Few Remarks on Loop Diagram Computations

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January 24, 2025

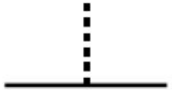
Outline

- Chiral Lagrangian: PV vs. PS Theories
- Equivalence including LF Zero Modes
- Fermion Characteristics - Self-Energy
- Ideas of Four-Fermion Operators
- Conclusion and Outlook

πN Lagrangian

- Chiral (pseudovector) Lagrangian

$$\mathcal{L}_{\pi N} = \frac{g_A}{2f_\pi}\,\bar{\psi}_N \gamma^\mu \gamma_5\, \vec{\tau}\cdot\partial_\mu \vec{\pi}\,\psi_N - \frac{1}{(2f_\pi)^2}\,\bar{\psi}_N \gamma^\mu\, \vec{\tau}\cdot(\vec{\pi}\times\partial_\mu\vec{\pi})\,\psi_N$$



$g_A = 1.267$
$f_\pi = 93$ MeV

→ lowest order approximation of chiral perturbation theory Lagrangian

→ *cf.* pseudoscalar Lagrangian

$$\mathcal{L}^{\rm PS}_{\pi N} = -g_{\pi NN}\,\bar{\psi}_N\, i\gamma_5 \vec{\tau}\cdot\vec{\pi}\,\psi_N \;+\sigma NN \text{ term}$$

Weinberg, *PRL* **18**, 88 (1967)

Relation between PV and PS Theories

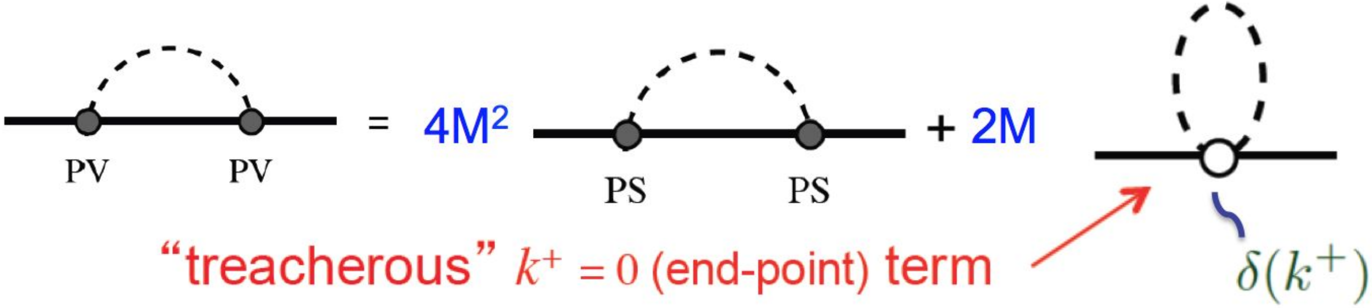
Self-Energy

$$\Sigma^{PV} = \frac{1}{2}\sum_s \overline{u}(p,s)\,\hat{\Sigma}^{PV}\,u(p,s) \qquad \hat{\Sigma}^{PV} = -i\left(\frac{2g_A}{f_\pi}\right)^2 \vec{\tau}\cdot\vec{\tau}\int\frac{d^4k}{(2\pi)^4}\frac{\not{k}\gamma_5(\not{p}-\not{k}+M)\gamma_5\not{k}}{D_\pi D_N}$$

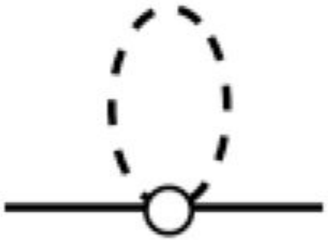
$$D_\pi = k^2 - m_\pi^2 + i\varepsilon \qquad D_N = (p-k)^2 - M^2 + i\varepsilon$$

$$\overline{u}(p)\not{k}\gamma^5\frac{1}{\not{p}-\not{k}-M}\gamma^5\not{k}\,u(p) = \overline{u}(p)[\not{k}-\not{p}+M]\gamma^5\frac{1}{\not{p}-\not{k}-M}\gamma^5[\not{k}-\not{p}+M]u(p)$$

$$= 4M^2\,\overline{u}(p)\gamma^5\frac{1}{\not{p}-\not{k}-M}\gamma^5 u(p) + 2M\,\overline{u}(p)u(p) + \overline{u}(p)\not{k}u(p)$$



$$I = \int d^2k \frac{1}{k^2 - m^2 + i\varepsilon}$$



Manifestly Covariant Calculation

$$I = -i\pi\left(\frac{2-n}{2} - \log\pi - \gamma + O(2-n) - \log\frac{m^2}{\mu^2}\right)_{n\to 2}$$

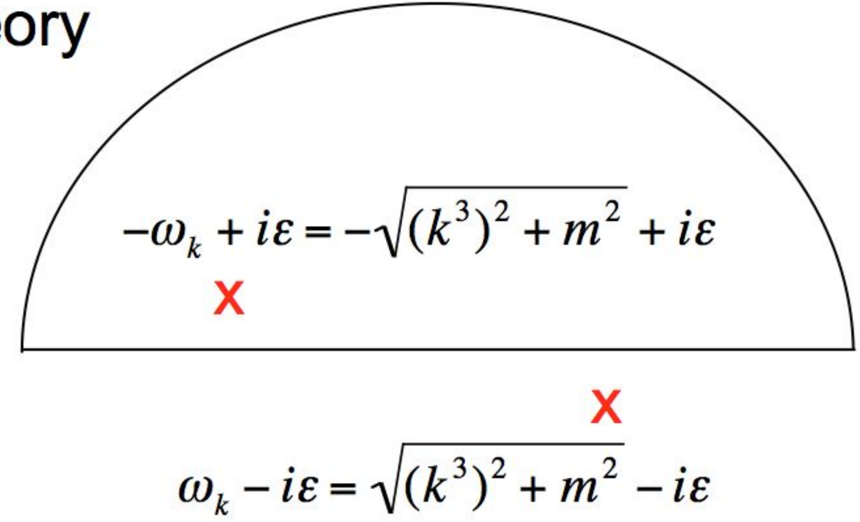
$$I_{LNA} = i\pi \log m^2$$

Time-Ordered Perturbation Theory

$$I = \int dk^3 dk^0 \frac{1}{(k^0 + \omega_k - i\varepsilon)(k^0 - \omega_k + i\varepsilon)}$$

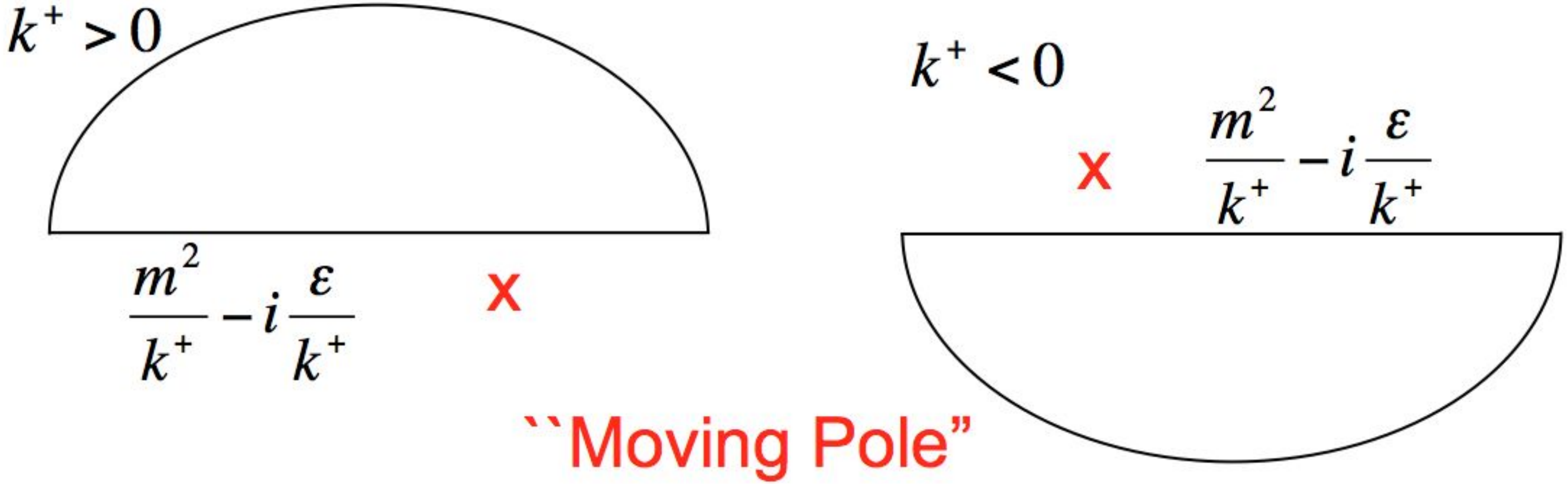
$$= -2i\pi \int_0^{\Lambda\to\infty} dk^3 \frac{1}{\sqrt{(k^3)^2 + m^2}}$$

$$= -2i\pi \log\left(\frac{2\Lambda}{m}\right)_{\Lambda\to\infty}$$



LFD

$$I=\frac{1}{2}\int dk^+dk^-\,\frac{1}{k^+k^- - m^2 + i\varepsilon}=\frac{1}{2}\int\frac{dk^+}{k^+}\int dk^-\,\frac{1}{k^- - \frac{m^2}{k^+} + i\frac{\varepsilon}{k^+}}$$



$$\int_{-\infty}^{+\infty} dk^- \frac{1}{(k^+k^- - m^2 + i\epsilon)^2} = \frac{2\pi i}{m^2}\delta(k^+)$$

Jan. 16, 2024.

Proof Define $I_\Lambda(k^+) \equiv \int_{-\Lambda}^{+\Lambda} dk^- \frac{1}{(k^+k^- - m^2 + i\epsilon)^2}$.

Now,

$$I_\Lambda(k^+) = \frac{1}{(k^+)^2}\int_{-\Lambda}^{+\Lambda} dk^- \frac{1}{\left(k^- - \frac{m^2 - i\epsilon}{k^+}\right)^2}$$

$$= \frac{1}{(k^+)^2}\left[-\frac{1}{k^- - \frac{m^2-i\epsilon}{k^+}}\right]_{-\Lambda}^{+\Lambda}$$

$$= \frac{1}{(k^+)^2}\left(-\frac{1}{\Lambda - \frac{m^2-i\epsilon}{k^+}} + \frac{1}{-\Lambda - \frac{m^2-i\epsilon}{k^+}}\right)$$

$$= -\frac{1}{k^+}\left(\frac{1}{k^+\Lambda - (m^2-i\epsilon)} + \frac{1}{k^+\Lambda + (m^2-i\epsilon)}\right)$$

$$= -\frac{\Lambda}{k^+\Lambda}\left(\frac{1}{k^+\Lambda - (m^2-i\epsilon)} + \frac{1}{k^+\Lambda + (m^2-i\epsilon)}\right)$$

$$= -\frac{\Lambda}{k^+\Lambda}\left(\frac{1}{k^+\Lambda + (m^2-i\epsilon)} + \frac{1}{k^+\Lambda - (m^2-i\epsilon)}\right).$$

Note here

$$-\frac{1}{2b}\left(\frac{1}{b+a} + \frac{1}{b-a}\right) = \frac{1}{2a}\left(\frac{1}{b+a} - \frac{1}{b-a}\right) = \frac{1}{2a}\left(\frac{1}{a+b} + \frac{1}{a-b}\right) = \frac{1}{(a+b)(a-b)}$$

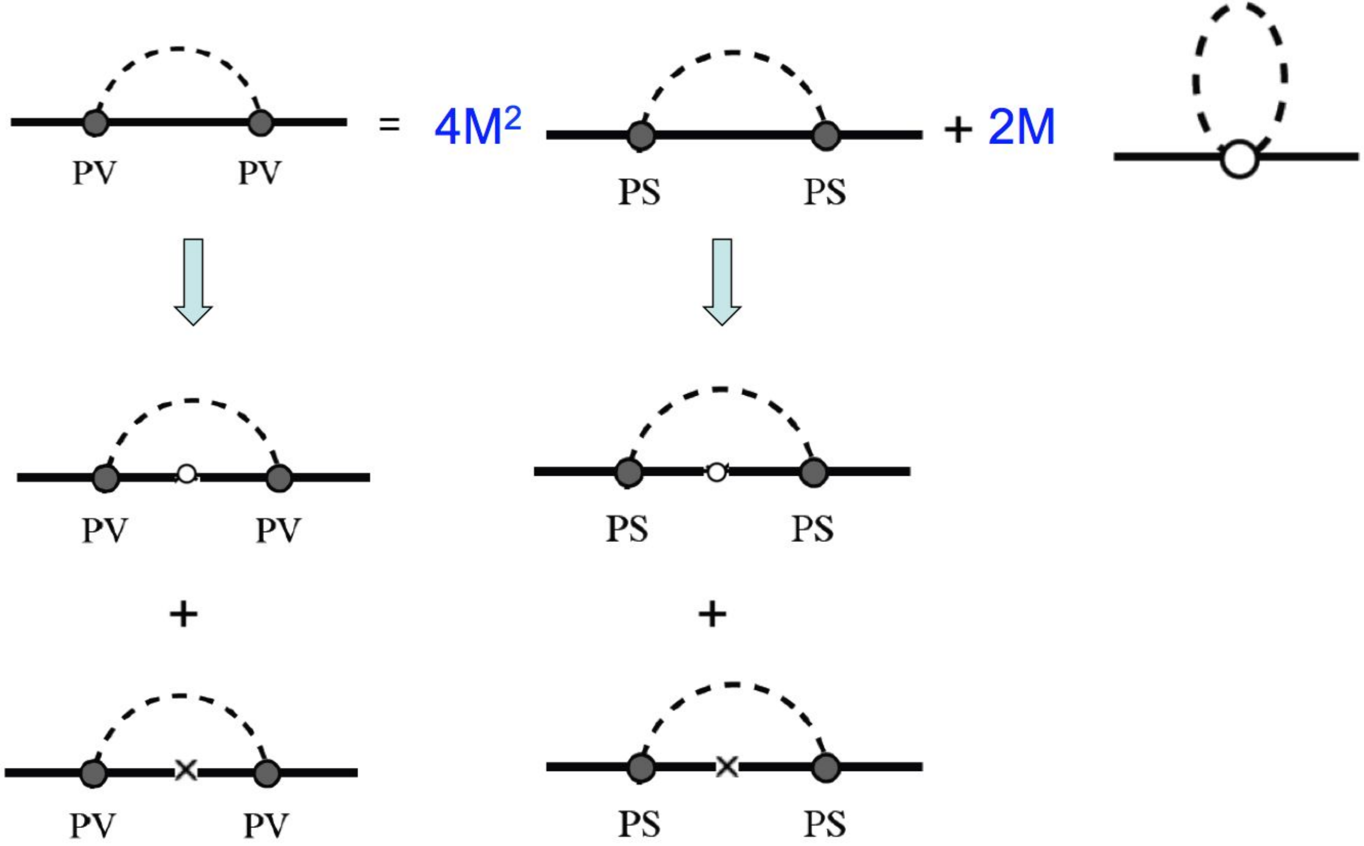
and thus we get (with $b = k^+\Lambda$, $a = m^2 - i\epsilon$)

$$I_\Lambda(k^+) = \frac{1}{m^2 - i\epsilon}\left(\frac{\Lambda}{k^+\Lambda + m^2 - i\epsilon} - \frac{\Lambda}{k^+\Lambda - m^2 + i\epsilon}\right)$$

and

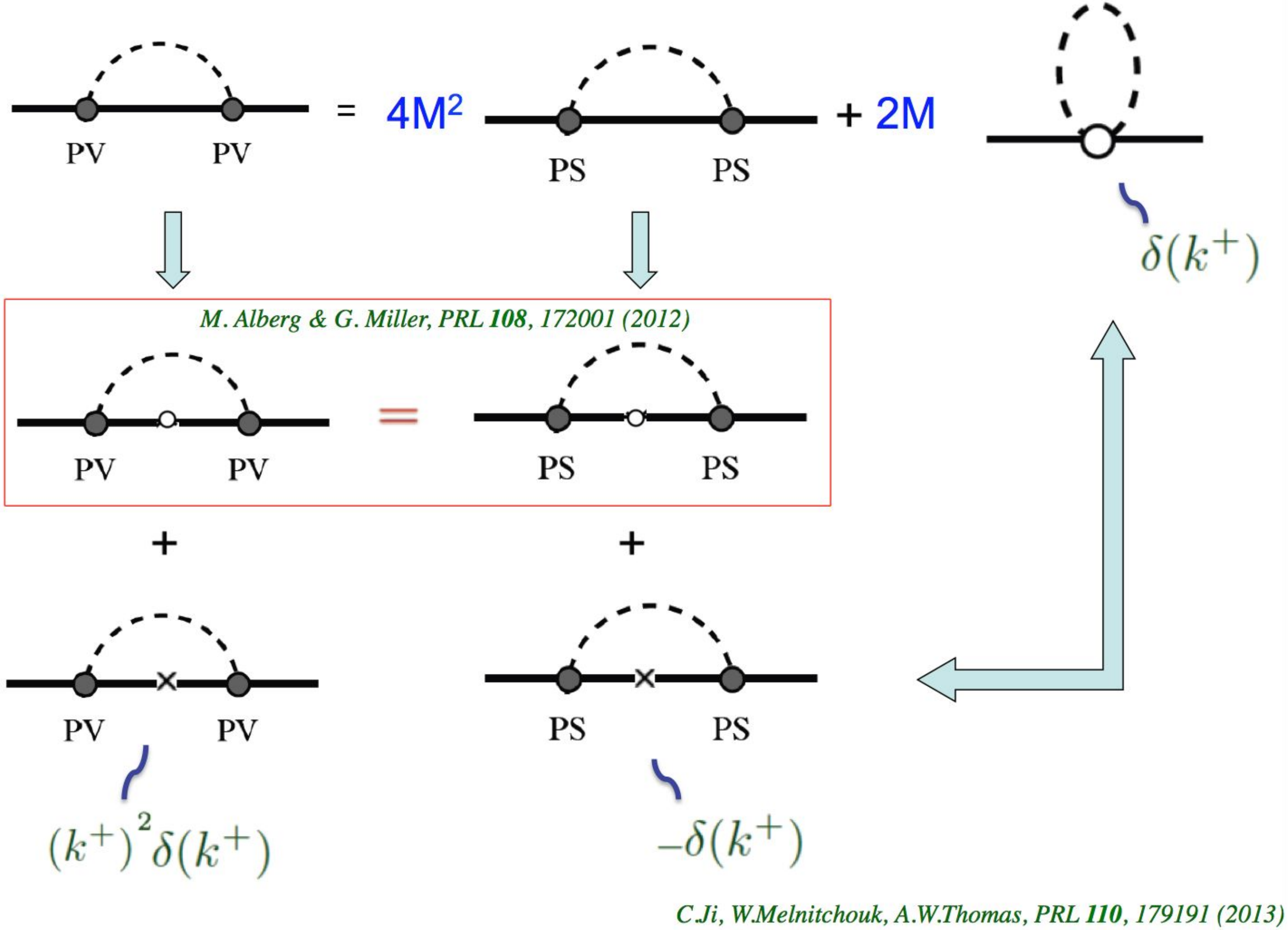
$$I_{\Lambda=\infty}(k^+) = \frac{1}{m^2 - i\epsilon}\left(\frac{1}{k^+ - i\epsilon} - \frac{1}{k^+ + i\epsilon}\right) = \frac{2\pi i}{m^2}\delta(k^+)$$

using $\frac{1}{k^+ - i\epsilon} = P\frac{1}{k^+} + i\pi\delta(k^+)$.



$$\frac{1}{\not{p} - M} = \frac{\sum_s u(p,s)\bar{u}(p,s)}{p^2 - M^2} + \frac{\gamma^+}{2p^+}$$

S.-J.Chang and T.-M.Yan, PRD, 1147 (1973)


PV PV
=
4M²
PS PS
+ 2M
δ(k⁺)
M. Alberg & G. Miller, PRL 108, 172001 (2012)
PV PV
=
PS PS
+
+
PV PV
PS PS
(k⁺)²δ(k⁺)
−δ(k⁺)
C.Ji, W.Melnitchouk, A.W.Thomas, PRL 110, 179191 (2013)

Fermion Propagator

Free Propagator

Interacting Propagator

$$S_f(p) = \frac{1}{\not p - m + i\varepsilon} \quad \Rightarrow \quad S(p) = \frac{1}{\not p - m - \sum(p) + i\varepsilon}$$

$$= \frac{F(p)}{\not p - M(p) + i\varepsilon}$$

$$\boxed{\Sigma(p) = \Sigma_s(p) + \Sigma_v(p)\not p}$$

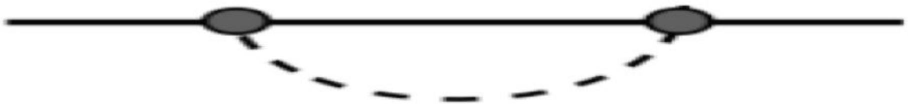
$F(p) = \left(1 - \Sigma_v(p)\right)^{-1}$ "Wave function renormalization factor"

$M(p) = \dfrac{m + \Sigma_s(p)}{1 - \Sigma_v(p)}$ "Renormalized fermion mass function"

Simple Example

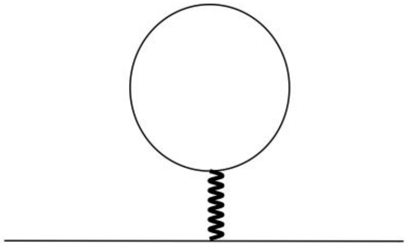
$$\Sigma = \frac{1}{2}\sum_s \bar{u}_i(p,s)\hat{\Sigma}_{ij}\, u_j(p,s) = \Sigma_S + \Sigma_V M$$

$$(\gamma_5)_{ik} \quad \frac{(\not{p} - \not{k} + M)_{k\ell}}{D_\pi D_N} \quad (\gamma_5)_{\ell j}$$



$$\hat{\Sigma}^{PS}_{ij} = \mathrm{N}\int \frac{d^4k}{(2\pi)^4}\frac{(\gamma_5)_{ik}(\not{p} - \not{k} + M)_{k\ell}(\gamma_5)_{\ell j}}{D_\pi D_N}$$

$$D_\pi = k^2 - m_\pi^2 + i\varepsilon \qquad D_N = (p-k)^2 - M^2 + i\varepsilon$$

$\bar{u}_i$  u_j

$$(\gamma_5)_{\ell j}(\gamma_5)_{ik} = \frac{1}{4}\delta_{ij}\delta_{\ell k} + \frac{1}{4}(\gamma_5)_{ij}(\gamma_5)_{\ell k} - \frac{1}{4}(\gamma^\alpha)_{ij}(\gamma_\alpha)_{\ell k}$$
$$+ \frac{1}{4}(\gamma^\alpha\gamma_5)_{ij}(\gamma_\alpha\gamma_5)_{\ell k} + \frac{1}{8}(\sigma^{\alpha\beta})_{ij}(\sigma_{\alpha\beta})_{\ell k}$$

$$
\begin{aligned}
&(\gamma_5)_{ik}(\not p - \not k + M)_{k\ell}(\gamma_5)_{\ell j} \\
&= \frac{Tr[\not p - \not k + M]}{4}\delta_{ij} + \frac{Tr[\gamma_5(\not p - \not k + M)]}{4}(\gamma_5)_{ij} - \frac{Tr[\gamma_\alpha(\not p - \not k + M)]}{4}(\gamma^\alpha)_{ij} \\
&+ \frac{Tr[\gamma_\alpha\gamma_5(\not p - \not k + M)]}{4}(\gamma^\alpha\gamma_5)_{ij} + \frac{Tr[\sigma_{\alpha\beta}(\not p - \not k + M)]}{8}(\sigma^{\alpha\beta})_{ij} \\
&= M\delta_{ij} + (k-p)_\alpha(\gamma^\alpha)_{ij}
\end{aligned}
$$

$$
\overline{u}(p)\hat{\Sigma}^{PS}u = \left[\mathrm{N}\int \frac{d^4k}{(2\pi)^4}\frac{M}{D_\pi D_N}\right]\overline{u}(p)u(p) + \left[\mathrm{N}\int \frac{d^4k}{(2\pi)^4}\frac{(k-p)_\alpha}{D_\pi D_N}\right]\overline{u}(p)\gamma^\alpha u(p)
$$

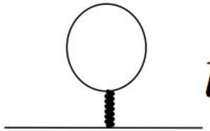
Σ_S Σ_V

$$(\gamma_5)_{\ell j}(\gamma_5)_{ik} = \frac{1}{4}\delta_{ij}\delta_{\ell k} + \frac{1}{4}(\gamma_5)_{ij}(\gamma_5)_{\ell k} - \frac{1}{4}(\gamma^\alpha)_{ij}(\gamma_\alpha)_{\ell k} + \frac{1}{4}(\gamma^\alpha\gamma_5)_{ij}(\gamma_\alpha\gamma_5)_{\ell k} + \frac{1}{8}(\sigma^{\alpha\beta})_{ij}(\sigma_{\alpha\beta})_{\ell k}$$

$(\gamma_5)_{ik}$ $(\gamma_5)_{\ell j}$

δ_{ik} $\delta_{\ell j}$

$$\delta_{\ell j}\delta_{ik} = \frac{1}{4}\delta_{ij}\delta_{\ell k} + \frac{1}{4}(\gamma_5)_{ij}(\gamma_5)_{\ell k} + \frac{1}{4}(\gamma^\alpha)_{ij}(\gamma_\alpha)_{\ell k} - \frac{1}{4}(\gamma^\alpha\gamma_5)_{ij}(\gamma_\alpha\gamma_5)_{\ell k} + \frac{1}{8}(\sigma^{\alpha\beta})_{ij}(\sigma_{\alpha\beta})_{\ell k}$$

$\bar{u}_i$  u_j

$$(\Sigma_S)^S = (\Sigma_S)^{PS}, (\Sigma_V)^S = -(\Sigma_V)^{PS}$$

PHYSICAL REVIEW D **87,** 093004 (2013)

Ideas of four-fermion operators in electromagnetic form factor calculations

Chueng-Ryong Ji,[1] Bernard L. G. Bakker,[2] Ho-Meoyng Choi,[3] and Alfredo T. Suzuki[1,4,*]

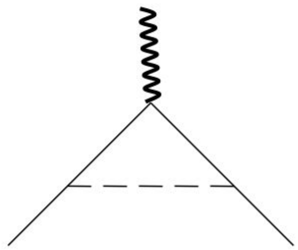
$$
\begin{aligned}
(O_\beta)_{ik}(O^\beta)_{\ell j} &= \mathrm{C}_S^\beta \delta_{ij}\delta_{\ell k} + \mathrm{C}_V^\beta (\gamma_\mu)_{ij}(\gamma^\mu)_{\ell k} \\
&\quad + \mathrm{C}_T^\beta (\sigma_{\mu\nu})_{ij}(\sigma^{\mu\nu})_{\ell k} \\
&\quad + \mathrm{C}_A^\beta (\gamma_\mu\gamma_5)_{ij}(\gamma^\mu\gamma_5)_{\ell k} + \mathrm{C}_P^\beta (\gamma_5)_{ij}(\gamma_5)_{\ell k} \\
&= \sum_\alpha \mathrm{C}_\alpha^\beta (\Omega_\alpha)_{ij}(\Omega^\alpha)_{\ell k}, \qquad (1)
\end{aligned}
$$

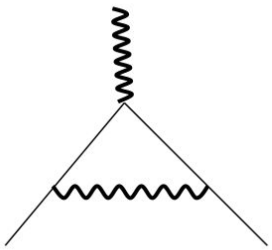
TABLE I. Fierz transformation coefficients of Eq. (1) [6].

	S	V	T	A	P
S	1/4	1/4	1/8	−1/4	1/4
V	1	−1/2	0	−1/2	−1
T	3	0	−1/2	0	3
A	−1	−1/2	0	−1/2	1
P	1/4	−1/4	1/8	1/4	1/4

[6] H. J. Weber, Ann. Phys. (N.Y.) **177**, 38 (1987).

Application to Form Factors



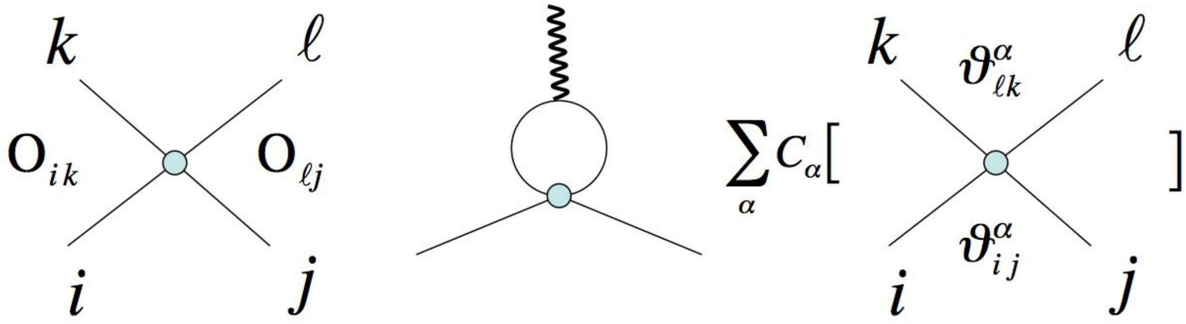


Nucleon Form Factors

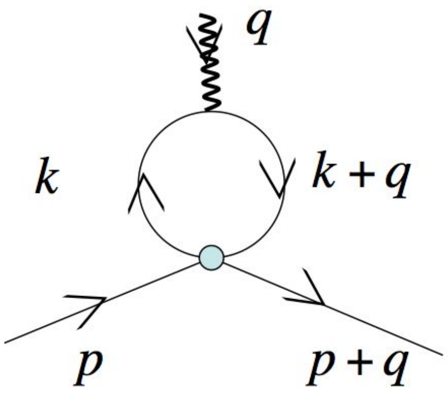
Electron Form Factors

Quark-Diquark Model

QED



Simple Example Calculation



$$\overline{u}(p+q)[J_\mu]u(p) = \sum_\alpha C_\alpha \Gamma_\mu^\alpha [\overline{u}_i \vartheta_{ij}^\alpha u_j]$$

$$\Gamma_\mu^\alpha = \int d^4k \frac{N_\mu^\alpha}{D_{k+q} D_k}$$

$$D_k = k^2 - m^2 + i\varepsilon$$

$$N_\mu^\alpha = Tr[(\not{k} + \not{q} + m)\gamma_\mu(\not{k} + m)\vartheta^\alpha]$$

$$\vartheta^\alpha = I\ ,\ \gamma_5\ ,\ \gamma_\nu\ ,\ \gamma_\nu\gamma_5\ ,\ \sigma_{\nu\delta}\ .$$

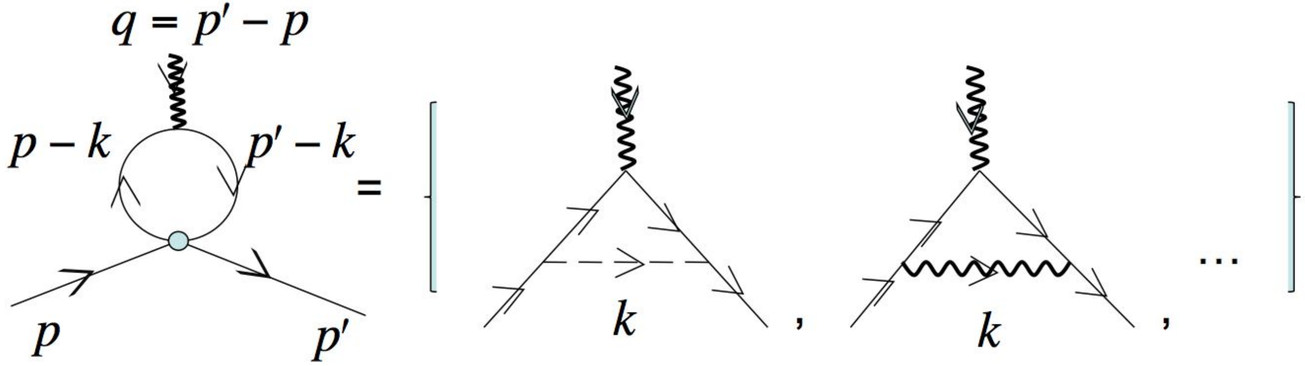
$$N_\mu^S = 4m(2k_\mu + q_\mu)\ , \quad N_\mu^P = 0\ ,$$

$$N_\mu^{V_\nu} = 4[(k+q)_\mu k_\nu + k_\mu (k+q)_\nu + \{m^2 - k\cdot(k+q)\}g_{\mu\nu}\ ,$$

$$N_\mu^{A_\nu} = -4i\varepsilon_{\mu\nu\beta\delta} q^\beta k^\delta\ , \quad N_\mu^{T_{\nu\delta}} = 4im(g_{\mu\nu}q_\delta - g_{\mu\delta}q_\nu)\ .$$

$$J_\mu = (q^2 g_{\mu\nu} - q_\mu q_\nu)\gamma^\nu \left[8\pi^2 i \int_0^1 dx\, x(1-x)\left(\frac{1}{\varepsilon} - \gamma - \frac{1}{2} + \log\frac{\mu^2}{\pi\Delta^2}\right)\right] + im\sigma_{\mu\nu}q^\nu \left[8\pi^2 i \int_0^1 dx\left(\frac{1}{\varepsilon} - \gamma - \frac{1}{2} + \log\frac{\mu^2}{\pi\Delta^2}\right)\right]$$

Momentum Dependent Four-Fermion Operator Example



$$\bar{u}(p')[J_\mu]u(p) = \sum_\alpha C_\alpha \Gamma^\alpha_\mu [\bar{u}_i \vartheta^\alpha_{ij} u_j]$$

$$\Gamma^\alpha_\mu = \int d^4k \frac{N^\alpha_\mu}{D_{p-k} D_{p'-k} D_k}$$

$$N^\alpha_\mu = Tr[(\not{k} + \not{q} + m)\gamma_\mu(\not{k} + m)\vartheta^\alpha]$$

$$\vartheta^\alpha = I\ ,\ \gamma_5\ ,\ \gamma_\nu\ ,\ \gamma_\nu\gamma_5\ ,\ \sigma_{\nu\delta}\ .$$

$$D_k = k^2 - m^2 + i\varepsilon$$

$$N_\mu^S = 4m(p_\mu + p'_\mu - 2k_\mu)\,, \quad N_\mu^P = 0\,,$$
$$N_\mu^{V_\nu} = 4[(p'-k)_\mu(p-k)_\nu + (p-k)_\mu(p'-k)_\nu + \{m^2 - (p-k)\cdot(p'-k)\}g_{\mu\nu}\,,$$
$$N_\mu^{A_\nu} = -4i\varepsilon_{\mu\nu\beta\delta}(p'-k)^\beta(p-k)^\delta\,, \quad N_\mu^{T_{\nu\delta}} = 4im(g_{\mu\nu}q_\delta - g_{\mu\delta}q_\nu)\,.$$

$$J_\mu = \int d^4k \frac{\sum_\alpha C_\alpha N_\mu^\alpha \vartheta^\alpha}{D_{p-k}D_{p'-k}D_k} = 2\times 4\int_0^1 dx \int_0^{1-x} dy \int d^4k' \frac{\mathrm{N}_\mu}{(k'^2 - \Delta^2)^3}$$

$$\Delta^2 = (x+y)m^2 + (1-x-y)m_X^2 - xyq^2 - (x+y)(1-x-y)M^2$$

$$\begin{aligned}
\mathrm{N}_\mu &= C_S m(1-x-y)(p+p')_\mu \\
&+ C_V[\{m^2 - k'^2 - (1-x-y)^2M^2 + (1-x-y+2xy)\frac{q^2}{2}\}\gamma_\mu \\
&+ 2k'_\mu \not{k}' + \frac{(1-x-y)^2}{2}(p+p')_\mu(\not{p}+\not{p}') - \frac{(1+x-y)(1-x+y)}{2}(p-p')_\mu(\not{p}-\not{p}')] \\
&+ iC_A\,\varepsilon_{\mu\nu\alpha\beta}\gamma^5\gamma^\nu(1-x-y)p^\alpha p'^\beta \quad + 2iC_T\, m\sigma_{\mu\nu}q^\nu
\end{aligned}$$

Reduction to F_1 and F_2

$$J_\mu = \gamma^\mu F_1(q^2) + i\frac{\sigma_{\mu\nu}q^\nu}{2M}F_2(q^2)$$

Gorden Decomposition and Extension

$$(p + p')_\mu \rightarrow \quad 2M\gamma_\mu - i\sigma_{\mu\nu}q^\nu$$

$$i\varepsilon_{\mu\nu\alpha\beta}\gamma^5\gamma^\nu p^\alpha p'^\beta \rightarrow \quad \frac{q^2}{2}\gamma_\mu - iM\sigma_{\mu\nu}q^\nu$$

$$J_\mu = \gamma^\mu F_1(q^2) + i\frac{\sigma_{\mu\nu}q^\nu}{2M}F_2(q^2)$$

$$F_i(q^2) = \mathrm{N}\int_0^1 dx \int_0^{1-x} dy \int \frac{d^d\kappa}{(2\pi)^d}\frac{N_i}{(\kappa^2+\Delta^2)^3} \quad (i=1,2)$$

$$\Delta^2 = (x+y)m^2 + (1-x-y)m_X^2 - xyq^2 - (x+y)(1-x-y)M^2$$

$$N_1 = 2Mm(1-x-y)C_S + (1-x-y)\frac{q^2}{2}C_A + \{m^2 + (1-x-y)^2M^2 + (1-x-y+2xy)\frac{q^2}{2} + (1-\frac{2}{d})\kappa^2\}C_V$$

$$N_2 = 4MmC_T - 2Mm(1-x-y)C_S - 2(1-x-y)^2M^2C_V - 2(1-x-y)M^2C_A$$

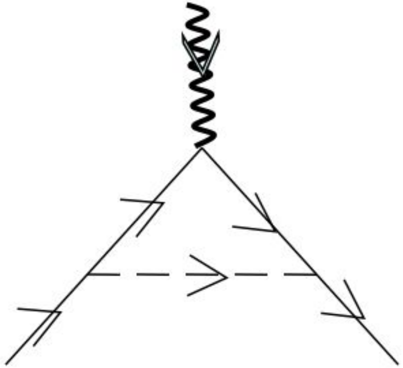
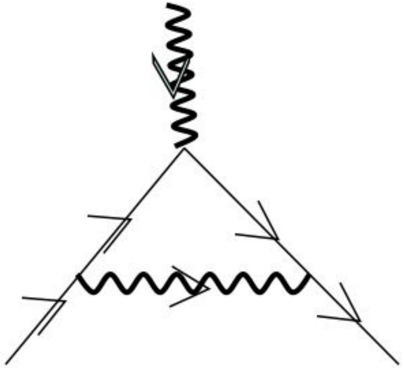
$$F_1(q^2) = \frac{g^2}{4\pi^2}\int_0^1 dx \int_0^{1-x} dy \left\{ \left(\frac{1}{\varepsilon} - \gamma - \frac{3}{2} + \log \frac{\mu^2}{\pi\Delta^2} \right) C_V \right.$$

$$+ \frac{2Mm(1-x-y)C_S + (1-x-y)\frac{q^2}{2}C_A}{\Delta^2}$$

$$\left. + \frac{\{m^2 + (1-x-y)^2 M^2 + (1-x-y+2xy)\frac{q^2}{2}\}C_V}{\Delta^2} \right\}$$

$$F_2(q^2) = \frac{g^2}{2\pi^2}\int_0^1 dx \int_0^{1-x} dy \left\{ \frac{2MmC_T - Mm(1-x-y)C_S}{\Delta^2} \right.$$

$$\left. - \frac{(1-x-y)^2 M^2 C_V + (1-x-y) M^2 C_A}{\Delta^2} \right\}$$

	C_S	C_P	C_V	C_A	C_T
	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{8}$
	1	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	0

$$F_1^{Scalar}(q^2) = \frac{g^2}{32\pi^2}\left(\frac{1}{\varepsilon} - \gamma - \frac{3}{2}\right)$$

$$+\frac{g^2}{16\pi^2}\int_0^1 dx \int_0^{1-x} dy \left[\ln\left(\frac{\mu^2}{\pi\Delta^2}\right) + \frac{\{m+(1-x-y)M\}^2 + xyq^2}{\Delta^2}\right]$$

$$F_2^{Scalar}(q^2) = \frac{g^2}{8\pi^2}\int_0^1 dx \int_0^{1-x} dy(x+y)\left\{\frac{mM+(1-x-y)M^2}{\Delta^2}\right\}$$

$$F_1^{QED}(q^2) = \frac{g^2}{16\pi^2}\left(\frac{1}{\varepsilon} - \gamma - \frac{3}{2}\right)$$

$$-\frac{g^2}{8\pi^2}\int_0^1 dx \int_0^{1-x} dy \left[\ln\left(\frac{\mu^2}{\pi\Delta^2}\right) + \frac{m^2\{(x+y)^2 - 2(1-x-y) + (1-x)(1-y)q^2\}}{\Delta^2}\right]$$

$$F_2^{QED}(q^2) = -\frac{g^2}{4\pi^2}\int_0^1 dx \int_0^{1-x} dy \left\{\frac{(x+y)(1-x-y)m^2}{\Delta^2}\right\}$$

All are equivalent!

$$J^\mu = \gamma^\mu F_1 + i\frac{\sigma^{\mu\nu}q_\nu}{2M}F_2$$ VT

$$= \gamma^\mu(F_1 + F_2) + \frac{(p+p')^\mu}{2M}F_2$$ VS

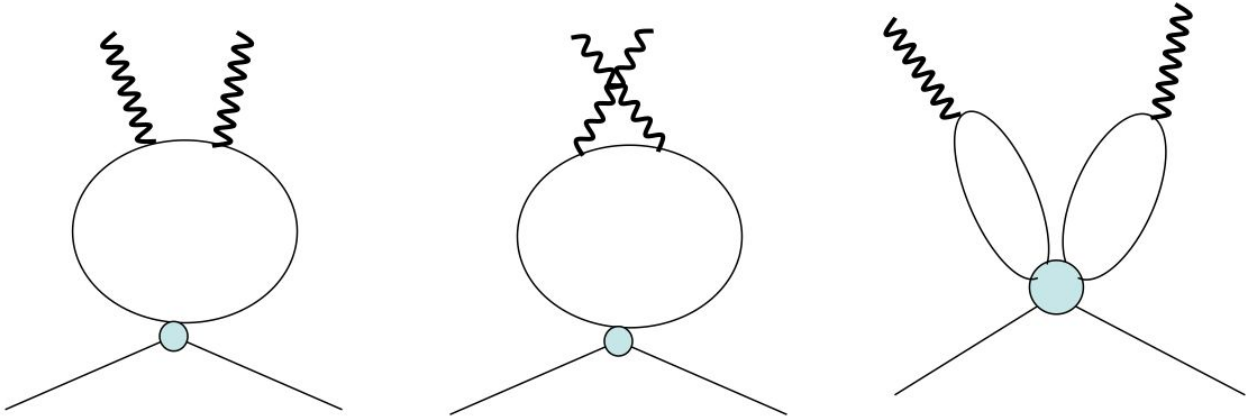
$$= \frac{(p+p')^\mu}{2M}\frac{4M^2F_1 + q^2F_2}{4M^2 - q^2} - i\varepsilon^{\mu\nu\alpha\beta}\gamma_5\gamma_\nu p_\alpha p'_\beta \frac{2(F_1+F_2)}{4M^2 - q^2}$$ SA

$$= \frac{(p+p')^\mu}{2M}F_1 + i\frac{\sigma^{\mu\nu}q_\nu}{2M}(F_1 + F_2)$$ ST

$$= \gamma^\mu(F_1 + \frac{q^2}{4M^2}F_2) - i\varepsilon^{\mu\nu\alpha\beta}\gamma_5\gamma_\nu p_\alpha p'_\beta \frac{F_2}{2M^2}$$ VA

$$= i\frac{\sigma^{\mu\nu}q_\nu}{2M}(\frac{4M^2}{q^2}F_1 + F_2) + i\varepsilon^{\mu\nu\alpha\beta}\gamma_5\gamma_\nu p_\alpha p'_\beta \frac{2F_1}{q^2}$$ TA

Two-Photon Application

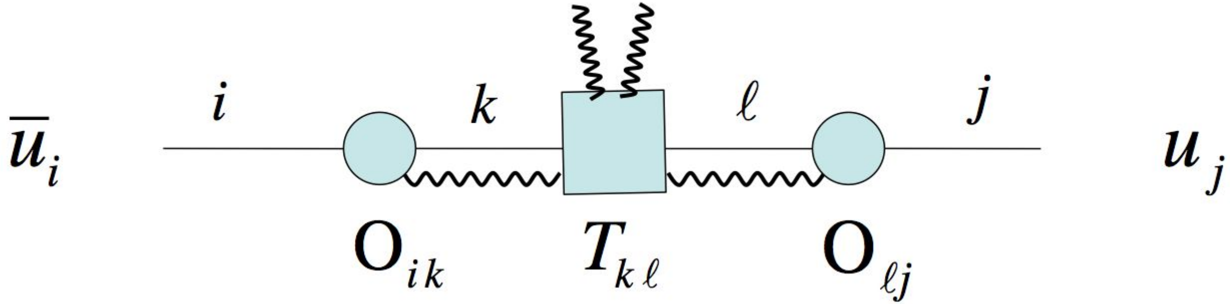


S-Channel | U-Channel | Six-Fermion Operator

"Handbag" | "Cat's ear"

Basic Idea

$$O_{ik}O_{\ell j} = \sum_\alpha C_\alpha \vartheta^\alpha_{ij}\vartheta^\alpha_{\ell k}$$



$$O_{ik}T_{k\ell}O_{\ell j} = \sum_\alpha C_\alpha \vartheta^\alpha_{ij}\vartheta^\alpha_{\ell k}T_{k\ell} = \sum_\alpha \vartheta^\alpha_{ij} C_\alpha Tr[\vartheta^\alpha T]$$

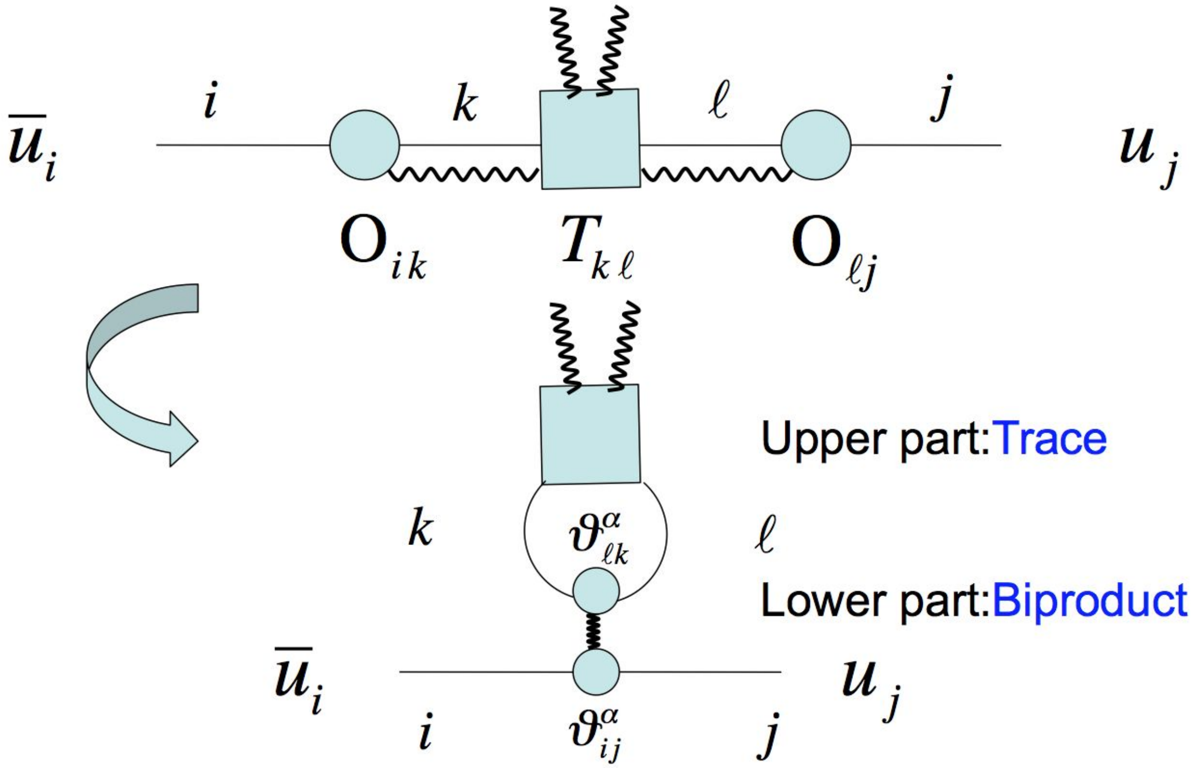
$$O_{ik}O_{\ell j} = \sum_\alpha C_\alpha \vartheta^\alpha_{ij}\vartheta^\alpha_{\ell k}$$

$$= C_S\delta_{ij}\delta_{\ell k} + C_P(\gamma_5)_{ij}(\gamma_5)_{\ell k} + C_V(\gamma_\alpha)_{ij}(\gamma^\alpha)_{\ell k}$$

$$+C_A(\gamma_\alpha\gamma_5)_{ij}(\gamma_\alpha\gamma_5)_{\ell k} + C_T(\sigma_{\alpha\beta})_{ij}(\sigma^{\alpha\beta})_{\ell k}$$

Basic Idea

$$O_{ik}\,O_{\ell j} = \sum_{\alpha} C_{\alpha} \vartheta^{\alpha}_{ij} \vartheta^{\alpha}_{\ell k}$$



Upper part:Trace

Lower part:Biproduct

Conclusion and Outlook

- Four-Fermion Idea provides an effective way to analyze hadronic processes.

  -Upper Part: Trace

  -Lower part: Biproduct

- Different processes may be described in a unified way.

  -Nucleon Form Factors, Electron Form Factors.

- Hadronic tensors of DVCS need further investigation incldiung Six-Fermion operators.