Generalized Parton Distributions of Photon and ρ meson

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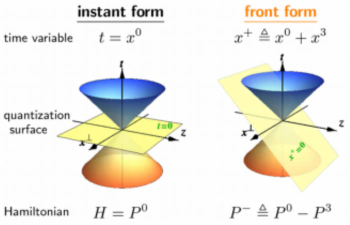
Content

Light Front Dynamics



$$p^\mu p_\mu = m^2 \Rightarrow \begin{cases} p^0 = \sqrt{\vec{p}^2 + m^2}, & \text{equal-time} \\ p^- = (\vec{p}_\perp^2 + m^2)/p^+, & \text{light-front} \end{cases}$$

- light-front energy: p^-

- momenta: (p^+, p^1, p^2), where $p^\mp = p^0 \mp p^3$
- LFWFs = equal-time WFs in IMF $\neq$ equal-time WFs in rest frame
- Light-front wavefunctions (LFWFs) are frame independent and provides intrinsic information of the structure of hadrons:

"*Hadron Physics without LFWFs is like Biology without DNA!*"

— Stanley J. Brodsky

- Boost invariant/frame independence: access to internal structure of hadrons.
- Direct access to partonic observables.
- Light-cone dominance in DIS/OPE
- Simplification of relativistic many-body dynamics

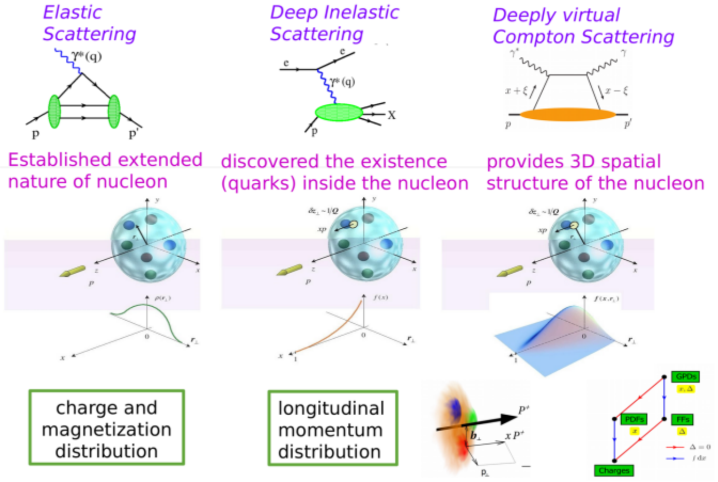
$$H_0^{IF} = \sum_i \sqrt{\vec{p}_i^2 + m_i^2} \leftrightarrow H_0^{LF} = \sum_i \frac{k_{\perp i}^2 + m_i^2}{x_i}$$

- Light-front physics underlines hadron structure measured in high-energy scattering experiments.
- Light front wave functions provide the full quantum information of hadrons.

Light-Front in QCD

- QCD provides a fundamental description of hadronic and nuclear structure and dynamics in terms of their quark and gluon degrees of freedom.
- One of the most outstanding problem of particle physics is to unravel the internal structure of the hadrons.
- Light-Front QCD is an ab initio approach to strongly interacting system. It is a Hamiltonian method, formulated in Minkowinski space rather than Euclidean space. The essential ingredient is Dirac's front form of Hamiltonian dynamics, where one quantize the theory at fixed-cone time $\tau = t + z/c$ rather than ordinary time t.

Probing 3D structure



Parton Distribution Functions (PDFs)

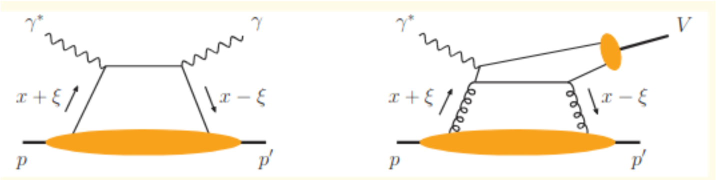
- PDFs were introduced by Feynman in 1969.
- PDFs $f(x)$ imparts an information about the probability of finding a parton carrying a longitudinal momentum fraction x inside the hadron.

But how partons are distributed in the plane transverse to the motion of hadron?

This missing information was then compensated in generalized parton distributions (GPDs).

Generalized Parton Distributions

- An essential tool to investigate hadron structure is the study of DIS, where individual quarks and gluons are resolved.



arXiv:1212.1701

- From parton densities one can extract the distribution of longitudinal momentum carried by the quarks, antiquarks and gluons.

Photon GPDs

- In standard model, photon is considered as one of the gauge boson and it is fascinating object in QCD studies.
- Photon has a rich structure which is still poorly explored.
- Photon structure function $F_2^{\gamma}(x, Q^2)$ can be interpreted as the momentum weighted sum of the quark distributions $q(x, Q^2)$ within the photon,
- Parton distribution of photon have been the subject of much work since the seminal papers by Witten and Klasen.

-Witten, Nucl. Phys. B 120 (1977) 189
-M. Klasen, Rev. Mod. Phys.74 (2002) 1221

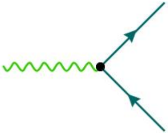
- Structure of photon can be accessed through DVCS process $\gamma^*\gamma \rightarrow \gamma\gamma$.

Photon GPDs

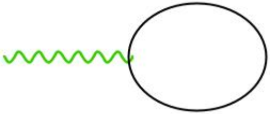
- the parton distributions in the photon have turned out to be of experimental importance in a number of accessible processes, both in $e^- e^+$ annihilation and photoproduction.
- Photon structure function, could be split up into two parts, a point like part and a hadronic part.

$$F_2(x, Q^2) = F_2^{PL}(x, Q^2) + F_2^{HAD}(x, Q^2)$$

- pointlike is calculable in QCD and hadron like is normally estimated using the Vector Meson Dominance (VMD), where the photon is represented as a sum of lowest mass vector meson states ρ, ω, ϕ.



(a) Pointlike Photon quark coupling



(b) VMD hadronic coupling (ρ, ω, ϕ)

- In QCD (light-front picture), the photon state can be expanded as:

$$|\gamma\rangle = |\gamma\rangle_{bare} + |q\bar{q}\rangle_{point-like} + \ldots\ldots + |\rho\rangle + |\omega\rangle + \ldots.$$

- **Point-like part**
- perturbative $\gamma \to q\bar{q}$
- wave function is calculable.
- Falls slowly (hard tail)
- **Hadronic part**
- $\gamma \to \rho, \omega, \phi$
- needs non-perturbative wave function
- Gaussian type, power-law wave functions

Work by Pire and his Colleagues

Available online at www.sciencedirect.com
ScienceDirect
Physics Letters B 645 (2007) 153–160
PHYSICS LETTERS B
www.elsevier.com/locate/physletb

Deeply virtual Compton scattering on a photon and generalized parton distributions in the photon

S. Friot [a], B. Pire [b,*], L. Szymanowski [b,c,d]

- Investigate deeply virtual Compton scattering on a photon target.
- Interpret results as a factorized amplitude with handbag diagrams.
- Propose anomalous generalized parton distributions for the photon.
- Identify $log(Q^2)$ dependence in GPDs and evolution equations.

Photon GPDs

- The correlator for the photon GPD can be written as

$$F^q = \int \frac{dy^-}{8\pi} e^{\frac{\iota x P^+ y^-}{2}} \langle \gamma(P'), \lambda' | \bar{\psi}(0)\gamma\psi(y^-)|\gamma(P), \lambda\rangle_{y^+=0},$$

where $\gamma(P)$ is a real photon state; P and P' are the initial and final momenta and λ and λ' are the helicities of the initial and final photon respectively.

- Here, we take $\Gamma = \gamma^+$, contributes for unpolarized photon.
- The two-particle Fock state expansion for photon ($J_z = \pm 1$) is defined as

$$|\gamma(P^+, \pm)\rangle = \int \frac{dx \; d^2\mathbf{k}_\perp}{16\pi^3} \Big[\psi^{\pm}_{\uparrow\uparrow}(x, \mathbf{k}_\perp)|xP^+, \mathbf{k}_\perp, \uparrow, \uparrow\rangle + \psi^{\pm}_{\uparrow\downarrow}(x, \mathbf{k}_\perp)$$

$$|xP^+, \mathbf{k}_\perp, \uparrow, \downarrow\rangle + \psi^{\pm}_{\downarrow\uparrow}(x, \mathbf{k}_\perp)|xP^+, \mathbf{k}_\perp, \downarrow, \uparrow\rangle + \psi^{\pm}_{\downarrow\downarrow}(x, \mathbf{k}_\perp)|xP^+, \mathbf{k}_\perp, \downarrow, \downarrow\rangle \Big]$$

- B. W. Xiao PRD 68 034020, M. Diehl Phys. Rep. 2003

- The light-front wave functions for photon with spin-up and spin-down are expressed as

$$\psi^{+}_{\uparrow\uparrow}(x,\mathbf{k}_\perp) = -\frac{\sqrt{2}\,m}{x(1-x)}\,\varphi_\gamma$$

$$\psi^{+}_{\uparrow\downarrow}(x,\mathbf{k}_\perp) = -\sqrt{2}\,\frac{k_1+ik_2}{1-x}\,\varphi_\gamma$$

$$\psi^{+}_{\downarrow\uparrow}(x,\mathbf{k}_\perp) = \sqrt{2}\,\frac{k_1+ik_2}{x}\,\varphi_\gamma$$

$$\psi^{+}_{\downarrow\downarrow}(x,\mathbf{k}_\perp) = 0$$

$$\psi^{-}_{\uparrow\uparrow}(x,\mathbf{k}_\perp) = 0$$

$$\psi^{-}_{\uparrow\downarrow}(x,\mathbf{k}_\perp) = -\sqrt{2}\,\frac{k_1+ik_2}{x}\,\varphi_\gamma$$

$$\psi^{-}_{\downarrow\uparrow}(x,\mathbf{k}_\perp) = \sqrt{2}\,\frac{k_1+ik_2}{1-x}\,\varphi_\gamma$$

$$\psi^{-}_{\downarrow\downarrow}(x,\mathbf{k}_\perp) = -\frac{\sqrt{2}\,m}{x(1-x)}\,\varphi_\gamma$$

where

$$\varphi(x,\mathbf{k}_\perp) = \frac{e_q}{\lambda^2 - \frac{{\mathbf{k}_\perp}^2+m^2}{x} - \frac{{\mathbf{k}_\perp}^2+m^2}{1-x}}$$

and where m and λ are constituent quark mass and photon mass respectively. Each configuration satisfies the spin sum rule $J^z = S^z_q + S^z_{\bar{q}} + l^z = 1$.

- Since photon is considered as the two-particle constituent, both the quark and antiquark GPDs will contribute.
- GPDs have the support in the interval $x \epsilon [-1, 1]$, which corresponds to DGLAP and ERBL regions.
- We restrict our calculations in DGLAP region. For quark and antiquark contributions, the respective regions $0 < x < 1$ and $-1 < x < 0$ are taken into account.
- The overlap representation of LFWFs is used to evaluate spin non-flip and spin flip GPDs of photon.

Non-zero skewness

$$H^q(x,\zeta,t) = \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}\Big[\psi^{*+}_{\uparrow\uparrow}(x',\mathbf{k}'_\perp)\psi^{+}_{\uparrow\uparrow}(x,\mathbf{k}_\perp) + \psi^{*+}_{\uparrow\downarrow}(x',\mathbf{k}'_\perp)\psi^{+}_{\uparrow\downarrow}(x,\mathbf{k}_\perp) + \psi^{*+}_{\downarrow\uparrow}(x',\mathbf{k}'_\perp)\psi^{+}_{\downarrow\uparrow}(x,\mathbf{k}_\perp) + \psi^{*+}_{\downarrow\downarrow}(x',\mathbf{k}'_\perp)\psi^{+}_{\downarrow\downarrow}(x,\mathbf{k}_\perp)\Big]$$

where $\mathbf{k}'_\perp = \mathbf{k}_\perp - \frac{1-x}{1-\zeta}\mathbf{\Delta}_\perp$ and $x' = \frac{x-\zeta}{1-\zeta}$.

$$H^q(x,\zeta,t) = \int \frac{d^2\mathbf{k}_\perp}{8\pi^3} e_q^2\Big[(1-x-x'+2xx')\Big\{\frac{1}{L_1}+\frac{1}{L_2} + (2\lambda^2 x'(1-x') - 2m^2 - (1-x')^2\mathbf{\Delta}_\perp^2)\frac{1}{L_1L_2}\Big\} + \frac{2m^2}{L_1L_2}\Big]$$

$$\begin{aligned} \mathbf{\Delta}_\perp^2 &= -(1-\zeta)t - 4M^2\zeta^2 \\ L_1 &= {\boldsymbol{k}_\perp}^2 + m^2 - \lambda^2 x(1-x) \\ L_2 &= (\boldsymbol{k}_\perp - (1-x)\mathbf{\Delta}_\perp)^2 + m^2 - \lambda^2 x(1-x) \end{aligned}$$

$$\int \frac{d^2\mathbf{k}_\perp}{L_1} = \int \frac{d^2\mathbf{k}_\perp}{\mathbf{k}_\perp{}^2 + m^2 - \lambda^2 x(1-x)}$$

$$I_{PV}(\Delta) = \int d^2\mathbf{k}_\perp \left(\frac{1}{k^2 + \Delta} - \frac{1}{k^2 + M^2} \right)$$

$$= \pi \log \frac{M^2}{m^2 - \lambda^2 x(1-x)}$$

After adding finite reference scale μ:

$$I_{reg} = \lim_{M\to\infty} \left[I_{PV}(\Delta; M) - I_{PV}(\mu^2; M) \right]$$

$$\int \frac{d^2\mathbf{k}_\perp}{L_1} = \int \frac{d^2\mathbf{k}_\perp}{L_2}$$

$$= \pi \log \left[\frac{\mu^2}{m^2 - \lambda^2 x(1-x)} \right]$$

Using the Feynmann parametrization technique

$$\int \frac{d^2\boldsymbol{k}_\perp}{L_1 L_2} = \int_0^1 d\alpha \int \frac{d^2\boldsymbol{k}_\perp}{\alpha L_1 + (1-\alpha)L_2}$$

$$= \pi \int_0^1 \frac{d\alpha}{\alpha(1-\alpha)(1-x')^2\boldsymbol{\Delta}_\perp^2 + m^2 - \alpha\lambda^2 x(1-x) - (1-\alpha)\lambda^2 x'(1-x')}$$

In the limit, $\zeta = t = \lambda = 0$, we get

$$H(x,0,0) = \frac{e^2}{4\pi^2}(1-2x+2x^2) log \frac{\mu^2}{m^2}$$

This results matches exactly with the result of PLB 645 (2007) 153

$$H_1^q(x,\xi,0) = \frac{N_C e_q^2}{4\pi^2}\left[\theta(x-\xi)\frac{x^2+(1-x)^2-\xi^2}{1-\xi^2} + \theta(\xi-x)\theta(\xi+x)\frac{x(1-\xi)}{\xi(1+\xi)} - \theta(-x-\xi)\frac{x^2+(1+x)^2-\xi^2}{1-\xi^2}\right]\log\frac{Q^2}{m^2},$$

Results

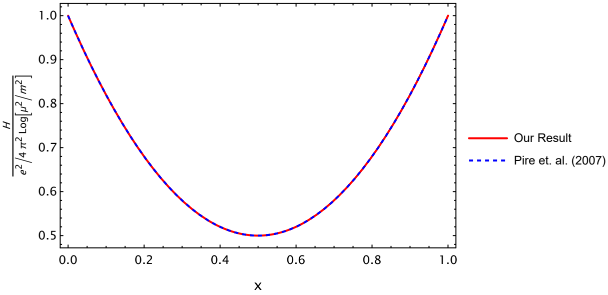


Figure 1: Result for the photon PDF. Good agreement with PLB 645 (2007).

Generalized Parton Distributions

- The five vector meson GPDs for the spin-one hadron are defined through the nonlocal matrix elements of the vector current on the LF as

$$V_{m_J,m'_J}(x,\xi,t) \triangleq \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p', J=1, m'_J | \bar{\psi}\left(-\frac{z^-}{2}\right)\gamma^+\psi\left(\frac{z^-}{2}\right) | p, J=1, m_J\rangle \Big|_{z^+=0,\mathbf{z}_\perp=\mathbf{0}_\perp}$$
$$= -(\epsilon'^*\cdot\epsilon)H_1(x,\xi,t) + \frac{(\epsilon\cdot n)(\epsilon'\cdot P)+(\epsilon'^*\cdot n)(\epsilon\cdot P)}{P\cdot n}H_2(x,\xi,t) - \frac{(\epsilon\cdot P)(\epsilon'^*\cdot P)}{2M^2}H_3(x,\xi,t)$$
$$+ \frac{(\epsilon\cdot n)(\epsilon'\cdot P)-(\epsilon'^*\cdot n)(\epsilon\cdot P)}{P\cdot n}H_4(x,\xi,t) + \left[4M^2\frac{(\epsilon\cdot n)(\epsilon'^*\cdot n)}{(P\cdot n)^2}\right]H_5(x,\xi,t).$$

-PRC 99, 035208 (2019)

$$H_1(x,\xi,t) = \frac{1}{3}\left[V_{0,0} - 2(\tau-1)V_{+,+} + 2\sqrt{2\tau}V_{+,0} + 2V_{+,-}\right]$$
$$H_2(x,\xi,t) = 2V_{+,+} - \frac{2}{\sqrt{2\tau}}V_{+,0}$$
$$H_3(x,\xi,t) = -\frac{V_{+,-}}{\tau}$$
$$H_4(x,\xi,t) = 0$$
$$H_5(x,\xi,t) = V_{0,0} - (1+2\tau)V_{+,+} + 2\sqrt{2\tau}V_{+,0} - V_{+,-}$$

- In the present work, we consider ρ meson in its leading state $|q\bar{q}\rangle$ in the light front quark model.

  - LFQM is quite successful in explaining the various electroweak properties of heavy mesons compared with experimental data.

    -PLB 349 393 (1995)
    -PRD 59 074015 (1999)
    -PRD 65 116001 (2002)

  - Successful in obtaining distribution amplitudes, decay constant and radiative decays for mesons.

    -PRD 75 034019 (2007)
    -PRD 68 054026 (2003)

  - In this model, meson is represented as

$$|M\rangle = \psi^M_{q\bar{q}} |q\bar{q}\rangle$$

$$\psi^M_{q\bar{q}} = \sqrt{\frac{\partial k_z}{\partial x}} \phi(x, \mathbf{k}_\perp) \mathcal{R}(x, \mathbf{k}_\perp, \lambda_q, \lambda_{\bar{q}})$$

  - Here $\phi(x, \mathbf{k}_\perp)$ is the radial wave function, $\frac{\partial k_z}{\partial x}$ is the Jacobian factor and $\mathcal{R}$ is the spin-orbit wave function obtained from the interaction-independent Melosh transformation .

- In terms of light-front Fock state decomposition, ρ meson can be defined as

$$\left|\psi_{\rho^+}\left(P^+, \vec{P}_\perp, S_z\right)\right\rangle = \int \frac{\mathrm{d}^2\vec{k}_\perp \mathrm{d}x}{16\pi^3}\varphi_{\rho^+}(x,\vec{k}_\perp) \times \bar{u}(k_1^+, k_1^-, \vec{k}_\perp, \lambda_1)\left(\gamma - \frac{k_1 - k_2}{\mathcal{M}+2m}\right)\cdot \epsilon_{S_z} v(k_2^+, k_2^-, -\vec{k}_\perp, \lambda_2) \left|xP^+, \vec{k}_\perp, \lambda_1, \lambda_2\right\rangle,$$

One can solve the above matrix element to get the following results

$$\begin{aligned}
\overline{u}_\uparrow(\gamma - \tfrac{k_1-k_2}{\mathcal{M}+2m})\cdot\epsilon_+ v_\uparrow &= -\frac{\sqrt{2}(m(\mathcal{M}+2m)+\vec{k}_\perp^2)}{\sqrt{x(1-x)}(\mathcal{M}+2m)},\\
\overline{u}_\uparrow(\gamma - \tfrac{k_1-k_2}{\mathcal{M}+2m})\cdot\epsilon_+ v_\downarrow &= -\frac{\sqrt{2}(k_1^+ + m)k^R}{\sqrt{x(1-x)}(\mathcal{M}+2m)},\\
\overline{u}_\downarrow(\gamma - \tfrac{k_1-k_2}{\mathcal{M}+2m})\cdot\epsilon_+ v_\uparrow &= \frac{\sqrt{2}(k_2^+ + m)k^R}{\sqrt{x(1-x)}(\mathcal{M}+2m)},\\
\overline{u}_\downarrow(\gamma - \tfrac{k_1-k_2}{\mathcal{M}+2m})\cdot\epsilon_+ v_\downarrow &= \frac{\sqrt{2}(k^R)^2}{\sqrt{x(1-x)}(\mathcal{M}+2m)},
\end{aligned}$$

for the ρ^+ meson with spin projection $S_z = +1$,

$$\begin{aligned}
\overline{u}_\uparrow(\gamma - \tfrac{k_1-k_2}{\mathcal{M}+2m})\cdot\epsilon_0 v_\uparrow &= -\frac{(k_2^+ - k_1^+)k^L}{\sqrt{x(1-x)}(\mathcal{M}+2m)},\\
\overline{u}_\uparrow(\gamma - \tfrac{k_1-k_2}{\mathcal{M}+2m})\cdot\epsilon_0 v_\downarrow &= -\frac{m(\mathcal{M}+2m)+2\vec{k}_\perp^2}{\sqrt{x(1-x)}(\mathcal{M}+2m)},\\
\overline{u}_\downarrow(\gamma - \tfrac{k_1-k_2}{\mathcal{M}+2m})\cdot\epsilon_0 v_\uparrow &= -\frac{m(\mathcal{M}+2m)+2\vec{k}_\perp^2}{\sqrt{x(1-x)}(\mathcal{M}+2m)},\\
\overline{u}_\downarrow(\gamma - \tfrac{k_1-k_2}{\mathcal{M}+2m})\cdot\epsilon_0 v_\downarrow &= -\frac{(k_1^+ - k_2^+)k^R}{\sqrt{x(1-x)}(\mathcal{M}+2m)},
\end{aligned}$$

for spin projection $S_z = 0$

$$\begin{aligned}
\overline{u}_\uparrow(\gamma - \tfrac{k_1-k_2}{\mathcal{M}+2m})\cdot\epsilon_- v_\uparrow &= \frac{\sqrt{2}(k^L)^2}{\sqrt{x(1-x)}(\mathcal{M}+2m)},\\
\overline{u}_\uparrow(\gamma - \tfrac{k_1-k_2}{\mathcal{M}+2m})\cdot\epsilon_- v_\downarrow &= -\frac{\sqrt{2}(k_2^+ + m)k^L}{\sqrt{x(1-x)}(\mathcal{M}+2m)},\\
\overline{u}_\downarrow(\gamma - \tfrac{k_1-k_2}{\mathcal{M}+2m})\cdot\epsilon_- v_\uparrow &= \frac{\sqrt{2}(k_2^+ + m)k^L}{\sqrt{x(1-x)}(\mathcal{M}+2m)},\\
\overline{u}_\downarrow(\gamma - \tfrac{k_1-k_2}{\mathcal{M}+2m})\cdot\epsilon_- v_\downarrow &= -\frac{\sqrt{2}(m(\mathcal{M}+2m)+\vec{k}_\perp^2)}{\sqrt{x(1-x)}(\mathcal{M}+2m)},
\end{aligned}$$

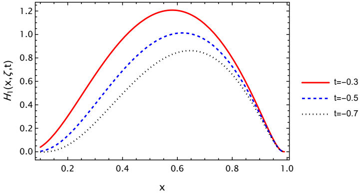
for spin projection $S_z = -1$

-arXiv:0706.2018

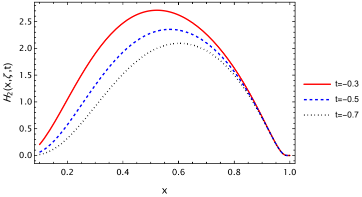
- Solving the matrix elements, will results in

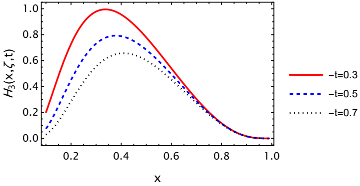
$$V_{S'_z,S_z} = \sum_{\lambda_1\lambda_2}\int \frac{d^2\boldsymbol{k}_\perp}{16\pi^3}\psi^*_{S'_z}(x',\boldsymbol{k}'_\perp)\psi_{S'_z}(x,\boldsymbol{k}_\perp)$$

Currently not writing the overlap expressions! (too lengthy!!)



1. GPDs results for fixed value of skewness=0.1 with different values of $-t$.
2. PDF sum rule verified!!





- Calculations are going on to get the results in the transverse position space.

What is Transverse Charge Density?

- A truly impressive level of experimental technique, effort, and ingenuity has been brought to measuring the electromagnetic form factors of the proton and neutron.
- These quantities are probability amplitudes that the nucleon can absorb a given amount of momentum and remain in the ground state.
- Should be important sources of information about the nucleon charge and magnetization densities.
- The textbook interpretation of these form factors is that their Fourier transforms are measurements of the charge and magnetization densities.
- But the initial and final nucleons have different momentum, and therefore different wave functions. This is because the relativistic boost operator that transforms a nucleon at rest into a moving one changes the wave function in a manner that depends on the momentum of the nucleon.

- The presence of different wave functions of the initial and final nucleons invalidates a probability or density interpretation.
- A proper determination of a charge density requires that the quantity be related to the square of a wave function.
- Proper determination of charge density requires measurement of matrix elements of density operators

$$\rho(x^-, \mathbf{b}_\perp) = J^+(x^-, \mathbf{b}_\perp) = \sum_q e_q \bar{q}(x^-, \mathbf{b}_\perp)\gamma^+ q(x^-, \mathbf{b}_\perp)$$

$$\rho(x^-, \mathbf{b}_\perp) = \langle p^+, R = 0, \lambda | \sum_q q_+^\dagger(x^-, \mathbf{b}_\perp) q_+(x^-, \mathbf{b}_\perp) | p^+, R = 0, \lambda \rangle$$

-D.E. Soper, PRD 15 (1977) 1141

- In the Drell-Yan frame, no momentum is transferred in the plus direction, so information regarding the x^- dependence of the density is not accessible

$$\rho(b) = \int \frac{QdQ}{2\pi} F_1(Q^2) J_0(Qb)$$

-Miller, Ann. Rev. Nucl. Part. Sci. 2010, 60:1-25

Transverse charge densities for proton and neutron

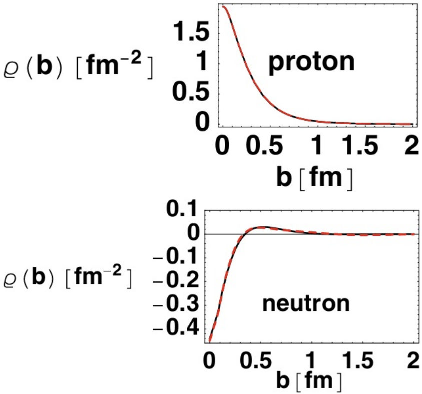


Figure 2: Proton and neutron charge densities. -G Miller PRL 99 112001

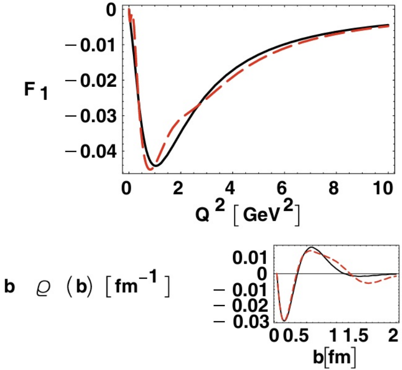


Figure 3: F_1 for neutron and long range structure of the charge density.

$$\rho(b) = \int \frac{QdQ}{2\pi} J_0(Qb) \frac{G_E(Q^2) + \tau G_M(Q^2)}{1+\tau}; \tau = \frac{Q^2}{4M^2}$$

Electromagnetic form factor of the ρ meson

- Form factors of meson can be expressed as

$$\langle P', \Lambda' | J^\mu | P, \Lambda \rangle = -\epsilon^*_{\Lambda'} \cdot \epsilon_\Lambda (P+P')^\mu F_1(Q^2) + (\epsilon^\mu_{\Lambda'} q \cdot \epsilon^*_{\Lambda'} - \epsilon^{*\mu}_{\Lambda'} q \cdot \epsilon_\Lambda) F_2(Q^2) + \frac{(\epsilon^*_{\Lambda'} \cdot q)(\epsilon_\Lambda \cdot q)}{2M^2} (P+P')^\mu F_3(Q^2),$$

-Berger, PRL 87 142302

$q = p - p'$ and $\epsilon_h[\epsilon'_h]$ is the polarization vector of the initial[final] meson with physical mass M_v

- The co-variant form factors of spin-1 hadron can be determined by the plus component of current

$$I^+_{h'h}(0) = \langle P', h' | J^+ | P, h \rangle$$

- Current matrix element is constrained by the invariance under the LF parity and time reversal and therefore reduce to four elements

$$I^+_{++}, I^+_{+-}, I^+_{+0} \text{ and } I^+_{00}$$

- In practical computation, instead of Lorentz invariant form factors $F_i(Q^2)$, the $G_C(Q^2)$ physical charge, $G_M(Q^2)$ magnetic and $G_Q(Q^2)$ quadrupole form factors are often used.

$$G_C = F_1 + \frac{2}{3}\kappa G_Q$$
$$G_M = -F_2$$
$$G_Q = F_1 + F_2 + (1+\kappa)F_3$$
$$\kappa = \frac{Q^2}{4M_v^2}$$

- At zero momentum transfer

$$eG_C(0) = e, \quad \text{(Charge)}$$
$$eG_M(0) = 2M_v\mu, \quad \text{(Magnetic Moment)}$$
$$-eG_Q(0) = M_v^2 Q. \quad \text{(Quadrupole Moment)}$$

- In literature, there are two types of prescription are available, for example Grach and Kondrayutak (GK) and Brodsky and Hiller (BH).

$$G_C^{GK} = \frac{1}{2P^+}\left[\frac{(3-2\kappa)}{3}I^+_{++} + \frac{4\kappa}{3}\frac{I^+_{+0}}{\sqrt{2\kappa}} + \frac{1}{3}I^+_{+-}\right]$$

$$G_M^{GK} = \frac{2}{2P^+}\left[I^+_{++} - \frac{1}{\sqrt{2\kappa}}I^+_{+0}\right]$$

$$G_Q^{GK} = \frac{1}{2P^+}\left[-I^+_{++} + 2\frac{I^+_{+0}}{\sqrt{2\kappa}} - \frac{I^+_{+-}}{\kappa}\right]$$

$$G_C^{BH} = \frac{1}{2P^+(1+2\kappa)}\left[\frac{(3-2\kappa)}{3}I^+_{00} + \frac{16\kappa}{3}\frac{I^+_{+0}}{\sqrt{2\kappa}} + \frac{2}{3}(2\kappa-1)I^+_{+-}\right]$$

$$G_M^{BH} = \frac{2}{2P^+(1+2\kappa)}\left[I^+_{00} + \frac{2\kappa-1}{\sqrt{2\kappa}}I^+_{+0} - I^+_{+-}\right]$$

$$G_Q^{BH} = \frac{-1}{2P^+(1+2\kappa)}\left[I^+_{00} - 2\frac{I^+_{+0}}{\sqrt{2\kappa}} + I^+_{+-}\frac{1+\kappa}{\kappa}\right]$$

- However, in this work we choose **BH-prescription** as it has (0,0) component which gives the most dominant contribution in the high momentum perturbative QCD (PQCD) region.

- In LFQM, the physical form factors are obtained from the $I^{+}_{\Lambda'\Lambda}$ which is defined as

$$I^{+}_{\Lambda'\Lambda} = \int \frac{dx}{2(1-x)} \int d^2\mathbf{k}_\perp \sqrt{\frac{\partial k'_z}{\partial x}\frac{\partial k_z}{\partial x}}\,\phi^*(x,\mathbf{k}_{\perp f})\phi(x,\mathbf{k}_{\perp i})\frac{(S^{+}_{\Lambda'\Lambda})}{M_{oi}M_{of}}$$

- $S^{+}_{\Lambda'\Lambda}$ is defined in PRD 70 053015.
- Radial wave function is defined as

$$\phi(x,\mathbf{k}^2) = \sqrt{\frac{1}{\pi^3/2\beta^3}}\,exp(-\mathbf{k}^2/2\beta^2)$$

$$\mathbf{k}^2 = {\mathbf{k}_\perp}^2 + k_z^2, k_z = (x-1/2)M_o$$

$$M_{oi}^2 = M_{of}^2 = M_o^2 = \frac{{\mathbf{k}_\perp}^2 + m^2}{x(1-x)}$$

- The model parameters used in this study are m=0.22 GeV, β =0.3659 GeV and M_v =0.77 GeV.
- These model parameters were obtained from the linear confining potential of QCD motivated Hamiltonian in LFQM.

Transverse charge density and Helicity form factors

- Charge density in transverse plane as a standard interpretation can be obtained by two- dimensional Fourier transform of form factor

$$\rho^{\rho}_{\lambda}(b) = \int \frac{d^2\mathbf{q}_\perp}{(2\pi)^2} e^{-i\mathbf{q}_\perp\cdot\mathbf{b}_\perp} G^{+}_{\lambda'\lambda}(Q^2),$$
$$= \int_0^\infty \frac{dQ\, Q}{2\pi} J_0(Qb) G^{+}_{\lambda'\lambda}(Q^2),$$

$G^{+}_{\lambda'\lambda}(Q^2)$ is the form factor related with the matrix elements of electromagnetic current $J^+(0)$ sandwich between two ρ meson states.

Here λ and λ' are the initial and final ρ meson states respectively.

$$\langle P^+, \frac{\mathbf{q}_\perp}{2}, \lambda' | J^+(0) | P^+, -\frac{\mathbf{q}_\perp}{2}, \lambda \rangle = 2P^+ G^{+}_{\lambda'\lambda}(Q^2)$$

- One can define the helicity- conserving form factor(G^{+}_{++} ,G^{+}_{00}) and helicity non-conserving form factors (G^{+}_{0+}, G^{+}_{-+}) respectively, in terms of G_C, G_M and G_Q

$$G^{+}_{++} = \frac{1}{1+\kappa}\left[G_C + G_M + \frac{\kappa}{3}G_Q\right],$$

$$G^{+}_{00} = \frac{1}{1+\kappa}\left[(1-\kappa)G_C + 2\kappa G_M - \frac{2\kappa}{3}(1+2\kappa)G_Q\right],$$

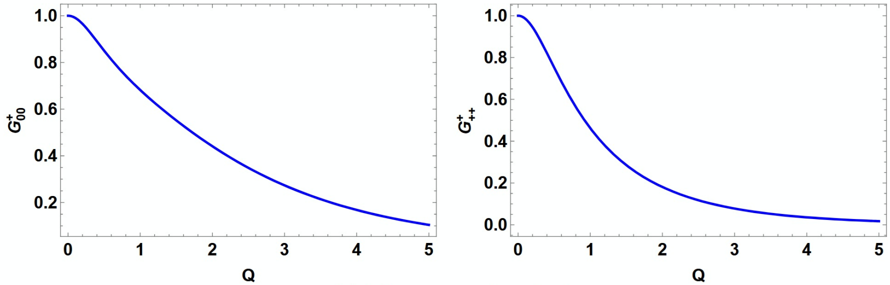
$$G^{+}_{0+} = -\frac{\sqrt{2\kappa}}{1+\kappa}\left[G_C - \frac{1}{2}(1-\kappa)G_M + \frac{\kappa}{3}G_Q\right],$$

$$G^{+}_{-+} = \frac{\kappa}{1+\kappa}\left[G_C - G_M - \left(1+\frac{2\kappa}{3}\right)G_Q\right]$$

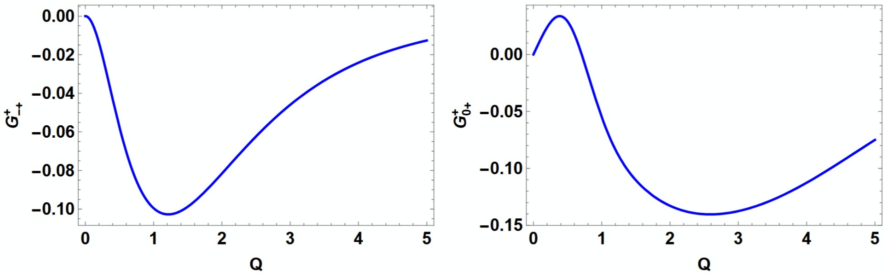
-PRD 70 053015 (2004)

Here, again $\kappa = Q^2/4M_v^2$

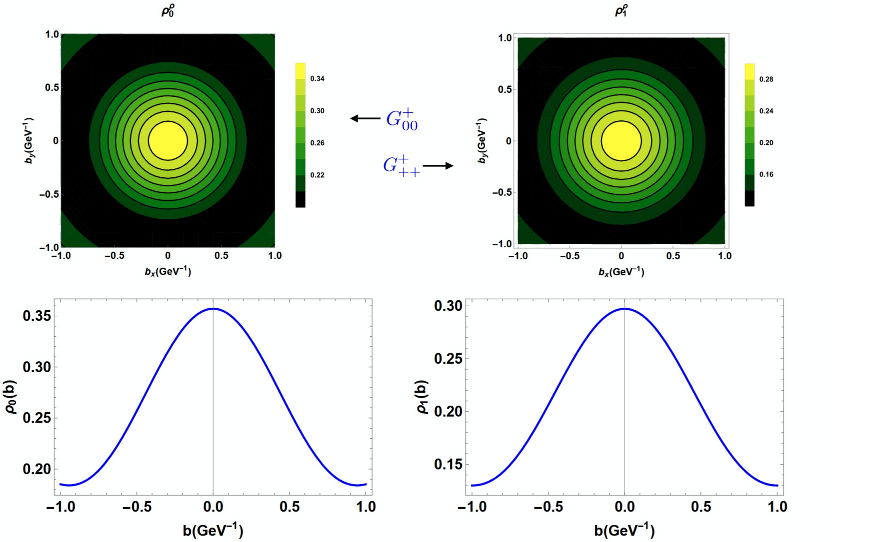
- G_C, G_M and G_Q are already calculated by C.R. Ji in PRD 70 053015, we have extracted the helicity form factors and charge density from it.



Helicity conserve form factors



Results for unpolarized ρ meson (PRD 99 014039 (2019)



Transversely Polarized ρ meson

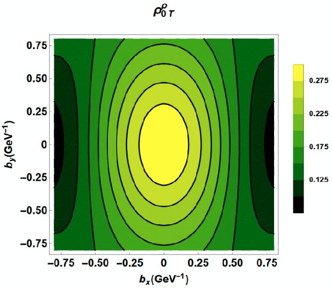
- We also consider the transversely polarized ρ meson state which provides information about dipole and quadrupole moments.
- Transverse charge density for transversely polarized meson can be defined as
$$\rho^{\rho}_{s_\perp T}(\mathbf{b}_\perp) = \int \frac{d^2\mathbf{q}_\perp}{(2\pi)^2} e^{-i\mathbf{q}_\perp \cdot \mathbf{b}_\perp} \frac{1}{2P^+} \langle P^+, \frac{\mathbf{q}_\perp}{2}, s_\perp | J^+ | P^+, -\frac{\mathbf{q}_\perp}{2}, s_\perp \rangle,$$
- Here $s_\perp$ is the meson spin projection along the transverse polarization direction $S_\perp = \cos\phi\ \hat{x} + \sin\phi\ \hat{y}$ and for $s_\perp = 0, 1$

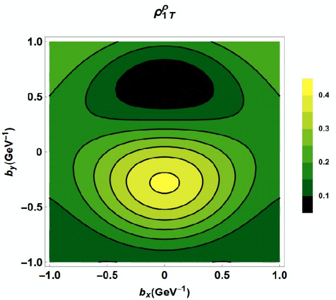
$$\rho_{0T}(b) = \int_0^\infty \frac{dQ\ Q}{2\pi} \left[J_0(bQ) G^+_{++} + \cos 2(\phi_b - \phi_s) J_2(bQ) G^+_{+-} \right]$$

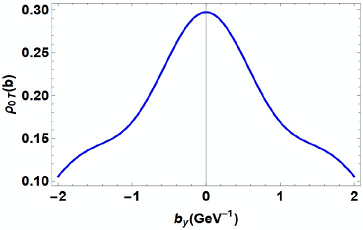
$$\rho_{1T}(b) = \int_0^\infty \frac{dQ\ Q}{2\pi} \left[\frac{J_0(bQ)}{2} (G^+_{++} + G^+_{00}) + \sin(\phi_b - \phi_s) J_1(bQ) \sqrt{2} G^+_{0+} - \cos 2(\phi_b - \phi_s) J_2(bQ) \frac{G^+_{-+}}{2} \right]$$

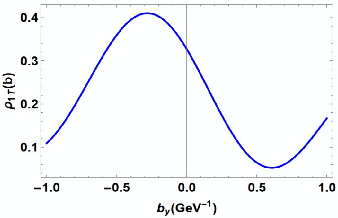
-Carlson and Vanderhaeghen, PRL 100 032004 (2008)

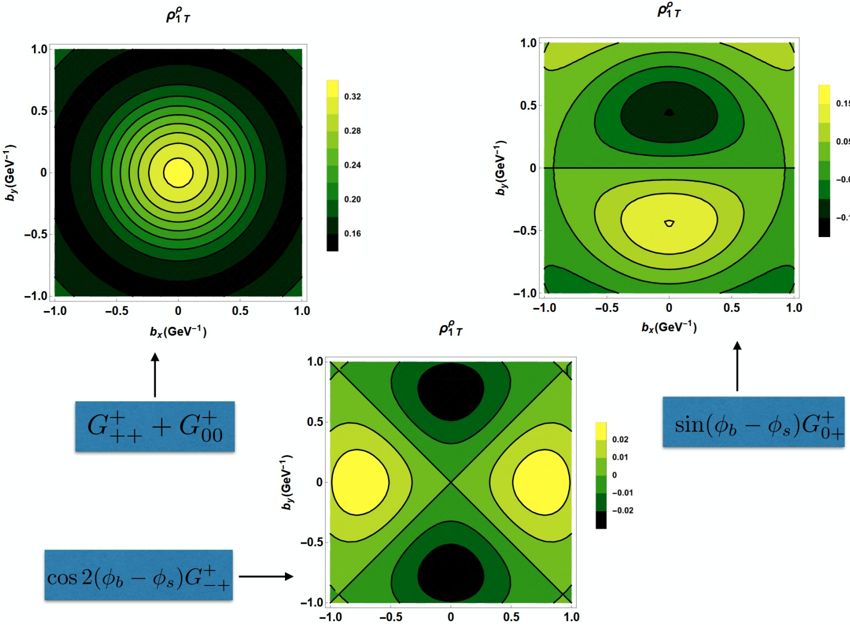
EPJA 41 1 (2009)


ρ^{ρ}_{0T}
0.275
0.225
0.175
0.125
0.75
0.50
0.25
0
−0.25
−0.50
−0.75
b_y(GeV^{-1})
−0.75 −0.50 −0.25 0 0.25 0.50 0.75
b_x(GeV^{-1})


ρ^{ρ}_{1T}
0.4
0.3
0.2
0.1
1.0
0.5
0
−0.5
−1.0
b_y(GeV^{-1})
−1.0 −0.5 0 0.5 1.0
b_x(GeV^{-1})


0.30
0.25
0.20
0.15
0.10
ρ_{0T}(b)
−2 −1 0 1 2
b_y(GeV^{-1})


0.4
0.3
0.2
0.1
ρ_{1T}(b)
−1.0 −0.5 0.0 0.5 1.0
b_y(GeV^{-1})



ρ^{ρ}_{1T}
$b_y(\mathrm{GeV}^{-1})$
$b_x(\mathrm{GeV}^{-1})$
1.0
0.5
0
−0.5
−1.0
0.32
0.28
0.24
0.20
0.16
ρ^{ρ}_{1T}
0.15
0.05
−0.05
−0.15
ρ^{ρ}_{1T}
0.02
0.01
0
−0.01
−0.02
$G^{+}_{++} + G^{+}_{00}$
$\sin(\phi_b - \phi_s) G^{+}_{0+}$
$\cos 2(\phi_b - \phi_s) G^{+}_{-+}$

Conclusions

- We have calculated the photon GPDs and PDF from the overlap of light-front wave functions.
- Currently working on the non-zero skewness case, to obtain results in longitudinal position space.
- Calculations are also done for the ρ meson GPDs for non zero skewness.
- Results for the transverse charge densities for the ρ meson obtained for the first time in LFQM.