QCD, ‘tHooft Model and the Light-Front Quark Model

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Outline

- **AI overviews of QCD**
- **‘t Hooft model as a toy QCD**
- **Dirac’s Forms of Relativistic Dynamics**
- **Quark Mass Gap Solution**
- **Quark-Antiquark Bound State Equation**
- **Link to the Light-Front Quark Model**
- **Bakamjian-Thomas Construction of Bound States**
- **Self-Consistency of Light-Front Quark Model**
- **Conclusions and Outlook**

AI overviews of QCD

Quantum Chromodynamics (QCD) is interesting because it describes the strong nuclear force, one of the fundamental forces of nature and explains how quarks and gluons interact to form hadrons like protons and neutrons. Its study reveals the inner workings of matter and the forces that govern the universe at its most basic level.

Key Words:

Strong Interaction, Quarks and gluons, Color Confinement, Asymptotic Freedom, Analog to Quantum Electrodynamics, Chiral symmetry breaking, Quark-gluon plasma, ...

Some studies have linked confinement and chiral symmetry breaking to QCD monopoles which are topological objects that can emerge in certain gauge conditions. These monopoles may play a role in both confinement and chiral symmetry breaking.

AI Overview

In Quantum Chromodynamics (QCD), the theory describing the strong force, **magnetic monopoles are theorized to play a role, particularly in the context of quark-gluon plasma and confinement**. While not directly observed as fundamental particles, QCD monopoles are often treated as quasiparticles or emergent phenomena within the theory, especially in extreme conditions like high temperatures.

In summary: While magnetic monopoles are not fundamental particles in QCD, they are proposed as quasiparticles or emergent phenomena that play a significant role in phenomena like quark confinement, quark-gluon plasma properties, and jet quenching in heavy-ion collisions. Research from Stony Brook University shows that studies of monopoles in lattice QCD and heavy-ion experiments have provided insights into their behavior and influence on QCD.

Recent Progress in Understanding the Role of Monopoles in QCD

A Dissertation presented

by

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December 2018

Hadronic structure on the light-front I
Instanton effects and quark-antiquark effective potentials

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Phys.Rev.D 107 (2023) 3, 034023 • e-Print: 2110.15927 [hep-ph]

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Hadronic structure on the light-front IX .
Orbital-spin-isospin wave functions of baryons

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Phys.Rev.D 108 (2023) 9, 094033 • e-Print: 2308.14694 [hep-ph]

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Large N_C QCD in 1+1 dim. ('tHooft Model)

$$\mathcal{L} = -\frac{1}{4} F^a_{\hat{\mu}\hat{\nu}} F^{\hat{\mu}\hat{\nu}a} + \bar{\psi}(i\gamma^{\hat{\mu}} D_{\hat{\mu}} - m)\psi$$

$$D_{\hat{\mu}} = \partial_{\hat{\mu}} - igA^a_{\hat{\mu}} t_a$$

$$F^a_{\hat{\mu}\hat{\nu}} = \partial_{\hat{\mu}} A^a_{\hat{\nu}} - \partial_{\hat{\nu}} A^a_{\hat{\mu}} + g f^{abc} A^b_{\hat{\mu}} A^c_{\hat{\nu}}$$

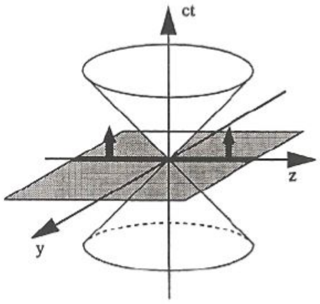
'tHooft Coupling $\lambda = \frac{g^2 (N_c - 1/N_c)}{4\pi}$ and mass m

$$g \to 0, N_C \to \infty; \lambda \to finite$$

Short List of LFD vs. IFD References

- G.'tHooft, NPB75,461(74) - LFD
- Y.Frishman, et al., PRD15(75) - Interpol Gauges IFD&LFD
- I.Bars&M.Green, PRD17,537(78) - IFD(formulation)
- A.Zhitnitsky, PLB165,405(85) - LFD(chiral sym breaking)
- M.Li, et al., JPG13, 915(87) - IFD(rest frame)
- K.Hornbostel, Ph.D. Dissertation(88) - LFD(DLCQ)
- M.Burkardt, PRD53,933(96) - LFD(vacuum condensates)
- Y.Kalashnikov&A.Nefed'ev,Phys.-Usp.45,346('02) - IFD(rev)
- Y. Jia, et al., JHEP11, 151('17) - IFD(moving frame)
- Y. Jia, et al., PRD98, 054011('18) - IFD(quasi-PDFs)
- B.Ma&C.Ji, PRD104,036004('21) - Link IFD&LFD

Dirac's Proposition for Relativistic Dynamics



The instant form

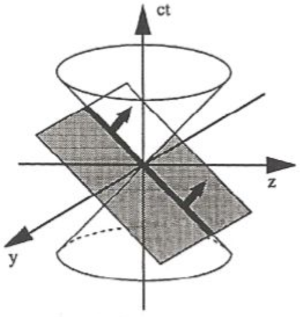
1949

Equal t | *Equal τ*

$$p^0 \leftrightarrow p^- = p^0 - p^3$$

$$(p^1, p^2) \leftrightarrow \vec{p}_\perp$$

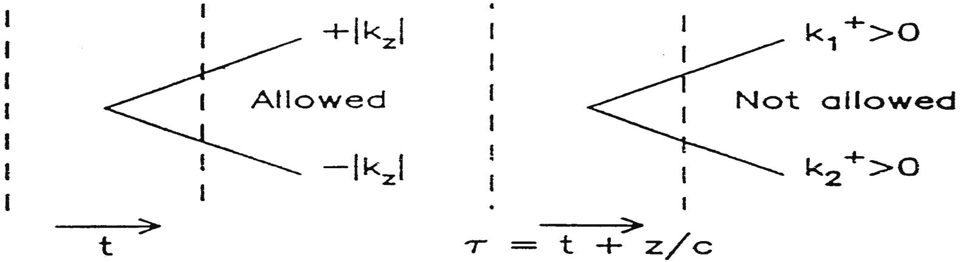
$$p^3 \leftrightarrow p^+ = p^0 + p^3$$



The front form

Energy-Momentum Dispersion Relations

$$p^0 = \sqrt{\vec{p}^{\,2} + m^2} \qquad p^- = \frac{\vec{p}_\perp^{\,2} + m^2}{p^+}$$



(a) (b)

Except zero-modes

$$k_1^+ = k_2^+ = 0$$

IFD

Instant Form Dynamics

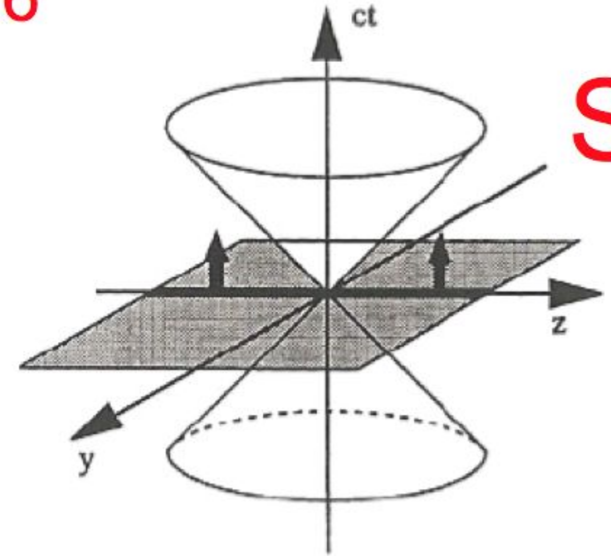
LFD

Light-Front Dynamics

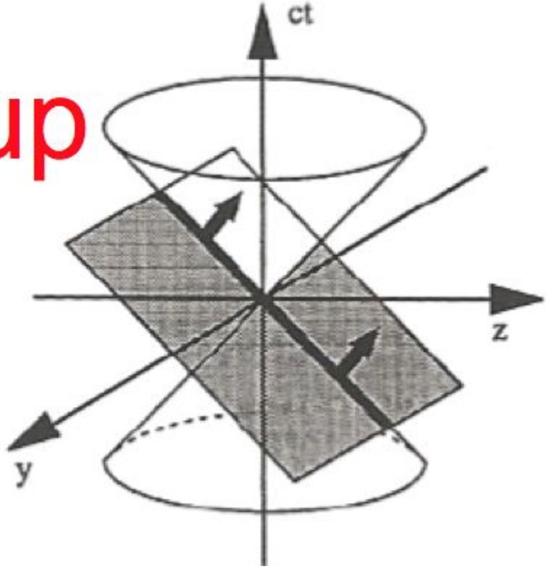
How many generators leave the time surface invariant?

6

7



Stability Group



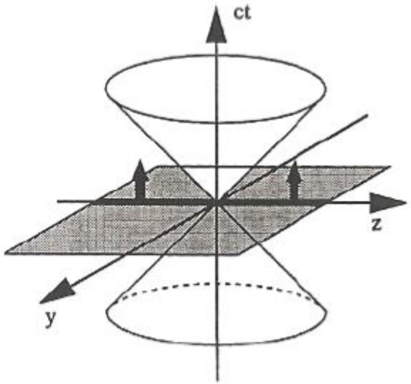
IFD

Instant Form Dynamics

LFD (maximum)

Light-Front Dynamics

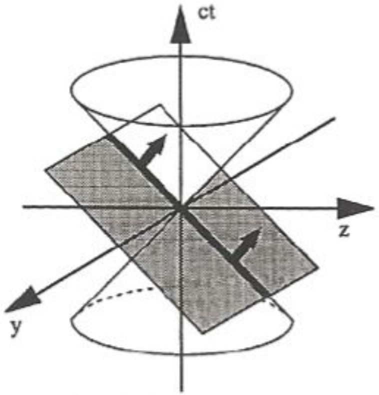
Can IFD and LFD be linked?



The instant form

1949

Yes, they can!



The front form

Traditional approach evolved from NR dynamics

Close contact with Euclidean space

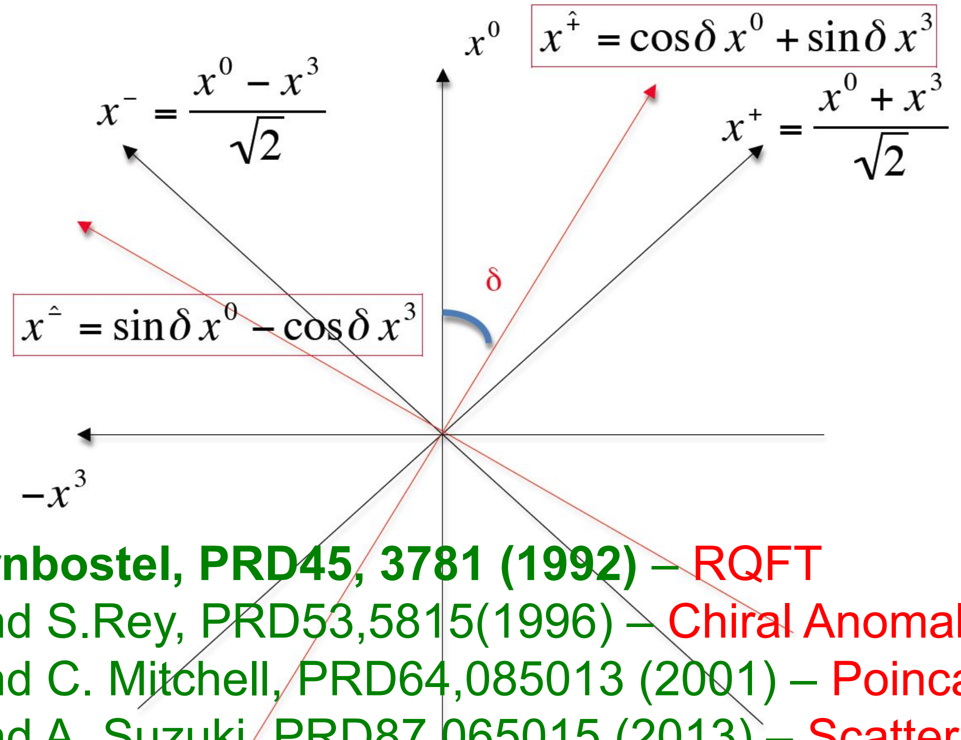
T-dept QFT, LQCD, IMF, etc.

Innovative approach for relativistic dynamics

Strictly in Minkowski space

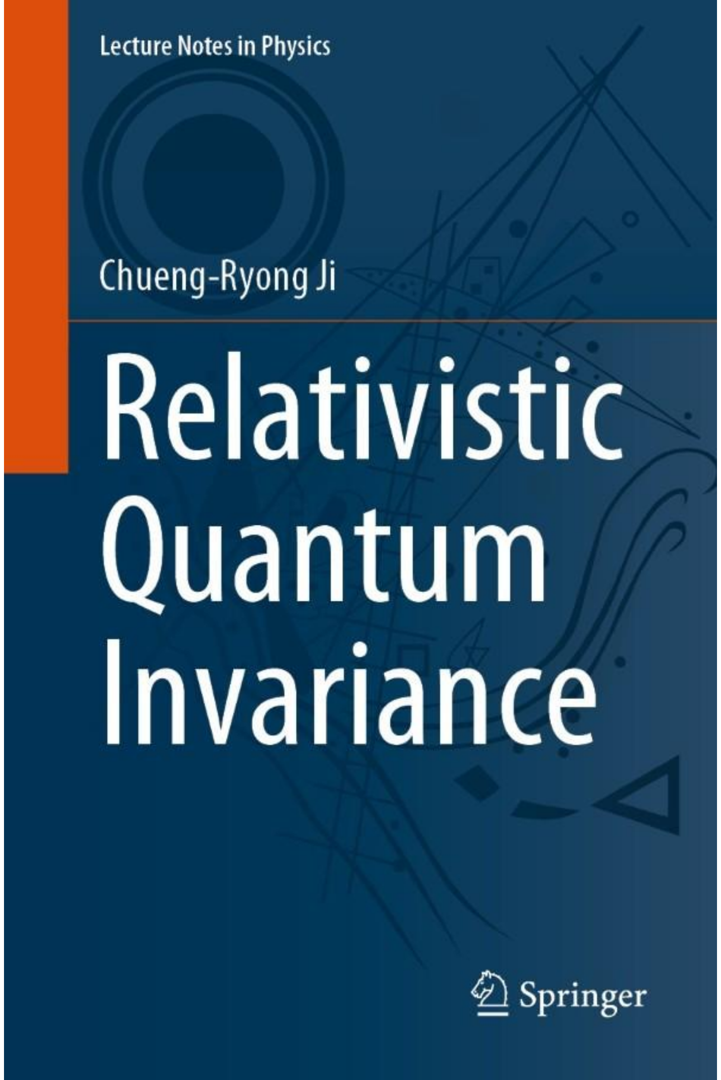
DIS, PDFs, DVCS, GPDs, etc.

Interpolation between IFD and LFD



$$(IFD) \quad 0 \le \delta \le \frac{\pi}{4} \quad (LFD)$$
$$1 \ge \quad C \equiv \cos(2\delta) \quad \ge 0$$

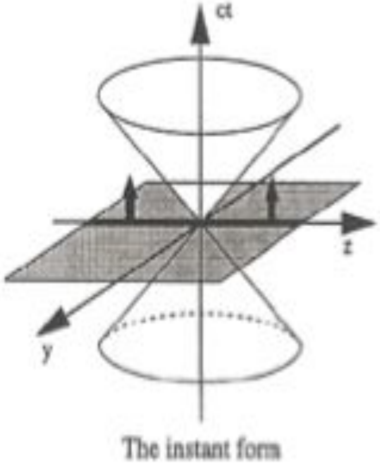
K. Hornbostel, PRD45, 3781 (1992) – RQFT
C.Ji and S.Rey, PRD53,5815(1996) – Chiral Anomaly
C.Ji and C. Mitchell, PRD64,085013 (2001) – Poincare Algebra
C.Ji and A. Suzuki, PRD87,065015 (2013) – Scattering Amps
C.Ji, Z. Li and A. Suzuki, PRD91, 065020 (2015) – EM Gauges
Z.Li, M. An and C.Ji, PRD92, 105014 (2015) – Spinors
C.Ji, Z.Li, B.Ma and A.Suzuki, PRD98, 036017(2018) – QED
B.Ma and C.Ji, PRD104, 036004(2021) – QCD_{1+1}

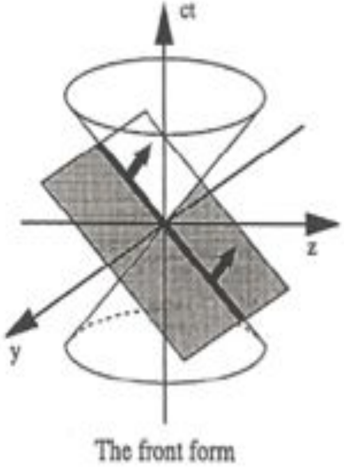


Lecture Notes in Physics (LNP, Vol. 1012), Springer Nature (2023).

Interpolation between

IFD and LFD





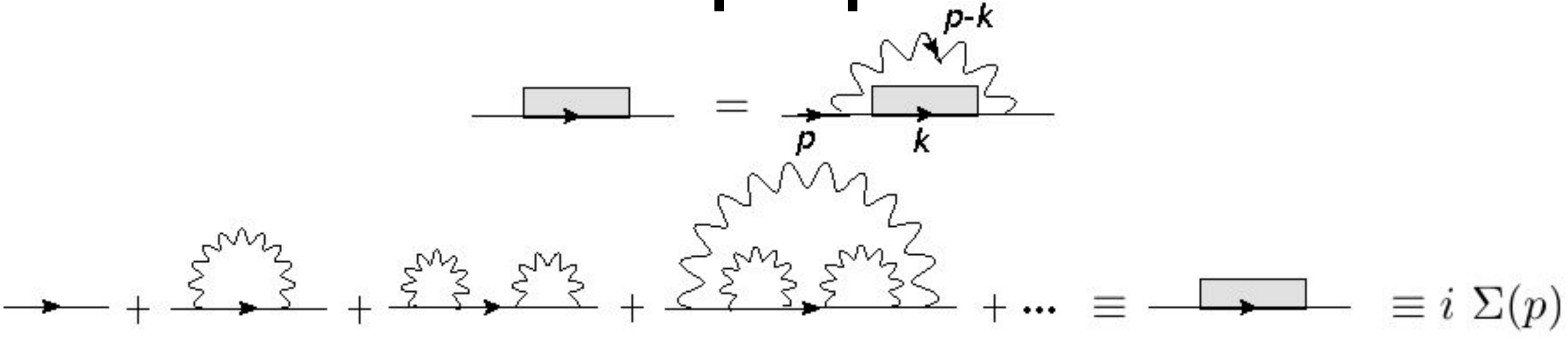
Interpolating Axial Gauge

$$A^a_{\hat{-}} = 0$$

$$A^a_1 = 0 \qquad\qquad A^+_a = 0$$

$$\mathcal{L} = \frac{1}{2}(\partial_{\hat{-}} A^a_{\hat{+}})^2 + \bar{\psi}(i\gamma^{\hat{+}} D_{\hat{+}} + i\gamma^{\hat{-}}\partial_{\hat{-}} - m)\psi$$

Mass Gap Equation



$$\Sigma(p_{\hat{-}}) = i\frac{\lambda}{2\pi}\int \frac{dk_{\hat{-}}\, dk_{\hat{+}}}{(p_{\hat{-}} - k_{\hat{-}})^2}\gamma^{\hat{+}} \frac{1}{\not{k} - m - \Sigma(k_{\hat{-}}) + i\epsilon}\gamma^{\hat{+}}$$

Fermion Propagator

Free Propagator → **Interacting Propagator**

$$S_f(p) = \frac{1}{\not{p} - m + i\varepsilon}$$

$$S(p) = \frac{1}{\not{p} - m - \Sigma(p) + i\varepsilon}$$

$$= \frac{F(p)}{\not{p} - M(p) + i\varepsilon}$$

$$\boxed{\Sigma(p) = \Sigma_s(p) + \Sigma_v(p)\not{p}}$$

$$F(p) = \left(1 - \Sigma_v(p)\right)^{-1}$$ "Wave function renormalization factor"

$$M(p) = \frac{m + \Sigma_s(p)}{1 - \Sigma_v(p)}$$ "Renormalized fermion mass function"

Energy-Momentum Dispersion Relation

Free particle

$$E = \sqrt{p_z^2 + m^2}$$

$$\theta_f = \tan^{-1}(p_z / m)$$

$$\beta = p_z / E$$

$$= \sin\theta_f$$

$$= \tanh\eta$$

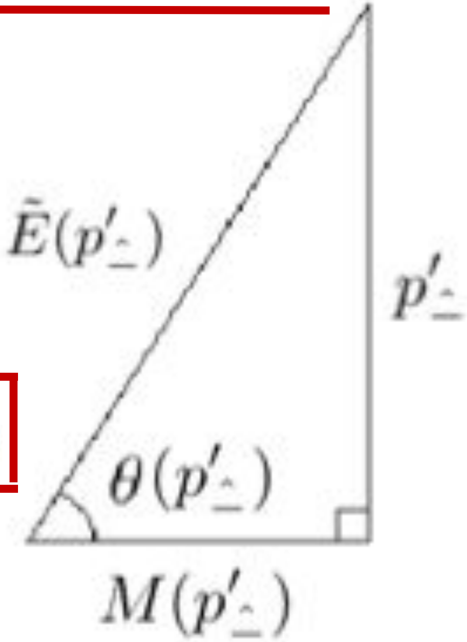
Interacting particle

$$\frac{F(p'_{\hat{-}})E(p'_{\hat{-}})}{\sqrt{C}} = \sqrt{p'^2_{\hat{-}} + M(p'_{\hat{-}})^2} \equiv \tilde{E}(p'_{\hat{-}})$$

$$\theta(p'_{\hat{-}}) = \theta_f(p'_{\hat{-}}) + 2\zeta(p'_{\hat{-}})$$

$$\begin{pmatrix} b^i(p'_{\hat{-}}) \\ d^{+i}(p'_{\hat{-}}) \end{pmatrix} = \begin{pmatrix} \cos\zeta(p'_{\hat{-}}) & -\sin\zeta(p'_{\hat{-}}) \\ \sin\zeta(p'_{\hat{-}}) & \cos\zeta(p'_{\hat{-}}) \end{pmatrix} \begin{pmatrix} b^i_f(p'_{\hat{-}}) \\ d^{+i}_f(p'_{\hat{-}}) \end{pmatrix}$$

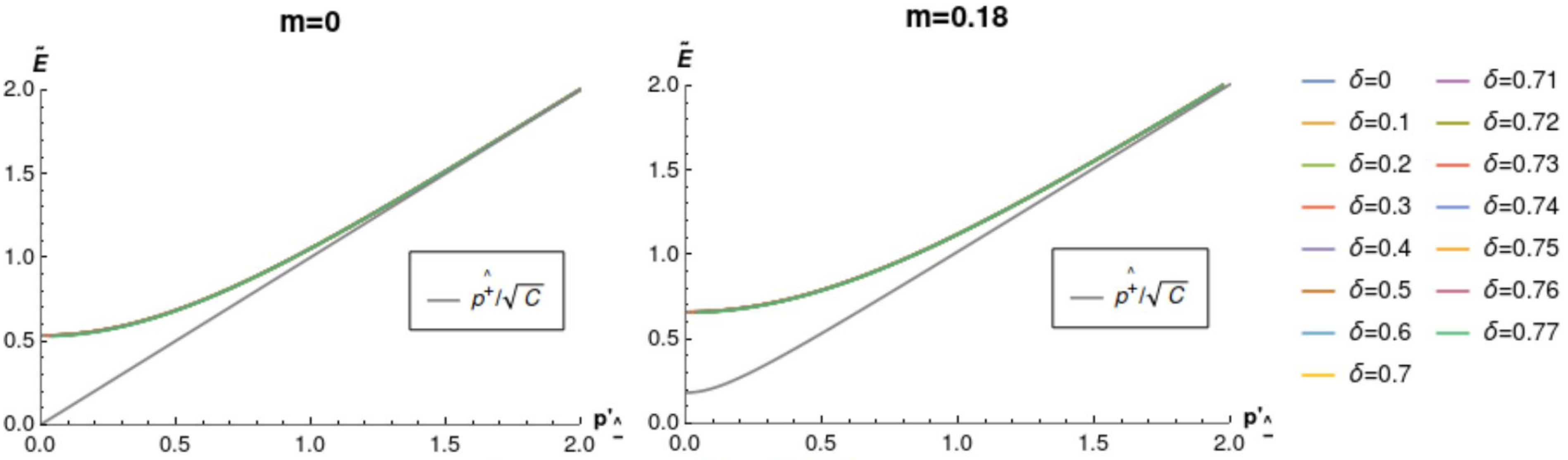
$b^i_f |0>=0, d^i_f |0>=0$ **vs.** $b^i |\Omega>=0, d^i |\Omega>=0$



Interpolation

$$(E, p_z) \Rightarrow (p^{\hat{+}} / \sqrt{C}, p_{\hat{-}} / \sqrt{C} \equiv p'_{\hat{-}})$$

Mass Gap Solutions



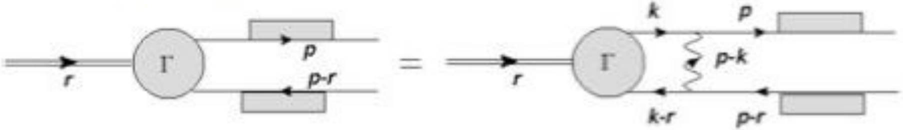
$$\tilde{E}(0) = \frac{F(0)E(0)}{\sqrt{\mathbb{C}}} = M(0)$$

m	0	0.045	0.18	0.749	1.00	2.11	4.23
$M(0)$	0.532778	0.563644	0.659112	1.10105	1.31167	2.30969	4.34358
$F(0)$	−0.495173	−0.584175	−0.987673	4.11079	2.17976	1.22134	1.05526

$$m \lesssim 0.56$$

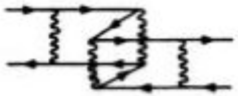
BOUND-STATE EQUATION

$$\Gamma(r,p) = \frac{i\lambda}{2\pi}\int \frac{dk_{\hat{-}}\,dk_{\hat{+}}}{(p_{\hat{-}} - k_{\hat{-}})^2} S(p)\gamma^{\hat{+}}\Gamma(r,k)\gamma^{\hat{+}}S(p-r)$$



$$\left[-r_{\hat{+}} + \frac{-\mathbb{S}p_{\hat{-}} + E(p_{\hat{-}})}{\mathbb{C}} + \frac{\mathbb{S}(p_{\hat{-}} - r_{\hat{-}}) + E(p_{\hat{-}} - r_{\hat{-}})}{\mathbb{C}}\right]\hat{\phi}_+(r_{\hat{-}}, p_{\hat{-}})$$
$$= \lambda \fint \frac{dk_{\hat{-}}}{(p_{\hat{-}} - k_{\hat{-}})^2}\left[C(p_{\hat{-}}, k_{\hat{-}}, r_{\hat{-}})\hat{\phi}_+(r_{\hat{-}}, k_{\hat{-}}) - S(p_{\hat{-}}, k_{\hat{-}}, r_{\hat{-}})\hat{\phi}_-(r_{\hat{-}}, k_{\hat{-}})\right],$$

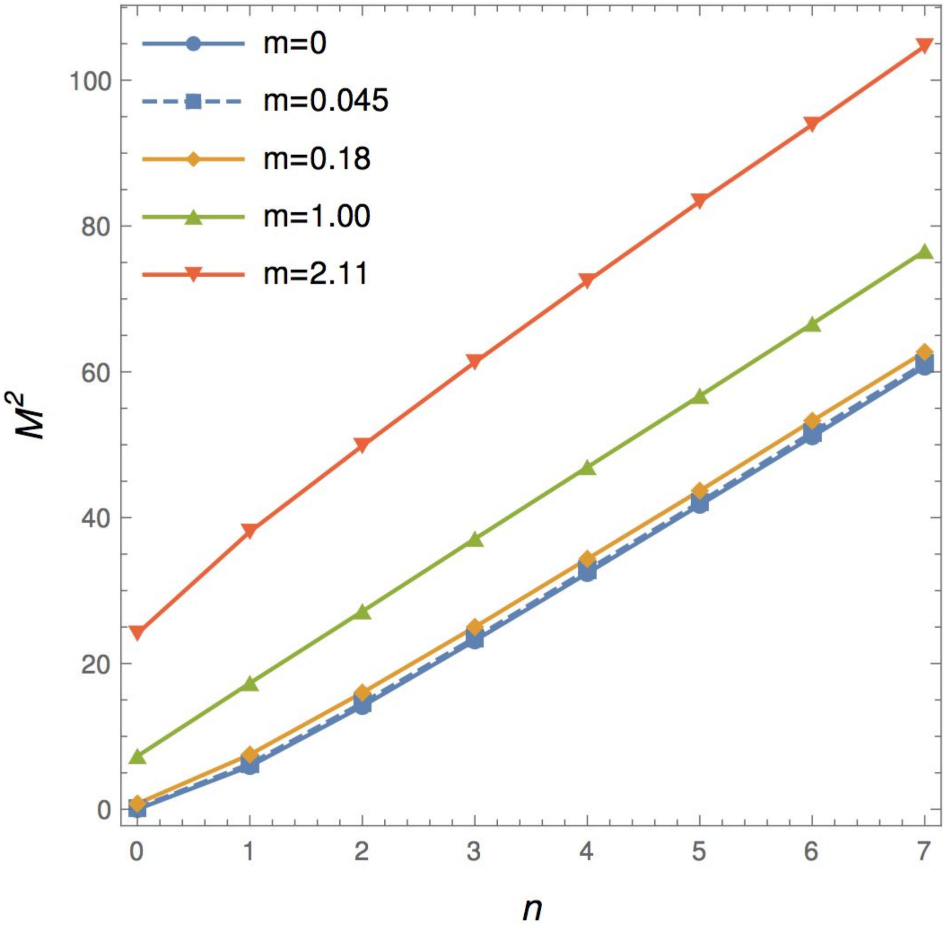
$$\left[r_{\hat{+}} + \frac{-\mathbb{S}(p_{\hat{-}} - r_{\hat{-}}) + E(p_{\hat{-}} - r_{\hat{-}})}{\mathbb{C}} + \frac{\mathbb{S}p_{\hat{-}} + E(p_{\hat{-}})}{\mathbb{C}}\right]\hat{\phi}_-(r_{\hat{-}}, p_{\hat{-}})$$
$$= \lambda \fint \frac{dk_{\hat{-}}}{(p_{\hat{-}} - k_{\hat{-}})^2}\left[C(p_{\hat{-}}, k_{\hat{-}}, r_{\hat{-}})\hat{\phi}_-(r_{\hat{-}}, k_{\hat{-}}) - S(p_{\hat{-}}, k_{\hat{-}}, r_{\hat{-}})\hat{\phi}_+(r_{\hat{-}}, k_{\hat{-}})\right].$$



LFD $$\left[\mathcal{M}^2 - \frac{m^2 - 2\lambda}{x} - \frac{m^2 - 2\lambda}{1-x}\right]\phi(x) = -2\lambda \fint_0^1 \frac{dy}{(x-y)^2}\phi(y)$$

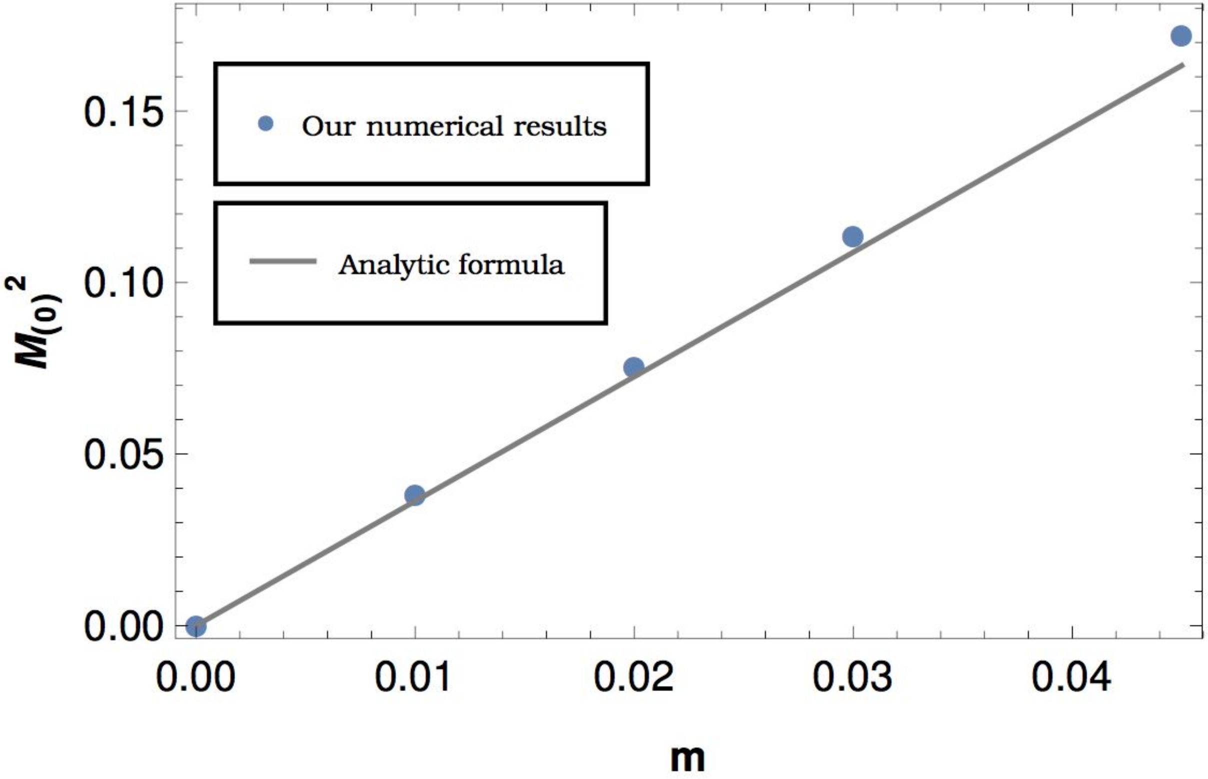
Meson Spectroscopy

- G.'tHooft, NPB75, 461(74) - LFD



- M.Li, et al., JPG13, 915(87) - IFD (rest frame)
- Y. Jia, et al., JHEP11, 151('17) - IFD (moving frame)

Gell-Mann - Oaks - Renner Relation

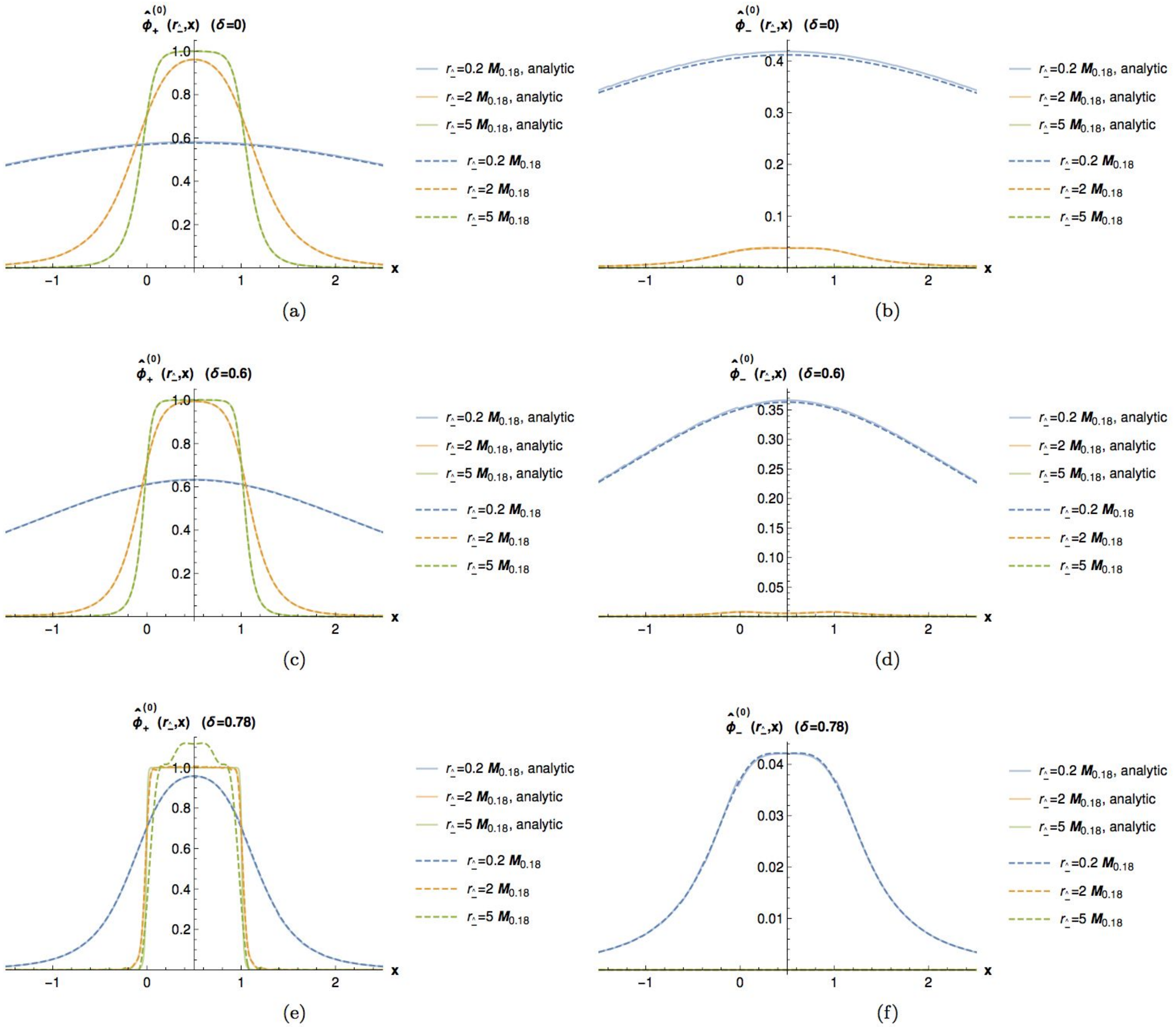


$$\mathcal{M}_\pi^2 = -\frac{4m < \bar{\psi}\psi >}{f_\pi^2} = \sqrt{\frac{8\pi^2 m^2 \lambda}{3}}$$

$$f_\pi = \sqrt{N_c/\pi}$$

Meson Ground-state Wave-function for m=0 case

$$\hat{\phi}^{(0)}_{\pm}(r_{\underline{\wedge}}, p_{\underline{\wedge}}) = \frac{1}{2}\left(\cos\frac{\theta(r_{\underline{\wedge}} - p_{\underline{\wedge}}) - \theta(p_{\underline{\wedge}})}{2} \pm \sin\frac{\theta(r_{\underline{\wedge}} - p_{\underline{\wedge}}) + \theta(p_{\underline{\wedge}})}{2}\right)$$



Parton Distribution Functions (PDFs)

$$q_n(x) = \int_{-\infty}^{+\infty} \frac{d\xi^-}{4\pi} e^{-ixP^+\xi^-}$$

$$\times \langle P_n^-, P^+ | \bar{\psi}(\xi^-)\gamma^+ \mathcal{W}[\xi^-, 0]\psi(0) | P_n^-, P^+ \rangle_C,$$

$$\mathcal{W}[\xi^-, 0] = \mathcal{P}\left[\exp\left(-ig_s \int_0^{\xi^-} d\eta^- A^+(\eta^-)\right)\right]$$

A^+=0 Gauge in LFD

Quasi-PDFs

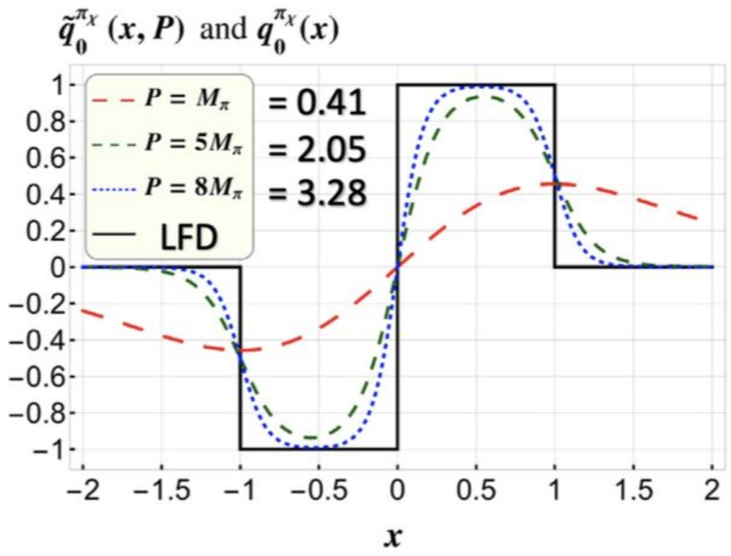
$$\tilde{q}_{(n)}(r_{\hat{-}}, x) = \int_{-\infty}^{+\infty} \frac{dx^{\hat{-}}}{4\pi} \mathrm{e}^{ix^{\hat{-}} r_{\hat{-}}}$$

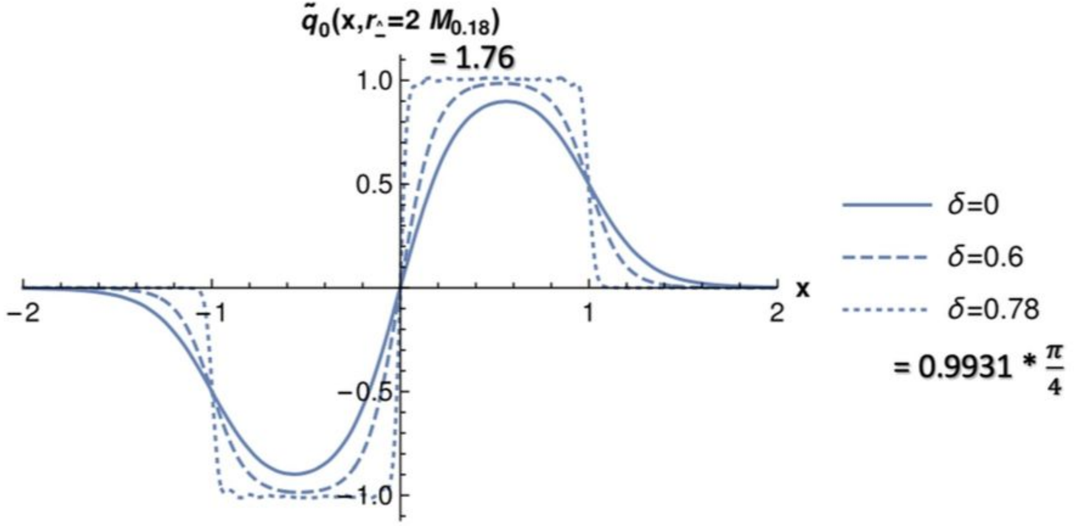
$$\times < r^{\hat{+}}_{(n)}, r_{\hat{-}} \mid \bar{\psi}(x^{\hat{-}}) \, \gamma_{\hat{-}} \, \mathcal{W}[x^{\hat{-}}, 0] \, \psi(0) \mid r^{\hat{+}}_{(n)}, r_{\hat{-}} >_C,$$

$$\mathcal{W}[x^{\hat{-}}, 0] = \mathcal{P}\left[\exp\left(-ig \int_0^{x^{\hat{-}}} dx'^{\hat{-}} A_{\hat{-}}(x'^{\hat{-}})\right)\right]$$

Interpolating dynamics

Y. Jia, et al., PRD98, 054011('18)
- IFD (quasi-PDFs)





B.Ma&C.Ji,PRD104,036004('21)
- Interpolating Dynamics

Bakamjian-Thomas Construction

B.Bakamjian and L.H.Thomas, Phys.Rev.92,1300(1953)

B.Keister and W.Polyzou, Adv.Nucl.Phys.20,225(1991)

$$[P^i, K^j] = i\delta_{ij} H$$

$$\{H, \mathbf{P}, \mathbf{J}, \mathbf{K}\} \Rightarrow \{M, \mathbf{P}, \mathbf{j}_c, \mathbf{X}_c\} \Rightarrow \{M_0, \mathbf{P}_0, \mathbf{j}_{c0}, \mathbf{X}_{c0}\} \Rightarrow \{M, \mathbf{P}_0, \mathbf{j}_{c0}, \mathbf{X}_{c0}\}$$

$$H = \sqrt{M^2 + \mathbf{P}^2};$$

$$\mathbf{K} = -\frac{1}{2}\{H, \mathbf{X}_c\}_+ - \frac{\mathbf{P} \times \mathbf{j}_c}{H + M};$$

$$\mathbf{J} = \mathbf{X}_c \times \mathbf{P} + \mathbf{j}_c.$$

$$P_0^\mu := P_1^\mu \otimes I_2 + I_1 \otimes P_2^\mu;$$

$$\mathbf{K}_0 := \mathbf{K}_1 \otimes I_2 + I_1 \otimes \mathbf{K}_2;$$

$$\mathbf{J}_0 := \mathbf{J}_1 \otimes I_2 + I_1 \otimes \mathbf{J}_2.$$

$$M := M_0 + V$$

$$\left[V, \{\mathbf{P}_0, \mathbf{j}_{c0}, \mathbf{X}_{c0}\}\right] = 0$$

Bakamjian-Thomas Construction in LFD

$$\{P^-, P^+, \mathbf{P}_\perp, \mathbf{E}_\perp, K^3, \mathbf{J}_\perp, J^3\} \Rightarrow \{M, P^+, \mathbf{P}_\perp, \mathbf{E}_\perp, K^3, \mathbf{j}_f\}$$

$$M^2 = P^+P^- - \mathbf{P}_\perp^2;\; j_f^3 = \frac{1}{P^+}[P^+J^3 - \hat{\mathbf{z}}\cdot(\mathbf{P}_\perp \times \mathbf{E}_\perp)];$$

$$\mathbf{j}_{f\perp} = \frac{1}{M}\left[-\tfrac{1}{2}(P^+ - P^-)(\hat{\mathbf{z}}\times\mathbf{E}_\perp) + \hat{\mathbf{z}}\times\mathbf{P}_\perp K^3 + P^+\mathbf{J}_\perp - \frac{\mathbf{P}_\perp}{P^+}[P^+J^3 - \hat{\mathbf{z}}\cdot(\mathbf{P}_\perp\times\mathbf{E}_\perp)]\right]$$

$$\{M_0, P_0^+, \mathbf{P}_{\perp 0}, \mathbf{E}_{\perp 0}, K_0^3, \mathbf{j}_{f0}\} \Leftrightarrow \{M, P^+, \mathbf{P}_\perp, \mathbf{E}_\perp, K^3, \mathbf{j}_f\}$$

$$M := M_0 + V$$

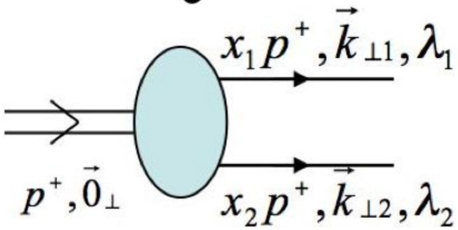
$$[\mathbf{E}_\perp, V]_- = [K^3, V]_- = [\mathbf{j}_{f0}, V]_- = [\mathbf{P}_\perp, V]_- = [P^+, V]_- = 0$$

Light-Front Quark Model(LFQM)

$$|Meson\rangle = \psi_{q\bar{q}}|q\bar{q}\rangle + \psi_{q\bar{q}g}|q\bar{q}g\rangle + \ldots$$
$$\approx \Psi_{Q\bar{Q}}|Q\bar{Q}\rangle,$$

where

$$|Q\rangle = \psi_q^Q|q\rangle + \psi_{qg}^Q|qg\rangle + \ldots$$
$$|\bar{Q}\rangle = \psi_{\bar{q}}^{\bar{Q}}|\bar{q}\rangle + \psi_{\bar{q}g}^{\bar{Q}}|\bar{q}g\rangle + \ldots$$



$$P^- = p_Q^- + p_{\bar{Q}}^-$$

$$M_0^2 = \frac{m_Q^2 + \boldsymbol{k}_\perp^2}{x} + \frac{m_{\bar{Q}}^2 + \boldsymbol{k}_\perp^2}{1-x}$$

Noninteracting "on-mass" shell Q & $\bar{Q}$ representation

$$\Psi_{Q\bar{Q}}(x_i, \vec{k}_{\perp i}, \lambda_i) = \Phi(x_i, \vec{k}_{\perp i})\chi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Radial → Φ; Spin-Orbit → χ

The interaction between $Q\bar{Q}$

includes Coulomb, Confinement, Spin-Spin, Spin-Orbit interactions.

$$M := M_0 + V_{Q\bar{Q}}$$

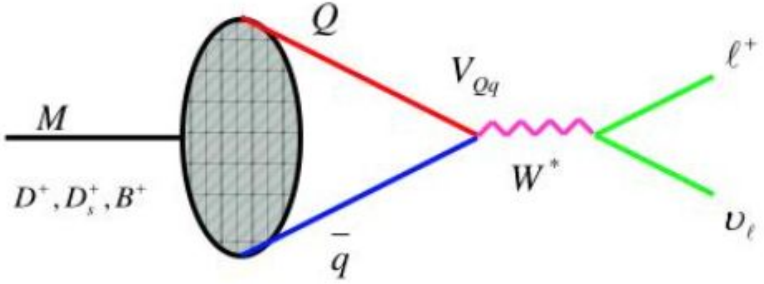
Interaction independent
Melosh transformation

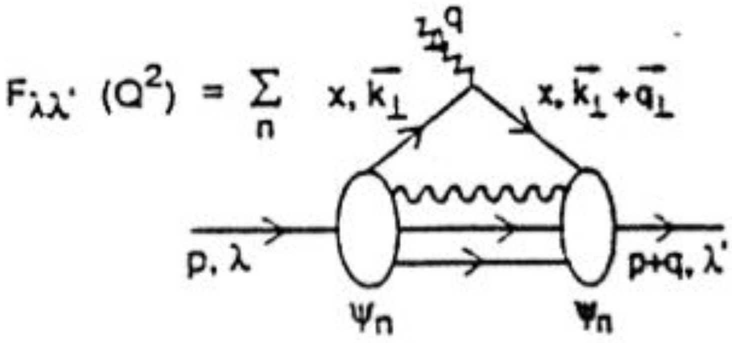
$$J^{PC} = 0^{++}(f_0, a_0, \ldots)$$
$$0^{-+}(\pi, K, \eta, \eta', \ldots)$$
$$1^{--}(\rho, K^*, \omega, \phi, \ldots)$$

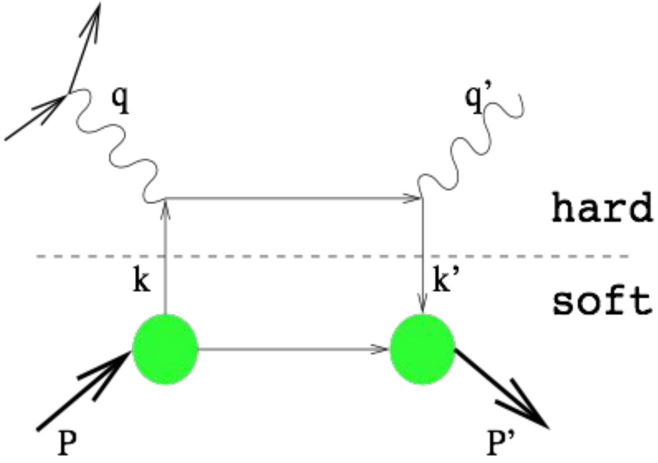
PRD59, 074015(99); PLB460, 461(99) by HMC and CRJ;
PRC92, 055203(2015) by HMC, CRJ, Z. Li, and H. Ryu
PRD106, 014009(2022) by A. J. Arifi, HMC, and CRJ
HMC and CRJ, PRD110, 014006(2024)

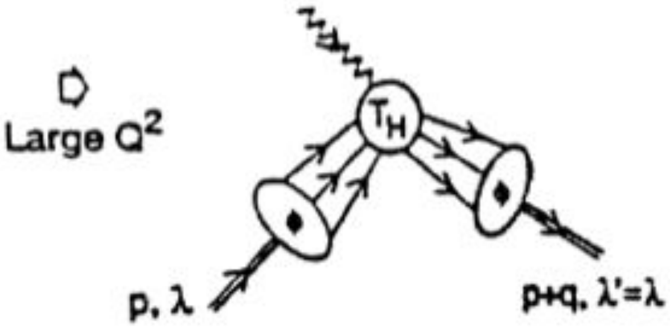
H.J. Melosh: PRD 9, 1095(1974)

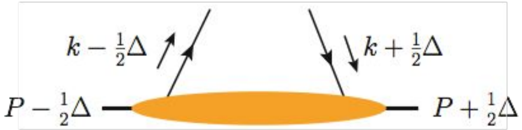
Two-point, Three-point and Four-point functions





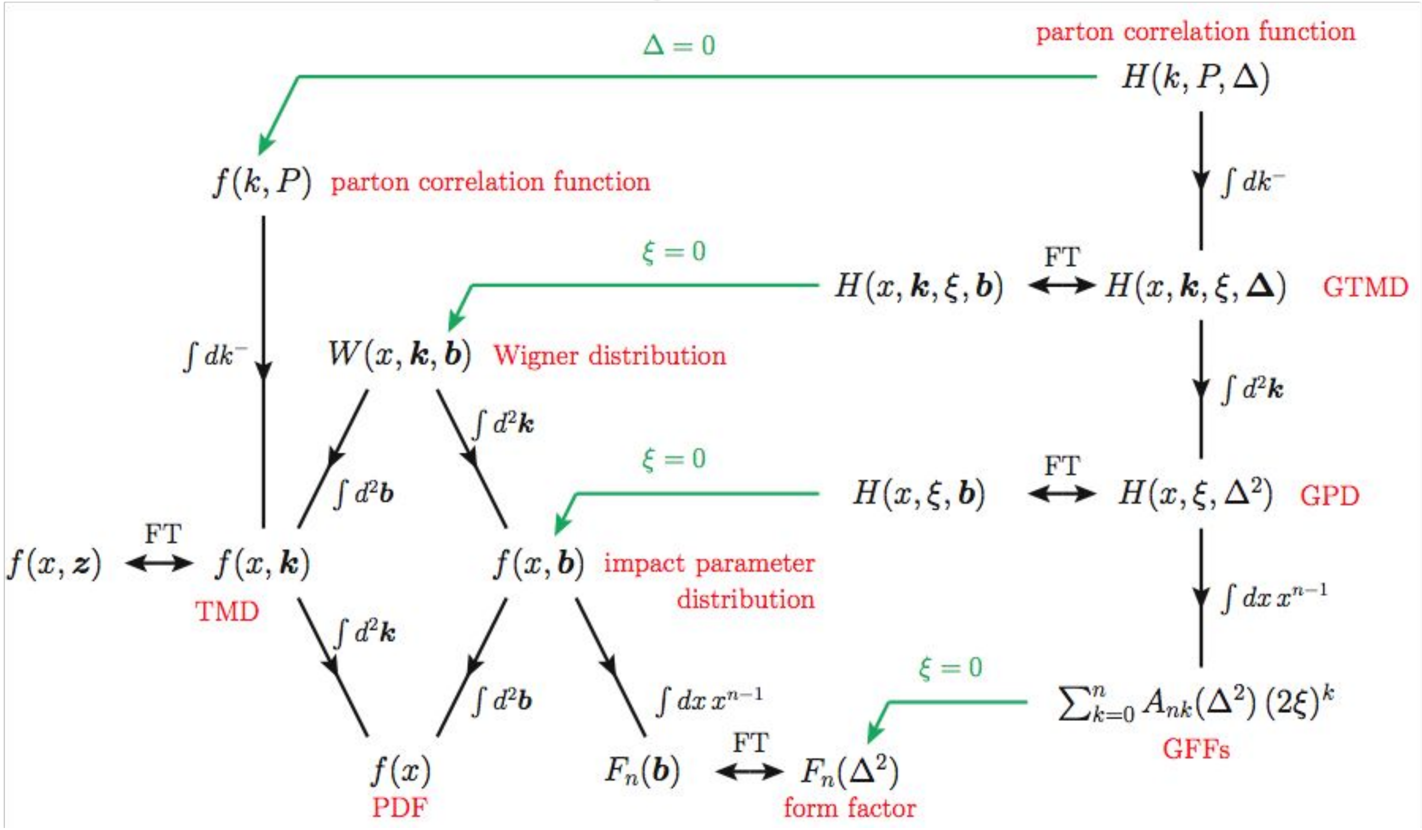






M.Diehl, EPJA 52, 149 (2016)

$$H(k, P, \Delta) = (2\pi)^{-4} \int \mathrm{d}^4 z \, e^{izk} \left\langle p\left(P + \frac{1}{2}\Delta\right) \middle| \bar{q}\left(-\frac{1}{2}z\right) \Gamma q\left(\frac{1}{2}z\right) \middle| p\left(P - \frac{1}{2}\Delta\right) \right\rangle$$



Electromagnetic Form Factor

$$F_\pi^{(\mu)}(Q^2)=\int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \frac{\phi(x,\mathbf{k}_\perp)\phi'(x,\mathbf{k}'_\perp)}{\sqrt{m^2+\mathbf{k}_\perp^2}\sqrt{m^2+\mathbf{k}'^2_\perp}} \mathcal{O}^{(\mu)}_{\mathrm{LFQM}}$$

$F_\pi^{(\mu)}$	$\mathcal{O}^{(\mu)}_{\mathrm{LFQM}}$	$\mathcal{H}^{(\mu)}_{(\uparrow\to\uparrow)+(\downarrow\to\downarrow)}$	$\mathcal{H}^{(\mu)}_{(\uparrow\to\downarrow)+(\downarrow\to\uparrow)}$
$F_\pi^{(+)}$	$\mathbf{k}_\perp\cdot\mathbf{k}'_\perp+m^2$	$\mathbf{k}_\perp\cdot\mathbf{k}'_\perp+m^2$	0
$F_\pi^{(\perp)}$	$\mathbf{k}_\perp\cdot\mathbf{k}'_\perp+m^2$	$\mathbf{k}_\perp\cdot\mathbf{k}'_\perp+m^2$	0
$F_\pi^{(-)}$	$\frac{2(1-x)\mathbf{q}_\perp^2 M_0^2(\mathbf{k}_\perp\cdot\mathbf{k}'_\perp+m^2+\mathbf{q}_\perp\cdot\mathbf{k}'_\perp)}{x[2M_0'^2\mathbf{q}_\perp^2+\mathbf{q}_\perp^4+(M_0^2-M_0'^2)^2]}$	$\frac{2\mathbf{q}_\perp^2\{(\mathbf{k}_\perp\cdot\mathbf{k}'_\perp+m^2)(\mathbf{k}_\perp^2+\mathbf{k}_\perp\cdot\mathbf{q}_\perp+m^2)+(1-x)(\mathbf{k}_\perp\times\mathbf{q}_\perp)^2\}}{x^2[2M_0'^2\mathbf{q}_\perp^2+\mathbf{q}_\perp^4+(M_0^2-M_0'^2)^2]}$	$\frac{2\mathbf{q}_\perp^2\{(1-x)m^2\mathbf{q}_\perp^2\}}{x^2[2M_0'^2\mathbf{q}_\perp^2+\mathbf{q}_\perp^4+(M_0^2-M_0'^2)^2]}$

$$F_\pi^{(+)} = F_\pi^{(\perp)} = F_\pi^{(-)}$$

See the link between the covariant field theory and the LFQM in HMC and CRJ, PRD110, 014006(2024).

Conclusions and Outlook

- **Link between QCD and LFQM may be feasible as exemplified by the mass gap solution in the ‘t Hooft model interpolation between IFD and LFD.**
- **LF ZMs appear essential in understanding the constituent mass in LFQM.**
- **BT construction provides the self-consistency of LFQM which assures the component and frame independence of the physical observables.**
- **Looking forward to studying monopoles in QCD with the interpolation between IFD and LFD.**

Extended Wick Rotation

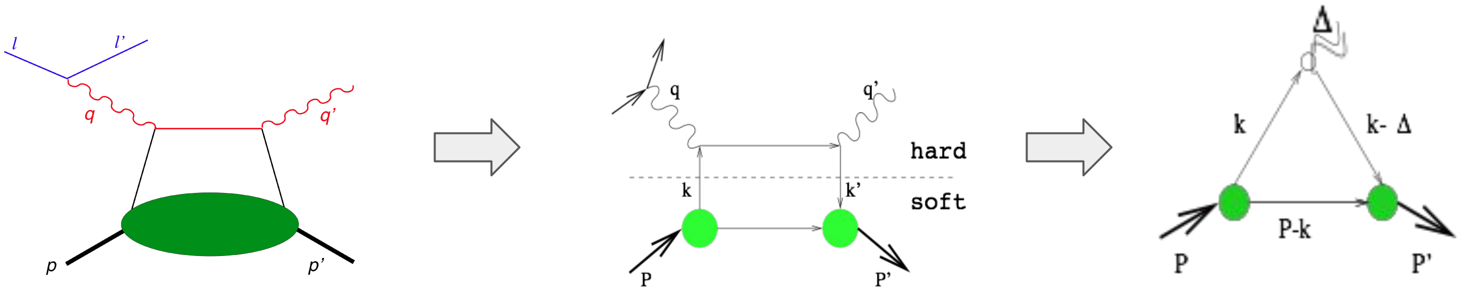
$$p^0 \rightarrow \tilde{P}^0 = ip^0 \quad (\delta = 0)$$

$$For \quad 0 < \delta < \pi / 4,$$

$$p^{\hat{+}} / \sqrt{C} \rightarrow \tilde{P}^{\hat{+}} / \sqrt{C} = ip^{\hat{+}} / \sqrt{C}.$$

Correspondence to Euclidean Space

$$p'^2_{\underline{\hat{}}} = p^2_{\underline{\hat{}}} / C \leftrightarrow -\tilde{P}^2$$



$$\langle P|J^{+}|P\rangle = 2P^{+}\int dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}\,|\phi(x,\mathbf{k}_\perp)|^2$$

$$f_1^q(x,\mathbf{k}_\perp) = \frac{1}{16\pi^3}|\phi(x,\mathbf{k}_\perp)|^2 \qquad \int dx \int d^2\mathbf{k}_\perp f_1^q(x,\mathbf{k}_\perp) = \int dx f_1^q(x) = 1$$

$$\langle P|J^{\perp}|P\rangle = \int dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}\,|\phi(x,\mathbf{k}_\perp)|^2\left(-\frac{2\mathbf{k}_\perp}{x}\right)$$

$$2\mathbf{k}_\perp f_3^q(x,\mathbf{k}_\perp) = \frac{1}{16\pi^3}|\phi(x,\mathbf{k}_\perp)|^2\left(-\frac{2\mathbf{k}_\perp}{x}\right) \qquad x f_3^q(x,\mathbf{k}_\perp) = -f_1^q(x,\mathbf{k}_\perp)$$

$$f(x) = \int d^2p_T f(x,p_T) \qquad 2\int dx f_4^q(x) = \int dx f_1^q(x) = 1$$