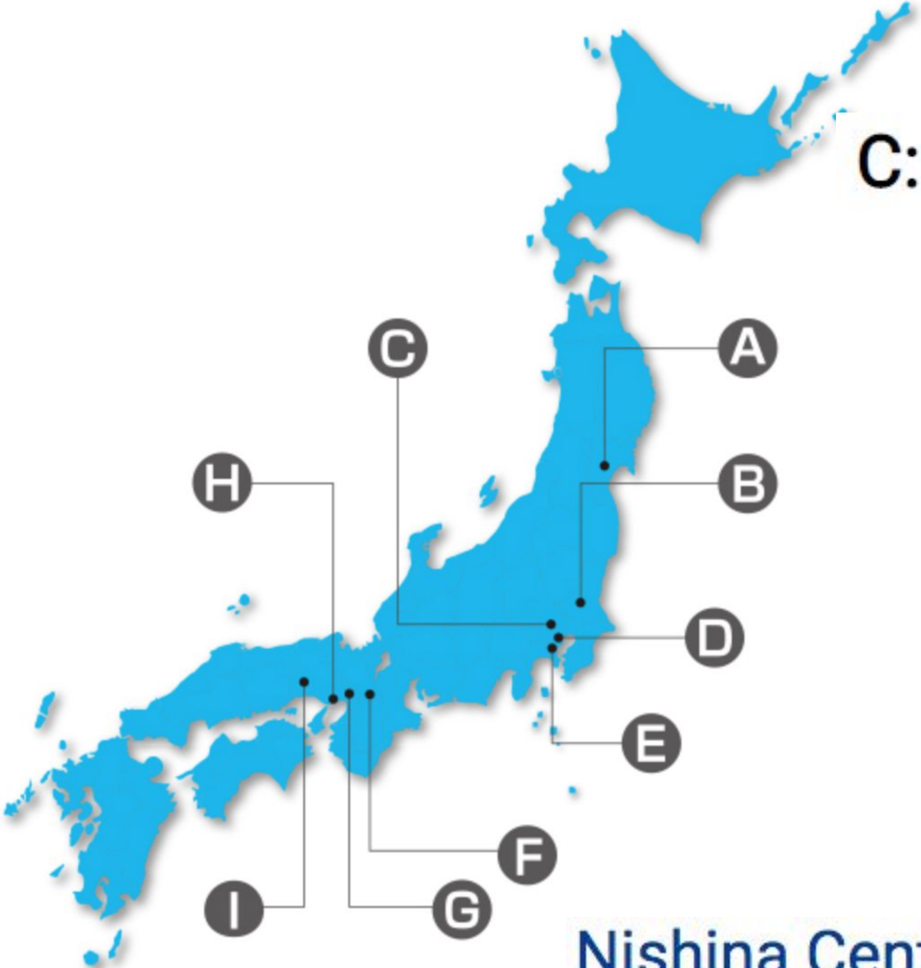
RIKEN Seminar
“Link between QCD and the Light-Front Quark Model”

Chueng-Ryong Ji

North Carolina State University

Group Meeting, September 20, 2024



C: Wako Branch / Headquarters

Nishina Center for Accelerator-Based Science

About RIKEN

Japanese Page

RIKEN, a National Research and Development Agency, is Japan's largest comprehensive research institution renowned for high-quality research in a diverse range of scientific disciplines. Founded in 1917, initially as a private research foundation, RIKEN has grown rapidly in size and scope, today encompassing a network of world-class research centers and institutes across Japan.

RIKEN Nishina Center for Accelerator-Based Science

Few-body Systems in Physics Laboratory

Director: Emiko Hiyama (D.Sc.)

Japanese Page

Research Summary

In our laboratory, we are applying accurate few-body problem calculational method to various fields such as hypernuclear physics, unstable nuclear physics and hadron physics. As a result, we are getting new understanding by solving three- and four-body problem accurately. Especially, we are researching hypernuclear physics. The hypernucleus is composed of a hyperon, neutrons and protons. The research purpose in our laboratory is to understand interaction between hyperon and nucleon in unified way by studying the structure of the hypernuclei from the view points of three- and four-body problems. Recently, we have succeeded in developing our method up to five-body problem.

Core members

Pascal Naidon	Senior Research Scientist
Makoto Oka	Senior Visiting Scientist
Nodoka Yamanaka	Contract Researcher
Lucas Happ	Special Postdoctoral Researcher
Ahmad Jafar Arifi	Special Postdoctoral Researcher
Shuhei Ohno	Junior Research Associate
Jinniu Hu	Visiting Scientist
Ying Zhang	Visiting Scientist

From: Pascal Naidon <pascal@riken.jp>
Subject: [fbsp-all:946] Today's seminar by Chueng-Ryong Ji - Link between QCD and the Light-Front Quark Model
Date: September 13, 2024 at 9:00:00 AM GMT+9
To: "fbsp-all@ribf.riken.jp" <fbsp-all@ribf.riken.jp>

Dear all,

Here is the link for today's seminar at 10:00 Japan time
https://riken-jp.zoom.us/j/95280420813?pwd=AU3cfad3bfP6kbVPPx7jaoboGScHOc.1
ID: 952 8042 0813
Code: aWHiRbuZ5m

____

Date and time: Friday 13 September 2024 10:00 Japan time
Speaker: **Chueng-Ryong Ji** (North Carolina State University)
Title: **Link between QCD and the Light-Front Quark Model**

Abstract:

I will present a mass gap solution of the 1+1D QCD in the large Nc limit known as the ‘tHooft model to discuss a link between QCD and the Light-Front Quark Model (LFQM). I will illuminate the interpolation between the instant form dynamics and the light-front dynamics and discuss its utility in the computation of the parton distribution function (PDF). I will then illustrate the Bakamjian-Thomas construction of the LFQM exemplifying the recent resolution of the light-front zero-mode issue raised about a decade ago regarding the pion transverse momentum distributions (TMDs) beyond the leading twist.


Sign in
View
Ahmad Jafar Arifi
Ahmad Jafar Arifi
crji
Pascal Naidon
Lucas Happ
Lucas Happ
nodoka yamanaka
nodoka yamanaka
Kan Sakamoto
Kan Sakamoto
C
Connor Donovan
Shuhei Ohno (YCU, RIKEN)
Shuhei Ohno (YCU, RIKEN)
Chueng Ryong Ji's seminar
Today
Messages addressed to "Meeting Group Chat" will also appear in the meeting group chat in Team Chat
MN
Makiko Nio 9:30 PM
Sorry. I leave now. I am attending another workshop.
1
MO
Makoto Oka 10:41 PM
I have to leave now. Thank you again for the nice talk.
Who can see your messages?
To: Meeting group chat
Message Chueng Ryong Ji's seminar
Audio
Video
8
Participants
Chat
React
Share
Apps
Whiteboards
More
Leave

Link between QCD and the Light-Front Quark Model

Chueng-Ryong Ji
North Carolina State University

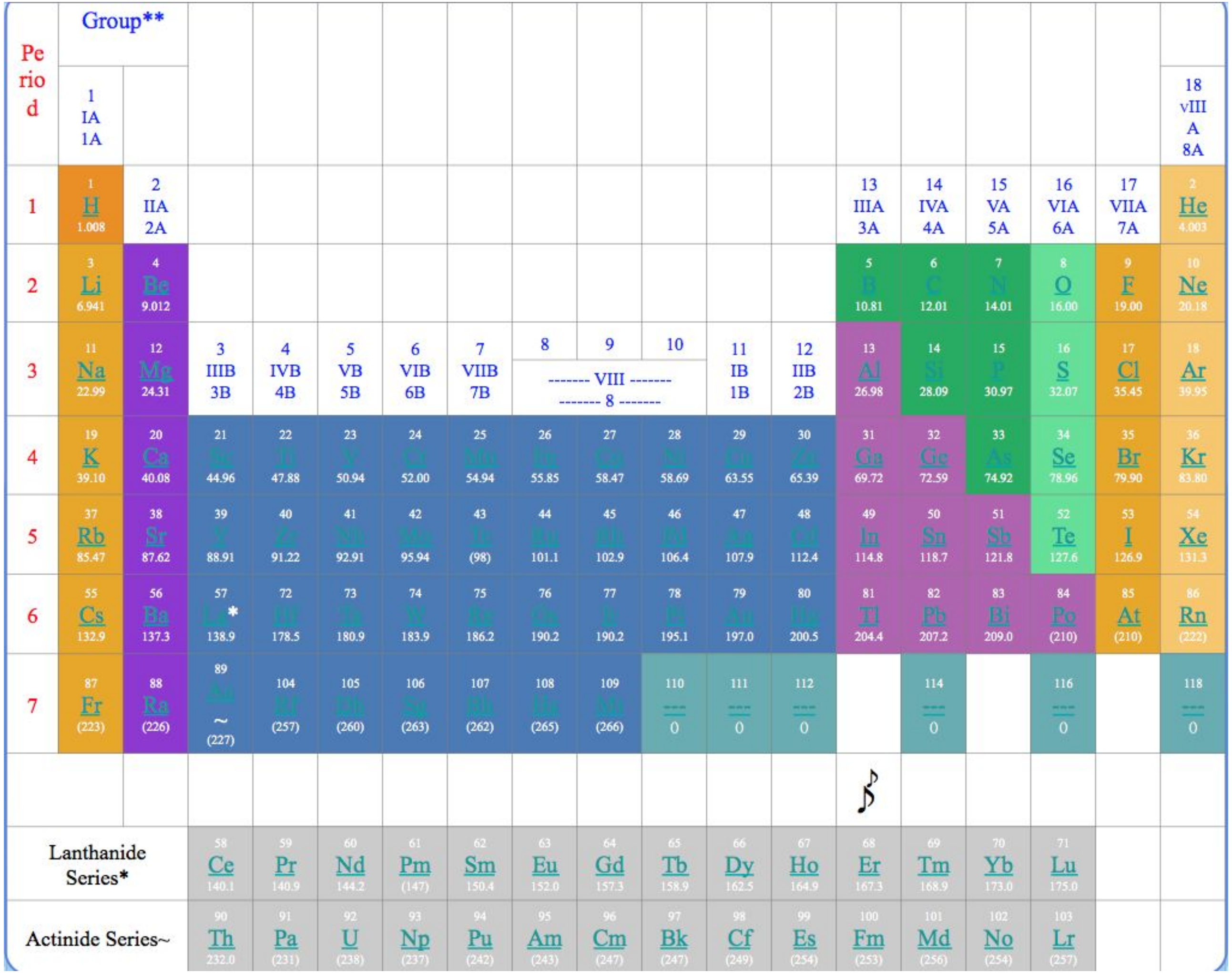
THEORY SEMINAR

RIKEN

September 12, 2024

Outline

- Motivation
- ‘t Hooft model as a toy QCD
- Quark mass gap solution
- Quark-Antiquark bound state equation
- Link to the Light-Front Quark Model
- BT Construction for Self-Consistent LFQM
- Example of LF zero-mode issues:
  Pion TMDs beyond leading twist
- Resolution in LFQM
- Conclusions and Outlook

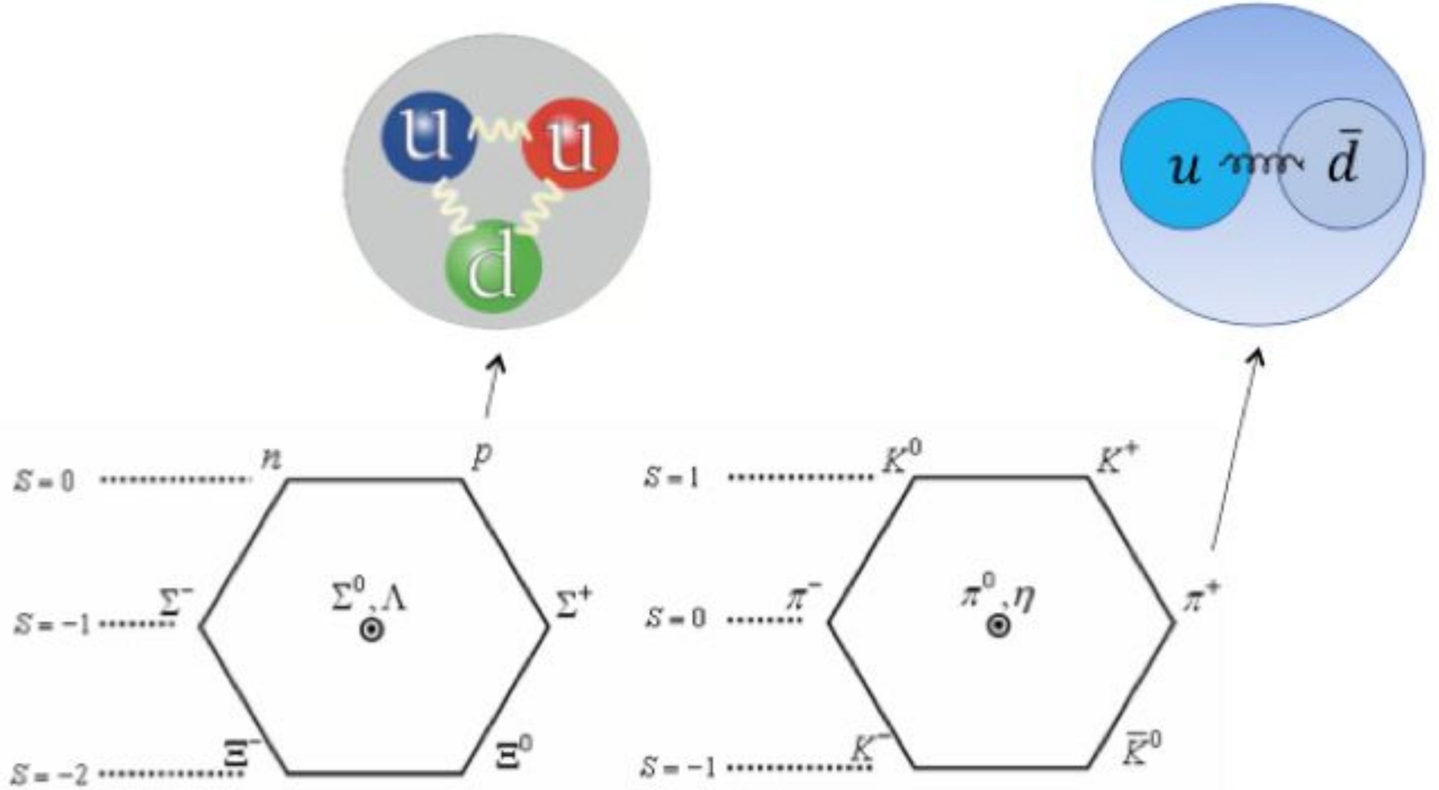


Atomic Model is well understood in QED.

$$Z\alpha < 1 \rightarrow Z < 1/\alpha \approx 137$$

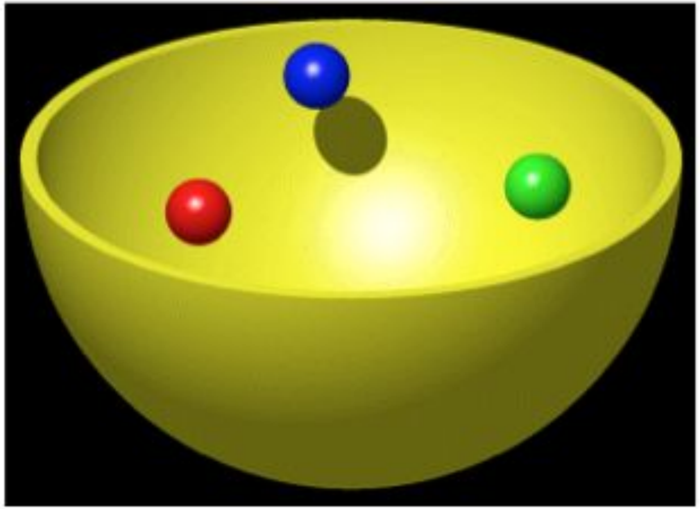
Link between Atomic Model and QED is robust.

How do we understand the Quark Model in Quantum Chromodynamics?

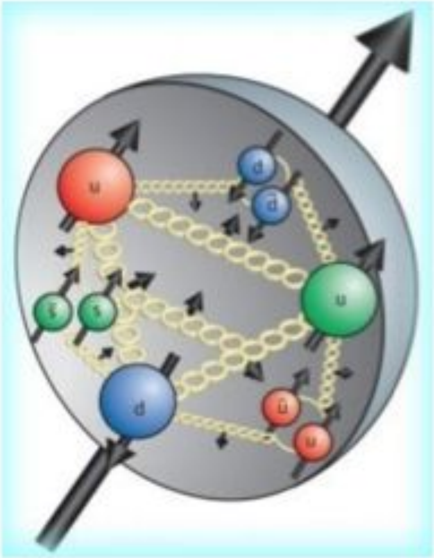


$$M_p = 938.272046 \pm 0.000021\, MeV$$

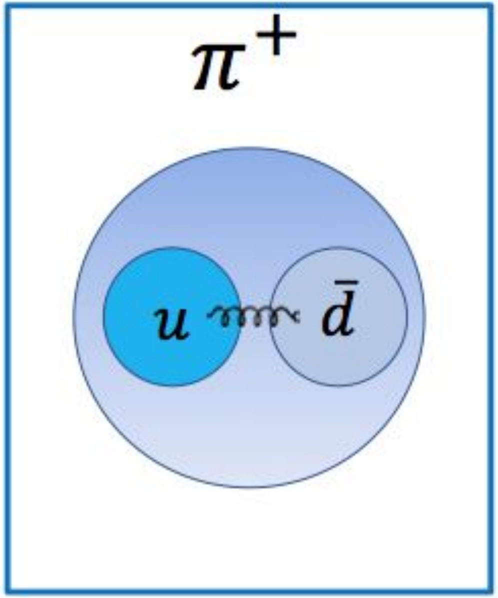
$$M_n = 939.565379 \pm 0.000021\, MeV$$



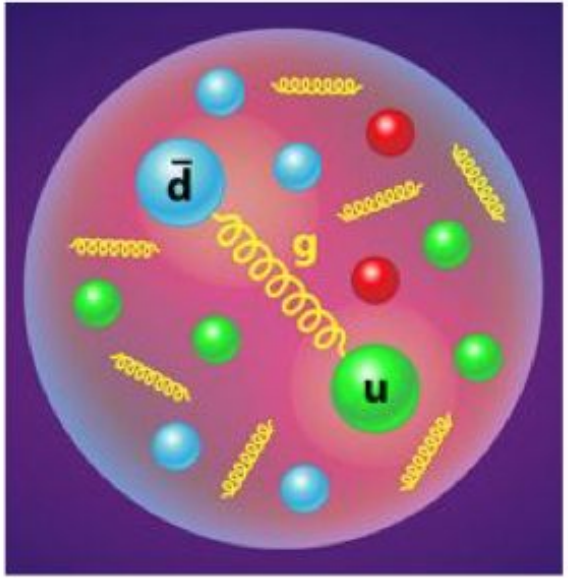
$\Longrightarrow$



$$m_u = 2.3^{+0.7}_{-0.5}\ MeV \quad ; \quad m_d = 4.8^{+0.7}_{-0.3}\ MeV$$



vs.



Constituent Quark Model

$$M = m_1 + m_2 + A\frac{\vec{s}_1 \cdot \vec{s}_2}{m_1 m_2}$$

$$m_u = m_d = 310 MeV/c^2$$

$$A = \left(\frac{2m_u}{\hbar}\right)^2 160 MeV/c^2$$

Quantum Chromodynamics

Isospin symmetry

Chiral symmetry

$$\mathrm{SU(2)_R \times SU(2)_L}$$

Spontaneous symmetry breakdown

Goldstone Bosons

$$F_\pi^2 M_\pi^2 = -(m_u + m_d)\langle 0|\bar{u}u|0\rangle$$

Effective field theory

Large N_C QCD in 1+1 dim. ('tHooft Model)

$$\mathcal{L} = -\frac{1}{4} F^a_{\hat{\mu}\hat{\nu}} F^{\hat{\mu}\hat{\nu}a} + \bar{\psi}(i\gamma^{\hat{\mu}} D_{\hat{\mu}} - m)\psi$$

$$D_{\hat{\mu}} = \partial_{\hat{\mu}} - igA^a_{\hat{\mu}} t_a$$

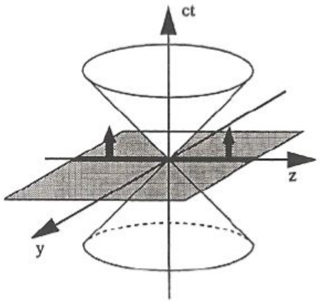
$$F^a_{\hat{\mu}\hat{\nu}} = \partial_{\hat{\mu}} A^a_{\hat{\nu}} - \partial_{\hat{\nu}} A^a_{\hat{\mu}} + gf^{abc} A^b_{\hat{\mu}} A^c_{\hat{\nu}}$$

'tHooft Coupling $\lambda = \frac{g^2 (N_c - 1/N_c)}{4\pi}$ and mass m

$$g \to 0, N_C \to \infty; \lambda \to finite$$

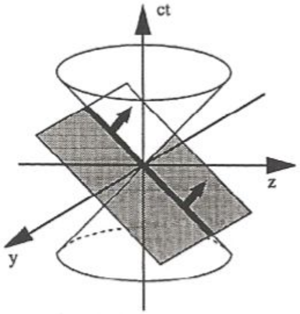
Dirac's Proposition for Relativistic Dynamics

1949



The instant form

Equal t		*Equal τ*
p^0	$\leftrightarrow$	$p^- = p^0 - p^3$
(p^1, p^2)	$\leftrightarrow$	$\vec{p}_\perp$
p^3	$\leftrightarrow$	$p^+ = p^0 + p^3$

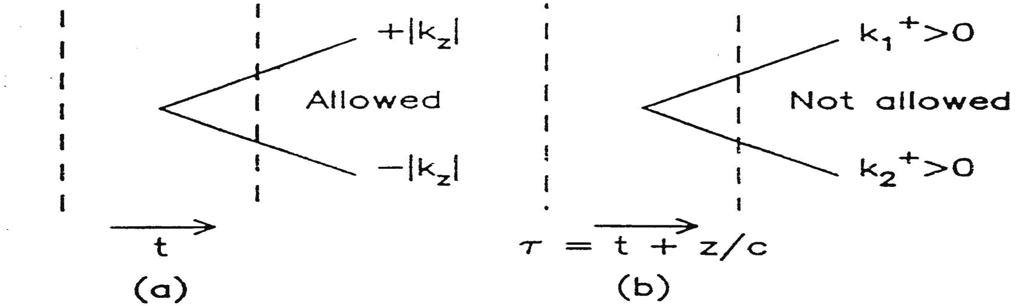


The front form

Energy-Momentum Dispersion Relations

$$p^0 = \sqrt{\vec{p}^{\,2} + m^2}$$

$$p^- = \frac{\vec{p}_\perp^{\,2} + m^2}{p^+}$$



Except zero-modes

$$k_1^+ = k_2^+ = 0$$

IFD

Instant Form Dynamics

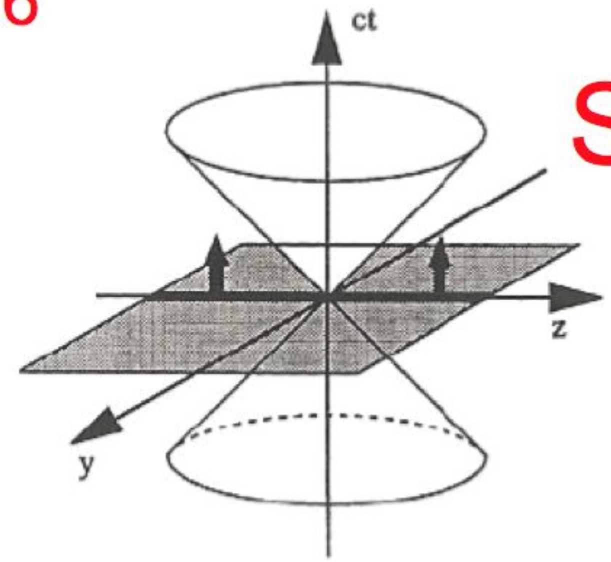
LFD

Light-Front Dynamics

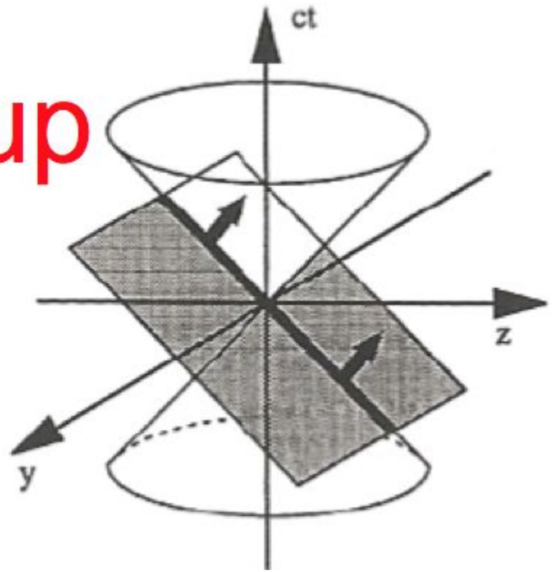
How many generators leave the time surface invariant?

6

7



Stability Group



IFD

Instant Form Dynamics

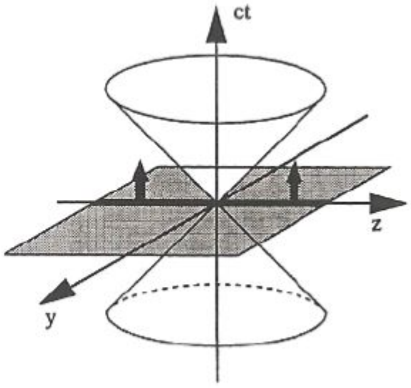
LFD (maximum)

Light-Front Dynamics

Short List of LFD vs. IFD References

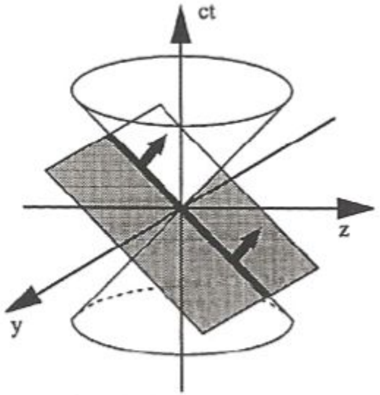
- G.'tHooft, NPB75,461(74) - LFD
- Y.Frishman, et al., PRD15(75) - Interpol Gauges IFD&LFD
- I.Bars&M.Green, PRD17,537(78) - IFD(formulation)
- A.Zhitnitsky, PLB165,405(85) - LFD(chiral sym breaking)
- M.Li, et al., JPG13, 915(87) - IFD(rest frame)
- K.Hornbostel, Ph.D. Dissertation(88) - LFD(DLCQ)
- M.Burkardt, PRD53,933(96) - LFD(vacuum condensates)
- Y.Kalashnikov&A.Nefed'ev,Phys.-Usp.45,346('02) - IFD(rev)
- Y. Jia, et al., JHEP11, 151('17) - IFD(moving frame)
- Y. Jia, et al., PRD98, 054011('18) - IFD(quasi-PDFs)
- B.Ma&C.Ji, PRD104,036004('21) - Link IFD&LFD

Can IFD and LFD be linked?



1949

Yes, they can!



Traditional approach evolved from NR dynamics

Close contact with Euclidean space

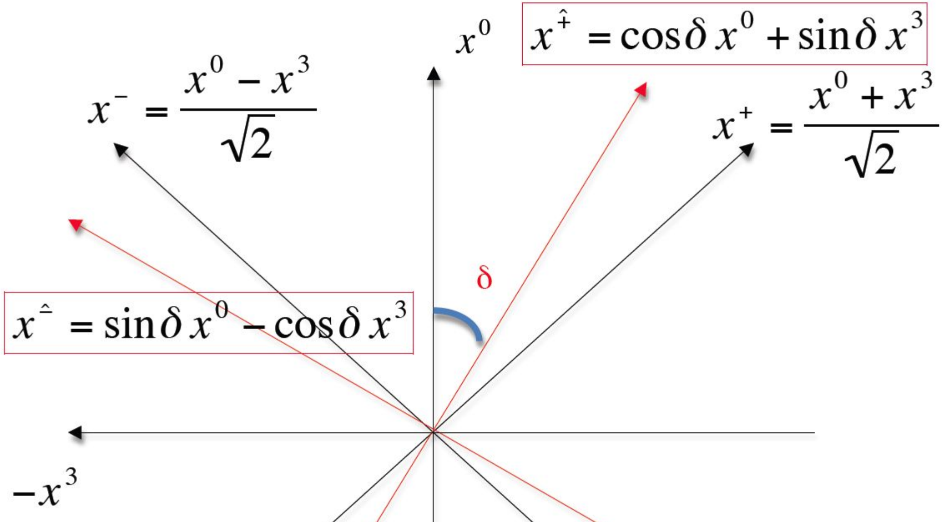
T-dept QFT, LQCD, IMF, etc.

Innovative approach for relativistic dynamics

Strictly in Minkowski space

DIS, PDFs, DVCS, GPDs, etc.

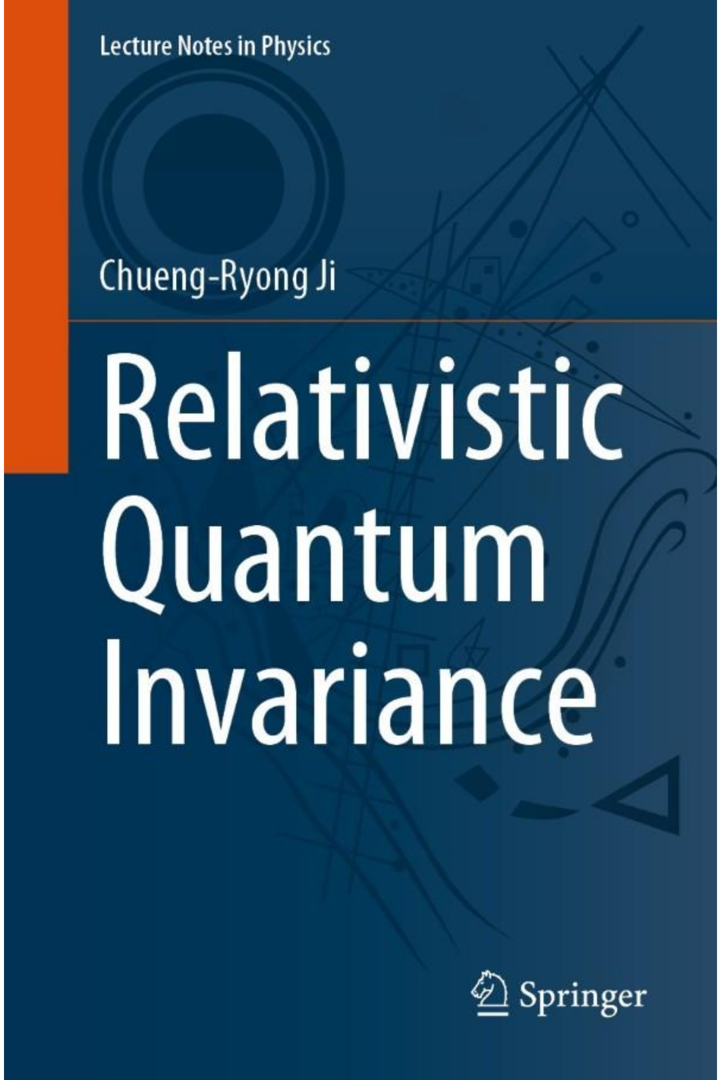
Interpolation between IFD and LFD



$$(IFD)\quad 0 \le \delta \le \frac{\pi}{4} \quad (LFD)$$

$$1 \ge \quad C \equiv \cos(2\delta) \quad \ge 0$$

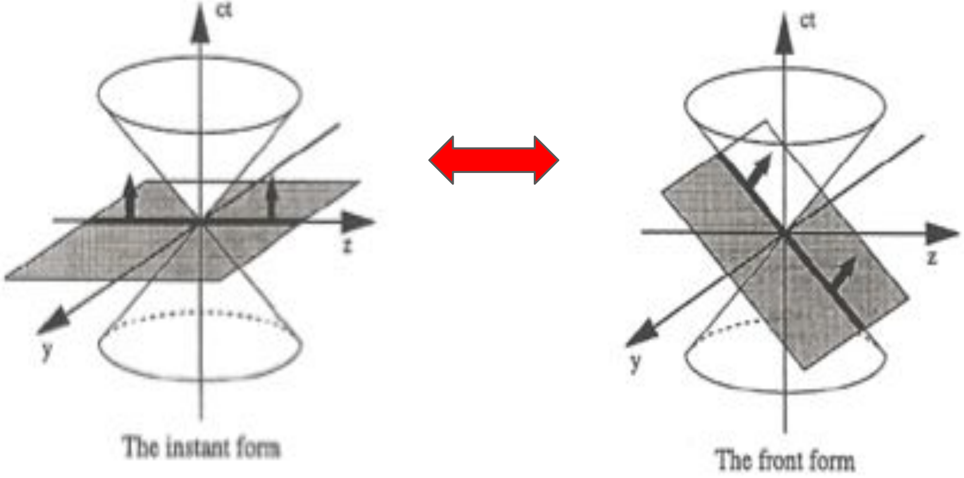
K. Hornbostel, PRD45, 3781 (1992) – RQFT
C.Ji and S.Rey, PRD53,5815(1996) – Chiral Anomaly
C.Ji and C. Mitchell, PRD64,085013 (2001) – Poincare Algebra
C.Ji and A. Suzuki, PRD87,065015 (2013) – Scattering Amps
C.Ji, Z. Li and A. Suzuki, PRD91, 065020 (2015) – EM Gauges
Z.Li, M. An and C.Ji, PRD92, 105014 (2015) – Spinors
C.Ji, Z.Li, B.Ma and A.Suzuki, PRD98, 036017(2018) – QED
B.Ma and C.Ji, arXiv:2105.09388v1[hep-ph], PRD104, 036004(2021) – QCD_{1+1}



Lecture Notes in Physics (LNP, Vol. 1012), Springer Nature (2023).

Interpolation between

IFD and LFD



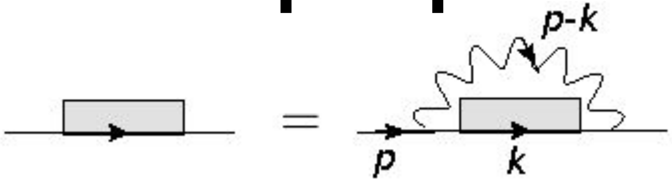
Interpolating Axial Gauge

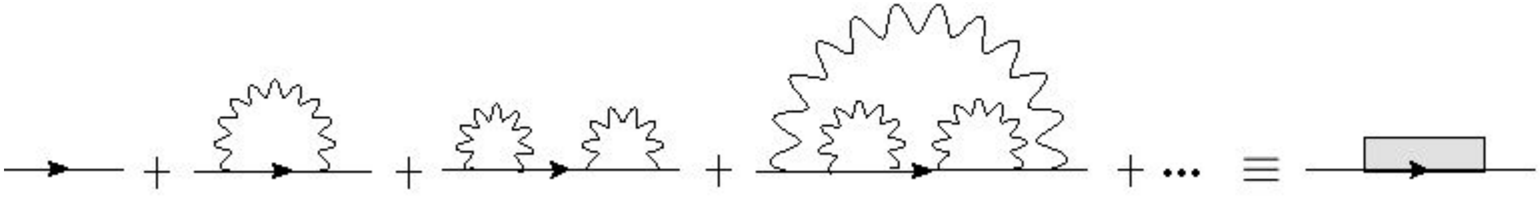
$$A^a_{\hat{-}} = 0$$

$$A^a_1 = 0 \qquad\qquad A^+_a = 0$$

$$\mathcal{L} = \frac{1}{2}(\partial_{\hat{-}} A^a_{\hat{+}})^2 + \bar{\psi}(i\gamma^{\hat{+}} D_{\hat{+}} + i\gamma^{\hat{-}}\partial_{\hat{-}} - m)\psi$$

Mass Gap Equation





$$\equiv i\,\Sigma(p)$$

$$\Sigma(p_{\hat{-}}) = i\frac{\lambda}{2\pi}\int \frac{dk_{\hat{-}}\,dk_{\hat{+}}}{(p_{\hat{-}} - k_{\hat{-}})^2}\gamma^{\hat{+}}\frac{1}{\not{k} - m - \Sigma(k_{\hat{-}}) + i\epsilon}\gamma^{\hat{+}}$$

Fermion Propagator

Free Propagator

$$S_f(p) = \frac{1}{\not p - m + i\varepsilon}$$

$$\boxed{\Sigma(p) = \Sigma_s(p) + \Sigma_v(p)\not p}$$

Interacting Propagator

$$S(p) = \frac{1}{\not p - m - \sum(p) + i\varepsilon}$$

$$= \frac{F(p)}{\not p - M(p) + i\varepsilon}$$

$$F(p) = \left(1 - \Sigma_v(p)\right)^{-1}$$ "Wave function renormalization factor"

$$M(p) = \frac{m + \Sigma_s(p)}{1 - \Sigma_v(p)}$$ "Renormalized fermion mass function"

Energy-Momentum Dispersion Relation

Free particle | **Interacting particle**

$$E = \sqrt{p_z^2 + m^2}$$

$$\theta_f = \tan^{-1}(p_z / m)$$

$$\beta = p_z / E$$

$$= \sin\theta_f$$

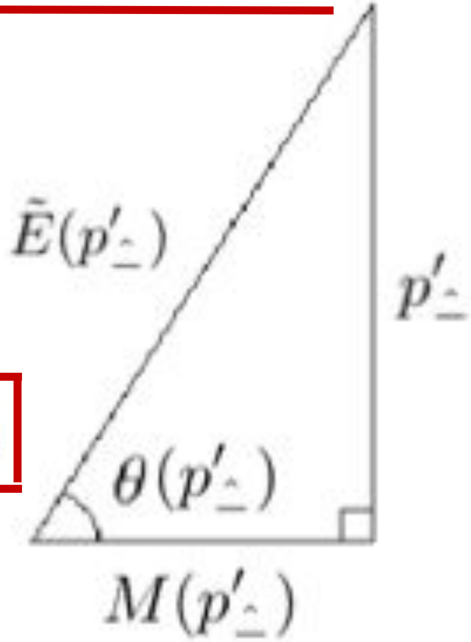
$$= \tanh\eta$$

$$\frac{F(p'_{\hat{-}})E(p'_{\hat{-}})}{\sqrt{C}} = \sqrt{p'^2_{\hat{-}} + M(p'_{\hat{-}})^2} \equiv \tilde{E}(p'_{\hat{-}})$$

$$\theta(p'_{\hat{-}}) = \theta_f(p'_{\hat{-}}) + 2\zeta(p'_{\hat{-}})$$

$$\begin{pmatrix} b^i(p'_{\hat{-}}) \\ d^{+i}(p'_{\hat{-}}) \end{pmatrix} = \begin{pmatrix} \cos\zeta(p'_{\hat{-}}) & -\sin\zeta(p'_{\hat{-}}) \\ \sin\zeta(p'_{\hat{-}}) & \cos\zeta(p'_{\hat{-}}) \end{pmatrix} \begin{pmatrix} b^i_f(p'_{\hat{-}}) \\ d^{+i}_f(p'_{\hat{-}}) \end{pmatrix}$$

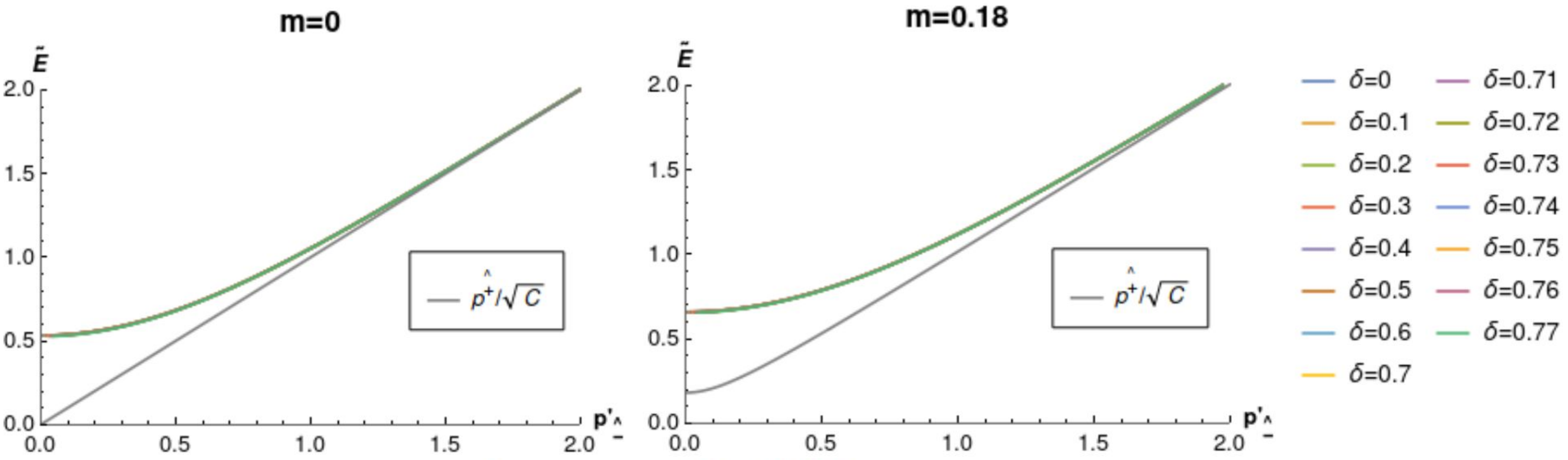
$b^i_f \,|0> = 0, d^i_f \,|0> = 0$ **vs.** $b^i \,|\Omega> = 0, d^i \,|\Omega> = 0$



Interpolation

$$(E, p_z) \Rightarrow (p^{\hat{+}} / \sqrt{C}, p_{\hat{-}} / \sqrt{C} \equiv p'_{\hat{-}})$$

Mass Gap Solutions



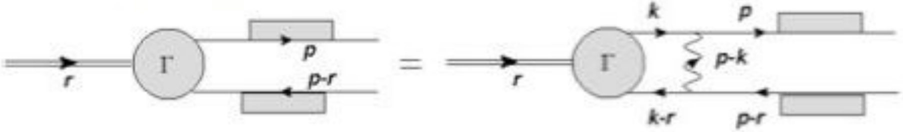
$$\tilde{E}(0) = \frac{F(0)E(0)}{\sqrt{\mathbb{C}}} = M(0)$$

m	0	0.045	0.18	0.749	1.00	2.11	4.23
$M(0)$	0.532778	0.563644	0.659112	1.10105	1.31167	2.30969	4.34358
$F(0)$	−0.495173	−0.584175	−0.987673	4.11079	2.17976	1.22134	1.05526

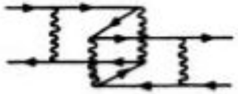
$$m \lesssim 0.56$$

BOUND-STATE EQUATION

$$\Gamma(r,p) = \frac{i\lambda}{2\pi}\int \frac{dk_{\hat{-}}\,dk_{\hat{+}}}{(p_{\hat{-}}-k_{\hat{-}})^2} S(p)\gamma^{\hat{+}}\Gamma(r,k)\gamma^{\hat{+}}S(p-r)$$



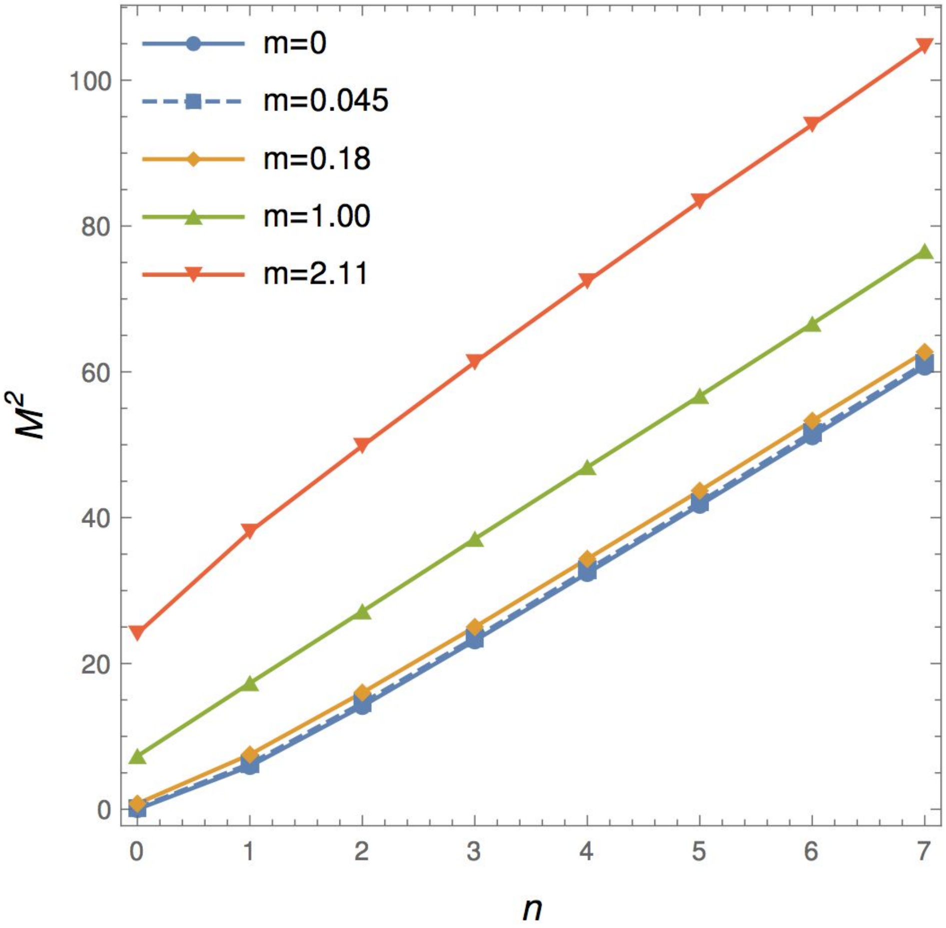
$$\left[-r_{\hat{+}} + \frac{-\mathbb{S}p_{\hat{-}} + E(p_{\hat{-}})}{\mathbb{C}} + \frac{\mathbb{S}(p_{\hat{-}}-r_{\hat{-}}) + E(p_{\hat{-}}-r_{\hat{-}})}{\mathbb{C}}\right]\hat{\phi}_+(r_{\hat{-}},p_{\hat{-}})$$
$$= \lambda \fint \frac{dk_{\hat{-}}}{(p_{\hat{-}}-k_{\hat{-}})^2}\left[C(p_{\hat{-}},k_{\hat{-}},r_{\hat{-}})\hat{\phi}_+(r_{\hat{-}},k_{\hat{-}}) - S(p_{\hat{-}},k_{\hat{-}},r_{\hat{-}})\hat{\phi}_-(r_{\hat{-}},k_{\hat{-}})\right],$$
$$\left[r_{\hat{+}} + \frac{-\mathbb{S}(p_{\hat{-}}-r_{\hat{-}}) + E(p_{\hat{-}}-r_{\hat{-}})}{\mathbb{C}} + \frac{\mathbb{S}p_{\hat{-}} + E(p_{\hat{-}})}{\mathbb{C}}\right]\hat{\phi}_-(r_{\hat{-}},p_{\hat{-}})$$
$$= \lambda \fint \frac{dk_{\hat{-}}}{(p_{\hat{-}}-k_{\hat{-}})^2}\left[C(p_{\hat{-}},k_{\hat{-}},r_{\hat{-}})\hat{\phi}_-(r_{\hat{-}},k_{\hat{-}}) - S(p_{\hat{-}},k_{\hat{-}},r_{\hat{-}})\hat{\phi}_+(r_{\hat{-}},k_{\hat{-}})\right].$$



LFD $$\left[\mathcal{M}^2 - \frac{m^2-2\lambda}{x} - \frac{m^2-2\lambda}{1-x}\right]\phi(x) = -2\lambda \fint_0^1 \frac{dy}{(x-y)^2}\phi(y)$$

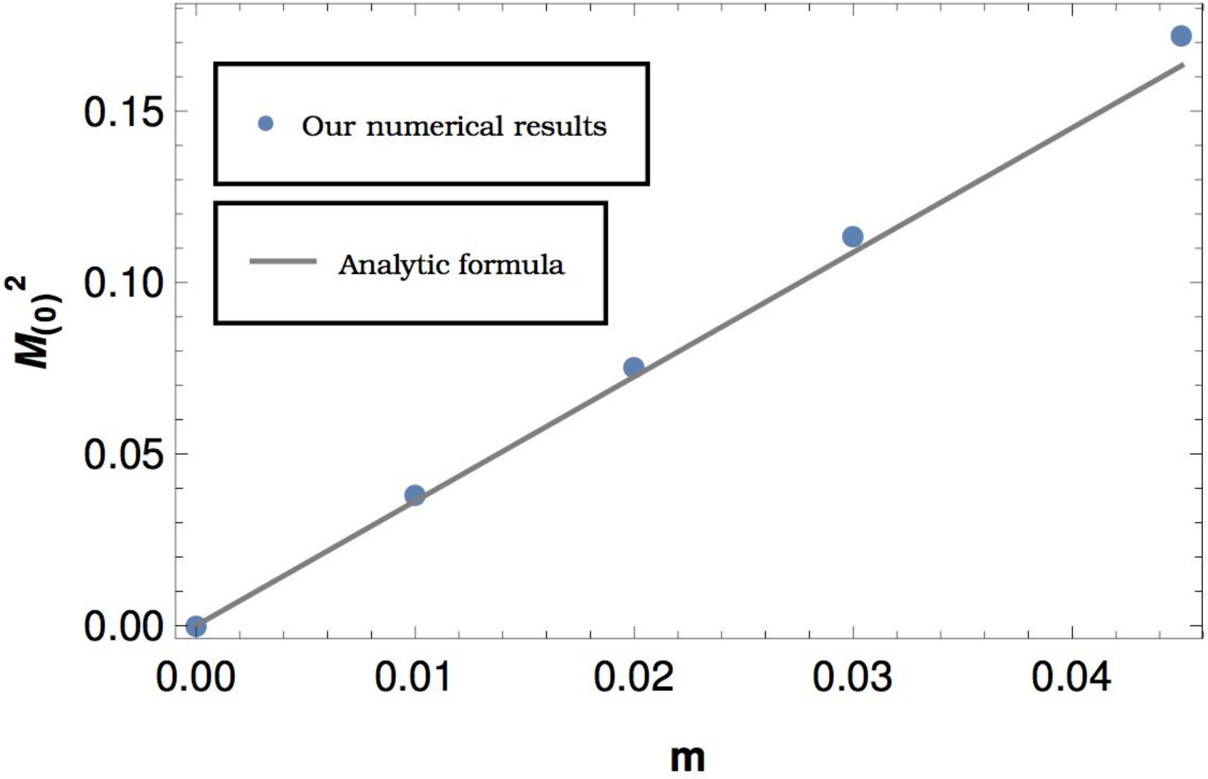
Meson Spectroscopy

- G.'tHooft, NPB75, 461(74) - LFD



- M.Li, et al., JPG13, 915(87) - IFD (rest frame)
- Y. Jia, et al., JHEP11, 151('17) - IFD (moving frame)

Gell-Mann - Oaks - Renner Relation

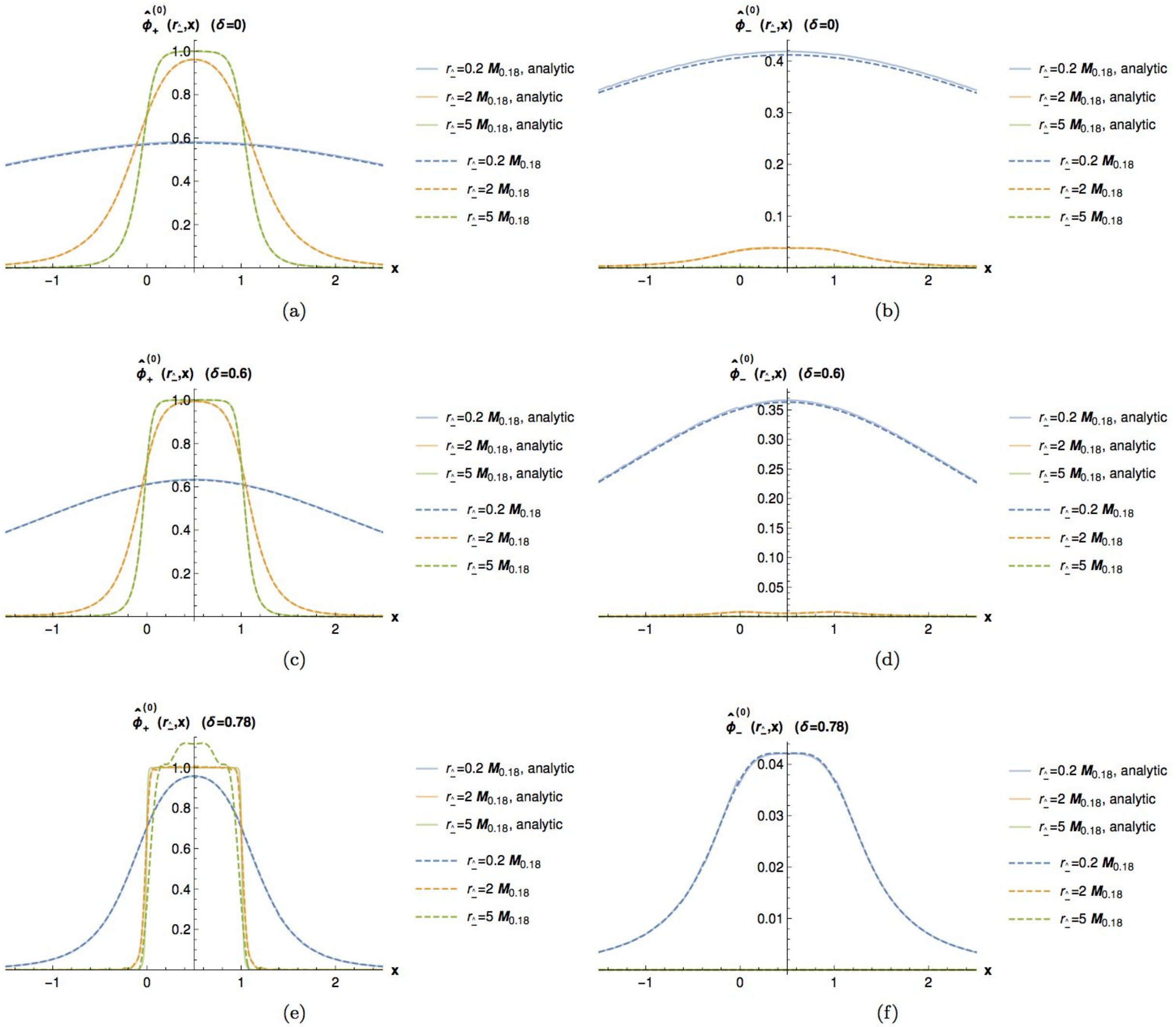


$$\mathcal{M}_\pi^2 = -\frac{4m < \bar{\psi}\psi >}{f_\pi^2} = \sqrt{\frac{8\pi^2 m^2 \lambda}{3}}$$

$$f_\pi = \sqrt{N_c/\pi}$$

Meson Ground-state Wave-function for m=0 case

$$\hat{\phi}^{(0)}_{\pm}(r_{\hat{-}}, p_{\hat{-}}) = \frac{1}{2}\left(\cos\frac{\theta(r_{\hat{-}} - p_{\hat{-}}) - \theta(p_{\hat{-}})}{2} \pm \sin\frac{\theta(r_{\hat{-}} - p_{\hat{-}}) + \theta(p_{\hat{-}})}{2}\right)$$



Parton Distribution Functions (PDFs)

$$q_n(x) = \int_{-\infty}^{+\infty} \frac{d\xi^-}{4\pi} e^{-ixP^+\xi^-}$$

$$\times \langle P_n^-, P^+ | \bar{\psi}(\xi^-)\gamma^+ \mathcal{W}[\xi^-, 0]\psi(0) | P_n^-, P^+ \rangle_C,$$

$$\mathcal{W}[\xi^-, 0] = \mathcal{P}\left[\exp\left(-ig_s \int_0^{\xi^-} d\eta^- A^+(\eta^-)\right)\right]$$

$A^+=0$ Gauge in LFD

Quasi-PDFs

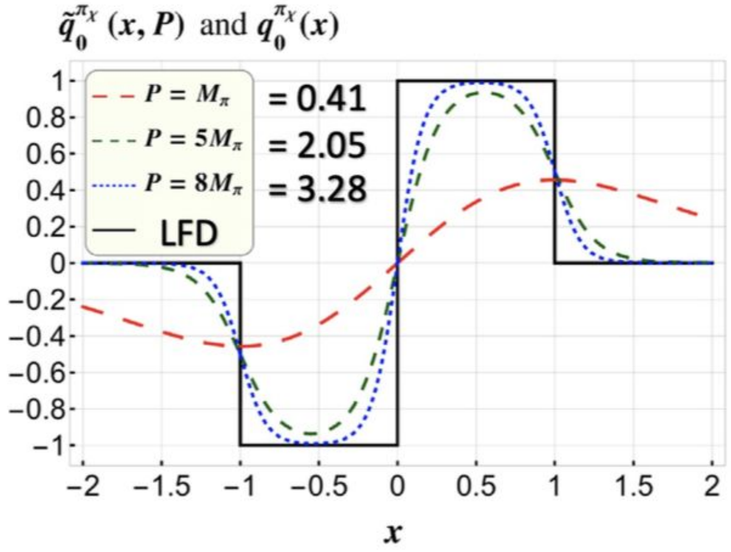
$$\tilde{q}_{(n)}(r_{\hat{-}}, x) = \int_{-\infty}^{+\infty} \frac{dx^{\hat{-}}}{4\pi} \, \mathrm{e}^{ix^{\hat{-}} r_{\hat{-}}}$$

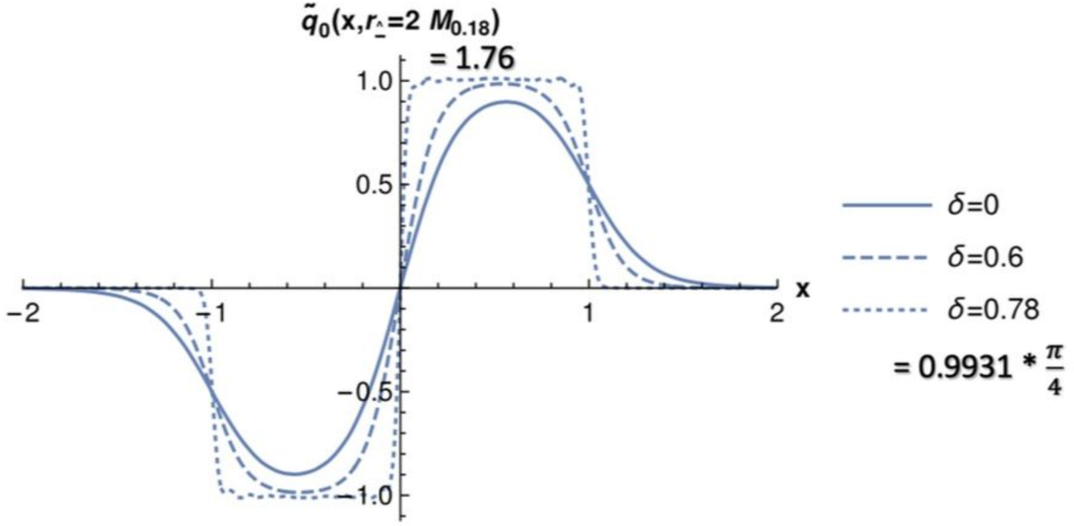
$$\times < r^{\hat{+}}_{(n)}, r_{\hat{-}} \mid \bar{\psi}(x^{\hat{-}}) \, \gamma_{\hat{-}} \, \mathcal{W}[x^{\hat{-}}, 0] \, \psi(0) \mid r^{\hat{+}}_{(n)}, r_{\hat{-}} >_C,$$

$$\mathcal{W}[x^{\hat{-}}, 0] = \mathcal{P}\left[\exp\left(-ig \int_0^{x^{\hat{-}}} dx'^{\hat{-}} A_{\hat{-}}(x'^{\hat{-}})\right)\right]$$

Interpolating dynamics

Y. Jia, et al., PRD98, 054011('18)
- IFD (quasi-PDFs)





B.Ma&C.Ji,PRD104,036004('21)
- Interpolating Dynamics

Bakamjian-Thomas Construction

B.Bakamjian and L.H.Thomas, Phys.Rev.92,1300(1953)

B.Keister and W.Polyzou, Adv.Nucl.Phys.20,225(1991)

$$[P^i, K^j] = i\delta_{ij} H$$

$$\{H, \mathbf{P}, \mathbf{J}, \mathbf{K}\} \Rightarrow \{M, \mathbf{P}, \mathbf{j}_c, \mathbf{X}_c\} \Rightarrow \{M_0, \mathbf{P}_0, \mathbf{j}_{c0}, \mathbf{X}_{c0}\} \Rightarrow \{M, \mathbf{P}_0, \mathbf{j}_{c0}, \mathbf{X}_{c0}\}$$

$$H = \sqrt{M^2 + \mathbf{P}^2};$$

$$\mathbf{K} = -\frac{1}{2}\{H, \mathbf{X}_c\}_+ - \frac{\mathbf{P} \times \mathbf{j}_c}{H + M};$$

$$\mathbf{J} = \mathbf{X}_c \times \mathbf{P} + \mathbf{j}_c.$$

$$P_0^\mu := P_1^\mu \otimes I_2 + I_1 \otimes P_2^\mu;$$

$$\mathbf{K}_0 := \mathbf{K}_1 \otimes I_2 + I_1 \otimes \mathbf{K}_2;$$

$$\mathbf{J}_0 := \mathbf{J}_1 \otimes I_2 + I_1 \otimes \mathbf{J}_2.$$

$$M := M_0 + V$$

$$\left[V, \{\mathbf{P}_0, \mathbf{j}_{c0}, \mathbf{X}_{c0}\}\right] = 0$$

Bakamjian-Thomas Construction in LFD

$$\{P^-, P^+, \mathbf{P}_\perp, \mathbf{E}_\perp, K^3, \mathbf{J}_\perp, J^3\} \Rightarrow \{M, P^+, \mathbf{P}_\perp, \mathbf{E}_\perp, K^3, \mathbf{j}_f\}$$

$$M^2 = P^+P^- - \mathbf{P}_\perp^2;\; j_f^3 = \frac{1}{P^+}[P^+J^3 - \hat{\mathbf{z}}\cdot(\mathbf{P}_\perp \times \mathbf{E}_\perp)];$$

$$\mathbf{j}_{f\perp} = \frac{1}{M}\left[-\tfrac{1}{2}(P^+ - P^-)(\hat{\mathbf{z}}\times\mathbf{E}_\perp) + \hat{\mathbf{z}}\times\mathbf{P}_\perp K^3 + P^+\mathbf{J}_\perp - \frac{\mathbf{P}_\perp}{P^+}[P^+J^3 - \hat{\mathbf{z}}\cdot(\mathbf{P}_\perp\times\mathbf{E}_\perp)]\right]$$

$$\{M_0, P_0^+, \mathbf{P}_{\perp 0}, \mathbf{E}_{\perp 0}, K_0^3, \mathbf{j}_{f0}\} \Leftrightarrow \{M, P^+, \mathbf{P}_\perp, \mathbf{E}_\perp, K^3, \mathbf{j}_f\}$$

$$M := M_0 + V$$

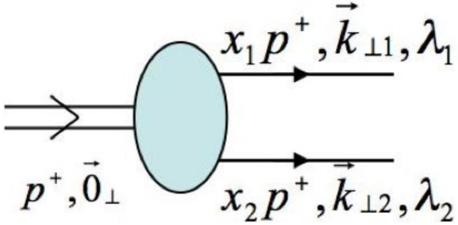
$$[\mathbf{E}_\perp, V]_- = [K^3, V]_- = [\mathbf{j}_{f0}, V]_- = [\mathbf{P}_\perp, V]_- = [P^+, V]_- = 0$$

Light-Front Quark Model(LFQM)

$$|Meson\rangle = \psi_{q\bar{q}}|q\bar{q}\rangle + \psi_{q\bar{q}g}|q\bar{q}g\rangle + \ldots$$
$$\approx \Psi_{Q\bar{Q}}|Q\bar{Q}\rangle,$$

where

$$|Q\rangle = \psi_q^Q|q\rangle + \psi_{qg}^Q|qg\rangle + \ldots$$
$$|\bar{Q}\rangle = \psi_{\bar{q}}^{\bar{Q}}|\bar{q}\rangle + \psi_{\bar{q}g}^{\bar{Q}}|\bar{q}g\rangle + \ldots$$



$$P^- = p_Q^- + p_{\bar{Q}}^-$$

$$M_0^2 = \frac{m_Q^2 + \boldsymbol{k}_\perp^2}{x} + \frac{m_{\bar{Q}}^2 + \boldsymbol{k}_\perp^2}{1-x}$$

Noninteracting "on-mass" shell Q & $\bar{Q}$ representation

$$\Psi_{Q\bar{Q}}(x_i, \vec{k}_{\perp i}, \lambda_i) = \Phi(x_i, \vec{k}_{\perp i})\chi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Radial

Spin-Orbit

The interaction between $Q\bar{Q}$

includes Coulomb, Confinement, Spin-Spin, Spin-Orbit interactions.

$$M := M_0 + V_{Q\bar{Q}}$$

Interaction independent Melosh transformation

$J^{PC} = 0^{++}(f_0, a_0, \ldots)$

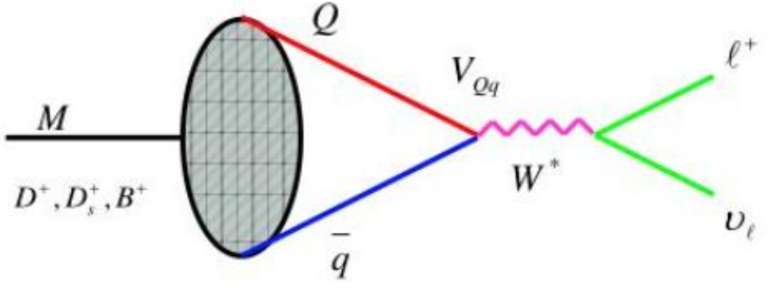
$0^{-+}(\pi, K, \eta, \eta', \ldots)$

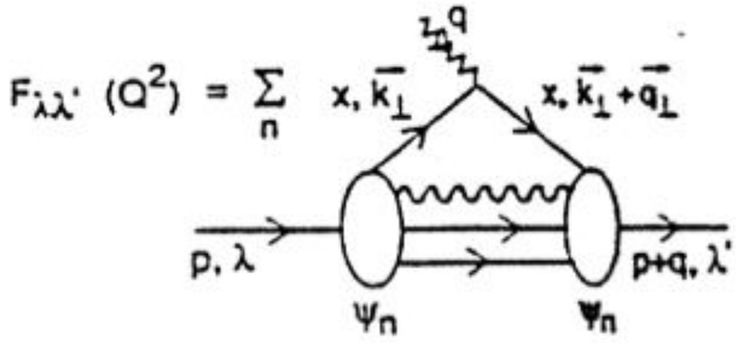
$1^{--}(\rho, K^*, \omega, \phi, \ldots)$

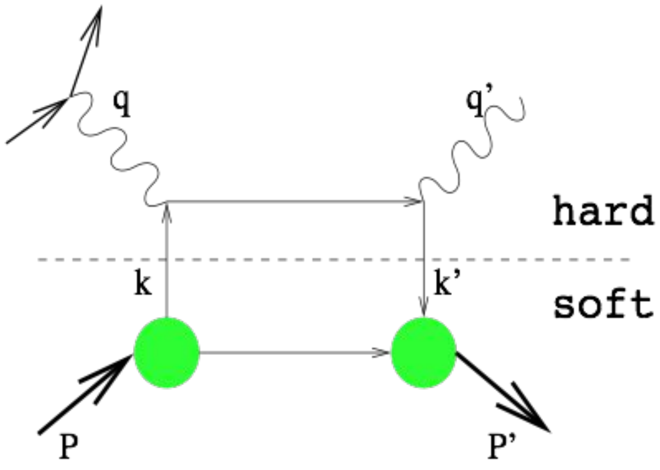
PRD59, 074015(99); PLB460, 461(99) by HMC and CRJ;
PRC92, 055203(2015) by HMC, CRJ, Z. Li, and H. Ryu
PRD106, 014009(2022) by A. J. Arifi, HMC, and CRJ
HMC and CRJ, PRD110, 014006(2024)

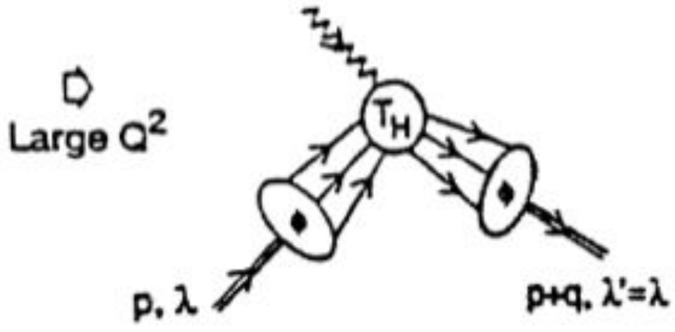
H.J. Melosh: PRD 9, 1095(1974)

Two-point, Three-point and Four-point functions

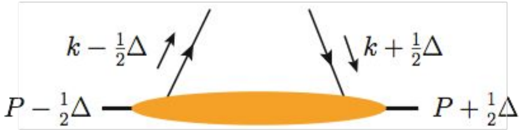




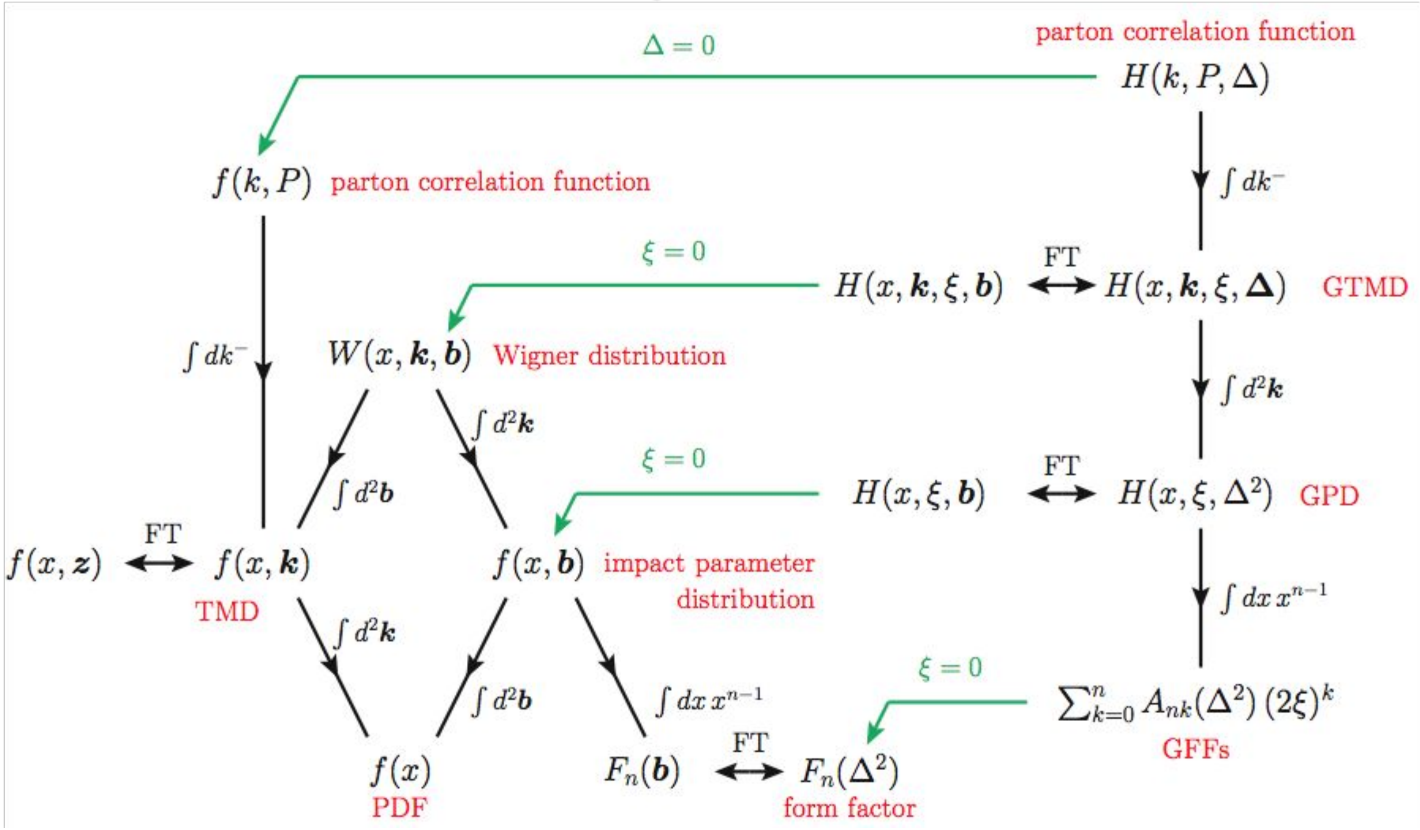




M.Diehl, EPJA 52, 149 (2016)



$$H(k,P,\Delta) = (2\pi)^{-4} \int \mathrm{d}^4 z\, e^{izk} \left\langle p\left(P+\frac{1}{2}\Delta\right) \middle| \bar{q}\left(-\frac{1}{2}z\right) \Gamma q\left(\frac{1}{2}z\right) \middle| p\left(P-\frac{1}{2}\Delta\right)\right\rangle$$



$$\int \frac{[dz]}{2(2\pi)^3} e^{ip\cdot z} \langle P|\bar{\psi}(0)\gamma^+ \psi(z)|P\rangle|_{z^+=0} = f_1^q(x, p_T),$$

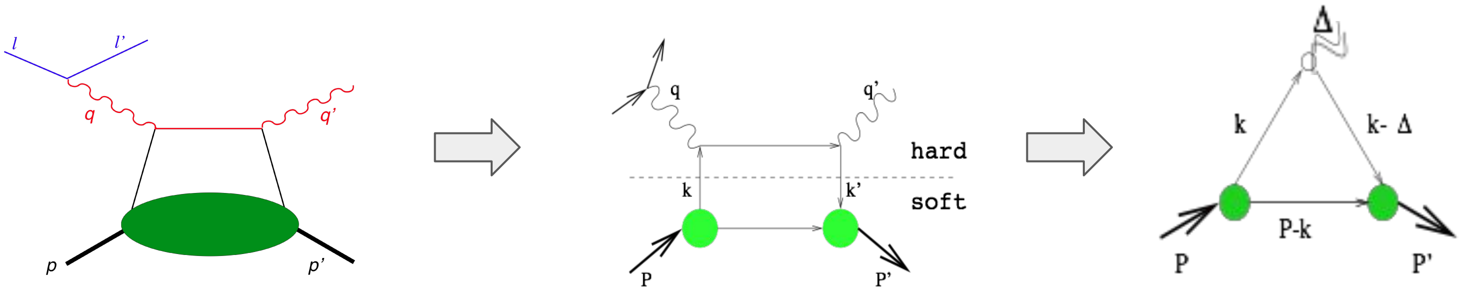
$$\int \frac{[dz]}{2(2\pi)^3} e^{ip\cdot z} \langle P|\bar{\psi}(0)\gamma_T^j \psi(z)|P\rangle|_{z^+=0} = \frac{p_T^j}{P^+} f_3^q(x, p_T),$$

$$\int \frac{[dz]}{2(2\pi)^3} e^{ip\cdot z} \langle P|\bar{\psi}(0)\gamma^- \psi(z)|P\rangle|_{z^+=0} = \left(\frac{m_\pi}{P^+}\right)^2 f_4^q(x, p_T),$$

where $[dz] = dz^- d^2 z_T$

$$\int \frac{\mathrm{d}z^- \mathrm{d}^2 z_T}{2(2\pi)^3} e^{ip\cdot z} \langle P|\overline{\psi}(0)\ \mathbb{1}\ \psi(z)|P\rangle|_{z^+=0}$$

$$= \frac{m_\pi}{P^+} e^q(x, p_T),$$



$$\langle P|J^+|P\rangle = 2P^+ \int dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \, |\phi(x,\mathbf{k}_\perp)|^2$$

$$f_1^q(x,\mathbf{k}_\perp) = \frac{1}{16\pi^3}|\phi(x,\mathbf{k}_\perp)|^2 \qquad \int dx \int d^2\mathbf{k}_\perp f_1^q(x,\mathbf{k}_\perp) = \int dx f_1^q(x) = 1$$

$$\langle P|J^\perp|P\rangle = \int dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \, |\phi(x,\mathbf{k}_\perp)|^2 \left(-\frac{2\mathbf{k}_\perp}{x}\right)$$

$$2\mathbf{k}_\perp f_3^q(x,\mathbf{k}_\perp) = \frac{1}{16\pi^3}|\phi(x,\mathbf{k}_\perp)|^2 \left(-\frac{2\mathbf{k}_\perp}{x}\right) \qquad x f_3^q(x,\mathbf{k}_\perp) = -f_1^q(x,\mathbf{k}_\perp)$$

$$f(x) = \int d^2p_T f(x,p_T) \qquad 2\int dx f_4^q(x) = \int dx f_1^q(x) = 1$$

Eur. Phys. J. C (2016) 76:415
DOI 10.1140/epjc/s10052-016-4257-8

THE EUROPEAN PHYSICAL JOURNAL C

CrossMark

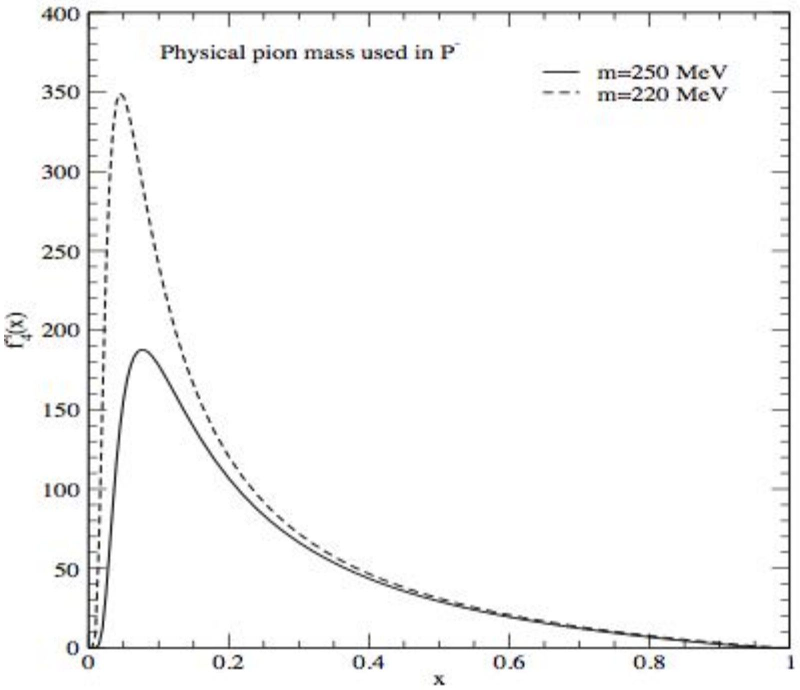
Regular Article - Theoretical Physics

Transverse pion structure beyond leading twist in constituent models

C. Lorcé[1], B. Pasquini[2,3,a], P. Schweitzer[4,5]

[1] Centre de Physique Théorique, École polytechnique, CNRS, Université Paris-Saclay, 91128 Palaiseau, France
[2] Dipartimento di Fisica, Università degli Studi di Pavia, Pavia, Italy
[3] Istituto Nazionale di Fisica Nucleare, Sezione di Pavia, Pavia, Italy
[4] Department of Physics, University of Connecticut, Storrs, CT 06269, USA
[5] Institute for Theoretical Physics, Tübingen University, Auf der Morgenstelle 14, 72076 Tübingen, Germany

"In particular, relations involving the twist-4 unpolarized TMD f_4^q are not satisfied for the pion, confirming the results obtained in the nucleon case. A fully consistent description of $f_4^q(x)$ in light-front formalism requires the inclusion of zero modes or higher Fock states which go beyond the scope of the LFCM. Due to the academic character of the twist-4 function f_4^q this is of no relevance for practical applications."



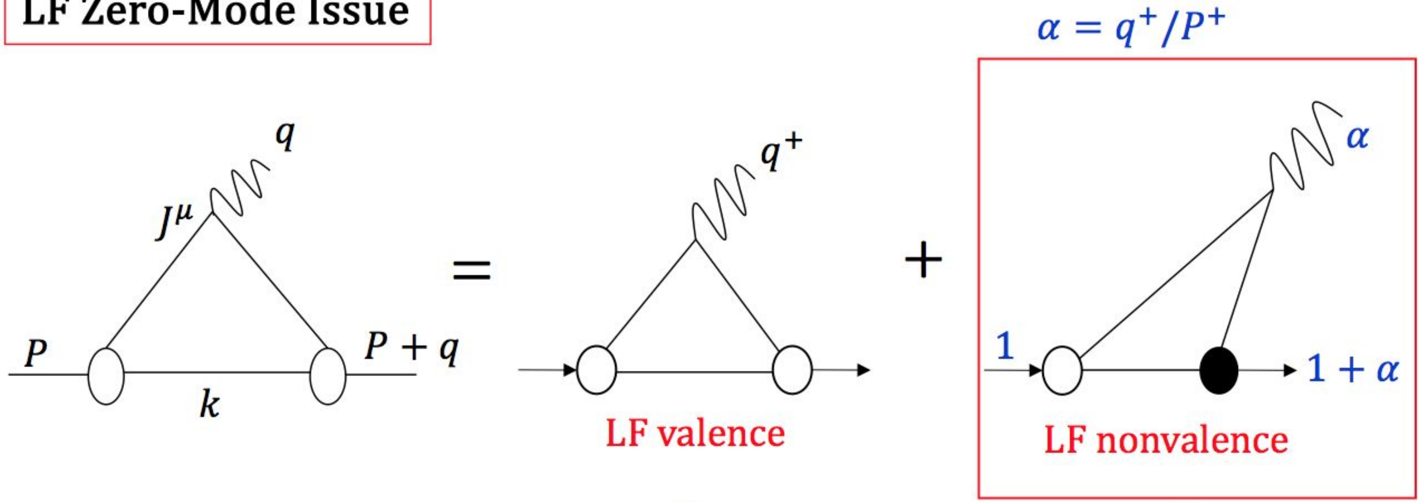
$$\int dx f_4^q(x) = 48.58 \text{ for } m = 0.25 \text{ GeV},$$
$$= 66.45 \text{ for } m = 0.22 \text{ GeV},$$

which are notably different from the expected value of 1/2. The authors of [4, 5] attributed this discrepancy to inadequate estimation of the zero-mode contribution to the J^- current in the computation of $f_4^q(x)$.

[4] C. Lorcé, B. Pasquini, and P. Schweitzer, Unpolarized transverse momentum dependent parton distribution functions beyond leading twist in quark models, JHEP **01**, 103 (2015).

[5] C. Lorcé, B. Pasquini, and P. Schweitzer, Transverse pion structure beyond leading twist in constituent models, Eur. Phys. J. C **76**, 415 (2016).

LF Zero-Mode Issue



$$\lim_{\alpha \to 0} \int_{\alpha}^{1} dx(\ldots)$$

$$\lim_{\alpha \to 0} \int_{0}^{\alpha} dx(\ldots)$$

☞ if J^- current is used

$$\lim_{\alpha \to 0} \int_{0}^{\alpha} dx(\ldots) \neq 0 \quad \text{: LF Zero-Mode !}$$

☞ One has to take into account of the zero mode when use J^- current!

New Development of including the LF Zero-Mode in the LFQM

$$\langle P'|\bar{q}\ \gamma^\mu\ q|\mathrm{P}\rangle = \wp^\mu\ F_P(q^2), \qquad \wp^\mu = (P+P')^\mu - q^\mu\ \frac{(M^2-M'^2)}{q^2}$$

In $q^+ = 0$ frame,

$$\langle P'|\bar{q}\ \gamma^\mu\ q|\mathrm{P}\rangle = \int_0^1 \mathrm{d}p_1^+ \int \frac{\mathrm{d}^2\mathbf{k}_\perp}{16\pi^3}\ \phi'(x,\mathbf{k}'_\perp)\phi(x,\mathbf{k}_\perp)\ \sum_{\lambda's} \mathcal{R}^\dagger_{\lambda_2\bar{\lambda}} \left[\frac{\bar{u}_{\lambda_2}(p_2)}{\sqrt{x_2}}\gamma^\mu \frac{u_{\lambda_1}(p_1)}{\sqrt{x_1}}\right] \mathcal{R}_{\lambda_1\bar{\lambda}},$$

Apply $P^- = p_q^- + p_{\bar{q}}^-$ (i.e. $M^2 \to M_0^2$)

$$F_P^{(\mu)}(Q^2) = \int_0^1 \mathrm{d}p_1^+ \int \frac{\mathrm{d}^2\mathbf{k}_\perp}{16\pi^3}\ \phi'(x,\mathbf{k}'_\perp)\phi(x,\mathbf{k}_\perp)\ \frac{1}{\wp^\mu}\ \sum_{\lambda's} \mathcal{R}^\dagger_{\lambda_2\bar{\lambda}} \left[\frac{\bar{u}_{\lambda_2}(p_2)}{\sqrt{x_2}}\gamma^\mu \frac{u_{\lambda_1}(p_1)}{\sqrt{x_1}}\right] \mathcal{R}_{\lambda_1\bar{\lambda}},$$

Electromagnetic Form Factor

$$\mathcal{J}^{\mu}_{\rm em} \equiv \mathcal{P}^{\mu} F_{\rm em}(q^2) = \left[(P+P')^{\mu} - q^{\mu}\frac{(M^2 - M'^2)}{q^2}\right] F_{\rm em}(q^2)$$

$$\mathcal{J}^{\mu}_{\rm em} = \int_0^1 dp_1^+ \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}\, \phi'(x,\mathbf{k}'_\perp)\phi(x,\mathbf{k}_\perp) \sum_{\lambda' s} h^{\mu}_{\lambda_1\bar{\lambda}\to\lambda_2\bar{\lambda}}$$

$$h^{\mu}_{\lambda_1\bar{\lambda}\to\lambda_2\bar{\lambda}} \equiv \mathcal{R}^{\dagger}_{\lambda_2\bar{\lambda}} \left[\frac{\bar{u}_{\lambda_2}(p_2)}{\sqrt{p_2^+}}\gamma^{\mu}\frac{u_{\lambda_1}(p_1)}{\sqrt{p_1^+}}\right] \mathcal{R}_{\lambda_1\bar{\lambda}}$$

$$F^{(\mu)}_{\pi}(Q^2) = \int_0^1 dp_1^+ \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}\, \phi'(x,\mathbf{k}'_\perp)\phi(x,\mathbf{k}_\perp) \sum_{\lambda' s} h^{\mu}_{\lambda_1\bar{\lambda}\to\lambda_2\bar{\lambda}} \Big/ \mathcal{P}^{\mu}$$

$$\mathcal{P}^{\mu} \equiv (P+P')^{\mu} - q^{\mu}({M_0}^2 - {M_0'}^2)/q^2$$

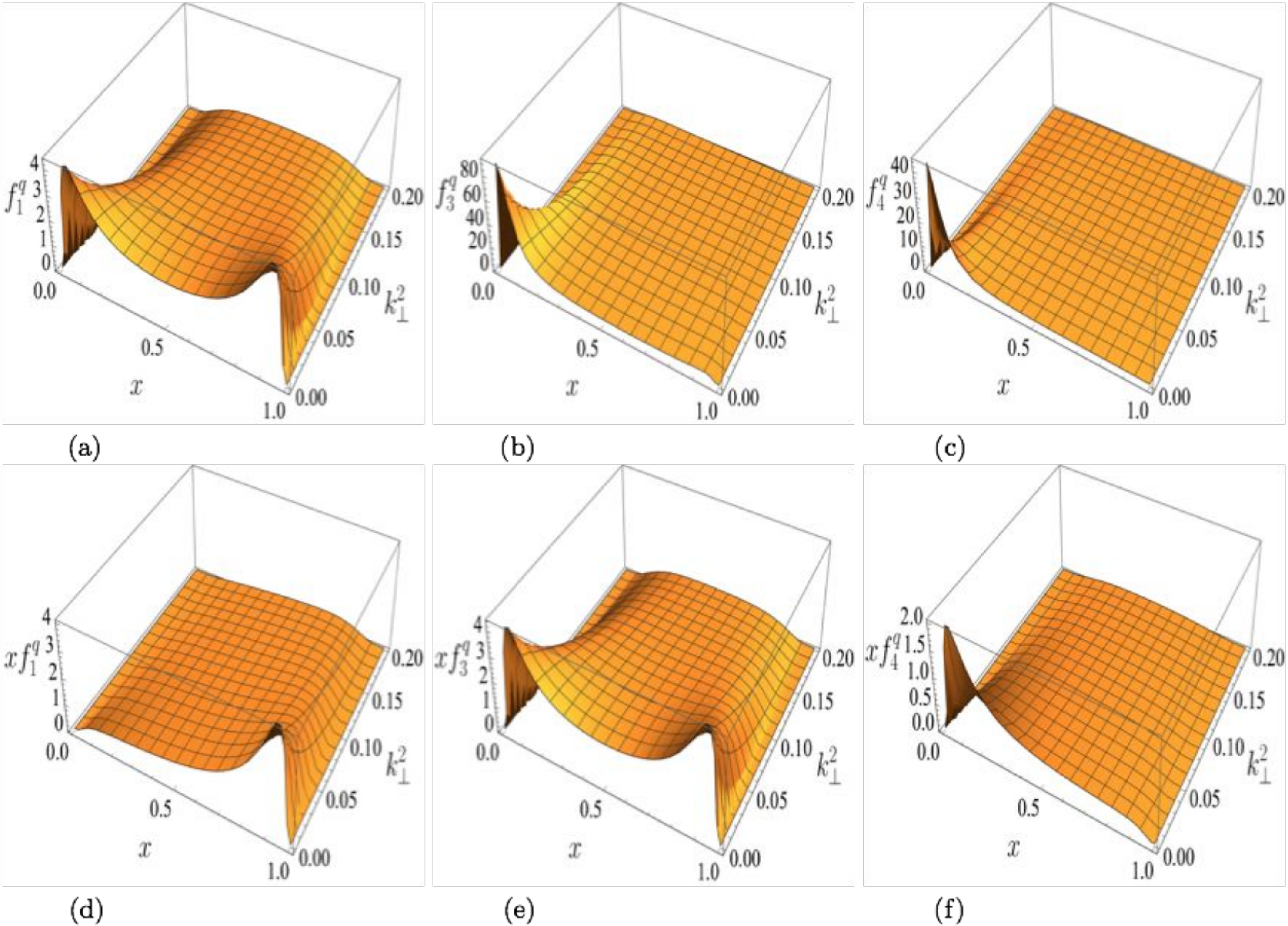
Electromagnetic Form Factor

$$F_\pi^{(\mu)}(Q^2) = \int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \frac{\phi(x,\mathbf{k}_\perp)\phi'(x,\mathbf{k}'_\perp)}{\sqrt{m^2+\mathbf{k}_\perp^2}\sqrt{m^2+\mathbf{k}'^2_\perp}} \mathcal{O}^{(\mu)}_{\rm LFQM}$$

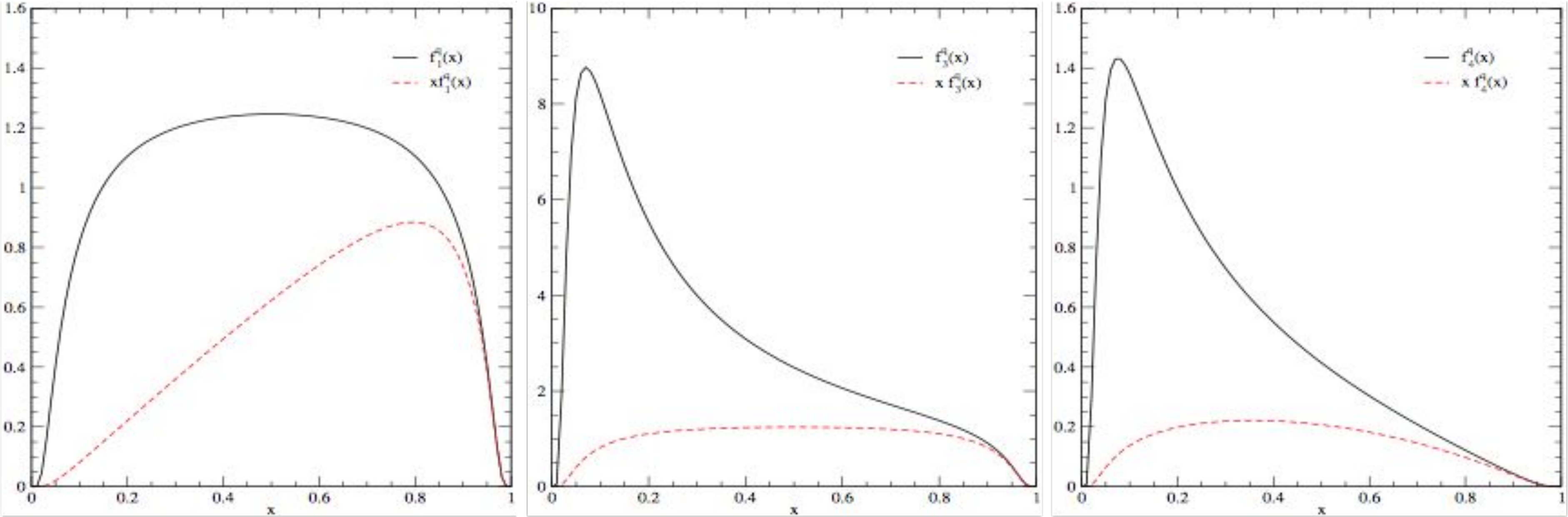
$F_\pi^{(\mu)}$	$\mathcal{O}^{(\mu)}_{\rm LFQM}$	$\mathcal{H}^{(\mu)}_{(\uparrow\to\uparrow)+(\downarrow\to\downarrow)}$	$\mathcal{H}^{(\mu)}_{(\uparrow\to\downarrow)+(\downarrow\to\uparrow)}$
$F_\pi^{(+)}$	$\mathbf{k}_\perp\cdot\mathbf{k}'_\perp + m^2$	$\mathbf{k}_\perp\cdot\mathbf{k}'_\perp + m^2$	0
$F_\pi^{(\perp)}$	$\mathbf{k}_\perp\cdot\mathbf{k}'_\perp + m^2$	$\mathbf{k}_\perp\cdot\mathbf{k}'_\perp + m^2$	0
$F_\pi^{(-)}$	$\frac{2(1-x)\mathbf{q}_\perp^2 M_0^2(\mathbf{k}_\perp\cdot\mathbf{k}'_\perp+m^2+\mathbf{q}_\perp\cdot\mathbf{k}'_\perp)}{x[2M_0'^2\mathbf{q}_\perp^2+\mathbf{q}_\perp^4+(M_0^2-M_0'^2)^2]}$	$\frac{2\mathbf{q}_\perp^2\{(\mathbf{k}_\perp\cdot\mathbf{k}'_\perp+m^2)(\mathbf{k}_\perp^2+\mathbf{k}_\perp\cdot\mathbf{q}_\perp+m^2)+(1-x)(\mathbf{k}_\perp\times\mathbf{q}_\perp)^2\}}{x^2[2M_0'^2\mathbf{q}_\perp^2+\mathbf{q}_\perp^4+(M_0^2-M_0'^2)^2]}$	$\frac{2\mathbf{q}_\perp^2\{(1-x)m^2\mathbf{q}_\perp^2\}}{x^2[2M_0'^2\mathbf{q}_\perp^2+\mathbf{q}_\perp^4+(M_0^2-M_0'^2)^2]}$

$$F_\pi^{(+)} = F_\pi^{(\perp)} = F_\pi^{(-)}$$

See the link between the covariant field theory and the LFQM in HMC and CRJ, PRD110, 014006(2024).



f_1^q
f_3^q
f_4^q
$x f_1^q$
$x f_3^q$
$x f_4^q$
x
$k_\perp^2$
0.0
0.5
1.0
0.00
0.05
0.10
0.15
0.20
(a)
(b)
(c)
(d)
(e)
(f)



$$2\int dx f_4^q(x) = \int dx f_1^q(x) = 1 \qquad x f_3^q(x, \mathbf{k}_\perp) = -f_1^q(x, \mathbf{k}_\perp)$$

Modulo the sign of $f_3^q(x, \mathbf{k}_\perp)$

Conclusions and Outlook

- Link between QCD and LFQM may be feasible as exemplified by the mass gap solution in the ‘t Hooft model interpolation between IFD and LFD.
- LF ZMs appear essential in understanding the constituent mass in LFQM.
- The issue of LF ZMs in LFQM computation of the pion twist 4 TMD is resolved by the consistency with the BT construction of the LFQM.
- Self-consistent LFQM assures the Component and Frame Independence of the physical observables.
- Meson structure studies of LFQM provide useful tools to study the nucleon structures via the convolution with the splitting functions computed by the chiral effective theory.

Back-up

Conformal Symmetry in IFD

1D

	P_0	$\mathfrak{K}_0$	D
P_0	0	$2iD$	iP_0
$\mathfrak{K}_0$	$-2iD$	0	$-i\mathfrak{K}_0$
D	$-iP_0$	$i\mathfrak{K}_0$	0

$$P_0 = i\partial_t$$

$$D = it\partial_t$$

$$\mathfrak{K}_0 = it^2\partial_t$$

2D

	P_0	$\mathfrak{K}_0$	D	$-P_3$	$\mathfrak{K}_3$	K^3
P_0	0	$2iD$	iP_0	0	$2iK^3$	$-iP_3$
$\mathfrak{K}_0$	$-2iD$	0	$-i\mathfrak{K}_0$	$-2iK^3$	0	$-i\mathfrak{K}_3$
D	$-iP_0$	$i\mathfrak{K}_0$	0	iP_3	$i\mathfrak{K}_3$	0
$-P_3$	0	$2iK^3$	$-iP_3$	0	$2iD$	iP_0
$\mathfrak{K}_3$	$-2iK^3$	0	$-i\mathfrak{K}_3$	$-2iD$	0	$-i\mathfrak{K}_0$
K^3	iP_3	$i\mathfrak{K}_3$	0	$-iP_0$	$i\mathfrak{K}_0$	0

Work in progress with Hariprashad Ravikumar et.al.@NCSU group meetings

Conformal Symmetry in LFD

1D

	P_+	$\mathfrak{K}_+$	D_+
P_+	0	$2\sqrt{2}iD_+$	$\sqrt{2}iP_+$
$\mathfrak{K}_+$	$-2\sqrt{2}iD_+$	0	$-\sqrt{2}i\mathfrak{K}_+$
D_+	$-\sqrt{2}iP_+$	$\sqrt{2}i\mathfrak{K}_+$	0

2D

	P_+	$\mathfrak{K}_+$	D_+	P_-	$\mathfrak{K}_-$	D_-
P_+	0	$2\sqrt{2}iD_+$	$\sqrt{2}iP_+$	0	0	0
$\mathfrak{K}_+$	$-2\sqrt{2}iD_+$	0	$-\sqrt{2}i\mathfrak{K}_+$	0	0	0
D_+	$-\sqrt{2}iP_+$	$\sqrt{2}i\mathfrak{K}_+$	0	0	0	0
P_-	0	0	0	0	$2\sqrt{2}iD_-$	$\sqrt{2}iP_-$
$\mathfrak{K}_-$	0	0	0	$-2\sqrt{2}iD_-$	0	$-\sqrt{2}i\mathfrak{K}_-$
D_-	0	0	0	$-\sqrt{2}iP_-$	$\sqrt{2}i\mathfrak{K}_-$	0

$$P_\pm = \frac{P_0 \pm P_3}{\sqrt{2}}, \mathfrak{K}_\pm = \frac{\mathfrak{K}_0 \mp \mathfrak{K}_3}{\sqrt{2}}, \text{ and } D_\pm = \frac{D \mp K^3}{\sqrt{2}}$$

Why Light-Front?

- Distinguished Vacuum Property
- Maximum Number of Kinematic Operators
- Distinguished Conformal Symmetry
  (Work in progress @ NCSU group meetings)
  Length Contraction and Time Dilation are tied with the Spacetime Conformal Symmetry most naturally in LFD.

Extended Wick Rotation

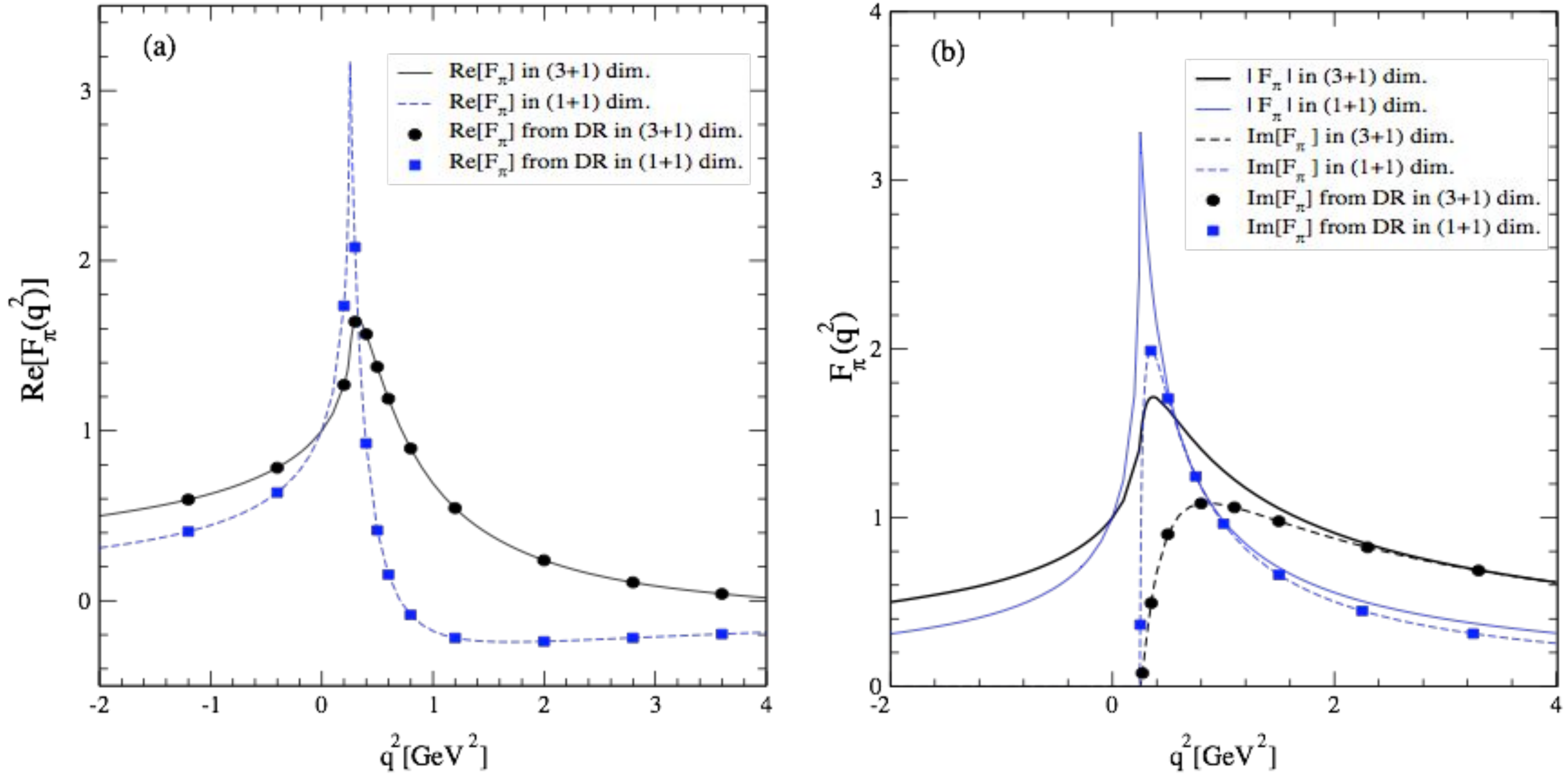
$$p^0 \to \tilde{P}^0 = ip^0 \quad (\delta = 0)$$

$For \quad 0 < \delta < \pi / 4,$

$$p^{\hat{+}} / \sqrt{C} \to \tilde{P}^{\hat{+}} / \sqrt{C} = ip^{\hat{+}} / \sqrt{C}\,.$$

Correspondence to Euclidean Space

$$p'^2_{\hat{-}} = p^2_{\hat{-}} / C \leftrightarrow -\tilde{P}^2$$



Yongwoo Choi, H.-M.Choi, C.-R. Ji and Y. Oh, PRD103,076002(2021)