Nucleon Structure Studies in Chiral Effective Theory

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Jeju Island
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Outline

- Global QCD Analysis with Chiral EFT
- Link between Chiral EFT and QCD
- Theoretical Inputs for Global Analyses
- Light-Front Quark Model (LFQM) and QCD
- Convolution of Chiral EFT with LFQM
- Conclusions and Outlook

Featured in Physics

First Monte Carlo Global QCD Analysis of Pion Parton Distributions

P. C. Barry, N. Sato, W. Melnitchouk, and Chueng-Ryong Ji (Jefferson Lab Angular Momentum (JAM) Collaboration)

Phys. Rev. Lett. **121**, 152001 (2018) – Published 10 October 2018

Physics Synopsis: More Gluons in the Pion

https://journals.aps.org/collections/50-years-QCD

Teaser: A combined analysis of collider data finds that the gluon contribution to the pion is three times larger than earlier estimates.

physicsworld

PARTICLE AND NUCLEAR RESEARCH UPDATE

Gluons account for much more pion momentum than previously thought

19 Oct 2018

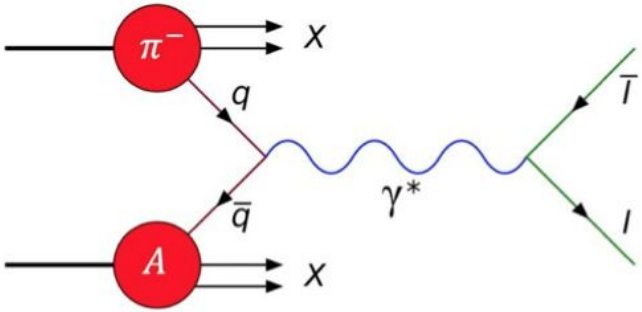
Pion Properties

- Lightest bound state composed of quarks, antiquarks, and gluons
- Masses: $m_{\pi^\pm} = 139.57\ \text{MeV}, m_{\pi^0} = 134.977\ \text{MeV}$
- Lifetimes: $\tau_{\pi^\pm} = 2.603\times10^{-8}\ \text{s}\ , \tau_{\pi^0} = 8.52\times10^{-17}\ \text{s}$

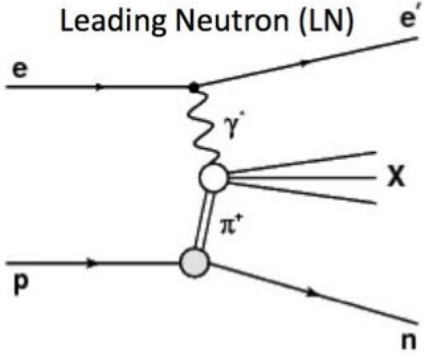
Charged pions decay via weak interaction

Neutral pions decay via electromagnetic interaction, *i.e.* $\pi^0 \rightarrow \gamma\gamma$

Drell-Yan (DY)



Leading Neutron (LN)



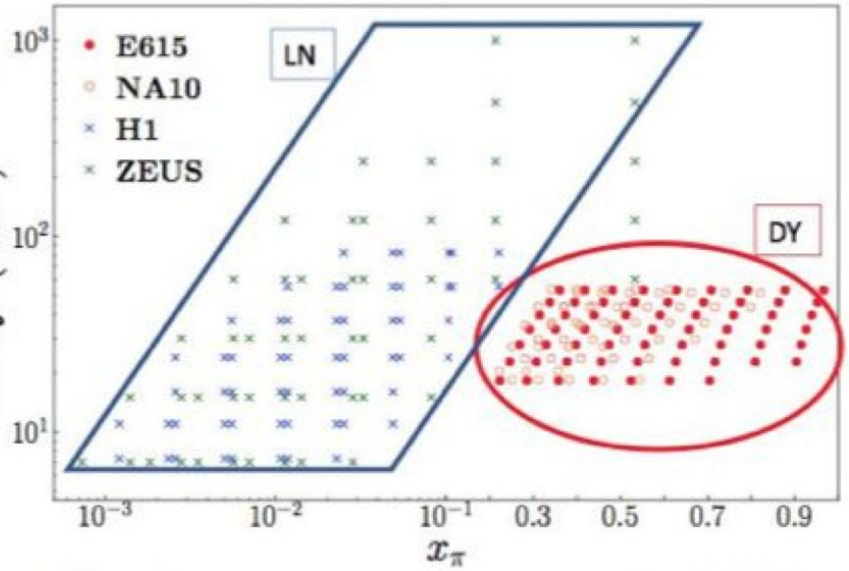
Datasets vs. Kinematics

- Large x_π -- Drell-Yan (DY)
- Small x_π -- Leading Neutron (LN)
- Not much data overlap
- In DY:

$$x_\pi = \frac{1}{2}\left(x_F + \sqrt{x_F^2 + 4\tau}\right)$$

- In LN:

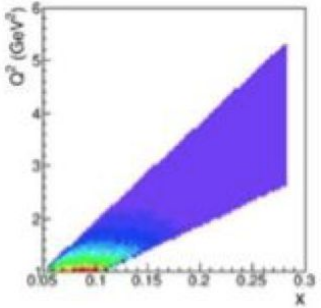
$$x_\pi = x_B/\bar{x}_L$$



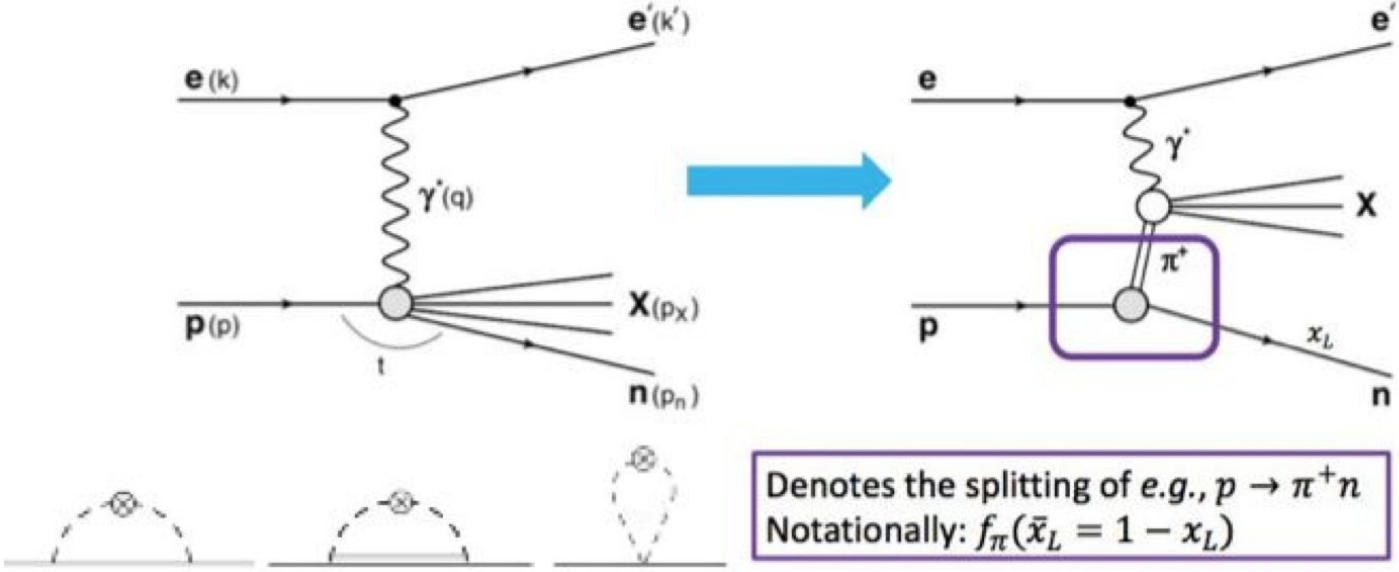
- JLab can reach much smaller Q^2 and larger x range than in the HERA

EIC Impact on Pion PDFs

- $s = 5400$ GeV^2, 1.2% systematic uncertainty, integrated $\mathcal{L} = 100\text{fb}^{-1}$



Convolution with Chiral Effective Theory



Denotes the splitting of *e.g.*, $p \to \pi^+ n$
Notationally: $f_\pi(\bar{x}_L = 1 - x_L)$

$$(\bar{d} - \bar{u})(x) = \frac{2}{3} \int_x^1 \frac{dy}{y} f_\pi(y)\ \bar{q}^\pi(x/y)$$

pion light-cone momentum distribution in nucleon

Chiral effective theory

- Effective low-energy theory of pions & nucleons

$$\mathcal{L}_{\pi N} = \frac{g_A}{2f_\pi}\,\bar{\psi}_N \gamma^\mu \gamma_5\, \vec{\tau}\cdot\partial_\mu\vec{\pi}\,\psi_N - \frac{1}{(2f_\pi)^2}\,\bar{\psi}_N \gamma^\mu\, \vec{\tau}\cdot(\vec{\pi}\times\partial_\mu\vec{\pi})\,\psi_N$$

$g_A = 1.267$

$f_\pi = 93$ MeV

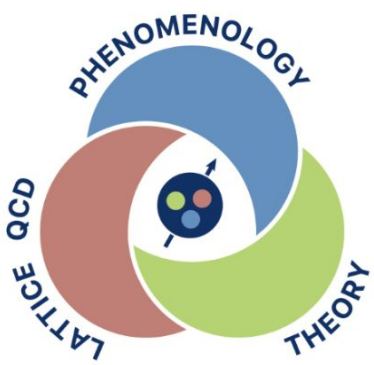
→ lowest order approximation of chiral perturbation theory Lagrangian

→ *cf.* pseudoscalar (PS) Lagrangian

$$\mathcal{L}^{\rm PS}_{\pi N} = -g_{\pi NN}\,\bar{\psi}_N\, i\gamma_5 \vec{\tau}\cdot\vec{\pi}\,\psi_N \;+ \sigma NN \text{ term}$$

Weinberg, PRL ***18****, 88 (1967)*

gives the classic "Sullivan" result
– full PV theory more complicated!



Supported by DOE topical collaboration program (DE-SC0023646)

Press & Highlights [https://qgtcollab.github.io/highlights.html]

2025

- Quantum calculations provide a sharper image of subatomic stress

2024

- Nucleon GPDs in nonlocal chiral effective theory
- Visualization of mass distribution within hadrons
- Physicists Determine Distribution of Strong Force inside Proton

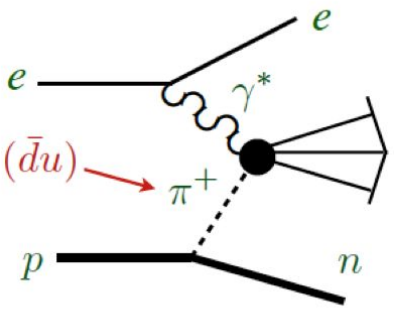
2023

- Theory Offers a High-Resolution View of Quarks Inside Protons
- Gravitational form factors of the proton revealed by theory and experiment
- Shedding Light on Shadow Generalized Parton Distributions
- The proton as seen with a finite speed of light

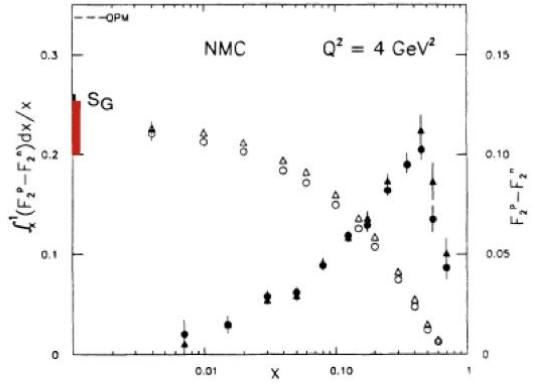
Flavor asymmetry

- Large flavor asymmetry in proton sea suggests important role of chiral symmetry in high-energy reactions

→ Sullivan process in DIS



Sullivan, PRD 5, 1732 (1972)



$$\int_0^1 \frac{dx}{x}(F_2^p - F_2^n) = \frac{1}{3} - \frac{2}{3}\int_0^1 dx\,(\bar{d} - \bar{u})$$
$$= 0.235(26)$$

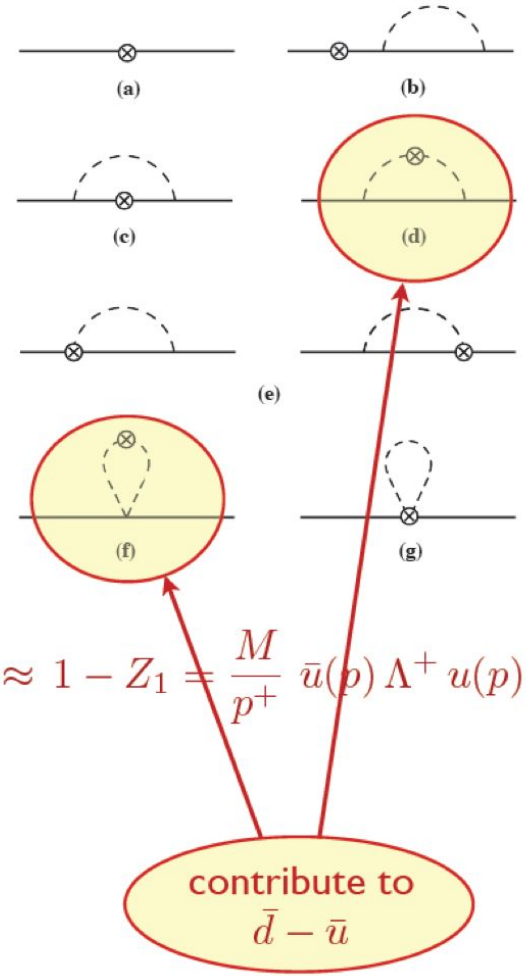
NMC, PRD 50, 1 (1994)

$$\bar{d} > \bar{u}$$

Flavor asymmetry

- Pion cloud corrections to electromagnetic N coupling

  → N rainbow (c), π rainbow (d), Kroll-Ruderman (e), π bubble (f), π tadpole (g)



- Vertex renormalization

$$(Z_1^{-1} - 1)\,\bar{u}(p)\,\gamma^\mu\,u(p) = \bar{u}(p)\,\Lambda^\mu\,u(p)$$

→ taking "+" components: $Z_1^{-1} - 1 \approx 1 - Z_1 = \dfrac{M}{p^+}\,\bar{u}(p)\,\Lambda^+\,u(p)$

→ $e.g.$ for N rainbow contribution,

$$\Lambda^N_\mu = -\frac{\partial\hat{\Sigma}}{\partial p^\mu}$$

Connection with QCD

$$(\bar{d}-\bar{u})(x) = \frac{2}{3}\int_x^1 \frac{dy}{y}\, f_\pi(y)\; \bar{q}^\pi(x/y)$$

$$f_\pi(y) = \frac{3g^2_{\pi NN}}{16\pi^2}\, y \int dt \frac{-t\; \mathcal{F}^2_{\pi NN}(t)}{(t-m_\pi^2)^2}$$

→ *model-independent* leading nonanalytic (LNA) behavior *consistent with Chiral Symmetry of QCD*.

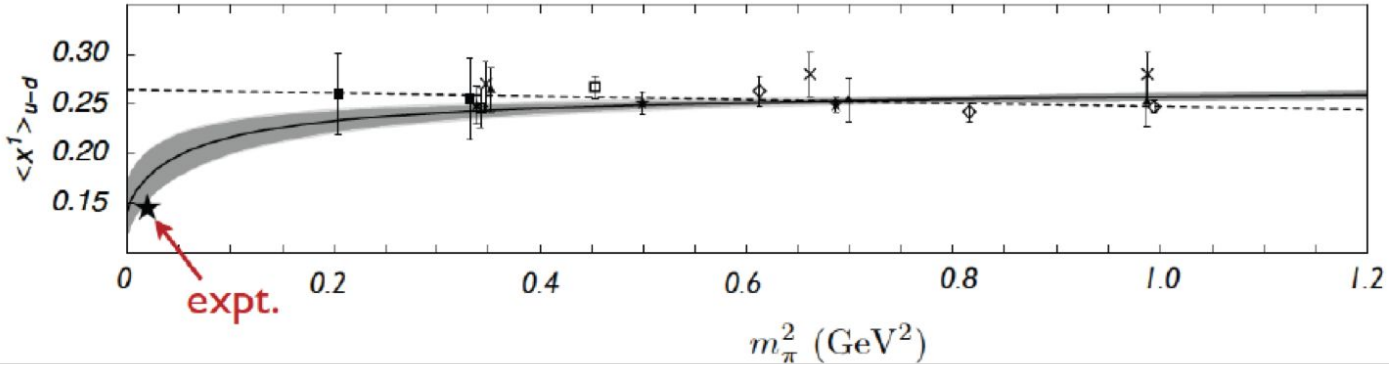
$$\langle x^0 \rangle_{\bar{d}-\bar{u}} \equiv \int_0^1 dx(\bar{d}-\bar{u})$$

$$m_\pi^2 f_\pi^2 = -2m_q < \bar{q}q >$$

$$= \frac{2}{3}\int_0^1 dy\, f_\pi(y) = \frac{2g_A^2}{(4\pi f_\pi)^2} m_\pi^2 \log(m_\pi^2/\mu^2) \; + \; \text{analytic terms}$$

- Nonanalytic behavior vital for chiral extrapolation of lattice data

*Thomas, Melnitchouk, Steffens PRL **85**, 2892 (2000)*



LNA of $\bar{D} - \bar{U} \equiv \int_0^1 dx(\bar{d} - \bar{u})$

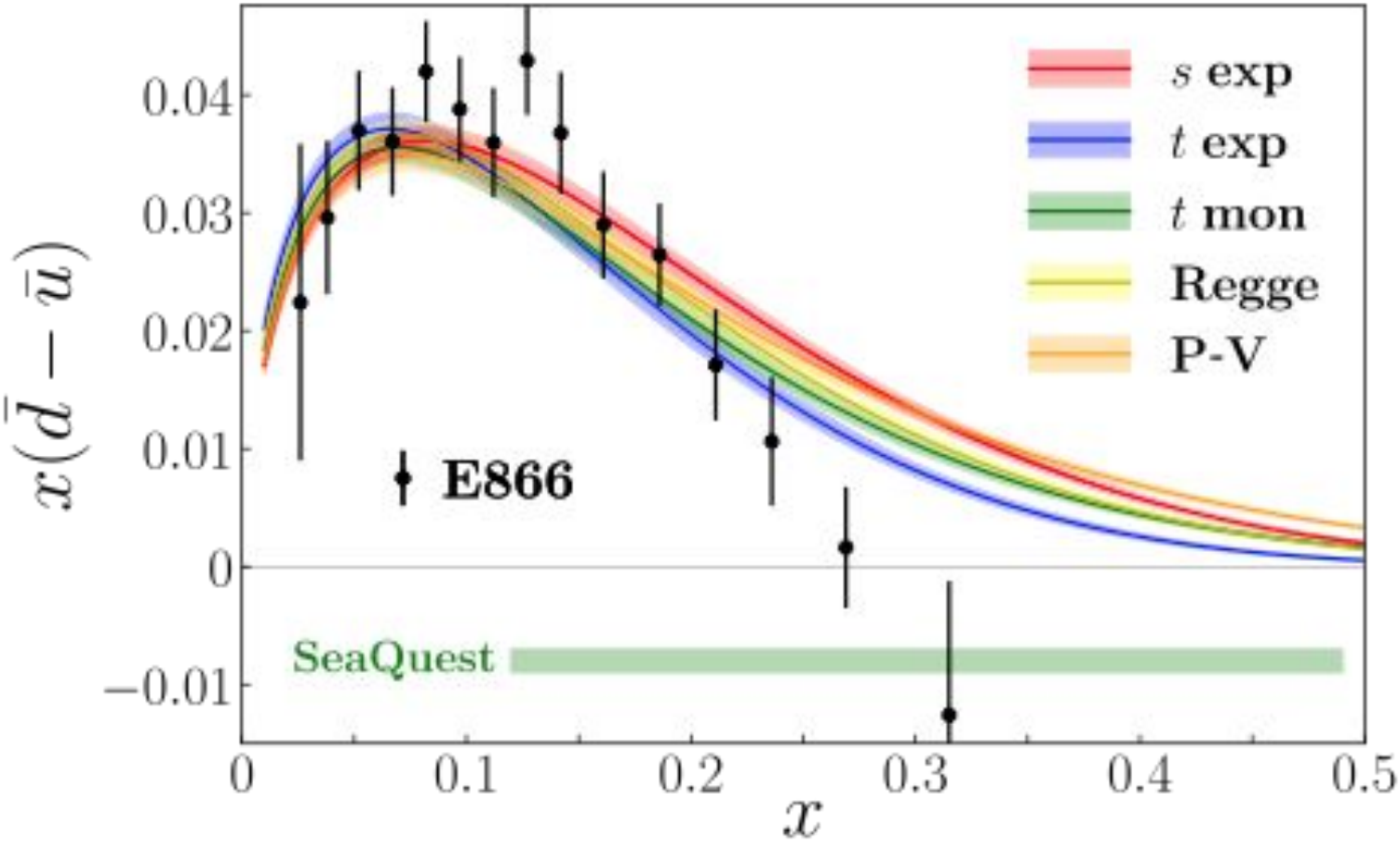
C.Ji, W.Melnitchouk,A.W.Thomas,
Phys.Rev.D 88 (2013) 7, 076005

$$(\bar{D}-\bar{U})_{LNA} = \frac{3g_A^2+1}{2(4\pi f_\pi)^2} m_\pi^2 \log m_\pi^2$$

$$= \frac{4g_A^2+(1-g_A^2)}{2(4\pi f_\pi)^2} m_\pi^2 \log m_\pi^2$$

"on-shell" contribution

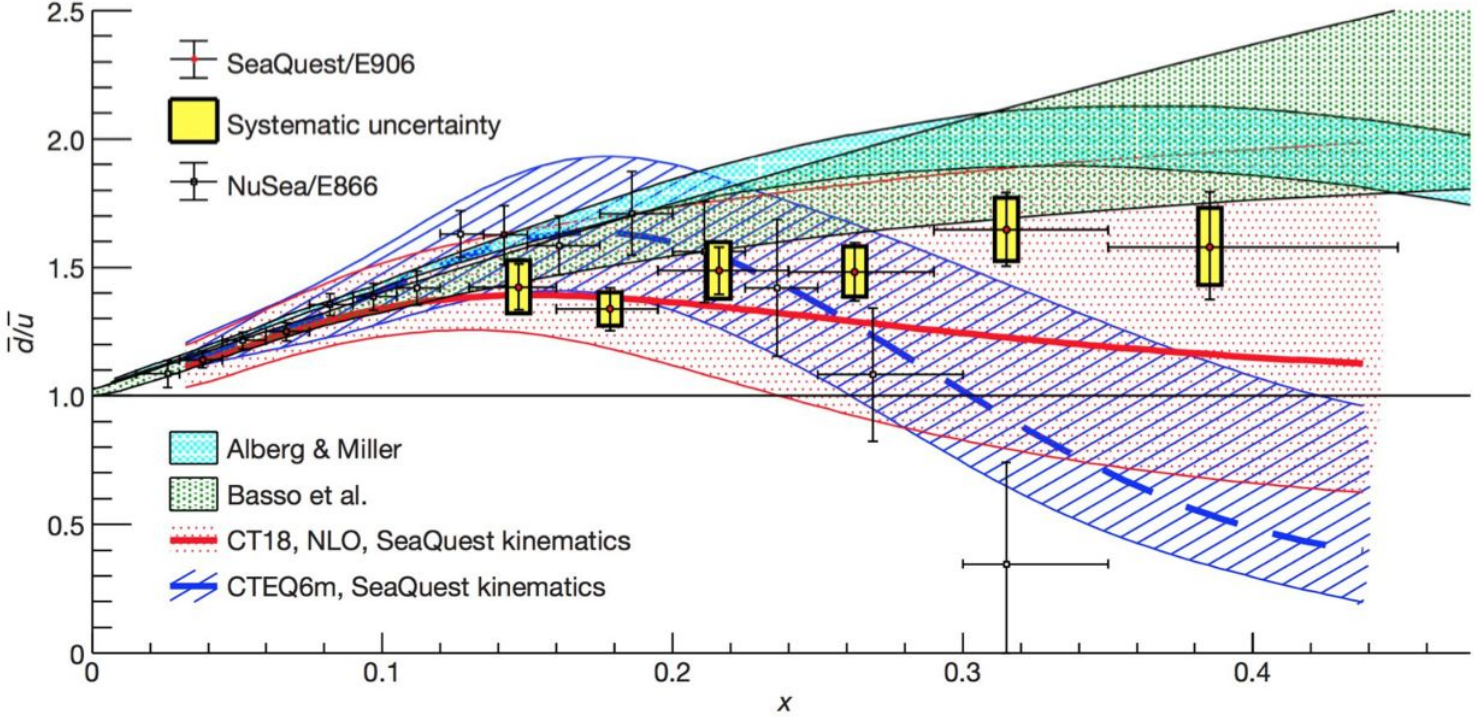
δ-function contribution

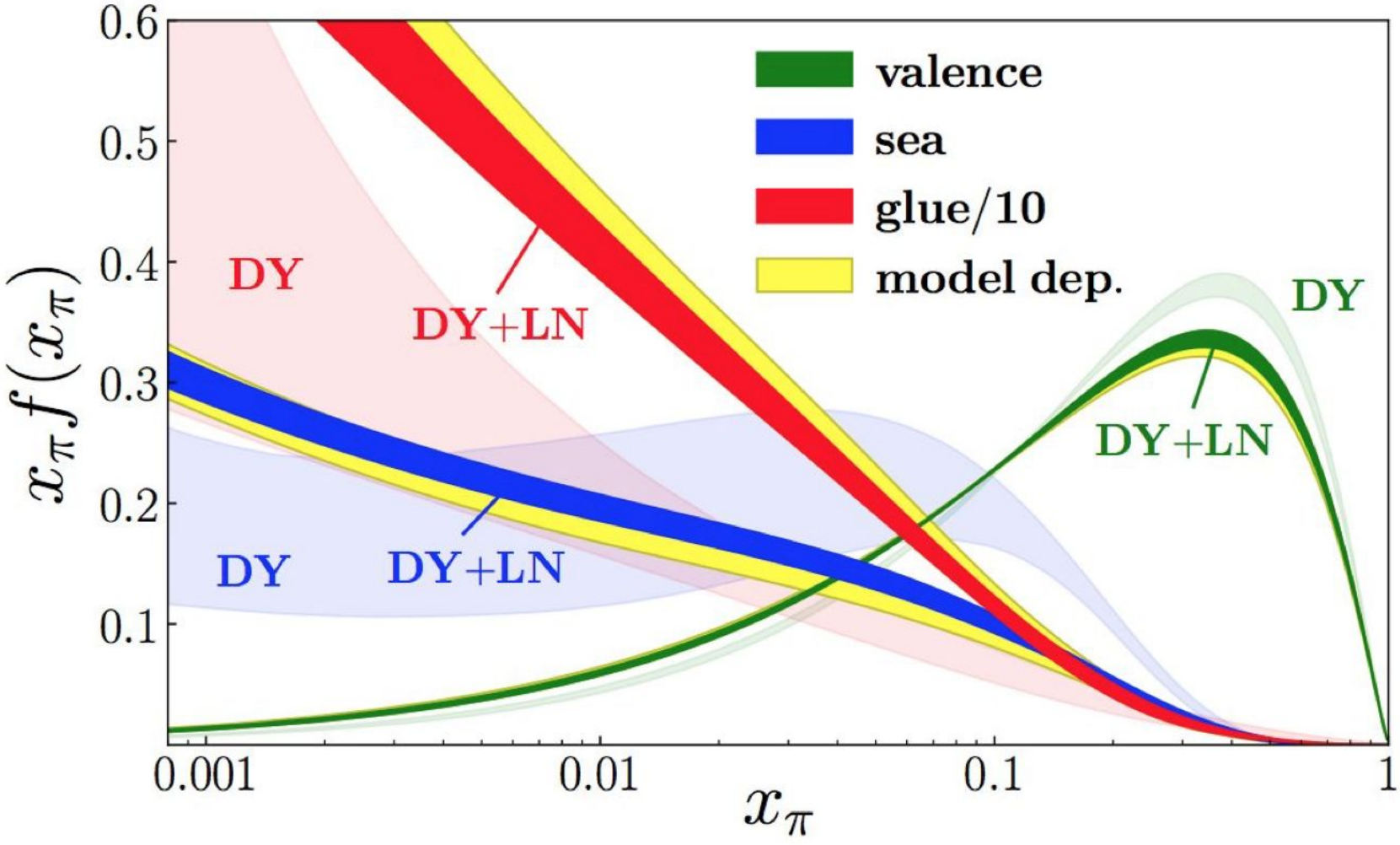


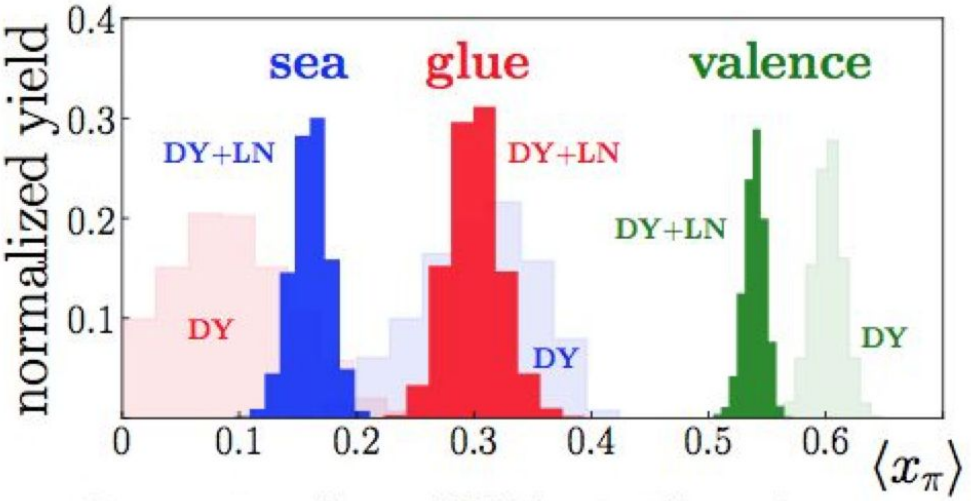
P. Barry, N. Sato, W. Melnitchouk, C.Ji, PRL121, 152001(2018)

Article

The asymmetry of antimatter in the proton

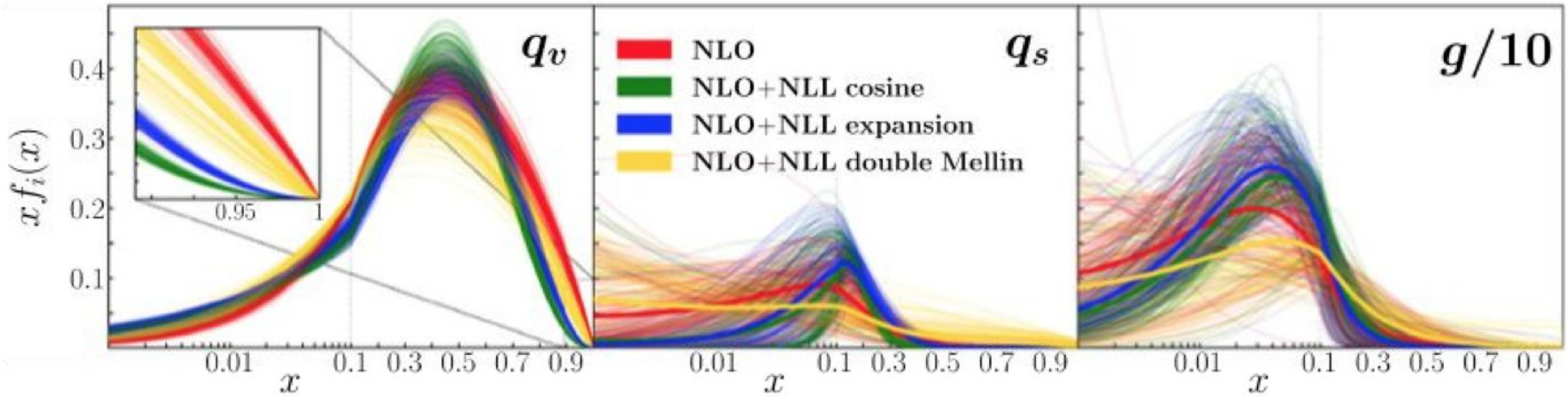


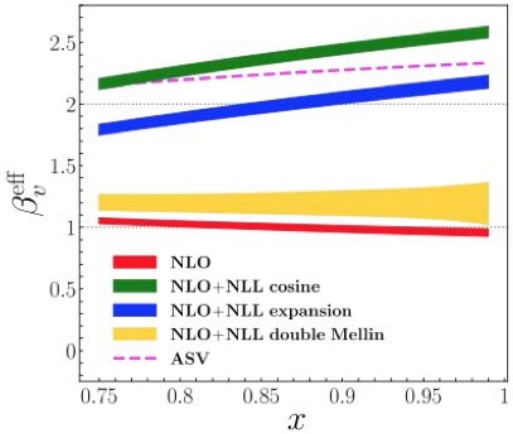

valence
sea
glue/10
model dep.
DY
DY+LN
DY
DY+LN
DY
DY+LN
$x_\pi f(x_\pi)$
x_π
0.6
0.5
0.4
0.3
0.2
0.1
0.001
0.01
0.1
1



- Constraints from HERA significantly increase $\langle x_\pi^g \rangle$.
  The role of the glue is more important than suggested by DY alone
- In contrast, the strength of the sea is reduced
- Due to momentum sum rule $\langle x_\pi^{\text{valence}} \rangle$ decreases

$$f_i(x,\mu_0;a_i) = N_i x^{\alpha_i}(1-x)^{\beta_i}(1+\gamma_i x^2)$$





Resummation method	$\langle x \rangle_v$	$\langle x \rangle_s$	$\langle x \rangle_g$
NLO	0.53(2)	0.14(4)	0.34(6)
NLO + NLL cosine	0.47(2)	0.14(5)	0.39(6)
NLO + NLL expansion	0.46(2)	0.16(5)	0.38(6)
NLO + NLL double Mellin	0.46(3)	0.15(7)	0.40(5)

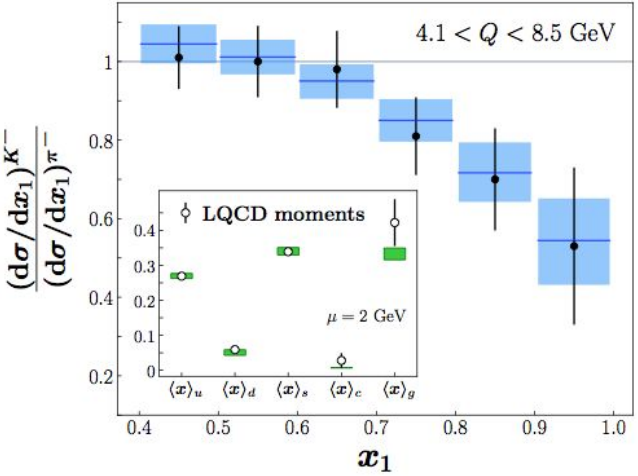
P. Barry, C. Ji, N. Sato, W. Melnitchouk,, PRL127, 232001(2021)

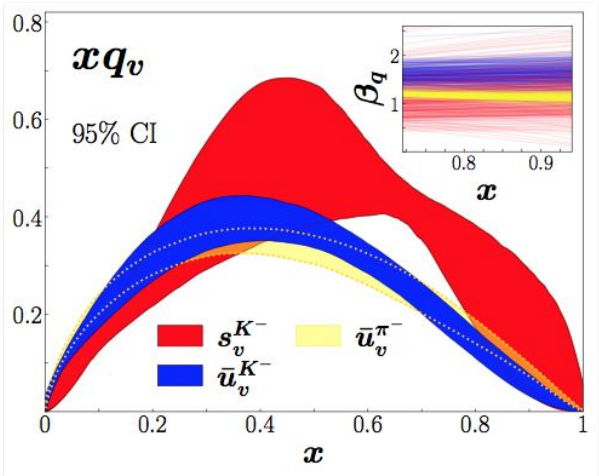
JLAB-THY-25-4569

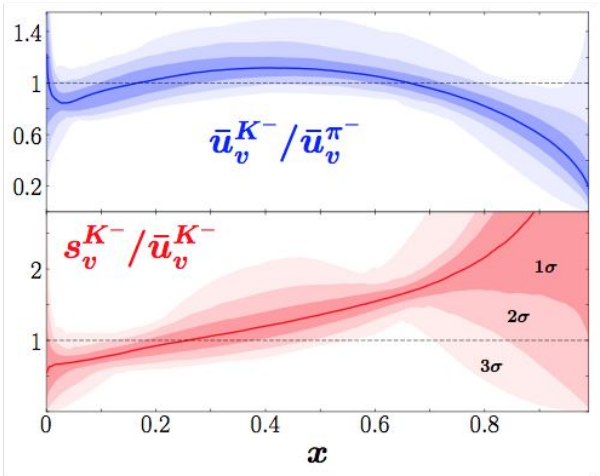
First simultaneous global QCD analysis of kaon and pion parton distributions with lattice QCD constraints

P. C. Barry,[1] Chueng-Ryong Ji,[2] W. Melnitchouk,[3] N. Sato,[3] and Fernanda Steffens[4, 5]

arXiv:2510.11979v1 [hep-ph] 13 Oct 2025

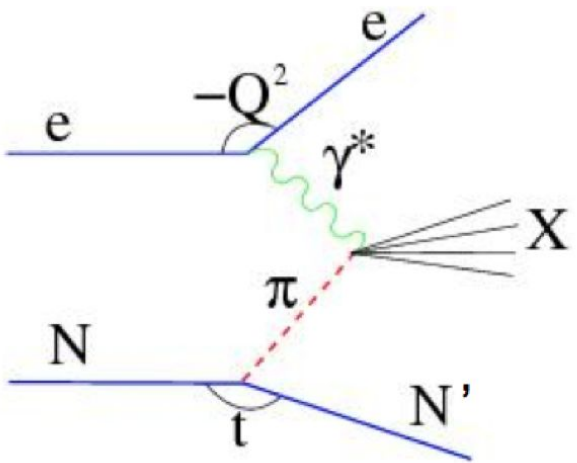






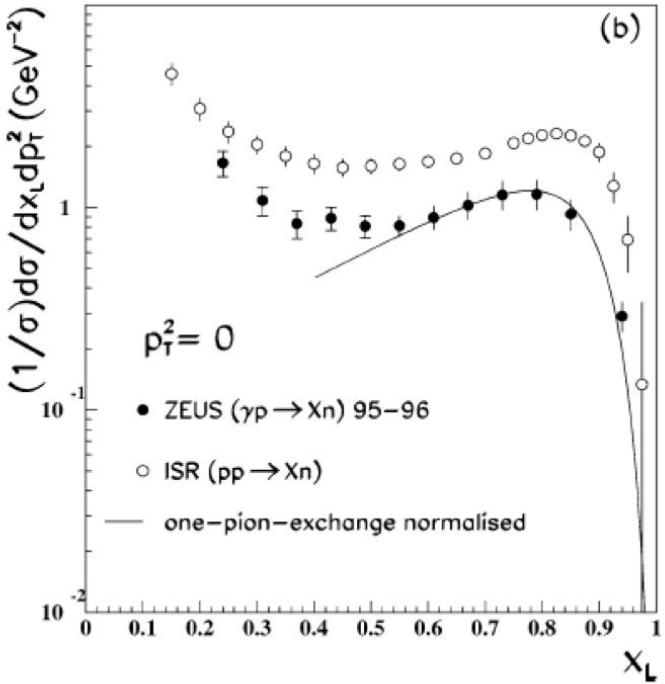
Measurement of Tagged Deep Inelastic Scattering (TDIS)

C.Keppel (Contact person)



$$e + p(or\ n) \rightarrow e' + p + X$$

$$e + D \rightarrow e' + p + p + X$$



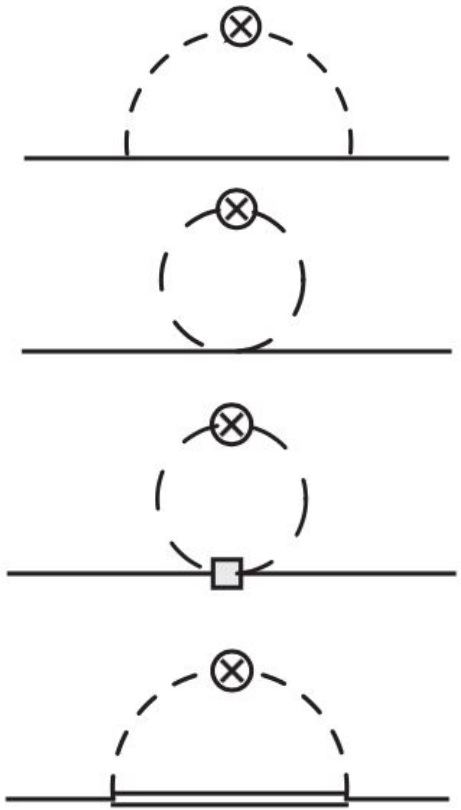
Leading neutron production in e^+p collisions at HERA

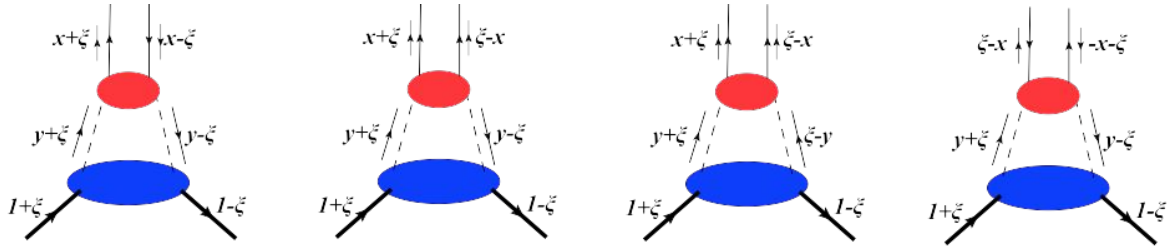
ZEUS Collaboration, NPB 637 (2002) 3–56

Nonlocal chiral contributions to GPDs of the proton at nonzero skewness:

arXiv:2406.03412v1 [hep-ph]; PRD110, 054049 (2024)

Z. Gao, F. He, C.-R. Ji, W. Melnitchouk, Y. Salamu, and P. Wang





$$H_q^{(\mathrm{rbw})}(x,\xi,t) = \begin{cases} \displaystyle\int_x^1 \frac{\mathrm{d}y}{y} f_{\phi B}^{(\mathrm{rbw})}(y,\xi,t)\, H_{q/\phi}\Big(\frac{x}{y},\frac{\xi}{y},t\Big), & [\xi < x < y] \\ \displaystyle\int_\xi^1 \frac{\mathrm{d}y}{y} f_{\phi B}^{(\mathrm{rbw})}(y,\xi,t)\, H_{q/\phi}\Big(\frac{x}{y},\frac{\xi}{y},t\Big), & [x < \xi < y] \\ \displaystyle\int_{-\xi}^{\xi} \frac{\mathrm{d}y}{2y} f_{\phi B}^{(\mathrm{rbw})}(y,\xi,t) \frac{1}{\pi}\int_{s_0}^{\infty} \mathrm{d}s\, \frac{\mathrm{Im}\Phi_{q/\phi}\big(\frac{1}{2}(1+\frac{x}{\xi}),\frac{1}{2}(1+\frac{y}{\xi}),s\big)}{s-t+i\epsilon}, & [|x|,|y| < \xi] \\ \displaystyle\int_{-x}^1 \frac{\mathrm{d}y}{y} f_{\phi B}^{(\mathrm{rbw})}(y,\xi,t)\, H_{q/\phi}\Big(\frac{x}{y},\frac{\xi}{y},t\Big), & [\xi < -x < y < 1] \end{cases}$$

Theoretical inputs

- **The valence quark GPD in pion**

Valence PDF in pion

M. Aicher, A. Schafer and W. Vogelsang, PRL105(2018)

Pion form factor

$$H_\pi^q(x,t) = q_v^\pi(x)\, F_\pi(t),$$

Parameterization

$$F_\pi(t) = \frac{1}{1 - t/\Lambda_\pi^2}.$$

M. Diehl, T. Feldmann , R. Jakob and P. Kroll, EPJC39 (2015)

Profile function

$$H_{q/\pi}(x,\xi,t)$$

$$= \int_{-1}^{1} \mathrm{d}\beta \int_{-1+|\beta|}^{1-|\beta|} \mathrm{d}\alpha \delta(x-\beta-\xi\alpha) h_b(\beta,\alpha) H_{q/\pi}(\beta,0,t)$$

$$+ \frac{\xi}{|\xi|} D_{q/\pi}\left(\frac{x}{\xi},t\right)\theta(\xi-|x|)$$

$$h_b(\beta,\alpha) = \frac{\Gamma(2b+2)}{2^{2b+1}\Gamma^2(b+1)} \frac{[(1-|\beta|)^2-\alpha^2]^b}{(1-|\beta|)^{2b+1}}$$

$$D_{q/\pi}(z,t) = \frac{15}{4} z(1-z^2) D_{q/\pi}(t)$$

A. V. Radyushkin, Phys. Lett. B **449**, 81 (1999).
I. V. Musatov and A. V. Radyushkin, Phys. Rev. D **61**, 074027 (2000).

Glimpse of Link between QCD and LFQM

B.Ma and C. Ji, PRD104, 036004 (2021) ; INPC2025

Large N_C QCD in 1+1 dim. ('tHooft Model)

$$\mathcal{L} = -\frac{1}{4} F^a_{\hat{\mu}\hat{\nu}} F^{\hat{\mu}\hat{\nu}\,a} + \bar{\psi}(i\gamma^{\hat{\mu}} D_{\hat{\mu}} - m)\psi$$

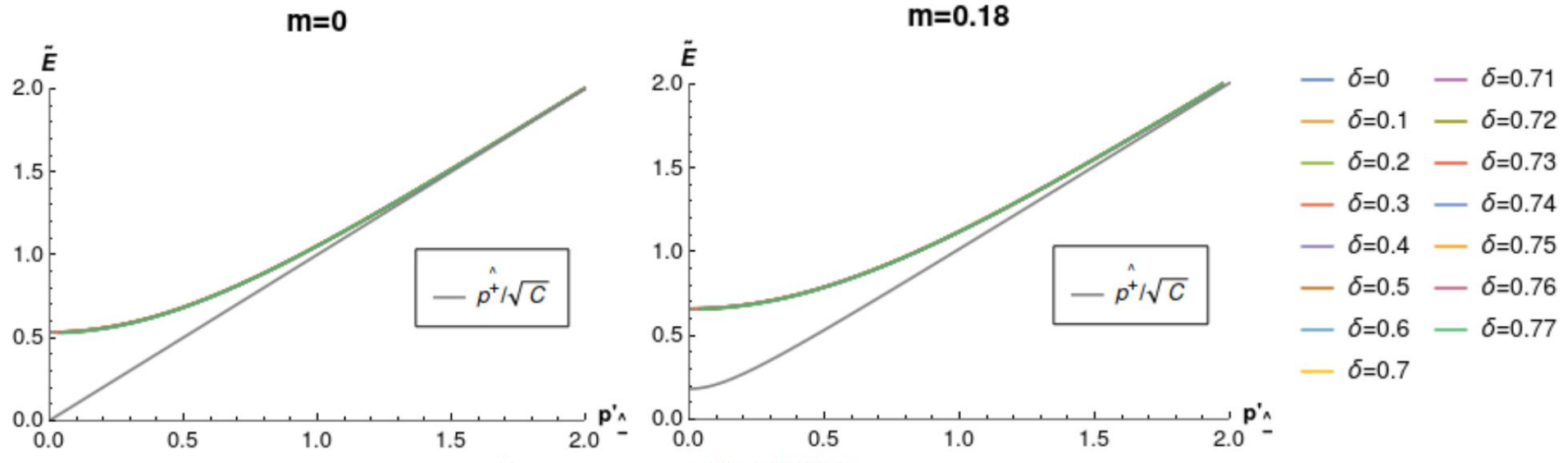
$$D_{\hat{\mu}} = \partial_{\hat{\mu}} - igA^a_{\hat{\mu}} t_a$$

$$F^a_{\hat{\mu}\hat{\nu}} = \partial_{\hat{\mu}} A^a_{\hat{\nu}} - \partial_{\hat{\nu}} A^a_{\hat{\mu}} + g f^{abc} A^b_{\hat{\mu}} A^c_{\hat{\nu}}$$

'tHooft Coupling $\lambda = \dfrac{g^2 (N_c - 1/N_c)}{4\pi}$ and mass m

$$g \to 0, N_C \to \infty; \lambda \to finite$$

Mass Gap Solutions



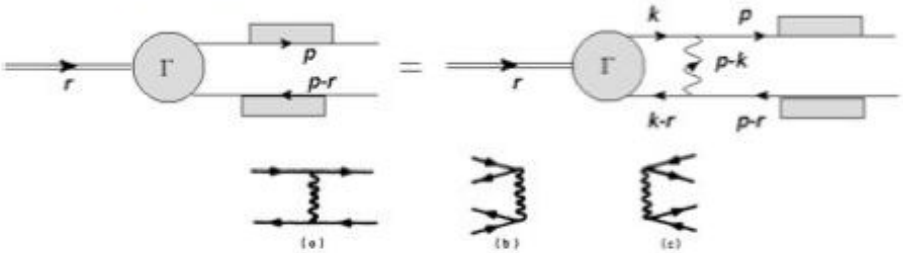
$$\tilde{E}(0) = \frac{F(0)E(0)}{\sqrt{\mathbb{C}}} = M(0)$$

m	0	0.045	0.18	0.749	1.00	2.11	4.23
$M(0)$	0.532778	0.563644	0.659112	1.10105	1.31167	2.30969	4.34358
$F(0)$	−0.495173	−0.584175	−0.987673	4.11079	2.17976	1.22134	1.05526

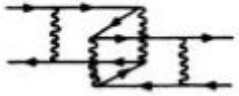
$$m \lesssim 0.56$$

BOUND-STATE EQUATION

$$\Gamma(r,p) = \frac{i\lambda}{2\pi}\int \frac{dk_- dk_+}{(p_- - k_-)^2} S(p)\gamma^+\Gamma(r,k)\gamma^+ S(p-r)$$



$$\left[-r_+ + \frac{-\mathbb{S}p_- + E(p_-)}{\mathbb{C}} + \frac{\mathbb{S}(p_- - r_-) + E(p_- - r_-)}{\mathbb{C}}\right]\hat{\phi}_+(r_-,p_-)$$
$$= \lambda\int \frac{dk_-}{(p_- - k_-)^2}\left[C(p_-,k_-,r_-)\hat{\phi}_+(r_-,k_-) - S(p_-,k_-,r_-)\hat{\phi}_-(r_-,k_-)\right],$$
$$\left[r_+ + \frac{-\mathbb{S}(p_- - r_-) + E(p_- - r_-)}{\mathbb{C}} + \frac{\mathbb{S}p_- + E(p_-)}{\mathbb{C}}\right]\hat{\phi}_-(r_-,p_-)$$
$$= \lambda\int \frac{dk_-}{(p_- - k_-)^2}\left[C(p_-,k_-,r_-)\hat{\phi}_-(r_-,k_-) - S(p_-,k_-,r_-)\hat{\phi}_+(r_-,k_-)\right].$$



LFD

$$\left[\mathcal{M}^2 - \frac{m^2-2\lambda}{x} - \frac{m^2-2\lambda}{1-x}\right]\phi(x) = -2\lambda\int_0^1 \frac{dy}{(x-y)^2}\phi(y)$$

Bakamjian-Thomas Construction

B.Bakamjian and L.H.Thomas, Phys.Rev.92,1300(1953)

B.Keister and W.Polyzou, Adv.Nucl.Phys.20,225(1991)

$$[P^i, K^j] = i\delta_{ij} H$$

$$\{H, \mathbf{P}, \mathbf{J}, \mathbf{K}\} \Rightarrow \{M, \mathbf{P}, \mathbf{j}_c, \mathbf{X}_c\} \Rightarrow \{M_0, \mathbf{P}_0, \mathbf{j}_{c0}, \mathbf{X}_{c0}\} \Rightarrow \{M, \mathbf{P}_0, \mathbf{j}_{c0}, \mathbf{X}_{c0}\}$$

$$H = \sqrt{M^2 + \mathbf{P}^2};$$

$$\mathbf{K} = -\frac{1}{2}\{H, \mathbf{X}_c\}_+ - \frac{\mathbf{P} \times \mathbf{j}_c}{H + M};$$

$$\mathbf{J} = \mathbf{X}_c \times \mathbf{P} + \mathbf{j}_c.$$

$$P_0^\mu := P_1^\mu \otimes I_2 + I_1 \otimes P_2^\mu;$$

$$\mathbf{K}_0 := \mathbf{K}_1 \otimes I_2 + I_1 \otimes \mathbf{K}_2;$$

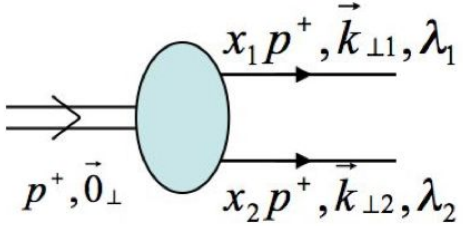
$$\mathbf{J}_0 := \mathbf{J}_1 \otimes I_2 + I_1 \otimes \mathbf{J}_2.$$

$$M := M_0 + V$$

$$\left[V, \{\mathbf{P}_0, \mathbf{j}_{c0}, \mathbf{X}_{c0}\}\right] = 0$$

Light-Front Quark Model(LFQM)

$$|Meson\rangle = \psi_{q\bar{q}}|q\bar{q}\rangle + \psi_{q\bar{q}g}|q\bar{q}g\rangle + \ldots$$
$$\approx \Psi_{Q\bar{Q}}|Q\bar{Q}\rangle,$$



$$P^- = p_Q^- + p_{\bar{Q}}^-$$

$$M_0^2 = \frac{m_Q^2 + \boldsymbol{k}_\perp^2}{x} + \frac{m_{\bar{Q}}^2 + \boldsymbol{k}_\perp^2}{1-x}$$

where

$$|Q\rangle = \psi_q^Q|q\rangle + \psi_{qg}^Q|qg\rangle + \ldots$$
$$|\bar{Q}\rangle = \psi_{\bar{q}}^{\bar{Q}}|\bar{q}\rangle + \psi_{\bar{q}g}^{\bar{Q}}|\bar{q}g\rangle + \ldots$$

Noninteracting "on-mass" shell Q & $\bar{Q}$ representation

$$\Psi_{Q\bar{Q}}(x_i, \vec{k}_{\perp i}, \lambda_i) = \Phi(x_i, \vec{k}_{\perp i})\chi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Radial — Spin-Orbit

The interaction between $Q\bar{Q}$

includes Coulomb, Confinement,
Spin-Spin, Spin-Orbit interactions.

$$M := M_0 + V_{Q\bar{Q}}$$

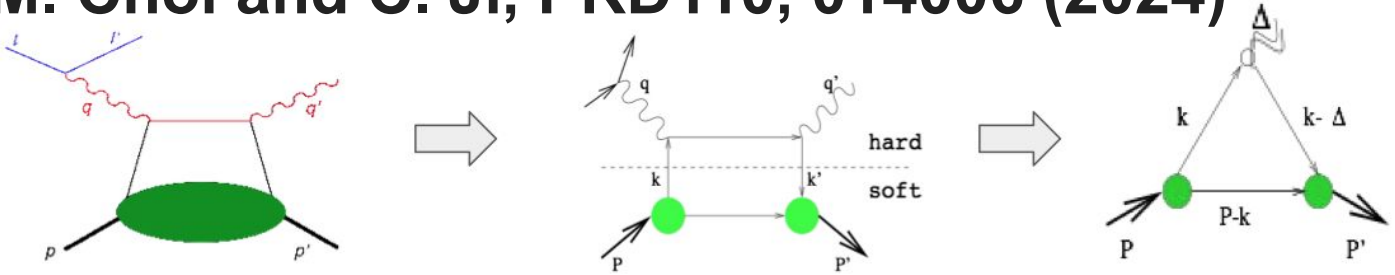
PRD59, 074015(99); PLB460, 461(99) by HMC and CRJ;
PRC92. 055203(2015) by HMC. CRI. Z. Li. and H. Ryu
PRD106, 014009(2022) by A. J. Arifi, HMC, and CRJ
HMC and CRJ, PRD110, 014006(2024)

Interaction independent
Melosh transformation

$$J^{PC} = 0^{++}(f_0, a_0, \ldots)$$
$$0^{-+}(\pi, K, \eta, \eta', \ldots)$$
$$1^{--}(\rho, K^*, \omega, \phi, \ldots)$$

H.J. Melosh: PRD 9, 1095(1974)

H.-M. Choi and C. Ji, PRD110, 014006 (2024)



$$\langle P|J^+|P\rangle = 2P^+ \int dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \, |\phi(x,\mathbf{k}_\perp)|^2$$

$$f_1^q(x,\mathbf{k}_\perp) = \frac{1}{16\pi^3}|\phi(x,\mathbf{k}_\perp)|^2 \qquad \int dx \int d^2\mathbf{k}_\perp f_1^q(x,\mathbf{k}_\perp) = \int dx f_1^q(x) = 1$$

$$\langle P|J^\perp|P\rangle = \int dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \, |\phi(x,\mathbf{k}_\perp)|^2 \left(-\frac{2\mathbf{k}_\perp}{x}\right)$$

$$2\mathbf{k}_\perp f_3^q(x,\mathbf{k}_\perp) = \frac{1}{16\pi^3}|\phi(x,\mathbf{k}_\perp)|^2 \left(-\frac{2\mathbf{k}_\perp}{x}\right) \qquad x f_3^q(x,\mathbf{k}_\perp) = -f_1^q(x,\mathbf{k}_\perp)$$

$$f(x) = \int d^2p_T f(x,p_T) \qquad 2\int dx f_4^q(x) = \int dx f_1^q(x) = 1$$

Off-shell pion properties: electromagnetic form factors and light-front wave functions

Jurandi Leão,[1,2] J. P. B. C. de Melo,[1] T. Frederico,[3] Ho-Meoyng Choi,[4] and Chueng-Ryong Ji[5]

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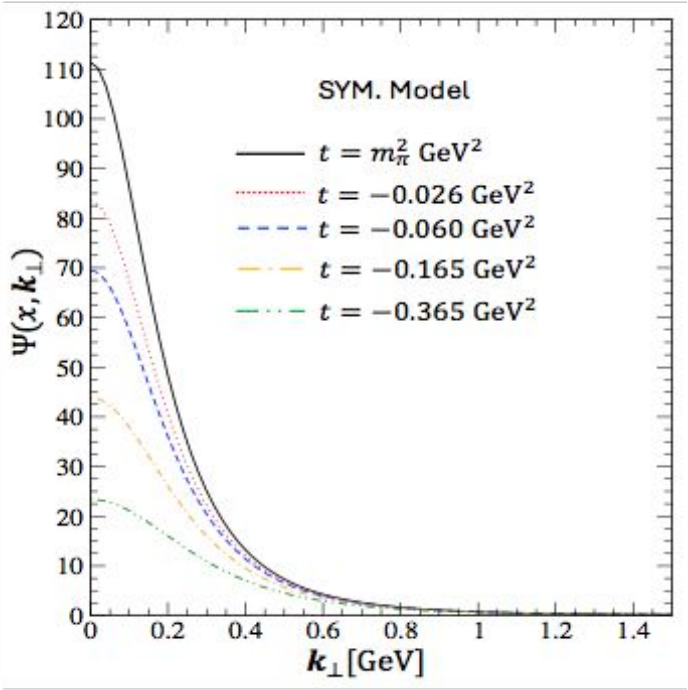

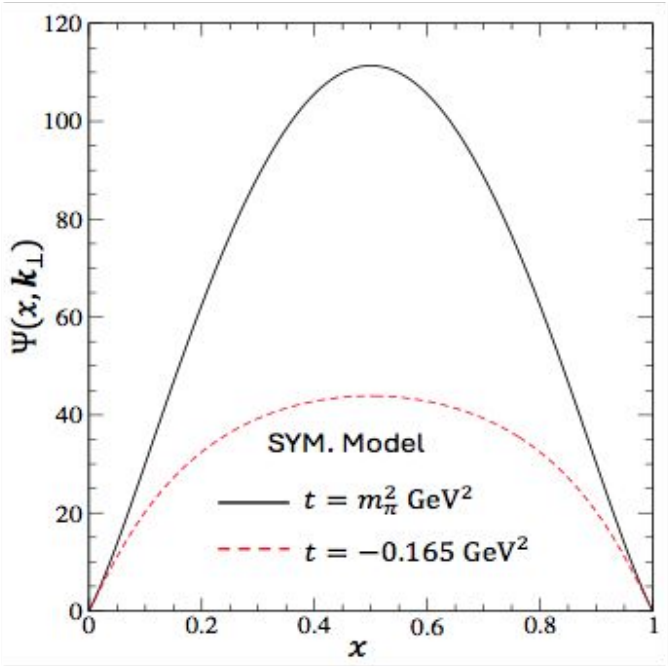

Conclusions and Outlook

- Meson structure studies of LFQM provide useful tools to study the nucleon structures via the convolution with the splitting functions computed by the Chiral EFT.
- Analysis of LNA behaviors in Chial EFT provides a useful link to QCD.
- Link between QCD and LFQM seems also feasible as exemplified by the mass gap solution in the ‘t Hooft model interpolation between IFD and LFD.
- Convoluting Chiral EFT with LFQM appears useful for the preparation of a comprehensive analysis of potential impacts for hadron structure studies envisioned in future facilities including JLab, EIC and AMBER.