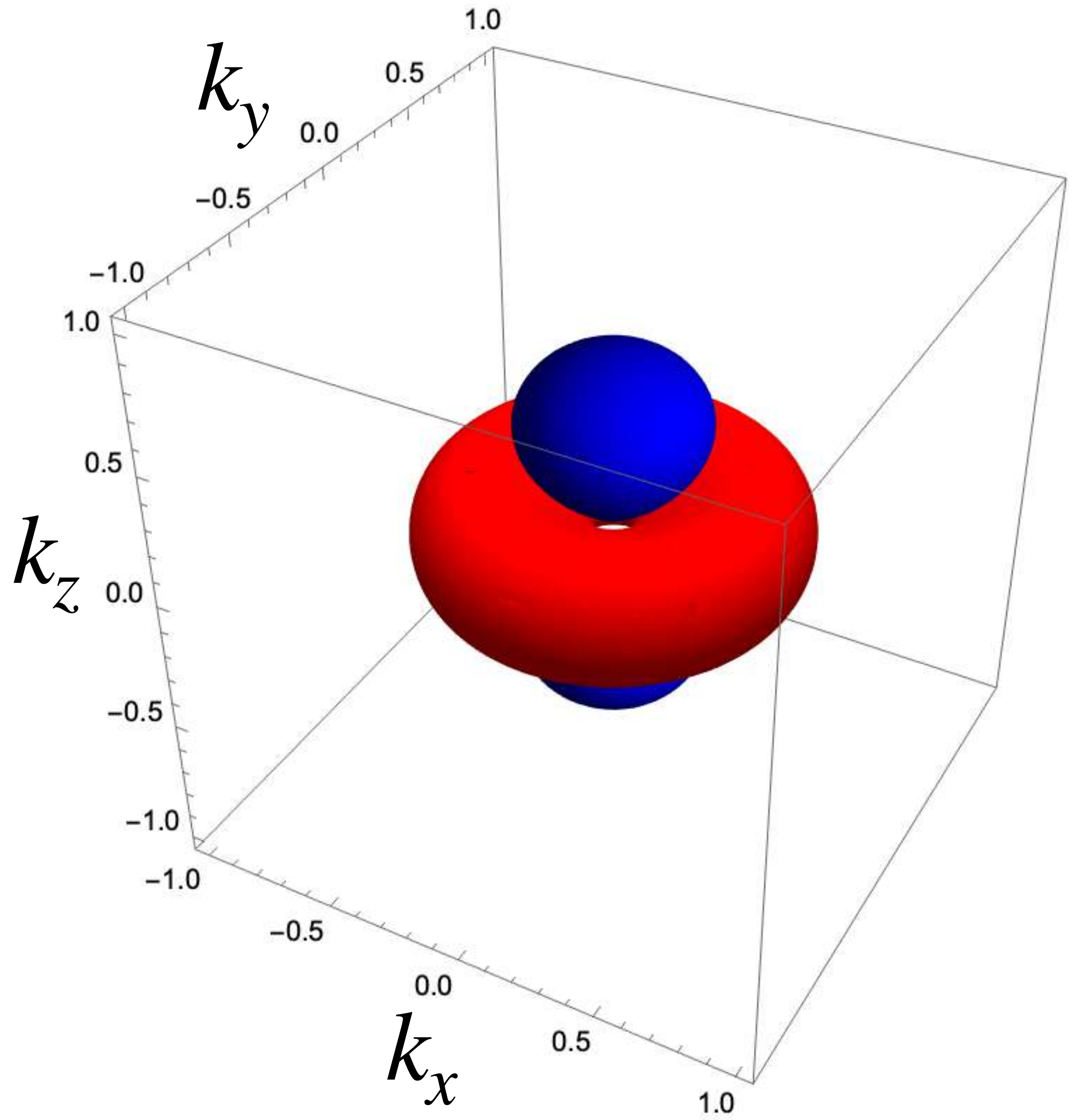
Different polarizations

$\mathcal{F}$	$\mathcal{G}$	$\epsilon(J_z)$	$H_{\uparrow\uparrow}$	$H_{\uparrow\downarrow}$	$H_{\downarrow\uparrow}$	$H_{\downarrow\downarrow}$	$\mathcal{O}$
	$\gamma^{(+,\perp)}$	$\epsilon(0)$	0	$m+\frac{2\mathbf{k}_\perp^2}{\mathcal{D}_0}$	$m+\frac{2\mathbf{k}_\perp^2}{\mathcal{D}_0}$	0	$2m+\frac{4\mathbf{k}_\perp^2}{\mathcal{D}_0}$
f_V	γ^-	$\epsilon(0)$	0	$m+\frac{2\mathbf{k}_\perp^2}{\mathcal{D}_0}$	$m+\frac{2\mathbf{k}_\perp^2}{\mathcal{D}_0}$	0	$2m+\frac{4\mathbf{k}_\perp^2}{\mathcal{D}_0}$
	$\gamma^{(\perp,-)}$	$\epsilon(+1)$	$M_0-\frac{(M_0+m)\mathbf{k}_\perp^2}{x_1x_2M_0\mathcal{D}_0}$	$\frac{x_1(x_1M_0+m)\mathbf{k}_\perp^2}{x_1x_2M_0\mathcal{D}_0}$	$\frac{x_2(x_2M_0+m)\mathbf{k}_\perp^2}{x_1x_2M_0\mathcal{D}_0}$	0	$M_0-\frac{2\mathbf{k}_\perp^2}{\mathcal{D}_0}$

The difference

$$\mathcal{O}_V(0)-\mathcal{O}_V(+1)=\frac{2}{D_0}(k_\perp^2-2k_z^2)$$

Degree of anisotropy: $\varepsilon=\frac{2k_z^2}{k_\perp^2}-1$



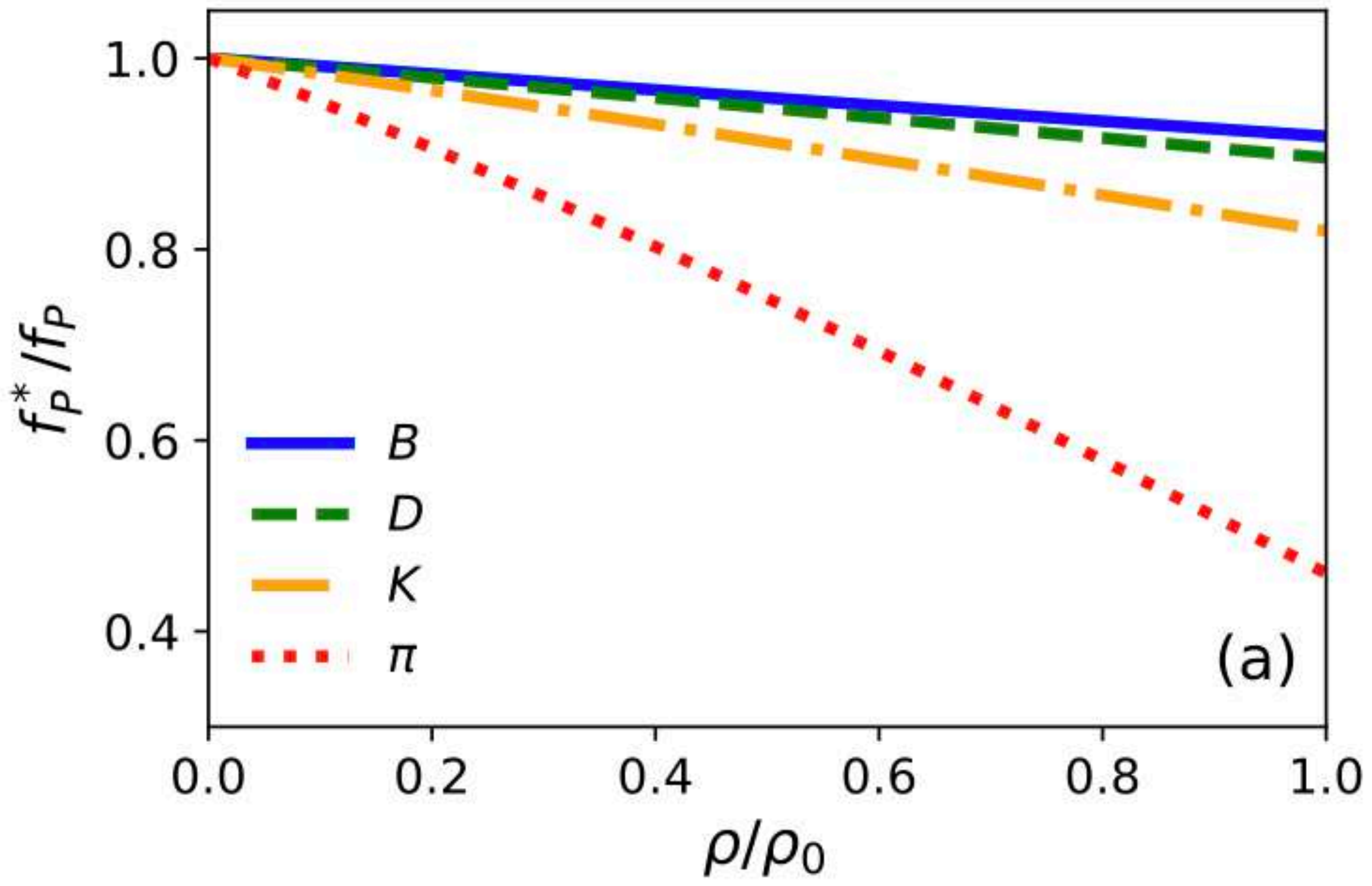
Key ingredient:

$$M\to M_0(x,k_\perp)$$

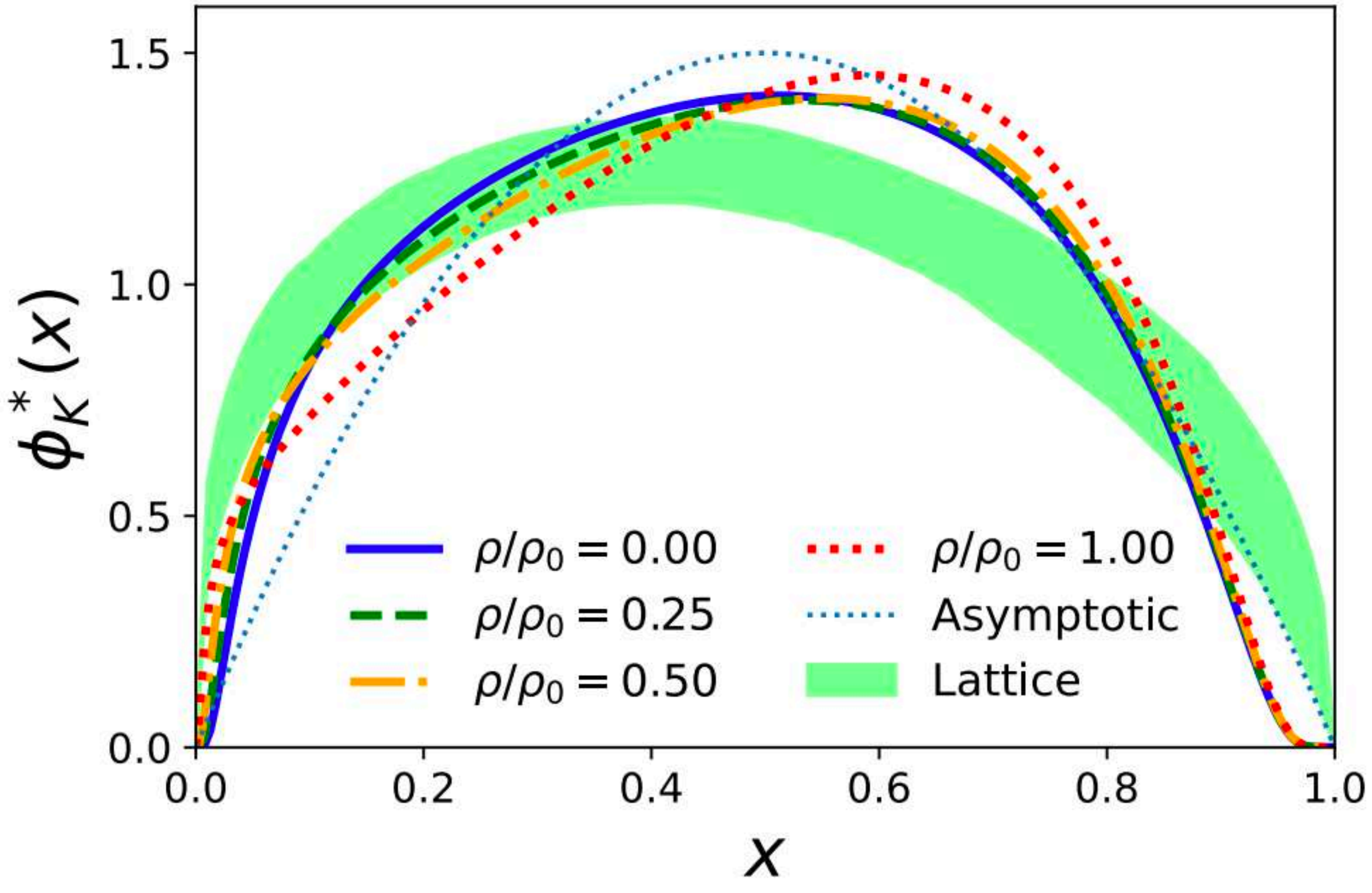
LFWFs in nuclear matter

- EMC effect: nuclear modification of nucleon structure function.
- Effective light-quark mass is smaller in nuclear medium: [Quark-meson coupling]

In-medium decay constant.



In-medium distribution amplitude.



PRD107, 114010 (2023).

Summary

Summary

- Nonperturbative effect is a central issue in hadron physics.
- Integral part of hadron physics:
  - Experiment, phenomenology, theory
- Constructing hadrons as relativistic bound states still posses a challenge.
  - Model can cover the low-energy and high-energy processes.
- Increase an accuracy of the model of hadrons and get a consistency.
- Our recent approaches include:
  - Gaussian expansion method
  - Light-front quark model

Let's discuss more!

Thank you!

In-medium LFWFs

Quark-Meson Coupling Model (Part 1)

- **Relativistic mean-field model**

$$\mathcal{L}_{\mathrm{QMC}} = \mathcal{L}_{\mathrm{nucleon}} + \mathcal{L}_{\mathrm{meson}} + \mathcal{L}_{\mathrm{int}},$$

$$\mathcal{L}_{\mathrm{int}} = \tilde{g}_\sigma^N(\hat{\sigma})\bar{\psi}\psi\hat{\sigma} - g_\omega^N\hat{\omega}^\mu\bar{\psi}\gamma_\mu\psi,$$

$$\mathcal{L} = \bar{\psi}[i\partial\!\!\!/ - m_N^*(\hat{\sigma}) - g_\omega^N\hat{\omega}^\mu\gamma_\mu]\psi + \mathcal{L}_{\mathrm{meson}},$$

$$m_N^*(\hat{\sigma}) = m_N - \tilde{g}_\sigma^N(\hat{\sigma})\hat{\sigma}.$$

$$\tilde{g}_\sigma^N(\sigma) = g_\sigma^N C_N(\sigma), \quad C_N(\sigma) = \frac{S_N(\sigma)}{S_N(\sigma=0)}.$$

=> The quark-meson coupling

$$g_\sigma^N = \tilde{g}_\sigma^N(\sigma=0) = 3g_\sigma^q S_N(\sigma=0),$$

$$g_\omega^N = 3g_\omega^q,$$

=> Vector and scalar meson fields

$$\omega = \frac{g_\omega^N\rho}{m_\omega^2}, \qquad \sigma = \frac{g_\sigma^N\rho_s}{m_\sigma^2}C_N(\sigma),$$

=> Vector and scalar density

$$\rho = \frac{4}{(2\pi)^3}\int d^3\mathbf{k}\Theta(k_F - k) = \frac{2k_F^3}{3\pi^2},$$

$$\rho_s = \frac{4}{(2\pi)^3}\int d^3\mathbf{k}\Theta(k_F - k)\frac{m_N^*(\sigma)}{\sqrt{m_N^{*2}(\sigma) + k^2}},$$

=> Total energy per nucleon

$$\frac{E_{\mathrm{tot}}}{A} = \frac{1}{\rho}\left[\frac{4}{(2\pi)^3}\int d^3\mathbf{k}\Theta(k_F - k)\sqrt{m_N^{*2}(\sigma) + k^2} + \frac{1}{2}g_\sigma^N C_N(\sigma)\sigma\rho_s + \frac{1}{2}g_\omega^N\omega\rho\right].$$

Quark-Meson Coupling Model (Part 2)

- **MIT bag model**

$$V_\sigma^q = g_\sigma^q \sigma, \qquad V_\omega^q = g_\omega^q \omega,$$

=> Static solution for the ground state quark

$$\psi(z) = \psi(r)\exp\{-i\varepsilon^* t/R^*\},$$

$$\psi(z) = \frac{N e^{-i\varepsilon^* t/R^*}}{\sqrt{4\pi}} \begin{pmatrix} j_0(x_q^* r/R^*) \\ i\beta_q^* j_1(x_q^* r/R^*)\boldsymbol{\sigma}\cdot\hat{r} \end{pmatrix} \chi_m,$$

$$\begin{pmatrix} \varepsilon_q^* \\ \varepsilon_{\bar{q}}^* \end{pmatrix} = \Omega_q^* \pm R^* V_\omega^q, \qquad \Omega_q^* = \sqrt{x_q^{*2} + (m_q^* R^*)^2},$$

$$j_0(x_q^*) = \beta_q^* j_1(x_q^*), \qquad \beta_q^* = \sqrt{\frac{\Omega_q^* - m_q^* R^*}{\Omega_q^* + m_q^* R^*}}.$$

=> Quark EoM in the presence of meson mean-fields

$$[i\partial\!\!\!/ - (m_q - V_\sigma^q) \mp \gamma^0 V_\omega^q] \begin{pmatrix} \psi_q(z) \\ \psi_{\bar{q}}(z) \end{pmatrix} = 0,$$

$$[i\partial\!\!\!/ - m_Q] \begin{pmatrix} \psi_Q(z) \\ \psi_{\bar{Q}}(z) \end{pmatrix} = 0,$$

=> Nucleon mass & radius

$$m_N^*(\sigma) = \frac{3\Omega_q^* - Z_N}{R^*} + \frac{4\pi R^{*3}}{3} B, \qquad \left.\frac{\mathrm{d}m_N^*(R^*)}{\mathrm{d}R^*}\right|_{R^*=R_N^*} = 0,$$

=> Scalar polarizability

$$S_N(\sigma) = \int_0^{R^*} \mathrm{d}^3\mathbf{r}\,\bar{\psi}(r)\psi(r), \quad = \frac{\Omega_q^*/2 + m_q^* R^*(\Omega_q^* - 1)}{\Omega_q^*(\Omega_q^* - 1) + m_q^* R^*/2}.$$

Meson structure & property in medium

- Modified quark properties

  => The light quark effective mass is modified

  by scalar potential $m_q^* = m_q - V_\sigma^q$

  => The light quark energy is modified

  by vector potential

  $E_q^* = E_q + V_\omega^q$ and $E_{\bar{q}}^* = E_{\bar{q}} - V_\omega^q$

- The total energy of meson,

$$P^{*0} = \begin{cases} E_M^*, & \text{for } (q\bar{q}), \\ E_M^* + V_\omega^q, & \text{for } (q\bar{Q}), \\ E_M^* - V_\omega^q, & \text{for } (Q\bar{q}), \end{cases}$$

  vector potential only appear for unequal quark mass.

- Momentum fraction x is also modified

  => For equal quark mass,

$$x \to \tilde{x}^* = \frac{p_q^{*+} + V_\omega^q}{P^{*+}} = x^* + \frac{V_\omega^q}{P^{*+}},$$

- The decay constant for equal quark mass only depends on scalar potential

$$f_M^* = 2\sqrt{6} \int_{-\frac{V_\omega^q}{P^{*+}}}^{1-\frac{V_\omega^q}{P^{*+}}} dx^* \int \frac{d^2\mathbf{k}_\perp}{2(2\pi)^3} \frac{\Phi(\tilde{x}^*, \mathbf{k}_\perp)}{\sqrt{\mathcal{A}(\tilde{x}^*)^2 + \mathbf{k}_\perp^2}} \mathcal{O}_M(\tilde{x}^*, \mathbf{k}_\perp).$$

$$f_M^* = 2\sqrt{6} \int_0^1 d\tilde{x}^* \int \frac{d^2\mathbf{k}_\perp}{2(2\pi)^3} \frac{\Phi(\tilde{x}^*, \mathbf{k}_\perp)}{\sqrt{\mathcal{A}(\tilde{x}^*)^2 + \mathbf{k}_\perp^2}} \mathcal{O}_M(\tilde{x}^*, \mathbf{k}_\perp).$$

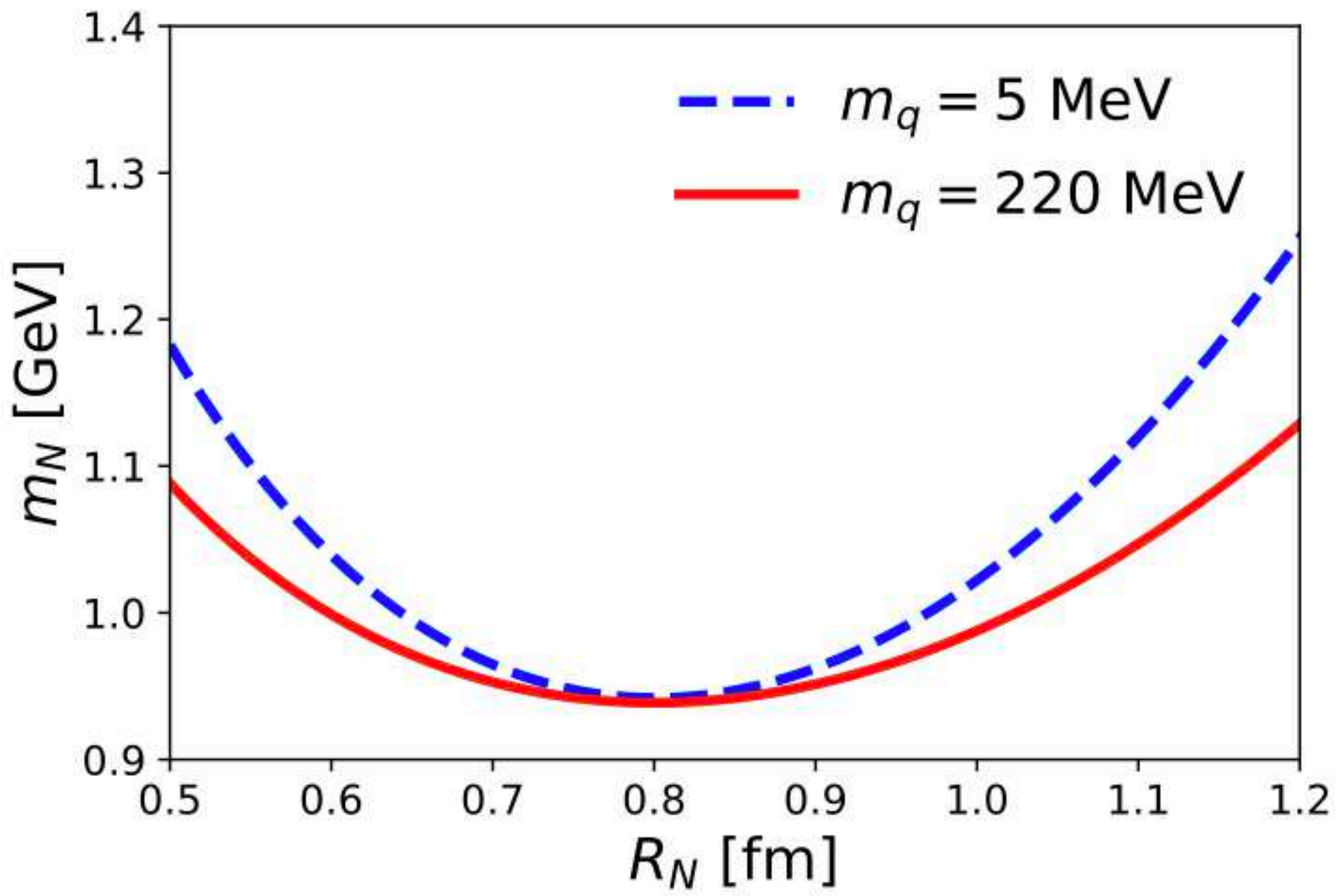
LFQM for meson properties in free space

m_q	m_s	m_c	m_b	$\beta_{q\bar{q}}$	$\beta_{q\bar{s}}$	$\beta_{q\bar{c}}$	$\beta_{q\bar{b}}$
0.22	0.45	1.8	5.2	0.3659	0.3886	0.4679	0.5266

	M_{expt} [MeV]	M_{theo} [MeV]	f_{expt} [MeV]	f_{theo} [MeV]
π	135	135	130	130
ρ	770	770	216	247
K	498	478	156	162
K^*	892	850	217	256
D	1865	1836	206	197
D^*	2007	1998	...	239
B	5279	5235	188	171
B^*	5325	5315	...	186

- Mesons in free space, we adopt the LFQM
  => Choi & Ji [PRD59, 074015 (1999)]
- In this model, the meson mass and decay constant in free space
  => reasonable agreement with data
- The quark mass and β parameters are fixed in free space.
  => The β parameters are assumed to be the same in medium.

MIT Bag parameter



m_q [MeV]	$B^{1/4}$ [MeV]	Z_N	x_q	$S_N(\sigma = 0)$
5	170	3.295	2.052	0.483
220	148	4.327	2.368	0.609

- The nucleon mass in Bag model

$$m_N^*(\sigma) = \frac{3\Omega_q^* - Z_N}{R^*} + \frac{4\pi R^{*3}}{3}B,$$

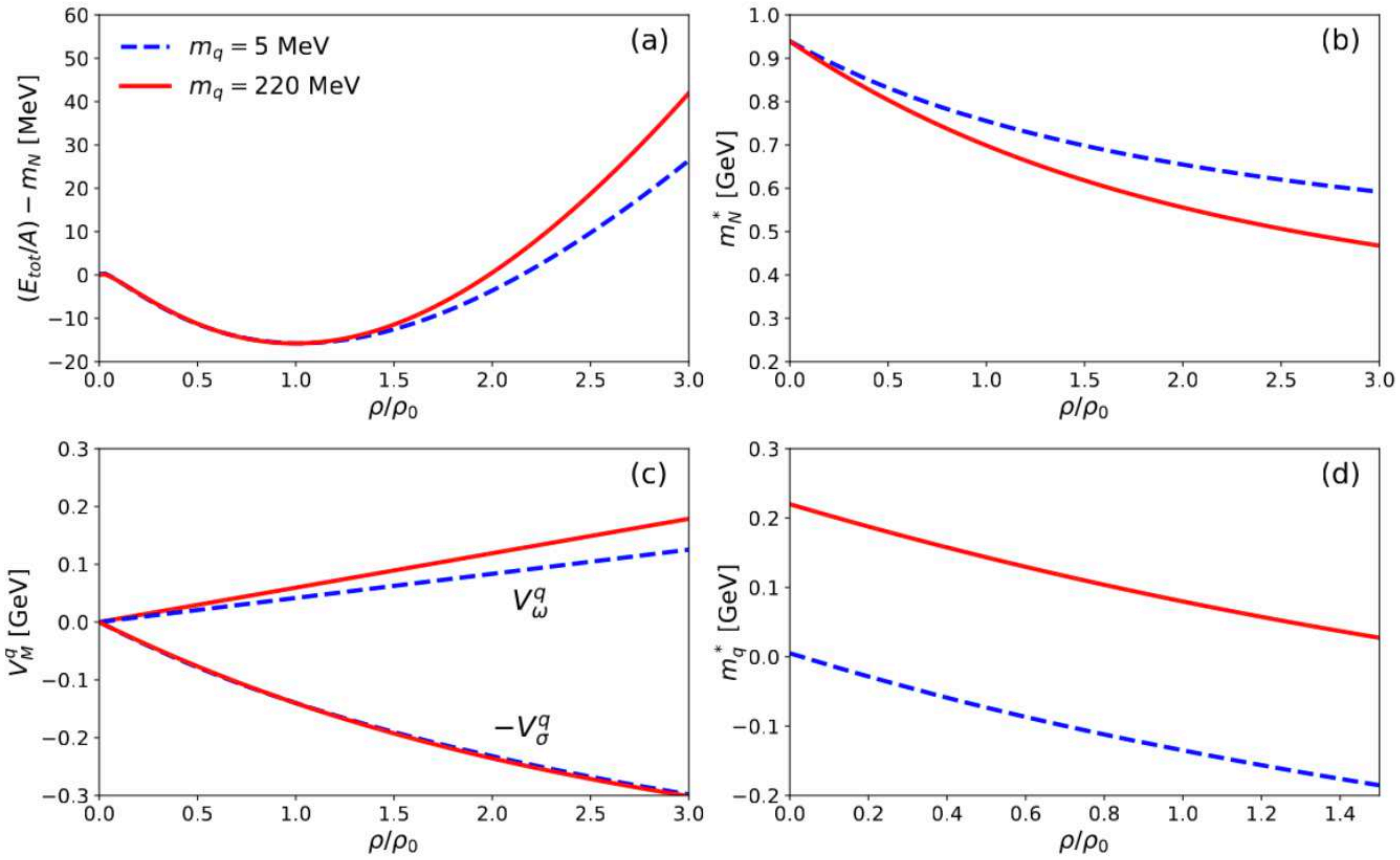
and minimized at

$$\left.\frac{\mathrm{d}m_N^*(R^*)}{\mathrm{d}R^*}\right|_{R^*=R_N^*} = 0,$$

- Bag parameters are fitted to
  => Nucleon mass: 939 MeV
  => Nucleon radius: 0.8 fm
- If we use the constituent quark, the bag parameters are not far different.

Nuclear Equation of State (EoS)

m_q [MeV]	$(g_\sigma^N)^2/4\pi$	$(g_\omega^N)^2/4\pi$	m_N^* [MeV]	K [MeV]
5	5.39	5.30	755	279
220	6.40	7.57	699	321

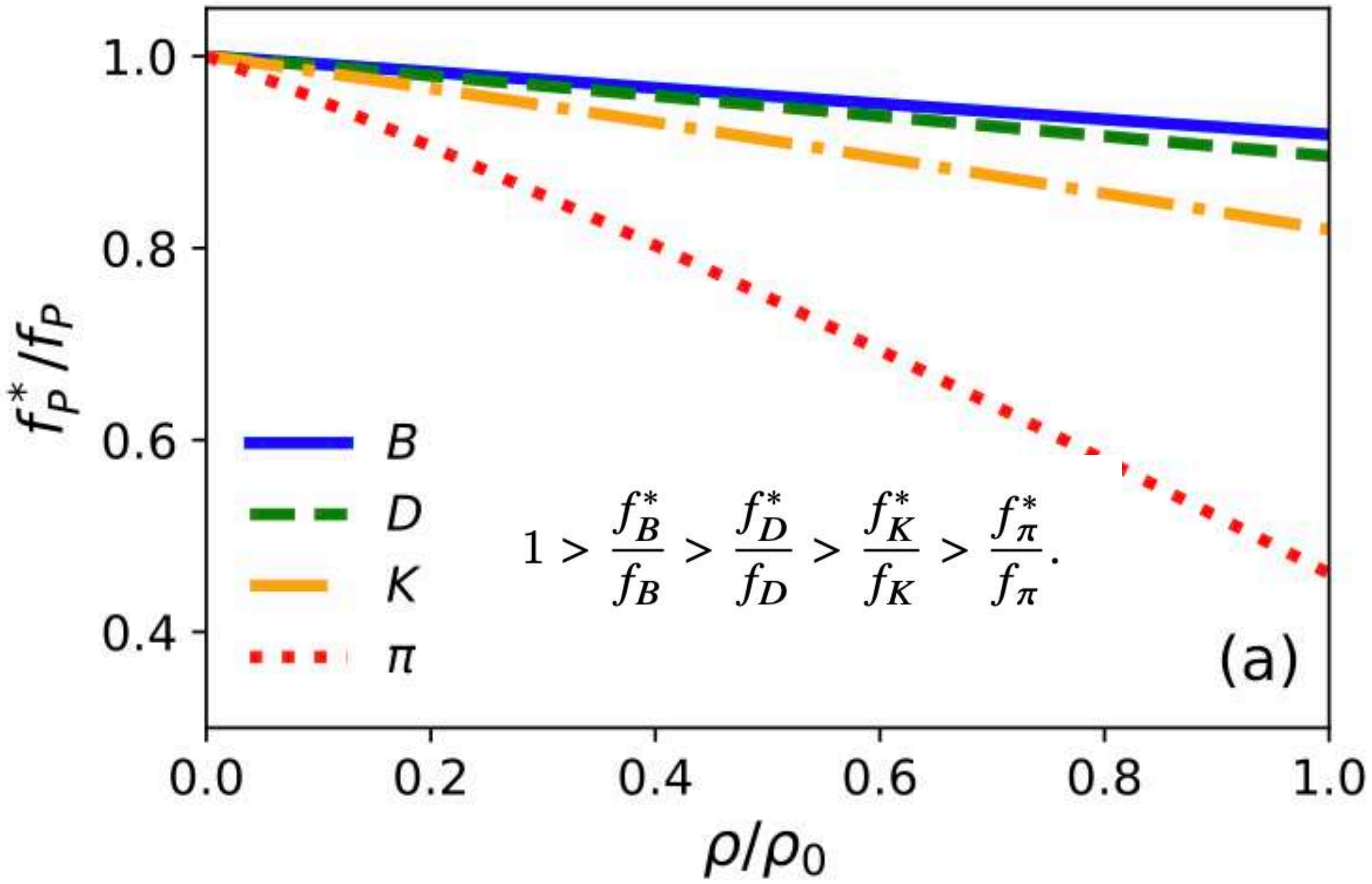


- The quark-meson couplings are fitted to
  => Binding energy -15.7 MeV
  => At saturation density

- Larger quark mass
  => Larger incompressibility K
  => Smaller effective nuclear mass
  => Similar scalar potential
  => Stronger vector potential

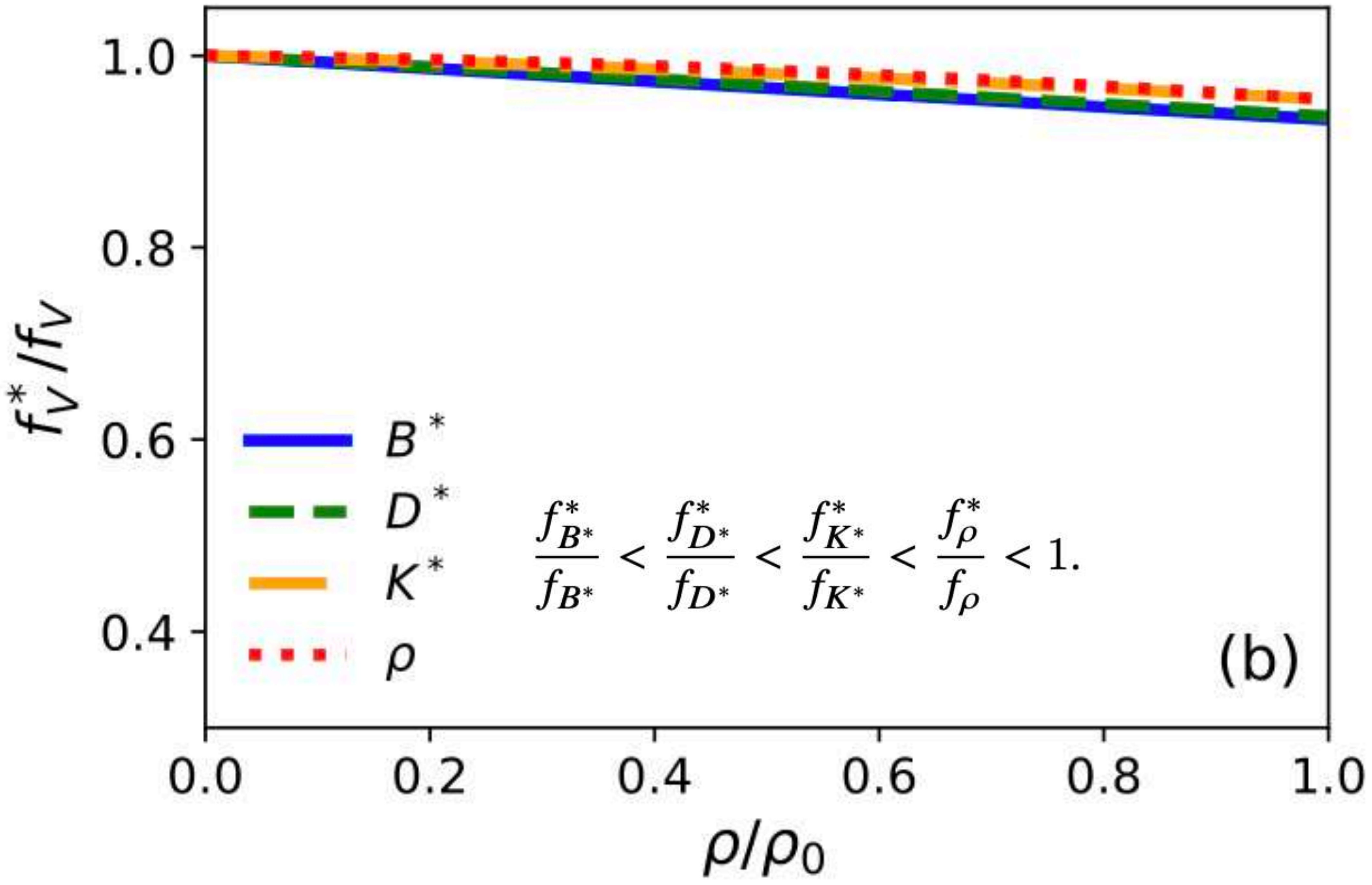
Ratio of in-medium decay constants

Pseudoscalar meson



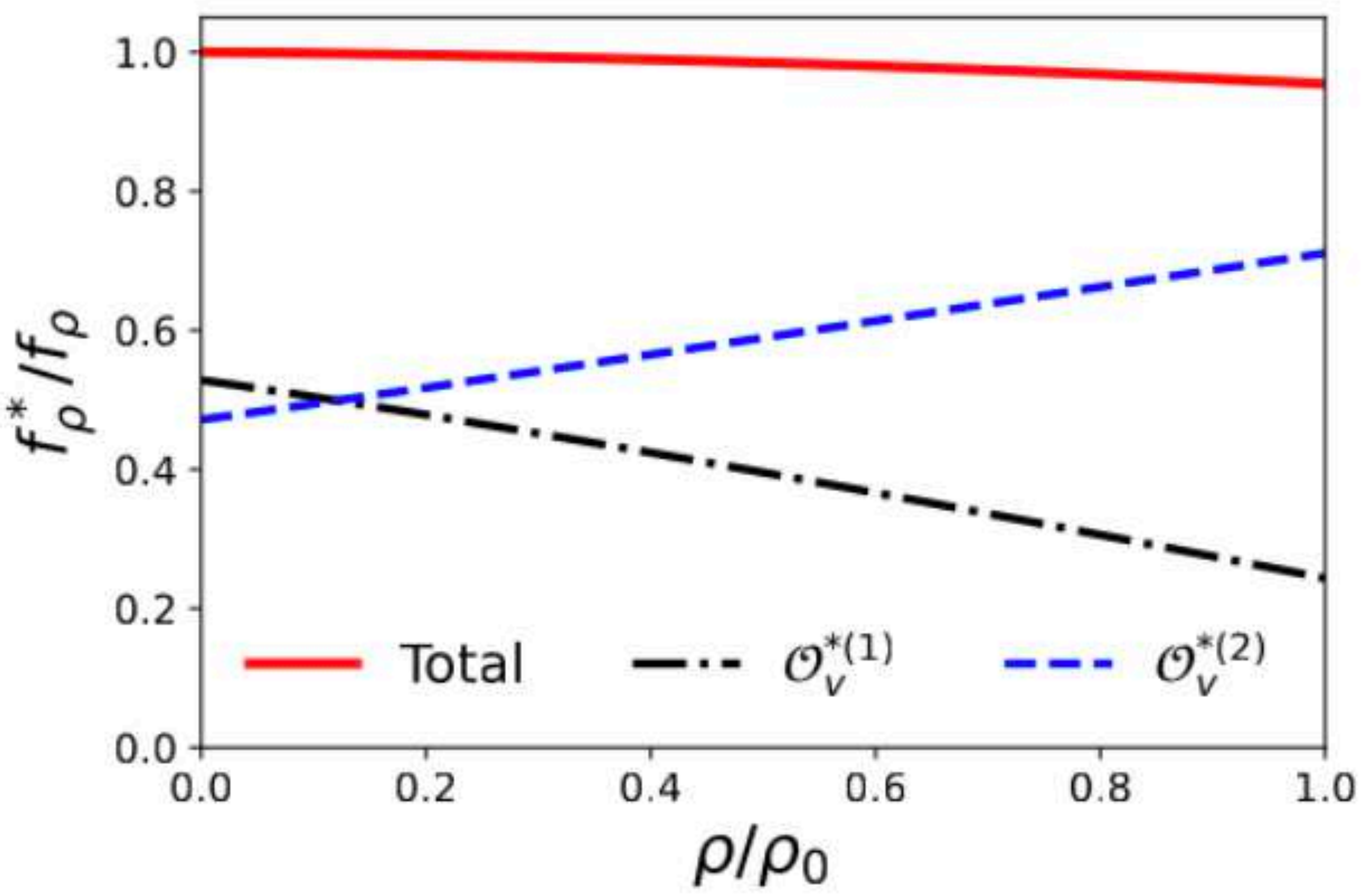
- Pion decay constant is reduced significantly
  => Our result is consistent with previous BSA.
  => rather smaller compared to pionic atom experiment.

Vector meson



- Vector meson decay constant are nearly unmodified.
- The rho meson decay constant is least reduced in medium.
  => Opposite ratio hierarchy

In-medium f_V^*/f_V decrease more slowly



- **Decay constant formula in LFQM**

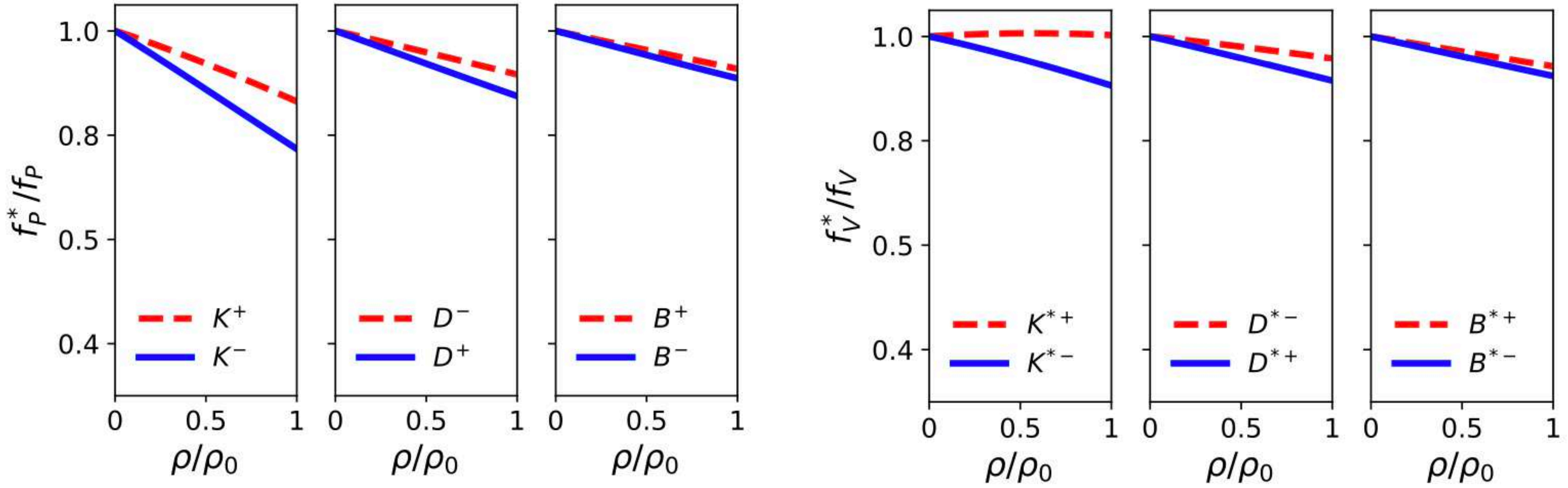
$$\mathcal{F} = \sqrt{6}\int_0^1 \mathrm{d}x \int \frac{\mathrm{d}^2\mathbf{k}_\perp}{16\pi^3} \frac{\phi(x,\mathbf{k}_\perp)}{\sqrt{m^2+\mathbf{k}_\perp^2}} \mathcal{O}(x,\mathbf{k}_\perp),$$

$$O_{\mathrm{P}}^* = m_q^*,$$

$$O_{\mathrm{V}}^* = m_q^* + \frac{2\mathbf{k}_\perp^2}{M_0^* + 2m_q^*}.$$

- The second term increases in medium
  => competing with the first term.
- Without the second term, the behavior will be the same as those of pseudoscalar meson.

Effect of nuclear vector potential

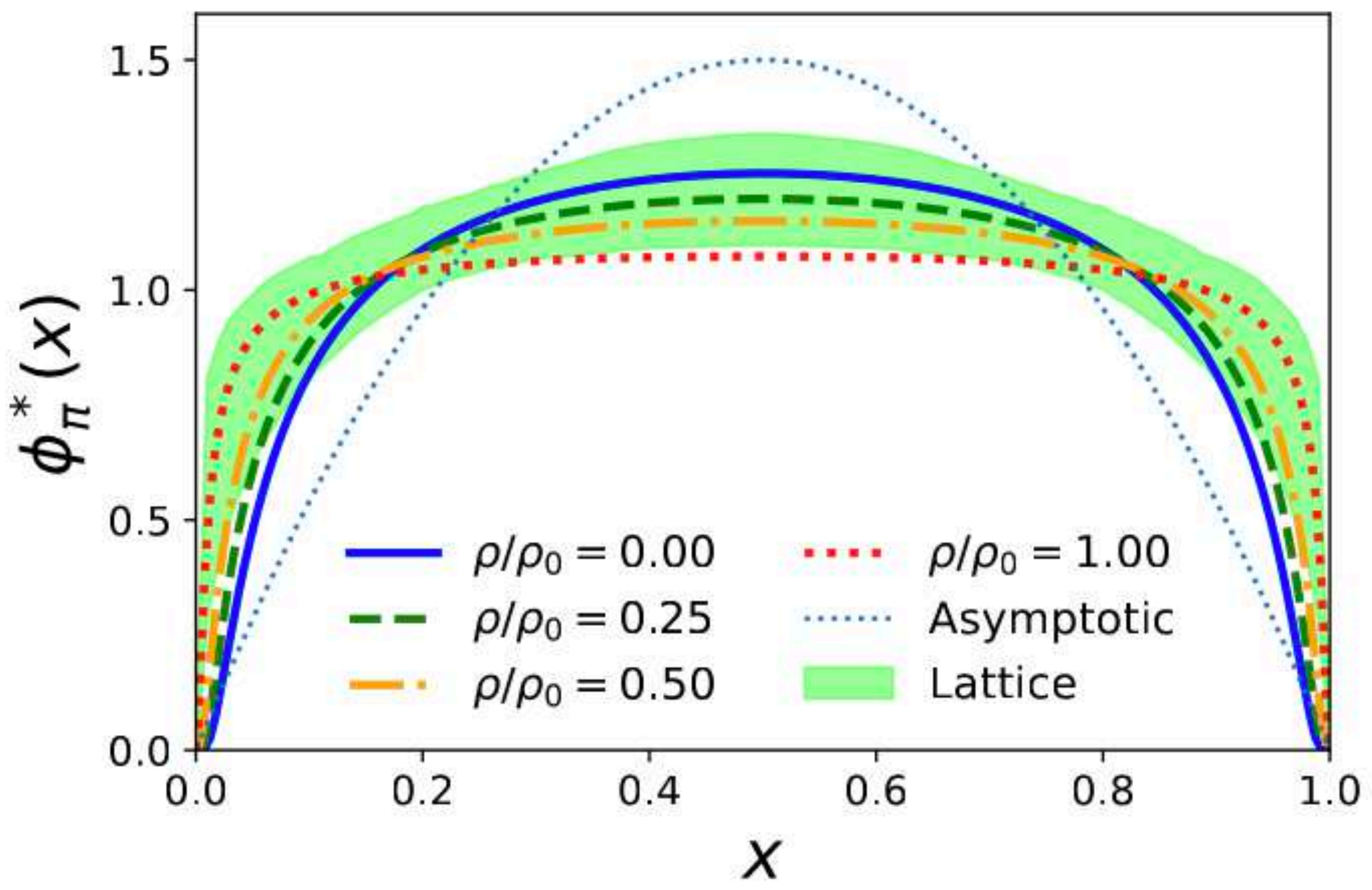


$$f_M^* = 2\sqrt{6}\int_0^1 d\tilde{x}^* \int \frac{d^2\mathbf{k}_\perp}{2(2\pi)^3}\left(1 \pm \frac{V_\omega^q}{P^{*+}}\right) \times \frac{\Phi(\tilde{x}^*, \mathbf{k}_\perp)}{\sqrt{\mathcal{A}(\tilde{x}^*)^2 + \mathbf{k}_\perp^2}} \mathcal{O}_M(\tilde{x}^*, \mathbf{k}_\perp).$$

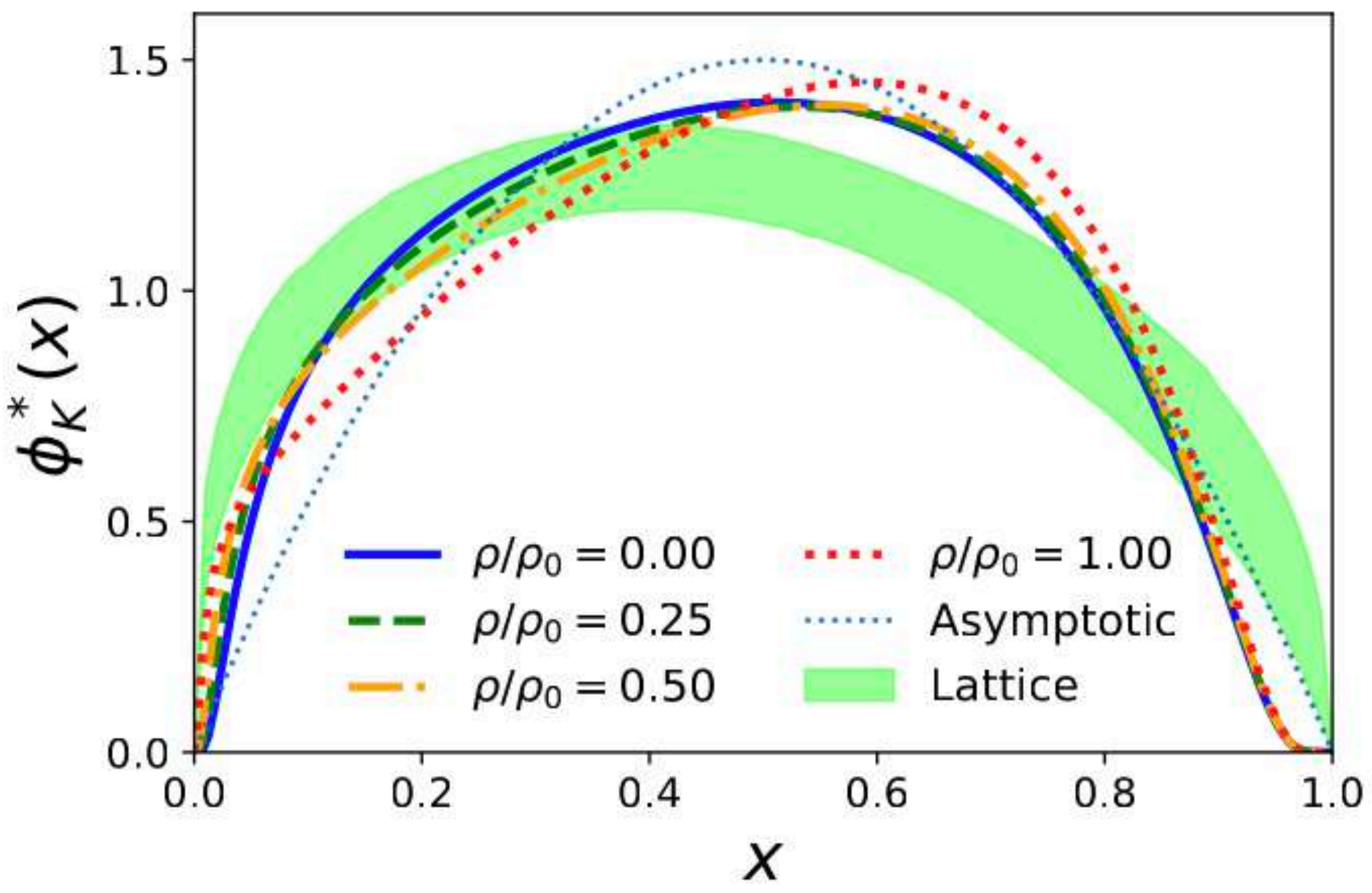
- The V_ω^q effect only appear for mesons with different quark content.
- The difference due to V_ω^q effect is getting smaller for heavier quark.

In medium DAs of π and K mesons

Lattice data, Phys. Rev. Lett. 129, 132001 (2022).



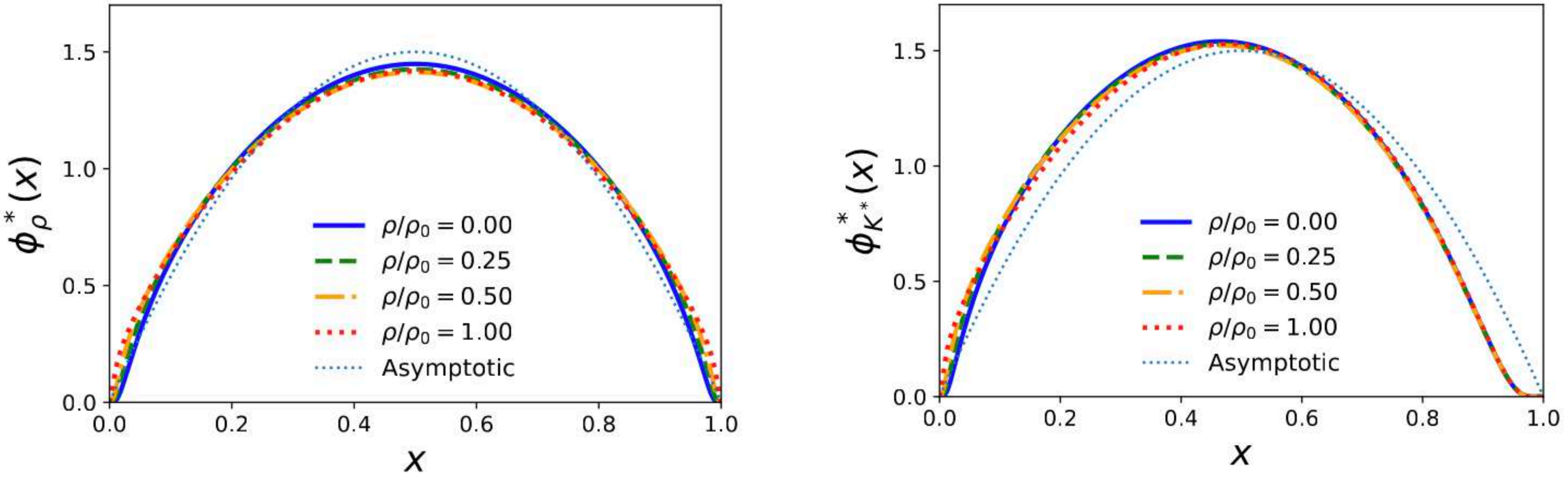
Light quark (u or d) carries momentum fraction x.



- In free space, the pion DA is consistent with Lattice data.
- The pion DA becomes flatten as the nuclear density increases (quark mass decreases).

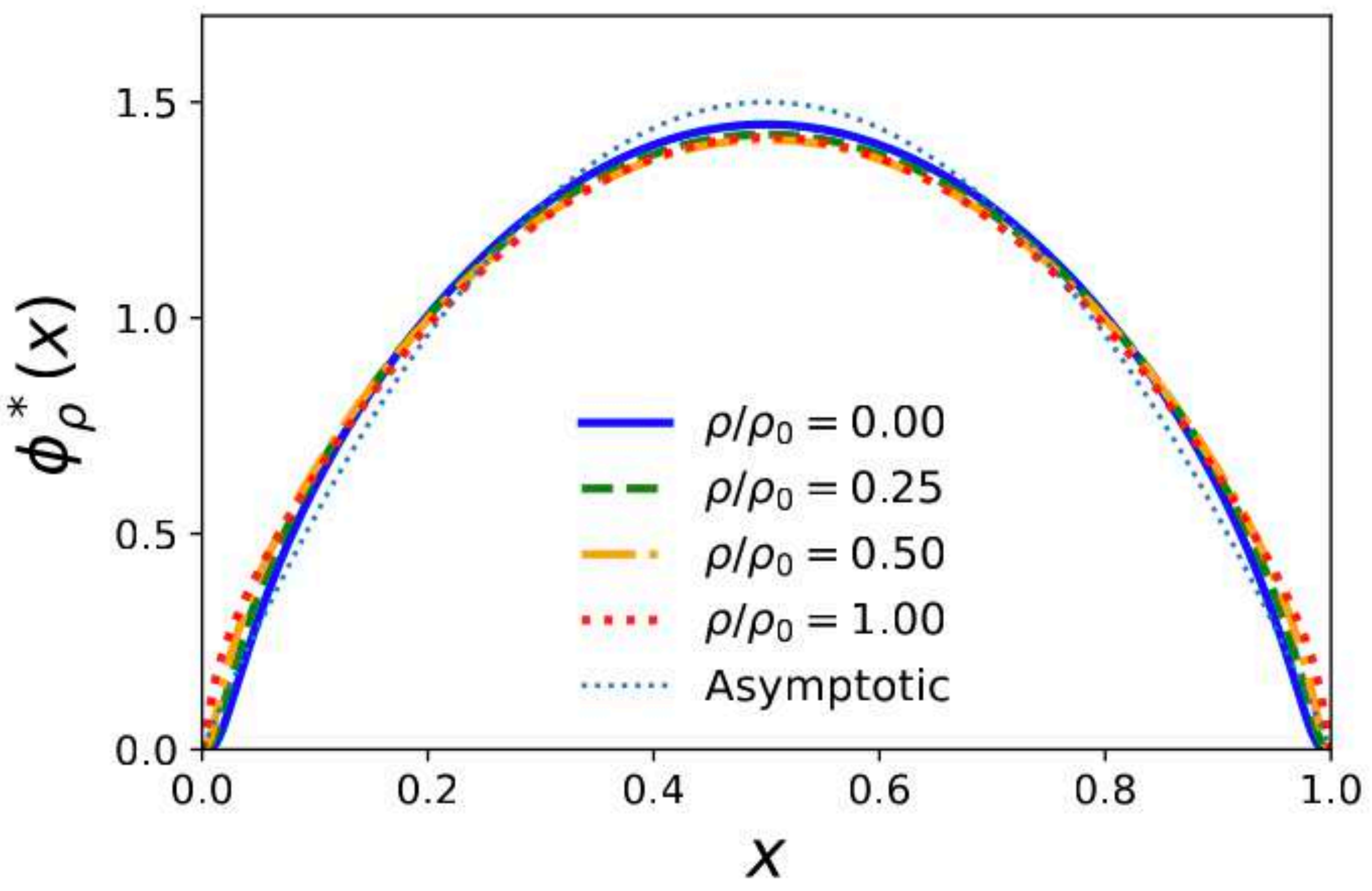
- The Kaon DA (x > 0.5) disagrees with the lattice data —> Possible cause: using a simple Gaussian WF or SU(3) flavor symmetry breaking.
- The Kaon DA decreases much faster near $x = m_1/(m_1 + m_2)$

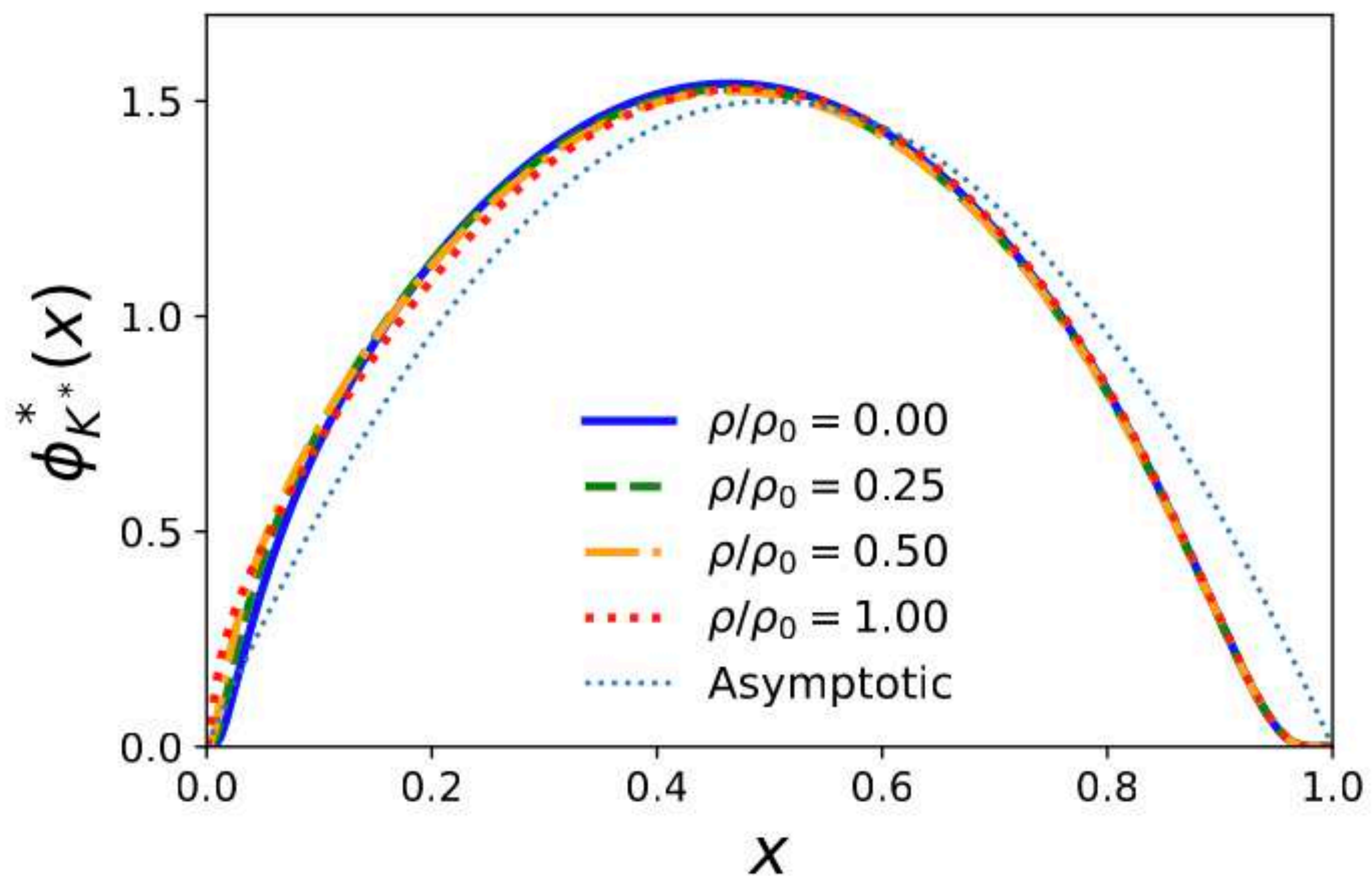
In-medium DAs of ρ and K^* mesons



- While the ρ DA is similar to Asymptotic result, the K^* DA is slightly shifted to the smaller x.
- The ρ and K^* DAs are moderately modified in medium.
- Moderate modification of vector meson DAs
  ==> the small reduction of decay constants.

In-medium DAs of ρ and K^* mesons

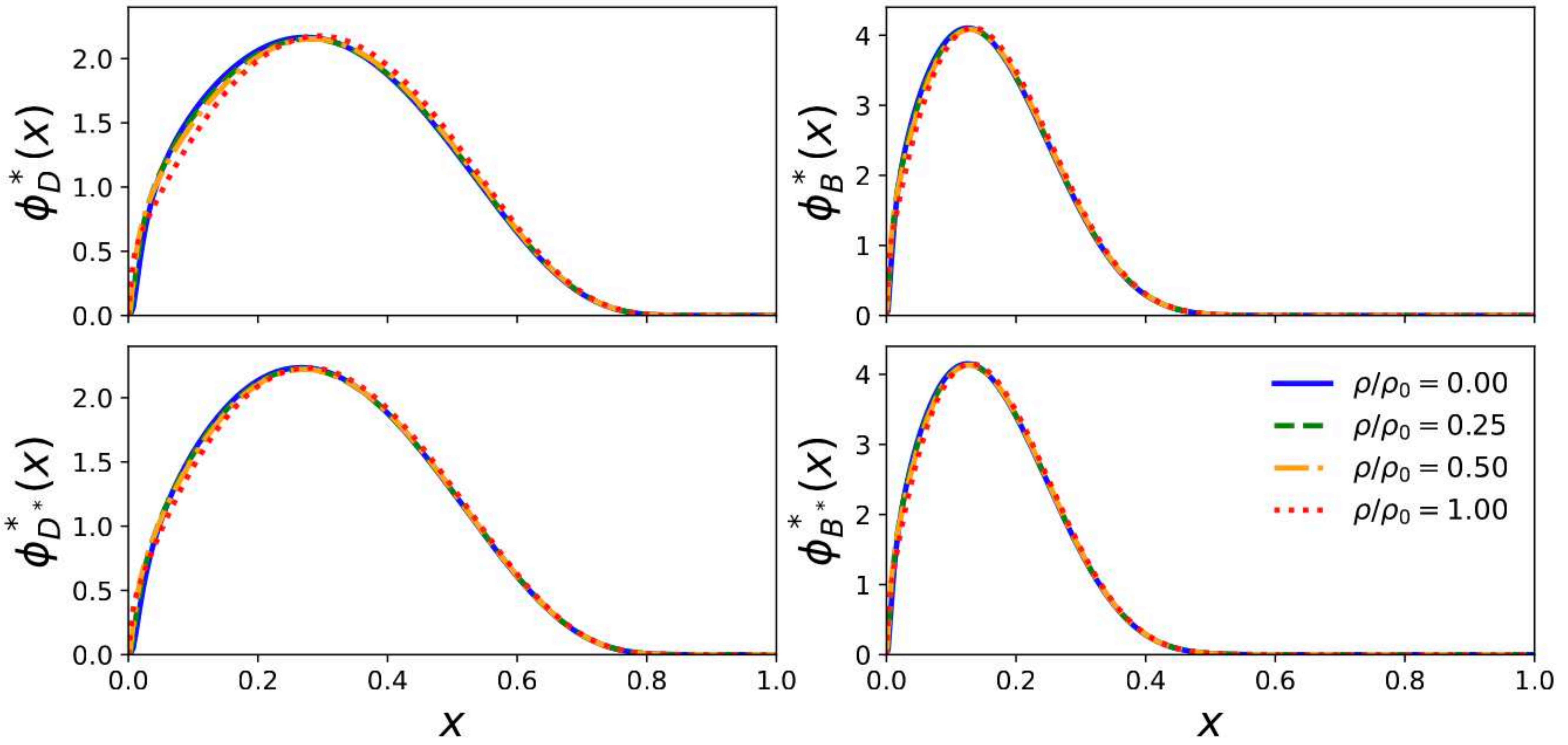




- While the ρ DA is similar to Asymptotic result, the K^* DA is slightly shifted to the smaller x.
- The ρ and K^* DAs are moderately modified in medium.

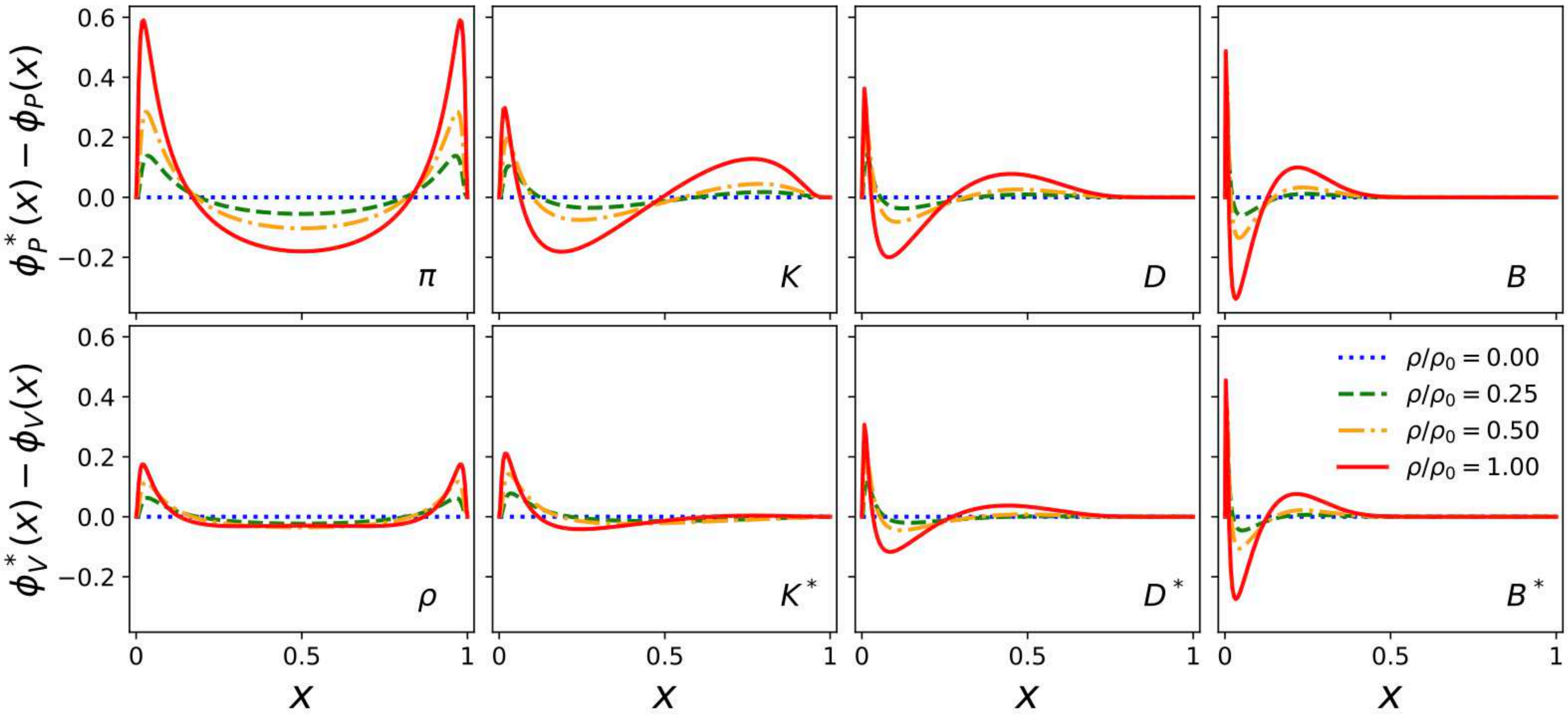
- Moderate modification of vector meson DAs
  ==> the small reduction of decay constants.

In-medium DAs of heavy-light mesons



- The DAs of heavy-light mesons are nearly unmodified in medium.

Difference of DAs in medium & in free space



- Here, the effect of in-medium modifications are more evident for each mesons.

  —> Maximum reduction and enchantments & Smearing of the peak near $x = 1$.