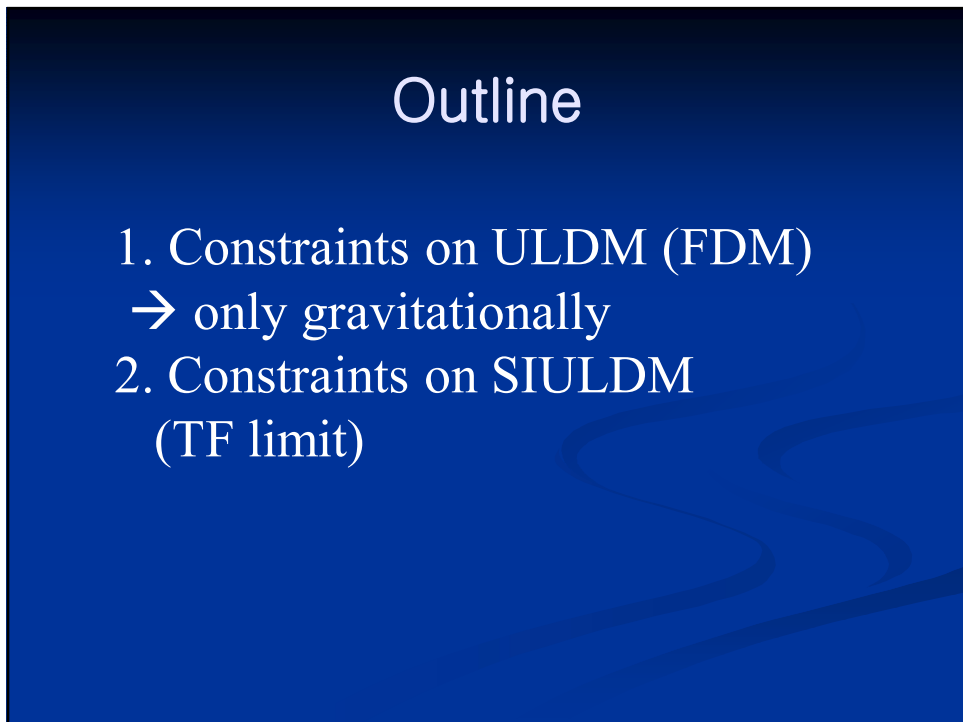
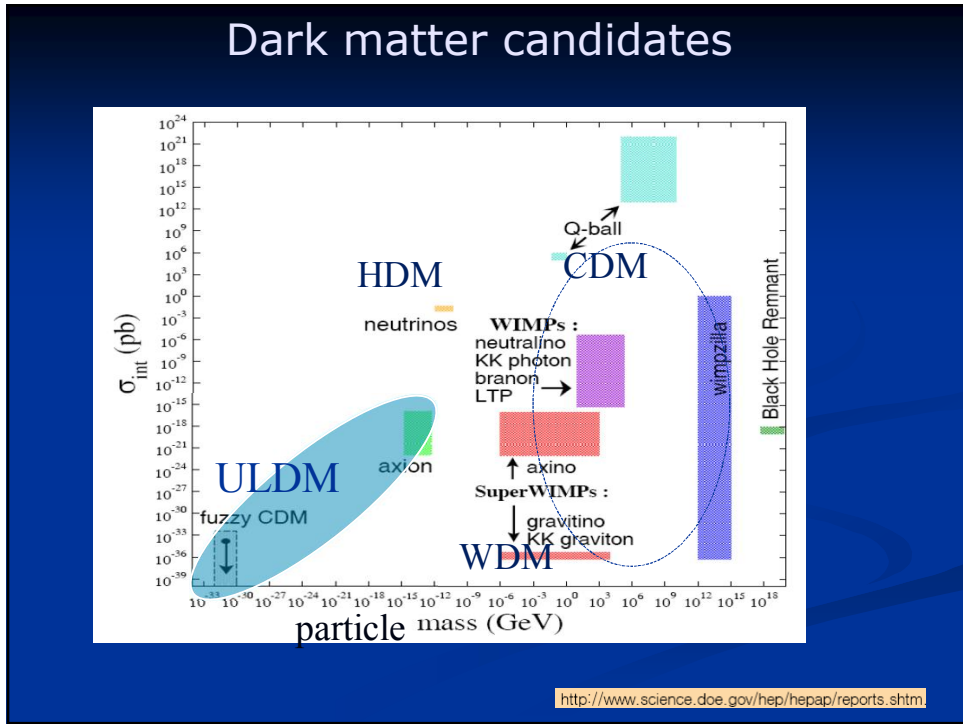


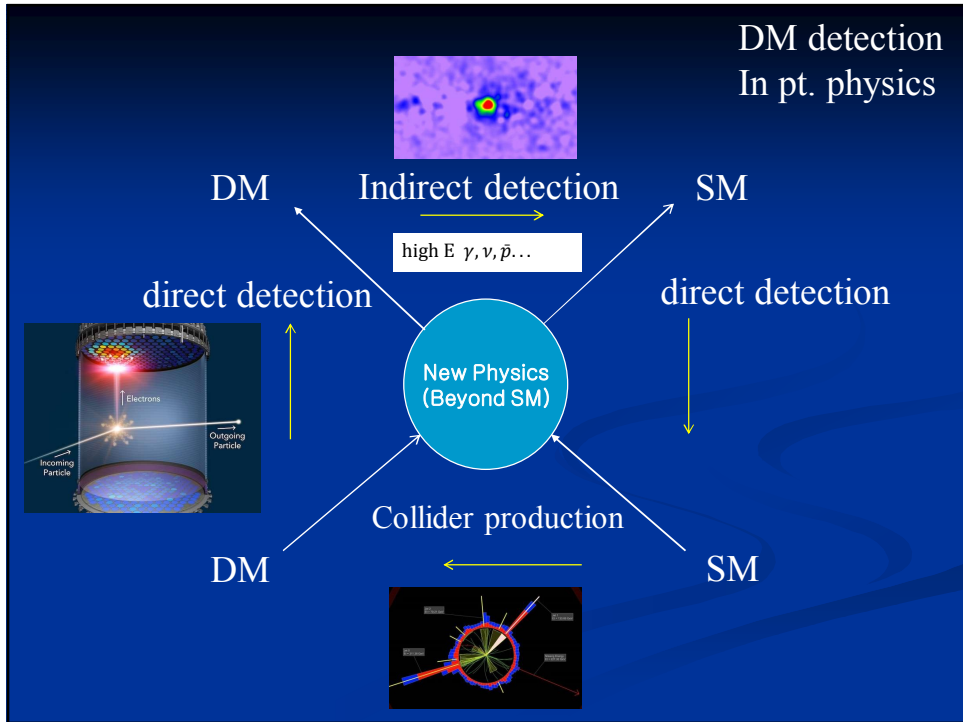
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4

RESEARCH ARTICLE | PHYSICS

COSINE-100 full dataset challenges the annual modulation signal of DAMA/LIBRA

NELSON CARLIN, JAE YOUNG CHO, JAE JIN CHOI, SEONHO CHOI, ANTHONY C. EZERIBE, LUIS EDUARDO FRANCA, CHANG HYON HA, IN SIK HAHN, SOPHIA J. HOLLICK, J.-I., AND GYUN HO YU, +49 authors

SCIENCE ADVANCES • 3 Sep 2025 • Vol 11, Issue 36 • DOI: 10.1126/sciadv.adv6503

205



COSINE-100


No direct, or indirect detection so far

5

Challenges for Λ CDM

cf) 2105.05208

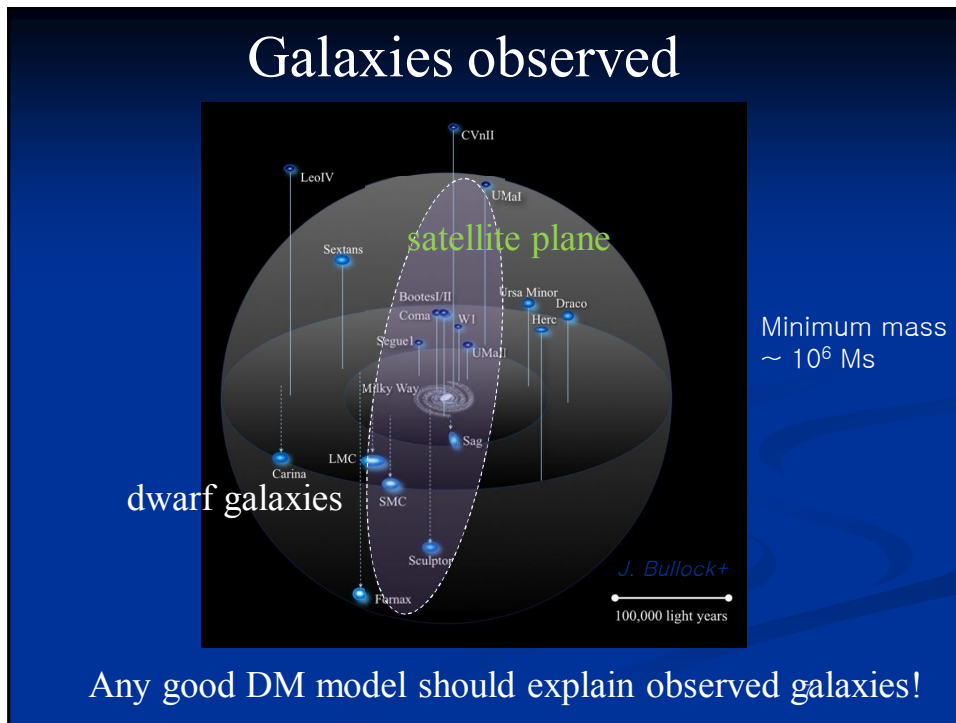
- Λ CDM was very successful but is becoming non-standard?
- 1. **Small scale crisis** (on galactic scale and below)
predicts too many small structures not observed
- 2. **Hubble parameter tension $\sim 5\sigma$** :
 H_0 mismatch between Planck estimation and SN
- 3. S_8 tension $\sim 2-3\sigma$: $S_8 \equiv \sigma_8 \sqrt{\Omega_m/0.3}$, $z < 2$, Mpc
matter density fluctuation amplitude mismatch
between Planck estimation and WL & Cluster
- 4. **DE is not Λ ? $\sim 4\sigma$** (ex, DESI, AP test)
- 5. Too early BHs and galaxies (Webb)
- 6. Cluster collision (collision speed & DM-star offset)
- 7. Li problem, Cosmic birefringence
...etc



JADES-GS-z14-0

→ ULDM model might address some of these tensions

6



7

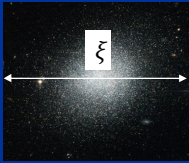
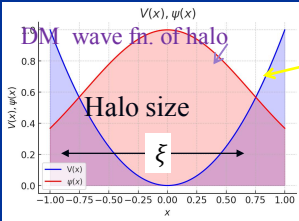
Solutions to Small scale problems

- CDM : $m \sim \text{GeV}$ (can't solve the problems)
→ Baryon physics (SN, BH jets,...)
- WDM : $m \sim \text{keV}$
→ Catch 22 problem (cusp)
Suppression of the power spectrum
prohibiting the formation of the dwarf galaxy
- SIDM: $\sigma/m \sim 0.5\text{-}1 \text{ cm}^2/\text{g}$
→ velocity dependent?
- ULDM: $m \sim 10^{-22} \text{ eV}$
→ Lyman alpha favors $m > 10^{-21} \text{ eV}$?

8

ULDM

- Galactic DM halo is a **BEC made of ultralight scalar particles**
- Quantum pressure** (from **uncertainty principle**) prevents collapse like atoms
- Galaxy size \sim de Broglie wavelength of DM particles \sim kpc
 $\rightarrow m \sim 10^{-22}$ eV
- Small $m \rightarrow$ high # density \rightarrow overlap of wave fn. \rightarrow classical wave

Self-gravitating potential well V

Schrodinger-Poisson (SP)

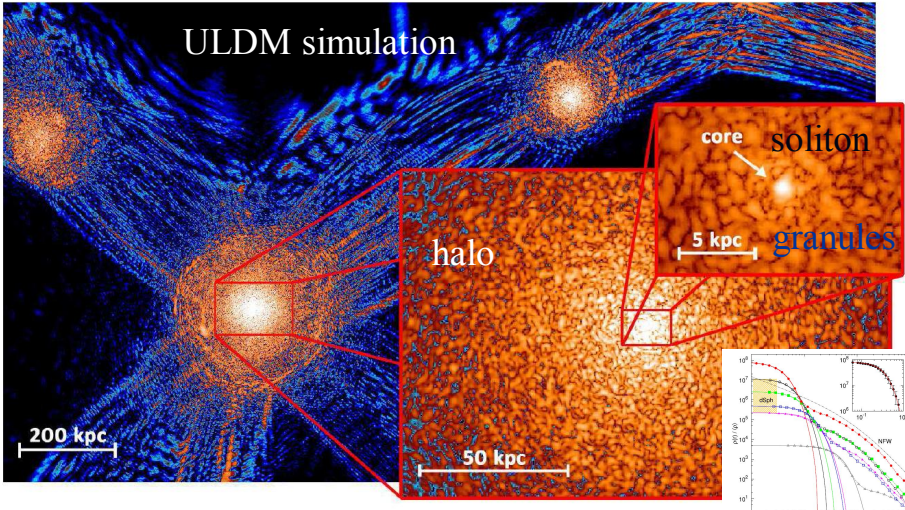
$$\begin{cases} i\hbar\partial_t\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + mV\psi + int. \\ \nabla^2V = 4\pi G(\rho_d + \rho_v) \text{ visible}, & \rho_d = m|\psi|^2 \end{cases}$$

ULDM SF $\phi(t, x) = \frac{1}{\sqrt{2m}} [e^{-imt}\psi(t, x) + e^{imt}\psi^*(t, x)]$

9

9

ULDM simulation



core size \sim granule size \sim typical length

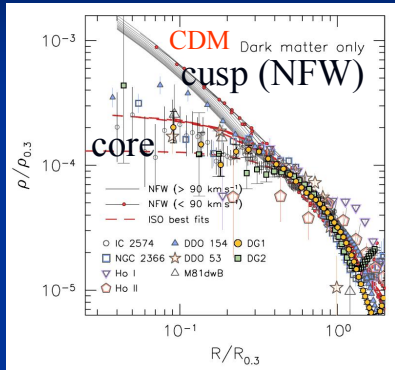
Schive et al , Nature physics 2014

10

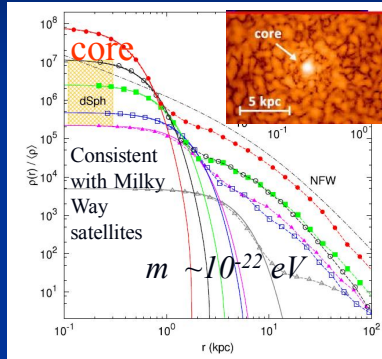
10

Core/Cusp problem of CDM

Observation



ULDM simulation →
Core ~ de Broglie wave len.



Schive et al, Nature physics 2014

Density profile from rotation curves of small galaxies strongly **disfavors** CDM
→ ULDM well explains the core profile!

11

Linear pert. Of ULDM

FDM has only 2 parameters m and bg density ρ_0 (or F)
(+ λ for ϕ^4 self-interacting ULDM)

a =scale factor

Nonrelativistic →

$$i\hbar\left(\frac{\partial\psi}{\partial t} + \frac{3}{2}H\psi\right) = -\frac{\hbar^2}{2ma^2}\Delta\psi + mV\psi + \frac{\lambda|\psi|^2\psi}{2m^2} \text{ self-int}$$

Madelung representation

$$\text{perturbation with } \psi = \sqrt{\rho}e^{iS}, \quad v \equiv \frac{\hbar}{ma}\nabla S \Rightarrow$$

$$\begin{cases} \partial_t \rho + 3H\rho + \frac{1}{a}\nabla \cdot (\rho v) = 0 \\ \partial_t v + \frac{1}{a}v \cdot \nabla v + Hv + \frac{1}{\rho a}\nabla p + \frac{1}{a}\nabla V + \frac{\hbar^2}{2m^2 a^3}\nabla \left(\frac{\Delta\sqrt{\rho}}{\sqrt{\rho}}\right) = 0 \end{cases}$$

Density contrast (k space)

$$\text{perturbation } \delta = \delta_k = \delta\rho/\rho_0$$

$$\Rightarrow \partial_t^2 \delta + 2H\partial_t \delta + \left(\frac{\hbar^2 k^2}{4m^2 a^2} + c_s^2 \right) \frac{k^2}{a^2} - 4\pi G \rho_0 \delta = 0$$

Hubble drag   Quantum Pressure   gravity

Quantum Jeans length

$$\lambda_{QJ} = \frac{2\pi}{k_J} a = \pi^{3/4} \hbar^{1/2} (G\rho_0 m^2)^{-1/4} \propto 1/\sqrt{mH} \sim 10 \text{ kpc}$$

- CDM-like on super-galactic scale (for a small $k < k_J$)
 - Suppress sub-galactic structure (for a large $k > k_J$)
- ULDM is an ideal alternative to CDM

12

12

Other cosmological Constraints

- BEC phase transition before nucleosynthesis: $m < 10^2 \text{ eV}$
- field oscillation before equality $m > 10^{-28} \text{ eV}$
- Maximum mass of galaxies from BS theory
 - spiral $1.04 \times 10^{12} M_\odot < O(1) M_p^2/m \rightarrow m < O(1) 1.28 \times 10^{-22} \text{ eV}$
 - elliptical $1 \times 10^{13} M_\odot < O(1) M_p^2/m \rightarrow m < O(1) 1.28 \times 10^{-23} \text{ eV}$
- Ly α forest $m > 10^{-21} \text{ eV}$
- high-redshift galaxy luminosity $\rightarrow m > 1.2 \times 10^{-22} \text{ eV}$
- Stella subpopulations in Fornax $\rightarrow m < 1.1 \times 10^{-22} \text{ eV}$
- Ultra-faint dSphs $\rightarrow m \sim 3.7\text{-}5.6 \times 10^{-22} \text{ eV}$

fiducial value $m \sim 10^{-22} \text{ eV}$

13

Scales from ULDM

$$\lambda_Q(z) = \frac{2\pi}{k} = \left(\frac{\pi^3 \hbar^2}{Gm^2 \bar{\rho}(z)} \right)^{1/4} = 71.75 \text{ kpc} \left(\frac{m}{10^{-22} \text{ eV}} \right)^{-1/2} \left(\frac{\bar{\rho}}{10^{-7} M_\odot / \text{pc}^3} \right)^{-1/4} \propto (1+z)^{-3/4},$$

$$M_Q(z) = \frac{4\pi}{3} \left(\frac{\lambda_Q}{2} \right)^3 \bar{\rho} = \frac{\pi^{13}}{6} \left(\frac{\hbar}{G^{1/2} m} \right)^{3/2} \bar{\rho}(z)^{1/4} = 1.93 \times 10^7 M_\odot \left(\frac{m}{10^{-22} \text{ eV}} \right)^{-3/2} \left(\frac{\bar{\rho}}{10^{-7} M_\odot / \text{pc}^3} \right)^{1/4} \propto (1+z)^{3/4},$$

$$X_c = \left[R_{99} = 9.95 \left(\frac{\hbar}{m} \right)^2 \frac{1}{GM} = 8.5 \text{ kpc} \left(\frac{10^{-22} \text{ eV}}{m} \right)^2 \frac{10^8 M_\odot}{M}, \quad t_c = \frac{1}{\sqrt{G\bar{\rho}}}, \right]$$

$$a_c = x_c/t_c^2 = 0.0044 G^3 m^4 M^3 / \hbar^4 = 8.38 \times 10^{-14} \text{ meter/s}^2 \left(\frac{m}{10^{-22} \text{ eV}} \right)^4 \left(\frac{M}{10^8 M_\odot} \right)^3 \simeq \sqrt{\frac{\hbar}{m}} (G\bar{\rho})^{3/4},$$

$$v_c \equiv x_c/t_c = 0.21 GMm/\hbar = 4.71 \text{ km/s} \left(\frac{M}{10^8 M_\odot} \right) \left(\frac{m}{10^{-22} \text{ eV}} \right) \simeq \sqrt{\frac{\hbar}{m}} (G\bar{\rho})^{1/4},$$

$$\Sigma = 0.01 \frac{G^2 m^4 M^3}{\hbar^4} = 1.38 M_\odot / \text{pc}^2 \left(\frac{m}{10^{-22} \text{ eV}} \right)^4 \left(\frac{M}{10^8 M_\odot} \right)^4 \propto (1+z)^3$$

Ji& Lee JFOPA 2412.10285

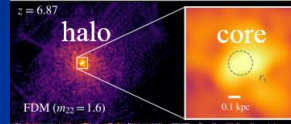
14

core-halo relation of ULDM

Q. Jeans mass

$$M_{QJ}^0 = \frac{4\pi}{3} \rho(z) \lambda_{QJ}^3(z) = 1.204 \times 10^8 \left(\frac{1+z}{m_{22}^2} \right)^{3/4} \left(\frac{\Omega_{dm}}{0.27} \right)^{1/4} \left(\frac{h}{0.7} \right)^{1/2} M_{\odot}$$

$\sim M_h$ halo mass



DM core size

$$r_s = 0.135 m_{22}^{-1} \left(\frac{\zeta'(z)}{\zeta'(7)} \right)^{-1} \left(\frac{M_h}{10^{11} M_{\odot}} \right)^{-1/3} \text{ kpc}$$

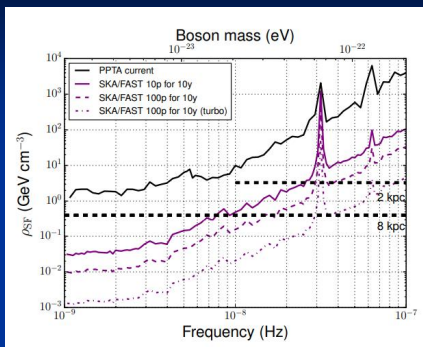
DM core mass

$$M_s = 1.68 \times 10^9 m_{22}^{-1} \left(\frac{\zeta'(z)}{\zeta'(7)} \right) \left(\frac{M_h}{10^{11} M_{\odot}} \right)^{1/3} M_{\odot} \sim \text{Mass gap}$$

where $m_{22} \equiv \frac{m}{10^{-22} \text{ eV}}$, $\zeta' = (1+z)^{1/2} \zeta(z)^{1/6}$
 with $\zeta(z) = (18\pi^2 + 82(\Omega_m(z) - 1) - 39(\Omega_m(z) - 1)^2) / \Omega_m(z) \sim 180$ at $z \geq 1$

15

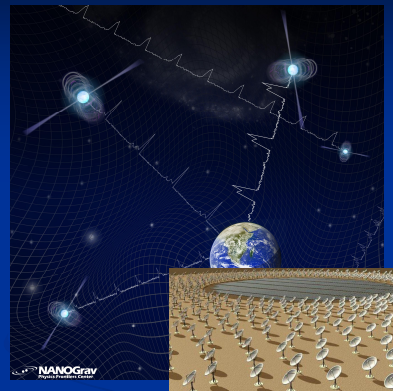
GW background detected by pulsar timing array



1810.03227

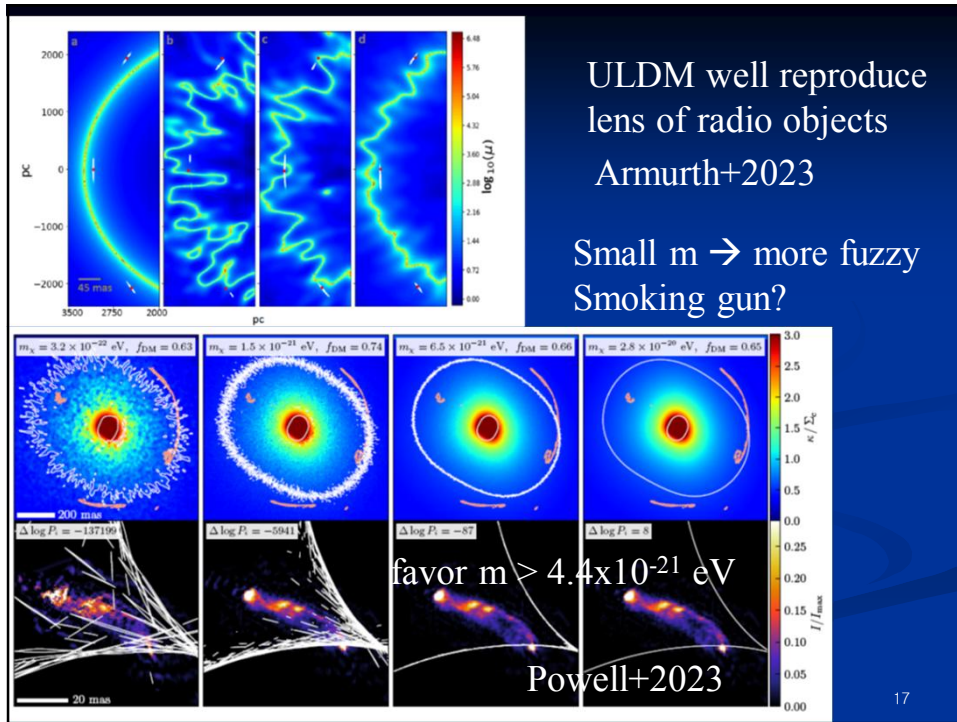
ULDM has
 intrinsic osc time scale
 $1/m \sim \text{yrs}$

$$\omega = \frac{1}{2.5 \text{ months}} \frac{m}{10^{-22} \text{ eV}}$$

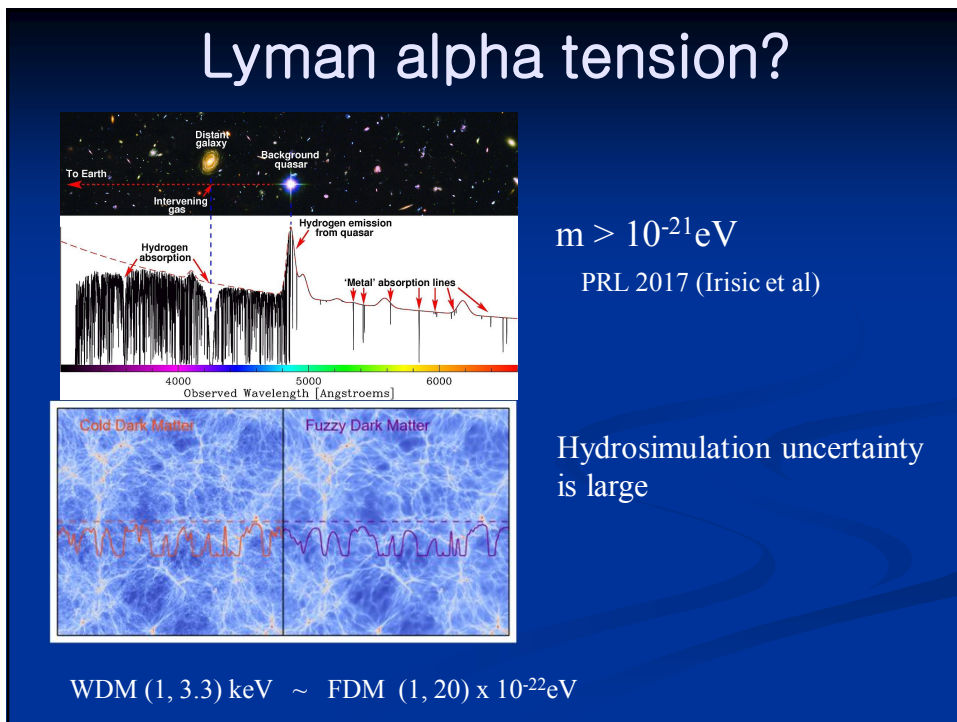


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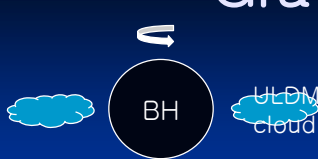


17



18

Gravitational atom



$(g^{\alpha\beta}\nabla_\alpha\nabla_\beta - \mu^2)\Phi(t, r) = 0$

ansatz $\Phi(t, r) = \frac{1}{\sqrt{2\mu}} [\psi(t, r)e^{-i\mu t} + \psi^*(t, r)e^{i\mu t}]$

Schrodinger equation with a Coulomb-like central potential $i\partial_t\psi(t, r) = \left(-\frac{1}{2\mu}\nabla^2 - \frac{\alpha}{r} + O(\alpha^2)\right)\psi(t, r)$

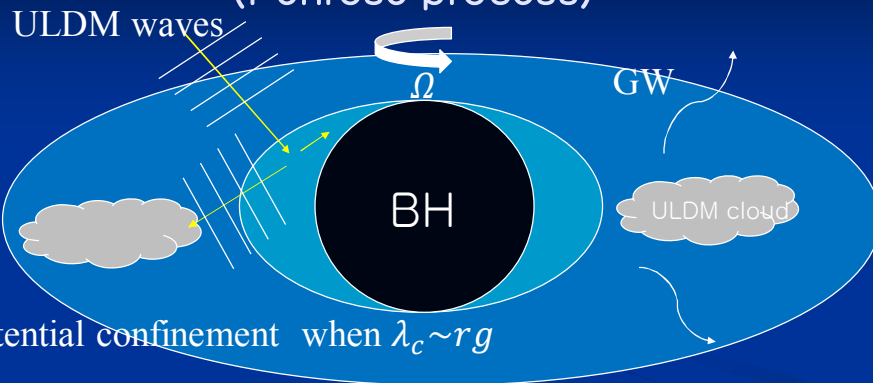
hydrogen atom like solution $\psi_{nlm}(t, r) = R_{nl}(r)Y_{lm}(\theta, \phi)e^{i(\mu - \omega_{nlm})t}$

gravitational fine structure constant $\alpha \equiv \frac{r_g}{\lambda_c} = \frac{GM\mu}{\hbar c} \approx 0.1 \left(\frac{M}{10^{11}M_\odot}\right) \left(\frac{\mu}{10^{-22}\text{eV}}\right) \ll 1$

Occupation # is huge, presence of horizon 19

19

Superradiance (Penrose process)

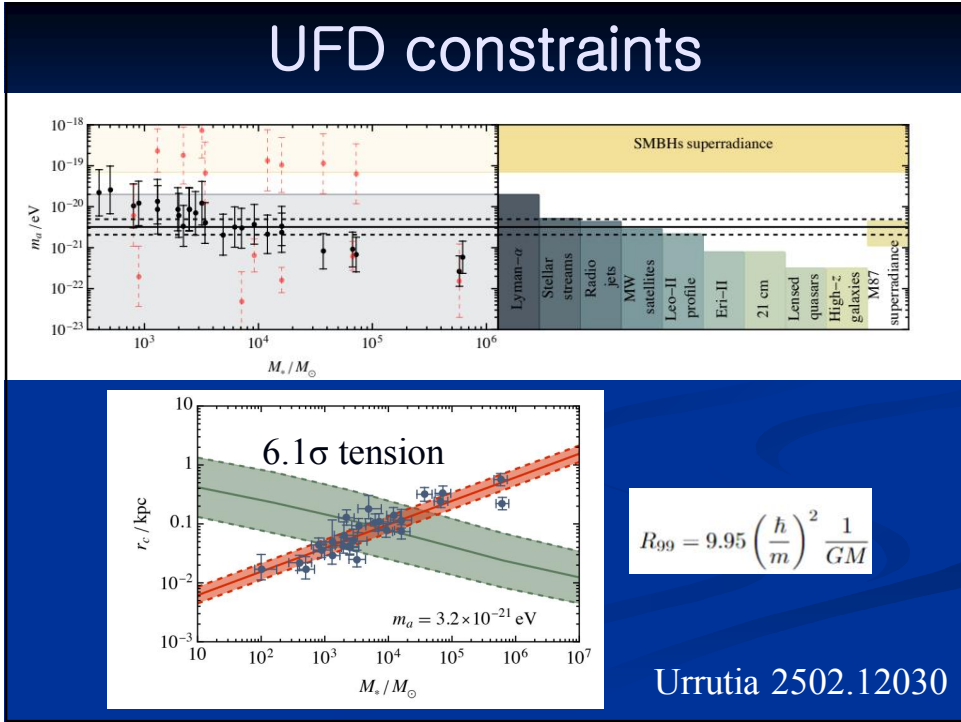


potential confinement when $\lambda_c \sim r_g$

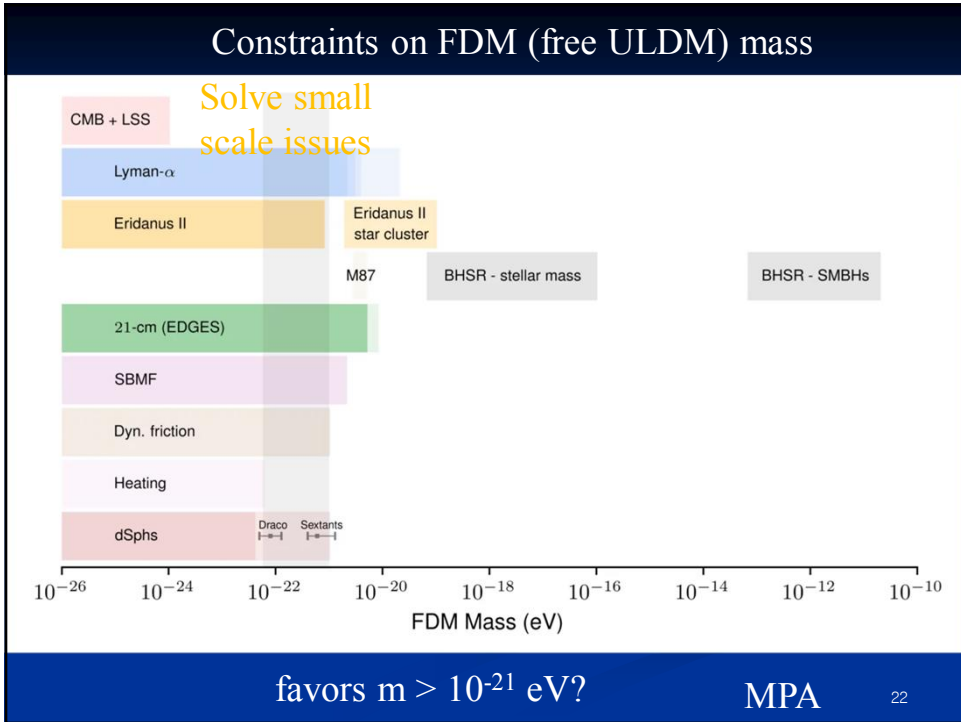
- growing mode if $\omega_{nlm} < m\Omega \rightarrow$ BH spin decreases
magnetic q. number
- can change GW patterns from a BH and BH binary

See Zhang & Yang 2018 20

20



21



22

Self-Interacting ULDM (SIULDM)

Lee and Koh (PRD 53, hep-ph/9507385)

Galactic DM halo is described by **coherent scalar field with self-interaction**

Action $S = \int \sqrt{-g} d^4x \left[\frac{-R}{16\pi G} - \frac{g^{\mu\nu}}{2} \phi_{;\mu}^* \phi_{;\nu} - \frac{m^2}{2} |\phi|^2 - \frac{\lambda}{4} |\phi|^4 \right]$

Metric $ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2 d\Omega$ Spherical.

Field $\phi(r, t) = (4\pi G)^{-\frac{1}{2}} \sigma(r) e^{-i\omega t}$

$\tilde{m} \equiv \frac{m}{\lambda^{\frac{1}{4}}}$

Exact ground state (soliton) $\sigma_* = \sqrt{\frac{\gamma_0 \text{Sin}(\sqrt{2}r_*)}{\sqrt{2}r_*}}$

$\Lambda \equiv \frac{\lambda m^2}{4\pi m^2}, \Lambda \gg 1$ (Newtonian & TF limit)
 New constant length scale $R_{TF} \approx \frac{\sqrt{\Lambda}}{m} = \sqrt{\frac{\pi \hbar^3 \lambda}{8cGm^4}} \sim \sqrt{\tilde{m}}$
 & mass scale $M_{\max} = \sqrt{\Lambda} \frac{m^2}{m}$

$r_* = r\Lambda^{-1/2}$

- Even tiny self-interaction drastically changes the scales!
- allows wider range for m to fit observations
- \tilde{m} determines all scales

23

23

Scales from SIULDM

Ji & Lee 2412.10285

$$\lambda_J = 2\pi\hbar\sqrt{\frac{a_s}{Gm^3}} = 2R_{TF} = \sqrt{\frac{\pi\hbar^3\lambda}{2cGm^4}} = 0.978 \text{ kpc} \left(\frac{\tilde{m}}{10\text{eV}}\right)^{-2},$$

$\mathbf{X_c} = R_{TF} = \pi\hbar\sqrt{\frac{a_s}{Gm^3}} = \sqrt{\frac{\pi\hbar^3\lambda}{8cGm^4}}$

$$t_c = \left(\frac{R_{TF}^3}{GM}\right)^{1/2} = \pi^{3/4} \left(\frac{\hbar^9\lambda^3}{512c^3G^5m^{12}}\right)^{1/4} \sqrt{\frac{1}{M}} \propto (1+z)^{-3/2}.$$

$$M_J(z) \equiv \frac{4\pi}{3} \left(\frac{\lambda_J}{2}\right)^3 \bar{\rho} = \frac{4\pi^4\hbar^3}{3} \left(\frac{a_s}{Gm^3}\right)^{3/2} \bar{\rho} = \frac{\pi^{5/2}}{\sqrt{288}} \left(\frac{\hbar^3\lambda}{cGm^4}\right)^{3/2} \bar{\rho} = 49 M_\odot \left(\frac{\tilde{m}}{10\text{eV}}\right)^{-6} \left(\frac{\bar{\rho}}{10^{-7}M_\odot/\text{pc}^3}\right) \propto (1+z)^3,$$

$$\rho_c = \frac{2\sqrt{2}M \left(\frac{cGm^4}{\lambda}\right)^{3/2}}{\hbar^9/2\pi^{3/2}} = 0.106 M_\odot/\text{pc}^3 \left(\frac{\tilde{m}}{10 \text{ eV}}\right)^6 \left(\frac{M}{10^8 M_\odot}\right) \propto (1+z)^3,$$

$$a_c = \mathbf{x}_c/t_c^2 = \frac{16cG^2m^4M}{\pi\hbar^3\lambda} = 1.163 \times 10^{-10} \text{ meter/s}^2 \left(\frac{\tilde{m}}{10\text{eV}}\right)^4 \left(\frac{M}{10^8 M_\odot}\right) \propto (1+z)^3$$

$$\Sigma = \frac{\hbar\pi^{3/2}\sqrt{\frac{h\lambda}{cGm^4}}\bar{\rho}}{6\sqrt{2}} = 5.124 M_\odot/\text{pc}^2 \left(\frac{\tilde{m}}{10\text{eV}}\right)^{-2} \left(\frac{\bar{\rho}}{10^{-2}M_\odot/\text{pc}^3}\right) = 104.4 M_\odot/\text{pc}^2 \left(\frac{\tilde{m}}{10\text{eV}}\right)^{-2} \left(\frac{M}{10^8 M_\odot}\right) \propto (1+z)^3$$

24

24

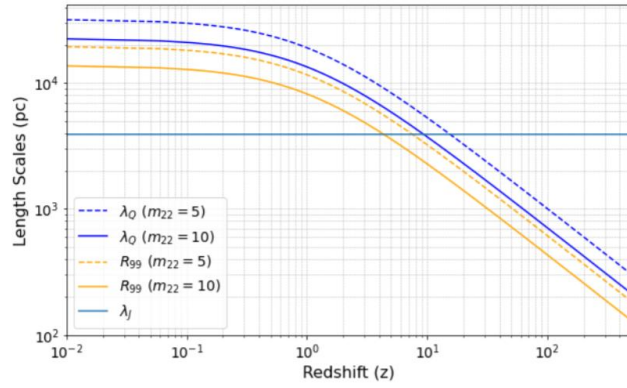


FIG. 1: The evolution of the typical length scales of galaxies versus redshift z . The upper two curves represent λ_Q for $m_{22} = m/10^{-22} \text{ eV} = 5$ and $m_{22} = 10$, respectively, while the lower two curves represent the corresponding R_{99} . The horizontal line represents $\lambda_J = 2R_{TF}$ in the TF limit with $\bar{m} = 5 \text{ eV}$. When $\lambda_J > \lambda_Q$, the size and mass of a dark matter halo formed at that time are governed by self-interaction-induced repulsion rather than quantum pressure, especially at high redshifts $z \gg 10$ and for larger mass $m \gtrsim 10^{-21} \text{ eV}$. We expect galaxy density perturbations to begin collapsing in the redshift range $O(10) \lesssim z \lesssim O(100)$.

Ji & Lee 2412.10285

25

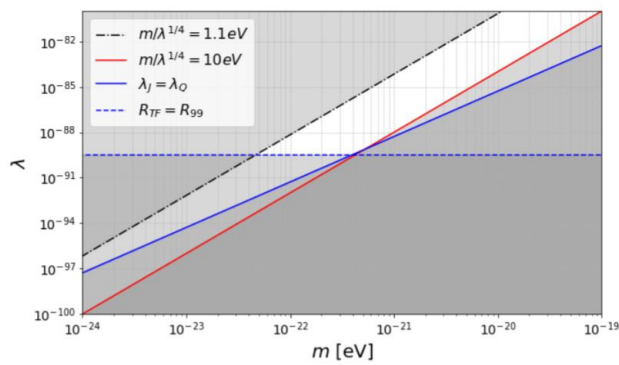
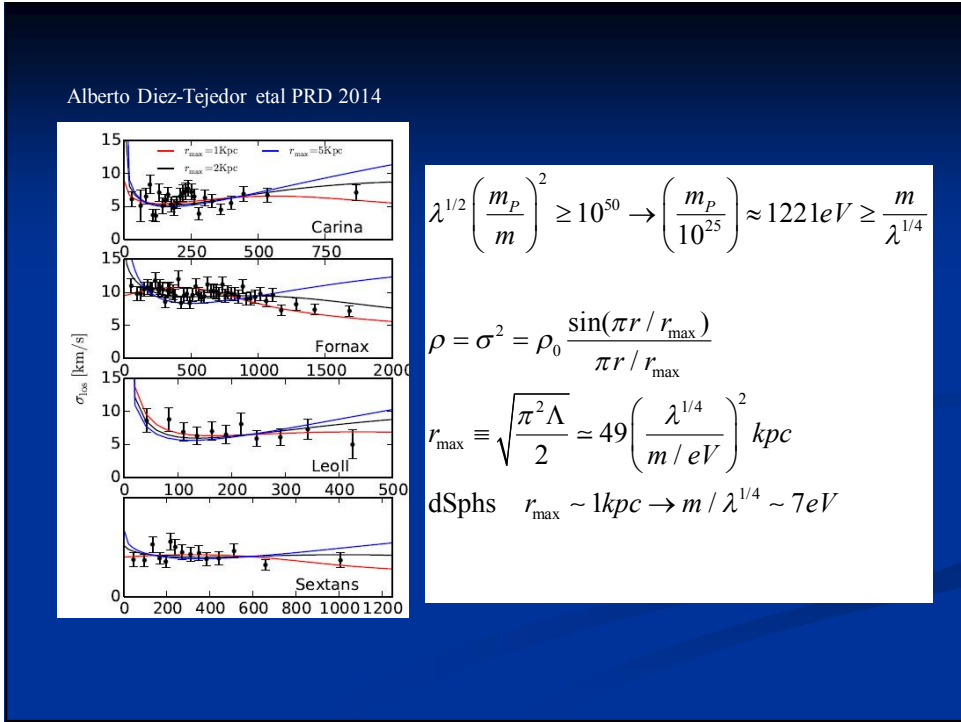
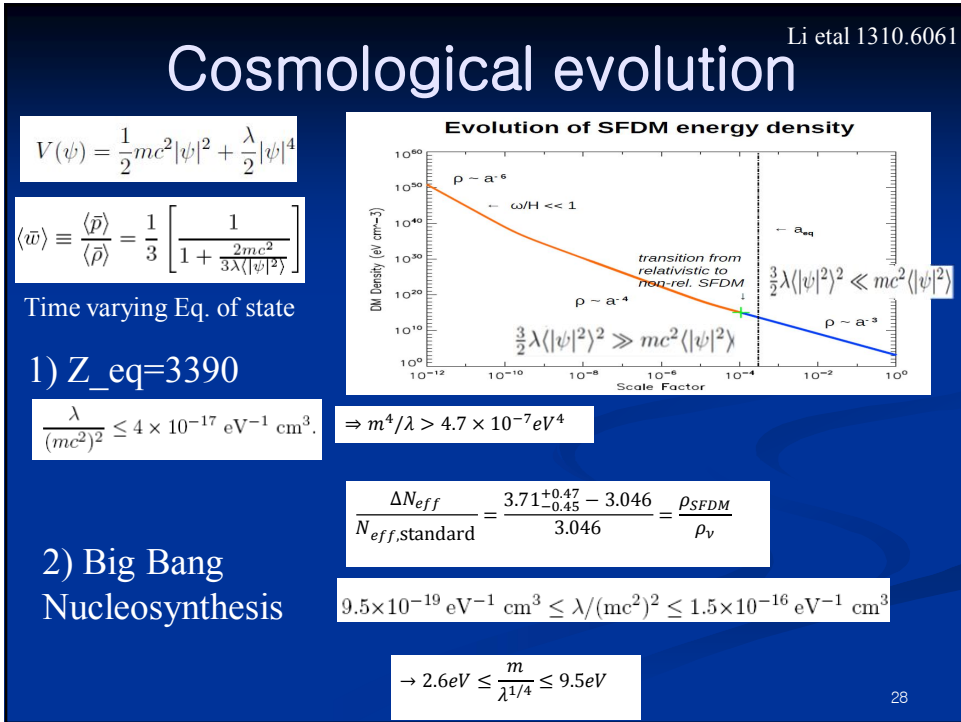


FIG. 2: Cosmological constraints on the mass m and coupling constant λ of ULDM are shown. The gray region denotes the parameter space excluded by current constraints. The dash-dot line corresponds to the bound from the dark matter density Ω_ϕ (Eq. (36)). The red solid line indicates the observational constraint from galaxies, $\bar{m} \lesssim 10 \text{ eV}$. The blue line marks the boundary where the condition $\lambda_J > \lambda_Q$ (Eq. (23)) holds at redshift $z = 100$. The dashed horizontal line represents the condition $R_{TF} > R_{99}$ for a soliton mass of $M = 10^8 M_\odot$.

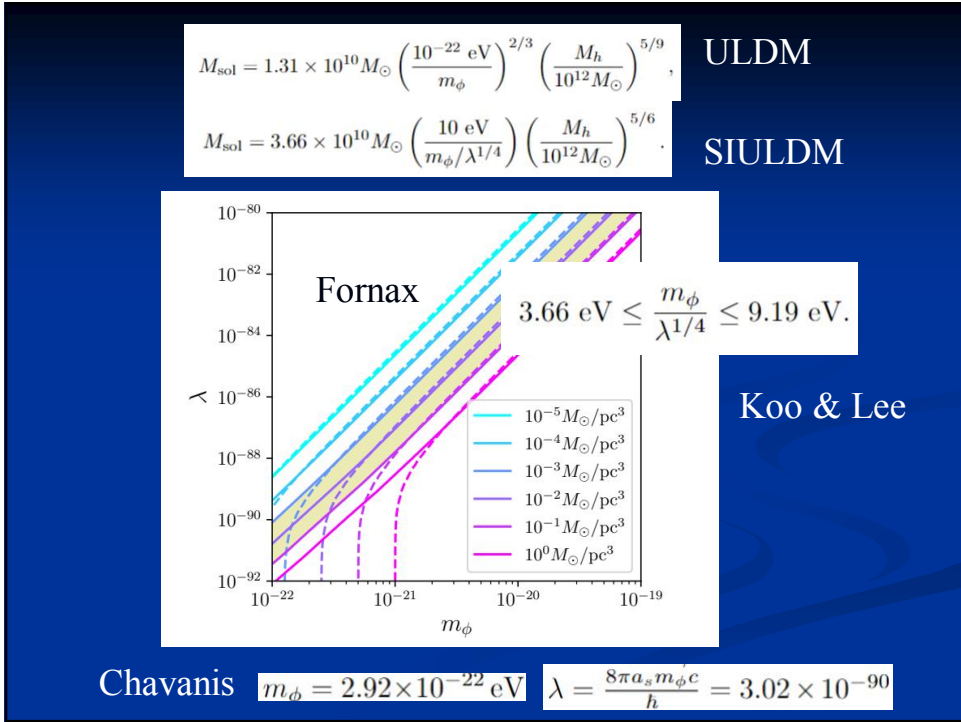
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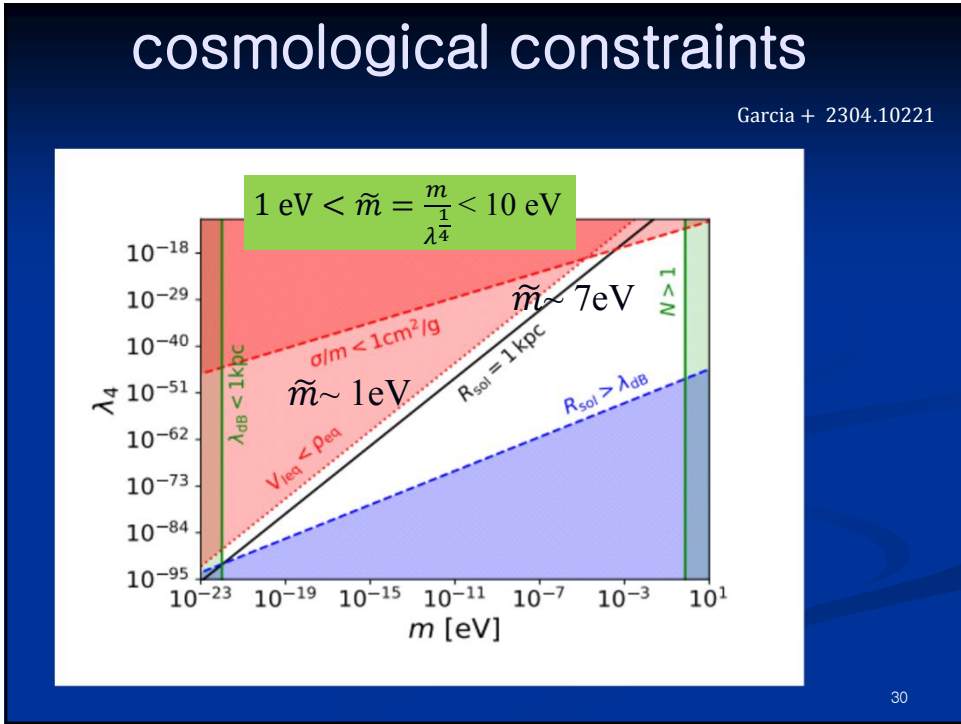
27



28



29



30

ULDM detection


ULDM makes coherent waves → if coupled to EM field
 → change in effective coupling constants (fine structure)
 → oscillation in frequency

sinusoidal modulation

$$\frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} + \frac{\varphi d_e}{4g^2} F_{\mu\nu} F^{\mu\nu} \quad \alpha(t) \approx \alpha [1 + d_e \varphi_0 \cos(\omega t + \delta)]$$

$$\rho_\varphi = \frac{1}{2} m^2 \varphi^2 = 0.3 \text{ GeV/cm}^3$$

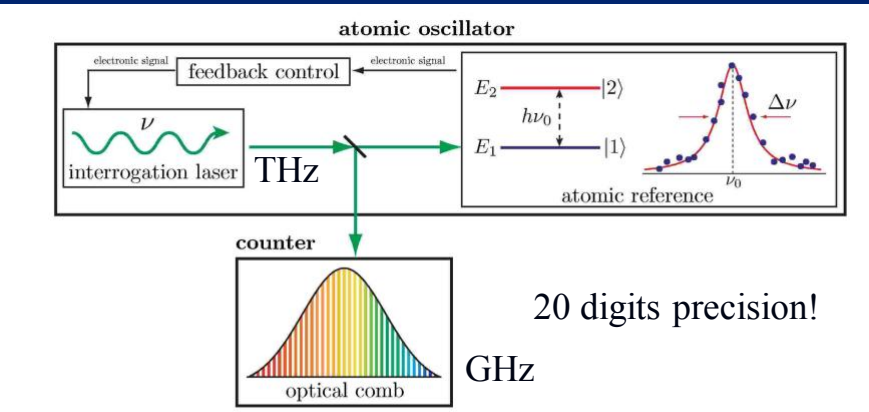
similar to 5th force

$$\omega = \frac{1}{2.5 \text{ months}} \frac{m}{10^{-22} \text{ eV}}$$


Arvanitaki et al., PRD 91

31

detection by atomic clock



atomic oscillator

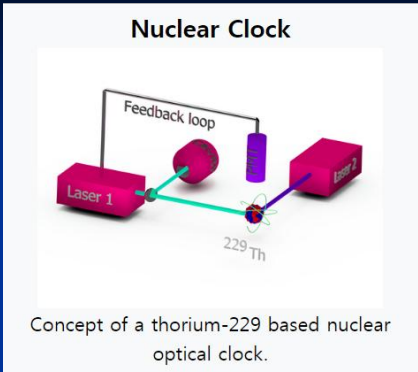
20 digits precision!
GHz

Fig. 2. – Schematic view of an optical atomic clock: the local oscillator (laser) is resonant with the atomic transition. A correction signal is derived from atomic spectroscopy that is fed back to the laser. An optical frequency synthesizer (optical frequency comb) is used to divide the optical frequency down to countable microwave or radio frequency signals.

arXiv:1401.2378v2

32

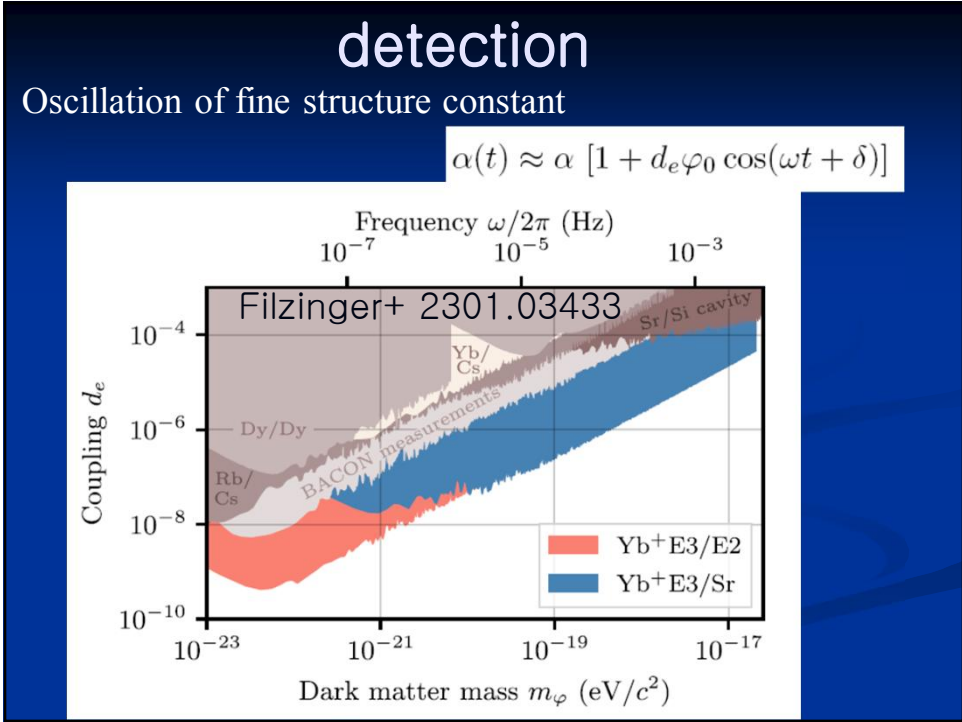
Nuclear Clock



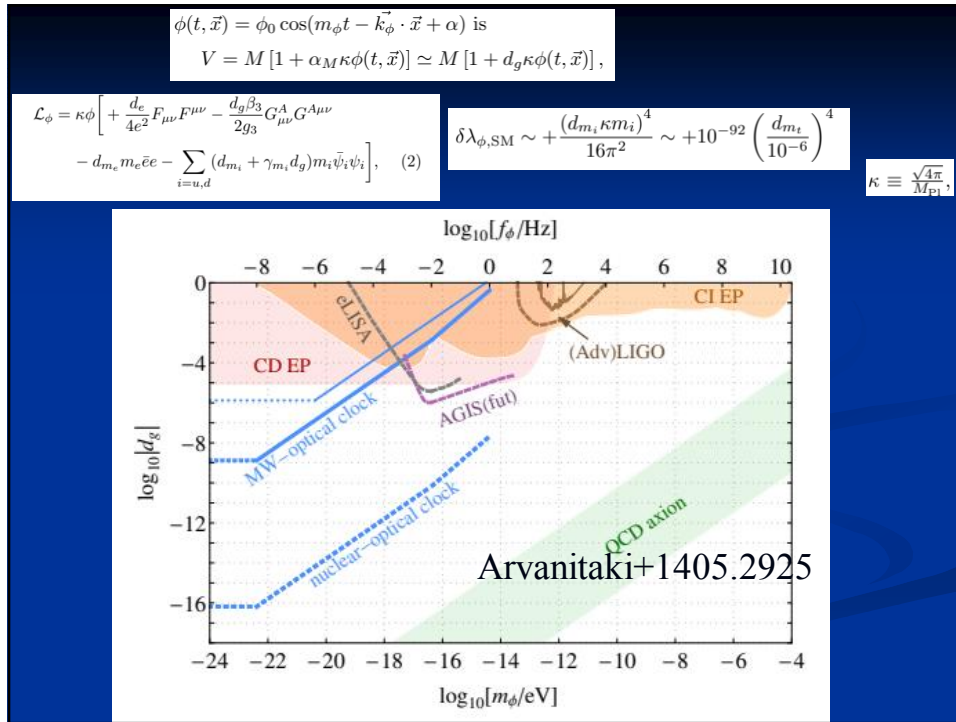
Concept of a thorium-229 based nuclear optical clock.

Thorium-229's uniquely low-energy nuclear isomer transition at about 8.3 eV (≈ 150 nm, $\sim 2 \times 10^{15}$ Hz) makes it the only nuclear excitation accessible to laser spectroscopy and thus a prime candidate for a nuclear optical clock.

33



34




35

Cross section

$\sigma(\phi\phi \rightarrow \phi\phi) = \lambda^2 / 128\pi m^2$

$\frac{\sigma}{m} = \frac{\lambda^2}{128\pi m^3} \approx 1 \text{ cm}^2 \text{ g}^{-1}$ for galactic cluster



a cosmological constraint

if $m \sim 10^{-22} \text{ eV} \rightarrow \lambda \sim 10^{-92}$

$\frac{m}{\lambda^{1/4}} \sim 0.1 \text{ eV}$

One loop correction

if gauge interaction, $g A_\mu \phi$

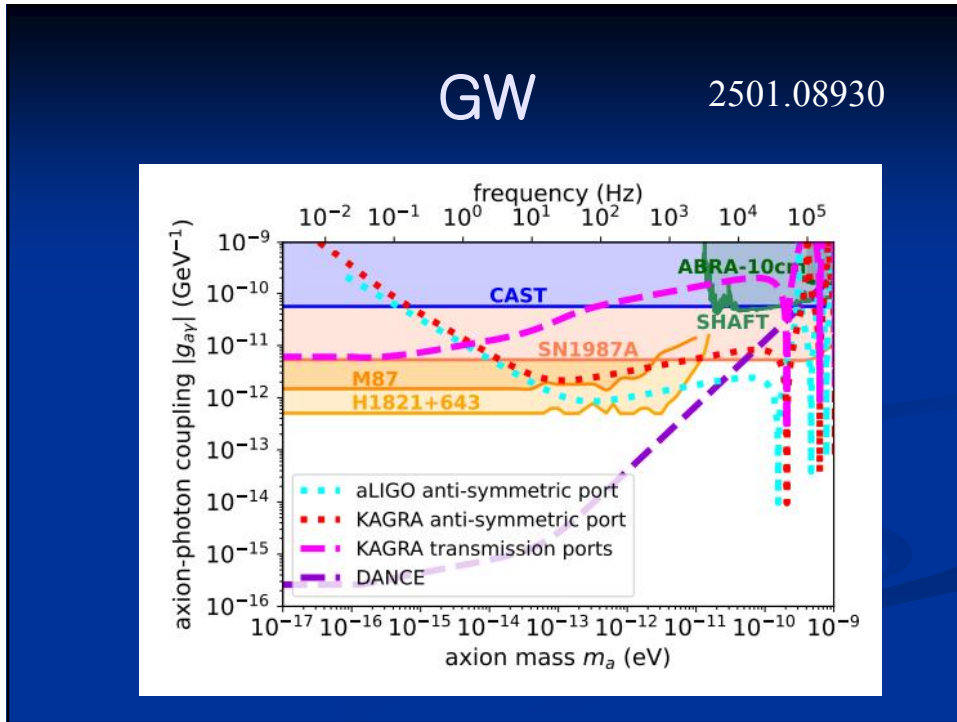
effective self-int. coupling $\lambda' \sim O\left(\frac{3g^4}{64\pi^2}\right)$

$\lambda' < \lambda_{obs} \Rightarrow g \leq 10^{-22.5} \Rightarrow d_e \sim g^2 \leq 10^{-45}$

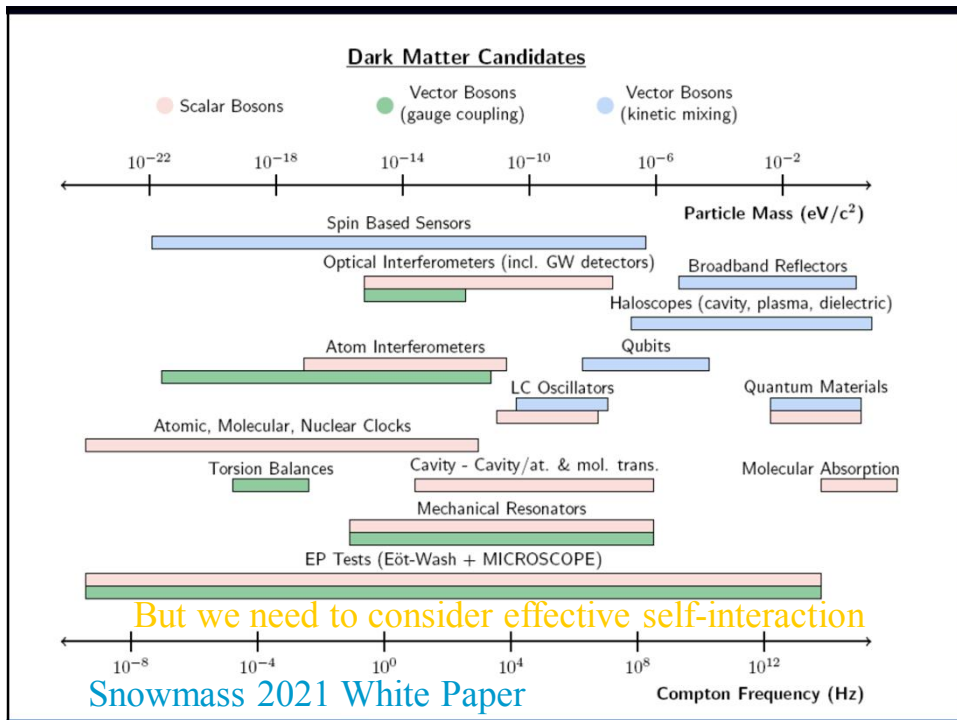
- gauge coupling should be extremely small
- hard to direct detect, if not impossible

in preparation 36

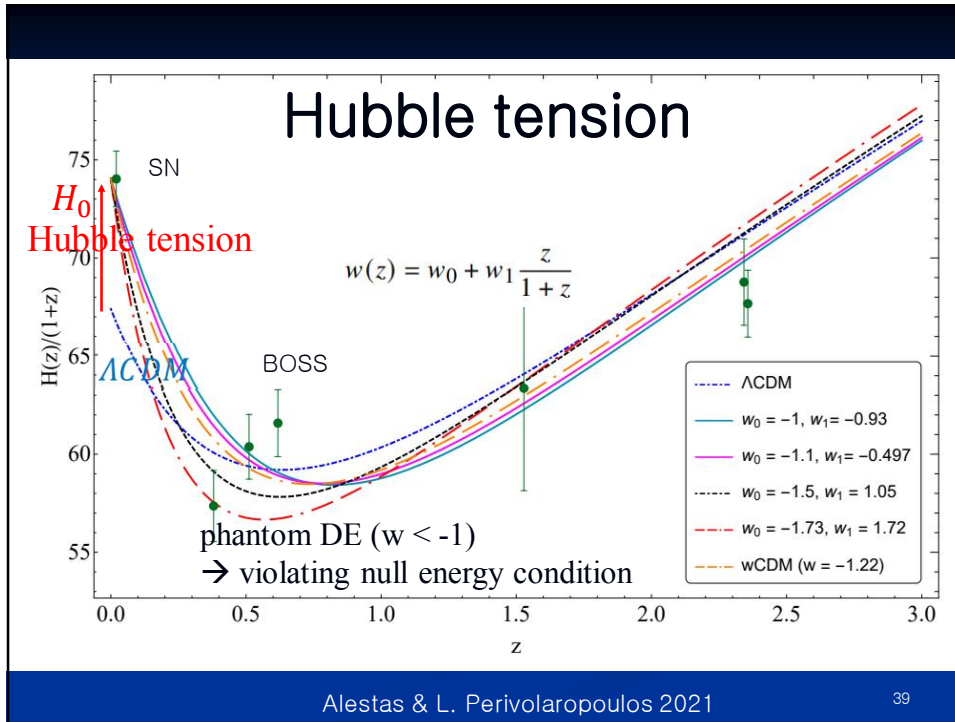
36



37



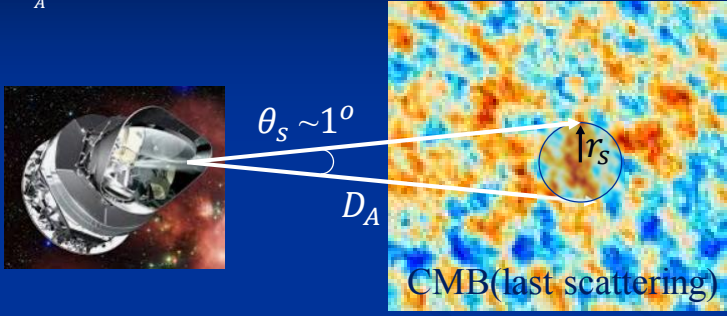
38



39

H_0 estimation from CMB

$\theta_s = \frac{r_s}{D_A}$ is a standard ruler with 0.04% precision. (fixed)



Sound Horizon

$$r_s = \frac{c}{\sqrt{3}H_{ls}} \int_{z_{ls}}^{\infty} \frac{dz}{[\rho(z)/\rho(z_{ls})]^{1/2} (1+R(z))^{1/2}}$$

Angular distance

$$D_A = \frac{c}{H_0} \int_0^{z_{ls}} \frac{dz}{[\rho(z)/\rho_0]^{1/2}} \sim \frac{1}{H_0}$$

r_s & D_A depend on expansion history (matter contents $\rho(z)$)

40

$$H_0 = \theta_s H_{ls} \frac{\int_{z_{ls}}^0 \frac{c dz}{[\rho(z)/\rho_0]^{1/2}}}{\int_{\infty}^{z_{ls}} \frac{c_s(z) dz}{[\rho(z)/\rho(z_{ls})]^{1/2}}}$$

late time (decrease $\rho(z)$)

early time (increase $\rho(z)$)

To increase H_0 we can ($\theta_s = \frac{r_s}{D_A}$ fixed)

- in early time solutions (decrease r_s to decrease D_A)
 - increase $\rho(z)$ (extra radiation, EDE) **just before ls**
 - but need more perturbation
 - worsen S_8 tension, ad hoc matter, coincidence?
- in late time solutions (r_s & D_A fixed, increase the integral)
 - decrease $\rho(z)$ by decaying DM after ls and/or increase DE to increase the integral
 - late evolution changes
 - tensions with BAO, SN; null energy violation; fine tuning

$$D_A = \frac{c}{H_0} \int_0^{z_{ls}} \frac{dz}{[\rho(z)/\rho_0]^{1/2}}$$

Do we need both solutions?

41

Our new proposal

$V(\phi) = \frac{m^2}{2} |\phi|^2 + \frac{\lambda}{4} |\phi|^4$

ϕ

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$V(\phi) \propto \phi^n$ decays with an equation of state $w = \frac{(n-2)}{(n+2)}$

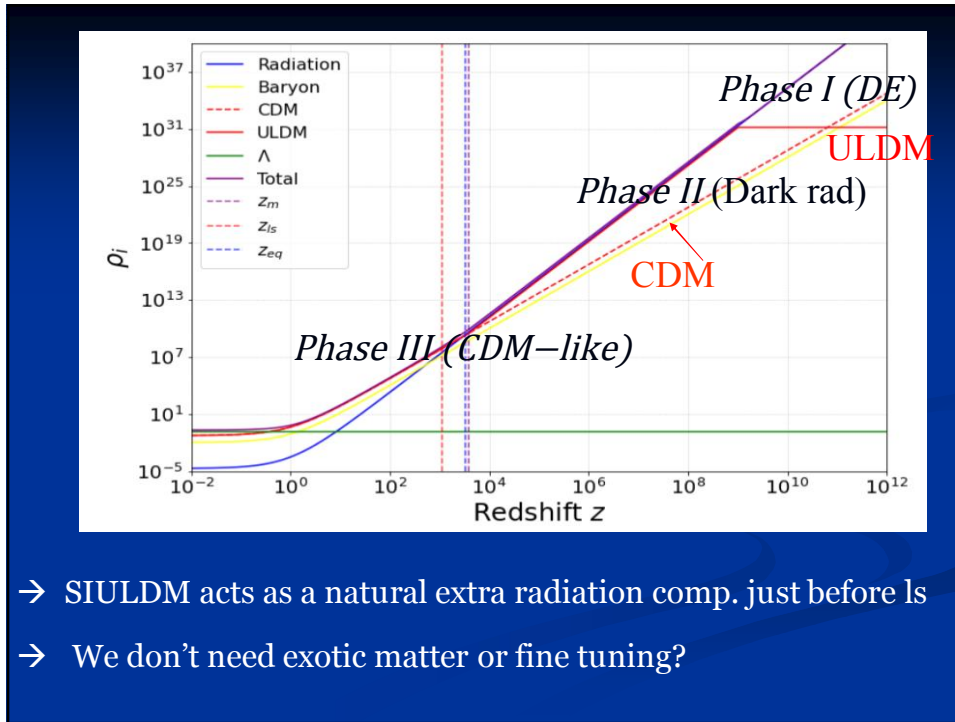
DE-like (slow-roll) for $z_{osc} < z$ (Phase I)

Dark rad-like ($\phi^4 > \phi^2$) or $z_m < z < z_{osc}$ (Phase II)

CDM ($\phi^4 < \phi^2$) or $z < z_m$ (Phase III)

SIULDM alone is enough!

42



43

$$\frac{H_0(ULDM)}{H_0(CDM)} \approx \frac{\int_{z_{ls}}^{\infty} \rho(CDM)(z)^{-1/2} \frac{dz}{\sqrt{1+R(z)}}}{\int_{z_{ls}}^{\infty} \rho(ULDM)(z)^{-1/2} \frac{dz}{\sqrt{1+R(z)}}}$$

$\rho(ULDM)(z) \propto \begin{cases} \sqrt{\omega_r(1+z)^4 + \omega_m(1+z)^3 + \omega_\Lambda} & \text{for } z < z_m \text{ (Phase III)} \\ \sqrt{\omega_r(1+z)^4 + \omega_\phi \frac{(1+z)^4}{1+z_m} + \omega_b(1+z)^3 + \omega_\Lambda} & \text{for } z_m < z < z_{osc} \text{ (Phase II)} \\ \sqrt{\omega_r(1+z)^4 + \omega_b(1+z)^3 + \omega_\phi \frac{(1+z_{osc})^4}{1+z_m} + \omega_\Lambda} & \text{for } z_{osc} < z \text{ (Phase I)} \end{cases}$

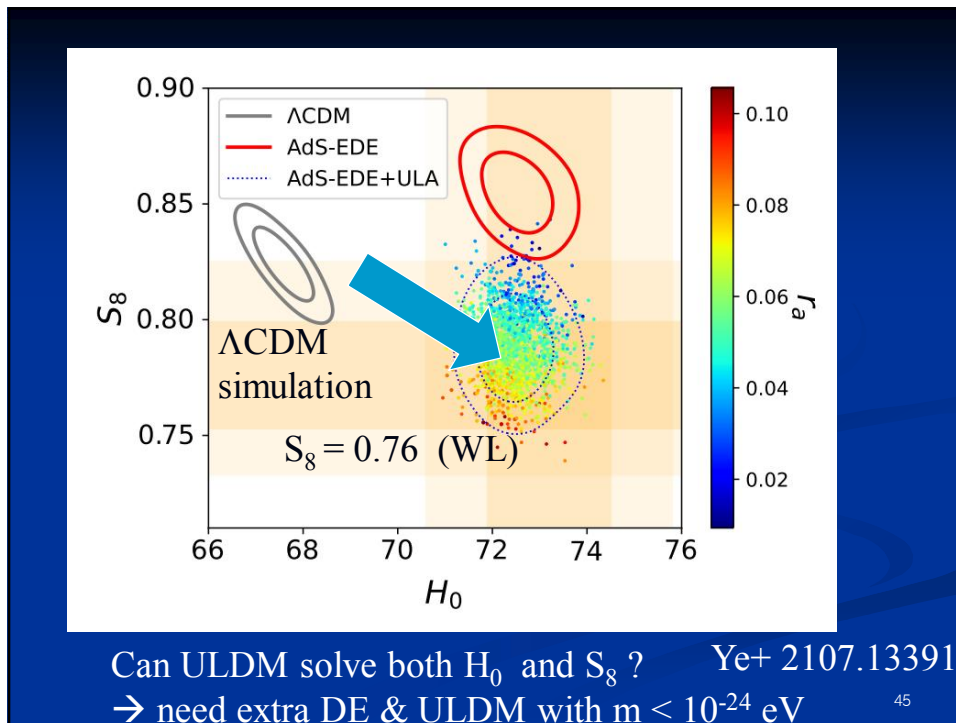
$\omega_i = h^2 \Omega_i$

where $\omega_b = 0.022, \omega_\phi = 0.119, \omega_r = 4.2 \times 10^{-5}, \omega_\Lambda = 0.314$ from Planck's Λ CDM values

Using the densities, we numerically obtain $\frac{H_0(ULDM)}{H_0(CDM)} \approx 1.049$ for $\tilde{m} = 0.9$

* \tilde{m} is near lower bound, input data assumed Λ CDM, age problem?

44



45

Conclusions

ULDM with $m \sim 10^{-21}$ eV or

self-interacting ULDM with $\frac{m}{\lambda^{1/4}} \sim 1$ eV

seems to be a viable alternative to CDM

46