Electron Structure & Mechanical Properties in Light-Front QED Model

Narinder Kumar
Doaba College, Jalandhar, India

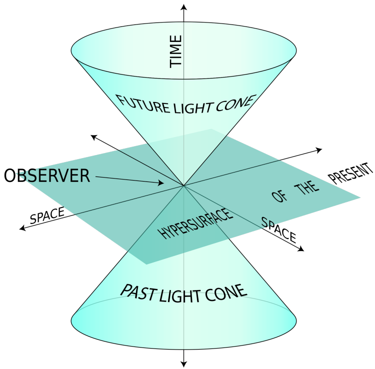
Based on N. Kumar & C. Mondal Nucl. Phys. B 934 226 (2018)
Jai More & N. Kumar (under preperation)

20 June 2025

अनुसंधान नेशनल रिसर्च फाउंडेशन
Anusandhan National Research Foundation

Table of contents

1. Light-Front Dynamics
2. Light-Front in QCD
3. Generalized Parton Distributions (GPDs)
4. Transverse Momentum Dependent Parton Distributions (TMDs)
5. Wigner Distributions
6. Light-Front QED Model
7. Wigner Distributions & GTMDs: Model Calculations
8. Gravitational Form Factors:Electron



-Wikipedia

- A light-cone is the path that a flash of light, emanating from a single event (localized to a single point in space and a single moment in time) and travelling in all directions, would take through space-time.

- Acc. to P.A.M. Dirac three forms of dynamics are possible.
- Instant Form ($x^0 = 0$)
- Point Form ($x_\mu x^\mu = a^2 > 0$)
- Front Form ($x^+ = x^0 + x^3 = 0$)
- One may set up a dynamical theory in which the dynamical variables refer to physical conditions on a front $x^+ = 0$. The resulting dynamics is called light-front dynamics, which Dirac called front-form.
- According to Dirac (1949) ".... the three-dimensional surface in space- time formed by a plane wave front advancing with the velocity of light. Such a surface will be called front for brevity ". An example of a light- front is given by the equation $x^+ = x^0 + x^3 = 0$.

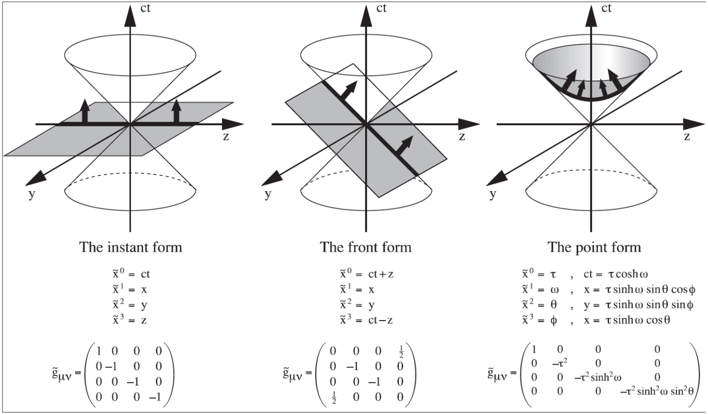


Figure: Dirac's Three Forms of Dynamics

-Phys.Rept. 301, 299(1998)

Why Light-Front Dynamics?

- A relativistic system is required to satisfy two principles: It is invariant under infinitesimal inhomogeneous Lorentz transformations (relativistic principle) and it is a Hamiltonian system (quantum mechanics).
- A dynamical system is characterized by ten generators of the Poincare group, which are determined by the Hamilton equations.
- In instant form, dynamical variables refer to some physical conditions at some instant of time. Invariant quantities are translations and rotations.
- In Point form, inhomogeneous Lorentz transformation is simple but all the four-vector momenta P^{μ} are dynamically dependent.
- In front form, out of the ten generators of Lorentz group only three are dynamically dependent (instead of four in other forms).
- Furthermore, there is no square root for the Hamiltonian which might simplify the dynamical structure since a square root relation between the energy and momentum cannot provide a simple picture of the bound state Schrödinger equation.

- Quantum Chromodynamics provides a fundamental description of hadronic and nuclear structure and dynamics in terms of their elementary quark and gluon degrees of freedom.
- One of the most outstanding problem of particle physics is to unravel the internal structure of hadrons such as proton and neutron in terms of their fundamental quark and gluon degrees of freedom.
- Light Front QCD (LFQCD) is an ab initio approach to strongly interacting system. It is like perturbative and lattice QCD directly connected to the QCD Lagrangian, but it is a Hamiltonian method, formulated in Minkowinski space rather than Euclidean space. The essential ingredient is Dirac's front form of Hamiltonian dynamics, where one quantize the theory at fixed light-cone time $\tau = t + z/c$ rather than ordinary time t.

Probing Hadron Structure

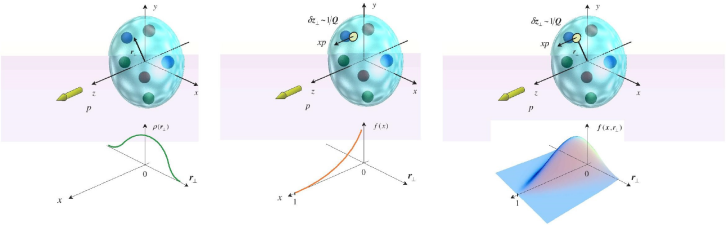


Figure: The pictorial comparison of form factors, parton distributions and zero skewness GPDs.

-1212.1701

- Form factors describe the transverse localization of partons in a fast moving nucleon, irrespective of their longitudinal momenta.
- Parton densities provides the probability to find partons of a given longitudinal momentum fraction x of the parent nucleon with transverse resolution 1/Q , no information on the transverse position of partons is accessible.

Generalized Parton Distributions (GPDs) [See M.Diehl, Phys. Rep. 388, 41 (2003)]

- In recent years it has become clear that appropriate exclusive scattering processes $\gamma^* p \to \gamma p$(DVCS) can provide missing information in PDFs encoded in GPDs.

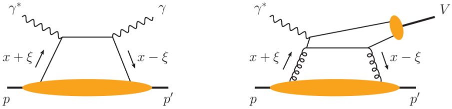


Image courtesy arXiv:1212.1701

- GPDs are much richer in content about the hadron structure than ordinary parton distributions.
- GPDs allows us to access partonic configurations with a given longitudinal momentum fraction, but also at specific location (transverse) inside the hadron.
- GPDs provide a 3-D picture of the partonic nucleon structure. From 3-D we meant that GPDs encode information on the distribution of partons both in the transverse plane and longitudinal direction.

GPDs continue...

- Unlike PDFs, GPDs have dependence on $x, \zeta, t = -\vec{\Delta}_\perp^2$.
- The Fourier transform of GPDs w.r.t impact parameter gives the impact parameter dependent parton distribution function (ipdpf).

$$\mathcal{H}, \mathcal{E}(x, \vec{b}_\perp) = \int \frac{d^2\vec{\Delta}_\perp}{(2\pi)^2} e^{-\iota \vec{\Delta}_\perp \cdot \vec{b}_\perp} H, E(x, 0, t).$$

-M. Burkardt Phys. Rev. D 62 071503 (2000)

-Int. Jou. Mod. Phys. A 18 173 (2002)

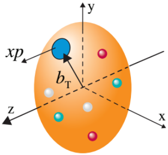


Image courtsey 1212.1701

- Thus, distribution of parton is obtained in position space i.e.. x and b. GPDs in impact parameter space as probability densities in 2 transverse coordinates and 1 longitudinal momentum.

Properties of GPDs

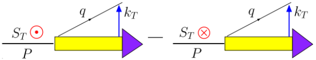
- Forward limit: ordinary parton distriutions
- $H(x, \xi = 0, t = 0) = f(x)$ -unpolarized quark distributions.
- $\tilde{H}(x, \xi = 0, t = 0) = \Delta f(x)$ -long. polarized quark distributions.
- $H_T(x, \xi = 0, t = 0) = h_1(x)$ trans. polarized quark distributions.
- $\int dx\ H(x, 0, t) = F_1(t)$ -Dirac Form Factor.
- $\int dx\ E(x, 0, t) = F_2(t)$ -Pauli Form Factor.
- $\int dx\ \tilde{H}(x, 0, t) = G_A(t)$ -Axial Form Factor.
- $\int dx\ \tilde{E}(x, 0, t) = G_P(t)$ -Pseudoscalar Form Factor.

Transverse Momentum Dependent Parton Distributions (TMDs)[See TMD Handbook arXiv:2304.03302]

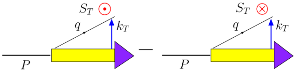
- But still we need information on the distribution of partons in momentum plane.
- Much more comprehensive picture of the nucleon structure can be obtained by considering GPDs as discussed above and by TMDs entering the description of various hard semi-inclusive reactions.
- TMDs can be accessed through semi-inclusive deep inelastic scattering (SIDIS) $l + N \to l + h + X$ or Drell-Yan process $p + N \to l^+ + l^- + X$.
- TMDs are of particular importance because they give rise to single spin asymmetries (SSAs).
- Single spin phenomena were measured by various experiments at Fermilab, RHIC as well as at COMPASS and HERMES collaboration. An important object in this context is the T-odd Sivers function, denoted by $f_{1T}^{\perp}$.

TMDs [See D. W. Sivers, Phys. Rev. D, 41, 83, 1990 & D. Boer and P. J. Mulders, Phys. Rev. D 57, 5780 ,1998]

- The Sivers distribution function $f_{1T}^{\perp}$ describes the difference between the momentum distributions of unpolarized quark inside the nucleons transversely polarized in opposite directions.



- There is another distribution function called as Boer-Mulders distribution function $h_1^{\perp}$. It describe the difference between the momentum distributions of the quarks transversly polarized in opposite directions inside the unpolarized nucleon.



Leading Twist TMDs (08 Independent TMDs)

- $f_1(x, \vec{p}_\perp)$ -Unpolarised quarks in an unpolarised nucleon (Unintegrated unpolarised distribution).
- $g_{1L}(x, \vec{p}_\perp)$ -correlate longitudinal spin of quark with longitudinal spin of nucleon (Unintegrated helicity distribution).
- $h_{1T}(x, \vec{p}_\perp)$ -correlate transverse spin of quark with transverse spin of nucleon (Unintegrated transverse distribution).
- $f_{1T}^{\perp}(x, \vec{p}_\perp)$ -Sivers function-correlate unpolarised quark with transversely polarised nucleon.
- $h_1^{\perp}(x, \vec{p}_\perp)$ -Boer-Mulders function- correlate transversely polarised quark with unpolarised nucleon.
- $g_{1T}^{\perp}(x, \vec{p}_\perp), h_{1L}^{\perp}(x, \vec{p}_\perp), h_{1T}^{\perp}(x, \vec{p}_\perp)$ - different double spin correlations

Leading Twist TMDs

Nucleon Spin   Quark Spin

Nucleon Polarization	Quark Polarization: Un-Polarized (U)	Quark Polarization: Longitudinally Polarized (L)	Quark Polarization: Transversely Polarized (T)
U	$f_1 =$		$h_1^{\perp} =$ — Boer-Mulders
L		$g_{1L} =$ — Helicity	$h_{1L}^{\perp} =$ —
T	$f_{1T}^{\perp} =$ — Sivers	$g_{1T}^{\perp} =$ —	$h_1 =$ — Transversity $h_{1T}^{\perp} =$ —

- Wigner distributions were first introduced by E. Wigner to study quantum corrections to classical statistical mechanics.

  E. Wigner Phys. Rev, 70 749 (1932)

- Strict interpretation of definite position and momentum fails for quantum particle due to uncertainty principle.
- Wigner distribution can normally have negative values: smoothing the Wigner distribution (Huismi distribution) results in a positive semi-definite function.
- correlations of quark momentum and position in the transverse plane as function of quark and nucleon polarizations.
- no known experiments can directly access them $\rightarrow$ requires phenomenological models.

- In QCD, Wigner distributions were first introduced by Xiangdong Ji

  -Phys. Rev. Lett. 91 062001 (2003)
- 5-D Wigner distributions can be studied in infinite-momentum frame or light-cone formalism.

  Lorce & Pasquini 2011
- Advantage: provides boost-invariant definition of Wigner Distributions.
- Integrating over transverse momentum, Wigner distributions reduce to the Fourier transform of GPDs.
- Integrating over transverse position $\vec{b}_\perp$, they reduces to TMDs.
- they are also related to orbital angular momentum carried by the quarks in the nucleon.

- Wigner distributions can be obtained from generalized transverse momentum dependent parton distributions (GTMDs) which are also called as mother distributions.
- GTMDs parameterize the off-forward transverse momentum dependent quark-quark correlator.

$$
\begin{aligned}
W^{\Gamma}(\vec{\Delta}_\perp, \vec{p}_\perp, x) &= \int \frac{dz^- d^2 z_\perp}{(2\pi)^2} e^{ip\cdot z} \langle P' | \bar{\psi}(-z/2) \Gamma \mathcal{W}_{[-\frac{z}{2},\frac{z}{2}]} \psi(z/2) | P \rangle \\
\rho^{\Gamma}(\vec{b}_\perp, \vec{p}_\perp, x) &= \int \frac{d^2 \vec{\Delta}_\perp}{(2\pi)^2} e^{-\iota \vec{\Delta}_\perp \cdot \vec{b}_\perp} W^{\Gamma}(\vec{\Delta}_\perp, \vec{p}_\perp, x) \\
\Gamma &= \gamma^+, \gamma^+\gamma_5, \iota\sigma^{+j}\gamma_5 (\text{leading-twist operators})
\end{aligned}
$$

-Stephan Meißner *et al* JHEP08(2009) 056

- From many years, it was not possible to access the GTMDs from experimental point of view.
- However, recently processes like exclusive double Drell-Yan process make it possible.

S. Bhattacharya *et. al.* Phys. Lett. B 771 396 (2017).

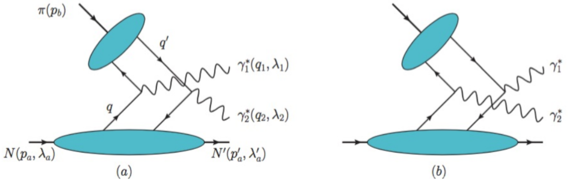


Figure: Leading order diagrams for DDY processes $\pi N \to \gamma_1^* \gamma_2^* N'$

- this process in sensitive to GTMDs in the ERBL (Efermov-Radyushkin-Brodsky-Lepage) region.

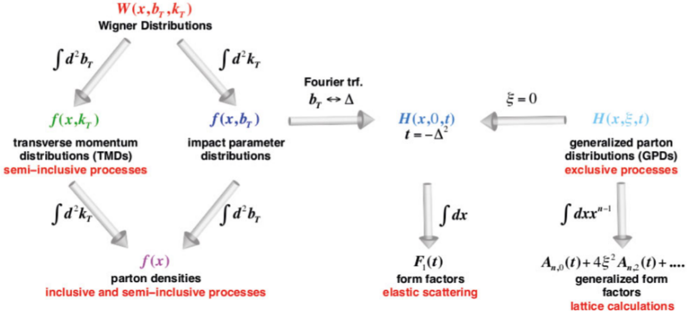


Figure: Connections between different quantities describing the distributions of quarks inside the nucleon.

-Image courtsey:1212.1701

Light-Front QED Model

- What is the shape of the electron?
- Most probably our answer is spherical but electron is a point particle having no underlying substructure.
- In quantum electrodynamics (QED) electron is considered as an elementary field.
- When we talk about the electron "structure", what we are actually doing is probing the fluctuations in the quantum theory.
- According to quantum theory, one can consider the fluctuations of electron into electron-photon pair i.e. $e \to e\gamma \to e$ with same quantum number.
- The virtual photon further broke up into virtual electrons and positrons with all possible combinations.

- Thus, as a result bare electron is no more an isolated particle but it is surrounded by virtual cloud of electrons, positrons and photons.
- Therefore, bare electron becomes a dressed electron and one can consider electrons, positrons and photons as composite particles in the original electron.
- The structure of the electron can be revealed when virtual cloud interacts with a probe, the parton content of the electron is resolved.
- The Schwinger one-loop radiative correction to the electron current in QED has played a historic role in the development of QFT.
- In the language of light-cone quantization, the electron anomalous magnetic moment $a_e = \frac{\alpha}{2\pi}$ is due to the one-fermion one-gauge boson Fock state component of the physical electron.
- An explicit calculation of the anomalous moment in this framework have been already done by Drell and Brodsky (Phys. Rev. D 22 (1980) 2236).

- We consider physical electron as a composite system comprising of a bare electron accompanied by its quantum fluctuations such as virtual photons, electron- positron pairs.

$$|e^-\rangle_{physical} = |e^-\rangle + |e^-\gamma\rangle + |e^-e^+e^-\rangle + ...$$

- We evaluate the results for the Wigner distribution of the physical electron by considering it as a two particle state (electron and photon). The two particle Fock state for an electron with $J_z = \pm\frac{1}{2}$ has four possible combinations.

$$\begin{aligned}
|e^-(P^+, \mathbf{P}_\perp = 0)\rangle_{2p}^{\uparrow,\downarrow} &= \int \frac{dx\ d^2\vec{p}_\perp}{16\pi^3\sqrt{x(1-x)}} \Big\{ \psi^{\uparrow,\downarrow}_{\frac{1}{2},+1} |\frac{1}{2}, +1; xP^+, \vec{p}_\perp\rangle + \psi^{\uparrow,\downarrow}_{\frac{1}{2},-1} |\frac{1}{2}, -1; xP^+, \vec{p}_\perp\rangle \\
&+ \psi^{\uparrow,\downarrow}_{-\frac{1}{2},+1} |-\frac{1}{2}, +1; xP^+, \vec{p}_\perp\rangle + \psi^{\uparrow,\downarrow}_{\frac{1}{2},-1} |\frac{1}{2}, -1; xP^+, \vec{p}_\perp\rangle \\
&+ \psi^{\uparrow,\downarrow}_{-\frac{1}{2},-1} |-\frac{1}{2}, -1; xP^+, \vec{p}_\perp\rangle \Big\}
\end{aligned}$$

- The above wave functions can be evaluated in the light-front QED perturbation theory

$$\begin{aligned}
\psi^{\uparrow}_{\frac{1}{2},+1}(x,\vec{p}_\perp) &= -\sqrt{2}\frac{-p^1+ip^2}{x(1-x)}\varphi, \\
\psi^{\uparrow}_{\frac{1}{2},-1}(x,\vec{p}_\perp) &= -\sqrt{2}\frac{p^1+ip^2}{1-x}\varphi, \\
\psi^{\uparrow}_{-\frac{1}{2},+1}(x,\vec{p}_\perp) &= -\sqrt{2}\left(M-\frac{m}{x}\right)\varphi, \\
\psi^{\uparrow}_{-\frac{1}{2},-1}(x,\vec{p}_\perp) &= 0.
\end{aligned}$$

$$\begin{aligned}
\psi^{\downarrow}_{\frac{1}{2},+1}(x,\vec{p}_\perp) &= 0, \\
\psi^{\downarrow}_{\frac{1}{2},-1}(x,\vec{p}_\perp) &= -\sqrt{2}\left(M-\frac{m}{x}\right)\varphi, \\
\psi^{\downarrow}_{-\frac{1}{2},+1}(x,\vec{p}_\perp) &= -\sqrt{2}\frac{-p^1+ip^2}{1-x}\varphi, \\
\psi^{\downarrow}_{-\frac{1}{2},-1}(x,\vec{p}_\perp) &= -\sqrt{2}\frac{p^1+ip^2}{x(1-x)}\varphi.
\end{aligned}$$

where

$$\varphi(x,\vec{p}_\perp) = \frac{e}{\sqrt{1-x}}\frac{1}{M^2-\frac{\vec{p}_\perp^2+m^2}{x}+\frac{\vec{p}_\perp^2+\mu^2}{1-x}}$$

-Brodsky *et. al.* NPB 593, 311 (2001).

- The co-efficients of φ are the matrix elements of $\frac{\bar{u}(p^+,p^-,\vec{p}_\perp)}{\sqrt{k^+}}\gamma.\epsilon^*\frac{u(P^+,P^-,\vec{P}_\perp)}{\sqrt{P^+}}$ which are the numerators of the wavefunctions corresponding to each constituent spin s^z configuration.

- Correlator function $W(\vec{\Delta}_\perp, \vec{p}_\perp, x)$ can be expressed in terms of LFWFs as

$$
\begin{aligned}
W^{[\gamma^+]}_{ss'}(\vec{\Delta}_\perp, \vec{p}_\perp, x) &= \frac{1}{16\pi^3} \sum_{\lambda'_1,\lambda_1,\lambda_2} \psi^{*s'}_{\lambda'_1\lambda_2} \; \chi^\dagger_{\lambda'_1} \; \chi_{\lambda_1} \; \psi^{s}_{\lambda_1\lambda_2}, \\
W^{[\gamma^+\gamma_5]}_{ss'}(\vec{\Delta}_\perp, \vec{p}_\perp, x) &= \frac{1}{16\pi^3} \sum_{\lambda'_1,\lambda_1,\lambda_2} \psi^{*s'}_{\lambda'_1\lambda_2} \; \chi^\dagger_{\lambda'_1} \sigma_3 \chi_{\lambda_1} \; \psi^{s}_{\lambda_1\lambda_2}, \\
W^{[\iota\sigma^{+j}\gamma_5]}_{ss'}(\vec{\Delta}_\perp, \vec{p}_\perp, x) &= \frac{1}{16\pi^3} \sum_{\lambda'_1,\lambda_1,\lambda_2} \psi^{*s'}_{\lambda'_1\lambda_2} \; \chi^\dagger_{\lambda'_1} \sigma_j \chi_{\lambda_1} \; \psi^{s}_{\lambda_1\lambda_2}
\end{aligned}
$$

Wigner Distributions: Definitions

- Wigner distributions of unpolarized internal electron in unpolarized physical electron is given by

$$\rho_{UU}(\vec{b}_\perp, \vec{p}_\perp, x) = \frac{1}{2}\left[\rho^{\gamma^+}(\vec{b}_\perp, \vec{p}_\perp, x; +\hat{e}_z) + \rho^{\gamma^+}(\vec{b}_\perp, \vec{p}_\perp, x; -\hat{e}_z)\right]$$

- Distortion due to longitudinal polarization of the physical electron state:

$$\rho_{LU}(\vec{b}_\perp, \vec{p}_\perp, x) = \frac{1}{2}\left[\rho^{[\gamma^+]}(\vec{b}_\perp, \vec{p}_\perp, x; +\hat{e}_z) - \rho^{[\gamma^+]}(\vec{b}_\perp, \vec{p}_\perp, x; -\hat{e}_z)\right]$$

- Distortion due to the longitudinal polarization of internal electron:

$$\rho_{UL}(\vec{b}_\perp, \vec{p}_\perp, x) = \frac{1}{2}\left[\rho^{[\gamma^+\gamma_5]}(\vec{b}_\perp, \vec{p}_\perp, x; +\hat{e}_z) + \rho^{[\gamma^+\gamma_5]}(\vec{b}_\perp, \vec{p}_\perp, x; -\hat{e}_z)\right]$$

- Distortion due to the correlation between the longitudinal polarized physical electron state and internal electron :

$$\rho_{LL}(\vec{b}_\perp, \vec{p}_\perp, x) = \frac{1}{2}\left[\rho^{[\gamma^+\gamma_5]}(\vec{b}_\perp, \vec{p}_\perp, x; +\hat{e}_z) - \rho^{[\gamma^+\gamma_5]}(\vec{b}_\perp, \vec{p}_\perp, x; -\hat{e}_z)\right]$$

Connection with GTMDs

- All the Wigner distributions are related to Fourier transforms of GTMDs (Def. in later slides)
- ρ_{UU} is related to F_{11}, ρUL is related to G_{11}, ρ_{LU} to F_{14} and ρ_{LL} to G_{14}
- ρ_{UU} can be considered as the mother distributions for the unpolarized GPD H and TMD f_1.
- ρ_{LL} is the mother distribution for the GPD $\tilde{H}$ and TMD g_{1L}.
- Connection with orbital angular momentum

$$L_z = \frac{1}{2}\int dx\left[x\{H(x,0,0)+E(x,0,0)-\tilde{H}(x,0,0)\}\right]$$

X.Ji PRL (1997)

$$H(x,0,t) = \int d^2k_\perp F_{11}$$

$$\tilde{H}(x,0,t) = \int d^2k_\perp G_{14}$$

- In our model $\rho_{LU} = \rho_{UL}$

$$\rho_{UU}(\vec{b}_\perp, \vec{p}_\perp) = \frac{4e^2}{16\pi^3}\int d\Delta_x d\Delta_y \int dx \cos(\Delta_x b_x + \Delta_y b_y) \left[\frac{1+x^2}{x^2(1-x)^2}(\vec{p}_\perp^2 - \frac{(1-x)^2\vec{\Delta}_\perp^2}{4}) + (M - \frac{m}{x})^2\right]\varphi^\dagger(\vec{p''}_\perp)\varphi(\vec{p'})$$

$$\rho_{LU}(\vec{b}_\perp, \vec{p}_\perp) = \frac{4e^2}{16\pi^3}\int d\Delta_x d\Delta_y \int dx \sin(\Delta_x b_x + \Delta_y b_y)\frac{\Delta_x p_y - \Delta_y p_x}{x^2(1-x)}(x^2-1) \varphi^\dagger(\vec{p''}_\perp)\varphi(\vec{p'})$$

$$\rho_{LL}(\vec{b}_\perp, \vec{p}_\perp) = \frac{4e^2}{16\pi^3}\int d\Delta_x d\Delta_y \int dx \cos(\Delta_x b_x + \Delta_y b_y) \left[\frac{1+x^2}{x^2(1-x)^2}(\vec{p}_\perp^2 - \frac{(1-x)^2\vec{\Delta}_\perp^2}{4}) - (M - \frac{m}{x})^2\right]\varphi^\dagger(\vec{p''}_\perp)\varphi(\vec{p'})$$

where

$$\varphi(\vec{p'}_\perp) = \frac{1}{\sqrt{1-x}}\frac{x(1-x)}{\vec{p'}^2_\perp - M^2x(1-x) + m^2(1-x) + \mu^2 x}$$

$$\varphi(\vec{p''}_\perp) = \frac{1}{\sqrt{1-x}}\frac{x(1-x)}{\vec{p''}^2_\perp - M^2x(1-x) + m^2(1-x) + \mu^2 x}$$

$$\vec{p'}_\perp = \vec{p}_\perp - (1-x)\frac{\vec{\Delta}_\perp}{2}$$

$$\vec{p''}_\perp = \vec{p}_\perp - (1-x)\frac{\vec{\Delta}_\perp}{2}$$

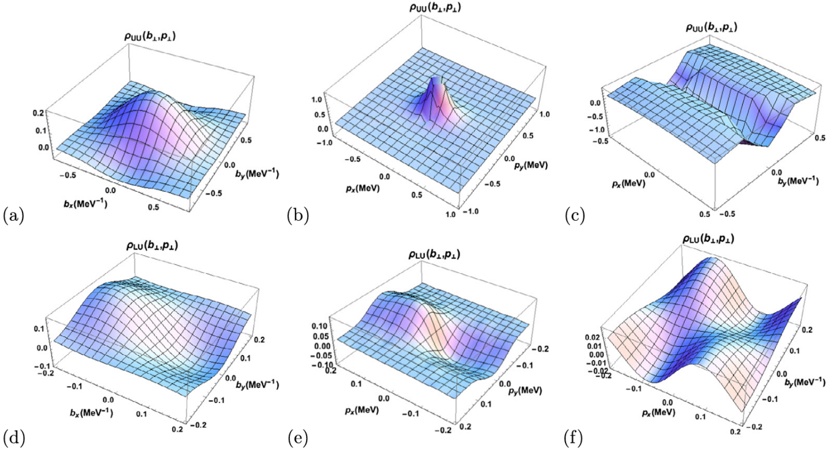


Fig. 3. (Color online.) Plots of Wigner distribution $\rho_{UU}(\mathbf{b}_\perp, \mathbf{p}_\perp)$ and $\rho_{LU}(\mathbf{b}_\perp, \mathbf{p}_\perp)$ for physical electron in impact-parameter plane with fixed transverse momentum $\mathbf{p}_\perp = 0.8$ MeV $\hat{e}_x$ (left panel), in momentum plane with fixed impact-parameter $\mathbf{b}_\perp = 0.8$ MeV^{-1} $\hat{e}_x$ (middle panel) and in mixed plane (right panel). The upper panel represents ρ_{UU} and the lower panel is for ρ_{LU}.

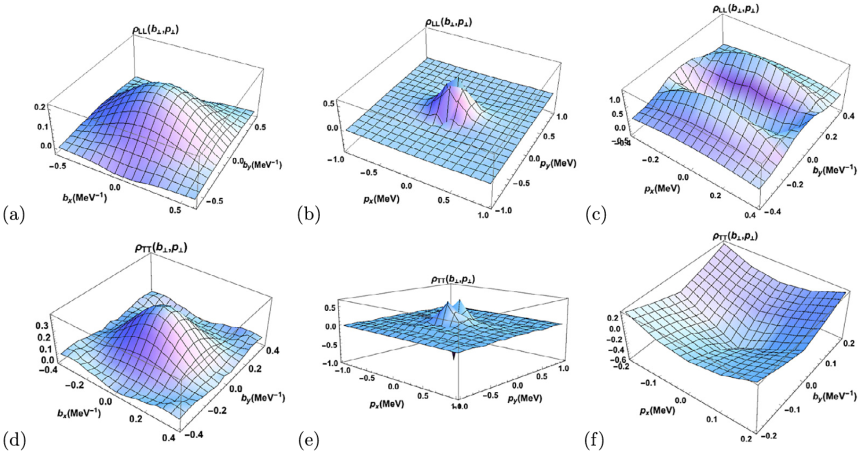


Fig. 5. (Color online.) Plots of Wigner distribution $\rho_{LL}(\mathbf{b}_\perp, \mathbf{p}_\perp)$ and $\rho_{TT}(\mathbf{b}_\perp, \mathbf{p}_\perp)$ for physical electron in impact-parameter plane with fixed transverse momentum $\mathbf{p}_\perp = 0.8$ MeV $\hat{e}_x$ (left panel), in momentum plane with fixed impact-parameter $\mathbf{b}_\perp = 0.8$ MeV^{-1} $\hat{e}_x$ (middle panel) and in mixed plane (right panel). The upper and panel represent ρ_{LL} and ρ_{TT}, respectively. For ρ_{TT}, the transverse polarizations of both the bare and the physical electron are along y-direction.

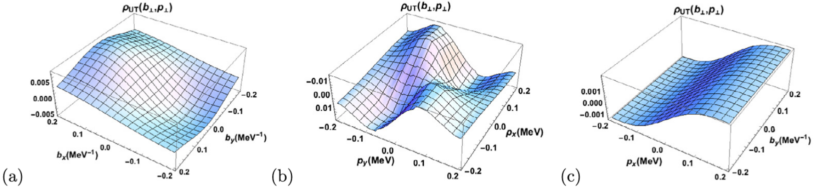


Fig. 4. (Color online.) Plots of Wigner distribution $\rho_{UT}(\mathbf{b}_\perp, \mathbf{p}_\perp)$ for physical electron in impact-parameter plane with fixed transverse momentum $\mathbf{p}_\perp = 0.8$ MeV $\hat{e}_x$ (left panel), in momentum plane with fixed impact-parameter $\mathbf{b}_\perp = 0.8$ MeV^{-1} $\hat{e}_x$ (middle panel) and in mixed plane (right panel). The transverse polarization of the bare electron or the physical electron is taken along x-direction.

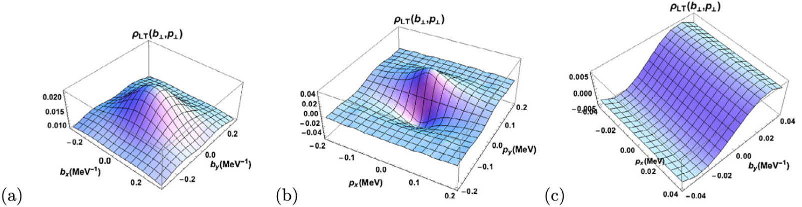


Fig. 6. (Color online.) Plots of Wigner distribution $\rho_{LT}(\mathbf{b}_\perp, \mathbf{p}_\perp)$ for physical electron in impact-parameter space with fixed transverse momentum $\mathbf{p}_\perp = 0.8$ MeV $\hat{e}_x$ and $\mathbf{b}_\perp = 0.8$ MeV^{-1} for ρ_{LT}. The transverse polarization of the bare electron or the physical electron is taken along x-direction.

GTMDs for Electron: Mother Distributions

- Generalized transverse momentum distributions which are known as the mother distributions of GPDs and TMDs can be extracted from different Wigner distributions.
- For the leading twist, the Wigner correlator, can be parametrized in terms of GTMDs as

$$W^{[\gamma^+]}_{\lambda\lambda'} = \frac{1}{2M}\,\bar{u}(p',\lambda')\Big[F_{1,1} + \frac{i\sigma^{i+}k^i_\perp}{P^+}\,F_{1,2} + \frac{i\sigma^{i+}\Delta^i_\perp}{P^+}F_{1,3} + \frac{i\sigma^{ij}k^i_\perp\Delta^j_\perp}{M^2}\,F_{1,4}\Big]u(p,\lambda)\,,$$

$$W^{[\gamma^+\gamma_5]}_{\lambda\lambda'} = \frac{1}{2M}\,\bar{u}(p',\lambda')\Big[-\frac{i\varepsilon^{ij}_\perp k^i_\perp\Delta^j_\perp}{M^2}\,G_{1,1} + \frac{i\sigma^{i+}\gamma_5 k^i_\perp}{P^+}G_{1,2} + \frac{i\sigma^{i+}\gamma_5\Delta^i_\perp}{P^+}\,G_{1,3} + i\sigma^{+-}\gamma_5\,G_{1,4}\Big]u(p,\lambda)\,,$$

$$W^{[i\sigma^{j+}\gamma_5]}_{\lambda\lambda'} = \frac{1}{2M}\,\bar{u}(p',\lambda')\Big[-\frac{i\varepsilon^{ij}_\perp k^i_\perp}{M}\,H_{1,1} - \frac{i\varepsilon^{ij}_\perp\Delta^i_\perp}{M}\,H_{1,2} + \frac{M\,i\sigma^{j+}\gamma_5}{P^+}\,H_{1,3} + \frac{k^j_\perp\,i\sigma^{k+}\gamma_5 k^k_\perp}{M\,P^+}\,H_{1,4} + \frac{\Delta^j_\perp\,i\sigma^{k+}\gamma_5 k^k_\perp}{M\,P^+}\,H_{1,5} + \frac{\Delta^j_\perp\,i\sigma^{k+}\gamma_5\Delta^k_\perp}{M\,P^+}\,H_{1,6} + \frac{k^j_\perp\,i\sigma^{+-}\gamma_5}{M}\,H_{1,7} + \frac{\Delta^j_\perp\,i\sigma^{+-}\gamma_5}{M}\,H_{1,8}\Big]u(p,\lambda),$$

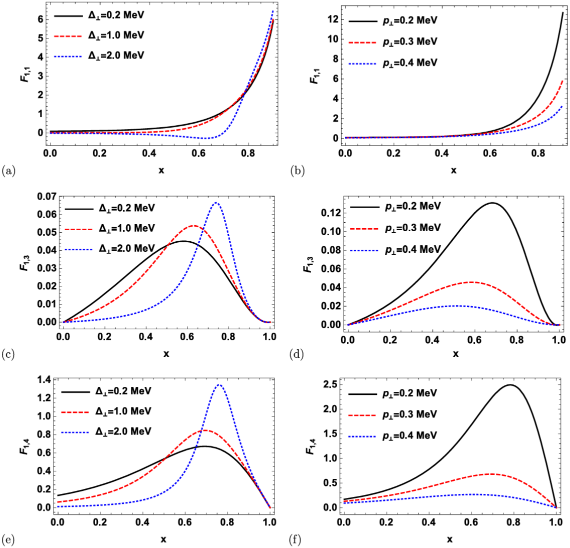


Fig. 15. (Color online.) Plots of GTMDs $F_{1,1}$, $F_{1,3}$ and $F_{1,4}$ with fixed value of $\mathbf{p}_\perp = 0.3$ MeV but with different values of $\mathbf{\Delta}_\perp$ (left panel) and fixed value of $\mathbf{\Delta}_\perp = 0.3MeV$ with different values of $\mathbf{p}_\perp$ (right panel).

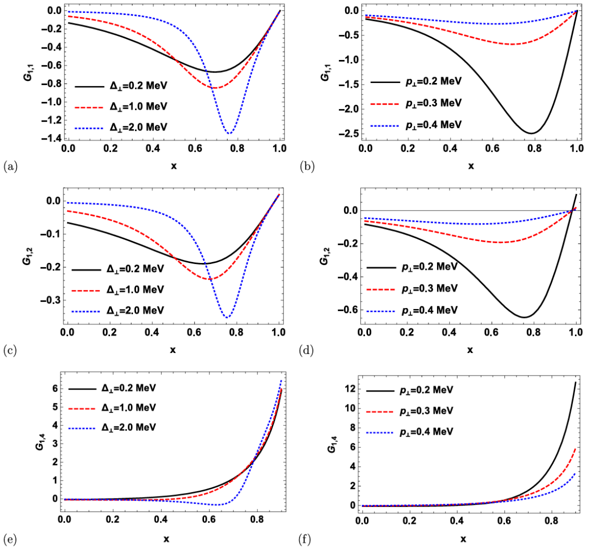


Fig. 16. (Color online.) Plots of GTMDs $G_{1,1}, G_{1,2}$ and $G_{1,4}$ with fixed value of $\mathbf{p}_\perp = 0.3$ MeV but with different values of $\mathbf{\Delta}_\perp$ (left panel) and fixed value of $\mathbf{\Delta}_\perp = 0.3 MeV$ with different values of $\mathbf{p}_\perp$ (right panel).

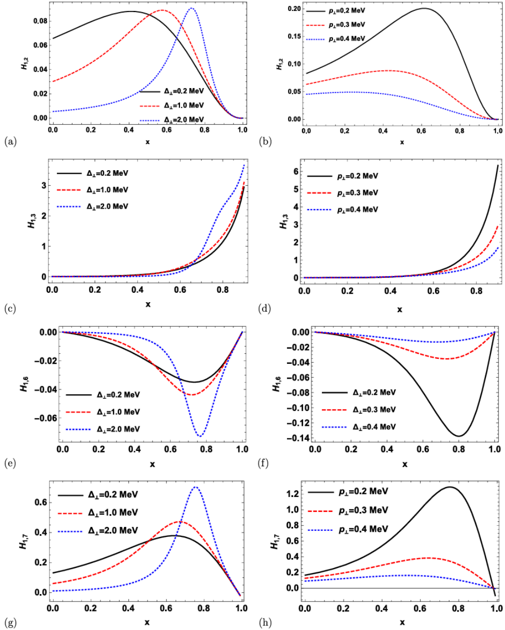


Fig. 17. (Color online.) Plots of GTMDs $H_{1,2}$, $H_{1,3}$, $H_{1,6}$ and $H_{1,7}$ with fixed value of $\mathbf{p}_\perp = 0.3$ MeV but with different values of $\mathbf{\Delta}_\perp$ (left panel) and fixed value of $\mathbf{\Delta}_\perp = 0.3$ MeV with different values of $\mathbf{p}_\perp$ (right panel).

- Before starting with electron, let us discuss about the basic questions about the proton.
- We know that proton makes up nearly 90% of the matter in the Universe. Elementary quarks contribute only few percent to proton mass.What is its origin of mass?
- Quarks hadronize and form proton as the Universe cooled below the Hagedorn temperature. What is the origin of confinement?
- The strong interaction is thought responsible for confinement. How are the forces distributed in space to make the proton a stable particle?

- The structure of the strongly interacting particles can be probed by the fundamental forces: electromagnetic, weak and gravity.

em:	$\partial_\mu J^\mu_{\rm em} = 0$	$\langle N' \vert J^\mu_{\rm em} \vert N\rangle$	$\longrightarrow$	$Q_{\rm prot}$	$=$	$1.602176487(40) \times 10^{-19}{\rm C}$
				$\mu_{\rm prot}$	$=$	$2.792847356(23)\mu_N$
weak:	PCAC	$\langle N' \vert J^\mu_{\rm weak} \vert N\rangle$	$\longrightarrow$	g_A	$=$	$1.2694(28)$
				g_p	$=$	$8.06(0.55)$
gravity:	$\partial_\mu T^{\mu\nu}_{\rm grav} = 0$	$\langle N' \vert T^{\mu\nu}_{\rm grav} \vert N\rangle$	$\longrightarrow$	$M_{\rm prot}$	$=$	$938.272013(23)\,{\rm MeV}/c^2$
				J	$=$	$\frac{1}{2}$
				D	$=$	**?**

Figure: P. Schwitzer et. al. hep-ph/1612.0672

- The D-term is the last unknown global property of the nucleon.

Probing the structure

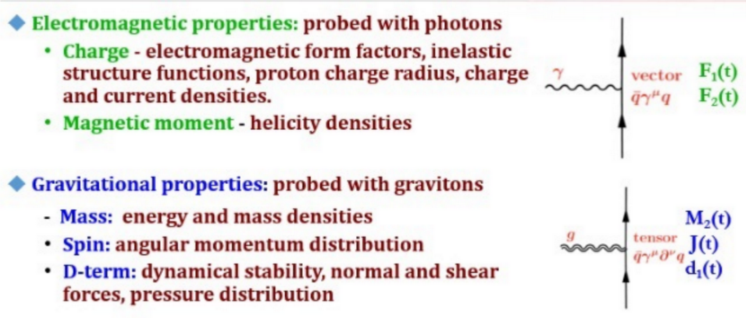


Figure: Kumano 2018

- Gravitational form factors are the matrix elements of the energy-momentum tensor.
- These objects describing the interaction with fermions probably first appeared in the seminal paper of Kobzarev and Okun in which the equivalence principle (EP) for spin motion was first identified.

L. Okun and I. Yu. Kobsarev, ZhETF, 43 (1962) 1904

- Note that as spin is essentially a quantum concept, this paper was probably (one of) the first discussions of the interaction of classical gravity with quantum objects.
- According to the EP, the anomalous gravitomagnetic moment (AGM), which is the gravitational analog of the anomalous magnetic moment, goes to zero.

- Vanishing of anomalous gravitomagnetic moment can be derived from the conservation of momentum and angular momentum.
- Later, energy-momentum structure form factors of particles were discussed by H. Pagels.

H. Pagels, Phys. Rev. 144(4), 1250 (1966)

- ". . . .. , there is very little hope of learning anything about the detailed mechanical structure of a particle, because of the extreme weakness of the gravitational interaction" (H. Pagels, 1966)

Are they experimentally accessible?

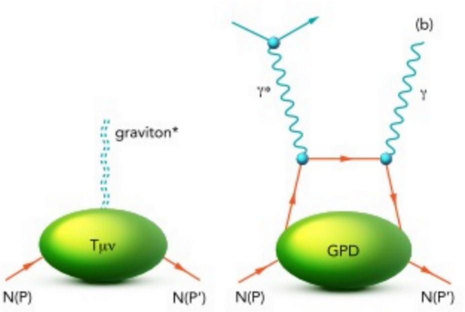


Figure: Burkert 2021

- Graviton colliders are not presently feasible but gravitational form factors ≈ second moment of unpolarized GPDs are constrained by hard exclusive processes.

- Recently Burkert et. al. reported measurement of the pressure distribution experienced by the quarks in the proton which is about 10^{35} Pascals from experiments JLAB (proton D-term extracted from DVCS).

Burkert *et. al.* Nature 557, 396 (2018).

- Kumano et. al. reported the first ever results for the gravitational form factors for the pion by analyzing the Belle data for two photon cross-section $\gamma^*\gamma \to \pi^0\pi^0$ by means of generalized distribution amplitudes (GDA).

Kumano, Song, Teryaev PRD 97 014020 (2018)

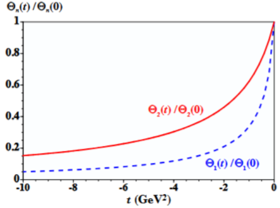


Figure: Kumano 2018

- The form factors of the energy-momentum tensor for a spin-$\frac{1}{2}$ particle are defined by

$$
\begin{aligned}
\langle P', s'| T^{\mu\nu} |P, s\rangle &= \bar{U}(P', s')\Big[- B(q^2)\frac{\bar{P}^{\mu}\bar{P}^{\nu}}{M} \\
&+ (A(q^2) + B(q^2))\frac{1}{2}(\gamma^{\mu}\bar{P}^{\nu} + \gamma^{\nu}\bar{P}^{\mu}) \\
&+ C(q^2)\frac{q^{\mu}q^{\nu} - q^2 g^{\mu\nu}}{M} + \bar{C}(q^2)\ M\ g^{\mu\nu}\Big] U(P, s)
\end{aligned}
$$

where $\bar{P}^{\mu} = \frac{1}{2}(P' + P)^{\mu}\bar{P}^{\mu} = \frac{1}{2}(P' + P)^{\mu}$is the average nucleon four momentum. $\bar{U}(P', s'), U(P, s)$ are the Dirac spinors and M is the mass of the target state.

- There exists constraints at zero-momentum transfer, which are related to mass and spin:

$$\begin{aligned} A(0) &= 1 \leftrightarrow \text{mass conservation} \\ J(0) &= \frac{1}{2} + \frac{1}{2}\mathrm{B}(0) \\ B(0) &= 0 \leftrightarrow \text{spin conservation} \end{aligned}$$

- B(0) is called "anomalous gravitomagnetic moment".

$$\sum \bar{c}(q^2) = 0 \text{ (To satisfy conservation law) } \partial^{\mu} T_{\mu\nu} = 0$$

- The transverse components of energy-momentum tensor define the stress tensor:

$$T^{ij} = \left(\frac{r^i r^j}{r^2} - \frac{1}{3}\delta^{ij} \right) s(r) + \delta^{ij} p(r)$$

- D-term is related to the distributions of shear forces inside the nucleon.
- *The word 'pressure' should not be taken literally in its usual sense in thermodynamics.
- *Conjecture: Stable hadrons must have a negative D term.

- By calculating the $++$ component of the energy-momentum tensor (J. More *et. al.*, PRD 105 056017 (2022)).

$$
\begin{aligned}
\mathcal{M}^{\mu\nu}_{S,S'} &= \frac{1}{2}\left[\langle P', S'|T^{\mu\nu}|P, S\rangle\right] \\
\mathcal{M}^{++}_{\uparrow,\uparrow} + \mathcal{M}^{++}_{\downarrow,\downarrow} &= 2(P^+)^2 A(q^2) \\
\mathcal{M}^{++}_{\uparrow,\downarrow} + \mathcal{M}^{++}_{\downarrow,\uparrow} &= \frac{\iota q_2}{M}(P^+)^2 B(q^2)
\end{aligned}
$$

- One can also extract the gravitational form factors $C(q^2)$ and $\bar{C}(q^2)$ by using the transverse component $(1,2)$ of the energy-momentum tensor

$$
\begin{aligned}
\mathcal{M}^{11}_{\uparrow,\downarrow} + \mathcal{M}^{11}_{\downarrow,\uparrow} - \mathcal{M}^{22}_{\uparrow,\downarrow} - \mathcal{M}^{22}_{\downarrow,\uparrow} &= \iota\left[\frac{B(q^2)}{4M} - \frac{C(q^2)}{M}\right]((q_1)^2 q_2 - (q_2)^3) \\
\mathcal{M}^{11}_{\uparrow,\downarrow} + \mathcal{M}^{11}_{\downarrow,\uparrow} + \mathcal{M}^{22}_{\uparrow,\downarrow} + \mathcal{M}^{22}_{\downarrow,\uparrow} &= \iota q_2\left[B(q^2)\frac{q^2}{4M} - C(q^2)\frac{3q^2}{M} + \bar{C}(q^2)2M\right]
\end{aligned}
$$

Recent work

Physics Letters B 820 (2021) 136501

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

The gravitational form factor $D(t)$ of the electron

Andreas Metz [a], Barbara Pasquini [b,c], Simone Rodini [b,c,*]

[a] Department of Physics, SERC, Temple University, Philadelphia, PA 19122, USA
[b] Dipartimento di Fisica, Università degli Studi di Pavia, I-27100 Pavia, Italy
[c] Istituto Nazionale di Fisica Nucleare, Sezione di Pavia, I-27100 Pavia, Italy

Physics Letters B 820 13650 (2021)

The gravitational form factors of the electron in quantum electrodynamics

Adam Freese[a], Andreas Metz[b], Barbara Pasquini[c,d], Simone Rodini[e]

[a] Department of Physics, University of Washington, Seattle, WA 98195, USA
[b] Department of Physics, SERC, Temple University, Philadelphia, PA 19122, USA
[c] Dipartimento di Fisica, Università degli Studi di Pavia, I-27100 Pavia, Italy
[d] Istituto Nazionale di Fisica Nucleare, Sezione di Pavia, I-27100 Pavia, Italy
[e] CPHT, CNRS, Ecole Polytechnique, Institut Polytechnique de Paris, Route de Saclay, 91128 Palaiseau, France

Physics Letters B 839, 137768 (2023)

Results

- Using the overlap of light-front wave functions, one can evaluate the gravitational form factor $A(q^2)$ and $B(q^2)$

$$A(q^2) = Z + \int \frac{dx\ d^2\vec{k}_\perp}{16\pi^3} x \Bigg[\Bigg(\psi^{\uparrow *}_{+\frac{1}{2},+1}(x,\vec{k}'_\perp)\psi^{\uparrow}_{+\frac{1}{2},+1}(x,\vec{k}_\perp) + \psi^{\uparrow *}_{+\frac{1}{2},-1}(x,\vec{k}'_\perp)\psi^{\uparrow}_{+\frac{1}{2},-1}(x,\vec{k}_\perp) + \psi^{\uparrow *}_{-\frac{1}{2},+1}(x,\vec{k}'_\perp)\psi^{\uparrow}_{-\frac{1}{2},+1}(x,\vec{k}_\perp) \Bigg) + \Bigg(\psi^{\downarrow *}_{+\frac{1}{2},+1}(x,\vec{k}'_\perp)\psi^{\downarrow}_{+\frac{1}{2},+1}(x,\vec{k}_\perp) + \psi^{\downarrow *}_{+\frac{1}{2},-1}(x,\vec{k}'_\perp)\psi^{\downarrow}_{+\frac{1}{2},-1}(x,\vec{k}_\perp) + \psi^{\downarrow *}_{-\frac{1}{2},+1}(x,\vec{k}'_\perp)\psi^{\downarrow}_{-\frac{1}{2},+1}(x,\vec{k}_\perp) \Bigg) \Bigg]$$

$$B(q^2) = \int \frac{dx\ d^2\vec{k}_\perp}{16\pi^3} x \Bigg[\Bigg(\psi^{\uparrow *}_{+\frac{1}{2},-1}(x,\vec{k}'_\perp)\psi^{\downarrow}_{+\frac{1}{2},-1}(x,\vec{k}_\perp) + \psi^{\uparrow *}_{-\frac{1}{2},+1}(x,\vec{k}'_\perp) \psi^{\downarrow}_{-\frac{1}{2},+1}(x,\vec{k}_\perp) \Bigg) + \Bigg(\psi^{\downarrow *}_{+\frac{1}{2},-1}(x,\vec{k}'_\perp)\psi^{\uparrow}_{+\frac{1}{2},-1}(x,\vec{k}_\perp) + \psi^{\downarrow *}_{-\frac{1}{2},+1}(x,\vec{k}'_\perp) \psi^{\uparrow}_{-\frac{1}{2},+1}(x,\vec{k}_\perp) \Bigg) \Bigg]$$

$$A_e(Q^2) = 1 + \frac{\alpha}{2\pi} \int \frac{dx}{(1-x)} \Bigg\{ 2x - \frac{x\left[\left(1+x^2\right)\left(2\mathcal{M}^2+Q\right) - 2(m-Mx)^2\right]}{Q} \frac{t_2}{t_1}$$
$$-(1+x^2)\left[\log\left(1+\frac{\lambda^2}{\mathcal{M}^2(1-x)^2}\right) - x\log\left(\frac{\lambda^2}{\mathcal{M}^2(1-x)^2}\right)\right]\Bigg\},$$

$$B_e(Q^2) = \frac{2\alpha M}{\pi} \frac{x^2(m-Mx)}{(1-x)} \frac{t_2}{Q^2 t_1}$$

where

- We choose the values of the masses as $M = 0.511 MeV$, $m = 0.5 MeV$ and $\mu = 0.02 MeV$.
- Here, we impose an UV-cutoff 'λ' on the $k_\perp$ integration.

$$A_e(0) + A_\gamma(0) = 1$$
$$B_e(0) + B_\gamma(0) = 0$$

$$
\begin{aligned}
D_e(q^2) &= \frac{2\alpha}{\pi}\frac{(m-Mx)M(-2+x)}{q^2}\Big[-2+t_1t_2\Big], \\
\bar{C}_e(q^2) &= \frac{\alpha}{4\pi}\frac{(m-Mx)}{M}\Bigg[-\frac{2t_2}{t_1}\left(\frac{4\mathcal{M}^2}{q^2}(-2+x)+\frac{2-3x}{(-1+x)}\right) \\
&\quad \left(-5+2x+(2x-3)\log\left(\frac{\lambda^2}{\mathcal{M}^2(x-1)^2}\right)+\right. \\
&\quad \left.\log\left(1+\frac{\lambda^2}{\mathcal{M}^2(x-1)^2}\right)\right)m\Bigg]
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M}^2 &= \frac{m^2}{1-x}-\frac{M^2x}{1-x}+\frac{\mu^2x}{(1-x)^2}, \\
t_1 &= \sqrt{1+\frac{4\mathcal{M}^2}{q^2}}, \\
t_2 &= \log\left(1+\frac{\left(1+t_1\right)q^2}{2\mathcal{M}^2}\right).
\end{aligned}
$$

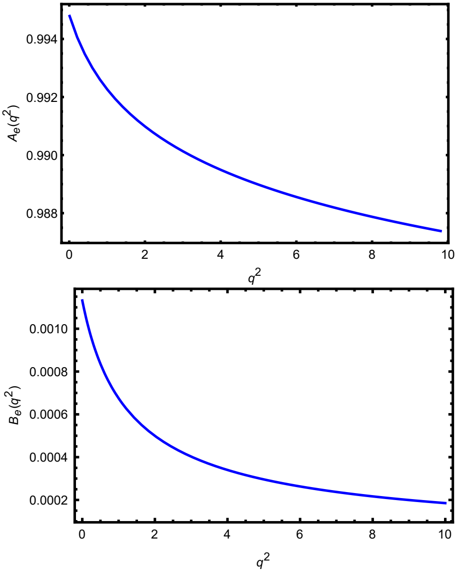


Figure: Electron gravitational form factors $A_e(q^2)$ and $B_e(q^2)$.

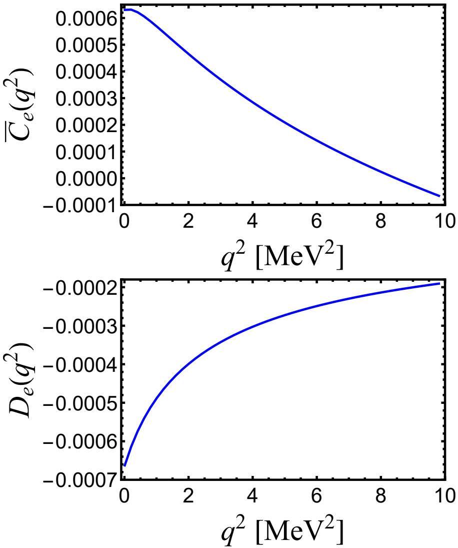
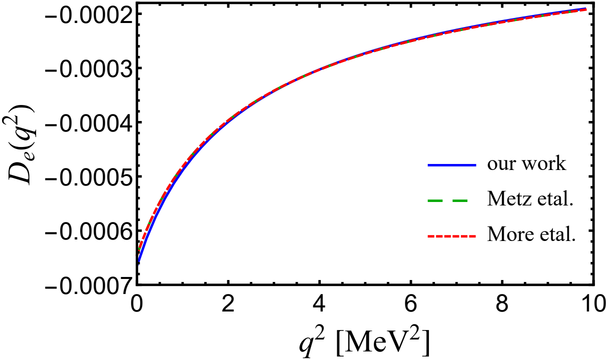


Figure: Electron gravitational form factors $\bar{C}_e(q^2)$ and $D_e(q^2)$.

Comparison with other models

- We compare our results with the dressed quark model(DQM)(in the limit $g^2 = 4\pi\alpha$) and with work of Metz *et. al.*.
- In the limit $M = m$ and zero photon mass our result for D-term matches with the results of DQM and Metz work.

- More *et. al.*PRD 105, 056017 (2022)
-Metz *et. al.* PLB 820, 136501 (2021)



- It is well known fact that energy-momentum tensor encodes the properties like mass and spin of the particle and it is also couples to the gravity.
- The globally unknown D-term offers many hidden mechanical properties of the particle like pressure and shear distributions.
- Other properties like energy density, normal and tangential forces are also associated with the D-term.
- The only experimental result for the pressure distribution is available only for the proton.
- Recently, D-term of the electron was studied by Metz*et. al.* PLB 820 136501 (2021) using Feynman diagrammatic approach and it was found to be divergent at $q^2 = 0$.
- To avoid the divergence they use non-zero photon mass.
- One can obtain the D-term from the transverse component of the energy-momentum tensor.

The expressions for the two-dimensional pressure and shear distributions for electron are

$$
\begin{aligned}
p_e(\vec{b}_\perp) &= \frac{1}{2M\ \vec{b}_\perp}\frac{d}{d\vec{b}_\perp}\left(\vec{b}_\perp\frac{d}{d\vec{b}_\perp}D_e(\vec{b}_\perp) - M\bar{C}(\vec{b}_\perp)\right),\\
s_e(\vec{b}_\perp) &= -\frac{\vec{b}_\perp}{M}\frac{d}{d\vec{b}_\perp}\left(\frac{1}{\vec{b}_\perp}\frac{d}{d\vec{b}_\perp}D_e(\vec{b}_\perp)\right)
\end{aligned}
$$

-Polyakov & Schweitzer, Int.J.Mod.Phys. A33 (2018) 1830025

$$
\begin{aligned}
F(\vec{b}_\perp) &= \frac{1}{(2\pi)^2}\int d^2q_\perp e^{i\vec{q}_\perp\cdot\vec{b}_\perp}\mathcal{F}(q^2),\\
&= \frac{1}{2\pi}\int_0^\infty q\ dq\ J_0(|q|\ |b|)\ \mathcal{F}(q^2),
\end{aligned}
$$

where $\mathcal{F}(q^2) = A, B, \overline{C}, D$ are the respective GFFs, J_0 is the zeroth order Bessel function, $\vec{b}_\perp$ is the impact parameter and M is the mass of the physical electron.

Pressure and Shear Distributions

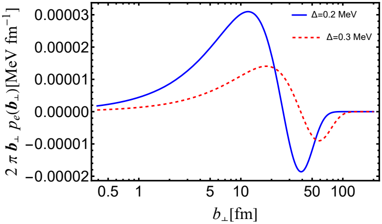


Figure: Pressure distribution (preliminary result).

- The numerical value of the Gaussian width decide the spread of distribution.
- For a stable system, a node is required in the pressure distribution and same is observed in our case.
- Further, positive pressure near the center represents the presence of the repulsive forces whereas negative pressure means presence of attractive forces which balance each other.

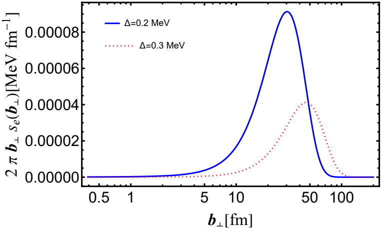


Figure: Shear distribution (preliminary result).

- For shear distributions, we observe that it remains positive in impact-parameter space and such behaviour is well observed in the stable hydrostatic systems.

Conclusions

- Wigner distributions of an electron are calculated which provides the multi-dimensional images of electron.
- We consider different polarization configurations.
- The transverse spin-spin correlations (not presented here) and GTMDs are also studied.
- Results provide rich and interesting information on the distribution of QED partons.
- We have discussed the gravitational form factors and mechanical properties of the electron.
- We evaluate the spin non-flip and spin flip gravitational form factors for the electron.
- We have also studied the D-term of the electron and compare it with the results of dressed quark model and Feynmann diagrammatic approach and they found to be consistent with our results.
- We have also calculated the pressure and shear distributions of the QED partons inside the electrons in the impact-parameter space.

- The qualitative behaviour of these forces is found to be consistent with the results of other phenomenological models of the nucleon.
- It would be interesting to see the contributions coming from the photon part which is currently under investigation.

1

Dr. Harleen Dahiya
(Associate Professor)
Group Leader

2

Dr. Nisha Dhiman

- Beyond SM
- B-meson and their decay

3

Dr. Shubham Sharma
(Post-Doc, MIPhT)

- Higher twist distribution functions
- Proton structure

4

Mr. Satyajit Puhan
(Research Scholar, SRF)

- Light and heavy mesons
- Leading and higher twist distributions
- Radially higher excited states

5

Ms. Navpreet Kaur
(Research Scholar, SRF)

- Low lying octet baryons
- Leading twist distributions
- In-medium meson properties

Figure: Hadron Physics Research Group

Thank You!