Self-interacting Ultralight dark matter

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Outline

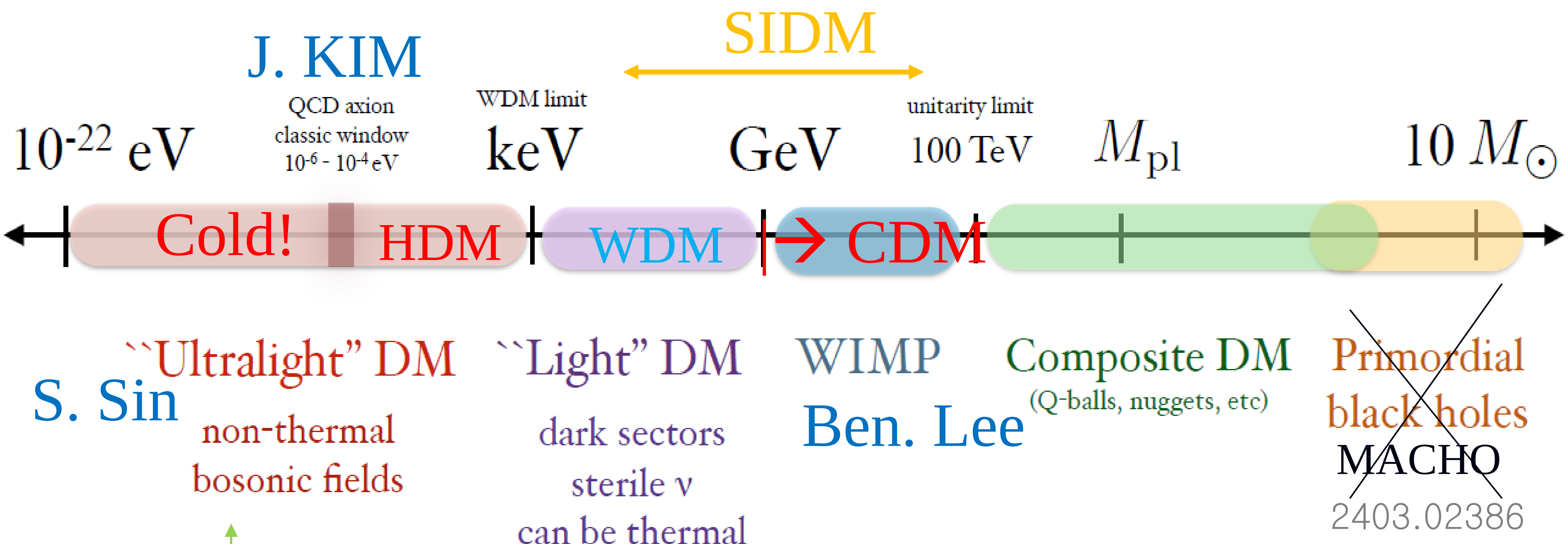
1. Brief review on ULDM models

2. Q. Scales of Fuzzy DM (free model)

3. Self-interacting ULDM and mysteries of Universe

Mass scale of dark matter

(not to scale) TASI lectures by Lin arXiv:1904.07915

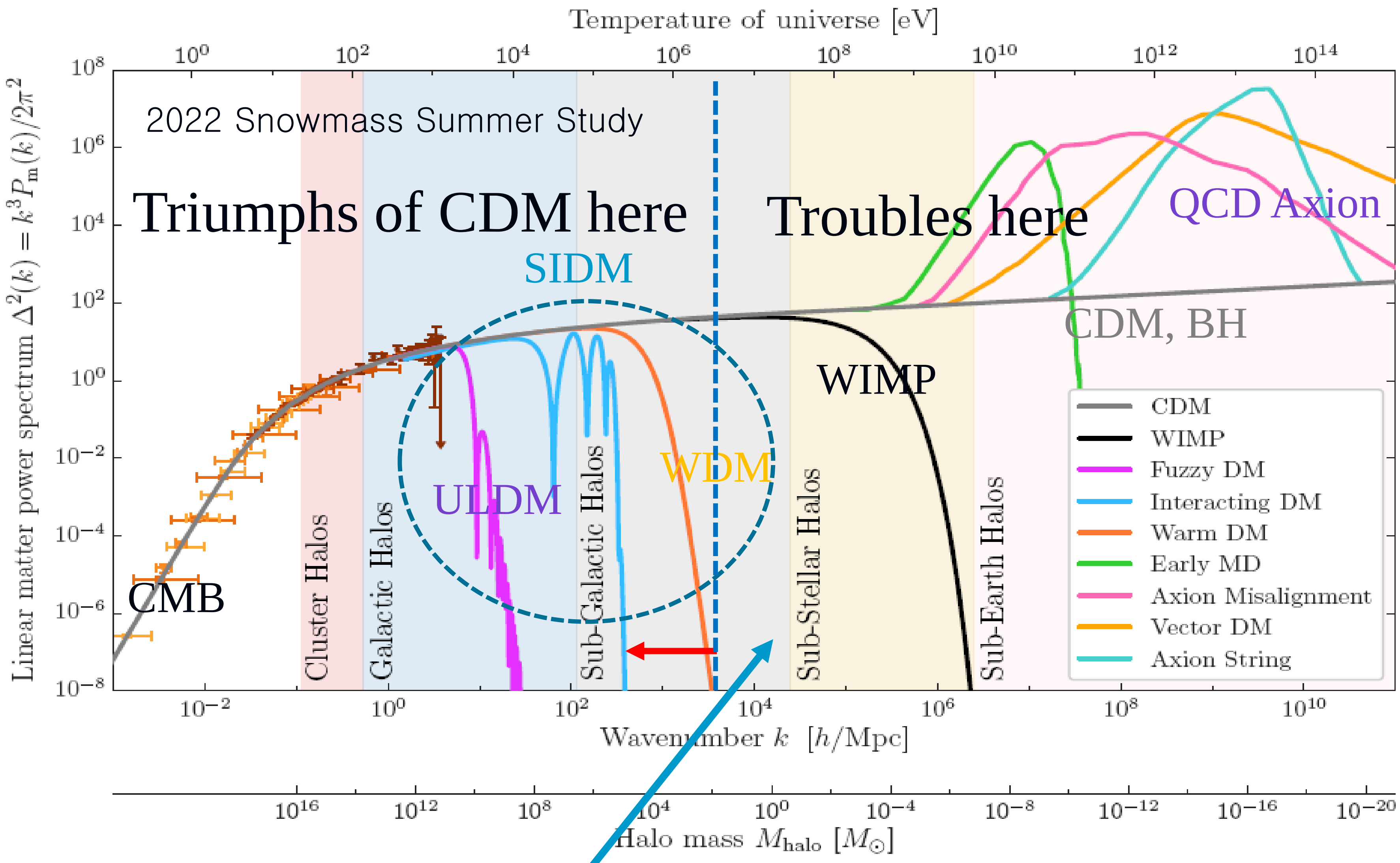


ULDM = Fuzzy, ULA, BEC, Wave, Scalar Field, ψ, Superfluid, quantum …

Compact objects in the mass range from $1.3 \times 10^{-5} M_\odot$ *to* $860\ M_\odot$ *cannot make up more than* 10% *of dark matter.* (2403.02386)

→ No DM star or planet observed in our galactic halo

Galaxies are DM dominated and seem to have ~ kpc size scale



No DM star or planet found so far → DM has kpc length scale?

Challenges for ΛCDM

cf) 2105.05208

- ΛCDM was very successful but is becoming non-standard?

1. Small scale crisis (on galactic scale and below)
   predicts too many small structures not observed

2. Hubble parameter tension ~5σ:
   H_0 mismatch between Planck estimation and SN

3. S_8 tension ~2-3σ : $S_8 \equiv \sigma_8\sqrt{\Omega_m/0.3}$
   matter density fluctuation amplitude
   mismatch between Planck estimation and WL & Cluster

4. Time varying EOS of DE (ex, DESI, AP test)

5. Too early BHs and galaxies (Webb)

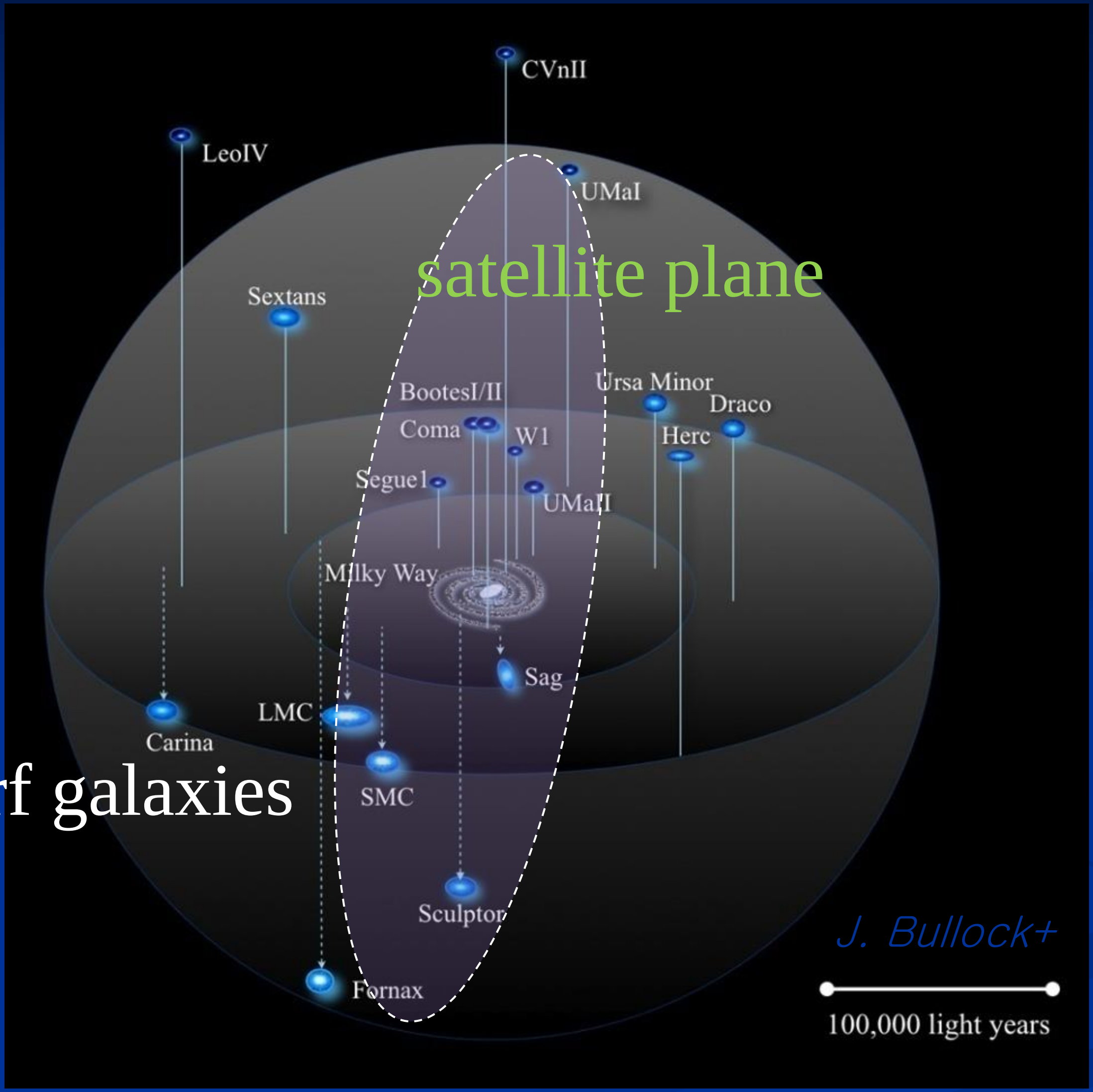
6. Cluster collision (collision speed & DM-star offset)

7. Li problem, Cosmic birefringence

…etc

→ Any good DM model should address these tensions

Galaxies observed



dwarf galaxies

Minimum mass
~ 10^6 Ms

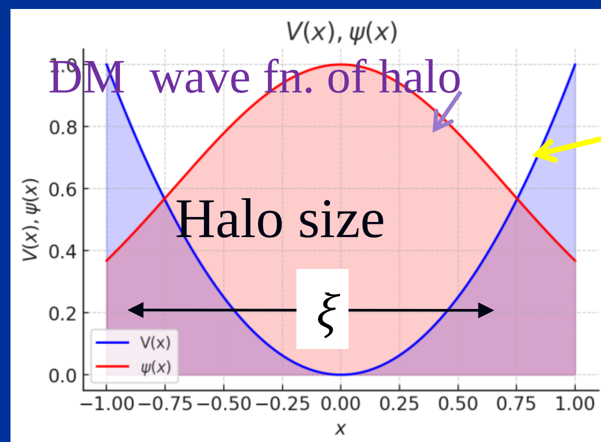
Any good DM model should explain observed galaxies

ULDM

- Galactic DM halo is a BEC made of ultralight scalar particles
- Quantum pressure (from uncertainty principle) prevents collapse
- Galaxy size ~ de Broglie wavelength of DM particles ~ kpc

→ m ~ 10^{-22} eV

- Small m → high # density → overlap of wave fn. → classical wave



Self-gravitating potential well V

Schrodinger -Poisson (SP)

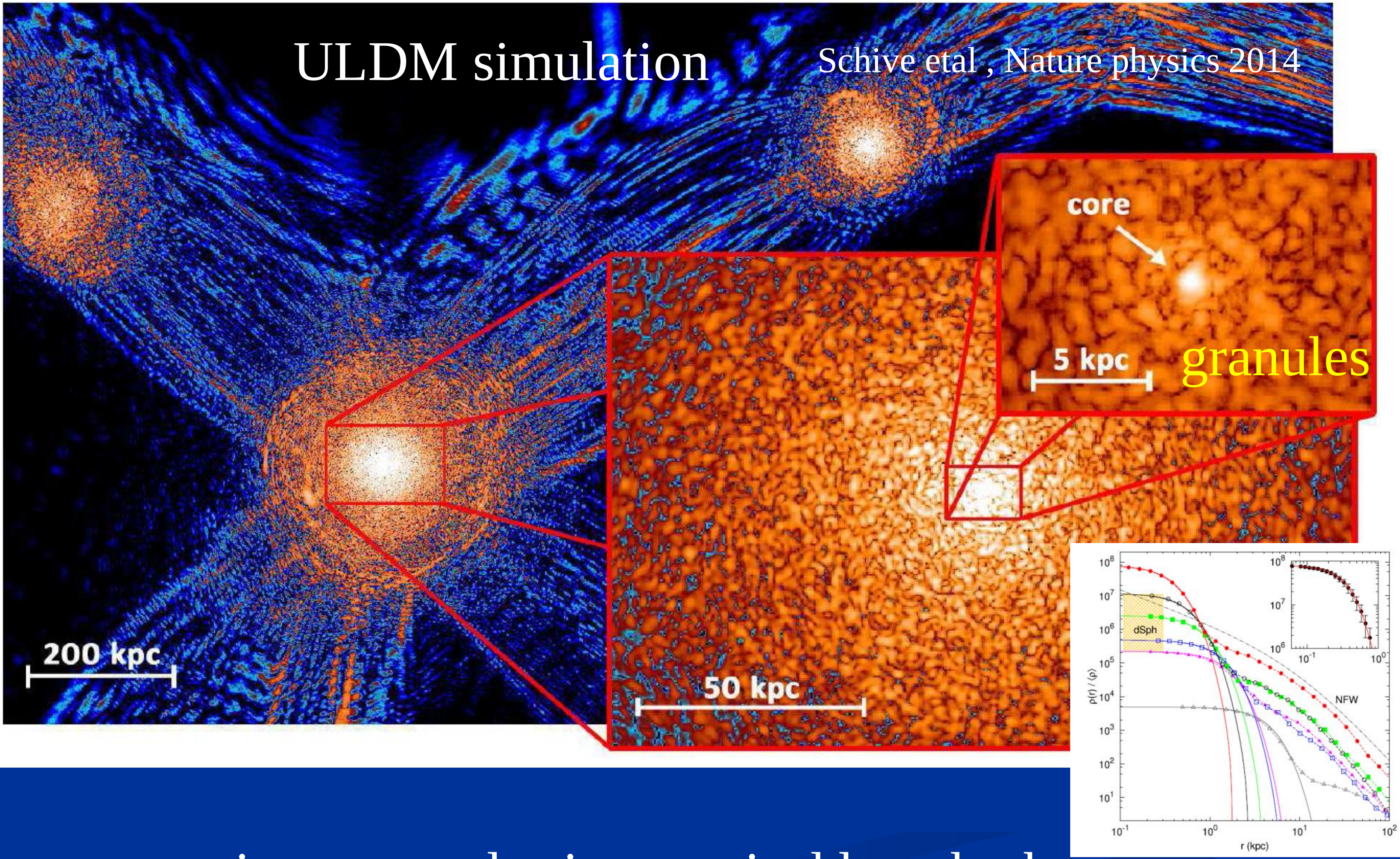
$$\begin{cases} i\hbar\partial_t\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + mV\psi + int. \\ \nabla^2 V = 4\pi G(\rho_d + \rho_v) \end{cases}$$

visible ; $\rho_d = m|\psi|^2$

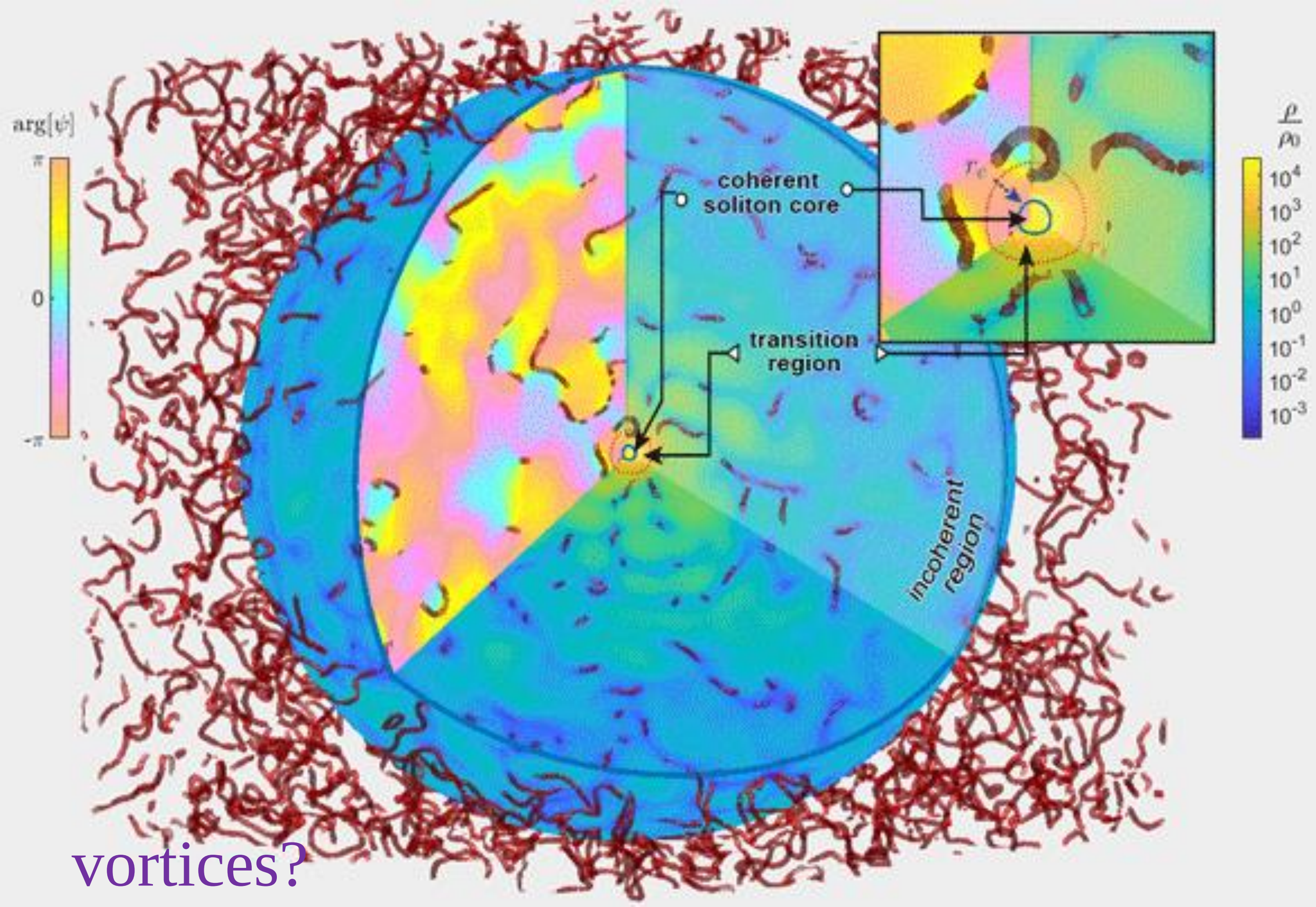
ULDM $\quad \phi(t,x) = \frac{1}{\sqrt{2m}}\left[e^{-imt}\psi(t,x) + e^{imt}\psi^*(t,x)\right]$

ULDM simulation

Schive etal , Nature physics 2014



- core size ~ granule size~ typical length~ kpc
- core profile nicely fits with dwarf galaxy observations



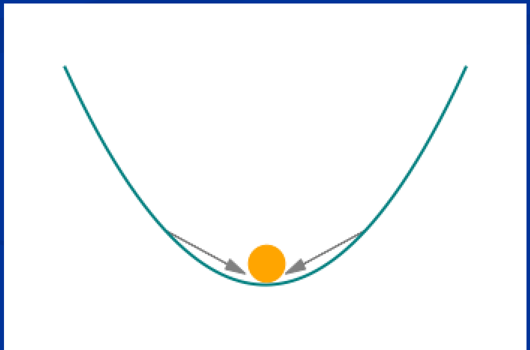
I-Kang Liu+ MNRAS

Features of ULDM

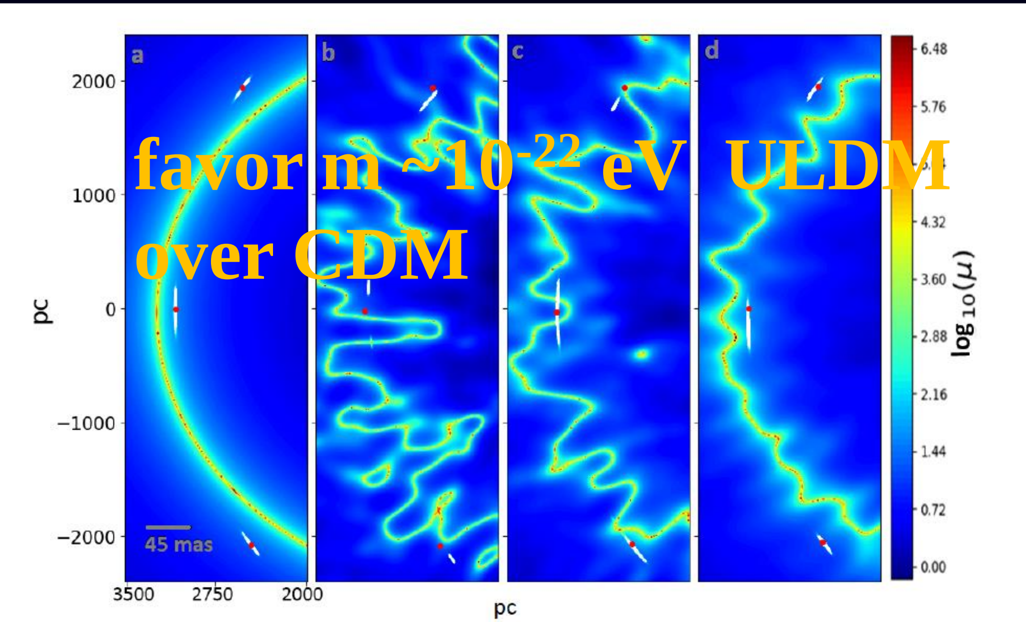
$$\phi(t,x) = \frac{1}{\sqrt{2m}}\left[e^{-imt}\psi(t,x) + e^{imt}\psi^*(t,x)\right]$$

fast (bg), slow (galaxy)

- Typical galaxy size ~ λ_{dB} ~ kpc
- wave nature → gravitational cooling
- small dynamical friction
- bg oscillation with m ~ nHz
- to explain DM density
  → GUT scale field value



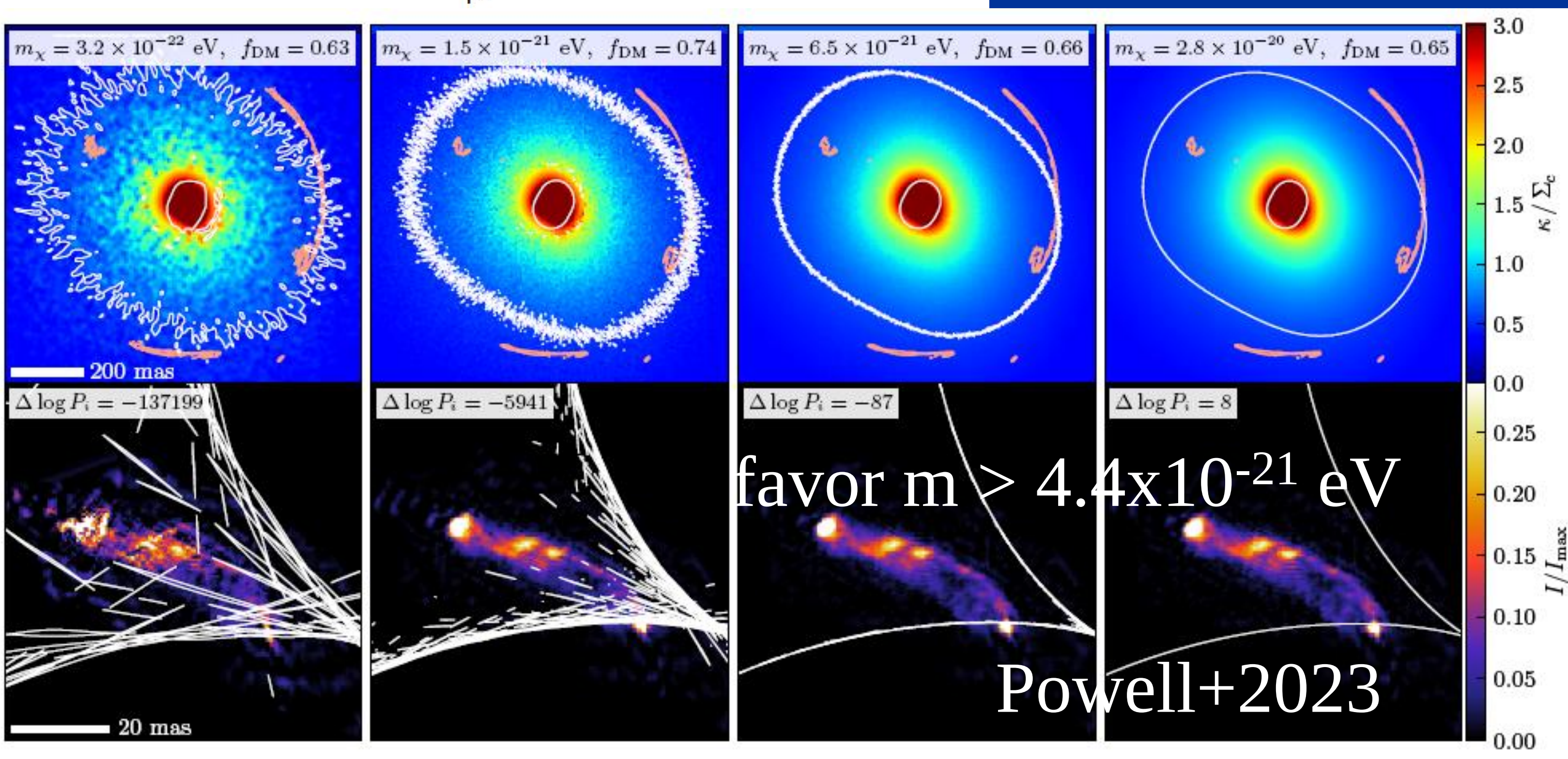
→ explain many mysteries of galaxies



favor m ~10^{-22} eV ULDM over CDM

ULDM well reproduce lens of radio objects

Armurth+2023



favor m > 4.4×10^{-21} eV

Powell+2023

Linear pert. Of ULDM

FDM has only 2 parameters m and bg density ρ_0
(+ λ for ϕ^4 self-interacting ULDM)

a=scale factor

Nonrelativistic →

$$i\hbar\left(\frac{\partial\psi}{\partial t}+\frac{3}{2}H\psi\right)=-\frac{\hbar^2}{2ma^2}\Delta\psi+mV\psi+\frac{\lambda|\psi|^2\psi}{2m^2}$$ self-int

Madelung representation

$$perturbation \quad \text{with } \psi=\sqrt{\rho}e^{iS}, \qquad v\equiv\frac{\hbar}{ma}\nabla S\Rightarrow$$

$$\begin{cases}\partial_t\rho+3H\rho+\frac{1}{a}\nabla\cdot(\rho v)=0\\ \partial_t v+\frac{1}{a}v\cdot\nabla v+Hv+\frac{1}{\rho a}\nabla p+\frac{1}{a}\nabla V+\frac{\hbar^2}{2m^2a^3}\nabla\left(\frac{\Delta\sqrt{\rho}}{\sqrt{\rho}}\right)=0\end{cases}$$

Quantum Pressure

perturbation $\delta=\delta_k=\delta\rho/\rho_0$

Density contrast (k space)

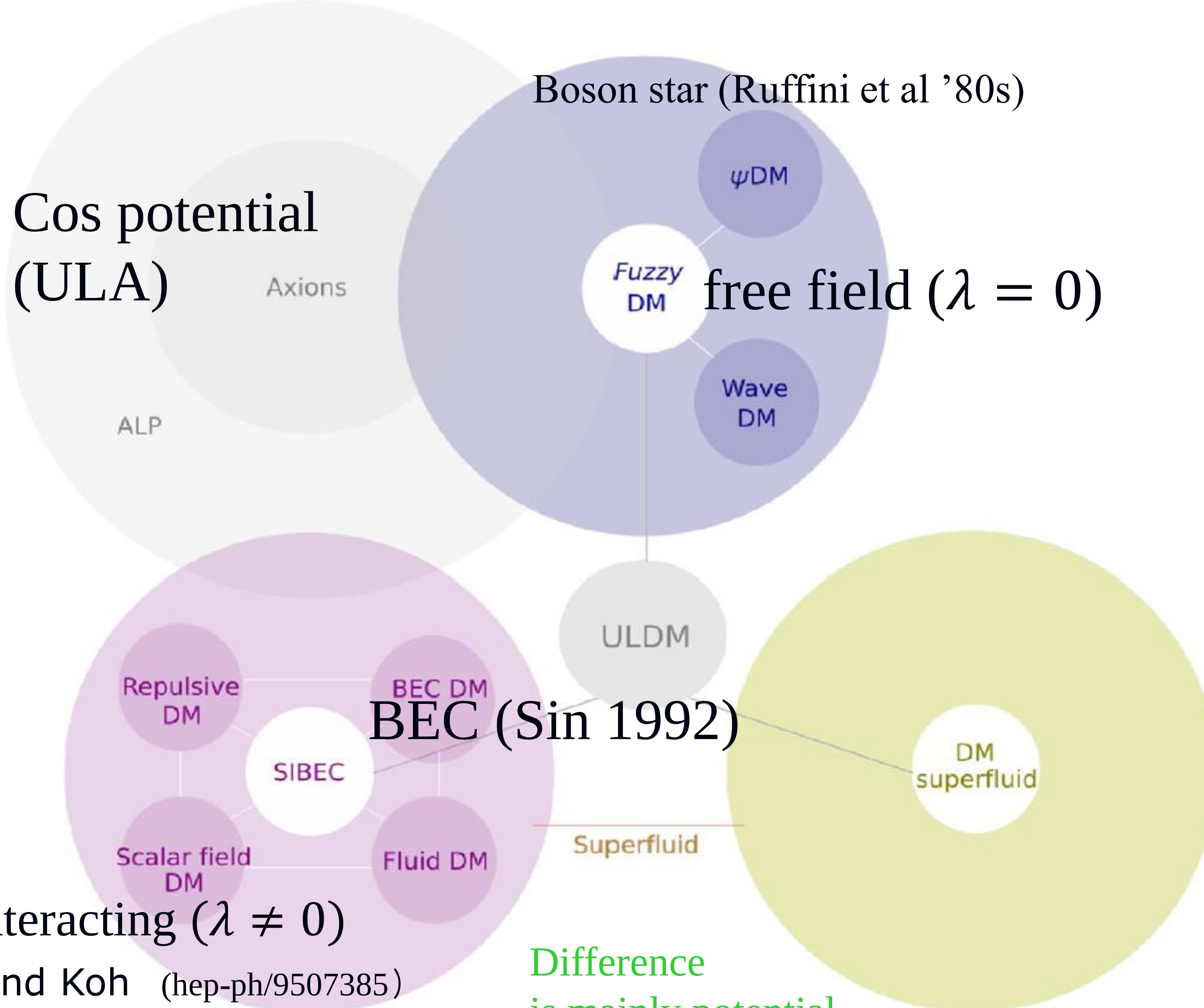
$$\Rightarrow\partial_t^2\delta+2H\partial_t\delta+\left(\left(\frac{\hbar^2k^2}{4m^2a^2}+c_s^2\right)\frac{k^2}{a^2}-4\pi G\rho_0\right)\delta=0$$

Hubble drag   gravity

Quantum Jeans length

$$\lambda_J=\frac{2\pi}{k_J}a=\pi^{3/4}\hbar^{1/2}(G\rho_0m^2)^{-1/4}\propto 1/\sqrt{mH}$$

- CDM-like on super-galactic scale (for a small $k < k_J$)
- Suppress sub-galactic structure (for a large $k > k_J$)



Boson star (Ruffini et al ’80s)

Cos potential (ULA)

free field ($\lambda = 0$)

BEC (Sin 1992)

Self-interacting ($\lambda \neq 0$)

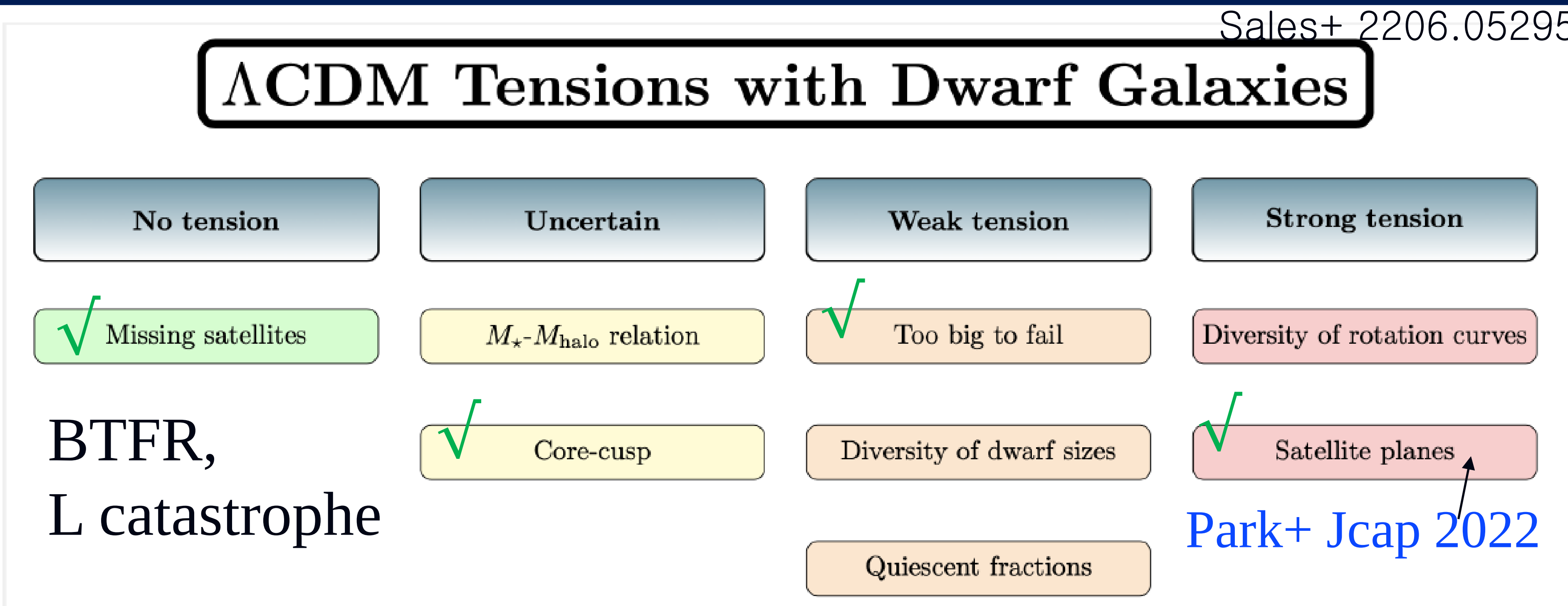
Lee and Koh (hep-ph/9507385)

Difference is mainly potential and name

Fig. 9 Map of the ULDM classes of models

Ferreira et al.

Some of small scale issues with CDM



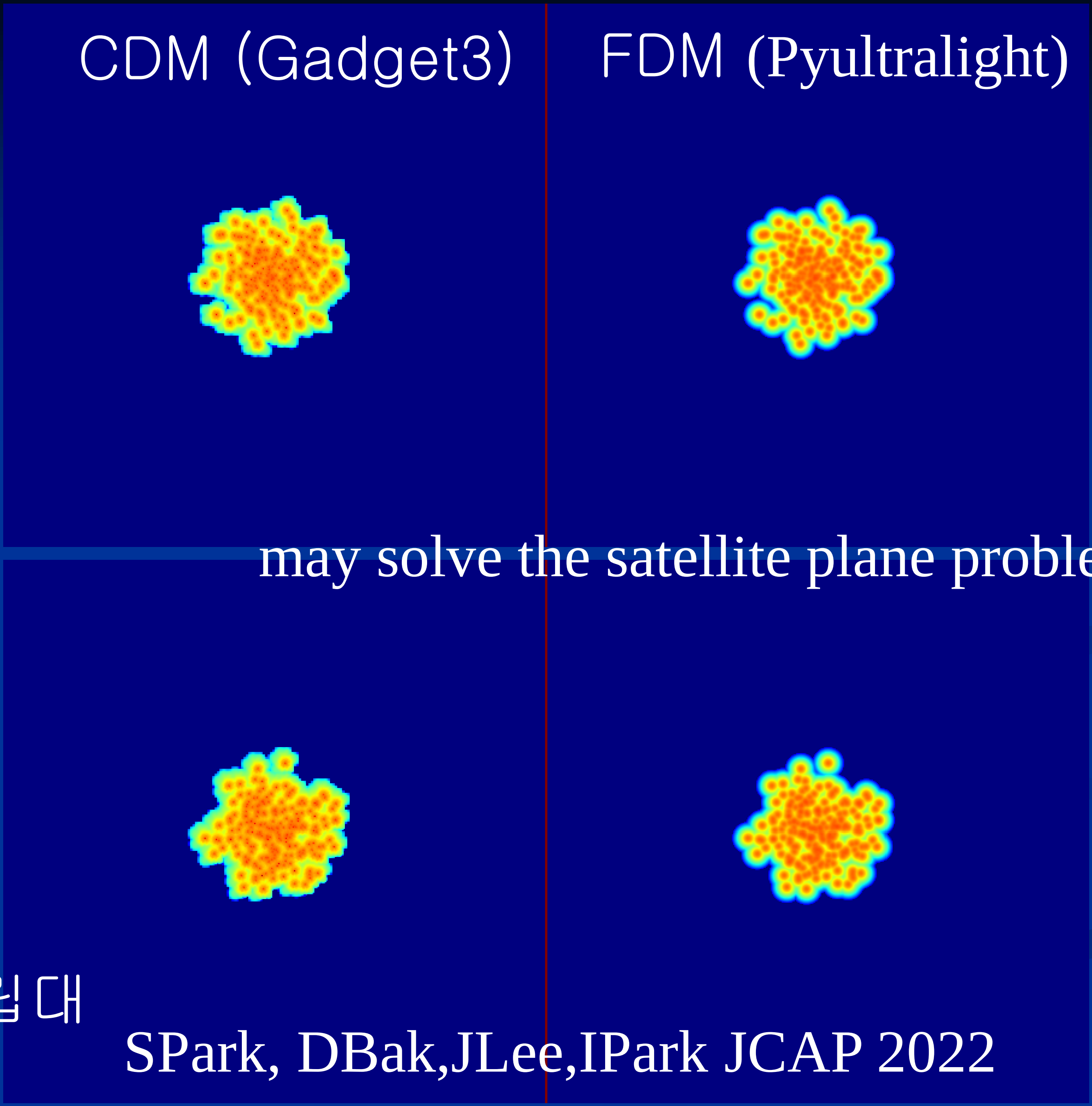
- Key problem is how to suppress small scale structures < dwarf galaxies.
  → we need a new CDM → ULDM with m~ 10^{-22} eV can solve many of these

- Still unsolved problems seem to be related to Baryon-DM relation

- Can baryon physics + precise numerical simulation + more observations save CDM?

x-z

x-y

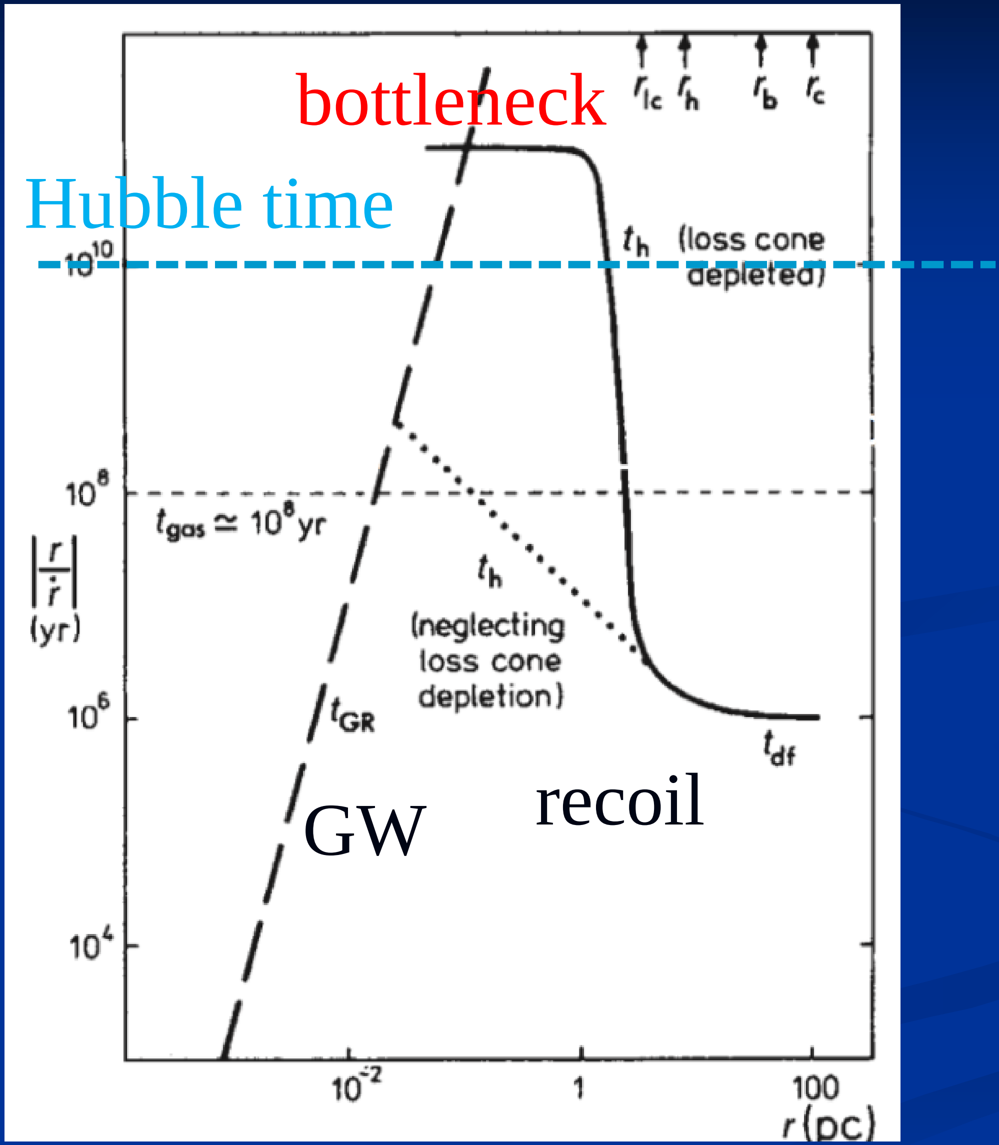


may solve the satellite plane problem

with 시립대

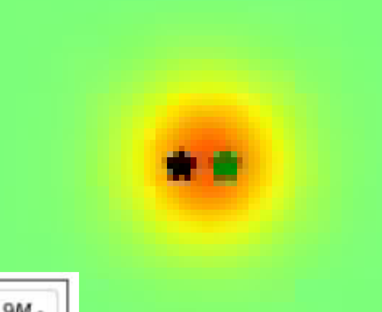
SPark, DBak,JLee,IPark JCAP 2022

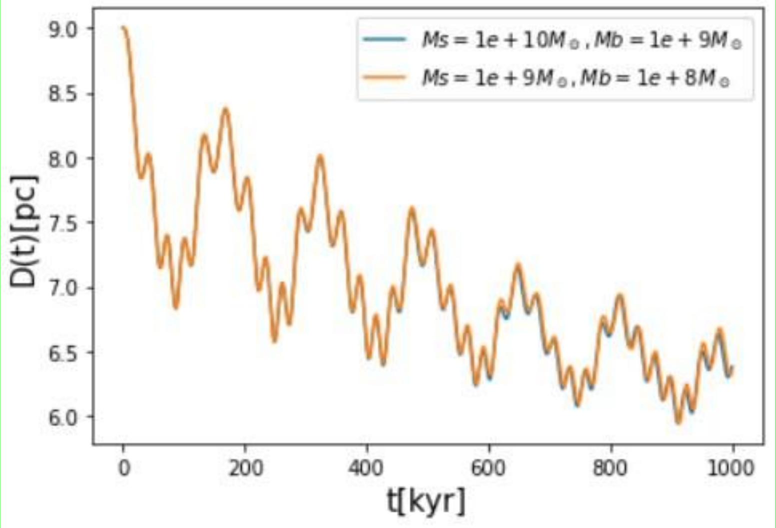
Final pc problem



Begelman+ (1980)

BH binary in a ULDM spike





may solve final pc problem
(Koo+ 2311.03412)

Typical scales of FDM

JLee 2310.01442

time dependent and functions of $\frac{\hbar}{m}$

1) time

$t_c \simeq (G\bar{\rho})^{-1/2}$: Hubble time of formation ~ dynamical time scale

2) length (q. Jeans length) → explain size evolution (JLee PLB 2016)

$$x_c = \lambda_{dB} = \left(\frac{\hbar}{m}\right)^2 \frac{1}{GM} = 854.8\, pc \left(\frac{10^{-22} eV}{m}\right)^2 \frac{10^8 M_\odot}{M} = \sqrt{\frac{\hbar}{m}} (G\bar{\rho})^{-1/4}$$

~ Gravitational Bohr radius ~ de Broglie wavelength

3) velocity

$$v_c \equiv x_c/t_c = GM\, m/\hbar = 22.4\, km/s \left(\frac{M}{10^8 M_\odot}\right)\left(\frac{m}{10^{-22} eV}\right) \simeq \sqrt{\frac{\hbar}{m}} (G\bar{\rho})^{1/4},$$

4) mass

$$M_Q = \frac{4}{3}\left(\frac{\lambda_Q}{2}\right)^3 \bar{\rho} = \frac{4}{3}\pi^{\frac{13}{4}} \left(\frac{\hbar}{G^{\frac{1}{2}} m}\right)^{\frac{3}{2}} \bar{\rho}(z)^{\frac{1}{4}} = 1.54 \times 10^8 M_\odot \left(\frac{m}{10^{-22} eV}\right)^{-3/2} \left(\frac{\bar{\rho}}{10^{-7} M_\odot/pc^3}\right)^{1/4}$$

also explain max. mass of galaxies ~ $10^{12} M_\odot$

5) Angular momentum

$$L_c = M x_c v_c = \hbar \frac{M}{m} = N\hbar, \text{ (L eigenstates?)}$$

$$= 1.1 \times 10^{96} \hbar \left(\frac{M}{10^8 M_\odot}\right)\left(\frac{10^{-22} eV}{m}\right) \simeq \frac{\left(\frac{\hbar}{m}\right)^{5/2} \bar{\rho}^{1/4}}{G^{3/4}}$$

6) acceleration → MOND (LKL, PLB 2019)

$$a_c = x_c/t_c^2 = G^3 m^4 M^3/\hbar^4$$

$$= 1.9 \times 10^{-11} meter/s^2 \left(\frac{m}{10^{-22} eV}\right)^4 \left(\frac{M}{10^8 M_\odot}\right)^3 \simeq \sqrt{\frac{\hbar}{m}} (G\bar{\rho})^{3/4}$$

cf) MOND scale $a_0 = 1.2 \times 10^{-10} meter/s^2$

7) potential V_c =1 gives Max. Galaxy mass M = $10^{12} M_\odot$

$$V_c = \frac{m^2}{\hbar^2}(4\pi G M)^2 = 8.8 \times 10^{-7} c^2 \sim \left(\frac{m}{10^{-22} eV}\right)^2 \left(\frac{M}{10^8 M_\odot}\right)^2$$ Nonrelativistic

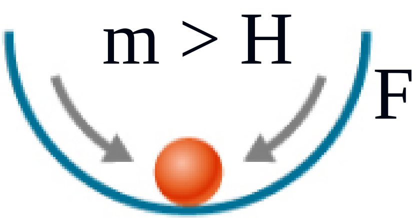
FDM gives typical scales of (dwarf) galaxies

ULA miracle

$$I = \int d^4x\sqrt{g}\left[\frac{1}{2}F^2 g^{\mu\nu}\partial_\mu a \partial_\nu a - \mu^4(1-\cos a)\right]$$

$$m = \frac{\mu^2}{F}$$

$$\ddot{a} + 3H\dot{a} + m^2 \sin a = 0$$



oscillation starts at $\mathrm{H} \sim \frac{T_{osc}^2}{M_P} = m$

MDE starts at $T_1 \sim 1eV \rightarrow \frac{\mu^4(DM)}{T_{osc}^4(rad)} \rightarrow \frac{\mu^4 T_{osc}}{T_{osc}^4 T_1} \sim 1$

$$F = \frac{\mu^2}{m} \sim \frac{M_P^{3/4} T_1^{1/2}}{m^{1/4}} \sim 10^{17}GeV \quad typical\ field\ value$$

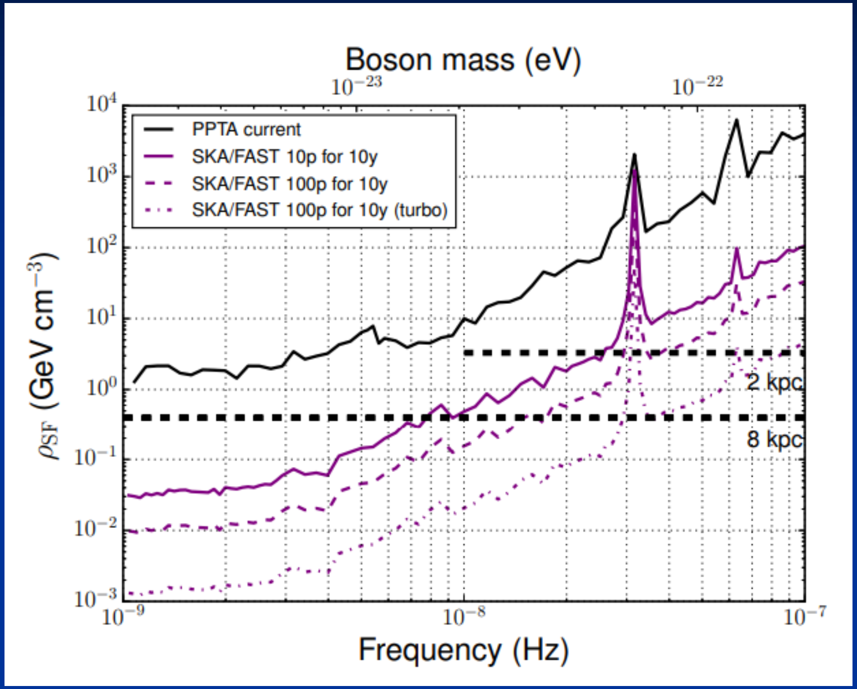
$\Omega_a \sim 0.1\left(\frac{F}{10^{17}GeV}\right)^2\left(\frac{m}{10^{-22}eV}\right)^{1/2}$ ULA miracle?

Hui et al 2017

ULDM naturally explains DM density with GUT scale.

This holds for generic ULDM with a quadratic pot.

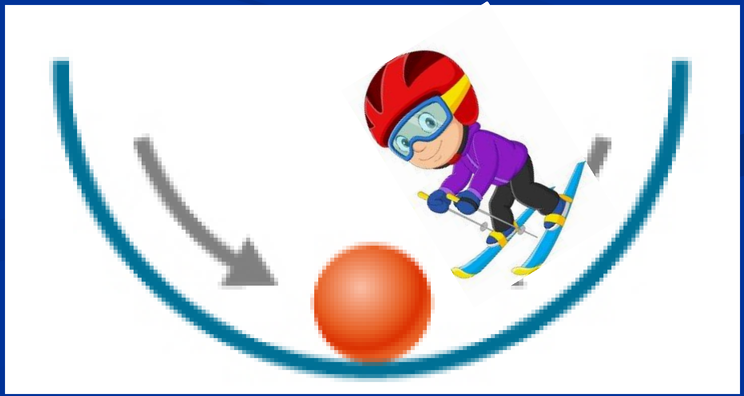
GW background detected by pulsar timing array



1810.03227

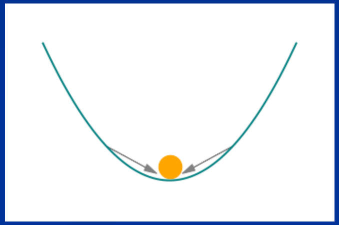
ULDM has
intrinsic osc time scale
period~1/m ~ yrs

$$\omega = \frac{1}{2.5 months} \frac{m}{10^{-22} eV}$$



Thermal history of FDM

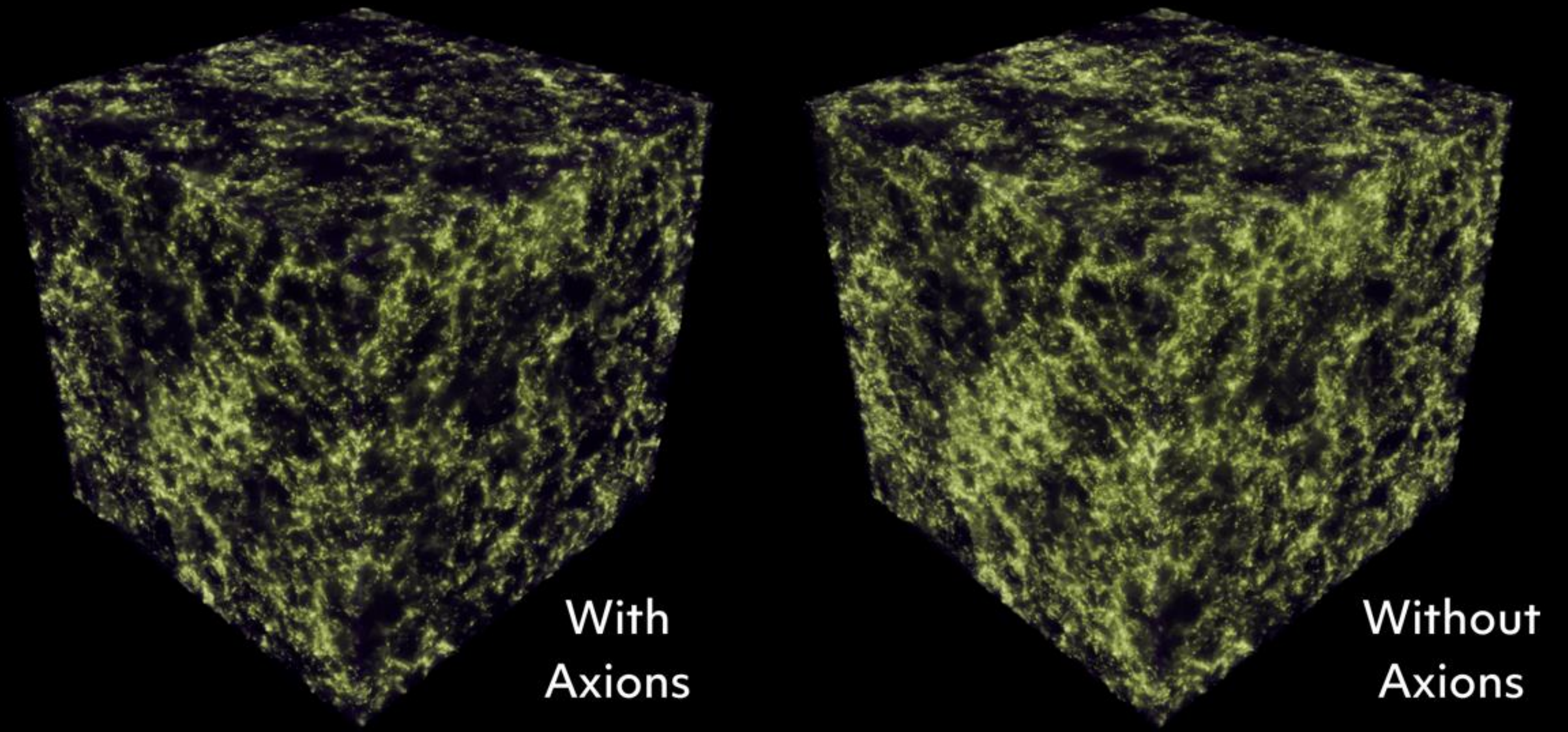
GUT	$F \sim \frac{M_P^{3/4}\, T_1^{1/2}}{m^{1/4}} \sim 10^{17}$ GeV
Oscillation starts	$T_{osc} \sim (M_P m)^{1/2} \sim 10^3$eV
mde	$T_1 \sim 1$ eV
Now	$T_0 \sim 10^{-4}$ eV



$$\Omega_\phi \sim 0.1 \left(\frac{F}{10^{17} GeV}\right)^2 \left(\frac{m}{10^{-22}\, eV}\right)^{1/2}$$

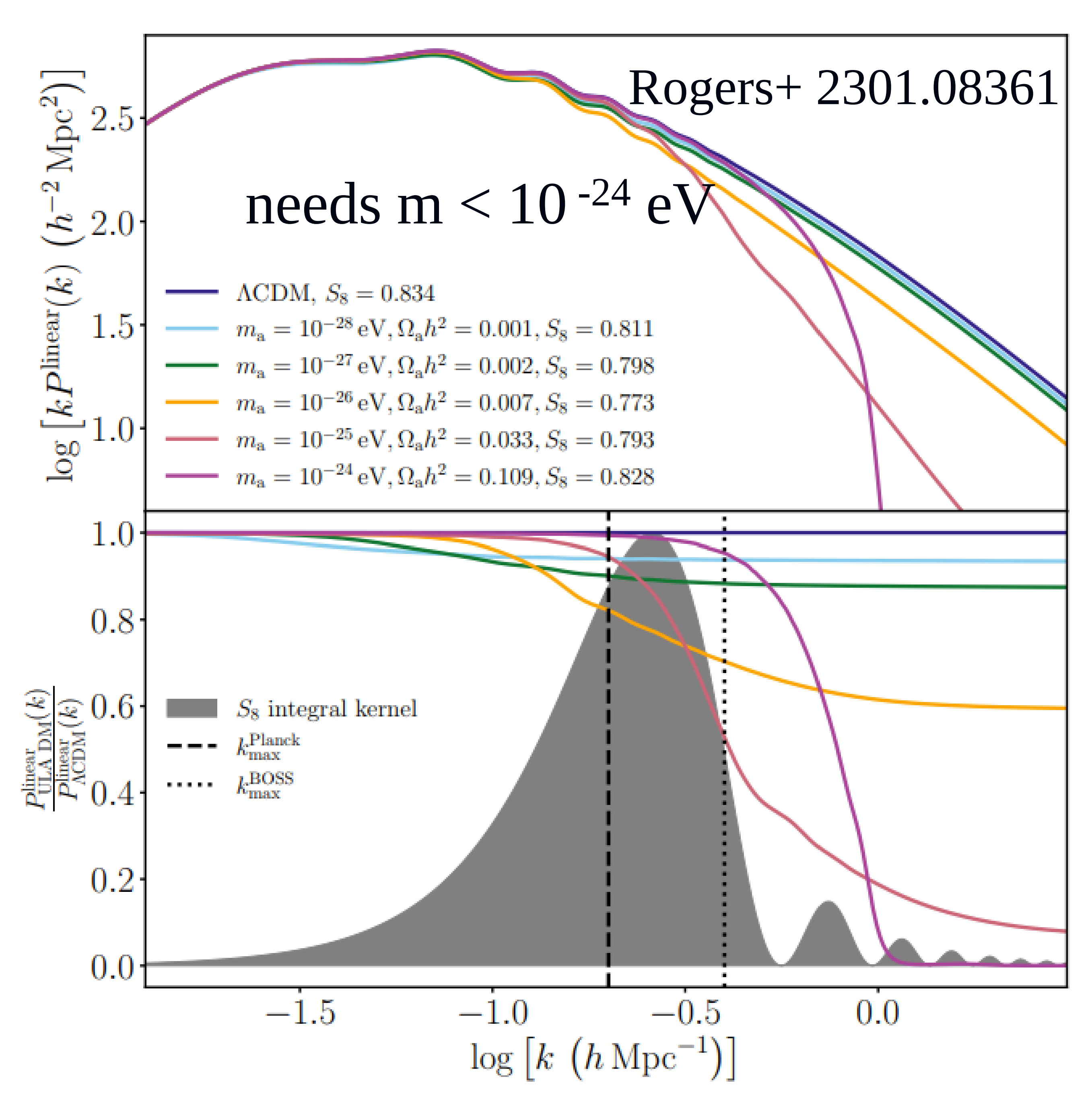
S_8=0.76 (WL)

=0.82 (CMB+ΛCDM)



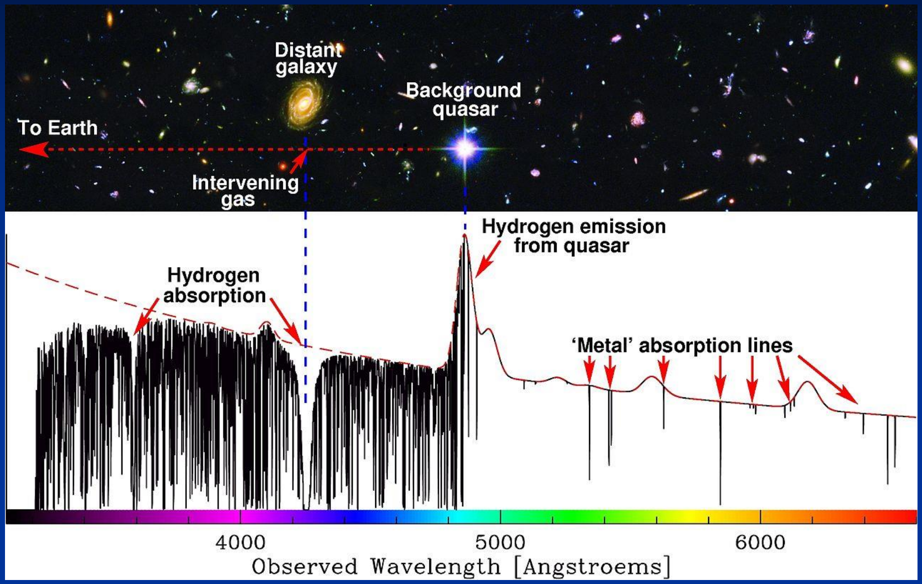
Alexander Spencer London

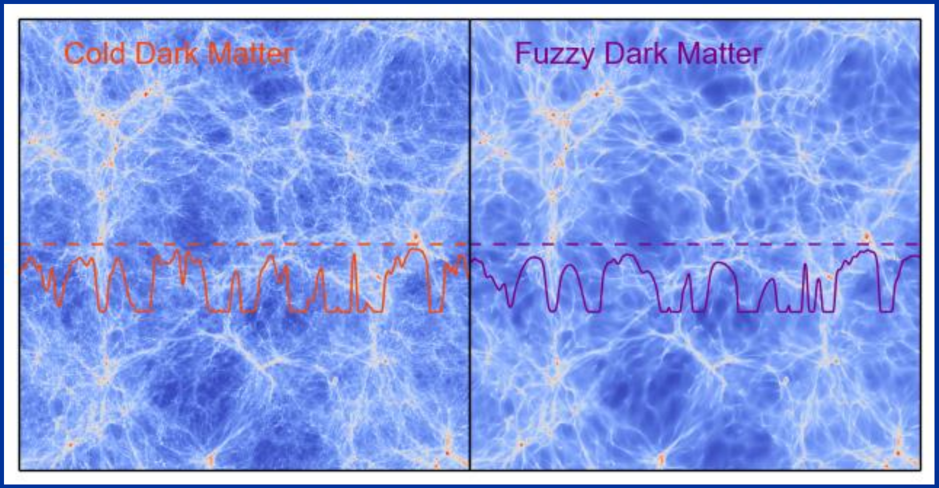
Can FDM solve S_8 tension?



Rogers+ 2301.08361
needs m < 10 -24 eV
ΛCDM, $S_8 = 0.834$
$m_a = 10^{-28}$ eV, $\Omega_a h^2 = 0.001$, $S_8 = 0.811$
$m_a = 10^{-27}$ eV, $\Omega_a h^2 = 0.002$, $S_8 = 0.798$
$m_a = 10^{-26}$ eV, $\Omega_a h^2 = 0.007$, $S_8 = 0.773$
$m_a = 10^{-25}$ eV, $\Omega_a h^2 = 0.033$, $S_8 = 0.793$
$m_a = 10^{-24}$ eV, $\Omega_a h^2 = 0.109$, $S_8 = 0.828$
$\log\left[k P^{\mathrm{linear}}(k)\ (h^{-2}\,\mathrm{Mpc}^2)\right]$
S_8 integral kernel
$k_{\mathrm{max}}^{\mathrm{Planck}}$
$k_{\mathrm{max}}^{\mathrm{BOSS}}$
$\frac{P^{\mathrm{linear}}_{\mathrm{ULA\ DM}}(k)}{P^{\mathrm{linear}}_{\Lambda\mathrm{CDM}}(k)}$
$\log\left[k\ (h\,\mathrm{Mpc}^{-1})\right]$

Lyman alpha tension?





$m > 10^{-21}$eV

PRL 2017 (Irisic et al)

Hydrosimulation uncertainty is large

WDM (1, 3.3) keV ~ FDM (1, 20) x 10^{-22}eV

Self-Interacting ULDM

Lee and Koh (PRD 53, hep-ph/9507385)

Galactic DM is described by coherent scalar field with self-interaction

Action

$$S = \int \sqrt{-g} d^4x \left[\frac{-R}{16\pi G} - \frac{g^{\mu\nu}}{2} \phi^*_{;\mu} \phi_{;\nu} - \frac{m^2}{2} |\phi|^2 - \frac{\lambda}{4} |\phi|^4 \right]$$

typical phi4 theory with gravity

Metric

$$ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2 d\Omega \quad \textit{Spherical.}$$

Field

$$\phi(r,t) = (4\pi G)^{-\frac{1}{2}} \sigma(r) e^{-i\omega t}$$

Stationary spherical

Exact ground state

$$\sigma_* = \sqrt{\frac{\gamma_0 \, \mathrm{Sin}(\sqrt{2} r_*)}{\sqrt{2} r_*}}$$

$$r_* = r\Lambda^{-1/2}$$

$\Lambda \equiv \frac{\lambda m_P^2}{4\pi m^2}$, $\Lambda >> 1$ (Newtonian & TF limit)

New length scale $R_{TF} \approx \frac{\sqrt{\Lambda}}{m} = \sqrt{\frac{\pi \hbar^3 \lambda}{8cGm^4}}$

& mass scale $M_{\max} = \sqrt{\Lambda} \frac{m_P^2}{m}$

Even tiny self-interaction drastically changes the scales!

→ allows wider range for m

→ constant length scale!

Merits of studying self-interacting ULDM

We can

- allow wider mass range

→avoid some tensions of FDM
- study direct, or indirect detection of ULDM
- calculate abundance
- understand particle model
- solve other mysteries like Hubble tension

Typical scales for selfinteracting ULDM

Lee & Ji 2412.10285

are functions of $\tilde{m} = \frac{m}{\lambda^{1/4}}$ ~ energy scale of ULDM

1) time

$t_c \simeq (G\bar{\rho})^{-1/2}$: Hubble time

2) length (Jeans length from self-int.)

$\lambda_J = 2\pi/k_J = \sqrt{\frac{\pi\hbar^3\lambda}{2cGm^4}} = 0.978 kpc \left(\frac{\tilde{m}}{10\text{eV}}\right)^{-2}$ $= 2R_{TF}$ t-indep. → $\tilde{m} \sim 10\text{eV}$

3) mass $M_J = \frac{4\pi}{3}\left(\frac{\lambda_J}{2}\right)^3 \bar{\rho} = \frac{\pi^{5/2}}{\sqrt{288}}\left(\frac{\hbar^3\lambda}{cGm^4}\right)^{3/2} \bar{\rho} = 4.908 \times 10^6 M_\odot \left(\frac{\tilde{m}}{10eV}\right)^{-6}\left(\frac{\bar{\rho}}{10^{-2}M_\odot/pc^3}\right)$

too small for $\tilde{m} \sim 10\text{eV}$

4) velocity $v_c \equiv x_c/t_c = \frac{2^{7/4}\left(\frac{cG^3m^1}{\hbar^3\lambda}\right)^{1/4}}{\pi^{1/4}}\sqrt{M} = 59.28\ \text{km/s}\left(\frac{M}{10^8 M_\odot}\right)^{1/2}\left(\frac{\tilde{m}}{10\text{eV}}\right)$

5) Angular momentum

$$L_c = M\mathbf{x}_c v_c = \left(\frac{32\pi G\hbar^3\lambda}{cm^4}\right)^{1/4} M^{3/2} = 3.375\times 10^{96}\hbar\left(\frac{M}{10^8 M_\odot}\right)^{3/2}\left(\frac{10\text{eV}}{\tilde{m}}\right)$$

6) acceleration

$$a_c = x_c/t_c^2 = \frac{16cG^2m^4M}{\pi\hbar^3\lambda} = 1.163\times 10^{-10}\ \text{meter/s}^2\left(\frac{\tilde{m}}{10\text{eV}}\right)^4\left(\frac{M}{10^8 M_\odot}\right)$$

cf) MOND scale $a_0 = 1.2\times 10^{-10} meter/s^2$

7) potential V_c =1 gives Max. Galaxy mass M ~ $10^{16} M_\odot$

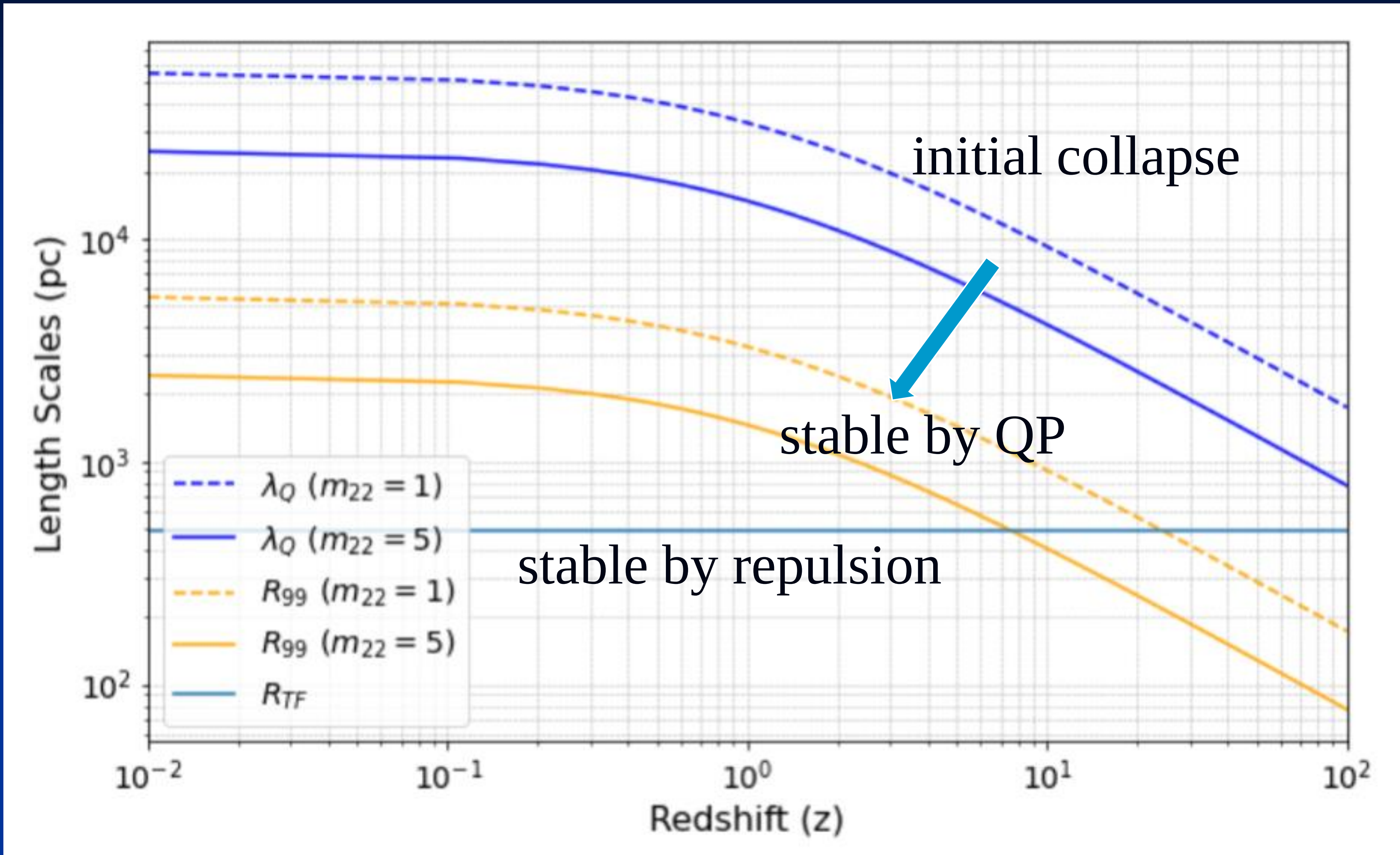
$$V_c = \frac{GM}{x_c} = \frac{GM\sqrt{\frac{2}{\pi}}}{\sqrt{\frac{\hbar^3\lambda}{cGm^4}}} = 4.888\times 10^{-9}c^2\left(\frac{\tilde{m}}{10\text{eV}}\right)^2\left(\frac{M}{10^8 M_\odot}\right)$$

8) density

$$\rho_c = \frac{2\sqrt{2}M\left(\frac{cGm^4}{\lambda}\right)^{3/2}}{\hbar^{9/2}\pi^{3/2}} = 0.106 M_\odot/pc^3\left(\frac{\tilde{m}}{10\text{eV}}\right)^6\left(\frac{M}{10^8 M_\odot}\right)$$

9) surface density observed $\Sigma_c = 10^{2.15\pm 0.2} M_\odot \text{pc}^{-2}$

$$\Sigma = \frac{\hbar\pi^{3/2}\sqrt{\frac{\hbar\lambda}{CGm^4}}\rho}{6\sqrt{2}} = 5.124 M_\odot/pc^2\left(\frac{\tilde{m}}{10eV}\right)^{-2}\left(\frac{\bar{\rho}}{10^{-2}M_\odot/pc^3}\right)$$



Length Scales (pc)
10^4
10^3
10^2
10^{-2}
10^{-1}
10^0
10^1
10^2
Redshift (z)
λ_Q ($m_{22} = 1$)
λ_Q ($m_{22} = 5$)
R_{99} ($m_{22} = 1$)
R_{99} ($m_{22} = 5$)
R_{TF}
initial collapse
stable by QP
stable by repulsion

some constraints

1) perturbative $\lambda < 1 \quad \rightarrow m \leq 10^3 eV$

2) max Mass $\lambda^{1/2}\left(\frac{m_P}{m}\right)^2 \geq 10^{50} \rightarrow$ $m^4 < \lambda \times 2.2 \times 10^{12} eV^4 \rightarrow$
$10^{-100} < \lambda$

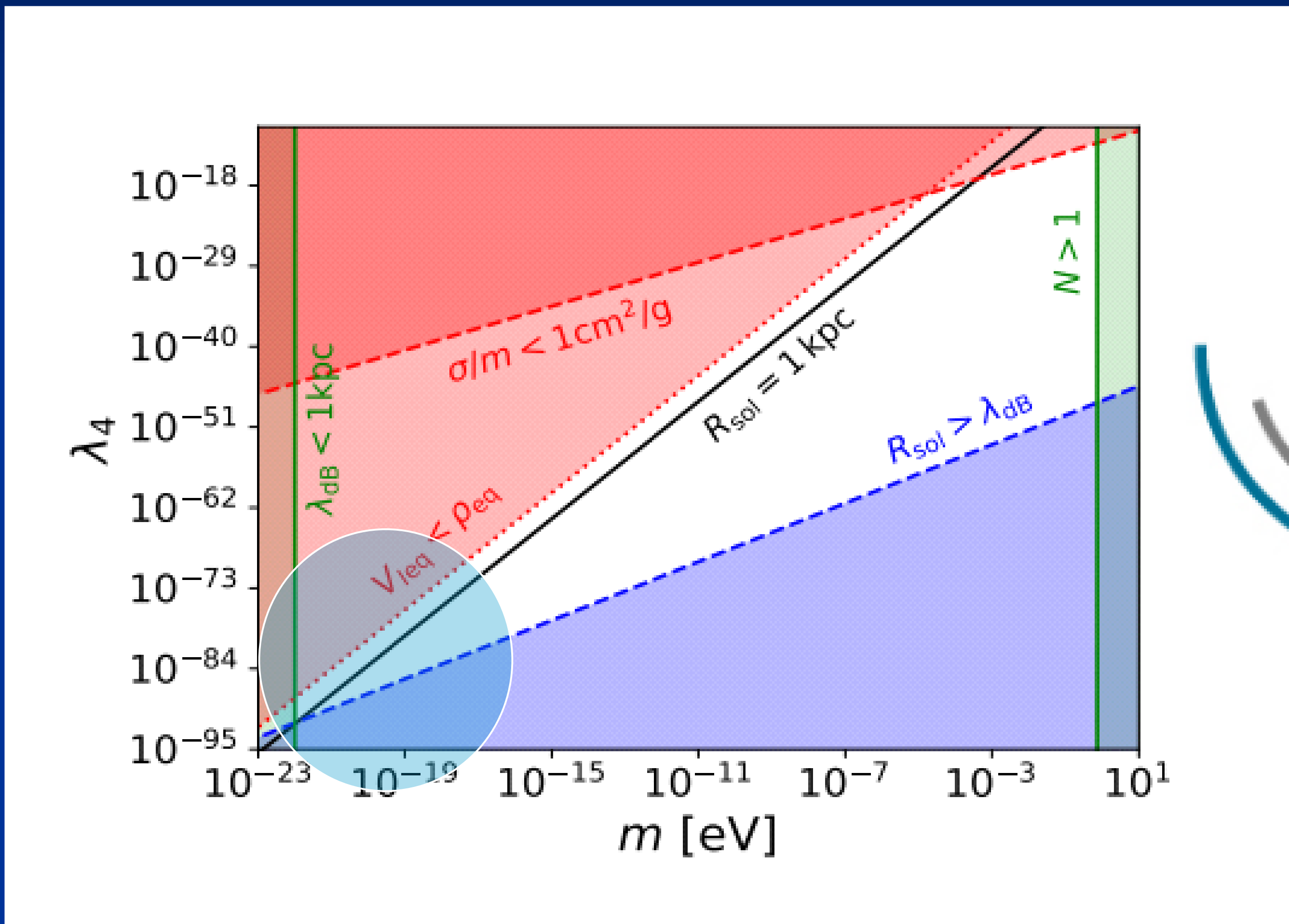
3) interparticle distance < Compton wavelength
$\rightarrow m \leq 10^{-2} eV$

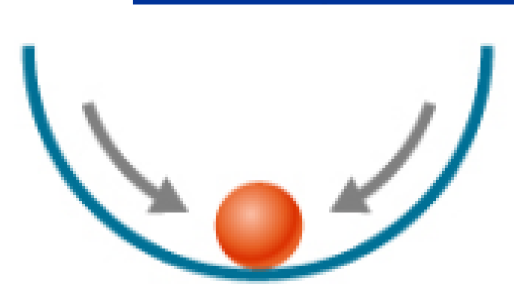
4) Core size $\lambda_J = 98\left(\frac{\lambda^{1/4}}{m}\right)^2 \text{kpc}$ ~kpc → $m/\lambda^{1/4}$~10 eV

5) act as DM at MR equality $m/\lambda^{1/4} > 1 \text{ eV}$

cosmological constraints

Garcia + 2304.10221





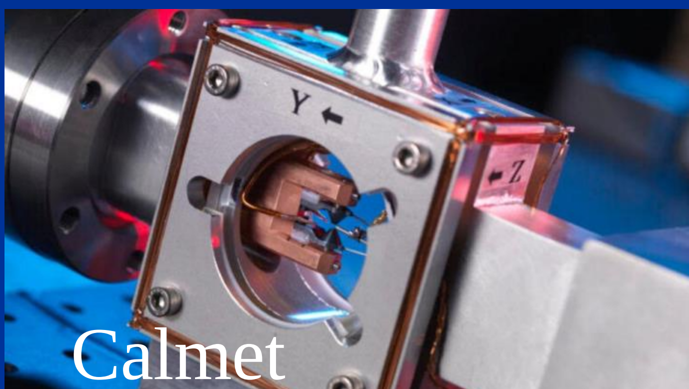
Detection

dilatonic coupling

$$\mathcal{L} \supset \varphi \frac{d_e}{4\mu_0} F_{\mu\nu}F^{\mu\nu} ,$$

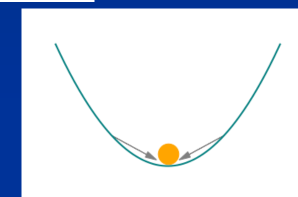
Using atomic clocks to detect ULDM by mimickin g time variations of fundamental constants

$$\alpha(t) \approx \alpha \, [1 + d_e \varphi_0 \cos(\omega t + \delta)]$$



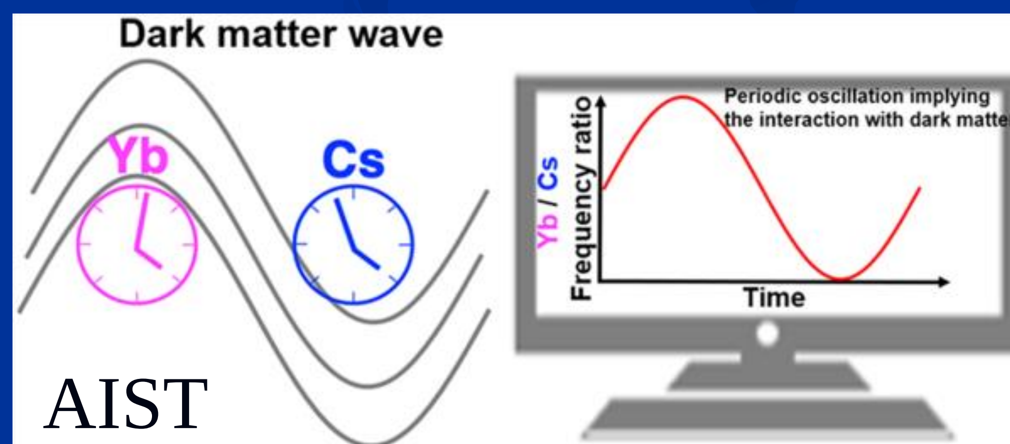
Calmet

Oscillation of fine structure constant



Due to nuclear and atomic structure Yb and Cs have different frequency dependency on α (special relativistic effects)

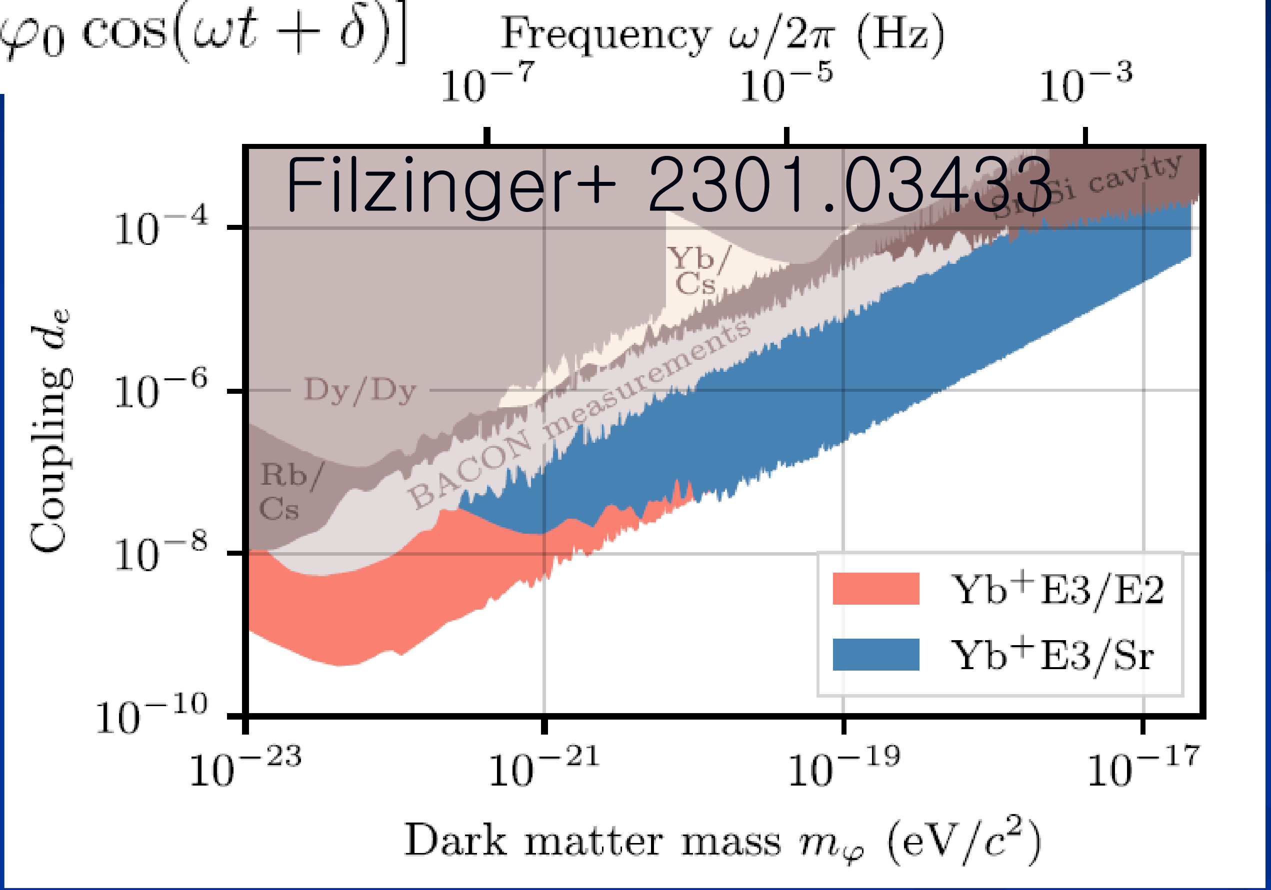
One-loop correction strongly constrain couplings



AIST

detection

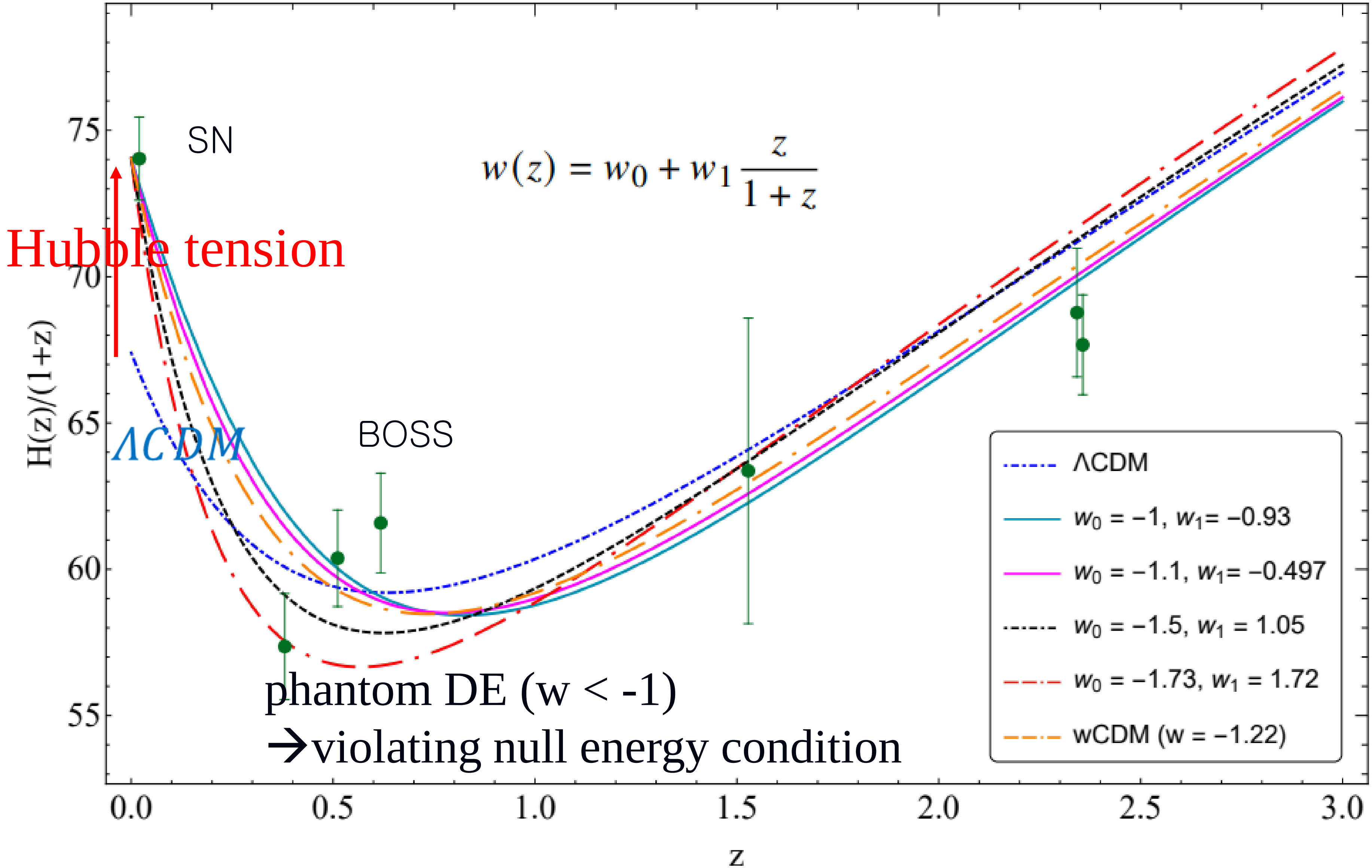
$$\alpha(t) \approx \alpha \left[1 + d_e \varphi_0 \cos(\omega t + \delta)\right]$$



$$\delta\lambda_{\phi,SM} \approx +\left((d_m \kappa m_i)^4\right)/(16\pi^2)$$

$$\approx 10^{(-92)} * \left((d_m)/\left(10^{(-6)}\right)\right)^4$$

Arvantiki 1405.2925



Alestas & L. Perivolaropoulos 2021

- standard ruler in the sky: distance travelled by sound wave until recombination.
- problem: only angular scale of sound horizon is accessible $\theta_s \equiv \frac{r_s(z_*)}{d_A(z_*)}$ **0.04% precision!**

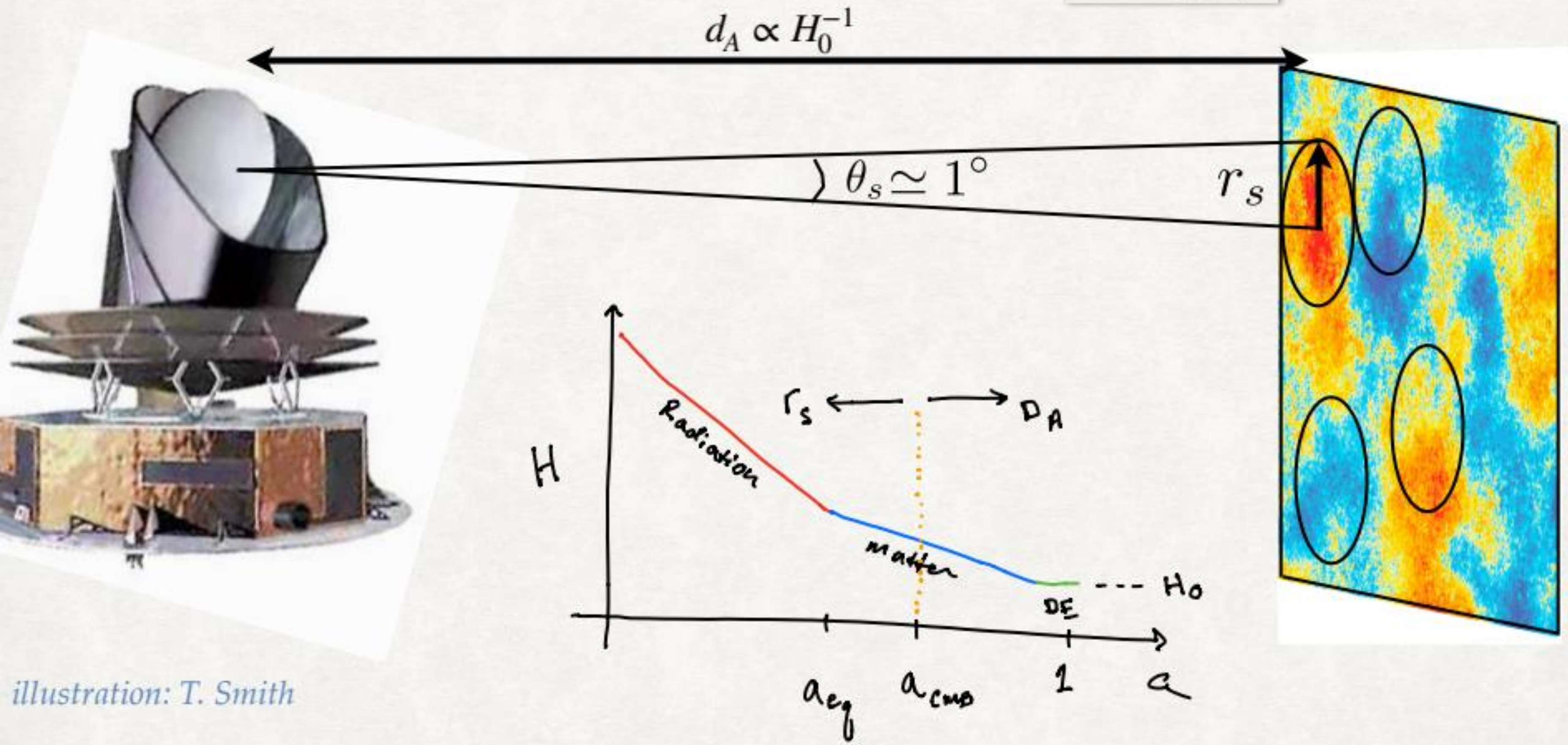
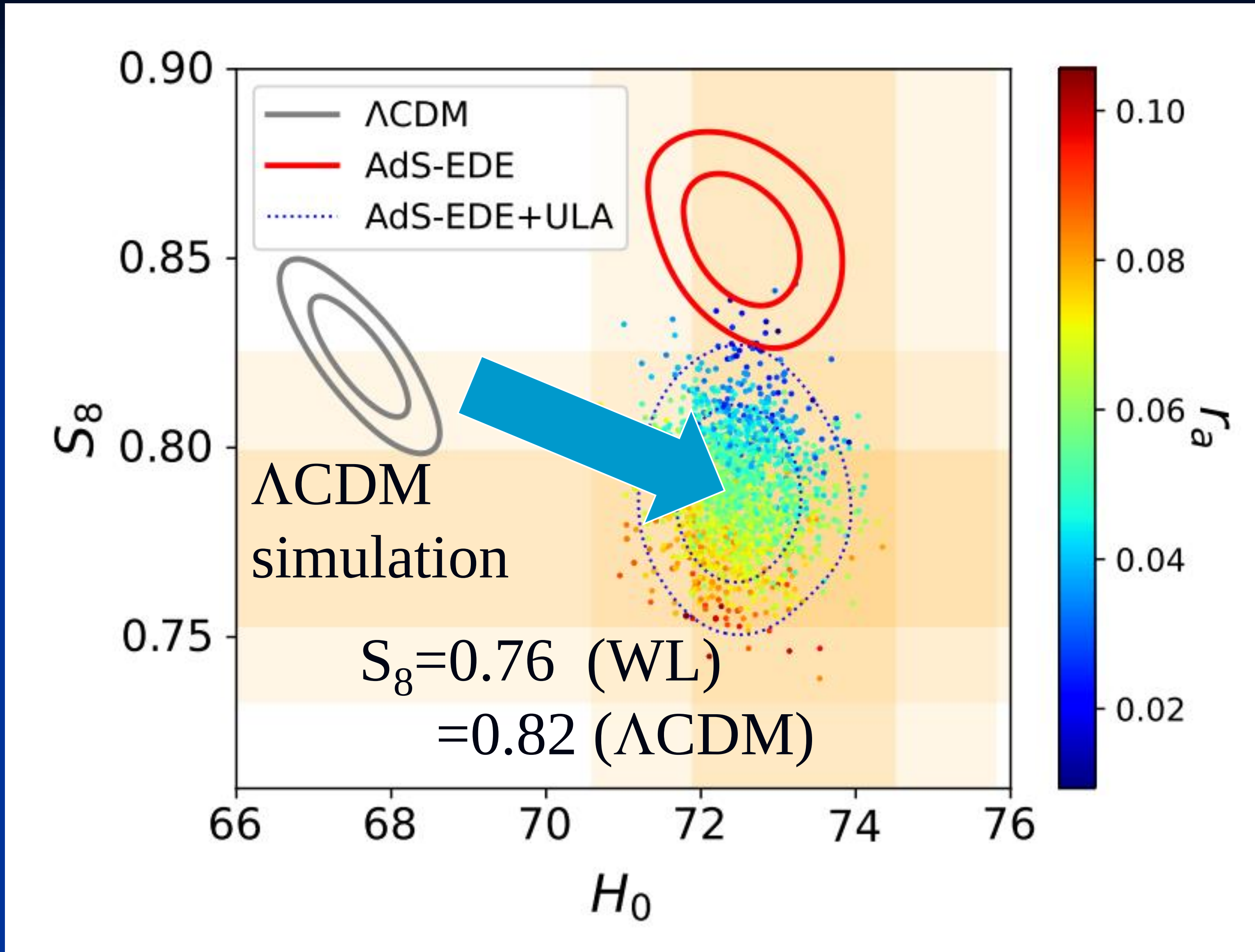
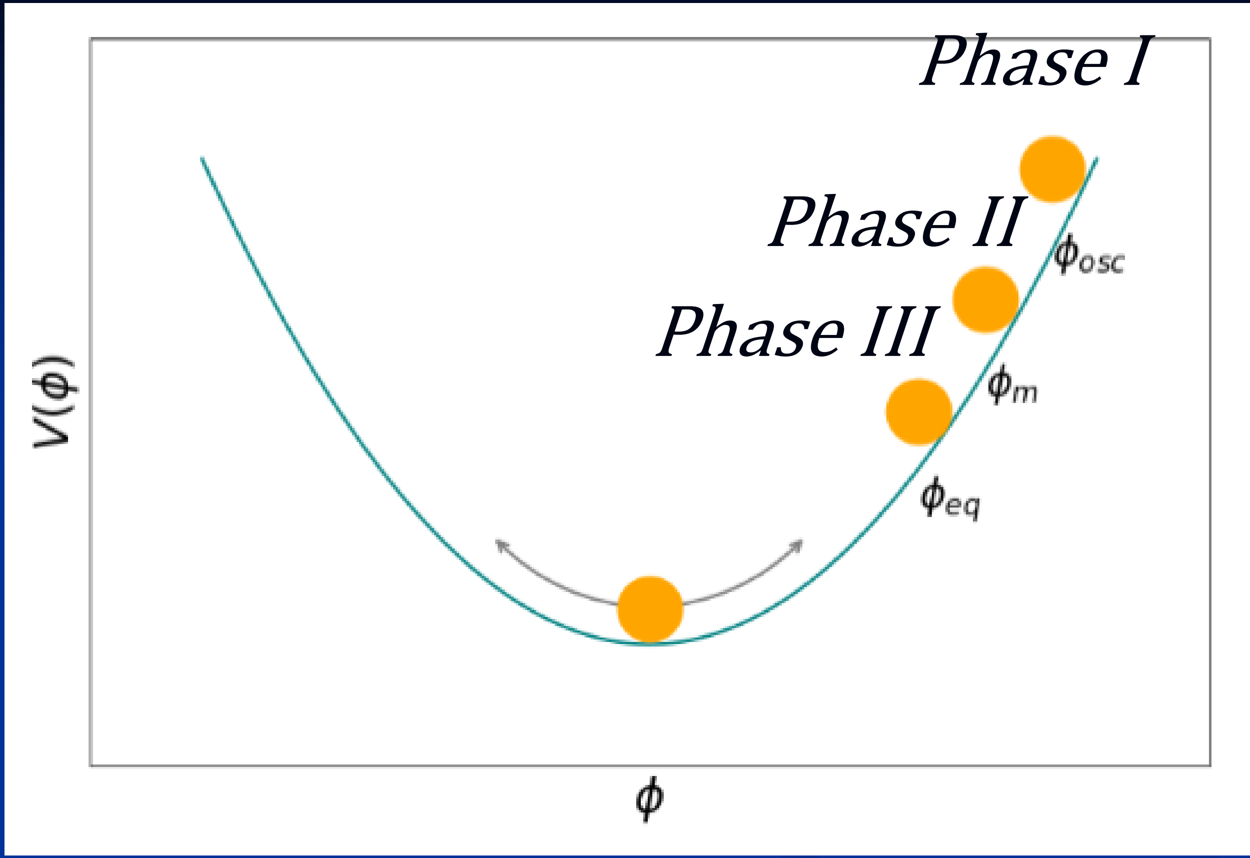


illustration: T. Smith



Ye+ 2107.13391

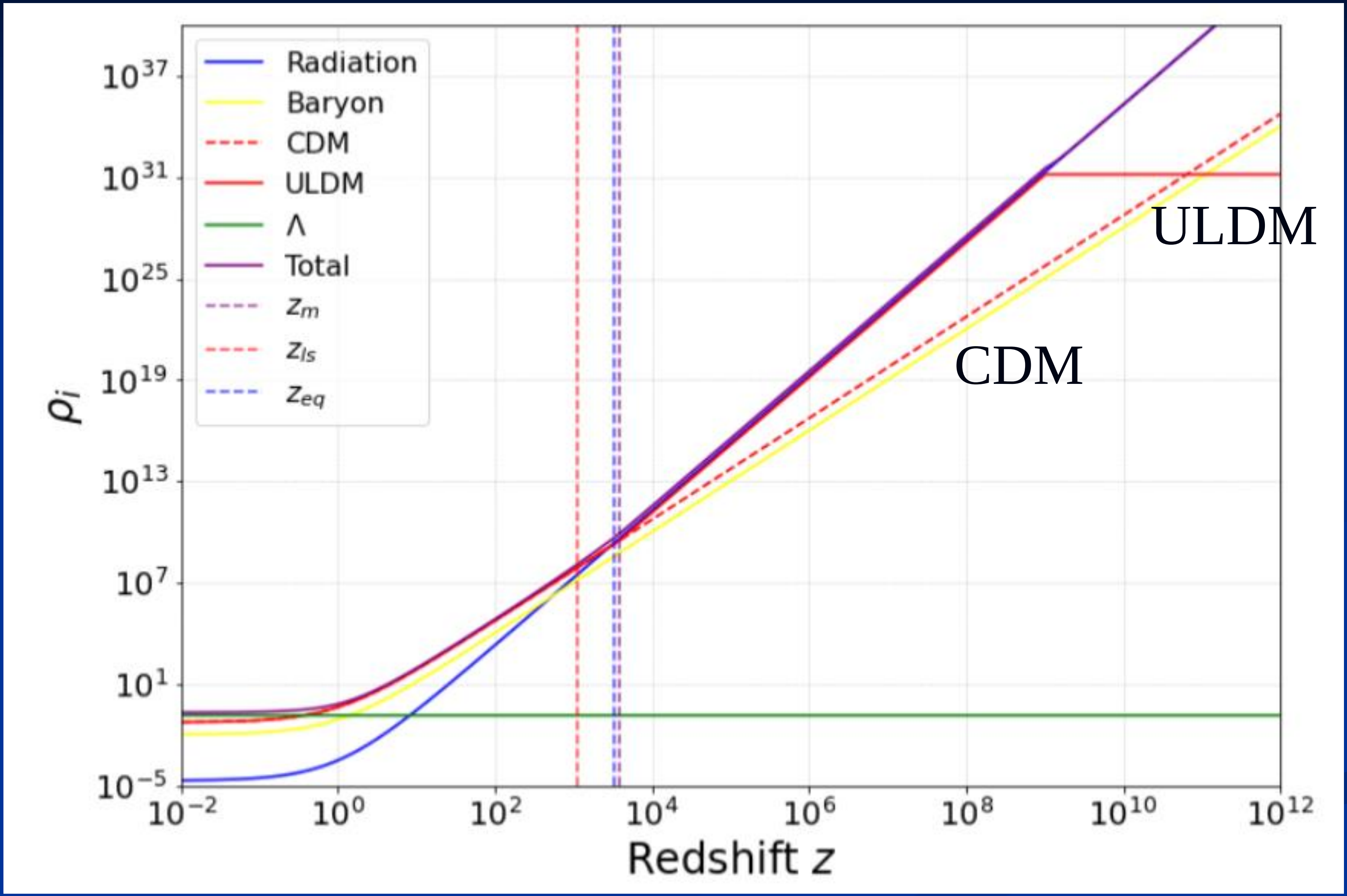
Can ULDM solve both H_0 and S_8 ?



$V(\phi) \propto \phi^n$ decays with an equation of state $w = \frac{(n-2)}{(n+2)}$

$$\begin{cases} CDM\ (\phi^4 < \phi^2)\quad or\ z < z_m\ (Phase\ III) \\ rad\text{-}like\ (\phi^4 > \phi^2)\quad or\ \ z_m < z < z_{osc}\ (Phase\ II) \\ DE - like\ (slow - roll)\ for\ \ z_{osc} < z\ (Phase\ I) \end{cases}$$

Jlee 2502.11568


Radiation
Baryon
CDM
ULDM
Λ
Total
z_m
z_{ls}
z_{eq}
ρ_i
Redshift z
ULDM
CDM

$$\theta_{\rm ls} = \frac{r_{\rm ls}}{D(z_{\rm ls})} = \frac{\int_{z_{\rm ls}}^{\infty} \frac{c_s(z)}{H(z)} dz}{\int_{0}^{z_{\rm ls}} \frac{c}{H(z)} dz}$$

$$r_s = \int_{z_{\rm ls}}^{\infty} \frac{c_s(z)}{H(z)} dz = \frac{c}{\sqrt{3} H_{\rm ls}} \int_{z_{\rm ls}}^{\infty} \frac{dz}{[\rho(z)/\rho(z_{\rm ls})]^{1/2} (1 + R(z))^{1/2}},$$

$$H_{ls} = 100 \text{ km s}^{-1} \text{Mpc}^{-1} \omega_r^{1/2} (1 + z_{ls})^2 \sqrt{1 + \frac{\omega_m}{(1 + z_{ls}) \omega_r}}$$

$$H_0 = \sqrt{3} H_{1\ s} \theta_s \frac{\int_0^{z_{\rm ls}} \left(\frac{\rho(z)}{\rho_0} \right)^{-1/2} dz}{\int_{z_{\rm ls}}^{\infty} \left(\frac{\rho(z)}{\rho(z_{\rm ls})} \right)^{-1/2} \frac{dz}{\sqrt{1 + R(z)}}}$$

$$\frac{H_0(ULDM)}{H_0(CDM)} \simeq \frac{\int_{z_{ls}}^{\infty} \rho(CDM)(z)^{-1/2} \frac{dz}{\sqrt{1+R(z)}}}{\int_{z_{ls}}^{\infty} \rho(ULDM)(z)^{-1/2} \frac{dz}{\sqrt{1+R(z)}}}$$

$$\rho(ULDM)(z) \propto \begin{cases} \sqrt{\omega_r(1+z)^4 + \omega_m(1+z)^3 + \omega_\Lambda} & for\, z < z_m \text{ (Phase III)} \\ \sqrt{\omega_r(1+z)^4 + \omega_\phi \frac{(1+z)^4}{1+z_m} + \omega_b(1+z)^3 + \omega_\Lambda} & for\, z_m < z < z_{osc} \text{ (Phase II)} \\ \sqrt{\omega_r(1+z)^4 + \omega_b(1+z)^3 + \omega_\phi \frac{(1+z_{osc})^4}{1+z_m} + \omega_\Lambda} & for\, z_{osc} < z \text{ (Phase I)} \end{cases}$$

where $\omega_b = 0.022, \omega_\phi = 0.119, \omega_r = 4.2 \times 10^{-5}, \omega_\Lambda = 0.314$ from Planck's Λ CDM values

Using the densities, we numerically obtain $\frac{H_0(\mathrm{ULDM})}{H_0(\mathrm{CDM})} \simeq 1.049$ for $\tilde{m} = 0.9$ and 1.00084 for $\tilde{m} = 5$

$$r_s = \int_{z_{1\ s}}^{\infty} \frac{c_s(z)}{H(z)} dz = \frac{c}{\sqrt{3} H_{1\ s}} \int_{z_{1\ s}}^{\infty} \frac{dz}{[\rho(z)/\rho(z_{\mathrm{ls}})]^{1/2}(1 + R(z))^{\quad 1/2}},$$

$$H_{ls} = 100 \text{ km s}^{-1}\text{Mpc}^{-1} \omega_r^{1/2} (1 + z_{ls})^2 \sqrt{1 + \frac{\omega_m}{(1 + z_{ls})\omega_r}}$$

$$H_0 = \sqrt{3} H_{1\ s} \theta_s \frac{\int_0^{z_{\mathrm{ls}}} \left(\frac{\rho(z)}{\rho_0}\right)^{-1/2} dz}{\int_{z_{\mathrm{ls}}}^{\infty} \left(\frac{\rho(z)}{\rho(z_{\mathrm{ls}})}\right)^{-1/2} \frac{dz}{\sqrt{1 + R(z)}}}$$

$$\rho_{eq}^{\phi} = \rho_{\mathrm{rad}}(T_{eq}) = \frac{\pi^2}{30} g_* T_{eq}^4,$$

$$\rho_{eq}^{\phi} = \rho_{osc}^{\phi} \left(\frac{T_m}{T_{osc}}\right)^4 \left(\frac{T_{eq}}{T_m}\right)^3 = \frac{\rho_{osc}^{\phi} T_m T_{eq}^3}{T_{osc}^4} = \frac{\phi_{osc}^2 T_m T_{eq}^3}{m_P^2}$$

$$\rightarrow \ \phi_{osc} = \sqrt{\frac{\rho_{eq}^{\phi} m_P^2}{T_m T_{eq}^3}}$$

$$\rho_m^{\phi} \simeq m^2 \, \phi_m^2 / 2 = \frac{m^4}{\lambda} = \widetilde{m}^4 = \rho_{eq}^{\phi} \left(\frac{T_m}{T_{eq}}\right)^3$$

$$\rightarrow \ T_m = \left(\frac{\widetilde{m}^4}{\rho_{eq}^{\phi}}\right)^{1/3} T_{eq}$$

Hierarchy of SIULDM

JLee 2410.02842

GUT	$T_c \simeq m/\sqrt{\lambda} = \tilde{m}^2/m \sim 10^{15}$ GeV
neutrino	$\tilde{m} \equiv m/\lambda^{1/4} \sim 10$ eV
	reverting Type I seesaw ⇩
EW	$T_{EW} \sim (T_c \tilde{m})^{1/2} \sim 10^3$ GeV
ULDM	m $\sim 10^{-22}$ eV, $\lambda \sim 10^{-92}$
	galaxy observations

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Neutrino mass

$$\mathcal{L}_{Yukawa} = -y\phi\bar{\nu^c}\nu$$ Majonara ν

real Majoron

$$m_\nu = 0.1\text{eV}\left(\frac{y}{10^{-25}}\right)\left(\frac{v}{10^{15}\text{GeV}}\right)$$

$$y = \frac{m_\nu\sqrt{\lambda}}{m_\phi} = 10^{-25}\left(\frac{m_\nu}{0.1\text{eV}}\right)\left(\frac{m_\phi}{10^{-22}\text{eV}}\right)^{-1}\left(\frac{\lambda}{10^{-92}}\right)^{1/2}$$

One-loop quantum correction from the Yukawa is $O(y^4) \simeq 10^{-8}\lambda \ll \lambda$

0.06 eV< Σm_ν <0.071 eV (DESI)

Conclusion

- *FDM with* $m \sim 10^{-22}$ *eV or*
  Self-interacting ULDM with $\frac{m}{\lambda^{1/4}} \sim 10eV$ *is consistent with many cosmological observations*

- *Tc is about GUT scale and SSB of ULDM can give neutrino masses and EW scale*

→Oscillation of ULDM can be detected by neutrino osc. and other experiments (GW, atomic clock)

ULDM possibly explain small scale issue, final pc, Hubble tension, S8, and many other mysteries