# Gluon Distribution Functions Inside a Proton From Light-Front Spectator Model

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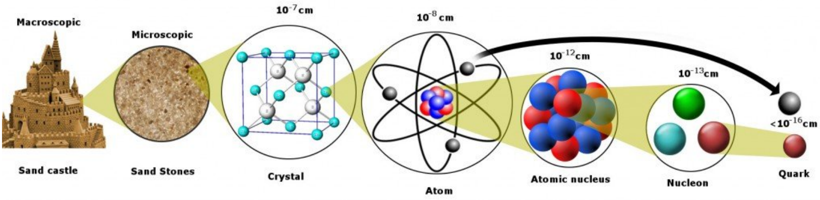
# Overview

1. Introduction
   Elastic scattering, DIS, SIDIS, Drell-Yan, DVCS etc...
2. Light-Front Quantum Chromodynamics
3. Light-Front spectator model
4. Distribution Functions
   - Parton Distribution Functions (PDFs)
   - Transverse Momentum Distributions (TMDs)
   - Generalized Parton Distributions (GPDs)
   - Generalized Transverse Momentum Distributions (GTMDs)
5. Conclusion

# The Standard Model

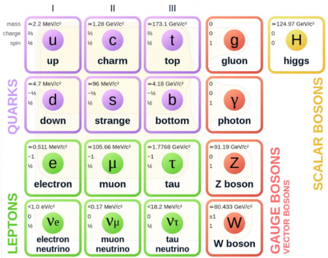
- Matter forms from fundamental particles through the formation of hadrons, which are composite particles made of quarks.



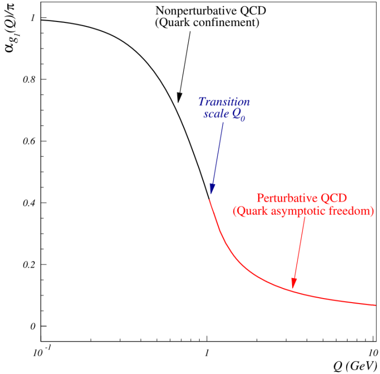
- The Standard Model of particle physics is one of the most successful theories in physics.
- Incomplete theory :
  i. Quantum gravity ?
  ii. Why 3 generations ?
  iii. Origin of $\nu$ masses ?
  iv. Hierarchy ?
  v. Matter-Antimatter Asymmetry ?
  vi. Dark matter & Energy ?

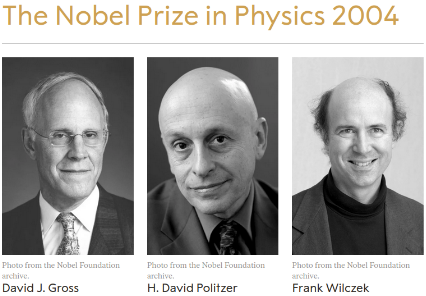
elementary particles force carriers

Fermions Bosons



- Quantum Chromodynamics (QCD) is the theory of strong interaction between quarks and gluons.
- It requiring nonperturbative methods to fully understand the **confinement** and **asymptotic freedom** phenomena.





The Nobel Prize in Physics 2004

Photo from the Nobel Foundation archive.
David J. Gross

Photo from the Nobel Foundation archive.
H. David Politzer

Photo from the Nobel Foundation archive.
Frank Wilczek

**Confinement** : The quarks and gluons are never observed as free particles ; they are always confined within hadrons.

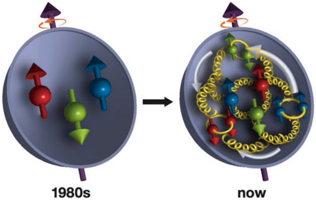
**Asymptotic Freedom** : At very short distances or high energies, the quarks and gluons to interact more weakly and behave almost like free particles.

- Non-Perturbative Methods : Lattice QCD, EFTs : ChPT, LFHQCD.

— H. Fritzsch et al., PLB 47 (1973), D. J. Gross and F. Wilczek, PRL 30 (1973), H. D. Politzer PRD 30 (1973), K. G. Wilson PRD 10 (1974)

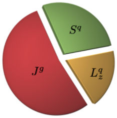
# The Proton Spin

- Nucleon spin decomposition in terms of quark and gluon degrees of freedom : "proton spin crisis".



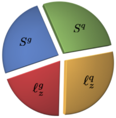
- There are two established approaches to look at the compositions of the proton spin :

Ji (kinetic EMT) Sum Rule



$$\frac{1}{2} = \underbrace{S^q(\mu) + L^q_z(\mu)}_{J^q} + J^g(\mu)$$

Jaffe-Manohar (canonical EMT) Sum Rule



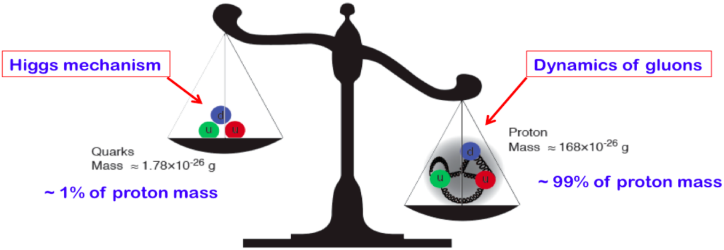
$$\frac{1}{2} = S^q(\mu) + \ell^q_z(\mu) + \ell^g_z(\mu) + S^g(\mu)$$

- Few more recent decompositions : Chen et al. Gauge-Invariant Decomposition, Wakamatsu's Decomposition.

— R.L. Jaffe and A. Manohar, NPB 337 (1990), X. Ji, PRL 78 (1997), Chen Sun Goldman, PRL 103 (2009), M. Wakamatsu, PRD 81 (2010)

# The Proton Mass

- Nucleon mass – dominates the mass of visible mass :



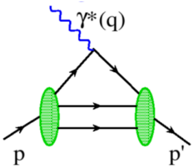
- X. Ji derived a decomposition of the nucleon mass into contributions from quark and gluon kinetic and potential energies, quark masses, and the trace anomaly.

$$M_p = \frac{\langle P|\int d^3x\, T^{00}|P\rangle}{\langle P|P\rangle}\bigg|_{\text{at rest}} = \overbrace{\underbrace{M_q}_{\text{Quark Energy}} + \underbrace{M_g}_{\text{Gluon Energy}}}^{\text{Relativistic motion}} + \overset{\chi\text{ Symmetry Breaking}}{\underset{\text{Quark Mass}}{M_m}} + \overset{\text{Quantum fluctuation}}{\underset{\text{Trace Anomaly}}{M_a}}$$

— X.-D. Ji, PRL (1995), PRD (1995), Pictures courtesy : J. Qui's Talk.

# Elastic Scattering : Nucleon properties

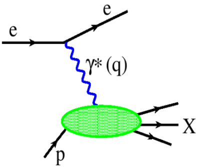
- Elastic scattering describe static properties, such as proton charge distribution and magnetic moment.



$$\langle N(p')|J^{\mu}(0)|N(p)\rangle = \bar{u}(p')\left[\gamma^{\mu}\underbrace{F_1(q^2)} + \frac{i\sigma^{\mu\nu}}{2m_N}q_{\nu}\underbrace{F_2(q^2)}\right]u(p)$$

Dynamical information of the partons, like orbital angular momentum are missing.

- Deep inelastic scattering describes the existence of quasi point like objects (partons) in the proton.

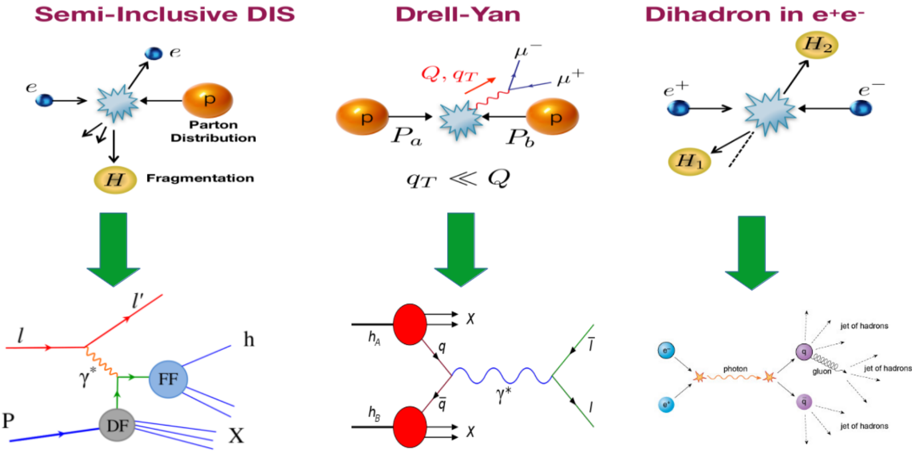


$$\underbrace{q(x)} = \frac{p^+}{4\pi}\int dy^- e^{ixp^+y^-} \times \langle p|\underbrace{\bar{\psi}_q(0)\mathcal{O}\psi_q(y)}|p\rangle\Big|_{y^+=\vec{y}_\perp=0}$$

PDFs : Probability distribution of quarks and gluons with longitudinal momentum fraction $x = p^+/P^+$.

— C. F. Perdrisat et al., Prog. Part. Nucl.Phys. 59 (2007), R. P. Feynman, PRL 23 (1969), J. D. Bjorken et al., Phys. Rev. 185 (1969)

# Semi Inclusive DIS, Drell-Yan



$$d\sigma \propto \hat{\sigma}_{eq\to e'q'} \otimes f_1 \widetilde{\otimes} D_{h/q'}$$

$$d\sigma \propto \hat{\sigma}_{q\bar{q}\to \ell\bar{\ell}} \otimes f_1 \widetilde{\otimes} f_1$$

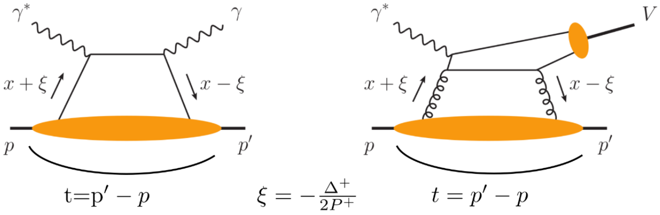
$$d\sigma \propto \hat{\sigma}_{e^+e^-\to ij} \otimes D_{h_1/i} \widetilde{\otimes} D_{h_2/j}$$

- HERMES, COMPASS, and JLab have studied SIDIS measurements, while EIC aim for high-precision SIDIS data.

— Collins (2011), TMD Handbook (2023), P.J. Mulders and R.D. Tangerman (1996), S. Arnold et al. (2009)

# Deeply Virtual Compton Scattering

- Exclusive processes like DVCS or vector meson productions (DVMP) are crucial to study the GPDs.



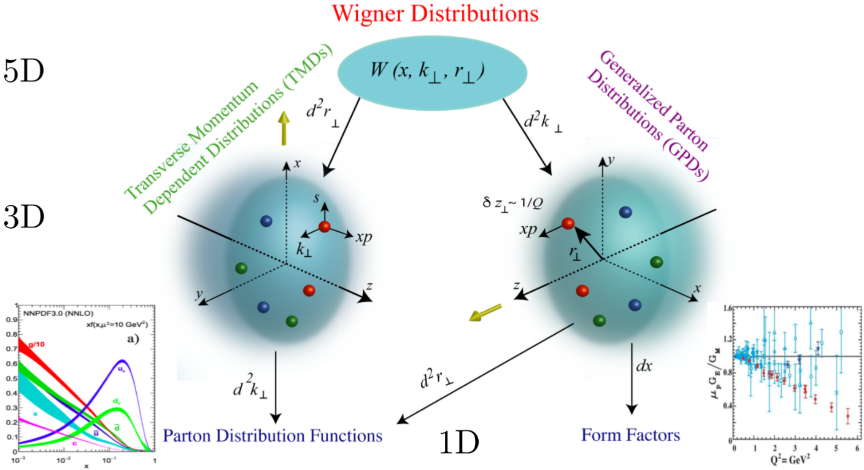
$x$ : longitudinal momentum fraction, $\xi$ : Skewness parameter, $t$ : transverse momentum transfer.

- GPDs encode the informations about the 3D spatial structure of the nucleon as well as the mass, spin and orbital angular momentum of the constituents.
- The Mellin moments of GPDs gives the EM and Gravitational Form Factors (GFFs).
- Many experimentas are going on (HERA H1, COMPASS, ZEUS collaborations, HERMES, JLab etc.) to gain insight into GPDs.

— M. Diehl (2003) ; A. V. Belitsky (2005) ; M. V. Polyakov (2001) ; M. Guidal (2013)

# GTMDs, Wigner Distributions & Hadron Tomography

- GTMDs gives the most complete informations on partonic structure of the nucleon $\Rightarrow$ "Mother Distributions".

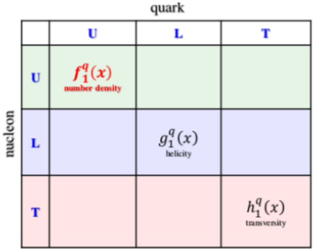


- Spatial Tomography of the Nucleon $\Rightarrow$ GPDs, & Momentum Tomography of the Nucleon $\Rightarrow$ TMDs.

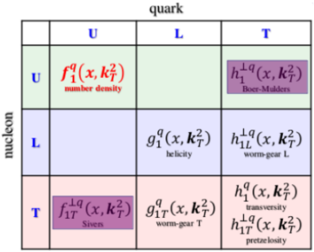
— J. Dudek et al., [EPJA, 2012] ; X. Ji [PRL, 2003] ; A. V. Belitsky, X.-d. Ji *et al.*. [PRD, 2004].

# Nucleon spin structure : collinear approach ↔ TMDs

- At leading twist there are : 8 proton TMDs : $6 \to$ T-even & $2 \to$ T-odd.

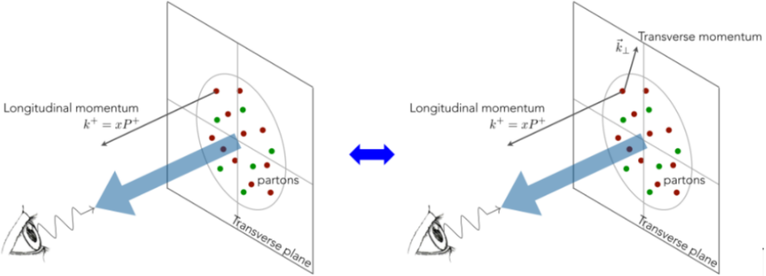


| | | quark | | |
|---|---|---|---|---|
| | | U | L | T |
| nucleon | U | $f_1^q(x)$<br>number density | | |
| | L | | $g_1^q(x)$<br>helicity | |
| | T | | | $h_1^q(x)$<br>transversity |



| | | quark | | |
|---|---|---|---|---|
| | | U | L | T |
| nucleon | U | $f_1^q(x, k_T^2)$<br>number density | | $h_1^{\perp q}(x, k_T^2)$<br>Boer-Mulders |
| | L | | $g_1^q(x, k_T^2)$<br>helicity | $h_{1L}^{\perp q}(x, k_T^2)$<br>worm-gear L |
| | T | $f_{1T}^{\perp q}(x, k_T^2)$<br>Sivers | $g_{1T}^q(x, k_T^2)$<br>worm-gear T | $h_1^q(x, k_T^2)$<br>transversity<br>$h_{1T}^{\perp q}(x, k_T^2)$<br>pretzelosity |

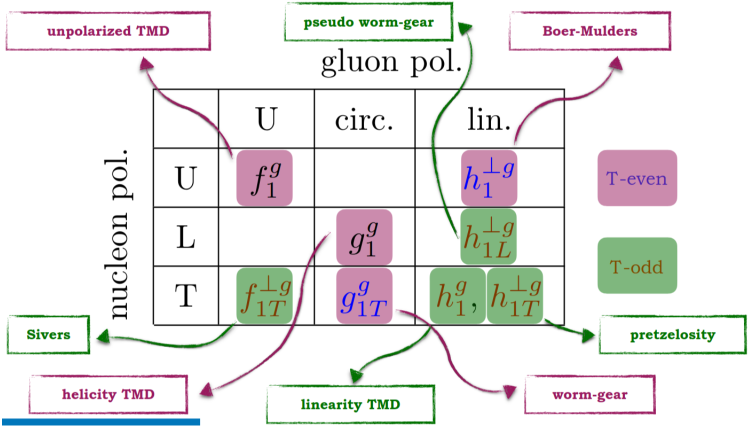
PDFs – universal (process independent) objects; T-odd PDFs – conditionally universal



Picture credit : Alessandro bacchetta

# Gluon TMDs

- There are eight gluon (four of them are T-even and four are T-odd) TMDs at lading twist.



| | U | circ. | lin. |
|---|---|---|---|
| U | $f_1^g$ | | $h_1^{\perp g}$ |
| L | | $g_1^g$ | $h_{1L}^{\perp g}$ |
| T | $f_{1T}^{\perp g}$ | $g_{1T}^g$ | $h_1^g, h_{1T}^{\perp g}$ |

— D. W. Sivers (1990) ; D. Boer and P. J. Mulders (1998) ; D. S. Hwang (2013) ; A. Bacchetta (2020).

# Light-Front QCD

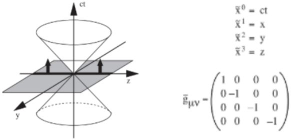
- Light Front QCD is an *ab initio* approach to study the strongly interacting system. [ —Dirac, 1949]
- Hamiltonian Formulation : $P^-P^+|\psi\rangle = M^2|\psi\rangle$, and has simplified Vacuum Structure.
- Light-Front coordinates : $x^\mu = (x^+, x^-, x^\perp)$ ; $k^\mu = (k^+, k^-, k^\perp)$ ;

## Instant form

- All measurements are made at fixed $t$ i.e. at $x^0 = 0$.
- Energy-momentum dispersion relation $p^0 = \sqrt{\vec{p}^2 + m^2}$.
- Vacuum is infinitely complex.

## Front form

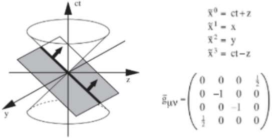
- All measurements are made at fixed light-cone time $x^+$ i.e. at $x^+ = x^0 + x^3 = 0$.
- Energy-momentum dispersion relation $p^- = \frac{\vec{p}_\perp^2 + m^2}{p^+}$.
- Vacuum is simple, as fluctuations are absent.



$\bar{x}^0 = ct$
$\bar{x}^1 = x$
$\bar{x}^2 = y$
$\bar{x}^3 = z$

$$\bar{g}_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

The instant form



$\bar{x}^0 = ct+z$
$\bar{x}^1 = x$
$\bar{x}^2 = y$
$\bar{x}^3 = ct-z$

$$\bar{g}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \frac{1}{2} & 0 & 0 & 0 \end{pmatrix}$$

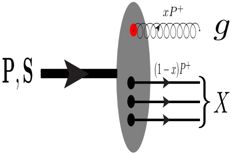
The front form

- The choice of light-front gauge $A^+ = 0$, eliminate certain degrees of freedom.

— P.A.M. Dirac (RMP, 1949) ; A. Harindranath (1996) ; Stanley J. Brodsky (2008)

# Light-Front spectator model

- In this model nucleons $(p = |g(uud)\rangle, n = |g(udd)\rangle)$ are considered as a bound state of an active gluon and a spin-1/2 spectator system.



- The two particle Fock-state expansion of nucleon with $J_z = \uparrow (\downarrow)$

$$|P, \uparrow (\downarrow)\rangle = \int \frac{dx d^2\mathbf{p}_\perp}{2(2\pi)^3\sqrt{x(1-x)}} \sum_{\lambda_g, \lambda_X} \psi^{\uparrow(\downarrow)}_{\lambda_g \lambda_X}(x, \mathbf{p}_\perp) |\lambda_g, \lambda_X; xP^+, \mathbf{p}_\perp\rangle$$

$\psi^{\uparrow(\downarrow)}_{\lambda_g \Lambda_X}(x, \mathbf{p}_\perp) \rightarrow$ The light-front wavefunctions corresponding two particle state.

$$\psi^{\uparrow(\downarrow)}_{\lambda_g \lambda_X}(x, \mathbf{p}_\perp) = N_g f(x, \mathbf{p}_\perp, \lambda_g, \lambda_X) \varphi(x, \mathbf{p}_\perp) \, ,$$

— D. Chakrabarti, T. Maji (2015) ; S. J. Brodsky, D. S. Hwang, BQ Ma, and Ivan Schmidt (2001)

- The Light Front Wave Functions :

For $J_z =\uparrow$,

$$\psi^{\uparrow}_{+1+\frac{1}{2}}(x,\mathbf{p}_\perp) = -\sqrt{2}\frac{(-p^1_\perp + ip^2_\perp)}{x(1-x)}\varphi(x,\mathbf{p}^2_\perp),$$

$$\psi^{\uparrow}_{+1-\frac{1}{2}}(x,\mathbf{p}_\perp) = -\sqrt{2}\left(M - \frac{M_X}{(1-x)}\right)\varphi(x,\mathbf{p}^2_\perp),$$

$$\psi^{\uparrow}_{-1+\frac{1}{2}}(x,\mathbf{p}_\perp) = -\sqrt{2}\frac{(p^1_\perp + ip^2_\perp)}{x}\varphi(x,\mathbf{p}^2_\perp),$$

$$\psi^{\uparrow}_{-1-\frac{1}{2}}(x,\mathbf{p}_\perp) = 0,$$

For $J_z =\downarrow$.

$$\psi^{\downarrow}_{+1+\frac{1}{2}}(x,\mathbf{p}_\perp) = 0,$$

$$\psi^{\downarrow}_{+1-\frac{1}{2}}(x,\mathbf{p}_\perp) = -\sqrt{2}\frac{(-p^1_\perp + ip^2_\perp)}{x}\varphi(x,\mathbf{p}^2_\perp),$$

$$\psi^{\downarrow}_{-1+\frac{1}{2}}(x,\mathbf{p}_\perp) = -\sqrt{2}\left(M - \frac{M_X}{(1-x)}\right)\varphi(x,\mathbf{p}^2_\perp),$$

$$\psi^{\downarrow}_{-1-\frac{1}{2}}(x,\mathbf{p}_\perp) = -\sqrt{2}\frac{(p^1_\perp + ip^2_\perp)}{x(1-x)}\varphi(x,\mathbf{p}^2_\perp).$$

- Modified soft-wall AdS/QCD wave function for two particle bound state :

$$\varphi(x,\mathbf{p}^2_\perp) = N_g\frac{4\pi}{\kappa}\sqrt{\frac{\log[1/(1-x)]}{x}}x^b(1-x)^a\exp\left[-\frac{\log[1/(1-x)]}{2\kappa^2x^2}\mathbf{p}^2_\perp\right]$$

- The AdS/QCD scale parameter $\kappa = 0.4$ GeV, and the model parameters $N_g,\ a,\ b$ are fitted with NNPDF3.0 unpolarized gluon PDFs at $Q_0 = 2$ GeV.

— T. Gutsche, V. E. Lyubovitskij, Ivan Schmidt, A. Vega (2014) ; Brodsky-Teramond (2009), D. Chakrabarti, T. Maji (2015)

# Parton Distribution Functions

- The integrated gluon-gluon correlation function :

$$\Phi^{g[ij]}(x;S) = \frac{1}{xP^+}\int \frac{d\xi^-}{2\pi}\, e^{ik\cdot\xi} \left\langle P;S \middle| F_a^{+j}(0)\ \mathcal{W}_{+\infty,ab}(0;\xi)\ F_b^{+i}(\xi) \middle| P;S \right\rangle \Big|_{\xi^+=0^+,\xi_\perp=0_\perp},$$

- Unpolarized gluon parton distribution function :

$$f_1^g(x) = 2N_g^2 x^{2b+1}(1-x)^{2a-2}\left[\kappa^2\frac{(1+(1-x)^2)}{\log[1/(1-x)]} + (M(1-x)-M_X)^2\right].$$

- The average longitudinal momentu of the gluon :

$$\langle x\rangle_g = \int_{0.001}^{1} dx\, x f_1^g(x)\,,$$

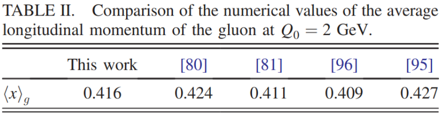
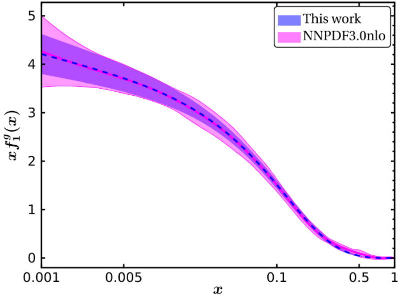


TABLE II. Comparison of the numerical values of the average longitudinal momentum of the gluon at $Q_0 = 2$ GeV.

| | This work | [80] | [81] | [96] | [95] |
|---|---|---|---|---|---|
| $\langle x\rangle_g$ | 0.416 | 0.424 | 0.411 | 0.409 | 0.427 |



- Considered 100 replica of the NNPDF3.0 data within the interval $0.001 < x < 1$ and excluded the small-x region due to large uncertainty.

— [80] A. Bacchetta *et al.*, EPJC 80 (2020), [81] Z. Lu and B.-Q. Ma, PRD 94(2016), [95] C. Alexandrou *et al.* ETM Collaboration, PRD 101 (2020), D. Chakrabarti *et al.* PRD 108 (2024)

- Gluon helicity parton distribution function ⇒ Gluon spin contribution to the total nucleon spin : $\Delta G = \int_0^1 dx g_{1L}^g(x)$

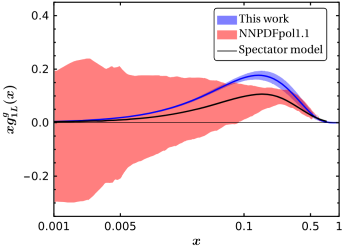
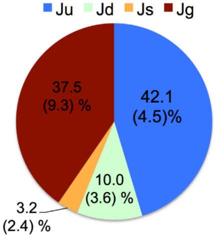
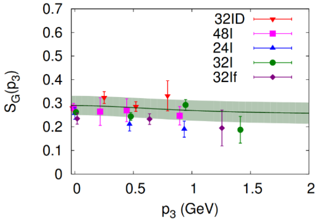


TABLE III. Comparison of the numerical values of the gluon spin contribution with the available data at $Q_0 = 2$ GeV.

| Gluon helicity | Central value | Our predictions |
|---|---|---|
| $\Delta G = \int_{0.05}^{0.3} dx \Delta g(x)$ | 0.20 [53] | $0.28^{+0.047}_{-0.037}$ |
| $\Delta G = \int_{0.05}^{0.2} dx \Delta g(x)$ | 0.23(6) [45] | $0.22^{+0.033}_{-0.024}$ |
| $\Delta G = \int_{0.05}^{1} dx \Delta g(x)$ | 0.19(6) [41] | $0.326^{+0.066}_{-0.050}$ |

- State-of-the-art lattice study on the proton spin :





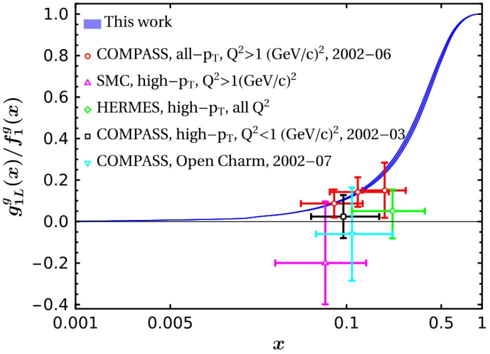
— [53] PHENIX Collaboration, PRL(2009), [45] NNPDF Collaboration, NPB(2014), [41] DSSV PRL 113(2014), X. Ji, F. Yuan and Y. Zhao, Nature Reviews Physics 3, 65 (2021)

- Our model results of gluon helicity asymmetry ratio :

$$\frac{g_{1L}^{g}(x)}{f_1^{g}(x)} = \frac{\left[\kappa^2 \frac{(1-(1-x)^2)}{\log[1/(1-x)]} + (M(1-x) - M_X)^2\right]}{\left[\kappa^2 \frac{(1+(1-x)^2)}{\log[1/(1-x)]} + (M(1-x) - M_X)^2\right]}$$

- The model-independent pQCD constraints on gluon helicity asymmetry ratio :

$$\lim_{x\to 0} \frac{g_{1L}^{g}(x)}{f_1^{g}(x)} = 0, \qquad \text{and} \qquad \lim_{x\to 1} \frac{g_{1L}^{g}(x)}{f_1^{g}(x)} = 1.$$



- The STAR and PHENIX collaborations measured the double-helicity asymmetry $A_{LL}$, finding data that supports a positive gluon spin contribution while ruling out the negative scenario.

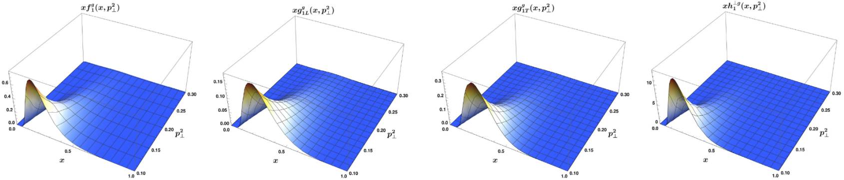
— D. de Florian *et al.*, [PRL, 2014], S. J. Brodsky *et al.* [PLB 1990], BG, D. Chakrabarti *et al.* [PRD, 2023].

# Gluon Transverse Momentum Distributions

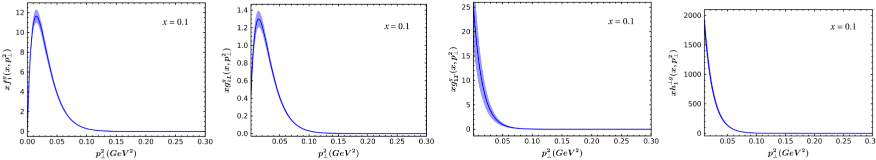
- In the light front formalism, the unintegrated gluon correlation function for leading twist gluon TMDs in the SIDIS process :

$$\Phi^{g[ij]}(x, \mathbf{p}_\perp; S) = \frac{1}{xP^+} \int \frac{d\xi^-}{2\pi} \frac{d^2\xi_\perp}{(2\pi)^2} e^{ik\cdot\xi} \left\langle P; S \right| F_a^{+j}(0)\ \mathcal{W}_{+\infty,ab}(0;\xi)\ F_b^{+i}(\xi) \left| P; S \right\rangle \Big|_{\xi^+=0^+} ,$$

- The gluon TMDs : $f_1^g$, $g_{1L}^g$, $g_{1T}^g$ and $h_1^{\perp g}$ are obtained as the overlap representation of the proton LFWFs.



- T-even gluon TMDs as a function of $\mathbf{p}_\perp^2$ at $x = 0.1$.

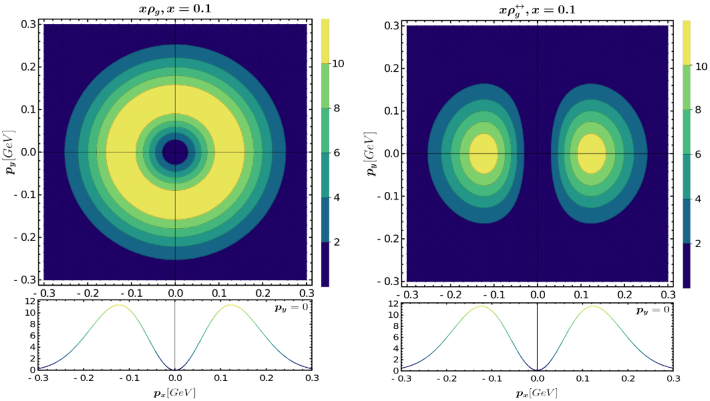


— A. Bacchetta *et al.* (EPJC, 2020) ; S. Meissner *et al.* (PRD, 2007), BLFQ Collaboration (PLB, 2024)

# Gluon densities in transverse momentum plane

- The unpolarized and linearly polarized gluon density in an unpolarized nucleon :

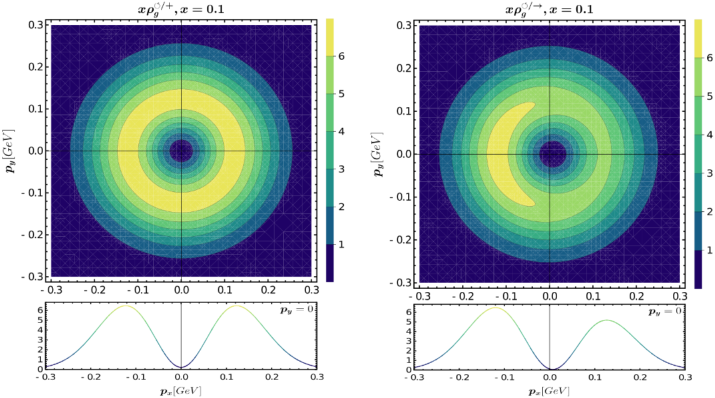
$$x\rho_g(x,p_x,p_y) = xf_1^g(x,\mathbf{p}_\perp^2)\,, \quad x\rho_g^{\leftrightarrow}(x,p_x,p_y) = \frac{1}{2}\left[xf_1^g(x,\mathbf{p}_\perp^2) + \frac{p_x^2-p_y^2}{2M^2}xh_1^{\perp g}(x,\mathbf{p}_\perp^2)\right]$$



— A. Bacchetta *et al.* (EPJC, 2020) ; D. Chakrabarti *et al.* (PRD, 2023), BLFQ Collaboration (PLB, 2024)

- The circularly polarized gluon density in a longitudinally and transversely polarized nucleon :

$$x\rho_g^{\circlearrowleft/+}(x, p_x, p_y) = \frac{1}{2}\left[x f_1^g(x, \mathbf{p}_\perp^2) + x g_{1L}^g(x, \mathbf{p}_\perp^2)\right], \qquad x\rho_g^{\circlearrowleft/\leftrightarrow}(x, p_x, p_y) = \frac{1}{2}\left[x f_1^g(x, \mathbf{p}_\perp^2) - \frac{p_x}{M} x g_{1T}^g(x, \mathbf{p}_\perp^2)\right]$$

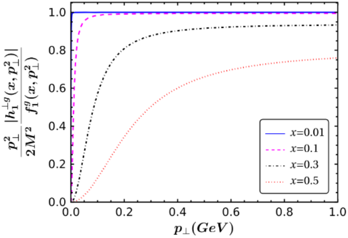


— A. Bacchetta *et al.* (EPJC, 2020) ; D. Chakrabarti *et al.* (PRD, 2023), BLFQ Collaboration (PLB, 2024)

# Positivity Bounds and Relations among Gluon TMDs

- Model-independent positivity bound relation :

$$
\begin{aligned}
&f_1^g(x,\mathbf{p}_\perp^2) > 0,\\
&f_1^g(x,\mathbf{p}_\perp^2) \geq |g_{1L}^g(x,\mathbf{p}_\perp^2)|\,,\\
&f_1^g(x,\mathbf{p}_\perp^2) \geq \frac{|\mathbf{p}_\perp|}{M}|g_{1T}^g(x,\mathbf{p}_\perp^2)|\,,\\
&f_1^g(x,\mathbf{p}_\perp^2) \geq \frac{|\mathbf{p}_\perp|^2}{2M^2}|h_1^{\perp g}(x,\mathbf{p}_\perp^2)|.
\end{aligned}
$$



Mulders-Rodrigues inequalities :

$$
f_1^g \geq \sqrt{[g_{1L}^g]^2 + \left[\frac{|\mathbf{p}_\perp|}{M}g_{1T}^g\right]^2}\,;\; f_1^g \geq \sqrt{[g_{1L}^g]^2 + \left[\frac{\mathbf{p}_\perp^2}{2M^2}h_1^{\perp g}\right]^2}\,;\; f_1^g \geq \sqrt{\left[\frac{|\mathbf{p}_\perp|}{M}g_{1T}^g\right]^2 + \left[\frac{\mathbf{p}_\perp^2}{2M^2}h_1^{\perp g}\right]^2}.
$$

- Generalized sum rule for all the T-even TMDs :

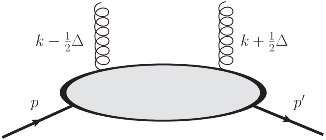
$$
\boxed{[f_1^g(x,\mathbf{p}_\perp^2)]^2 = [g_{1L}^g(x,\mathbf{p}_\perp^2)]^2 + \left[\frac{|\mathbf{p}_\perp|}{M}g_{1T}^g(x,\mathbf{p}_\perp^2)\right]^2 + \left[\frac{\mathbf{p}_\perp^2}{2M^2}h_1^{\perp g}(x,\mathbf{p}_\perp^2)\right]^2}
$$

— V. E. Lyubovitskij *et al.* (PRD, 2021) ; D. Chakrabarti *et al.* (PRD, 2023)

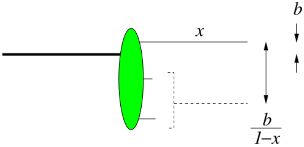
# Generalized Parton Distributions

- For the leading-twist gluon GPDs, the light-cone correlator :

$$F^{[\Gamma]}\left(x,\Delta,S,S'\right) = \frac{1}{P^+}\int \frac{dz^-}{2\pi} e^{ip.z}\left\langle P';S'\left| F^{+i}(-\tfrac{z}{2})\, F^{+i}(\tfrac{z}{2})\right| P;S\right\rangle\Big|_{z^+=0,z_\perp=0},$$



- Fourier transform of GPDs with respect to $\Delta_\perp$ yields the Impact Parameter Distributions (IPDs).



- IPDs provides a spatial probability distribution of partons within the hadron in the transverse plane.
- Within the forward limit $(\xi = 0, t = 0)$ : GPDs $\Rightarrow$ PDFs.

— M. Diehl (2003) ; A. V. Belitsky (2005) ; M. V. Polyakov (2001) ; M. Guidal (2013)

# Skewed ($\xi \neq 0$) GPDs

- Within the light-front gauge, there are four chiral-even and four are chiral odd GPDs at leading twist,

Chiral Even GPDs :

$$F^g(x,\Delta;\lambda,\lambda') = \frac{1}{2P^+}\bar{u}(p',\lambda')\left[H^g\,\gamma^+ + E^g\,\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}\right]u(p,\lambda),$$

$$\tilde{F}^g(x,\Delta;\lambda,\lambda') = \frac{1}{2P^+}\bar{u}(p',\lambda')\left[\tilde{H}^g\,\gamma^+\gamma_5 + \tilde{E}^g\,\frac{\gamma_5\Delta^+}{2M}\right]u(p,\lambda),$$

Chiral Odd GPDs :

$$\tilde{F}_T^{g,ij}(x,\Delta;\lambda,\lambda') = \mathbf{S}\,\frac{1}{2P^+}\,\frac{P^+\Delta^j - \Delta^+P^j}{2MP^+}\bar{u}(p',\lambda')\Bigg[H_T^g\,i\sigma^{+i}$$

$$+\tilde{H}_T^g\,\frac{P^+\Delta^i - \Delta^+P^i}{M^2} + E_T^g\,\frac{\gamma^+\Delta^i - \Delta^+\gamma^i}{2M} + \tilde{E}_T^g\,\frac{\gamma^+P^i - P^+\gamma^i}{M}\Bigg]u(p,\lambda),$$

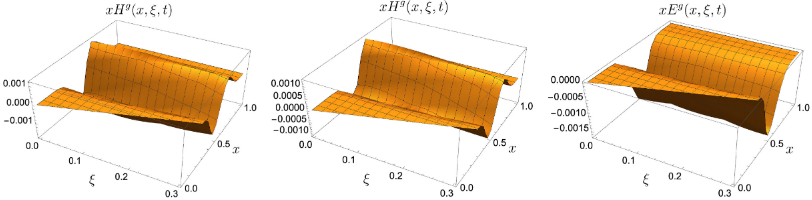
- Kinematical variables :

$$P = \frac{1}{2}(p+p'), \qquad \Delta = p' - p, \qquad \xi = \frac{p^+ - p'^+}{p^+ + p'^+},$$

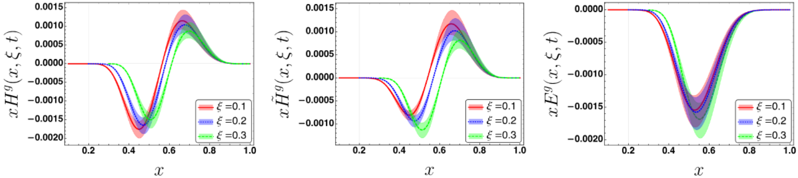
— M. Diehl, EPJC 19 (2001), B. Kriesten *et al.*, PRD 105 (2022), Xiangdong Ji *et al.* PRD 109 (2024), D. Chakrabarti *et al.*, PRD 96 (207).

# Skewed gluon GPDs

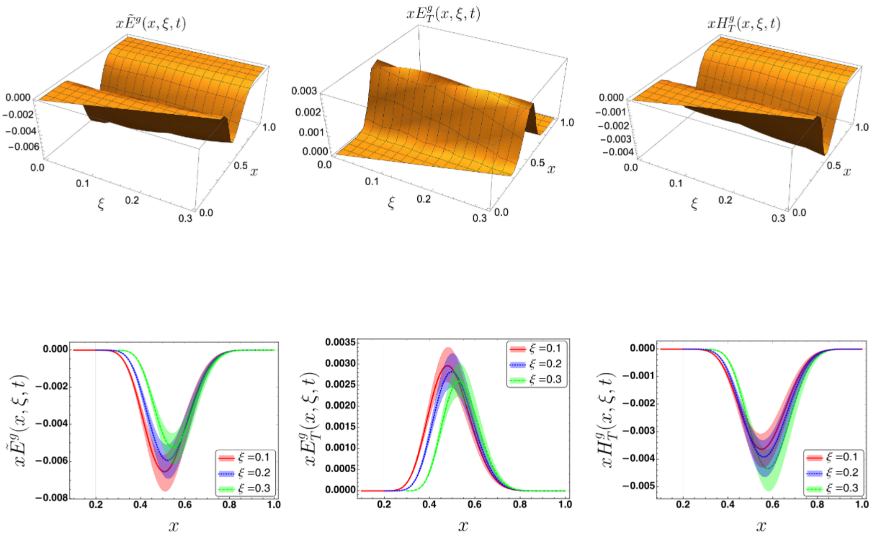
- 3D representation of gluon GPDs as a function of $x$ and $\xi$ for fixed transverse momentum transfer $-|t| = 3$ GeV$^2$.



- 2D representation of gluon GPDs as a function of $x$ for fixed $\xi$ and transverse momentum transfer $-|t| = 3$ GeV$^2$.



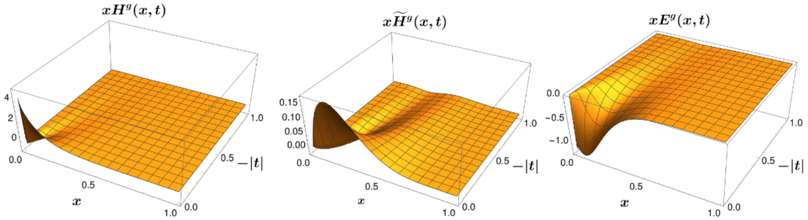
— D. Chakrabarti, BG *et al.*, [PRD 109, 2024]



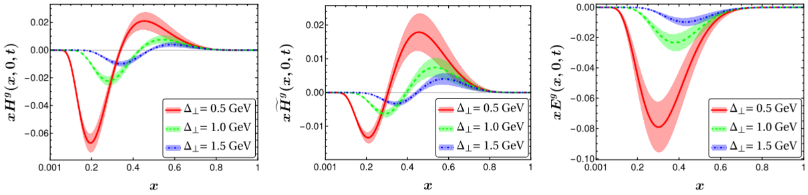
— D. Chakrabarti, BG *et al.*, [PRD 109, 2024]

# Non-skewed gluon GPDs

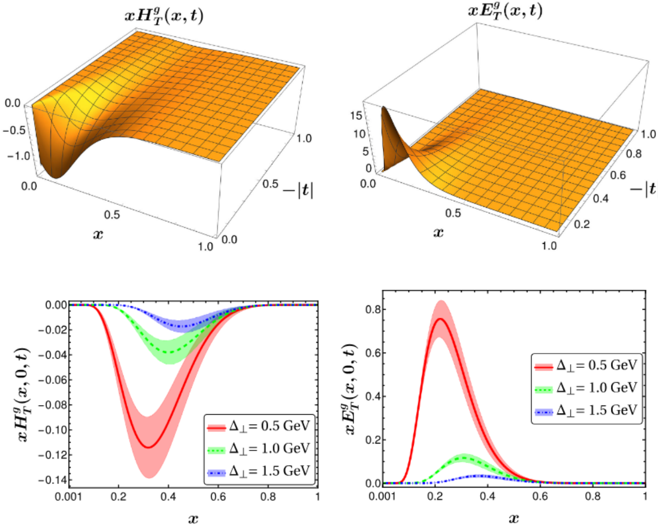
- 3D representation of zero skewed ($\xi = 0$) gluon GPDs as a function of $x$ and $-|t|$.



- 2D representation of zero skewed ($\xi = 0$) gluon GPDs as a function of $x$ at avrious transverse momentum transfers.



— D. Chakrabarti, BG *et al.*, [PRD 109, 2024], Z.Lu *et al.*, [PRD 108, 2023], J.P Vary *et al.* BLFQ, [PLB, 2023]



- Non-skewed gluon GPDs are consistent with similar kind of Spectator model results.

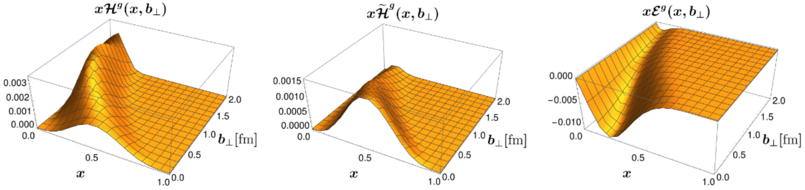
— D. Chakrabarti, BG *et al.*, [PRD 109, 2024], Z.Lu *et al.*, [PRD 108, 2023], J.P Vary *et al.* BLFQ, [PLB, 2023]

# Impact parameter parton distributions

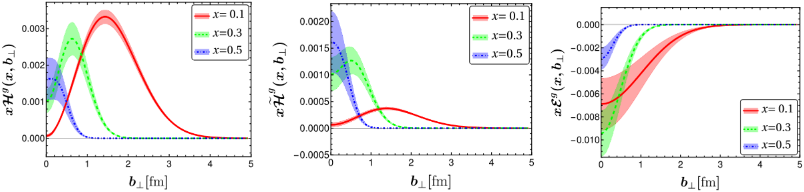
- IPDs can be obtained by taking the two-dimensional Fourier transformation of GPDs.

$$\chi(x, b_\perp) = \frac{1}{2\pi}\int \mathrm{d}^2\Delta e^{-i\Delta_\perp . b_\perp}\chi(x, \Delta_\perp)$$

- 3D representation of zero skewed ($\xi = 0$) gluon IPDs as a function of $x$ and $b_\perp$.

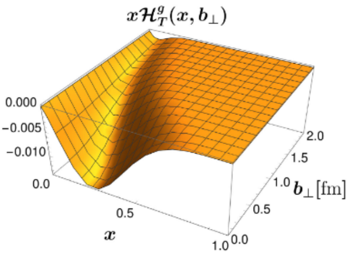


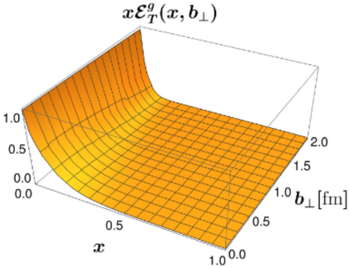
- In contrast to GPDs $H_g$, IPDs $\mathcal{H}_g$ have a probabilistic interpretation and follow positivity constraints.

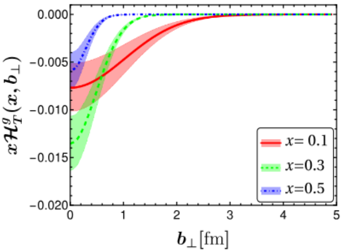


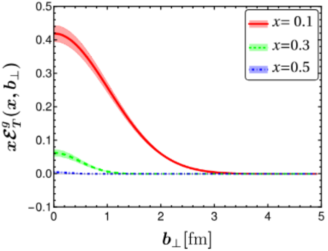
— D. Chakrabarti, BG *et al.*, [PRD 109, 2024], Z.Lu *et al.*, [PRD 108, 2023], J.P Vary *et al.* BLFQ, [PLB,

# Impact parameter parton distributions : Chiral odd









— D. Chakrabarti, BG *et al.*, [PRD 109, 2024], Z.Lu *et al.*, [PRD 108, 2023], J.P Vary *et al.* BLFQ, [PLB, 2023]

# Applications gluon of GPDs : Gluon kinetic OAM

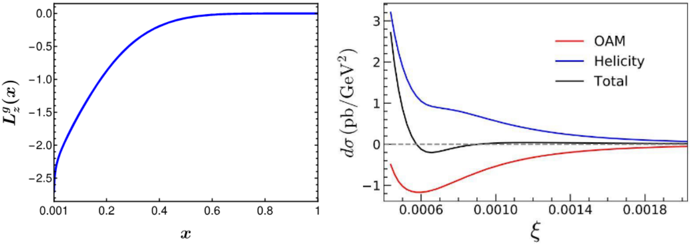
- Ji sum rule : Total gluon angular momentum ($J_z^g$) → Moments of the gluon GPDs $H_g$ and $E_g$.

$$J_z^g = \frac{1}{2}\int \mathrm{d}x x\Big[H_g(x,0,0) + E_g(x,0,0)\Big]$$

LFSPM : $J_z^g = 0.058$ ($\simeq$ 11%) $\sim$ BLFQ : $J_z^g = 0.066$ ($\simeq$ 13%). While, Lattice QCD predictions are $J_z^g = 0.189(46)(10)$ ($\simeq$ 38%).

- The gluon kinetic OAM in the light-cone gauge :

$$L_z^g = \int \mathrm{d}x\Big\{\frac{1}{2}x[H_g(x,0,0) + E_g(x,\xi=0,0)] - \tilde{H}_g(x,0,0)\Big\}$$

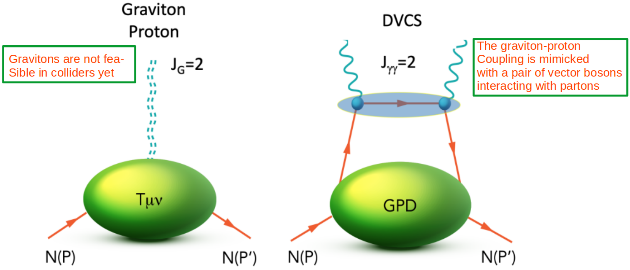


In our model, the kinetic OAM is $L_z^g = -0.42$, compared to $L_z^g = -0.12$ in a similar spectator model.

— S. Bhattacharya *et al.*, PRL 128 (2022), B. Gurjar *et al.* PRD 109 (2024), C. Tan *et al.* PRD 108 (2023)

# Gravitational Form Factors & GPDs

- Encoded informations : Spin, angular momentum and energy densities, mechanical properties : pressure and force distributions etc.



Gravitons are not fea-
Sible in colliders yet

The graviton-proton
Coupling is mimicked
with a pair of vector bosons
interacting with partons

- Parametrization of matrix element in terms of GFFs :

$$\langle P'|T_i^{\mu\nu}(0)|P\rangle = \bar{U}'\left[-B_i(q^2)\frac{\bar{P}^\mu\bar{P}^\nu}{M} + (A_i(q^2)+B_i(q^2))\frac{1}{2}(\gamma^\mu\bar{P}^\nu+\gamma^\nu\bar{P}^\mu)+ C_i(q^2)\frac{q^\mu q^\nu - q^2 g^{\mu\nu}}{M} + \bar{C}_i(q^2)Mg^{\mu\nu}\right]U$$

Momentum sum rule: $\sum_i A^i(0) = 1$

$$\mathrm{A}^i(t) = \int \mathrm{d}x x H^i(x,0,t).$$

Gravitomagnetic moment sum rule: $\sum_i B^i(0) = 0\,,$

$$\mathrm{B}^i(t) = \int \mathrm{d}x x E^i(x,0,t).$$

Spin sum rule: $\mathrm{J}^i = \frac{1}{2}[A^i(0)+B^i(0)]$

$$J^i = \frac{1}{2}\int \mathrm{d}x x[H^i(x,0,0)+E^i(x,0,0)]$$

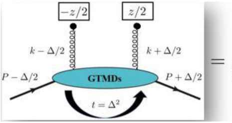
Mechanical properties: Shear and pressure

$$D(t) = 4C(t)$$

— Burkert *et al.*, [Rev. Mod. Phys. 95, 2023], X. Ji, [PRL 78, 1997], Polyakov, [Int.J.Mod.Phys.A 33 (2018)]

# Gluon GTMDs and Canonical OAM

- Parametrization of Gluon GTMD correlator.



$$= \frac{1}{2M}\,\bar{u}(p',\lambda')\left[\boldsymbol{F^g_{1,1}} + \frac{i\sigma^{i+}k^i_\perp}{P^+}\,\boldsymbol{F^g_{1,2}} + \frac{i\,\sigma^{i+}\Delta^i_\perp}{P^+}\,\boldsymbol{F^g_{1,3}} + \frac{i\sigma^{ij}k^i_\perp\Delta^j_\perp}{M^2}\,\boldsymbol{F^g_{1,4}}\right]u(p,\lambda)$$

- In the forward limit, i.e., at $\xi = 0, \Delta_\perp = 0$,

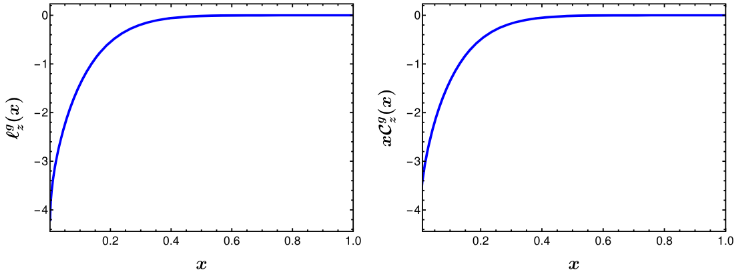
$$f^g_1(x) = \int d^2\mathbf{p}_\perp F^g_{1,1}(x,0,\mathbf{p}_\perp,0,0),\;\; \ell^g_z(x) = -\int d^2\mathbf{p}_\perp \frac{\mathbf{p}^2_\perp}{M^2} F^g_{1,4}(x,0,\mathbf{p}_\perp,0,0).$$

$$\Delta G = \int dx d^2\mathbf{p}_\perp G^g_{1,4}(x,0,\mathbf{p}_\perp,0,0).\;\; C^g_z(x) = \int d^2\mathbf{p}_\perp \frac{\mathbf{p}^2_\perp}{M^2} G^g_{1,1}(x,0,\mathbf{p}_\perp,0,0)$$

- Within the $\xi \to 0$ limit, only $F^g_{1,4}$ term contributes in the gluon OAM. While, $G^g_{1,1}$ and $G^g_{1,4}$ are related to the polarized GTMDs correlator.

— Meissner, Metz, Schlegel, [JHEP 2009], Lorc ´e, Pasquini, [JHEP 09, 2013]

- Gluon canonical orbital angular momentum $\ell_z^g$ and the spin-orbit correlation function $\mathcal{C}_z^g$,



- We found the gluon canonical OAM to be $\ell_z^g = -0.38$, consistent with the another spectator model result of $\ell_z^g = -0.33$, supporting the sign and magnitude reported in the literature.

- Spin-orbit correlation factor $\mathcal{C}_z^g$ also found to be negative in our model, which implies that the gluon spin and OAM are oriented in opposite directions.

— D. Chakrabarti, BG *et al.* [PRD 109, 2024], C. Tan, Z. Lu, [e-Print :2312.07997 [hep-ph]], Lorce & Pasquini [PRD 84, 2011], A. Mukherjee *et al.* [EPJC 78, 2018]

# Conclusion

- LF-QCD is a powerful tool to study the strongly interacting bound state systems like baryons and mesons.
- We utilized the NNPDF3.0nlo data for gluon unpolarized pdfs to fix the AdS/QCD model parameters.
- We obatin the gluon spin $\Delta G = 0.326^{+0.066}_{-0.050}$, which is consistent with PHENIX and NNPDF collaborations.
- Properly defining and measuring quark and gluon orbital angular momentum is complex and requires more work, both theoretically and experimentally.
- Spin can be decomposed in several ways : Frame-independent (Ji), IMF (Jaffe-Manohar), Chen *et al.* etc.
- DSA in exclusive dijet production is a unique observable to access the gluon OAM and helicity at EIC.
- GPDs and GTMDs provides an intuitive definition of the kinetic and canonical OAMs.
- Electron-Ion collider will dramatically alter our knowledge of the gluonic structure of hadrons and nuclei.

**Thanks for your attentation ! !**