Interpolating Scattering Amplitudes between Instant and Light-Front Dynamics

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APCTP SEMINAR

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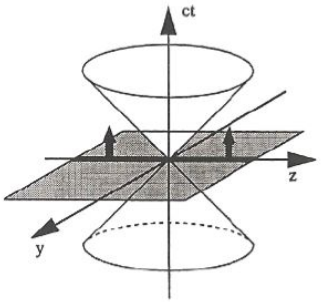
Outline

- Dirac's Proposition for Relativistic Dynamics
- Distinguished Features of LFD
- Scattering amplitudes linked between IFD and LFD
- Interpolating QED helicity scattering amplitudes
- QCD(1+1) in large Nc ('tHooft Model) interpolation
- Quasi-PDFs in IFD vs. PDFs in LFD
- Chiral effective convolution with meson PDFs
- Summary and Outlook

Dirac's Proposition for Relativistic Dynamics

$k_1^+ = k_2^+ = 0$



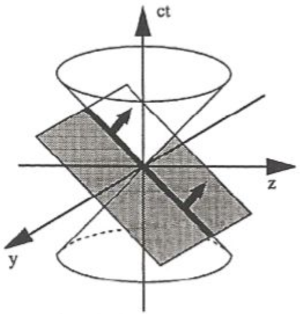
The instant form

Equal t **1949** *Equal τ*

$$p^0 \leftrightarrow p^- = p^0 - p^3$$

$$(p^1, p^2) \leftrightarrow \vec{p}_\perp$$

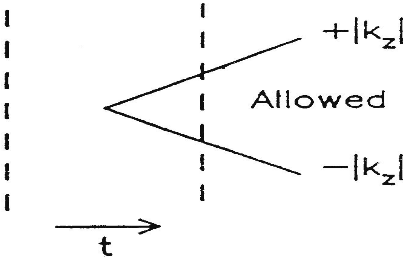
$$p^3 \leftrightarrow p^+ = p^0 + p^3$$



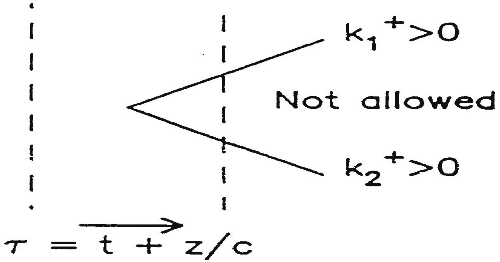
The front form

Energy-Momentum Dispersion Relations

$$p^0 = \sqrt{\vec{p}^{\,2} + m^2} \qquad p^- = \frac{\vec{p}_\perp^{\,2} + m^2}{p^+}$$



(a)



(b)

Except zero-modes

$k_1^+ = k_2^+ = 0$

IFD

Instant Form Dynamics

LFD

Light-Front Dynamics

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Quantum simulation: Cracking the exponential wall

Michael Kreshchuk, LBNL

September 23, 2022

Quantum Computers and Quantum Computation

Quantum Computer:
A highly controllable quantum system, which naturally stores superpositions of quantum states.

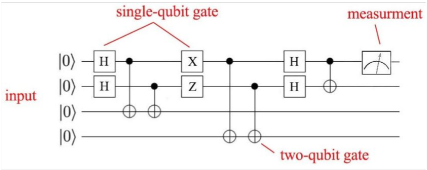
Basic example:

$$|\Psi_{phys}\rangle = \alpha|\uparrow\uparrow\downarrow\uparrow\downarrow \cdots\rangle + \beta|\uparrow\downarrow\downarrow\uparrow\uparrow \cdots\rangle + \cdots$$
$$|\Psi_{qubit}\rangle = \alpha|11010 \ldots\rangle + \beta|10011 \ldots\rangle + \cdots$$

$|\Psi_{phys}\rangle$ — state of a spin chain, state of a molecule in the second-quantized formalism, ect.

$|\Psi_{qubit}\rangle$ — state in the quantum computer.

Depending on hardware, one can implement 1-, 2-, or many-qubit gates, which are used to manipulate $|\Psi_{qubit}\rangle$ with local or non-local elementary operations.



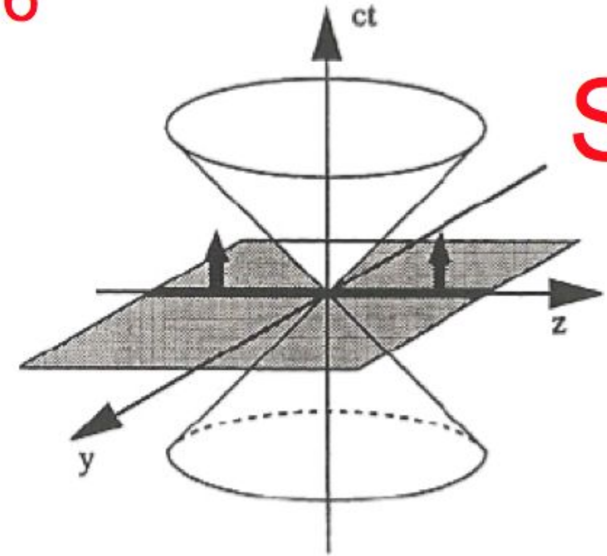
Quantum Simulation of LF QFT

For several reasons LF QFT is highly appealing as starting point for quantum simulation:

LF QFT Features	Advantages for Quantum Simulation
Linear EoM → few DOFs	Lower qubit count*
LF momentum > 0 → few DOFs are occupied	Lower qubit count**
Efficient basis choice → early truncation	Lower qubit count
Observables are easy to extract from the LFWF	Measurements are easy to design
Trivial vacuum	Good initial state is readily available,
Valence sector calculations give good results	and this state is easy to prepare

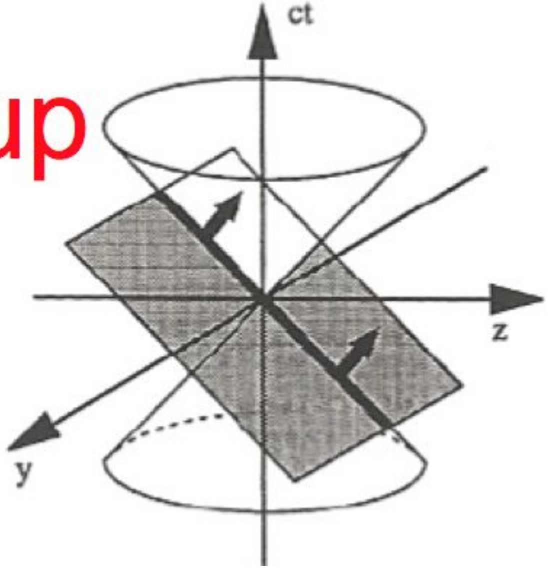
How many generators leave the time surface invariant?

6



Stability Group

7



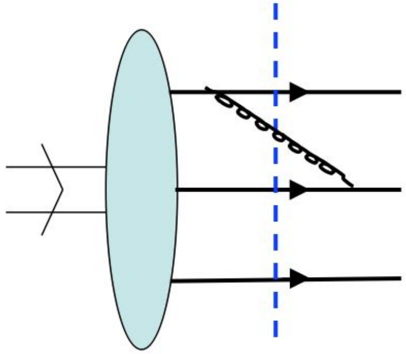
IFD

Instant Form Dynamics

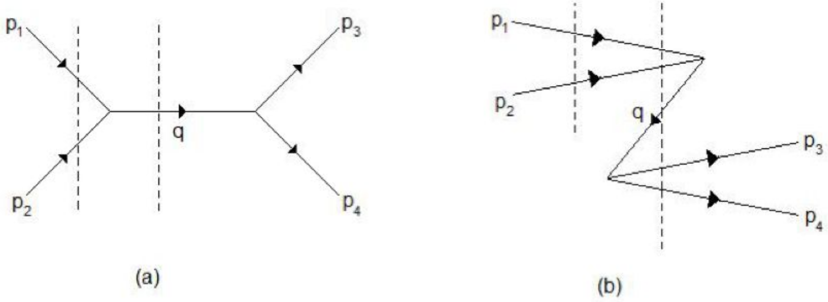
LFD (maximum)

Light-Front Dynamics

Physical Meaning of Stability Group



Equal-time Wavefunction

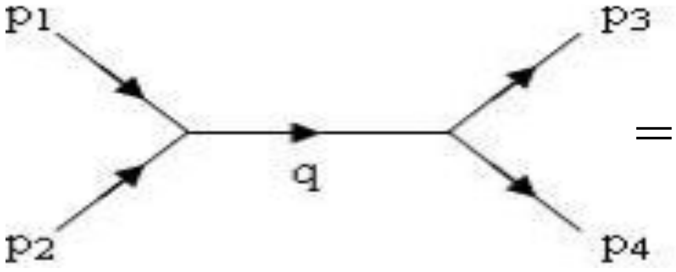


Time-ordered Scattering Amplitudes

Invariant under Stability Group Elements

Kinematic Transformations

"$e^+e^- \to \mu^+\mu^-$"



$$= \frac{1}{q^2 - m^2} = \frac{1}{s - m^2} \qquad q^2 = (p_1 + p_2)^2 \neq m^2$$

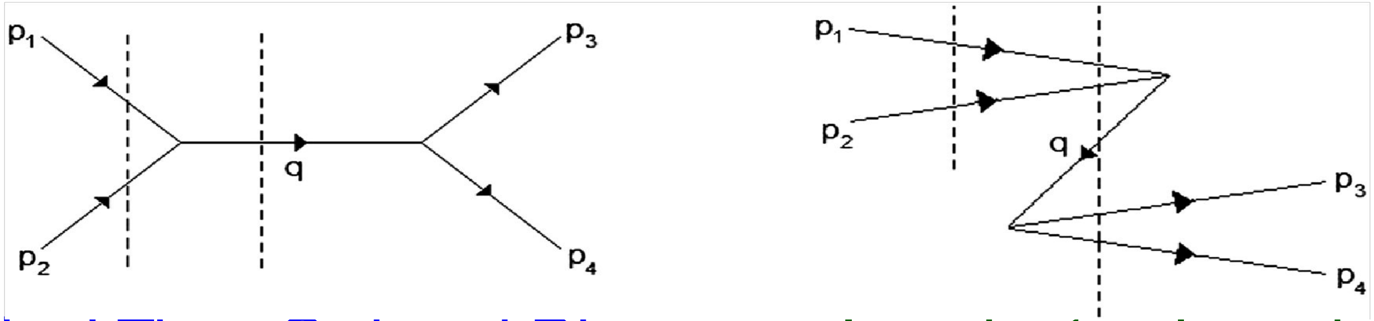
Four-momentum conservation but off-mass-shell

Feynman Diagram: Invariant under all 10 Poincaré generators

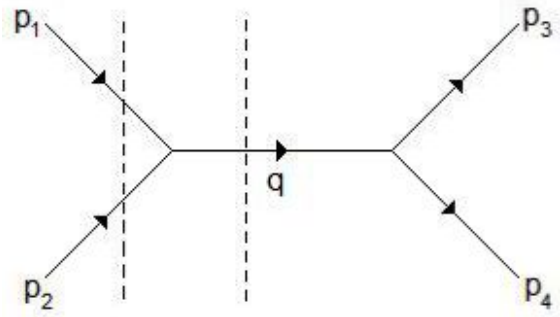
t⟶ (time evolution; time ordered process in QFT; Energy is not conserved within Δt)

$(\Delta E)(\Delta t) \sim \hbar$

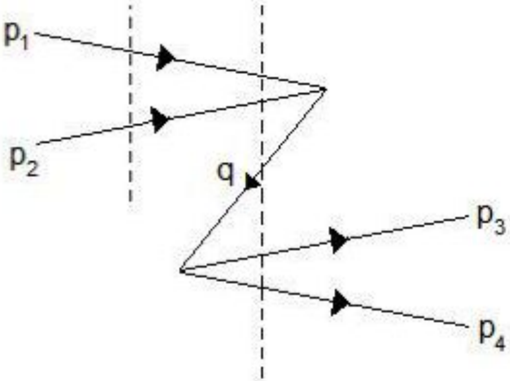
Three-momentum conservation but on-mass-shell



Individual Time-Ordered Diagrams: Invariant only under translation and rotation (6 kinematic generators)



(a)



(b)

$$\Sigma^a_{\rm IFD} = \frac{1}{2q^0}\left(\frac{1}{p_1^0+p_2^0-q^0}\right)$$

$$\Sigma^b_{\rm IFD} = -\frac{1}{2q^0}\left(\frac{1}{p_1^0+p_2^0+q^0}\right)$$

$$\Sigma^a_{IFD}+\Sigma^b_{IFD} = \frac{1}{2q^0}\left(\frac{1}{p_1^0+p_2^0-q^0}-\frac{1}{p_1^0+p_2^0+q^0}\right)$$

$$= \frac{1}{(p_1^0+p_2^0)^2-(q^0)^2}$$

$$= \frac{1}{\{(p_1^0+p_2^0)^2-(\vec{p}_1+\vec{p}_2)^2\}-\{(q^0)^2-\vec{q}^{\,2}\}}$$

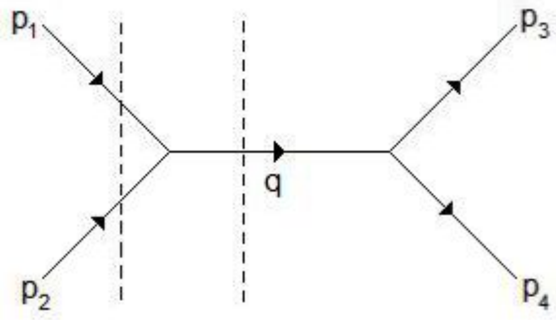
: Three-momentum conservation

$$= \frac{1}{(p_1+p_2)^2-q^2}$$

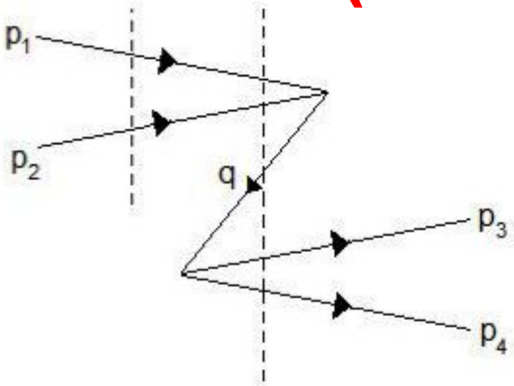
: $q^2 = m^2$; on -mass-shell

$$= \frac{1}{s-m^2}$$

Infinite Momentum Frame (IMF) Approach



(a)



(b)

$$\frac{1}{E_1 + E_2 - Eq}$$

$$-\frac{1}{Eq + E_3 + E_4}$$

$$= -\frac{1}{Eq + E_1 + E_2}$$

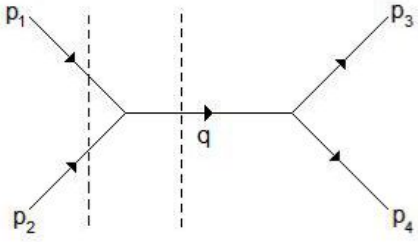
$$\rightarrow \quad 0$$

S.Weinberg, PR158,1638(1967)
"Dynamics at Infinite Momentum"

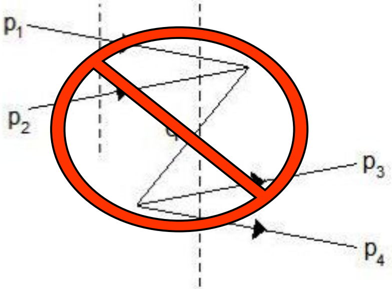
Note that this is still in the instant form (IFD).

However, in LFD, (b) drops for any reference frame (not just for IMF)

τ (= t+z/c) →



(a)



(b)

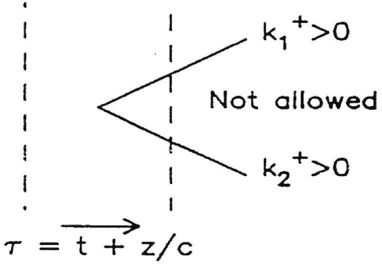
$$\Sigma^a_{LFD} + \Sigma^b_{LFD} = \frac{1}{q^+}\left(\frac{1}{p_1^- + p_2^- - q^-} + 0\right)$$

$$= \frac{1}{q^+\left(\frac{(p_1+p_2)^2 + (\vec{p}_{1\perp} + \vec{p}_{2\perp})^2}{(p_1+p_2)^+} - \frac{m^2 + \vec{q}_\perp^{\,2}}{q^+}\right)}$$

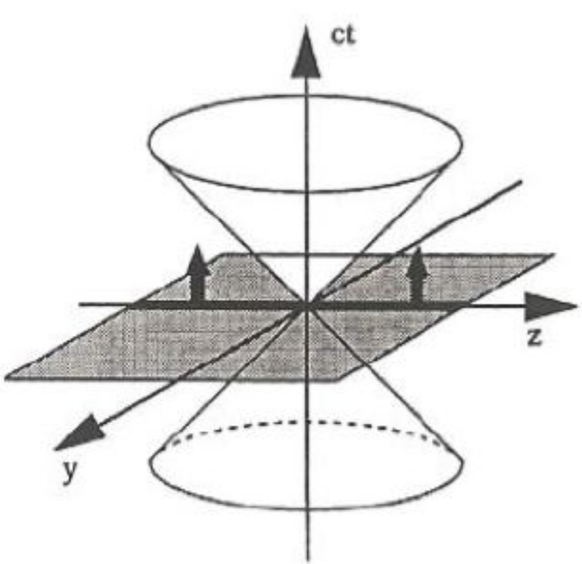
$$= \frac{1}{(p_1+p_2)^2 - m^2}$$

$$= \frac{1}{s - m^2}$$

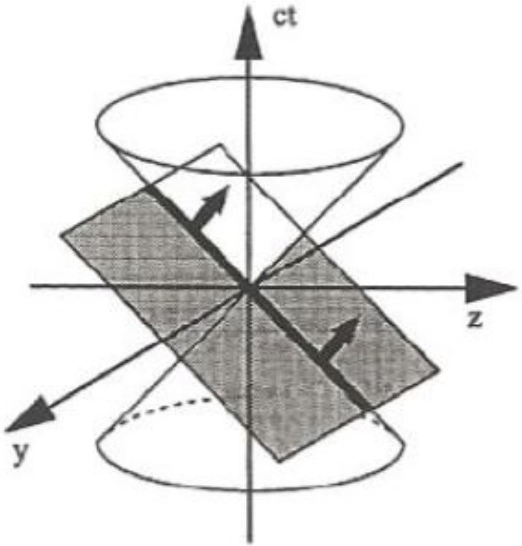
$$p^- = \frac{\vec{p}_\perp^{\,2} + m^2}{p^+}$$



Relativistic Quantum Invariance



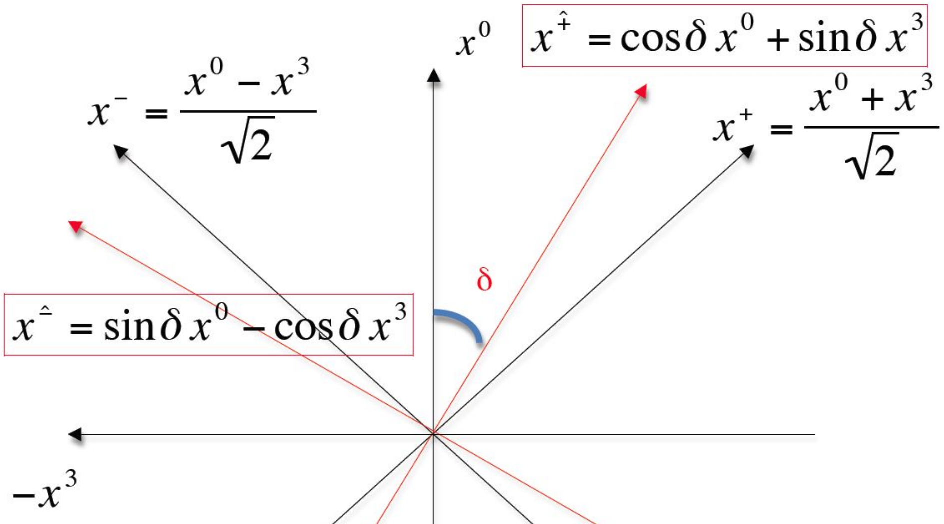
The instant form



The front form

Can IFD and LFD be linked?

Interpolation between IFD and LFD



$$(IFD)\quad 0 \le \delta \le \frac{\pi}{4} \quad (LFD)$$

$$1 \ge \quad C \equiv \cos(2\delta) \quad \ge 0$$

K. Hornbostel, PRD45, 3781 (1992) – RQFT

C.Ji and S.Rey, PRD53,5815(1996) – Chiral Anomaly

C.Ji and C. Mitchell, PRD64,085013 (2001) – Poincare Algebra

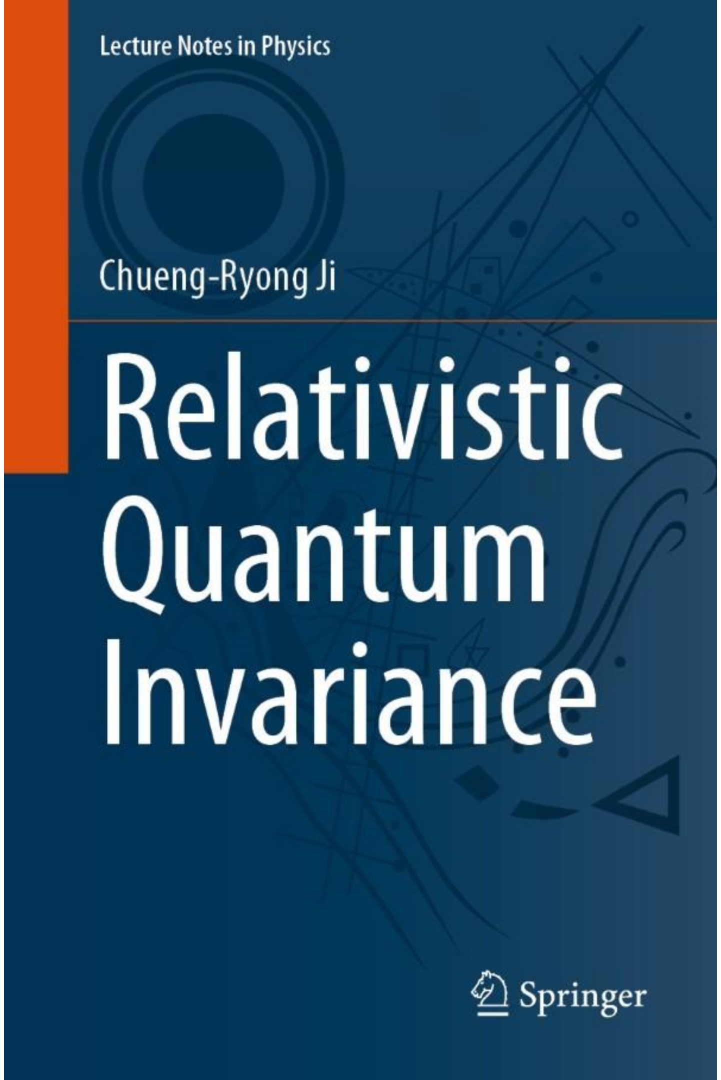
C.Ji and A. Suzuki, PRD87,065015 (2013) – Scattering Amps

C.Ji, Z. Li and A. Suzuki, PRD91, 065020 (2015) – EM Gauges

Z.Li, M. An and C.Ji, PRD92, 105014 (2015) – Spinors

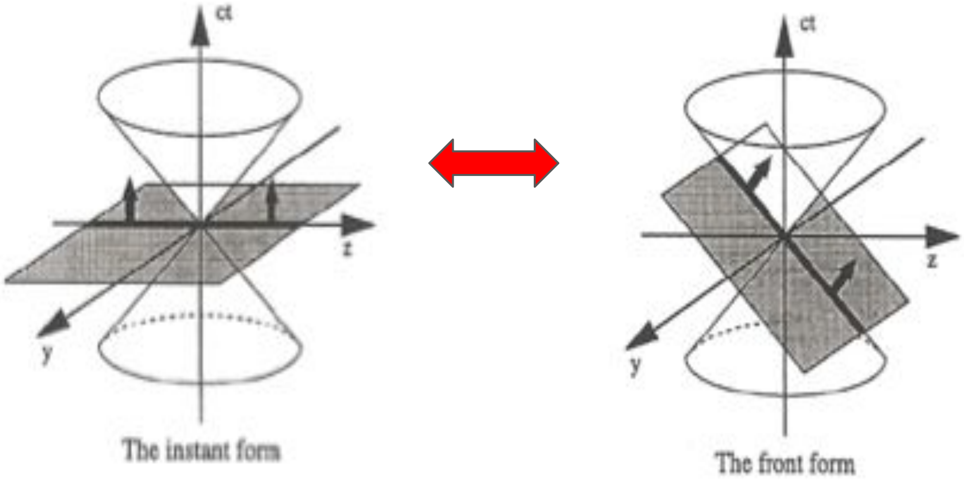
C.Ji, Z.Li, B.Ma and A.Suzuki, PRD98, 036017(2018) – QED

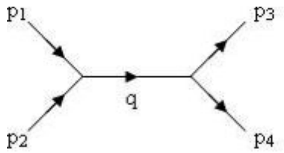
B.Ma and C.Ji, PRD104, 036004(2021) – QCD_{1+1}



Lecture Notes in Physics (LNP, Vol. 1012), Springer Nature (2023).

Interpolation between IFD and LFD

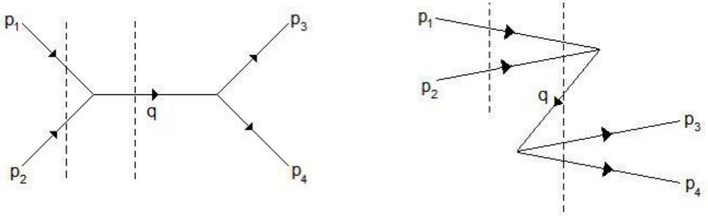




$$\delta = 0$$
$$p_0 = p^0$$
$$-p_3 = p^3$$

$$0 < \delta < \pi/4$$
$$p_{\hat{+}} = p^0 \cos\delta - p^3 \sin\delta$$
$$p_{\hat{-}} = p^0 \sin\delta + p^3 \cos\delta$$

$$\delta = \pi/4$$
$$p_+ = p^-$$
$$p_- = p^+$$



(a) (b)

$$\frac{1}{2q^0}\left(\frac{1}{p_1^0 + p_2^0 - q^0} - \frac{1}{p_1^0 + p_2^0 + q^0}\right)$$

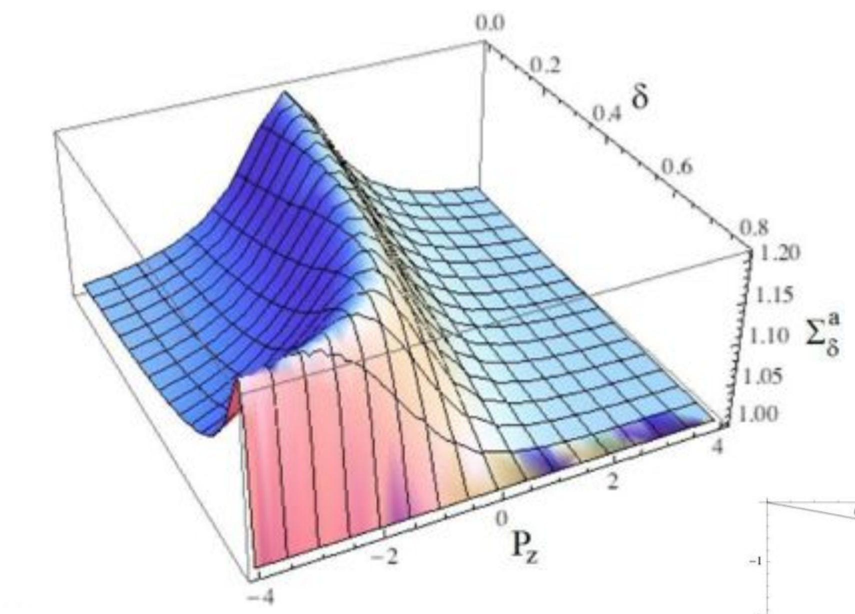
$$\frac{1}{2\omega_q}\left(\frac{1}{P_{\hat{+}} + \frac{\mathbb{S}q_{\hat{-}} - \omega_q}{\mathbb{C}}} - \frac{1}{P_{\hat{+}} + \frac{\mathbb{S}q_{\hat{-}} + \omega_q}{\mathbb{C}}}\right)$$

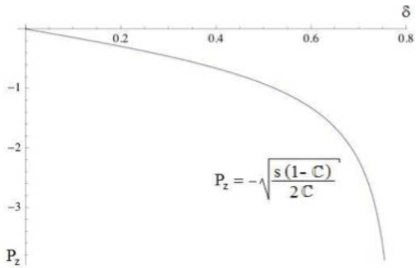
$$\omega_q = \sqrt{q_{\hat{-}}^2 + \mathbb{C}\left(\vec{q}_\perp^{\,2} + m^2\right)}$$

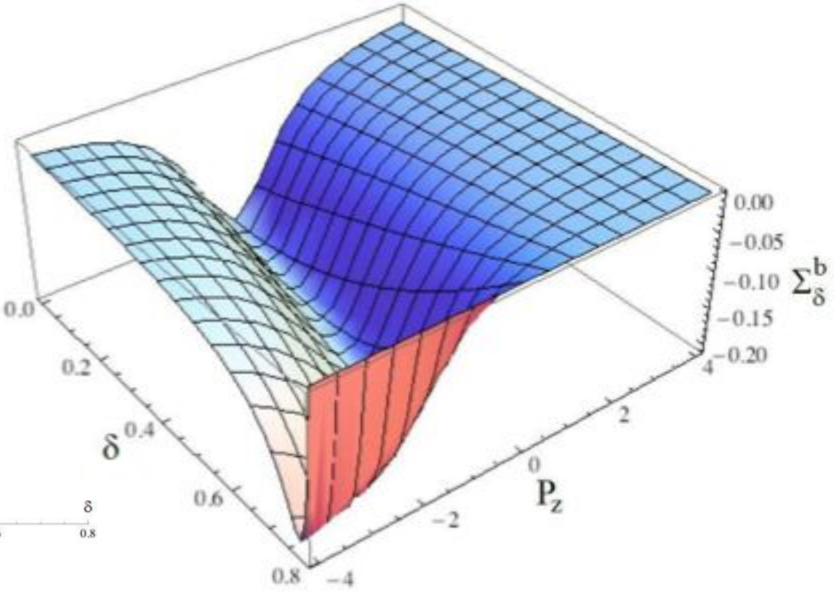
$$\mathbb{C} = \cos 2\delta$$
$$\mathbb{S} = \sin 2\delta$$

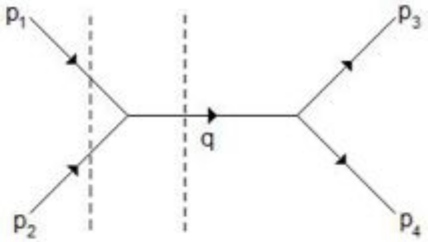
$$\frac{1}{P^+}\frac{1}{\left\{P^- - \frac{(\vec{P}_\perp^2 + m^2)}{2P^+}\right\}}$$

$$\frac{\mathbb{S}q_{\hat{-}} + \omega_q}{\mathbb{C}} \to \frac{2}{\mathbb{C}} - \frac{\vec{q}_\perp^{\,2} + m^2}{2q_{\hat{-}}} + \mathcal{O}(\mathbb{C})$$
$$\to \infty \quad \text{as } \mathbb{C} \to 0$$





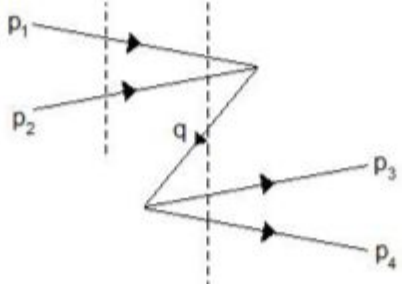




$\Sigma(a)+\Sigma(b)=1/(s-m^2)$; s=2 GeV^2, m=1GeV

J-shape peak & valley : $P_z = -\sqrt{\frac{s(1-C)}{2C}}$; $C = \cos(2\delta)$

As C → 0, $P^+ = P^0+P_z$ →0 leads to LF Zero-modes.



$$A^{\hat{+}} = 0, \ \partial_{\hat{-}} A_{\hat{-}} + \boldsymbol{\partial}_\perp \mathbf{A}_\perp \mathbb{C} = 0 \quad (\mathbb{C} = \cos 2\delta)$$

$\delta \to 0$ $(\mathbb{C} \to 1)$

$\delta \to \pi/4$ $(\mathbb{C} \to 0)$

C.Ji, Z. Li, and A. T. Suzuki, PRD91, 065020(2015)

$$A^0 = 0, \ \nabla \cdot \mathbf{A} = 0$$

Coulomb Gauge

$$A^+ = 0$$

Light-front Gauge

$$\sum_{\lambda=\pm} \epsilon^*_{\hat{\mu}}(\lambda)\epsilon_{\hat{\nu}}(\lambda) = -g_{\hat{\mu}\hat{\nu}} + \frac{(q\cdot n)(q_{\hat{\mu}} n_{\hat{\nu}} + q_{\hat{\nu}} n_{\hat{\mu}})}{\mathbf{q}_\perp^2 \mathbb{C} + q_{\hat{-}}^2} - \frac{\mathbb{C} q_{\hat{\mu}} q_{\hat{\nu}}}{\mathbf{q}_\perp^2 \mathbb{C} + q_{\hat{-}}^2} - \frac{q^2 n_{\hat{\mu}} n_{\hat{\nu}}}{\mathbf{q}_\perp^2 \mathbb{C} + q_{\hat{-}}^2}$$

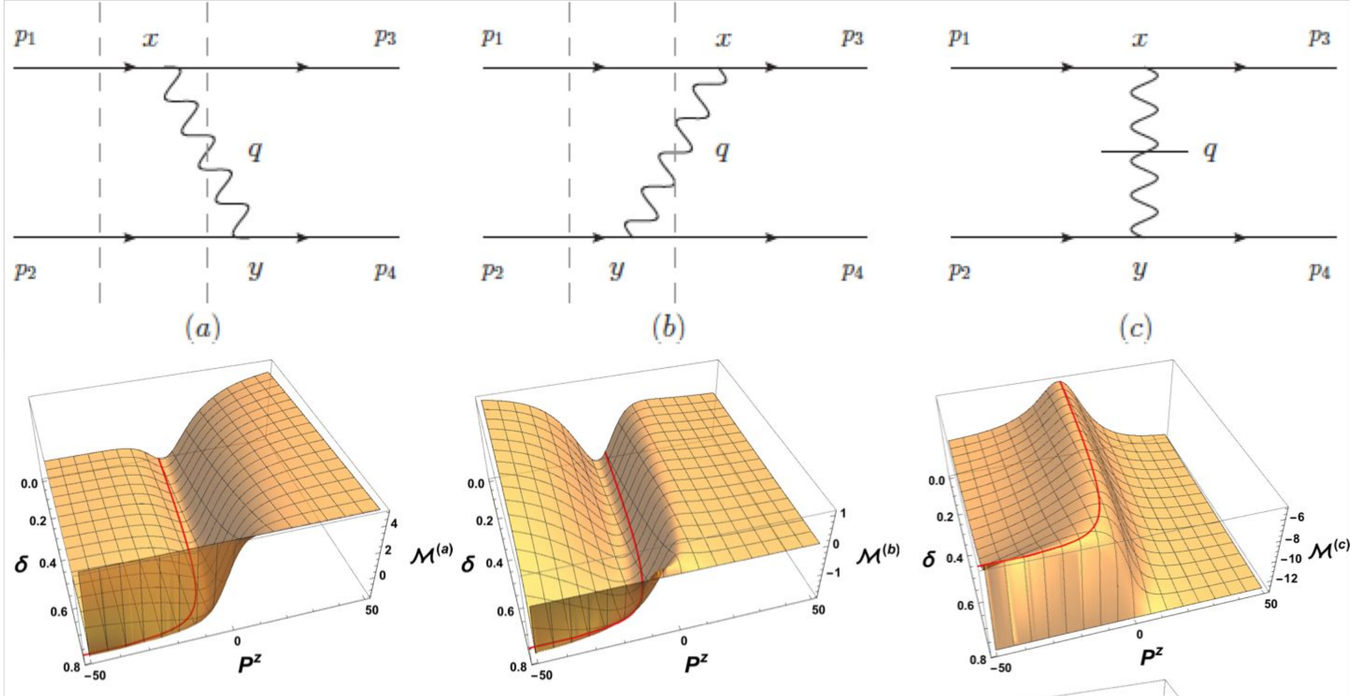
LFD

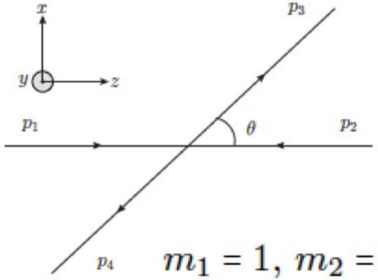
$$-g_{\mu\nu} + \frac{(q\cdot n)(q_\mu n_\nu + q_\nu n_\mu)}{(q\cdot n)^2} - \frac{q^2 n_\mu n_\nu}{(q\cdot n)^2}$$

P.Srivastava and S. Brodsky, PRD64,045006(2001)

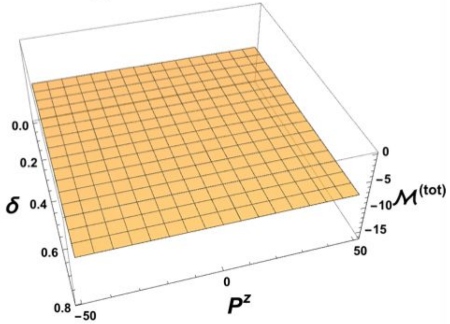
IFD

$$-\eta_{\mu\nu} + \frac{(q\cdot n)(q_\mu n_\nu + q_\nu n_\mu)}{(q\cdot n)^2 - q^2} - \frac{q_\mu q_\nu}{(q\cdot n)^2 - q^2} - \frac{q^2 n_\mu n_\nu}{(q\cdot n)^2 - q^2}$$





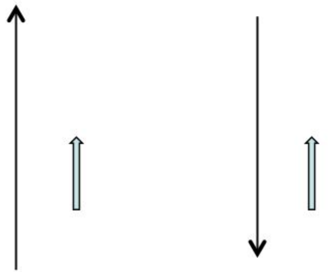
Total amplitude is independent of P^Z and δ as it must be.



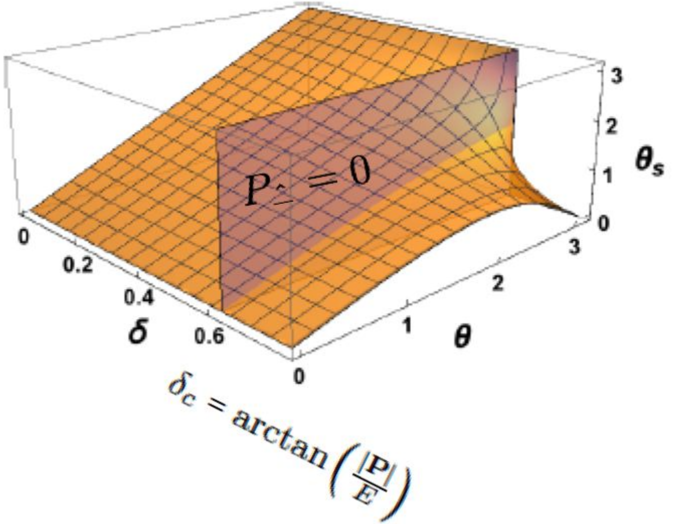
$m_1 = 1$, $m_2 = 2$, $p = 3$, and $\theta = \pi/3$.

Helicity

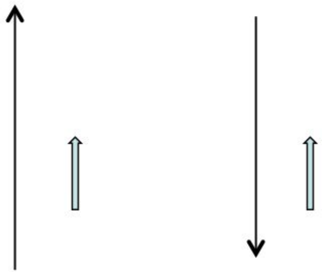
IFD



+1 -1



LFD

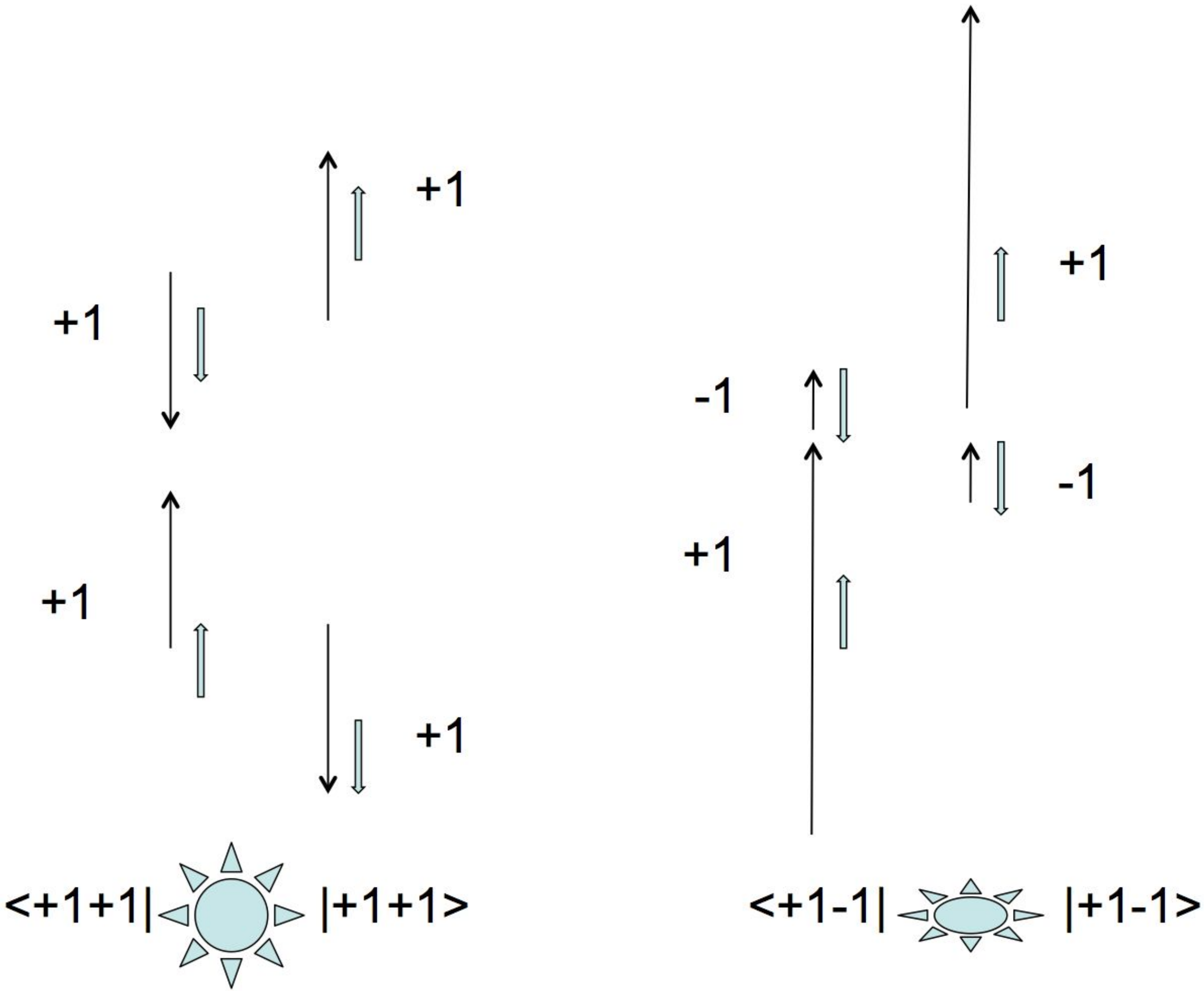


+1 +1

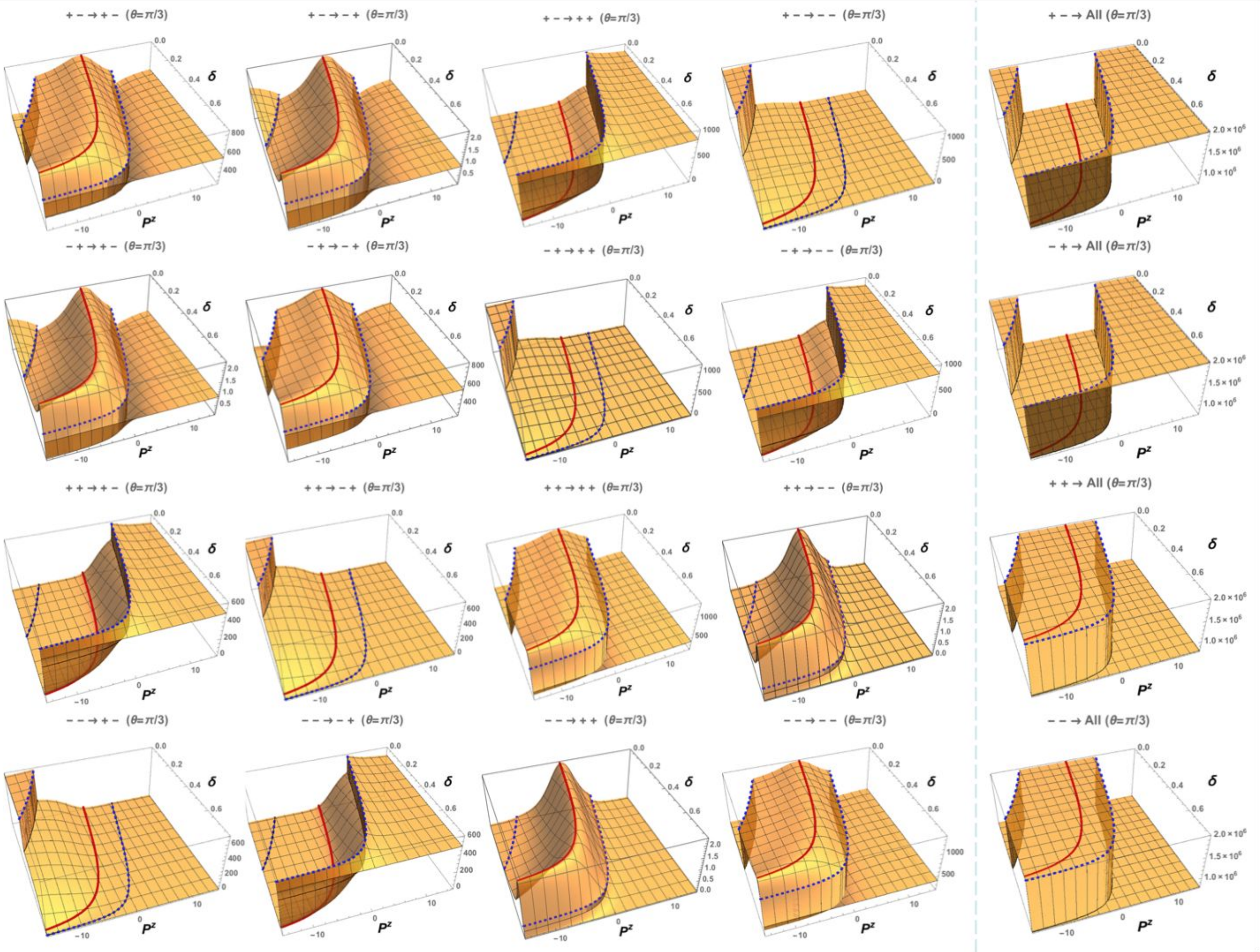
M. Jacob and G. Wick,
Ann. Phys., 7, 404 (1959)

C. Carlson and C.Ji,
PRD, 67, 116002 (2003)

K_z Dependent vs. K_z Independent



+1
+1
+1
+1
<+1+1|
|+1+1>
+1
-1
-1
+1
<+1-1|
|+1-1>


+ − → + − (θ=π/3)
+ − → − + (θ=π/3)
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− − → − + (θ=π/3)
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− − → − − (θ=π/3)
− − → All (θ=π/3)
δ
P^z

Large N_C QCD in 1+1 dim. ('tHooft Model)

$$\mathcal{L} = -\frac{1}{4} F^a_{\hat{\mu}\hat{\nu}} F^{\hat{\mu}\hat{\nu}a} + \bar{\psi}(i\gamma^{\hat{\mu}} D_{\hat{\mu}} - m)\psi$$

$$D_{\hat{\mu}} = \partial_{\hat{\mu}} - igA^a_{\hat{\mu}} t_a$$

$$F^a_{\hat{\mu}\hat{\nu}} = \partial_{\hat{\mu}} A^a_{\hat{\nu}} - \partial_{\hat{\nu}} A^a_{\hat{\mu}} + g f^{abc} A^b_{\hat{\mu}} A^c_{\hat{\nu}}$$

'tHooft Coupling $\lambda = \frac{g^2 (N_c - 1/N_c)}{4\pi}$ and mass m

$$g \rightarrow 0, N_C \rightarrow \infty; \lambda \rightarrow finite$$

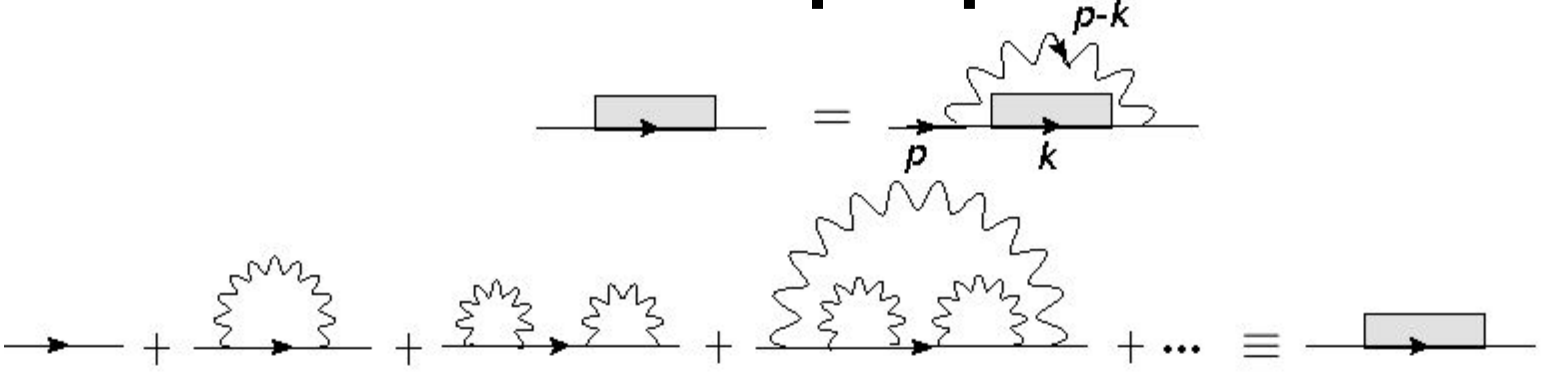
Interpolating Axial Gauge

$$A^a_{\hat{-}} = 0$$

$$A^a_1 = 0 \qquad\qquad A^+_a = 0$$

$$\mathcal{L} = \frac{1}{2}(\partial_{\hat{-}} A^a_{\hat{+}})^2 + \bar{\psi}(i\gamma^{\hat{+}} D_{\hat{+}} + i\gamma^{\hat{-}}\partial_{\hat{-}} - m)\psi$$

Mass Gap Equation



$\equiv i\ \Sigma(p)$

$$\Sigma(p_{\hat{-}}) = i\frac{\lambda}{2\pi}\int \frac{dk_{\hat{-}}\,dk_{\hat{+}}}{(p_{\hat{-}} - k_{\hat{-}})^2}\gamma^{\hat{+}}\frac{1}{\not{k} - m - \Sigma(k_{\hat{-}}) + i\epsilon}\gamma^{\hat{+}}$$

Fermion Propagator

Free Propagator

$$S_f(p) = \frac{1}{\not p - m + i\varepsilon}$$

$$\boxed{\Sigma(p) = \Sigma_s(p) + \Sigma_v(p)\not p}$$

Interacting Propagator

$$S(p) = \frac{1}{\not p - m - \sum(p) + i\varepsilon}$$

$$= \frac{F(p)}{\not p - M(p) + i\varepsilon}$$

$$F(p) = \left(1 - \Sigma_v(p)\right)^{-1}$$ "Wave function renormalization factor"

$$M(p) = \frac{m + \Sigma_s(p)}{1 - \Sigma_v(p)}$$ "Renormalized fermion mass function"

Energy-Momentum Dispersion Relation

Free particle

$$E = \sqrt{p_z^2 + m^2}$$

$$\theta_f = \tan^{-1}(p_z / m)$$

$$\beta = p_z / E$$

$$= \sin\theta_f$$

$$= \tanh\eta$$

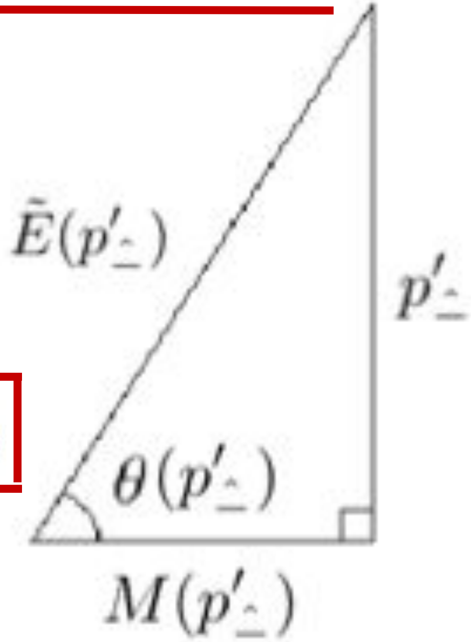
Interacting particle

$$\frac{F(p'_{\hat{-}})E(p'_{\hat{-}})}{\sqrt{C}} = \sqrt{p'^2_{\hat{-}} + M(p'_{\hat{-}})^2} \equiv \tilde{E}(p'_{\hat{-}})$$

$$\theta(p'_{\hat{-}}) = \theta_f(p'_{\hat{-}}) + 2\zeta(p'_{\hat{-}})$$

$$\begin{pmatrix} b^i(p'_{\hat{-}}) \\ d^{+i}(p'_{\hat{-}}) \end{pmatrix} = \begin{pmatrix} \cos\zeta(p'_{\hat{-}}) & -\sin\zeta(p'_{\hat{-}}) \\ \sin\zeta(p'_{\hat{-}}) & \cos\zeta(p'_{\hat{-}}) \end{pmatrix} \begin{pmatrix} b^i_f(p'_{\hat{-}}) \\ d^{+i}_f(p'_{\hat{-}}) \end{pmatrix}$$

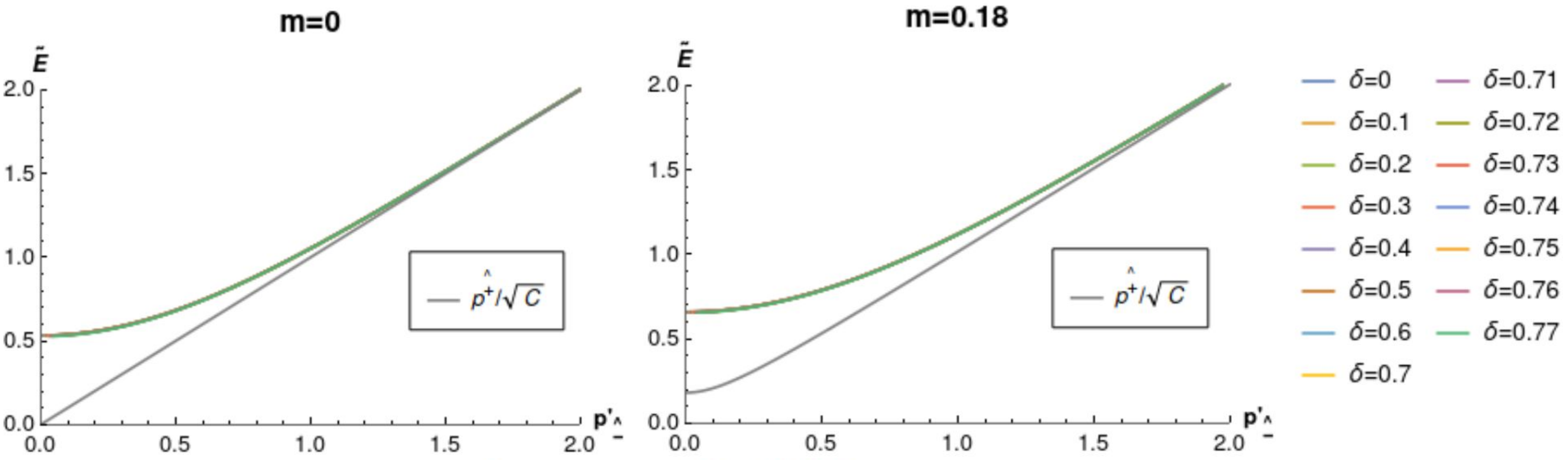
$b^i_f |0\rangle = 0, d^i_f |0\rangle = 0$ **vs.** $b^i |\Omega\rangle = 0, d^i |\Omega\rangle = 0$



Interpolation

$$(E, p_z) \Rightarrow (p^{\hat{+}} / \sqrt{C}, p_{\hat{-}} / \sqrt{C} \equiv p'_{\hat{-}})$$

Mass Gap Solutions



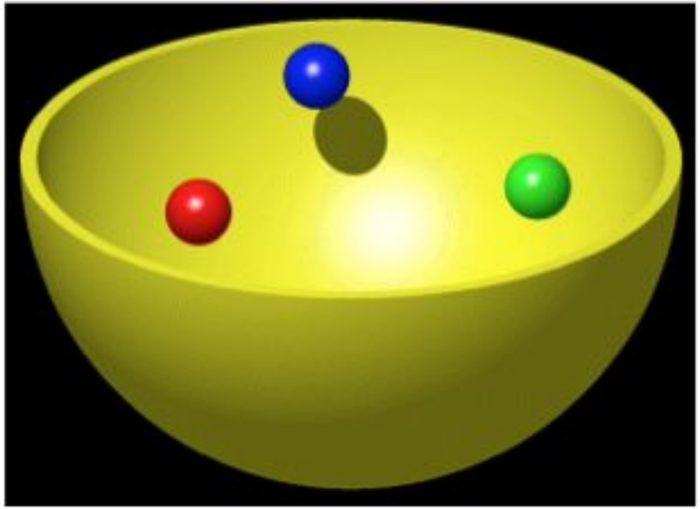
$$\tilde{E}(0) = \frac{F(0)E(0)}{\sqrt{\mathbb{C}}} = M(0)$$

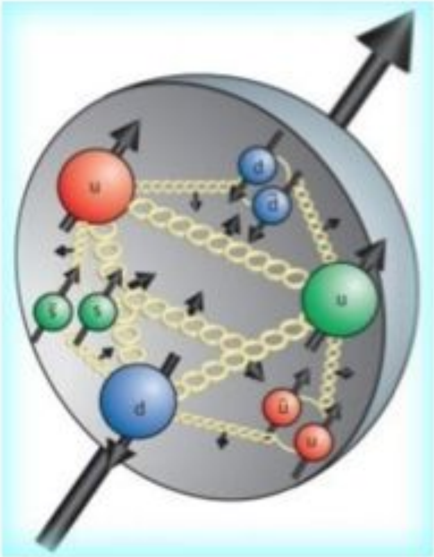
m	0	0.045	0.18	0.749	1.00	2.11	4.23
$M(0)$	0.532778	0.563644	0.659112	1.10105	1.31167	2.30969	4.34358
$F(0)$	−0.495173	−0.584175	−0.987673	4.11079	2.17976	1.22134	1.05526

$$m \lesssim 0.56$$

$$M_p = 938.272046 \pm 0.000021\,MeV$$

$$M_n = 939.565379 \pm 0.000021\,MeV$$

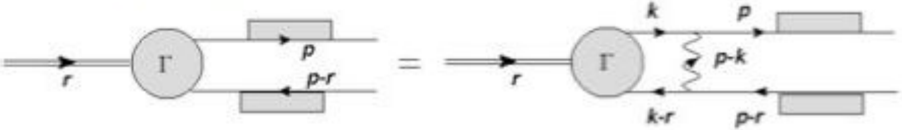




$$m_u = 2.3^{+0.7}_{-0.5}\,MeV \quad ; \quad m_d = 4.8^{+0.7}_{-0.3}\,MeV$$

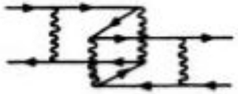
BOUND-STATE EQUATION

$$\Gamma(r,p) = \frac{i\lambda}{2\pi}\int \frac{dk_{\hat{-}}\,dk_{\hat{+}}}{(p_{\hat{-}} - k_{\hat{-}})^2} S(p)\gamma^{\hat{+}}\Gamma(r,k)\gamma^{\hat{+}}S(p-r)$$



$$\left[-r_{\hat{+}} + \frac{-\mathbb{S}p_{\hat{-}} + E(p_{\hat{-}})}{\mathbb{C}} + \frac{\mathbb{S}(p_{\hat{-}} - r_{\hat{-}}) + E(p_{\hat{-}} - r_{\hat{-}})}{\mathbb{C}}\right]\hat{\phi}_+(r_{\hat{-}}, p_{\hat{-}})$$
$$= \lambda \fint \frac{dk_{\hat{-}}}{(p_{\hat{-}} - k_{\hat{-}})^2}\left[C(p_{\hat{-}}, k_{\hat{-}}, r_{\hat{-}})\hat{\phi}_+(r_{\hat{-}}, k_{\hat{-}}) - S(p_{\hat{-}}, k_{\hat{-}}, r_{\hat{-}})\hat{\phi}_-(r_{\hat{-}}, k_{\hat{-}})\right],$$

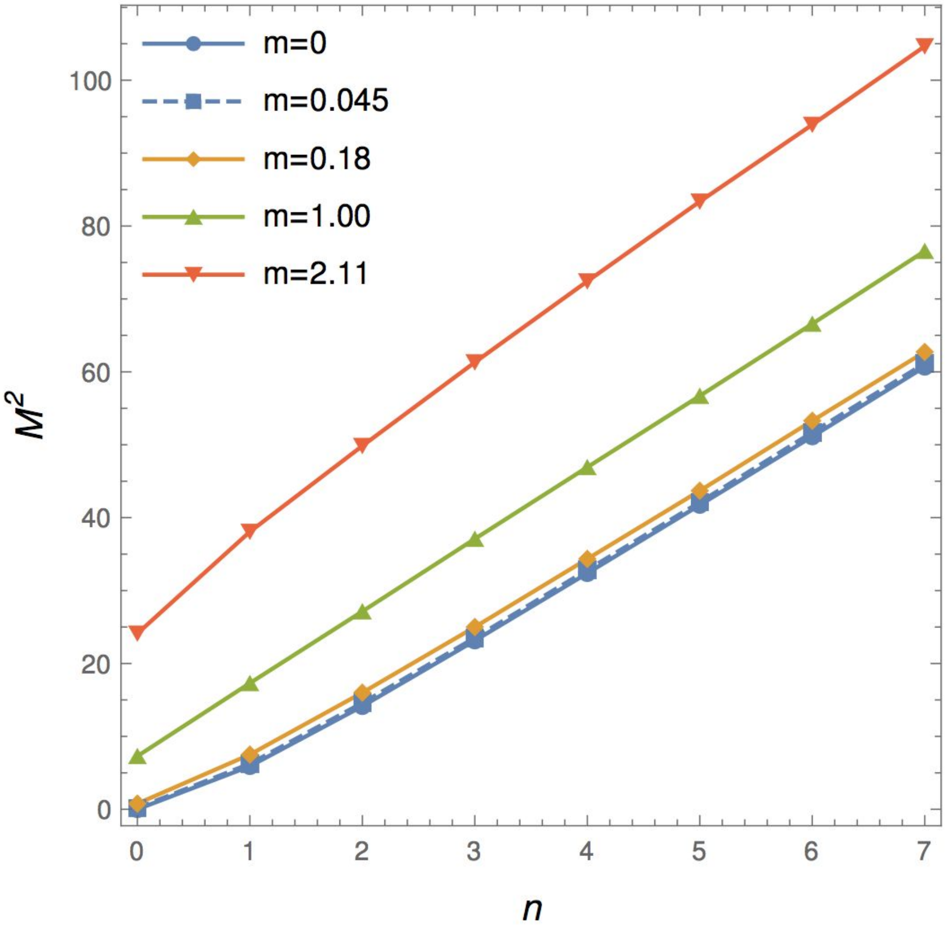
$$\left[r_{\hat{+}} + \frac{-\mathbb{S}(p_{\hat{-}} - r_{\hat{-}}) + E(p_{\hat{-}} - r_{\hat{-}})}{\mathbb{C}} + \frac{\mathbb{S}p_{\hat{-}} + E(p_{\hat{-}})}{\mathbb{C}}\right]\hat{\phi}_-(r_{\hat{-}}, p_{\hat{-}})$$
$$= \lambda \fint \frac{dk_{\hat{-}}}{(p_{\hat{-}} - k_{\hat{-}})^2}\left[C(p_{\hat{-}}, k_{\hat{-}}, r_{\hat{-}})\hat{\phi}_-(r_{\hat{-}}, k_{\hat{-}}) - S(p_{\hat{-}}, k_{\hat{-}}, r_{\hat{-}})\hat{\phi}_+(r_{\hat{-}}, k_{\hat{-}})\right].$$



LFD $$\left[\mathcal{M}^2 - \frac{m^2 - 2\lambda}{x} - \frac{m^2 - 2\lambda}{1-x}\right]\phi(x) = -2\lambda \fint_0^1 \frac{dy}{(x-y)^2}\phi(y)$$

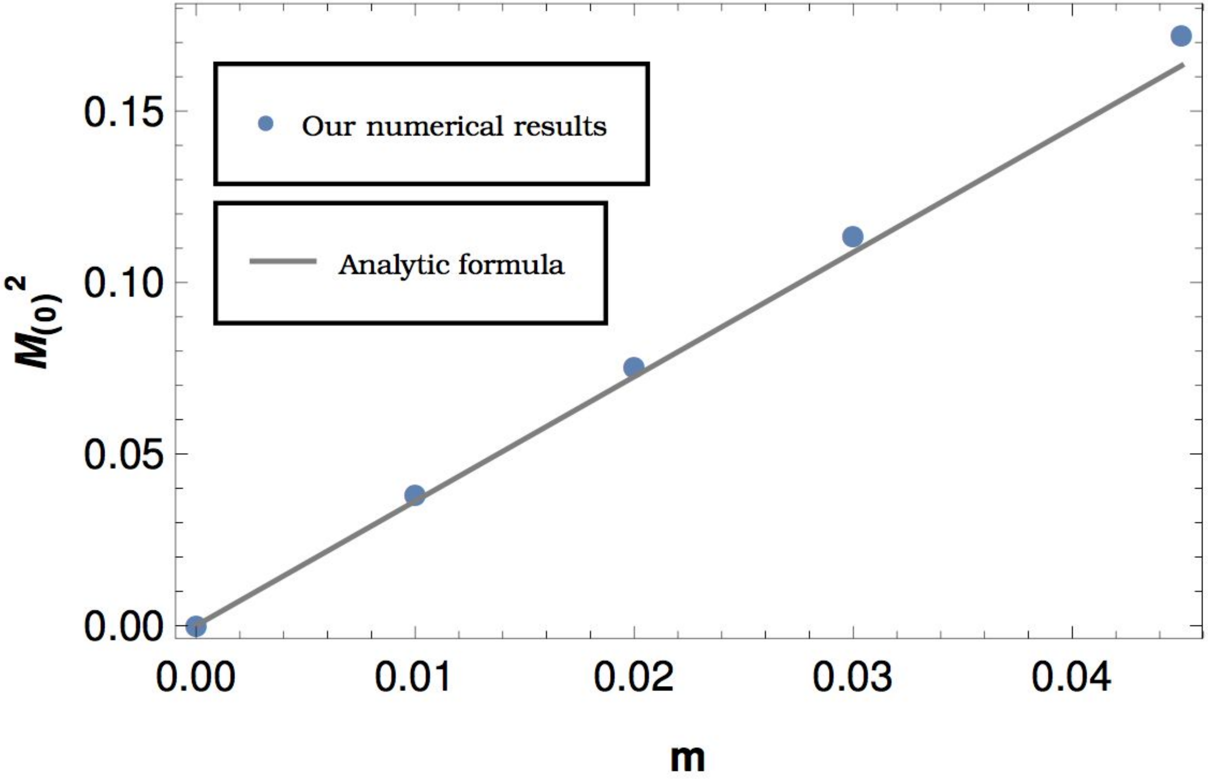
Meson Spectroscopy

- G.'tHooft, NPB75, 461(74) - LFD



- M.Li, et al., JPG13, 915(87) - IFD (rest frame)
- Y. Jia, et al., JHEP11, 151('17) - IFD (moving frame)

Gell-Mann - Oaks - Renner Relation

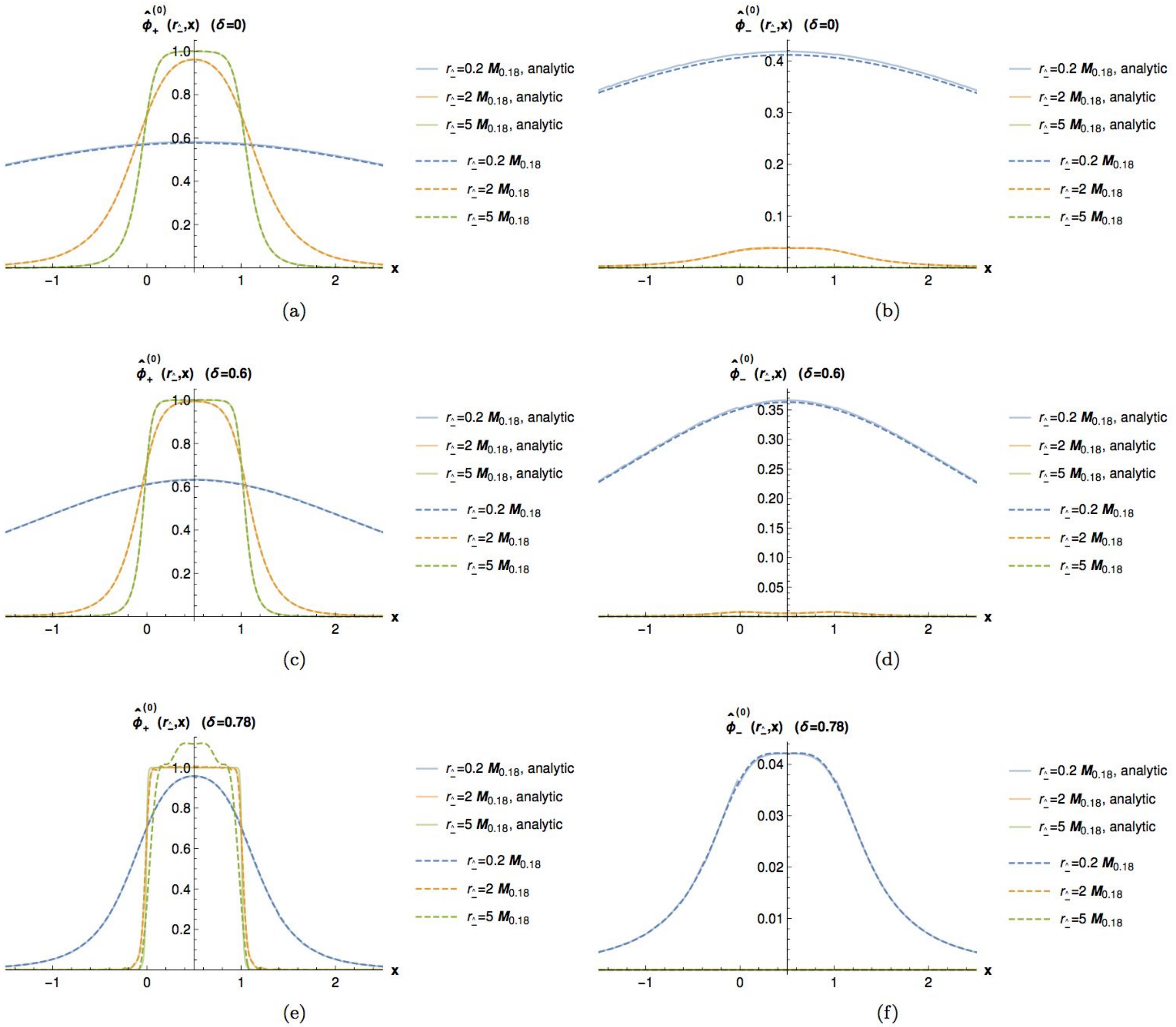


$$\mathcal{M}_\pi^2 = -\frac{4m < \bar{\psi}\psi >}{f_\pi^2} = \sqrt{\frac{8\pi^2 m^2 \lambda}{3}}$$

$$f_\pi = \sqrt{N_c/\pi}$$

Meson Ground-state Wave-function for m=0 case

$$\hat{\phi}^{(0)}_{\pm}(r_{\underline{\wedge}}, p_{\underline{\wedge}}) = \frac{1}{2}\left(\cos\frac{\theta(r_{\underline{\wedge}} - p_{\underline{\wedge}}) - \theta(p_{\underline{\wedge}})}{2} \pm \sin\frac{\theta(r_{\underline{\wedge}} - p_{\underline{\wedge}}) + \theta(p_{\underline{\wedge}})}{2}\right)$$



Parton Distribution Functions (PDFs)

$$q_n(x) = \int_{-\infty}^{+\infty} \frac{d\xi^-}{4\pi} e^{-ixP^+\xi^-}$$

$$\times \langle P_n^-, P^+ | \bar{\psi}(\xi^-)\gamma^+ \mathcal{W}[\xi^-, 0]\psi(0) | P_n^-, P^+ \rangle_C,$$

$$\mathcal{W}[\xi^-, 0] = \mathcal{P}\left[\exp\left(-ig_s \int_0^{\xi^-} d\eta^- A^+(\eta^-)\right)\right]$$

A^+=0 Gauge in LFD

Quasi-PDFs

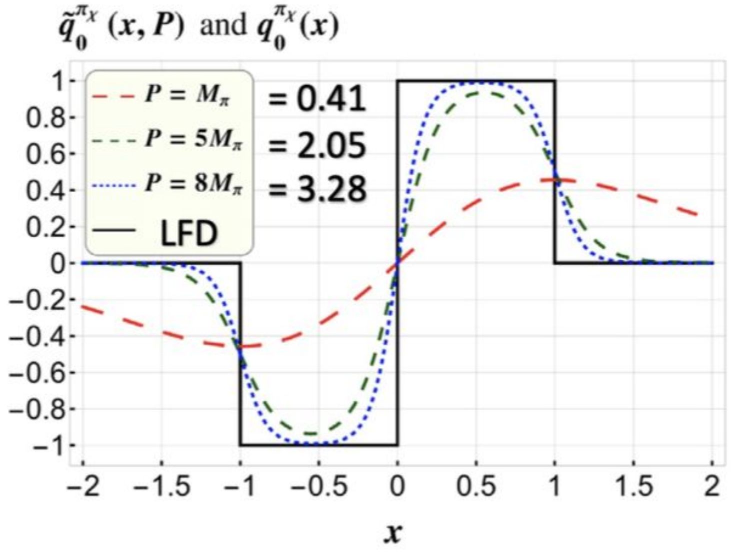
$$\tilde{q}_{(n)}(r_{\hat{-}}, x) = \int_{-\infty}^{+\infty} \frac{dx^{\hat{-}}}{4\pi} \, e^{ix^{\hat{-}} r_{\hat{-}}}$$

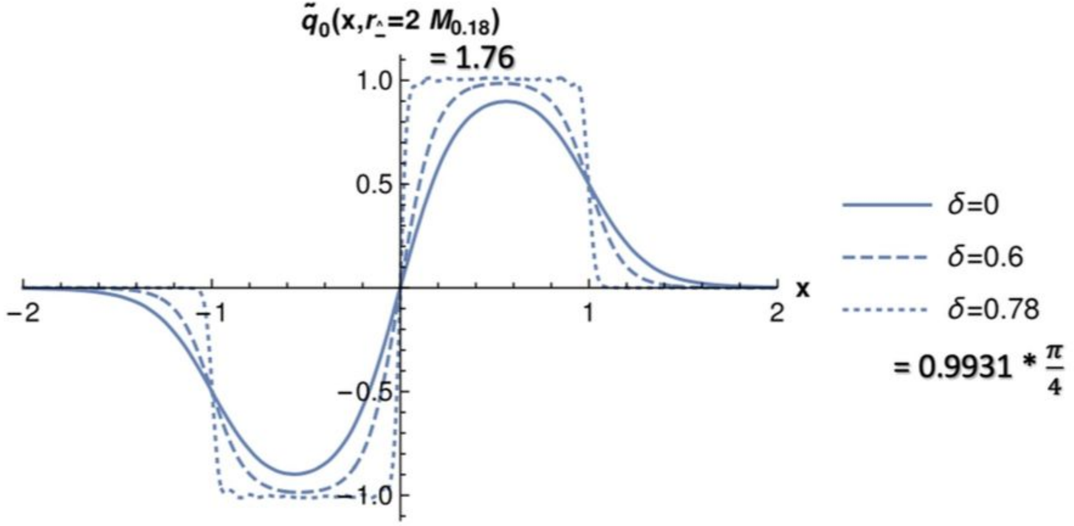
$$\times < r^{\hat{+}}_{(n)}, r_{\hat{-}} \mid \bar{\psi}(x^{\hat{-}}) \, \gamma_{\hat{-}} \, \mathcal{W}[x^{\hat{-}}, 0] \, \psi(0) \mid r^{\hat{+}}_{(n)}, r_{\hat{-}} >_C,$$

$$\mathcal{W}[x^{\hat{-}}, 0] = \mathcal{P}\left[\exp\left(-ig \int_0^{x^{\hat{-}}} dx'^{\hat{-}} A_{\hat{-}}(x'^{\hat{-}})\right)\right]$$

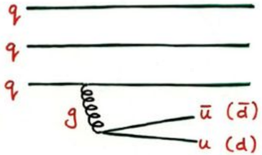
Interpolating dynamics

Y. Jia, et al., PRD98, 054011('18)
- IFD (quasi-PDFs)





B.Ma&C.Ji,PRD104,036004('21)
- Interpolating Dynamics



Are more anti-down quarks than anti-up quarks in the proton sea?

- *Motivation*: can one understand flavor asymmetries in the nucleon ($e.g. \bar{d} - \bar{u}$) from QCD?

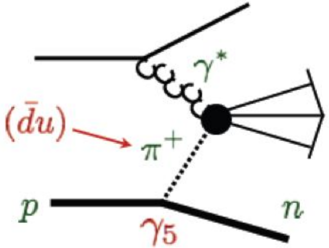
→ origin of 5-quark Fock components $|qqq\,\bar{q}q\rangle$ of nucleon

$$\int_0^1 dx\;(\bar{d}(x) - \bar{u}(x)) = 0.118 \pm 0.012$$

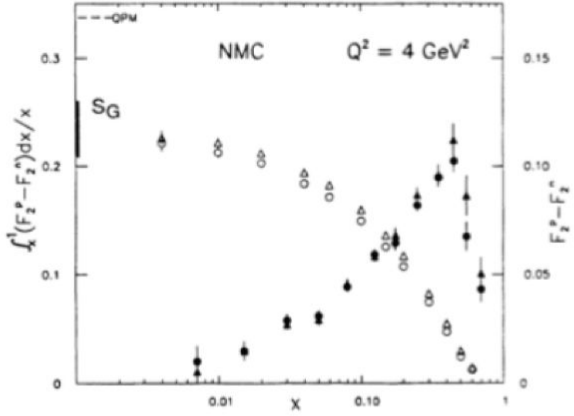
E866 (Fermilab), PRD ***64****, 052002 (2001)*

Sullivan process

$\bar{d} > \bar{u}$ ⟷ $\pi^+ n > \pi^- \Delta^{++}$



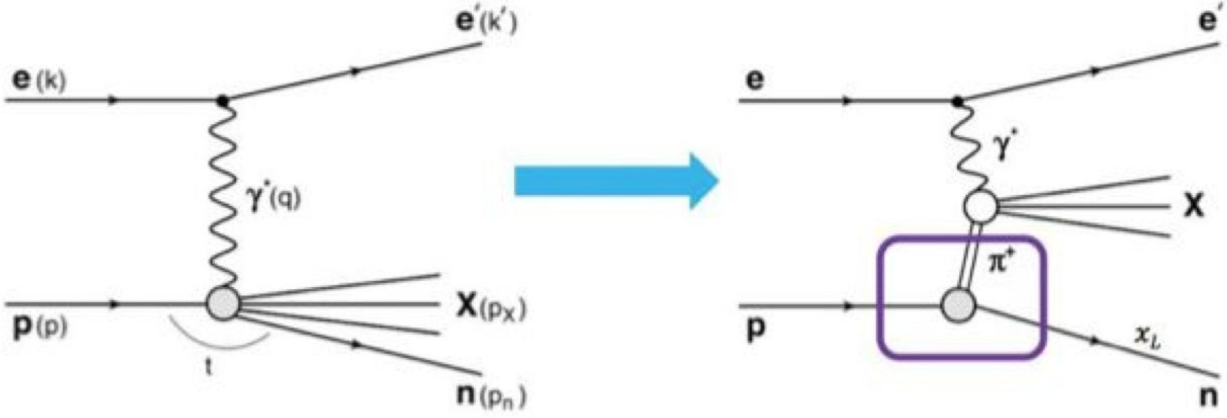
Sullivan, PRD ***5****, 1732 (1972)*
Thomas, PLB ***126****, 97 (1983)*



$$\int_0^1 \frac{dx}{x}(F_2^p - F_2^n) = \frac{1}{3} - \frac{2}{3}\int_0^1 dx\,(\bar{d} - \bar{u}) = 0.235(26)$$

New Muon Collaboration, PRD ***50****, 1 (1994)*

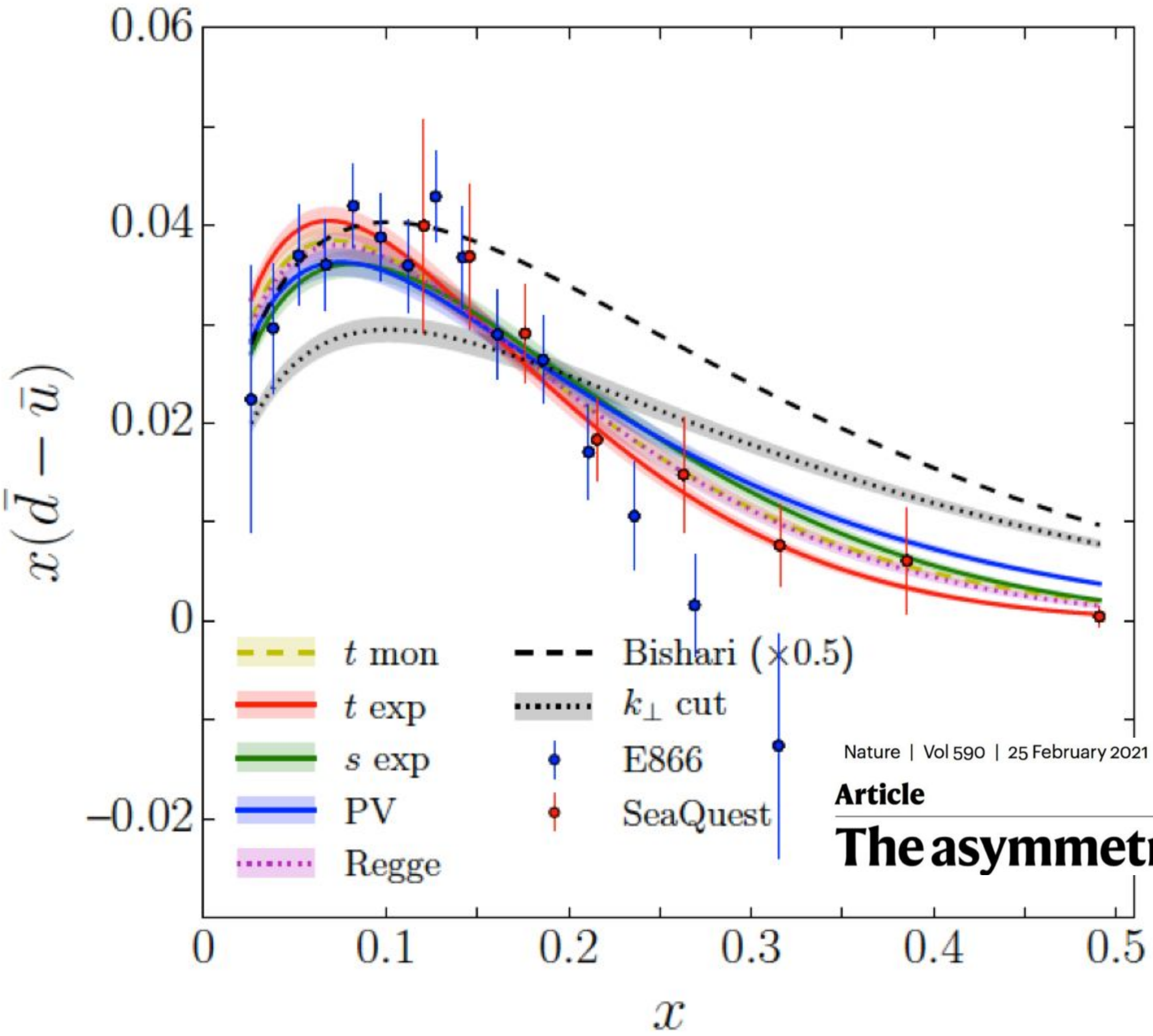
Convolution with Chiral Effective Theory



Denotes the splitting of *e.g.*, $p \to \pi^+ n$
Notationally: $f_\pi(\bar{x}_L = 1 - x_L)$

$$(\bar{d} - \bar{u})(x) = \frac{2}{3} \int_x^1 \frac{dy}{y}\, f_\pi(y)\ \bar{q}^\pi(x/y)$$

pion light-cone momentum distribution in nucleon



Nature | Vol 590 | 25 February 2021 | 561

Article

The asymmetry of antimatter in the proton

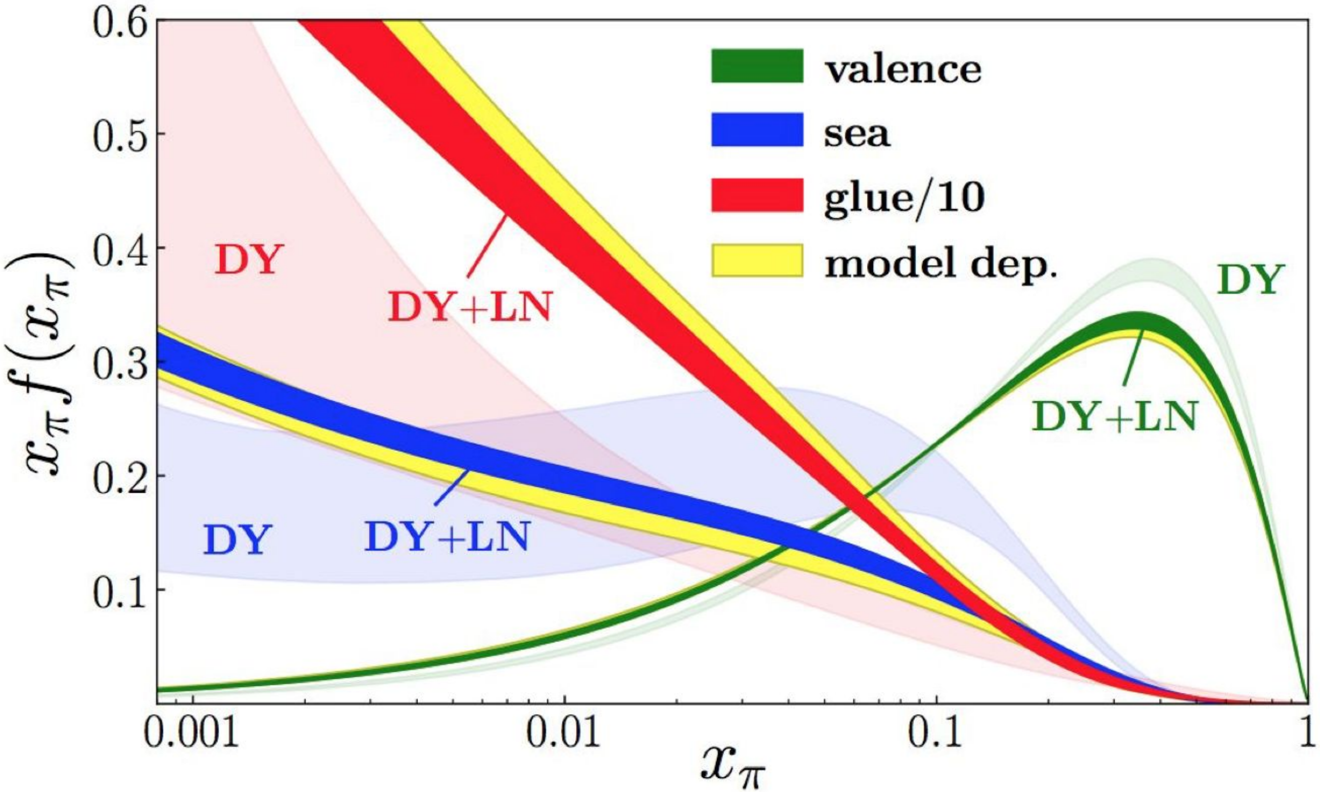
Featured in Physics

First Monte Carlo Global QCD Analysis of Pion Parton Distributions

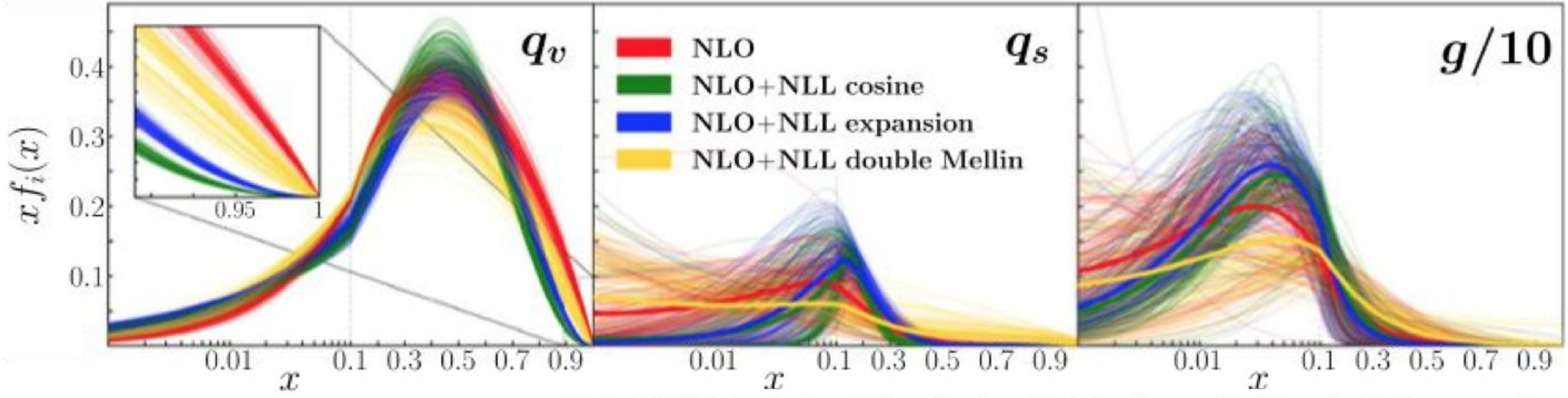
P. C. Barry, N. Sato, W. Melnitchouk, and Chueng-Ryong Ji (Jefferson Lab Angular Momentum (JAM) Collaboration)

Phys. Rev. Lett. **121**, 152001 (2018) – Published 10 October 2018

Physics Synopsis: More Gluons in the Pion

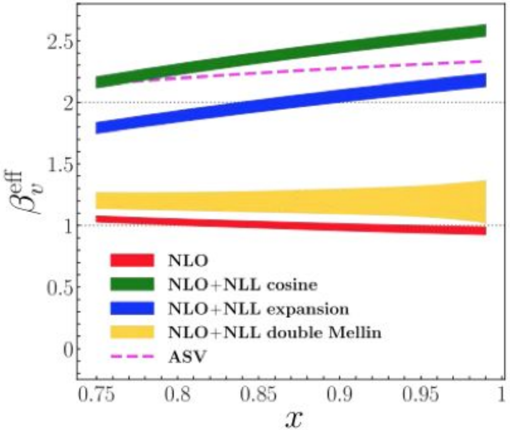


$$f_i(x, \mu_0; \boldsymbol{a}_i) = N_i x^{\alpha_i} (1-x)^{\beta_i} (1 + \gamma_i x^2)$$



Global QCD Analysis of Pion Parton Distributions with Threshold Resummation

P. C. Barry,[1] Chueng-Ryong Ji,[2] N. Sato,[1] and W. Melnitchouk[1]



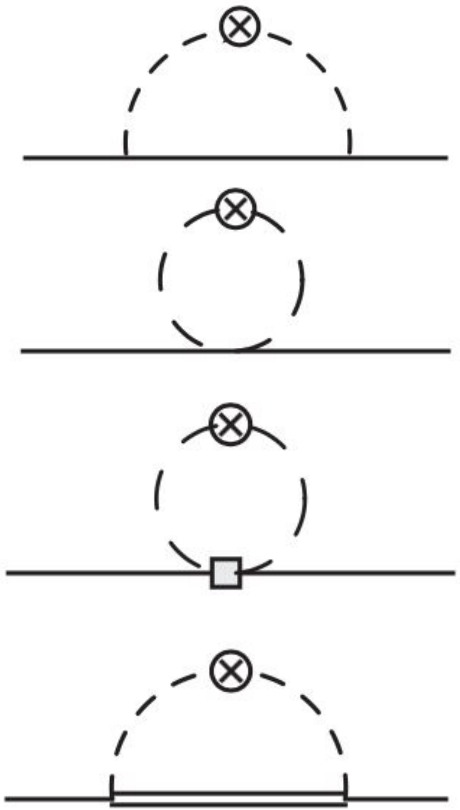
Resummation method	$\langle x \rangle_v$	$\langle x \rangle_s$	$\langle x \rangle_g$
NLO	0.53(2)	0.14(4)	0.34(6)
NLO + NLL cosine	0.47(2)	0.14(5)	0.39(6)
NLO + NLL expansion	0.46(2)	0.16(5)	0.38(6)
NLO + NLL double Mellin	0.46(3)	0.15(7)	0.40(5)

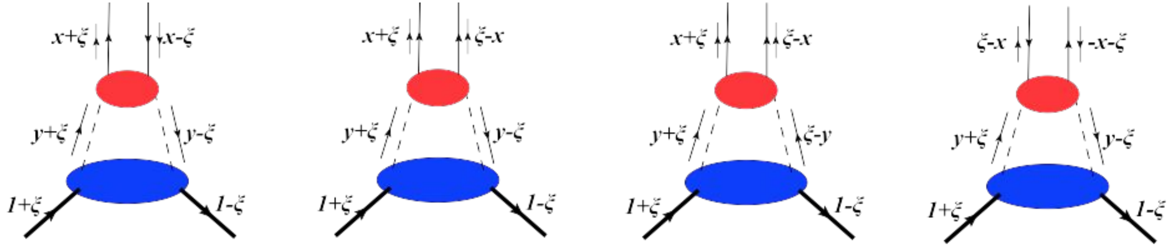
PHYSICAL REVIEW LETTERS **127,** 232001 (2021)

Nonlocal chiral contributions to GPDs of the proton at nonzero skewness:

arXiv:2406.03412v1 [hep-ph]; PRD110, 054049 (2024)

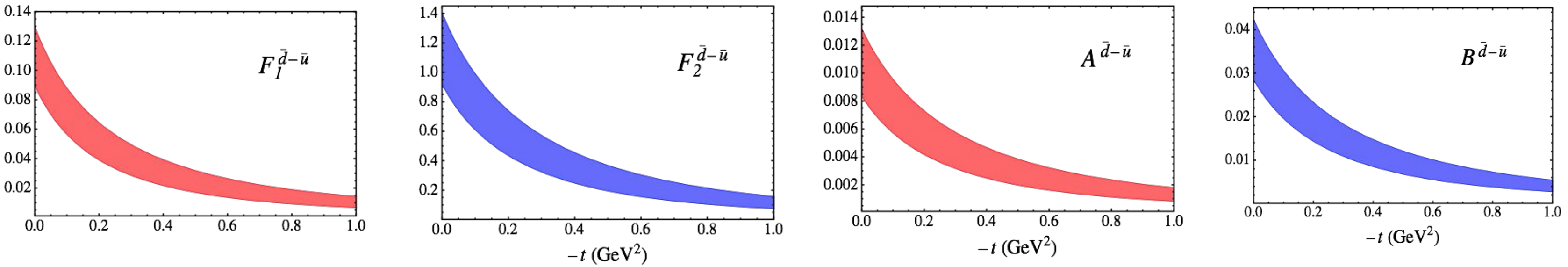
Z. Gao, F. He, C.-R. Ji, W. Melnitchouk, Y. Salamu, and P. Wang

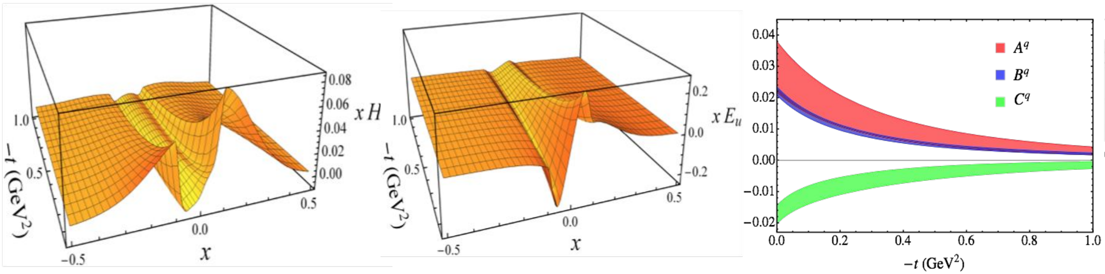




$$H_q^{(\mathrm{rbw})}(x,\xi,t)=\begin{cases}\displaystyle\int_x^1\frac{\mathrm{d}y}{y}f_{\phi B}^{(\mathrm{rbw})}(y,\xi,t)\,H_{q/\phi}\Big(\frac{x}{y},\frac{\xi}{y},t\Big), & [\xi<x<y]\\[2ex] \displaystyle\int_\xi^1\frac{\mathrm{d}y}{y}f_{\phi B}^{(\mathrm{rbw})}(y,\xi,t)\,H_{q/\phi}\Big(\frac{x}{y},\frac{\xi}{y},t\Big), & [x<\xi<y]\\[2ex] \displaystyle\int_{-\xi}^{\xi}\frac{\mathrm{d}y}{2y}f_{\phi B}^{(\mathrm{rbw})}(y,\xi,t)\frac{1}{\pi}\int_{s_0}^{\infty}\mathrm{d}s\,\frac{\mathrm{Im}\Phi_{q/\phi}\big(\frac{1}{2}(1+\frac{x}{\xi}),\frac{1}{2}(1+\frac{y}{\xi}),s\big)}{s-t+i\epsilon}, & [|x|,|y|<\xi]\\[2ex] \displaystyle\int_{-x}^1\frac{\mathrm{d}y}{y}f_{\phi B}^{(\mathrm{rbw})}(y,\xi,t)\,H_{q/\phi}\Big(\frac{x}{y},\frac{\xi}{y},t\Big), & [\xi<-x<y<1]\end{cases}$$

- **Nonlocal generalization of the effective Lagrangian provides systematic finite range regularization**
- **Nonzero skewness provides testing ground for the theory via polynomiality condition**
- **Extension to gravitational form factors, A, B and D=4C**





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Review

Nucleon form factors and parton distributions in nonlocal chiral effective theory

P. Wang [a,b,*], Fangcheng He [c], Chueng-Ryong Ji [d], W. Melnitchouk [e]

Conclusions and Outlook

- **Dirac's proposed LFD provides distinguished features useful for effective quantum computations of hadronic amplitudes.**
- **QED(3+1) and QCD(1+1) at large Nc have been interpolated between IFD and LFD providing useful hadron physics information based on Relativistic Quantum Invariance.**
- **Link between QCD and LFQM may be feasible as exemplified by the mass gap solution in the ‘t Hooft model interpolation between IFD and LFD.**
- **Meson structure studies of LFQM provide useful tools to study the nucleon structures via the convolution with the splitting functions computed by the chiral effective theory.**